PLSC 503 – Spring 2024
Regression Models for Ordinal Outcomes

April 15, 2024

Ordinal Data

Ordinal data are:

- Discrete: $Y \in \{1, 2, ...\}$
- *Grouped Continuous* Data
- *Assessed Ordered* Data

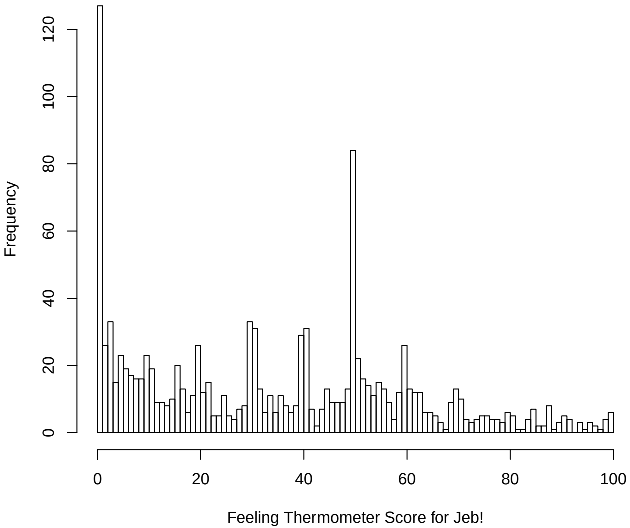
In general:

- Some things can be ordered, but shouldn't be
- Some things are ordered in some circumstances but not others
- Orderings can differ across applications

Ordinal vs. Continuous Response Models

"I'd like to get your feelings toward some of our political leaders and other people who are in the news these days. I'll read the name of a person and I'd like you to rate that person using something we call the feeling thermometer. Ratings between 50 and 100 degrees mean that you feel favorably and warm toward the person; ratings between 0 and 50 degrees mean that you don't feel favorably toward the person and that you don't care too much for that person. You would rate the person at the 50 degree mark if you don't feel particularly warm or cold toward the person."

Thermometer Scores for Jeb! (2016)



A Fake-Data Example

$$Y_i^* = 0 + 1.0X_i + u_i,$$

$$X_i \sim U[0, 10]$$

$$u_i \sim N(0, 1)$$

$$\begin{aligned} Y_{1i} &= 1 \quad \text{if} \quad Y_i^* < 2.5 \\ &= 2 \quad \text{if} \quad 2.5 \leq Y_i^* < 5 \\ &= 3 \quad \text{if} \quad 5 \leq Y_i^* < 7.5 \\ &= 4 \quad \text{if} \quad Y_i^* > 7.5 \end{aligned}$$

$$\begin{aligned} Y_{2i} &= 1 \quad \text{if} \quad Y_i^* < 2 \\ &= 2 \quad \text{if} \quad 2 \leq Y_i^* < 8 \\ &= 3 \quad \text{if} \quad 8 \leq Y_i^* < 9 \\ &= 4 \quad \text{if} \quad Y_i^* > 9 \end{aligned}$$

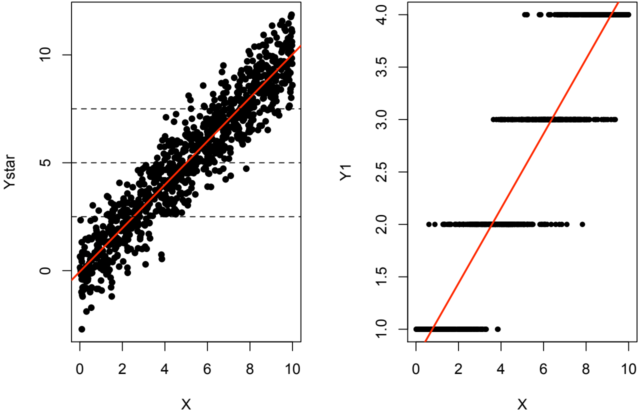
World's Best Regression

```
> summary(lm(Ystar~X))

Call:
lm(formula = Ystar ~ X)

Residuals:
   Min     1Q Median     3Q    Max
-3.006 -0.654 -0.049  0.643  3.298

Coefficients:
            Estimate Std. Error t value            Pr(>|t|)
(Intercept)  -0.0830     0.0609   -1.36                0.17
X             1.0110     0.0106   95.48 <0.0000000000000002 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.988 on 998 degrees of freedom
Multiple R-squared:  0.901,Adjusted R-squared:  0.901
F-statistic: 9.12e+03 on 1 and 998 DF,  p-value: <0.0000000000000002
```

Also A Pretty Good Regression

```
> summary(lm(Y1~X))

Call:
lm(formula = Y1 ~ X)

Residuals:
    Min      1Q  Median      3Q     Max
-1.2889 -0.2439  0.0158  0.2592  1.3968

Coefficients:
            Estimate Std. Error t value             Pr(>|t|)
(Intercept)  0.69979    0.02639    26.5 <0.0000000000000002 ***
X            0.35825    0.00459    78.0 <0.0000000000000002 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.428 on 998 degrees of freedom
Multiple R-squared:  0.859,Adjusted R-squared:  0.859
F-statistic: 6.09e+03 on 1 and 998 DF,  p-value: <0.0000000000000002
```

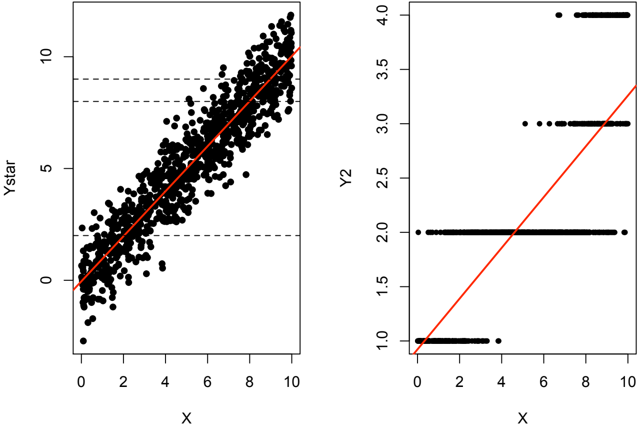
What That Looks Like



A Not-So-Good Regression

```
> summary(lm(Y2~X))

Call:
lm(formula = Y2 ~ X)

Residuals:
    Min      1Q  Median      3Q     Max
-1.3115 -0.3205 -0.0405  0.2914  1.4876

Coefficients:
            Estimate Std. Error t value            Pr(>|t|)
(Intercept)  0.88919    0.03069    29.0 <0.0000000000000002 ***
X            0.24383    0.00534    45.7 <0.0000000000000002 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.498 on 998 degrees of freedom
Multiple R-squared:  0.676,Adjusted R-squared:  0.676
F-statistic: 2.09e+03 on 1 and 998 DF,  p-value: <0.0000000000000002
```

What That Looks Like



Models for Ordinal Responses

$$Y_i^* = \mu + u_i$$

$$Y_i = j \text{ if } \tau_{j-1} \leq Y_i^* < \tau_j,\ j \in \{1, ...J\}$$

$$\begin{aligned}
Y_i &= 1 \quad \text{if} \quad -\infty \leq Y_i^* < \tau_1 \\
&= 2 \quad \text{if} \quad \tau_1 \leq Y_i^* < \tau_2 \\
&= 3 \quad \text{if} \quad \tau_2 \leq Y_i^* < \tau_3 \\
&= 4 \quad \text{if} \quad \tau_3 \leq Y_i^* < \infty
\end{aligned}$$

Ordinal Response Models: Probabilities

$$
\begin{aligned}
\Pr(Y_i = j) &= \Pr(\tau_{j-1} \leq Y^* < \tau_j) \\
&= \Pr(\tau_{j-1} \leq \mu_i + u_i < \tau_j)
\end{aligned}
$$

$$\mu_i = \mathbf{X}_i \boldsymbol{\beta}$$

$$
\begin{aligned}
\Pr(Y_i = j | \mathbf{X}, \boldsymbol{\beta}) &= \Pr(\tau_{j-1} \leq Y_i^* < \tau_j | \mathbf{X}) \\
&= \Pr(\tau_{j-1} \leq \mathbf{X}_i \boldsymbol{\beta} + u_i < \tau_j) \\
&= \Pr(\tau_{j-1} - \mathbf{X}_i \boldsymbol{\beta} \leq u_i < \tau_j - \mathbf{X}_i \boldsymbol{\beta}) \\
&= \int_{-\infty}^{\tau_j - \mathbf{X}_i \boldsymbol{\beta}} f(u_i) du - \int_{-\infty}^{\tau_{j-1} - \mathbf{X}_i \boldsymbol{\beta}} f(u_i) du \\
&= F(\tau_j - \mathbf{X}_i \boldsymbol{\beta}) - F(\tau_{j-1} - \mathbf{X}_i \boldsymbol{\beta})
\end{aligned}
$$

So for $Y \in \{1, 2, 3\}$:

$$\begin{aligned}
\Pr(Y_i = 1) &= F(\tau_1 - \mathbf{X}_i\boldsymbol{\beta}) - 0 \\
\Pr(Y_i = 2) &= F(\tau_2 - \mathbf{X}_i\boldsymbol{\beta}) - F(\tau_1 - \mathbf{X}_i\boldsymbol{\beta}) \\
\Pr(Y_i = 3) &= 1 - F(\tau_2 - \mathbf{X}_i\boldsymbol{\beta})
\end{aligned}$$

Possibilities: logit...

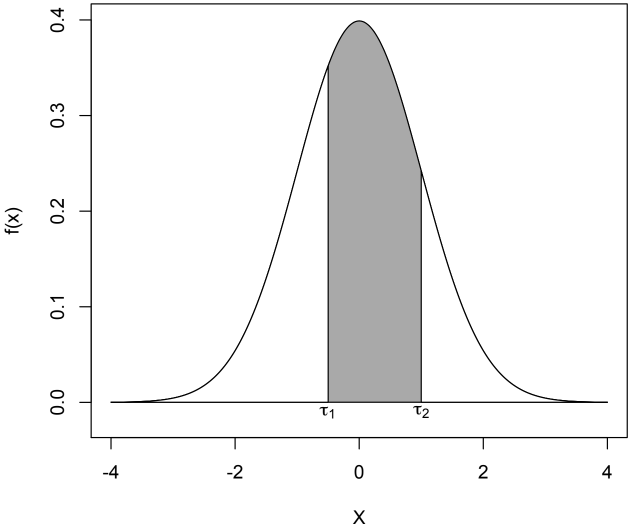
$$F(x) = \Lambda(x) = \frac{\exp(x)}{1 + \exp(x)}$$

...probit...

$$F(x) = \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) du$$

... etc.

What That Looks Like



Define:

$$\begin{aligned}\delta_{ij} &= 1 \text{ if } Y_i = j \\ &= 0 \text{ otherwise.}\end{aligned}$$

Likelihood:

$$L(Y|\mathbf{X}, \boldsymbol{\beta}, \tau) = \prod_{i=1}^{N}\prod_{j=1}^{J}[F(\tau_j - \mathbf{X}_i\boldsymbol{\beta}) - F(\tau_{j-1} - \mathbf{X}_i\boldsymbol{\beta})]^{\delta_{ij}}$$

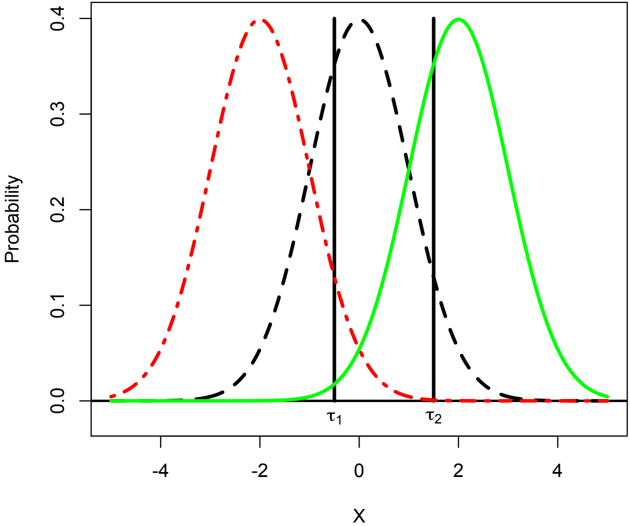
Log-Likelihood, probit:

$$\ln L(Y|\mathbf{X}, \boldsymbol{\beta}, \tau) = \sum_{i=1}^{N}\sum_{j=1}^{J}\delta_{ij}\ln[\Phi(\tau_j - \mathbf{X}_i\boldsymbol{\beta}) - \Phi(\tau_{j-1} - \mathbf{X}_i\boldsymbol{\beta})]$$

Log-Likelihood, logit:

$$\ln L(Y|\mathbf{X}, \boldsymbol{\beta}, \tau) = \sum_{i=1}^{N}\sum_{j=1}^{J}\delta_{ij}\ln[\Lambda(\tau_j - \mathbf{X}_i\boldsymbol{\beta}) - \Lambda(\tau_{j-1} - \mathbf{X}_i\boldsymbol{\beta})]$$

The Intuition



Requires:

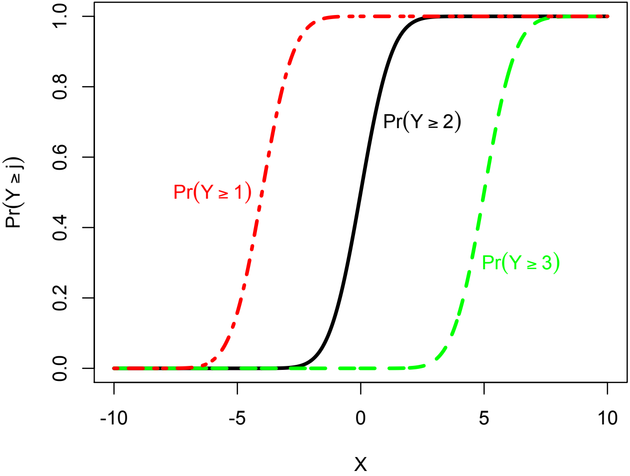
- (Usual) Assumption about $\sigma^2_{Y^*}$
- Identifying β_0 vs. the τs...
- Must either omit β_0 or drop one of the $J - 1$ τs
- In practice: Stata & R omit β_0

These models impose:

$$\frac{\partial \Pr(Y_i \geq j)}{\partial X} = \frac{\partial \Pr(Y_i \geq j')}{\partial X} \; \forall \; j \neq j'$$

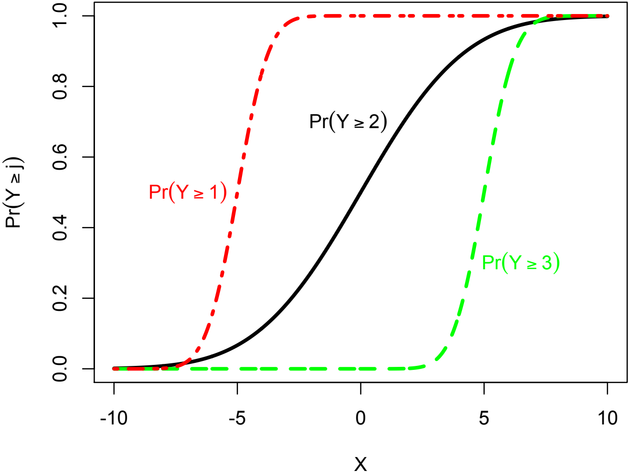
(aka "proportional odds"...)

Parallel Regressions Envisioned



$$\frac{\partial \Pr(Y_i \geq j)}{\partial X} \neq \frac{\partial \Pr(Y_i \geq j')}{\partial X} \; \forall \, j \neq j'$$

Nonparallel Regressions Envisioned



Estimation (in R)

Models can be fit using:

- `polr` (in `MASS`)
- `clm` (in `ordinal`)
- `ologit`/`oprobit` (in `Zelig`; calls `polr`)
- `vglm` (in `VGAM`)

Interpretation, etc.:

- `modelsummary` / `modelplot`
- `marginaleffects`
- Various tools in the `easystats` suite of packages

Yet Another SCOTUS Example!

The January 2018 TAPS Survey asked ≈1000 respondents "(D)o you approve or disapprove of the way the following are doing their jobs? – The U.S. Supreme Court." Choices were:

1 = "Strongly Disapprove" ($N = 97$)

2 = "Somewhat Disapprove" ($N = 276$)

3 = "Somewhat Approve" ($N = 581$)

4 = "Strongly Approve" ($N = 80$)

The data:

```
> describe(SCdf,skew=FALSE)
                 vars    n       mean        sd  median    min     max  range      se
ID                  1 1034 1062311.76 114597.13 1007385 989877 1358330 368453 3563.80
SCOTUSApproval      2 1034       2.62      0.76       3      1       4      3    0.02
TrumpApproval       3 1034       2.08      1.21       1      1       4      3    0.04
KnowChiefJustice    4 1034       0.79      0.41       1      0       1      1    0.01
Democrat            5 1034       0.36      0.48       0      0       1      1    0.01
GOP                 6 1034       0.33      0.47       0      0       1      1    0.01
Female              7 1034       0.46      0.50       0      0       1      1    0.02
White               8 1034       0.86      0.34       1      0       1      1    0.01
Black               9 1034       0.07      0.26       0      0       1      1    0.01
Education          10 1034      11.45      1.79      12      3      15     12    0.06
Age                11 1034      57.93     15.20      60     18     113     95    0.47
```

Ordered Logit (using polr)

```
> library(MASS)
> SC.logit<-polr(as.factor(SCOTUSApproval)~KnowChiefJustice+Democrat+
+                  GOP+Female+White+Black+Education+Age,data=SCdf)
> summary(SC.logit)

Call:
polr(formula = as.factor(SCOTUSApproval) ~ KnowChiefJustice +
    Democrat + GOP + Female + White + Black + Education + Age,
    data = SCdf)

Coefficients:
                    Value Std. Error t value
KnowChiefJustice  0.31692    0.15645   2.026
Democrat          0.08771    0.15149   0.579
GOP               0.49492    0.15311   3.233
Female            0.01301    0.12421   0.105
White            -0.07584    0.23822  -0.318
Black            -0.07732    0.32412  -0.239
Education         0.08349    0.03564   2.343
Age               0.00928    0.00409   2.266

Intercepts:
    Value  Std. Error t value
1|2 -0.443  0.515     -0.861
2|3  1.291  0.513      2.516
3|4  4.408  0.533      8.265

Residual Deviance: 2233.96
AIC: 2255.96
```

Ordered Logit (using `clm`)

```
> library(ordinal)
> SC.logit2<-clm(as.factor(SCOTUSApproval)~KnowChiefJustice+Democrat+
+                  GOP+Female+White+Black+Education+Age,data=SCdf)

> summary(SC.logit2)
formula: as.factor(SCOTUSApproval) ~ KnowChiefJustice + Democrat + GOP + Female + White + Black + Education + Age
data:    SCdf

 link  threshold nobs logLik   AIC     niter max.grad cond.H
 logit flexible  1034 -1116.98 2255.96 6(0)  5.02e-08 8.3e+05

Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
KnowChiefJustice  0.31692    0.15645    2.03   0.0428 *
Democrat          0.08771    0.15149    0.58   0.5626
GOP               0.49492    0.15311    3.23   0.0012 **
Female            0.01301    0.12421    0.10   0.9165
White            -0.07584    0.23822   -0.32   0.7502
Black            -0.07732    0.32412   -0.24   0.8115
Education         0.08349    0.03564    2.34   0.0192 *
Age               0.00928    0.00409    2.27   0.0234 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Threshold coefficients:
    Estimate Std. Error z value
1|2   -0.443      0.515   -0.86
2|3    1.291      0.513    2.52
3|4    4.408      0.533    8.27
```

```
> SC.probit<-polr(as.factor(SCOTUSApproval)~KnowChiefJustice+Democrat+
+                   GOP+Female+White+Black+Education+Age,data=SCdf,
+                   method="probit")
> summary(SC.probit)

Call:
polr(formula = as.factor(SCOTUSApproval) ~ KnowChiefJustice +
    Democrat + GOP + Female + White + Black + Education + Age,
    data = SCdf, method = "probit")

Coefficients:
                     Value Std. Error  t value
KnowChiefJustice  0.181021    0.09035  2.00349
Democrat          0.070178    0.08561  0.81970
GOP               0.249899    0.08681  2.87880
Female            0.000309    0.07045  0.00438
White            -0.029160    0.13700 -0.21285
Black            -0.039993    0.18195 -0.21980
Education         0.050138    0.02007  2.49763
Age               0.005261    0.00234  2.25112

Intercepts:
    Value  Std. Error t value
1|2 -0.245  0.291     -0.841
2|3  0.739  0.291      2.539
3|4  2.546  0.299      8.528

Residual Deviance: 2236.45
AIC: 2258.45
```

Logit / Probit Comparison

Ordered Regression Results

	Logit	Probit
$\hat{\tau}_1$	-0.443	-0.245
	(0.515)	(0.291)
$\hat{\tau}_2$	1.291*	0.739*
	(0.513)	(0.291)
$\hat{\tau}_3$	4.408*	2.546*
	(0.533)	(0.299)
Know Chief Justice	0.317*	0.181*
	(0.156)	(0.090)
Democrat	0.088	0.070
	(0.151)	(0.086)
GOP	0.495*	0.250*
	(0.153)	(0.087)
Female	0.013	0.000
	(0.124)	(0.070)
White	-0.076	-0.029
	(0.238)	(0.137)
Black	-0.077	-0.040
	(0.324)	(0.182)
Education	0.083*	0.050*
	(0.036)	(0.020)
Age	0.009*	0.005*
	(0.004)	(0.002)
Num.Obs.	1034	1034
AIC	2256.0	2258.4
BIC	2310.3	2312.8
RMSE	2.50	2.50

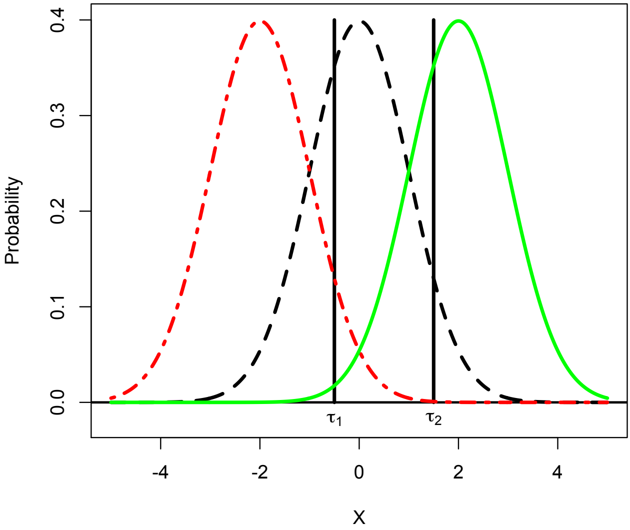
The marginal effects are:

$$\begin{aligned}\frac{\partial \Pr(Y=j)}{\partial X_k} &= \frac{\partial F(\hat{\tau}_{j-1} - \bar{\mathbf{X}}\hat{\beta})}{\partial X_k} - \frac{\partial F(\hat{\tau}_j - \bar{\mathbf{X}}\hat{\beta})}{\partial X_k} \\ &= \hat{\beta}_k[f(\hat{\tau}_{j-1} - \bar{\mathbf{X}}\hat{\beta}) - f(\hat{\tau}_j - \bar{\mathbf{X}}\hat{\beta})]\end{aligned}$$

So:

- $\text{sign}\left(\frac{\partial \Pr(Y=1)}{\partial X_k}\right) = \text{-sign}(\hat{\beta}_k)$
- $\text{sign}\left(\frac{\partial \Pr(Y=J)}{\partial X_k}\right) = \text{sign}(\hat{\beta}_k)$
- $\frac{\partial \Pr(Y=\ell)}{\partial X_k},\ \ell \in \{2, 3, ...J-1\}$ are non-monotonic

Marginal Effects, Illustrated



For a δ-unit change in X_k:

$$\begin{aligned} \mathrm{OR}_{X_k} &= \frac{\frac{\Pr(Y>j|\mathbf{X},X_k+\delta)}{\Pr(Y\leq j|\mathbf{X},X_k+\delta)}}{\frac{\Pr(Y>j|\mathbf{X},X_k)}{\Pr(Y\leq j|\mathbf{X},X_k)}} \\ &= \exp(\delta\hat{\beta}_k) \end{aligned}$$

Calculating Odds Ratios

```
> olreg.or <- function(model) {
+   coeffs <- coef(summary(SC.logit))
+   lci <- exp(coeffs[ ,1] - 1.96 * coeffs[ ,2])
+   or <- exp(coeffs[ ,1])
+   uci <- exp(coeffs[ ,1] + 1.96 * coeffs[ ,2])
+   lreg.or <- cbind(lci, or, uci)
+   lreg.or
+ }

> olreg.or(SC.logit)

                    lci     or    uci
KnowChiefJustice  1.010  1.373   1.87
Democrat          0.811  1.092   1.47
GOP               1.215  1.640   2.21
Female            0.794  1.013   1.29
White             0.581  0.927   1.48
Black             0.490  0.926   1.75
Education         1.014  1.087   1.17
Age               1.001  1.009   1.02
1|2               0.234  0.642   1.76
2|3               1.330  3.635   9.94
3|4              28.875 82.135 233.63
```

Odds Ratios via `modelsummary`

```
> modelsummary(list("Odds Ratios"=SC.logit),output="latex",stars=c("*"=0.05),exponentiate=TRUE)
```

Table of Odds Ratios

	Odds Ratios
$\hat{\tau}_1$	0.642
	(0.331)
$\hat{\tau}_2$	3.635*
	(1.865)
$\hat{\tau}_3$	82.135*
	(43.807)
KnowChiefJustice	1.373*
	(0.215)
Democrat	1.092
	(0.165)
GOP	1.640*
	(0.251)
Female	1.013
	(0.126)
White	0.927
	(0.221)
Black	0.926
	(0.300)
Education	1.087*
	(0.039)
Age	1.009*
	(0.004)
Num.Obs.	1034
AIC	2256.0
BIC	2310.3
RMSE	2.50

* $p < 0.05$

What do those things mean?

- `KnowChiefJustice`:
  - $\mathrm{OR} = \exp(0.317) = 1.37$
  - "The odds of a respondent rating the Court "Somewhat Disapprove" or better (versus "Strongly Disapprove") are about 37 percent higher for individuals who can correctly identify the Chief Justice than for those who cannot."
  - "The odds of respondents rating the Court "Somewhat Approve" or better (versus "Somewhat Disapprove" or "Strongly Disapprove") are also about 37 percent higher for individuals who can correctly identify the Chief Justice than for those who cannot."
- `Age`:
  - $\mathrm{OR} = \exp(0.009) = 1.01$
  - "A one-year increase in a respondent's age raises the odds of them rating the Court in a higher set of categories (versus all lower ones) by about one percent."
  - "A ten-year increase in a respondent's age yields an odds ratio of $\exp(10 \times 0.009) = \exp(0.09) = 1.09$, corresponding to an expected 9 percent increase in their approval rating of the Court."

Predicted probabilities (at $\bar{\mathbf{X}}$):

$$\Pr(\widehat{Y_i = j}|\mathbf{X}) = F(\hat{\tau}_j - \bar{\mathbf{X}}_i\hat{\beta}) - F(\hat{\tau}_{j-1} - \bar{\mathbf{X}}_i\hat{\beta})$$

Variable means/medians:

- `KnowChiefJustice` $= 1$, `Democrat` $= 0$, `GOP` $= 0$, `Female` $= 0$, `White` $= 1$, `Black` $= 0$, `Education` $= 11.4$, `Age` $= 57.9$.
- Yields:

$$\begin{aligned}
\sum_{k=1}^{K} \bar{\mathbf{X}}_k\hat{\beta}_k &= (0.32 \times 1) + (0.09 \times 0) + (0.49 \times 0) + (0.01 \times 0) - \\
&\quad (0.08 \times 1) - (0.08 \times 0) + (0.08 \times 11.4) + (0.01 \times 57.9) \\
&= 0.32 - 0.08 + 091 + 0.58 \\
&= \mathbf{1.73}.
\end{aligned}$$

Predicted Probabilities: "By Hand"

$$\begin{aligned}
\Pr(Y=1) &= \Lambda(-0.44 - 1.73) - 0 \\
&= \frac{\exp(-2.17)}{1+\exp(-2.17)} \\
&= \mathbf{0.10}.
\end{aligned}$$

$$\begin{aligned}
\Pr(Y=2) &= \Lambda(1.29 - 1.73) - \Lambda(-0.44 - 1.73) \\
&= \Lambda(-0.44) - \Lambda(-2.17) \\
&= 0.39 - 0.10 \\
&= \mathbf{0.29}.
\end{aligned}$$

$$\begin{aligned}
\Pr(Y=3) &= \Lambda(4.41 - 1.73) - \Lambda(1.29 - 1.73) \\
&= \Lambda(2.68) - \Lambda(-0.44) \\
&= 0.94 - 0.39 \\
&= \mathbf{0.55}.
\end{aligned}$$

$$\begin{aligned}
\Pr(Y=4) &= 1 - \Lambda(4.41 - 1.73) \\
&= 1 - \Lambda(2.68) \\
&= 1 - 0.94 \\
&= \mathbf{0.06}.
\end{aligned}$$

Changes in Predicted Probabilities

Changing `KnowChiefJustice`=0 $\rightarrow \sum_{k=1}^{K} \bar{\mathbf{X}}_k \hat{\beta}_k = 1.41$

- $\Pr(Y = 1) = \Lambda(-0.44 - 1.41) - 0 = \mathbf{0.14}$.
- $\Pr(Y = 2) = \Lambda(1.29 - 1.41) - \Lambda(-0.44 - 1.41) = 0.47 - 0.14 = \mathbf{0.33}$.
- $\Pr(Y = 3) = \Lambda(4.41 - 1.41) - \Lambda(1.29 - 1.41) = 0.95 - 0.47 = \mathbf{0.48}$.
- $\Pr(Y = 4) = 1 - 0.95 = \mathbf{0.05}$.

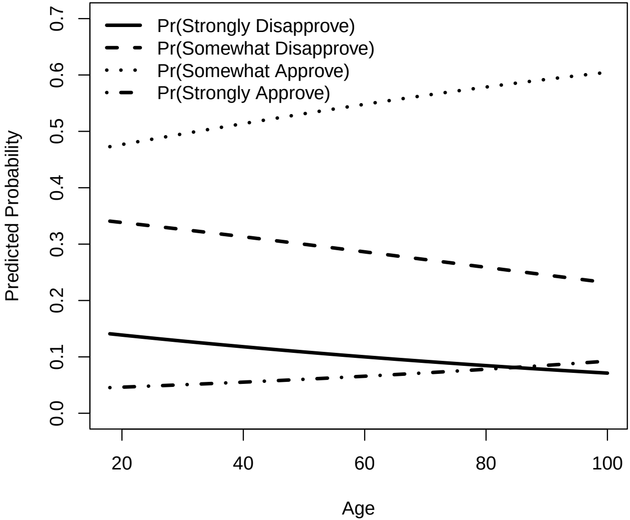
Changes in Probability

Outcome	Change in Probability
ΔPr(Strongly Disapprove)	0.04
ΔPr(Somewhat Disapprove)	0.04
ΔPr(Somewhat Approve)	-0.07
ΔPr(Strongly Approve)	-0.01

General idea: Plot predicted probabilities of each categorical outcome across different values of **X**...

- Can be category-specific or "cumulative"
- In-sample in `$fitted.values`
- Both `polr` and `clm` classes support `predict`, `confint`, etc.

Plot by Outcome

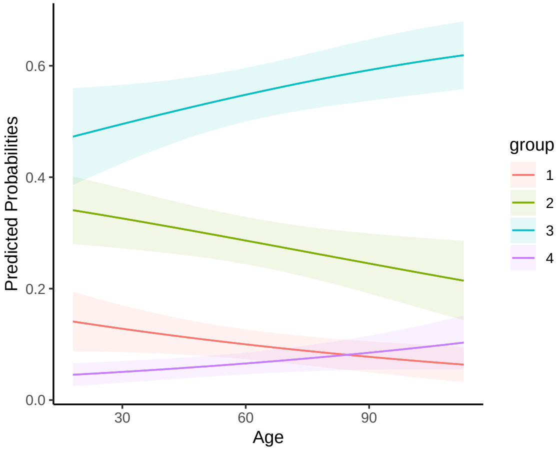


(How'd He Do That?)

```
> Sim<-data.frame(KnowChiefJustice=median(SCdf$KnowChiefJustice),
+                 Democrat=median(SCdf$Democrat),
+                 GOP=median(SCdf$GOP),
+                 Female=median(SCdf$Female),
+                 White=median(SCdf$White),
+                 Black=median(SCdf$Black),
+                 Education=mean(SCdf$Education),
+                 Age=seq(18,100,1))
>
> SC.hat<-predict(SC.logit,Sim,type='probs')
>
> plot(c(min(Sim$Age),max(Sim$Age)),c(0,0.7),type='n',
+      xlab="Age", ylab="Predicted Probability")
> lines(min(Sim$Age):max(Sim$Age),SC.hat[,1],lty=1,lwd=3)
> lines(min(Sim$Age):max(Sim$Age),SC.hat[,2],lty=2,lwd=3)
> lines(min(Sim$Age):max(Sim$Age),SC.hat[,3],lty=3,lwd=3)
> lines(min(Sim$Age):max(Sim$Age),SC.hat[,4],lty=4,lwd=3)
> legend("topleft",bty="n",lwd=3,lty=c(1:4),
+        legend=c("Pr(Strongly Disapprove)",
+                 "Pr(Somewhat Disapprove)",
+                 "Pr(Somewhat Approve)",
+                 "Pr(Strongly Approve)"))
```

Similar, using marginaleffects

```
p<-plot_predictions(SC.logit,condition=c("Age","group"),type="probs",vcov=TRUE) +
  theme_classic() +
  ylab("Predicted Probabilities")
```

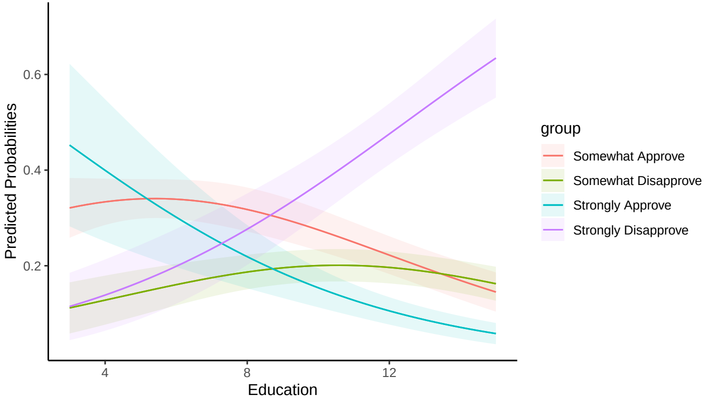


Stronger Effects: Trump Approval

```
> SCdf$Trump<-factor(SCdf$TrumpApproval,labels=c("Strongly Disapprove",
+                    "Somewhat Disapprove","Somewhat Approve","Strongly Approve"))
> DT.logit<-polr(Trump~Democrat+GOP+Female+
+                White+Black+Education+Age,data=SCdf)
> summary(DT.logit)

Call:
polr(formula = Trump ~ Democrat + GOP + Female + White + Black +
    Education + Age, data = SCdf)

Coefficients:
            Value Std. Error t value
Democrat  -2.0627    0.20001  -10.31
GOP        1.9099    0.15789   12.10
Female    -0.2995    0.13569   -2.21
White     -0.3221    0.27158   -1.19
Black     -0.6658    0.42985   -1.55
Education -0.2161    0.03795   -5.69
Age        0.0137    0.00445    3.08

Intercepts:
                                        Value   Std. Error t value
Strongly Disapprove|Somewhat Disapprove  -2.217   0.573      -3.868
Somewhat Disapprove|Somewhat Approve     -1.402   0.571      -2.454
Somewhat Approve|Strongly Approve         0.016   0.572       0.029

Residual Deviance: 1934.80
AIC: 1954.80
```

Predicted Probabilities

```
> p<-plot_predictions(DT.logit,condition=c("Education","group"),type="probs",vcov=TRUE) +
+   theme_classic() + ylab("Predicted Probabilities")
```



Variants / Extensions (also for PLSC 504...)

- *Generalized* models (relax parallel regressions; Brant (1990))
- *Heteroscedastic* models
- Varying τs (Maddala, Terza, Sanders)
- Models for "balanced" scales (Jones & Sobel)
- Compound Ordered Hierarchical Probit ("`chopit`") (Wand & King)
- "Zero-Inflated" Ordered Models (Hill, Bagozzi, Moore & Mukherjee)

- Latent class/mixture models (Winkelmann, etc.)