PLSC 503: "Multivariate Analysis for Political Research"

Exercise Six

The topic of the day is models with multiplicative interactions. This exercise has the customary two parts.

Part I

Consider a model like:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i, \tag{1}$$

where both Xs are continuous variables, and (to keep things simple) $N = 400$.

1. *Using simulations*, show that min $\left[\widehat{\text{s.e.}(\hat{\beta}_1)}\right]$ (that is, the minimum value of the standard error of the "direct effect" coefficient on X_1) occurs when $\bar{X}_2 = 0$.
2. Next, suppose that $\beta_0 = 3$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, and $\beta_3 = -0.5$, with $X_1 \sim U[-1, 1]$, $X_2 \sim U[-1, 1]$, Cov$(X_1, X_2) = 0$ and $u \sim$ i.i.d. $N(0, 1)$.
   (a) Again using simulations, illustrate and describe the bias in the estimates of β_0, β_1, β_3, and (correspondingly) ψ_1 that results from omitting the "direct effect" of X_2 – that is, from estimating a model of the form:

   $$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_3 X_{1i} X_{2i} + u_i \tag{2}$$

   Briefly describe why you found what you found.
   (b) Repeat the exercise in Part I.2.a, this time with $X_2 \sim U[2, 4]$. What differs? Why?
   (c) Repeat the exercise in Part I.2.a, this time increasing Cov(X_1, X_2) *gradually*. (Hint: An easy way to do this is to generate:

   $$\begin{aligned} X_2 &= X_1 + e_i, \text{with} \\ e_i &\sim \text{Uniform}[-\tau, \tau] \end{aligned}$$

   for various values of τ). What differs? Why?

Part II

The real-data part of this week's homework is also about interaction terms, and uses data on "Article 177" references to the European Court of Justice (ECJ). Article 177 references are cases in which national judges call upon the ECJ to issue a ruling in a case in which European Community law is material to the issue at hand. Since the vast majority of these cases deal with economic and trade issues, we'll examine the general effects of trade levels on the number of such references.

The data consist of annual observations on 15 EU nations for the period 1961-1995; missing data bring the total number of cases to 316. The primary variable of interest is the number of Article 177 references brought to the ECJ from that nation in that particular year (*Cases*). We'll examine the effect of three key variables on that outcome:

- *Trade* – The annual sum of imports plus exports, in billions of (constant) U.S. dollars.
- *Trade Balance* – The difference between annual imports and annual exports, in billions of (constant) U.S. dollars.
- *Centralization* – Governmental centralization, coded 1 if the state is a unitary system, and 0 if a federal/decentralized one.

Quasi-constructivist scholars (e.g., Stone Sweet and Brunell 1998, from which the data are drawn) would predict that higher overall levels of trade will increase the number of Article 177 claims. This general effect, however, could be mediated by two factors. First, the effect of overall trade levels on Article 177 references ought to be greater for states which have high negative trade imbalances (i.e., greater imports and fewer exports) than among those which have net trade surpluses. Second, one might expect that, because cases can be brought from any level of government, federal systems (with their higher number of potential "points of contact") will both (a) have absolutely higher numbers of references than will unified systems, and (b) will exhibit greater responsiveness to trade levels in the number of Article 177 references than will unitary systems.

Your assignment is simple: to test the above hypotheses, using OLS regression. Once you have done so, briefly write up your findings. Be sure to interpret your results both statistically and substantively, and to include discussion(s) of the uncertainty associated with your findings.

This homework is due (in the usual way) in electronic (PDF) form, at or before 11:59 p.m. EST on **Wednesday, March 13, 2024**. It is worth 50 possible points.