Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

BE Semester–III (NOV 2013)

Mathematics –III (CC301A)

(DIFFERENTIAL EQUATION & INTEGRAL TRANSFORM)

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work.**

Section I

Q.1 (i) Solve following differential equations: [5]
(a) $(x^2-y^2)dx=2xy\,dy$ (b) $(1+y^2)dx=(\tan^{-1}y-x)\,dy$

(ii) Find Laplace transform of (i) $f(t)=t\cos t$ (ii) $f(t)=e^{at}\cosh bt$ [5]

(iii) Solve partial differential equation $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}=\sin x\cos 2y$ [5]

OR

(iii) Solve differential equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{2x}\sin x.$ [5]

Q.2 (i) Solve by the method of undetermined coefficient [5]
$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^x\sin x.$

(ii) Obtain the Fourier series for the function $f(x)=2x+1$, $-\pi<x<\pi$ [5]

OR

(i) Solve by the method of variation of parameter [5]
$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$$

(ii) Find Z-transform of the function $f(k)=\cos\alpha k$. [5]

Q.3 (i) Using Convolution theorem, find $L^{-1}\left\{\frac{1}{s^2(s-1)}\right\}$. [5]

(ii) Solve Cauchy Euler differential equation [5]
$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=x^3$$

OR

(i) Find the Fourier transform of $f(t)=\begin{cases} e^{-at} & for\ t>0, \quad a>0 \\ 0 & for\ t<0, \quad a>0 \end{cases}$ [5]

(ii) Form PDE from z=y+f(x+ln, y). [5]

Section II

Q.4 (i) Find the orthogonal trajectories of the family of rectangular hyperbolas $x^2-y^2=a^2$. [5]

(ii) Find Laplace transform of $f(t)=(1-e^{-t})/t$. [5]

(iii) Solve $y'' + 4y' + 3y = 4e^{-x}$ given that y=0, y'=2 when x=0. [5]

OR

(iii) If Z{f(k)}=F(z) then Prove that [5]

(i) Z{k f(k)}=-z(d/dz)F(z) (ii) Z{a^k f(k)}=-F(z/a)

Q.5 (i) Solve by Exactness method $x^2e^ydx + \frac{x^3e^y}{3}dy = 0$ [5]

(ii) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = 3x^2$. [5]

OR

(i) Define Laplace transform and write any two properties of it. [5]

(ii) Find the Fourier cosine and sine transform of $f(x)=e^{-ax}$, a>0. [5]

Q.6 (i) Solve by the general method $\frac{d^2y}{dx^2} + y = cosec\ x$. [5]

(ii) Find the Fourier series of $f(x)=\begin{cases} x, & -\frac{\pi}{2}<x<-\frac{\pi}{2} \\ \pi-x, & \frac{\pi}{2}<x<\frac{3\pi}{2} \end{cases}$ [5]

OR

(i) Solve $xy^2p + x^2y\,q = -(x^2-y^2)z$ [5]

(ii) Solve (a) $L^{-1}\left[\frac{s+10}{s^2-s-2}\right]$ [5]

BEST OF LUCK

Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

BE Semester–III (NOV 2014)

Mathematics –III (CC301A)

(DIFFERENTIAL EQUATION & INTEGRAL TRANSFORM)

Max Marks: 70

Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are compulsory.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section I

Q.1 (i) Find Laplace transform of (i) $f(t)= e^{-3t}\, t^2 \sin 2t$ [5]

(ii) $f(t) = \begin{cases} 0, & 0 \le t \le 2 \\ 3, & when\; t \ge 2 \end{cases}$

(ii) Solve following differential equations: [5]

(a) $(x+y)y\,dx+(y-x)dy=0$ (b) $\frac{dv}{dx} = \left(\frac{x+y+1}{x+y+3}\right)^2$

(iii) Find the Fourier sine transform of $f(x)=e^{-2x}+e^{-3x}$, $x>0$ [5]

OR

(iii) Solve following Partial differential equation [5]

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+y}$$

Q.2 (i) Solve Cauchy Euler differential equation [5]

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$$

(ii) Find Z-transform of the function $f(k)=2^k\cos(3k+2)$ [5]

OR

(i) Solve by the method of variation of parameter [5]

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 3x^{3/2}e^{x}$$

(ii) Obtain the Fourier series for the function $f(x)=\sin ax$, $-\pi<x<\pi$. [5]

Q.3 (i) Form PDE from z=f(x+iy)+g(x-iy) [5]

(ii) Find the orthogonal trajectories of the cordioid r=a(1+cosθ) [5]

OR

(i) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = xe^{4x}$. [5]

(ii) Find $L^{-1}\left\{\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}\right\}$. [5]

Section II

Q.4 (i) Solve $y'' - 4y = 2e^{2t} + e^4$ given that t=0, y=0 by Laplace Transform. [5]

(ii) Find the Fourier half range cosine series of f(x)=c-x with period 2c. [5]

(iii) Solve $\frac{dy}{dx} + y = e^{-x}$, given that y(1)=4. [5]

OR

(iii) Solve px+qy=pq [5]

Q.5 (i) Solve by the method of undetermined coefficient [5]

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = e^{3x}.$$

(ii) Define Convolution theorem and evaluate $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$. [5]

OR

(i) Find Z-transform of following function [5]

$$f(k)=3(2^k)-4(3^k),\ k \geq 0$$

(ii) Compute L{f(t)} if $F(t)=\begin{cases} sint & 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases}$ extended periodically with period 2π. [5]

Q.6 (i) Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$. [5]

(ii) $Solve\ (D - 3D' - 2)^2 z = 2e^{2x}\tan(y + 3x)$ [5]

OR

(i) $y'' - 2ky' + k^2 = e^x, k \neq 1$ [5]

(ii) Using Fourier integral formula, show that: [5]

$$\int_0^\infty \frac{cos\lambda x}{\lambda^2 + a^2}\, d\lambda = \frac{\pi}{2a} e^{-ax}$$

BEST OF LUCK

Exam Seat No : ....

KADI SARVA VISHWAVIDYALAYA
B.E 3rd SEMESTER EXAMINATION (NOV 2015)

SUBJECT: Differential equations and Integral Transforms (Code: CC301A)

Date:27/11/2015 Time: 3 hour Marks: 70

Instruction:
1. Answer each section in separate Answer Sheet.
2. Use of scientific Calculator is permitted.
3. All questions are compulsory.
4. Indicate clearly,the option you attempted along with its respective question number.
5. Use the last page of main supplementary for rough work.

Section:1

Q.1 (a) Solve the differential equation $\frac{dy}{dx} = (4x+y)^2$. [05]

(b) By using convolution theorem.Find inverse Laplace transform of $\frac{1}{s^2(s+1)^2}$. [05]

(c) Solve the following partial differential equation. [05]

(i) $q(1+p) = pz$

(ii) $p+q = e^x + e^y$

OR

(c) Find fourier series to represent the function $f(x) = \pi^2 - x^2$ in the interval $-\pi \leq x \leq \pi$ and $f(x+2\pi) = f(x)$. [05]

Q.2 (a) Obtain Laplace transformation of following. [05]

(i) $L\left[e^{-4t}\int_0^t t\sin 3t dt\right]$.

(b) Solve the differential equation $(D^3 - 4D)y = 2\cosh^2(2x)$. [05]

OR

Q.2 (a) Solve the differential equation $(D^3+3D^2+2D)\, y = x^2+4x+8$ by using method of undetermined coefficients. [05]

(b) Find the orthogonal trajectories of the family of semicubical parabolas $ay^2 = x^3$. [05]

Q.3 (a) Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = \sin(x+3y)$ [05]

(b) Find the inverse $Z-$transform of $\frac{z^2}{(z-1)(z-\frac{1}{2})}$ for [05]

(i) $|z| > 1$

(ii) $|z| < \frac{1}{2}$

OR

Q.3 (a) Expand $f(x) = lx - x^2,\ 0 \leq x \leq l$ in a half rang *cosine* series. [05]

(b) Solve $\frac{dy}{dx} + y = \cos 2t,$ where $y(0) = 1$ by using laplace transform. [05]

Section:2

Q.4 (a) Find the following laplace inverse transform. [05]

(i) $L^{-1}\left[\frac{s-1}{s^2-6s+25}\right]$

(b) Define linear differential equation and also Solve $\frac{dy}{dx}+\frac{4x}{1+x^2}\,y=\frac{1}{(x^2+1)^3}$. [05]

(c) Find the fourier integral representation of the function $f(x)=\begin{cases}1 & ;\text{if } |x|<1\\ 0 & ;\text{if } |x|>1\end{cases}$ [05]

and hence evaluate following.

(i) $\int_0^\infty \frac{\sin\lambda cos\lambda x}{\lambda}$

(ii) $\int_0^\infty \frac{\sin\lambda}{\lambda}$

OR

(c) Solve $x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$ by Lagrange method. [05]

Q.5 (a) Find the $Z-$transform of the following function [05]

(i) $a^k\sin\alpha k,\;\; k\geq 0$

(b) Solve the differential equation $(D^3+3D^2-4D-12)y=12\,xe^{-2x}$ [05]

OR

Q.5 (a) Solve $u_{xx}+u_{yy}=0$ which satisfies the boundary condition $u(0,y)=u(a,y)=0$ for $0\leq y\leq b$ and $u(x,b)=0,\;\; u(x,0)=f(x)$ for $0\leq x\leq a$. [05]

(b) Evaluate the following [05]

(i) $L\left[t^4U(t-2)\right]$

(ii) $L^{-1}\left[\frac{e^{-3s}}{(s-2)^4}\right]$.

Q.6 (a) Solve the differential equation $(D^2+1)\,y=cosec(x)$ by using method of variation of parameters. [05]

(b) Define fourier series of function and also find the fourier transform of $f(x)=e^{-4x^2}$ [05]

OR

Q.6 (a) Solve the differential equation $x^2\frac{d^2y}{dx^2}+6x\frac{dy}{dx}+6y=x\log x$. [05]

(b) Define necessary condition for Exact differential equation and solve differential equation $(y^2e^{xy^2}+4x^3)dx+(2xye^{xy^2}-3y^2)dy=0$ [05]