PLSC 503 – Spring 2024
Binary Response Models, II

April 1, 2024

Running Example: House Vote on NAFTA (1993)

Response / Outcome

- `vote` – Whether (=1) or not (=0) the House member in question voted in favor of NAFTA.

Predictors

- `PropHisp` – The proportion of the House member's district who are of Latino/hispanic origin.
- `Democrat` – Whether the House member in question is a Democrat (=1) or a Republican (=0).
- `COPE` – The 1993 AFL-CIO (COPE) voting score of the member in question; the original variable ranges from 0 to 100, with higher scores indicating more pro-labor positions. Rescaled to range from 0 to 1.
- `DemXCOPE` – The multiplicative interaction of `Democrat` and `COPE`.

Model & Data

$$\begin{aligned}\Pr(\texttt{vote}_i = 1) \quad &= \quad f[\beta_0 + \beta_1(\texttt{PropHisp}_i) + \beta_2(\texttt{Democrat}_i) + \\ &\qquad \beta_3(\texttt{COPE}_i) + \beta_4(\texttt{Democrat}_i \times \texttt{COPE}_i) + u_i]\end{aligned}$$

```
> summary(NAFTA)

      Vote           PropHisp         Democrat           COPE          DemXCOPE
 Min.   :0.000   Min.   :0.000   Min.   :0.000   Min.   :0.000   Min.   :0.000
 1st Qu.:0.000   1st Qu.:0.010   1st Qu.:0.000   1st Qu.:0.170   1st Qu.:0.000
 Median :1.000   Median :0.030   Median :1.000   Median :0.810   Median :0.750
 Mean   :0.539   Mean   :0.088   Mean   :0.585   Mean   :0.602   Mean   :0.516
 3rd Qu.:1.000   3rd Qu.:0.100   3rd Qu.:1.000   3rd Qu.:1.000   3rd Qu.:1.000
 Max.   :1.000   Max.   :0.830   Max.   :1.000   Max.   :1.000   Max.   :1.000
```

Basic Model(s)

Logit:

$$\Pr(Y_i = 1) = \frac{\exp(\mathbf{X}_i\boldsymbol{\beta})}{1 + \exp(\mathbf{X}_i\boldsymbol{\beta})}$$

or probit:

$$\Pr(Y_i = 1) = \Phi(\mathbf{X}_i\boldsymbol{\beta})$$

Probit Estimates

```
> NAFTA.probit<-glm(Vote~PropHisp+Democrat+COPE+DemXCOPE,
                   NAFTA,family=binomial(link="probit"))
> summary(NAFTA.probit)

Call:
glm(formula = Vote ~ PropHisp + Democrat + COPE + DemXCOPE,
    family = binomial(link = "probit"), data = NAFTA)

Deviance Residuals:
   Min      1Q  Median      3Q     Max
-3.173  -0.677   0.362   0.764   1.817

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)    1.078      0.153    7.03  2.1e-12 ***
PropHisp       1.279      0.467    2.74   0.0062 **
Democrat       3.034      0.739    4.11  4.0e-05 ***
COPE          -2.201      0.440   -5.00  5.8e-07 ***
DemXCOPE      -2.888      0.903   -3.20   0.0014 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 598.99  on 433  degrees of freedom
Residual deviance: 441.06  on 429  degrees of freedom
AIC: 451.1

Number of Fisher Scoring iterations: 8
```

Logit Estimates

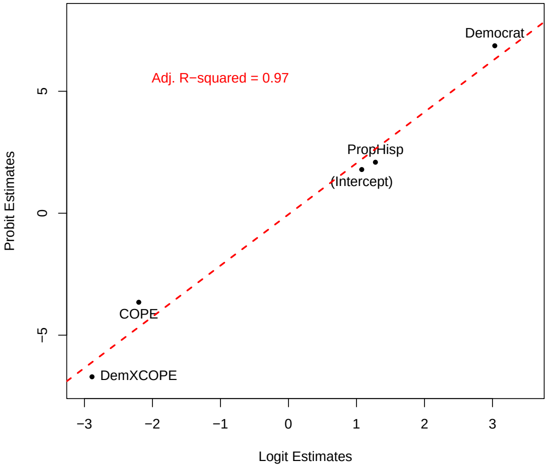
```
> NAFTA.fit<-glm(Vote~PropHisp+Democrat+COPE+DemXCOPE,
                        NAFTA,family=binomial)
> summary(NAFTA.fit)

Call:
glm(formula = Vote ~ PropHisp + Democrat + COPE + DemXCOPE,
    family = binomial, data = NAFTA)

Deviance Residuals:
   Min      1Q  Median      3Q     Max
-3.264  -0.650   0.310   0.728   1.818

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)    1.792      0.275    6.50  7.8e-11 ***
PropHisp       2.091      0.794    2.63  0.00846 **
Democrat       6.866      1.547    4.44  9.1e-06 ***
COPE          -3.650      0.760   -4.80  1.6e-06 ***
DemXCOPE      -6.705      1.820   -3.68  0.00023 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 598.99  on 433  degrees of freedom
Residual deviance: 436.83  on 429  degrees of freedom
AIC: 446.8

Number of Fisher Scoring iterations: 5

> # Equivalent to:
>
> fit<-glm(Vote~PropHisp+Democrat*COPE,NAFTA,family=binomial)
```

Models (table via `modelsummary`)

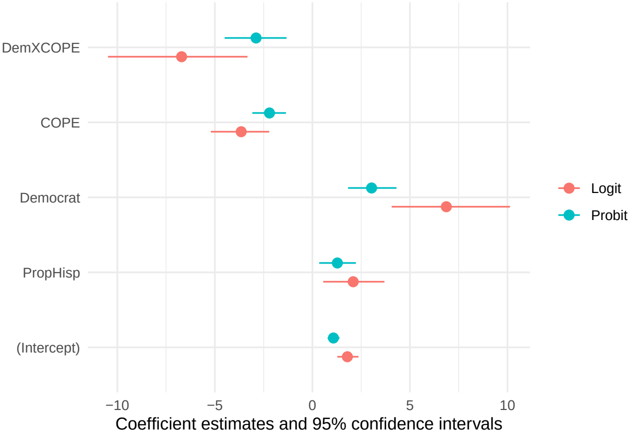
Table: Logits and Probits

	Logit	Probit
(Intercept)	1.792	1.078
	(0.275)	(0.153)
PropHisp	2.091	1.279
	(0.794)	(0.467)
Democrat	6.866	3.034
	(1.547)	(0.739)
COPE	-3.650	-2.201
	(0.760)	(0.440)
DemXCOPE	-6.705	-2.888
	(1.820)	(0.903)
Num.Obs.	434	434
AIC	446.8	451.1
BIC	467.2	471.4
Log.Lik.	-218.414	-220.532
F	26.622	30.723
RMSE	0.40	0.41

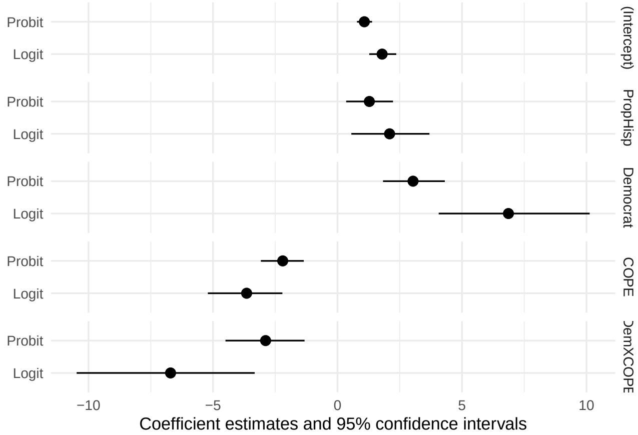
$\hat{\beta}_{\text{probit}}$ vs. $\hat{\beta}_{\text{logit}}$



Coefficient Plots (via `modelplot`)



Faceted Plot (also via `modelplot`)



Log-Likelihoods, "Deviance," etc.

- `R` / `glm` reports "deviances":
  - "Residual" deviance $= 2(\ln L_S - \ln L_M)$
  - "Null" deviance $= 2(\ln L_S - \ln L_N)$
  - stored in `object$deviance` and `object$null.deviance`
- So:

$$\begin{aligned} LR_{\boldsymbol{\beta}=\mathbf{0}} &= 2(\ln L_M - \ln L_N) \\ &= \text{“Null” deviance} - \text{“Residual” deviance} \end{aligned}$$

Example:

```
> LLR<-NAFTA.fit$null.deviance - NAFTA.fit$deviance

> LLR
[1] 162

> pchisq(LLR,4,lower.tail=FALSE)
[1] 5.04e-34
```

Interpretation: "Signs-n-Significance"

For both logit and probit:

- $\hat{\beta}_k > 0 \leftrightarrow \frac{\partial \Pr(Y=1)}{\partial X_k} > 0$
- $\hat{\beta}_k < 0 \leftrightarrow \frac{\partial \Pr(Y=1)}{\partial X_k} < 0$
- $\frac{\hat{\beta}_k}{\hat{\sigma}_k} \sim N(0,1)$

Interactions:

$$\hat{\beta}_{\texttt{COPE|Democrat=1}} \equiv \hat{\phi}_{\texttt{COPE}} = \hat{\beta}_3 + \hat{\beta}_4$$

$$\text{s.e.}(\hat{\beta}_{\texttt{COPE|Democrat=1}}) = \sqrt{\text{Var}(\hat{\beta}_3) + (\texttt{Democrat})^2 \text{Var}(\hat{\beta}_4) + 2\,(\texttt{Democrat})\,\text{Cov}(\hat{\beta}_3, \hat{\beta}_4)}$$

```
> NAFTA.fit$coeff[4]+NAFTA.fit$coeff[5]

 COPE
-10.4

> # z-statistic:
>
> (NAFTA.fit$coeff[4]+NAFTA.fit$coeff[5]) /
+   (sqrt(vcov(NAFTA.fit)[4,4] +
+   (1)^2*vcov(NAFTA.fit)[5,5] +
+   2*1*vcov(NAFTA.fit)[4,5]))
 COPE
-6.25

> # Square that, and it's a chi-square statistic:
>
> ((NAFTA.fit$coeff[4]+NAFTA.fit$coeff[5]) /
+   (sqrt(vcov(NAFTA.fit)[4,4] +
+           (1)^2*vcov(NAFTA.fit)[5,5] +
+           2*1*vcov(NAFTA.fit)[4,5])))^2
COPE
  39
```

(Or use car...)

```
> library(car)
> linearHypothesis(NAFTA.fit,"COPE+DemXCOPE=0")
Linear hypothesis test

Hypothesis:
COPE  + DemXCOPE = 0

Model 1: restricted model
Model 2: Vote ~ Democrat + PropHisp + COPE + DemXCOPE

  Res.Df Df Chisq    Pr(>Chisq)
1    430
2    429  1    39 0.00000000042 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

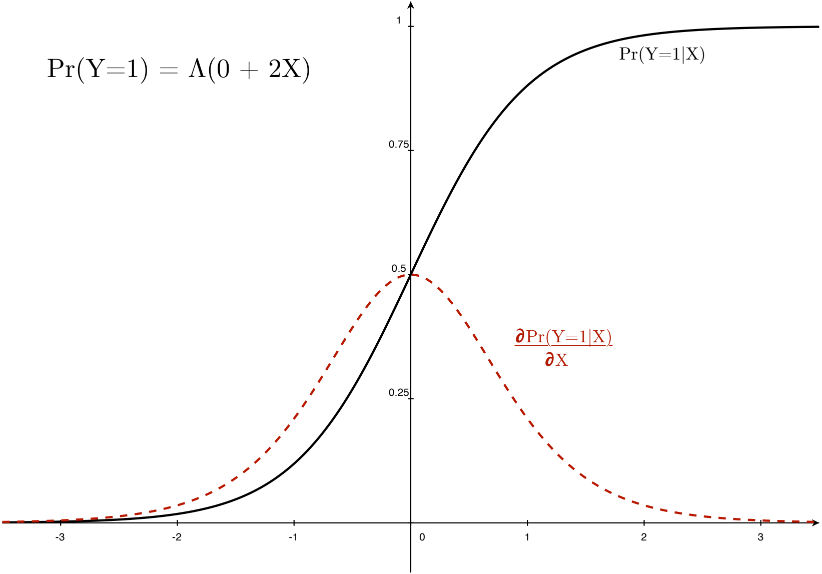
The *marginal effect* is:

$$\begin{aligned}
\frac{\partial \Pr(\hat{Y}_i = 1)}{\partial X_k} &= \frac{\partial F(\mathbf{X}_i \hat{\boldsymbol{\beta}})}{\partial X_k} \\
&= f(\mathbf{X}_i \hat{\boldsymbol{\beta}})\hat{\beta}_k \\
&= \Lambda(\mathbf{X}_i \hat{\boldsymbol{\beta}})[1 - \Lambda(\mathbf{X}_i \hat{\boldsymbol{\beta}})]\hat{\beta}_k \quad \text{(for logit) or} \\
&= \phi(\mathbf{X}_i \hat{\boldsymbol{\beta}})\hat{\beta}_k \quad \text{(for probit)}
\end{aligned}$$

Note that these depend on $\mathbf{X}\hat{\boldsymbol{\beta}}$, which means we either have to:

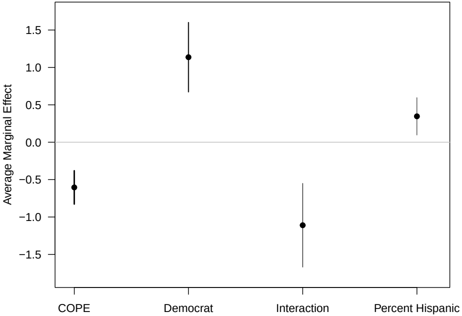
1. ...hold $\mathbf{X}\hat{\boldsymbol{\beta}}$ constant at some value(s), or
2. ...average over the actual values of $\mathbf{X}_i\hat{\boldsymbol{\beta}}$ observed in the data.

Marginal Effects Illustrated



```
> summary(margins(NAFTA.fit))
   factor     AME     SE       z      p   lower   upper
     COPE -0.6043 0.1139 -5.3048 0.0000 -0.8275 -0.3810
 Democrat  1.1366 0.2370  4.7953 0.0000  0.6720  1.6011
 DemXCOPE -1.1101 0.2858 -3.8836 0.0001 -1.6703 -0.5498
 PropHisp  0.3462 0.1280  2.7054 0.0068  0.0954  0.5970
```

Plotted:



Log-Odds of $Y = 1$ are linear in $\mathbf{X}$:

$$\ln \Omega(\mathbf{X}) = \ln \left[\frac{\frac{\exp(\mathbf{X}\boldsymbol{\beta})}{1+\exp(\mathbf{X}\boldsymbol{\beta})}}{1 - \frac{\exp(\mathbf{X}\boldsymbol{\beta})}{1+\exp(\mathbf{X}\boldsymbol{\beta})}} \right] = \mathbf{X}\boldsymbol{\beta}$$

That implies that:

$$\frac{\partial \ln \Omega}{\partial \mathbf{X}} = \boldsymbol{\beta}$$

OR for a one-unit change in X_k:

$$\frac{\Omega(X_k = \ell + 1)}{\Omega(X_k = \ell)} = \exp(\hat{\beta}_k)$$

OR for a δ-unit change in X_k:

$$\frac{\Omega(X_k = \ell + \delta)}{\Omega(X_k = \ell)} = \exp(\hat{\beta}_k \delta)$$

Also:

$$\text{Percentage Change in the Odds} = 100[\exp(\hat{\beta}_k \delta) - 1]$$

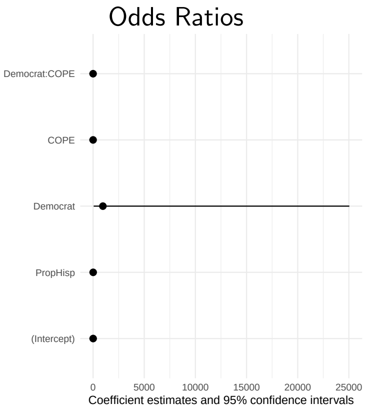
Odds Ratios Implemented

```
> P<-qnorm(0.975)

> lreg.or <- function(model)
+ {
+   coeffs <- coef(summary(model))
+   lowerCI <- exp(coeffs[ ,1] - P * coeffs[ ,2])
+   OR <- exp(coeffs[ ,1])
+   upperCI <- exp(coeffs[ ,1] + P * coeffs[ ,2])
+   lreg.or <- cbind(OR,lowerCI,upperCI)
+   lreg.or
+ }

> lreg.or(NAFTA.fit)
                   OR    lowerCI    upperCI
(Intercept)   5.99928  3.4965990    10.2933
PropHisp      8.09352  1.7068838    38.3770
Democrat    958.67832 46.1969511 19894.4757
COPE          0.02599  0.0058625     0.1152
DemXCOPE      0.00122  0.0000345     0.0434

> Or via -confint-...

> exp(cbind(OR=coef(NAFTA.fit),confint.default(NAFTA.fit)))
                   OR      2.5 %     97.5 %
(Intercept)   5.99928  3.4965990    10.2933
PropHisp      8.09352  1.7068838    38.3770
Democrat    958.67832 46.1969511 19894.4757
COPE          0.02599  0.0058625     0.1152
DemXCOPE      0.00122  0.0000345     0.0434
```

Odds Ratios via `modelsummary` / `modelplot`

Table: Odds Ratios

	(1)
(Intercept)	5.999
	(1.652)
PropHisp	8.094
	(6.427)
Democrat	958.678
	(1483.358)
COPE	0.026
	(0.020)
Democrat $\times$ COPE	0.001
	(0.002)
Num.Obs.	434
AIC	446.8
BIC	467.2
Log.Lik.	-218.414
F	26.622
RMSE	0.40



What Does This *Mean*?

```
> NAFTA.fit

Coefficients:
(Intercept)     PropHisp     Democrat         COPE     DemXCOPE
       1.79         2.09         6.87        -3.65        -6.71

> exp(cbind(OR=coef(NAFTA.fit),confint.default(NAFTA.fit)))
                   OR      2.5 %     97.5 %
(Intercept)   5.99928  3.4965990    10.2933
PropHisp      8.09352  1.7068838    38.3770
Democrat    958.67832 46.1969511 19894.4757
COPE          0.02599  0.0058625     0.1152
DemXCOPE      0.00122  0.0000345     0.0434
```

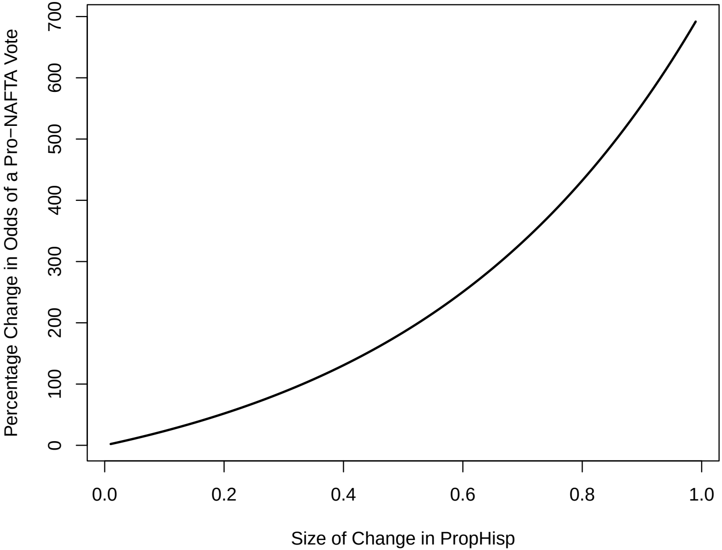
Consider `PropHispc`:

- A one-unit change (from 0 percent to 100 percent) in `PropHisp` corresponds to a $[(8.094 - 1) \times 100] =$ **709 percent expected increase in the odds** of a member of Congress voting in favor of NAFTA.
- A change of 0.10 (that is, a ten percentage-point increase) in the proportion of a member's district who is Hispanic corresponds to an **odds ratio** of:

$$\begin{aligned} \exp(2.09 \times 0.10) &= \exp(0.209) \\ &= 1.232 \end{aligned}$$

- This means that an increase of 0.10 in `PropHisp` corresponds to a $[(1.232 - 1) \times 100] =$ **23.2 percent expected increase in the odds** that a member of Congress would have voted in favor of NAFTA.
- For an increase of 0.20 (that is, 20 percentage points), the corresponding odds ratio and percent increase are 1.519 and 51.9 percent, respectively.

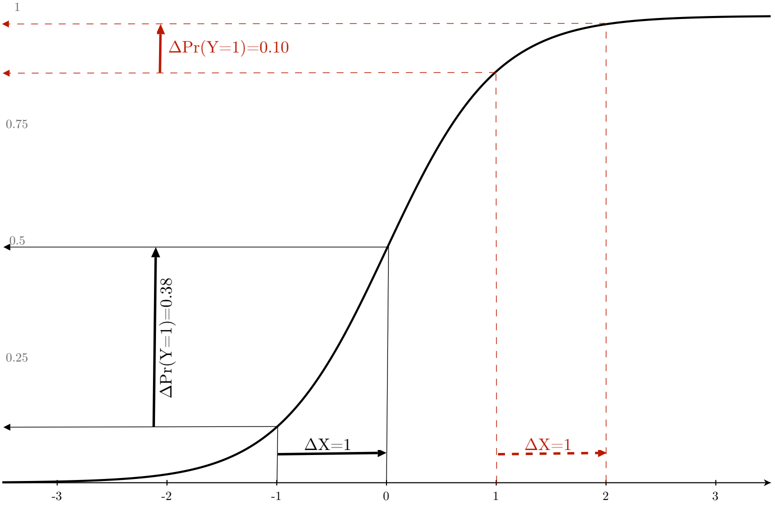
Percentage Change in Odds, by Δ`PropHisp`



Predicted probabilities:

$$\begin{aligned}
\widehat{\Pr(Y_i = 1)} &= F(\mathbf{X}_i\hat{\boldsymbol{\beta}}) \\
&= \frac{\exp(\mathbf{X}_i\hat{\boldsymbol{\beta}})}{1+\exp(\mathbf{X}_i\hat{\boldsymbol{\beta}})} \text{ for logit,} \\
&= \Phi(\mathbf{X}_i\hat{\boldsymbol{\beta}}) \text{ for probit.}
\end{aligned}$$

Predicted Probabilities Illustrated



Predicted Probabilities: Standard Errors

$$
\begin{aligned}
\text{Var}[\widehat{\Pr(Y_i = 1)}] &= \left[\frac{\partial F(\mathbf{X}_i \hat{\boldsymbol{\beta}})}{\partial \hat{\boldsymbol{\beta}}}\right]' \hat{\mathbf{V}} \left[\frac{\partial F(\mathbf{X}_i \hat{\boldsymbol{\beta}})}{\partial \hat{\boldsymbol{\beta}}}\right] \\
&= [f(\mathbf{X}_i \hat{\boldsymbol{\beta}})]^2 \mathbf{X}_i' \hat{\mathbf{V}} \mathbf{X}_i
\end{aligned}
$$

So,

$$
\text{s.e.}[\widehat{\Pr(Y_i = 1)}] = \sqrt{[f(\mathbf{X}_i \hat{\boldsymbol{\beta}})]^2 \mathbf{X}_i' \hat{\mathbf{V}} \mathbf{X}_i}
$$

Changes in $\widehat{\Pr(Y=1)}$:

$$\Delta \widehat{\Pr(Y=1)}_{\mathbf{X}_A \rightarrow \mathbf{X}_B} = \frac{\exp(\mathbf{X}_B \hat{\boldsymbol{\beta}})}{1+\exp(\mathbf{X}_B \hat{\boldsymbol{\beta}})} - \frac{\exp(\mathbf{X}_A \hat{\boldsymbol{\beta}})}{1+\exp(\mathbf{X}_A \hat{\boldsymbol{\beta}})}$$

or

$$= \Phi(\mathbf{X}_B \hat{\boldsymbol{\beta}}) - \Phi(\mathbf{X}_A \hat{\boldsymbol{\beta}})$$

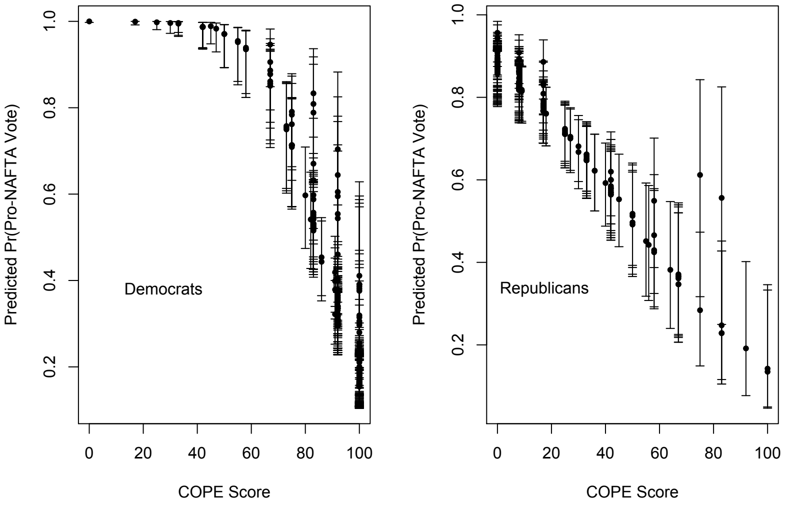
Standard errors obtainable via delta method, bootstrap, etc...

In-Sample Predictions

```
> preds<-NAFTA.fit$fitted.values

> hats<-predict(NAFTA.fit,se.fit=TRUE)
> hats
$fit
          1           2           3           4 ...
 9.01267619  7.25223902  6.11013844  5.57444635 ...
 ...
 $se.fit
        1         2         3         4 ...
1.5331506 1.2531475 1.1106989 0.9894208 ...


> XBUB<-hats$fit + (1.96*hats$se.fit)
> XBLB<-hats$fit - (1.96*hats$se.fit)
> plotdata<-cbind(as.data.frame(hats),XBUB,XBLB)
> plotdata<-data.frame(lapply(plotdata,binomial(link="logit")$linkinv))
```

```
...

> par(mfrow=c(1,2))
> library(plotrix)

> with(NAFTA,
+  plotCI(COPE[Democrat==1],plotdata$fit[Democrat==1],ui=plotdata$XBUB[Democrat==1],
+          li=plotdata$XBLB[Democrat==1],pch=20,xlab="COPE Score",ylab="Predicted
+          Pr(Pro-NAFTA Vote)"))

> with(NAFTA,
+  plotCI(COPE[Democrat==0],plotdata$fit[Democrat==0],ui=plotdata$XBUB[Democrat==0],
+          li=plotdata$XBLB[Democrat==0],pch=20,xlab="COPE Score",ylab="Predicted
+          Pr(Pro-NAFTA Vote)"))
```

In-Sample Predictions



Out-of-Sample Predictions

"Fake" data:

```
> sim.data<-data.frame(PropHisp=mean(NAFTA$PropHisp),Democrat=rep(0:1,101),
                       COPE=seq(from=0,to=1,length.out=101))
> sim.data$DemXCOPE<-sim.data$Democrat*sim.data$COPE
```

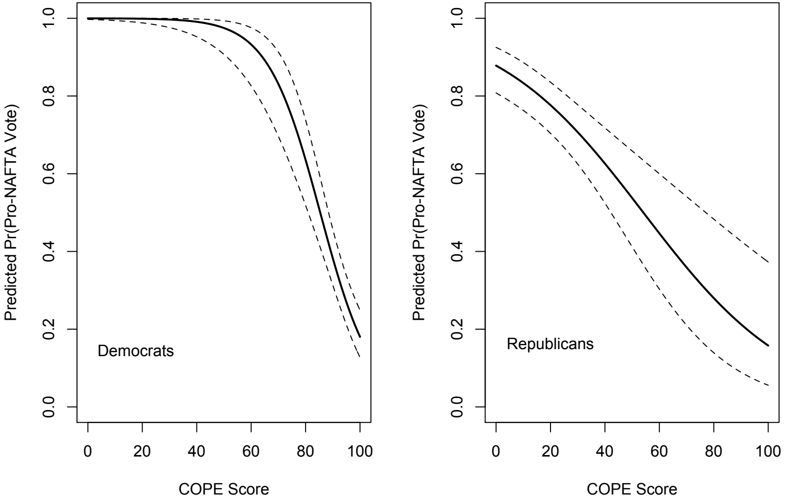
Generate predictions:

```
> OutHats<-predict(NAFTA.fit,se.fit=TRUE,newdata=sim.data)
> OutHatsUB<-OutHats$fit+(1.96*OutHats$se.fit)
> OutHatsLB<-OutHats$fit-(1.96*OutHats$se.fit)
> OutHats<-cbind(as.data.frame(OutHats),OutHatsUB,OutHatsLB)
> OutHats<-data.frame(lapply(OutHats,binomial(link="logit")$linkinv))
```

Plot:

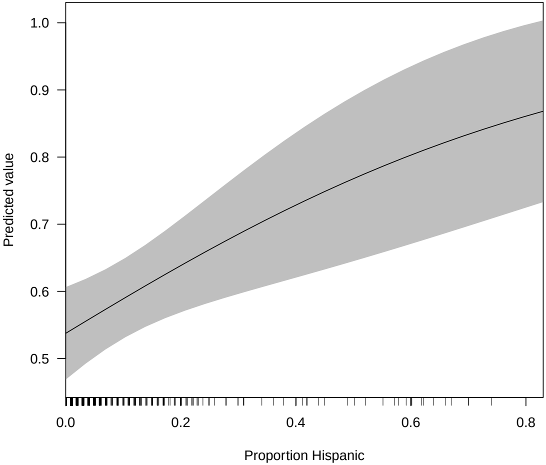
```
> both<-cbind(sim.data,OutHats)
> both<-both[order(both$COPE,both$Democrat),]
> bothD<-both[both$Democrat==1,]
> bothR<-both[both$Democrat==0,]

> par(mfrow=c(1,2))

> plot(bothD$COPE,bothD$fit,t="l",lwd=2,ylim=c(0,1),
      xlab="COPE Score",ylab="Predicted Pr(Pro-NAFTA Vote)")
> lines(bothD$COPE,bothD$OutHatsUB,lty=2)
> lines(bothD$COPE,bothD$OutHatsLB,lty=2)
> text(0.3,0.2,label="Democrats")

> plot(bothR$COPE,bothR$fit,t="l",lwd=2,ylim=c(0,1),
      xlab="COPE Score",ylab="Predicted Pr(Pro-NAFTA Vote)")
> lines(bothR$COPE,bothR$OutHatsUB,lty=2)
> lines(bothR$COPE,bothR$OutHatsLB,lty=2)
> text(0.7,0.9,label="Republicans")
```

Out-of-Sample Predictions



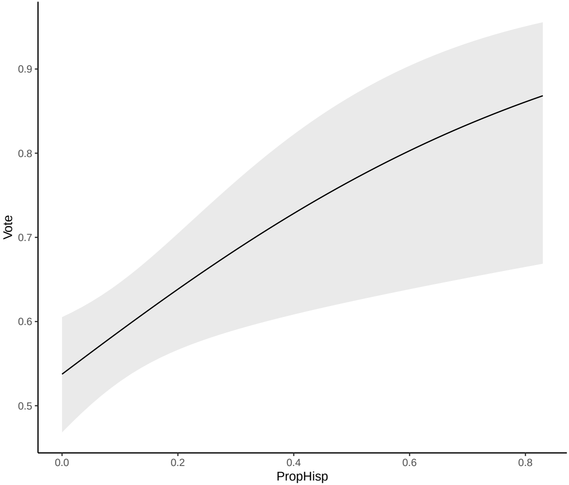
Single-Variable Example (using `cplot`)

```
> cplot(NAFTA.fit,"PropHisp",xlab="Proportion Hispanic")
```



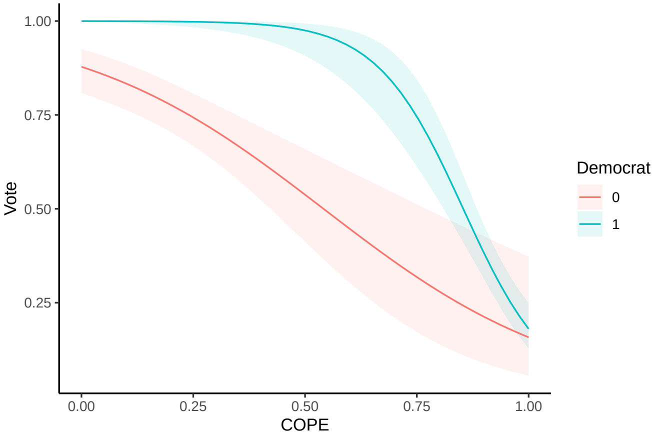
Same, using marginaleffects::plot_prediction

```
> plot_predictions(NAFTA.fit,condition="PropHisp") + theme_classic()
```



Interactive part, using `plot_prediction`

```
> plot_predictions(fit,condition=c("COPE","Democrat")) + theme_classic()
```



Goodness of Fit

Some alternatives....

- Pseudo-R^2 (no!)
- Proportional reduction in error (PRE) – a/k/a "accuracy"
- ROC curves.

Model Fit: Predictions

Suppose we assign:

$$\hat{Y}_i = 0 \quad \text{if} \quad \widehat{\Pr(Y_i = 1)} \leq \tau$$
$$\hat{Y}_i = 1 \quad \text{if} \quad \widehat{\Pr(Y_i = 1)} > \tau$$

This would then give us a "confusion matrix":

	Predicted $\hat{Y}_i$	
Actual Y_i	$\hat{Y}_i = 0$	$\hat{Y}_i = 1$
$Y_i = 0$	True Negative ("TN")	False Positive ("FP")
$Y_i = 1$	False Negative ("FN")	True Positive ("TP")

This means we have:

- Total actual negatives = TN + FP
- Total actual positives = TP + FN
- Number correctly predicted = TP + TN

Model Fit: PRE

Proportional Reduction in Error (PRE):

$$\text{PRE} = \frac{N_{MC} - N_{NC}}{N - N_{NC}}$$

- N_{NC} = number correctly predicted under the "null model,"
- N_{MC} = number correctly predicted under the estimated model,
- N = total number of observations.

PRE tells us how much (proportionally) better our model does at predicting Y in-sample than would a model that only contained an intercept.

PRE: Example

```
> Assume tau = 0.5...
>
> table(NAFTA.fit$fitted.values>0.5,nafta$vote==1)

       FALSE TRUE
  FALSE   148   49
  TRUE     52  185
```

$$\begin{aligned} \text{PRE} &= \frac{N_{MC} - N_{NC}}{N - N_{NC}} \\ &= \frac{(148 + 185) - 234}{434 - 234} \\ &= \frac{99}{200} \\ &= \mathbf{0.495} \end{aligned}$$

Chi-Square test:

```
> chisq.test(NAFTA.fit$fitted.values>0.5,NAFTA$Vote==1)

Pearson's Chi-squared test with Yates' continuity correction

data:  NAFTA.fit$fitted.values > 0.5 and NAFTA$Vote == 1
X-squared = 120, df = 1, p-value <2e-16
```

Related Ideas

Concepts:

- *Sensitivity* (or "true positive rate")
  - The proportion of all actual positives that were predicted correctly
  - Sensitivity $= \frac{\text{TP}}{\text{TP} + \text{FN}}$
- *Specificity* (or "true negative rate")
  - The proportion of all actual negatives that were predicted correctly
  - Specificity $= \frac{\text{TN}}{\text{TN} + \text{FP}}$
- False positive rate $= 1-$*Specificity*
- False negative rate $= 1-$*Sensitivity*

Varying τ

Suppose we set $\tau = 0.00001$. Then:

- We would essentially *always* predict $\hat{Y}_i = 1$, which means
- ...we would always correctly predict all the actual positives (maximize TPs), but
- ...we'd also always get every actual negative wrong (maximize FPs).

Similarly, if we set $\tau = 0.99999$. Then:

- We would essentially *always* predict $\hat{Y}_i = 0$, which means
- ...we would always correctly predict all the actual negatives (maximize TNs), but
- ...also always get every actual positive wrong (maximize FNs).

Values of τ between the extremes trade off true positives for false positives; as τ increases, we have fewer of the former and more of the latter.

```
> # Tau = 0.2:

> Hats02<-ifelse(NAFTA.fit$fitted.values>0.2,1,0)
> CrossTable(NAFTA$Vote,Hats02,prop.r=FALSE,prop.c=FALSE,
             prop.t=FALSE,prop.chisq=FALSE)

             | Hats02
  NAFTA$Vote |         0 |         1 | Row Total |
-------------|-----------|-----------|-----------|
           0 |        96 |       104 |       200 |
-------------|-----------|-----------|-----------|
           1 |         1 |       233 |       234 |
-------------|-----------|-----------|-----------|
Column Total |        97 |       337 |       434 |
-------------|-----------|-----------|-----------|

TPR = 233/234 = 0.996
FPR = 104/200 = 0.520


> # Tau = 0.8:

> Hats08<-ifelse(NAFTA.fit$fitted.values>0.8,1,0)
> CrossTable(NAFTA$Vote,Hats08,prop.r=FALSE,prop.c=FALSE,
             prop.t=FALSE,prop.chisq=FALSE)

             | Hats08
  NAFTA$Vote |         0 |         1 | Row Total |
-------------|-----------|-----------|-----------|
           0 |       178 |        22 |       200 |
-------------|-----------|-----------|-----------|
           1 |       123 |       111 |       234 |
-------------|-----------|-----------|-----------|
Column Total |       301 |       133 |       434 |
-------------|-----------|-----------|-----------|

TPR = 111/234 = 0.474
FPR = 178/200 = 0.890
```

“Receiver Operating Characteristic” (ROC) Curves

Now, imagine:

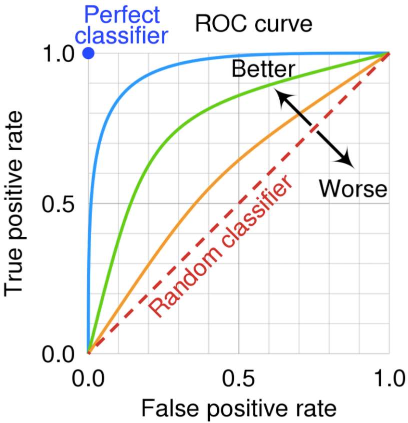
1. Fit a model
2. Choose a value of τ very near 0
3. Generate $\hat{Y}_i$s
4. Calculate and save the TPR and FPR for that value of τ
5. Increase τ by a very small amount
6. Go to (3), and repeat until τ is very close to 1.0

We could then plot the true positive rate vs. false positive rate (i.e., *Specificity* vs. 1 - *Sensitivity*)

ROC Curves (continued)

- If the model fits perfectly, it will have a 1.0 true positive rate, and a 0.0 false negative rate
- If the model fits no better than random chance, the curve defined by those points will be a diagonal line.
- (Intuition: If each prediction is no better than a (weighted) coin flip, the rate of true positives and false positives will increase together.)
- In between these extremes, better-fitting models will have curves that are closer to the upper-left corner

"AUROC": Area under the ROC curve
→ assessment of model fit

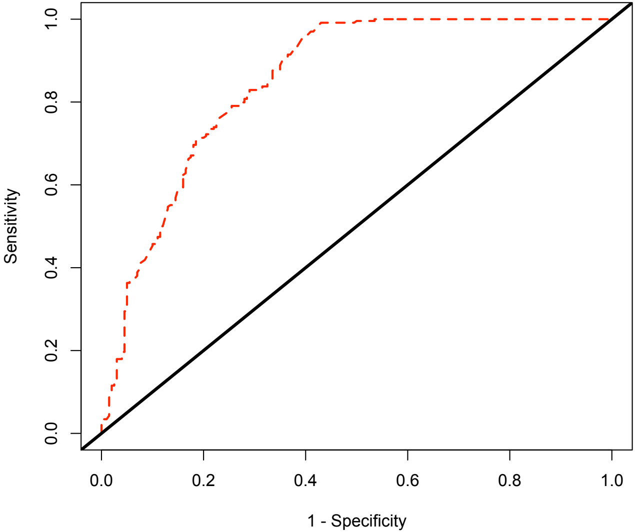


(Source)

ROC Curves Implemented

```
> library(ROCR)

> NAFTA.hats<-predict(NAFTA.fit,type="response")

> preds<-ROCR::prediction(NAFTA.hats,NAFTA$Vote)

> plot(performance(preds,"tpr","fpr"),lwd=2,lty=2,
       col="red",xlab="1 - Specificity",ylab="Sensitivity")
> abline(a=0,b=1,lwd=3)
```

ROC Curve: Example



Interpreting AUROC Curves

- Area under ROC $=$ 0.90-1.00 $\rightarrow$ Excellent (A)
- Area under ROC $=$ 0.80-0.90 $\rightarrow$ Good (B)
- Area under ROC $=$ 0.70-0.80 $\rightarrow$ Fair (C)
- Area under ROC $=$ 0.60-0.70 $\rightarrow$ Poor (D)
- Area under ROC $=$ 0.50-0.60 $\rightarrow$ Total Failure (F)

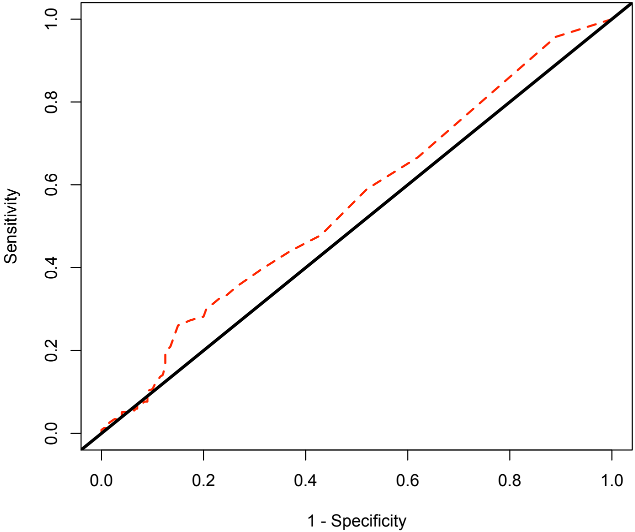
ROC Curve: A Poorly-Fitting Model

Model is:

$$\Pr(\texttt{vote}_i = 1) \quad = \quad f[\beta_0 + \beta_1(\texttt{PropHisp}_i) + u_i]$$

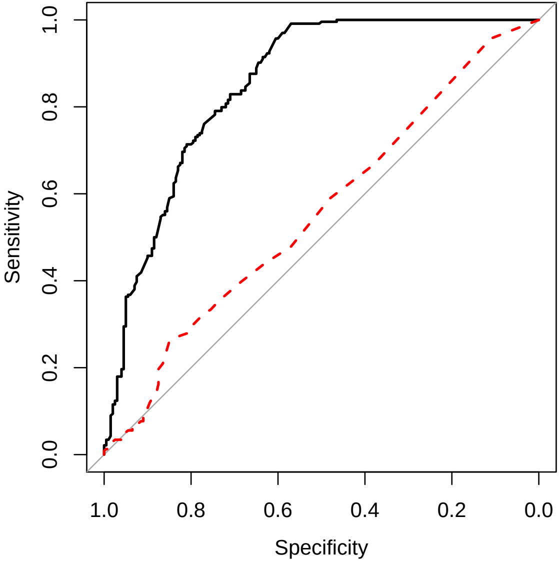
```
> NAFTA.bad<-with(NAFTA,
                glm(Vote~PropHisp,family=binomial(link="logit")))
> NAFTA.bad.hats<-predict(NAFTA.bad,type="response")
> bad.preds<-ROCR::prediction(NAFTA.bad.hats,NAFTA$Vote)

> plot(performance(bad.preds,"tpr","fpr"),lwd=2,lty=2,
     col="red",xlab="1 - Specificity",ylab="Sensitivity")
> abline(a=0,b=1,lwd=3)
```

Bad ROC!



```
> install.packages("pROC")
> library(pROC)

> GoodROC<-roc(NAFTA$Vote,NAFTA.hats,ci=TRUE)
> GoodAUC<-auc(GoodROC)
> BadROC<-roc(NAFTA$Vote,NAFTA.bad.hats,ci=TRUE)
> BadAUC<-auc(BadROC)

> GoodAUC
Area under the curve: 0.85

> BadAUC
Area under the curve: 0.556
```

Combined Plot



Useful Things

Model Fitting, etc.:

- `glm` (in base `stats`)
  - Binary responses = `family(binomial)`
  - Links: `logit`, `probit`, `cloglog`, `log`, `cauchit` (Cauchy)
- Some `easystats` packages:
  - `datawizard` (standardizing variables, etc.)
  - `correlation` (what the name says...)

Model Interpretation + Visualization:

- `modelsummary` (tables and plots of estimates, ORs, etc.)
- `marginaleffects` (generate and plot of predictions, etc.)
- `margins` (marginal effects)
- `ROCR`, `pROC` (generate / plot ROC curves, calculate AUROC)
- `easystats` packages:
  - `report` + `parameters` (tables, output, etc.)
  - `modelbased` + `effectsize` (substantive interpretation of models)
  - `performance` (model fit: R^2, AUROC, etc.)

Other Binary-Response Topics

Things we'll probably talk about later:

- Rare events and "separation"
- Binary responses in panel / longitudinal data
- Multilevel / hierarchical models for binary responses
- Models with (binary) sample selection
- Measurement models for binary outcomes (e.g., item response models)
- Semi- and non-parametric models (see, e.g., Horowitz and Savin 2001)
- "Heteroscedastic" models (where $\sigma_i^2 \neq \sigma^2 \, \forall \, i$) (see, e.g., Alvarez and Brehm 1995, 1997; Tutz 2018)
- "Bivariate" probit models, where

$$\{Y_{1i}, Y_{2i}\} \sim BVN(0, 0, 1, 1, \rho)$$