Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

BE Semester–IV (April 2016)

Mathematics –IV (CC401A)

(Complex Analysis and Numerical Analysis)

27.4.2016

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work.**

Section I

Q.1 (i) Find the complex number z if arg(z+1)=$\pi/6$ and arg(z-1)=$2\pi/3$. [5]

(ii) Determine the analytic function whose imaginary part is $v = e^x(x\cos y - y\sin y)$. [5]

(iii) Define Newton-Gregory forward difference interpolation formula and use it to find y(3.62) from the following table [5]

X:	3.60	3.65	3.70	3.75
Y:	36.598	38.475	40.447	42.521

OR

(iii) Show that f(z)=$|z|^2$ is not differentiable at any $z\neq 0$. [5]

Q.2 (i) Find the bilinear transformation which maps the point z= -2, 0, 2 onto the points w= ∞, $\frac{1}{2}$, $\frac{3}{4}$ respectively. [5]

(ii) State Cauchy's Residue theorem and find the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at z=1, 2, 3 and infinity and show that their sum is zero. [5]

OR

(i) By using Gauss's backward interpolation formula compute the population for the year 1976, from the following table; [5]

Year:	1931	1941	1951	1961	1971	1981
Population: (in lakhs)	12	15	20	27	39	52

(ii) Given that y= ln x and [5]

X:	4.0	4.2	4.4	4.6	4.8	5.0	5.2
Y:	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

Evaluate $\int_4^{5.2} \ln x\, dx$ by using Simpson's 3/8 rule by taking n=6.

Q.3 (i) Evaluate $\int_{|z|=2} \frac{e^z}{(z-3)(z-1)}dz$. [5]

(ii) State and prove necessary condition for a function to be analytic. [5]

OR

(i) Find the real root of the equation $x^4 + 2x^3 - x - 1$ using the Bisection method lying in the interval [0, 1]. [5]

(ii) Establish (i) $\mu^2 \equiv \frac{\delta^2}{4} + 1$ (ii) $\mu + \frac{\delta}{2} \equiv E^{1/2}$ [5]

Section II

Q.4 (i) Separate real and imaginary parts of $tan^{-1}(x + iy)$. [5]

(ii) Verify that the function $f(z)=z^2$ satisfies the Cauchy Riemann equations and determine the derivative $f'(z)$. [5]

(iii) Given log 100=2, log 101=2.0043, log 103=2.0128, log 104=2.0170. Find log 102. [5]

OR

(iii) The function $y=f(x)$ is given at the points (7, 3), (8,1), (9,1) and (10,9). Find the value of y for x=9.5 using Lagrange's interpolation formula. [5]

Q.5 (i) Find the power series expansion for the function $1/(z-3)$ in the following three regions: (a) $|z| < 3$ (b) $|z - 2| < 1$ (c) $|z| > 3$. [5]

(ii) State singularity and define all kind of singularities by giving the example for each. [5]

OR

(i) State Stirling's interpolation formula and use it to find y_{28} given: $y_{20}=49225$, $y_{25}=48316$, $y_{30}=47236$, $y_{35}=45926$, $y_{40}= 44306$. [5]

(ii) Using fourth order Runge-Kutta method, find an approximate value of y for x=0.2 in steps of 0.1, if $\frac{dy}{dx} = x + y^2$, given y=1 when x=0. [5]

Q.6 (i) Determine F(1) and F(5) if $F(\propto) = \oint_C \frac{3z^2-2z+1}{z-\propto}$, where C is an ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. [5]

(ii) Find the invariant points and the normal form of the bilinear transformation: $\frac{3z-4}{z-1}$. [5]

OR

(i) Apply Gauss Seidel iteration method to solve the following system of equations: $9x_1-2x_2+x_3=50$, $x_1+5x_2-3x_3=18$, $-2x_1+2x_2+7x_3=19$ up to 4^{th} iterations. [5]

(ii) Find the real root of the equation $x^3-3x-5=0$ correct to four places of decimals by Newton-Raphson method. [5]

BEST OF LUCK

Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,
Gandhinagar

BE Semester–IV (MAY 2014)
Mathematics –IV (CC401A)
(Complex Analysis and Numerical Analysis)

Dt: 10/05/2014

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work.**

Section I

Q.1 (i) Prove that the function $u(x, y) = x^3-3xy^2$ is harmonic and obtain its conjugate. [5]

(ii) Show that the product of all the values of $i^{2/3}$ is -1. [5]

(iii) Determine the poles and the residue at each pole for the function $\frac{z^2-2z}{(z+1)^2(z^2+1)}$. [5]

OR

(iii) Use Stirling's formula to find y_{28} given: $y_{20}=49225$, $y_{25}=48316$, $y_{30}=47236$, $y_{35}=45926$, $y_{40}=44306$. [5]

Q.2 (i) Find the image of the circle $(x-3)^2+y^2=4$ under the transformation $w=1/z$. [5]

(ii) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's 1/3 and 3/8 rule. Hence obtain the approximate value of π in each case. [5]

OR

(i) Expand $f(z)=\frac{z^2-z+1}{z^2-z-2}$ in the region (a) $1<|z|<2$ and (b) $|z|>2$ [5]

(ii) Find the root of equation $x^3+2x-8=0$ using Newton-Raphson method to 3 decimal places. [5]

Q.3 (i) Find the value of $\int_{|z|=1} \frac{\sin^6 z}{[z-\frac{\pi}{6}]^3}\,dz$ [5]

(ii) Use Bessel formula to find y_{25}, given that $y_{20}=24$, $y_{24}=32$, $y_{28}=35$, $y_{32}=40$. [5]

OR

(i) Evaluate $\int_c \bar{z}dz$ from z=0 to z=4+2i along the curve C given by (a) $z=t^2+it$ (b) the line from z=0 to z=2i and then the line from z=2i to z=4+2i. [5]

(ii) Using Runge-Kutta method, solve the equation dy/dx =x+y, with initial condition y(0)=1 from x=0.1 to x=0.4, when h=0.1. [5]

Section II

Q.4 (i) Find z if arg (z+1)=π/6 and arg (z-1)=2π/3. [5]

(ii) Given the following data, find f(x) as a polynomial in powers of (x-5) [5]

x :	0	2	3	4	7	9
f(x) :	4	26	58	112	466	922

(iii) Show that $f(z)=\sqrt{|xy|}$ is not differentiable even though CR equations are satisfied. [5]

OR

(iii) Evaluate $\int_c \frac{dz}{(z^2+1)(z^2-4)}$, C: $|z| = 1.5$. [5]

Q.5 (i) Find the bilinear transformation which maps the point z=0, -1, ∞ into the points w= -1, -i, 1. [5]

(ii) The population of a town in the decennial census was as given below. [5]

year :	1911	1921	1931	1941	1951	1961
population:	12	15	20	27	39	52

Estimate the population during the period from 1946 to 1948 by using Newton Gregory forward difference interpolation formula.

OR

(i) Find the radii of convergence of the following power series: [5]

(a) $\sum_{n=1}^{\infty}(1+\frac{1}{n^2})^{n^3}\ z^n$ (b) $\sum \frac{n(5+2i)^n}{3^n}\ (z-2i)^n$

(ii) Find a real root of $\cos x - xe^x=0$ by Secant method. [5]

Q.6 (i) Prove that all roots of $z^7-5z^3+12=0$ lie between circles $|z| = 1$ and $|z| = 2$. [5]

(ii) Apply Gauss Seidel iteration method to solve the following system of equations: $9x_1-2x_2+x_3=50$, $x_1+5x_2-3x_3=18$, $-2x_1+2x_2+7x_3=19$ up to 4th iterations. [5]

OR

(i) Prove that : (i) $\frac{\Delta^2 x^3}{Ex^3} = \frac{6}{(1+x)^2},\ if\ h=1$ (ii) $\delta \equiv \Delta E^{-1/2}$ [5]

(ii) Write the statements of (i) Cauchy Residue theorem [5]
(ii) Cauchy Integral Theorem (iii) Cauchy Integral Formula
(iv) Liouville's Theorem (v) Maximum Modulus Theorem

BEST OF LUCK

Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,
Gandhinagar
ATKT -November 2014
BE Semester–IV (CC401A)
(Complex Analysis and Numerical Analysis)

Dt: 1.11.14

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section I

Q.1 (i) Prove that $f(z) = \sqrt{z}$ is analytic everywhere except at z = 0. [5]

(ii) Express (1) $\sqrt{3} + i$ (2) $\sqrt{3} - i$ in polar form. [5]

(iii) State De Moiver's Theorem and evaluate $\left(1 + i\sqrt{3}\right)^{90} + \left(1 - i\sqrt{3}\right)^{90}$. [5]

OR

(iii) Prove the following (i) $E = 1 + \Delta$ (ii) $\Delta\nabla = \Delta - \nabla$ [5]

Q.2 (i) Find the bilinear transformation which maps the points z = 1,i,-1 into the points w = 0,1,∞. [5]

(ii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal Rule and Simpson's 1/3. Here h = 1. [5]

OR

(i) Classsify all the singularity for the function $f(z) = sec\left(\frac{1}{z-2}\right)$ at z =2. [5]

(ii) Solve the following system using Gauss Elimination Method : [5]
$2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16$

Q.3 (i) Determine a,b,c,d so that the function, [5]
$f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic.

(ii) From the following table [5]

x	5	7	11	13	17
F(x)	150	392	1452	2366	5202

Evaluate f(9) using Lagrange's interpolation formula.

OR

(i) Evaluate $\oint_C \frac{1}{z^4+z^3-2z^2} dz$, where C denotes the circle, $C = \{z/|z| = 3\}$ with positive orientation. [5]

(ii) Using Runge-Kutta method, solve the equation dy/dx =x+y, with initial condition y(0)=1 from x=0 to x=0.2, when h=0.2. [5]

Section II

Q.4 (i) Find the complex conjugate, norm and modulus of $z = \frac{1-i}{1+i}$. [5]

(ii) Using Newton's divided difference formula find the quadratic equation for the following data and hence find y(2). [5]

x	0	1	4
y	2	1	4

(iii) Show that $f(z) = \frac{1}{z-1}$ is differentiable everywhere except at z = 1. [5]

OR

(iii) Evaluate $\int_0^{2+i} z^2 dz$ along the line joining the points $z_1 = 0$ and $z_2 = 2 + i$. [5]

Q.5 (i) If w_1, w_2, w_3, w_4 are distinct images of z_1, z_2, z_3, z_4 under the transformation $w = \frac{az+b}{cz+d}\,(ad - bc \neq 0)$ then show that $\frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_4)(z_3-z_2)} = \frac{(w_1-w_2)(w_3-w_4)}{(w_1-w_4)(w_3-w_2)}$ [5]

(ii) Using Newton's Forward interpolation formula find y at x=8 from the following table: [5]

x	0	5	10	15	20	25
y	7	11	14	18	24	32

OR

(i) Expand $f(z) = \frac{1}{z}$ in Taylor's series about $z_0 = 1$. [5]

(ii) Use the Secant method to find the root of $x^2 - 4x - 10=0$ with initial estimates $x_1 = 4$ & $x_2 = 2$ upto four stages. [5]

Q.6 (i) Prove that all roots of $z^7-5z^3+12=0$ lie between circles $|z| = 1$ and $|z| = 2$. [5]

(ii) Evaluate $\int_0^1 e^{-x^2} dx$ by Gauss integration formula with n = 3. [5]

OR

(i) Find real root of the equation $x^3 - 3x - 5 = 0$ correct up to two decimal places by using Newton Raphson method. [5]

(ii) Write the statements of [5]

(i) Cauchy Residue theorem

(ii) Cauchy Integral Theorem

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Exam Seat no. …………

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

BE Semester–IV (April 2015)

Mathematics –IV (CC401A)

(Complex Analysis and Numerical Analysis)

Dt: 28.4.2015

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section I

Q.1 (i) Find z if $|z+i| = |z|$ and $\arg\left(\frac{z+i}{z}\right) = \frac{\pi}{4}$. [5]

(ii) Derive Cauchy-Riemann equations in polar form. [5]

(iii) Use Lagrange's interpolation formula to find y when x=10 given: [5]

x	5	6	9	11
y	12	13	14	16

OR

(iii) Represent the function f(z)= $\frac{1}{z^2+4z+3}$ in Laurent's series if [5]
(a) $1 < |z| < 3$ (b) $|z| > 3$.

Q.2 (i) Evaluate $\int_C \frac{z^2+z+1}{z-1}dz$ where C is contour (a) $|z| = 1$ (b) $|z| = 1/2$. [5]

(ii) Use Newton-Gregory forward difference interpolation formula to compute y(3.62) from the following table [5]

x	3.60	3.65	3.70	3.75
y	36.598	38.475	40.447	42.521

OR

(i) State Cauchy's Residue theorem and find residue of $\frac{z^2}{(z-1)(z-2)}$ at z=1, z=2. [5]

(ii) Using Taylor's Series method, find the solution of $\frac{dy}{dx} = 3x + y^2$ and y=1, x=0. Also find y at x=0.1, correct to four decimal places. [5]

Q.3 (i) Find the bilinear transformation which maps the point z=2,1,0 into the points w= 1,0, i. [5]

(ii) Evaluate $\int_0^1 \frac{dx}{1+x}$ using trapezoidal rule with h=0.125 [5]

OR

(i) Evaluate $\int_0^{1+2i} z^2 dz$ along the curve $2x^2=y$. [5]

(ii) Evaluate the integral $\int_c \tan z \, dz$ where C=$|z| = 2$. [5]

Section II

Q.4 (i) Prove that $\left(1+i\sqrt{3}\right)^{8}+\left(1-i\sqrt{3}\right)^{8}=-2^{8}$. [5]

(ii) Using divided difference formula, find f(x) as a polynomial in powers of (x-6) [5]

x :	-1	0	2	3	7	10
f(x) :	-11	1	1	1	141	561

(iii) Find p such that the function f(z) expressed in polar co-ordinates as $f(z)=r^2\cos 2\theta + ir^2 \sin p\theta$ is analytic. [5]

OR

(iii) Find the image of the triangular region bounded by the lines x=0, y=0 and x+y=1 in Z-plane under the transformation $w = ze^{i\pi/4}$. [5]

Q.5 (i) Determine the number of roots of the equation $z^8-4z^5+z^2-1=0$ that lie inside the circle $|z|=1$. [5]

(ii) Given that f(20)=14, f(24)=32, f(28)=35, f(32)=40. Find f(25) by Gauss interpolation formula. [5]

OR

(i) Find the radii of convergence of the following power series: [5]

(a) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\ (z-2i)^n$ (b) $\sum\left(\frac{i+n\sqrt{2}}{1+2in}\right) z^n$

(ii) Find root of $x^3-5x-7=0$ correct upto 3 decimal places by Secant method. [5]

Q.6 (i) By using Newton's method, find the root of x^4-x-10=0 which is near to x=2 correct to three places of decimals. [5]

(ii) Evaluate $\int_C \frac{2z-1}{z(z+1)(z-3)}dz, where\ C: |z|=2$ by Cauchy's Residue theorem. [5]

OR

(i) If $u=x^2-y^2, v=\frac{-y}{x^2+y^2}$ then show that both u and v are harmonic functions but f(z)=u+iv not satisfying C-R equations. [5]

(ii) Using Runge-Kutta 4th order method, find the value of y when x=1 given that y=1 when x=0 and that $\frac{dy}{dx}=\frac{y-x}{y+x}, h=1$. [5]

BEST OF LUCK

Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

ATKT Examination BE Semester–IV OCT - 2015

Mathematics –IV (CC401A)

(Complex Analysis and Numerical Analysis)

20·10·2015

Max Marks: 70

Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section I

Q.1 (i) Determine by Lagrange's formula the percentage of criminals under 35 years, when [5]

Age (under years)	25	30	40	50
% of number of criminals	52	67.3	84.1	94.4

(ii) Find the fifth root of unity. [5]

(iii) Using Newton's divided difference formula, find a polynomial function satisfying the following data and find $f(1)$. [5]

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

OR

(iii) Separate $(\sqrt{i})^{\sqrt{i}}$ into real and imaginary parts. [5]

Q.2 (i) Is the function $f(z) = \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$ continuous at $z = i$? [5]

(ii) Use the Simpson's $\frac{1}{3}$ rule with step length h = 0.5 to estimate $\int_0^1 \frac{1}{1+x} dx$. [5]

OR

(i) Discuss the analyticity of the function [5]
(i) $|z|^2$ And (ii) $z\bar{z}$ at any point z.

(ii) Find the largest Eigen value and corresponding eigenvector of $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ [5]

Q.3 (i) By using Newton's interpolation formula, find the values of f (22) from the following data. [5]

x	20	25	30	35	40	45
f(x)	354	332	291	260	231	204

(ii) Convert the following numbers into rectangular form [5]
(i) $10\angle -30^\circ$ (ii) $5e^{-\frac{\pi}{4}i}$

OR

(i) Given a triangle with vertices at 3 +4i, $-3-4i$ and –5i, find their images for the transformation [5]
(i) W = z +5i (ii) W = i z + (2-i)

(ii) Given $\frac{dy}{dx} = 1 + y^2$ where y = 0 when x = 0 find y(0.2) and y(0.4) using Runge Kutta Method. [5]

Section II

Q.4 (i) Solve the following system by Gauss-Elimination method with partial pivoting. [5]

$$-3x_1 + 6x_2 - 9x_3 = -46.73$$
$$x_1 - 4x_2 + 3x_3 = 19.57$$
$$2x_1 + 5x_2 - 7x_3 = -20.07$$

(ii) Prove that (i) $Im(iz) = Re(z)$ (ii) $Re(iz) = -Im(z)$ [5]

(iii) Evaluate $\int_c (x - y + ix^2)dz$, where c is along the real axis from $z = 0$ to $z = 1$ and then the line joining $z = 1$ to $z = 1 + i$. [5]

OR

(iii) Prove that : [5]
(i) $1+\mu^2\delta^2 = (1+\frac{1}{2}\delta^2)^2$ (ii) $\mu\delta = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$

Q.5 (i) Evaluate $\oint_C \frac{z-1}{(z+1)^2(z-2)}dz$; where C is $|z - i| = 2$. [5]

(ii) Find a real root of the equation $x^3 - 4x - 9 = 0$ by using Bisection Method. [5]

OR

(i) Use Taylor's series method to solve the equation and compare with the analytical solution $\frac{dy}{dx} = -xy$, $y(0) = 0$. [5]

(ii) Determine the Laurent's series expansion of $f(z) = \frac{1}{(z+1)(z+3)}$ valid for (i) $1 < |z| < 3$ (ii) $|z| > 3$. [5]

Q.6 (i) Find the radii of convergence of the following power series : [5]
(i) $\sum \frac{n!}{n^n} z^n$ $(ii) \sum \frac{(n+1)}{(n+2)(n+3)} z^n$

(ii) Determine the location and order of the zeros of the following functions: [5]
(i) $(z^4 - 1)^4$ (ii) $(z^2 + 1)(e^z - 1)$.

OR

(i) Find the residues at the singular points (each of its poles). [5]
(i) $\frac{\cos Z}{z^3}$ (ii) $\frac{z}{(z-1)\,(z-2)^2}$

(ii) Find the root of the equation $x^3 - 21x + 35 = 0$ correct up to two decimal places by Newton-Raphson method. [5]

BEST OF LUCK