Exam Seat no. …………

KADI SARVA VISHWAVIDHYALAYA,
Gandhinagar
November 2013
BE Semester–II Remedial (CC101B)
Mathematics –II (Linear Algebra and Vector Calculus) 1-1-2014

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section I

Q.1 (i) Solve by Gauss Elimination method the system: x-y+z=4, x+y+z=0, 2x+3y-z= -5 [5]

(ii) Let V be the set of all pairs (x, y) of real numbers over the field R of real numbers. Check whether V is a vector-space over R defined by the operations: [5]
$(x_1,y_1)+(x_2,y_2)=(x_1+x_2, y_1+y_2)$ and $k(x,y)=(k^2x, k^2y)$

(iii) Consider a linear transformation $T: R^2 \to R^2$ which consist of a reflection across Y-axis, followed by a shear in X-direction with factor 5 and then reflection across X-axis. Find the standard matrix. [5]

OR

(iii) Show that the subspace S={(1,1,1), (1,2,3), (2,-1,1)} of R^3 spans the entire vector space V. [5]

Q.2 (i) Find the Eigen value and Eigenvectors of the matrix [5]
$A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

(ii) Find the gradient of $\phi(x,y,z) = 2z^3 - 3(x^2+y^2)z + tan^{-1}(xz)$ at (1,1,1). [5]

OR

(i) If u= (1,2,2), v=(3,4,6), then prove that w=(5,8,10) is a linear combination of u and v. [5]

(ii) Find the work done by the force $\bar{F} = 3x^2i + (2xz - y)j + zk$ over the curve $0 \le t \le 1$ from (0,0,0) to (1,1,1). [5]

Q.3 (i) For the basis $S=\{v_1,v_2,v_3\}$ of R^3, where $v_1=(1,1,1)$, $v_2=(1,1,0)$ and $v_3=(1,0,0)$. Let $T: R^3 \to R^2$ be a linear transformation such that $T(v_1)=(1,0)$, $T(v_2)=(2,-1)$, $T(v_3)=(4,3)$. Find a formula for $T(x_1,x_2,x_3)$ and then use the formula to find $T(4,3,-2)$. [5]

(ii) Find the directional derivative of $\phi(x,y,z) = x^3 - xy^2 - z)$ at $(1,1,0)$ in the direction of $v=2i-3j+6k$. [5]

OR

(i) Consider the bases $S_1=\{u_1, u_2\}$ and $S_2=\{v_1, v_2\}$ where $u_1=(1,-1)$, $u_2=(0,6)$, $v_1=(2,1)$, $v_2=(-1,4)$ [5]
(i) Find the transition matrix from S_2 to S_1.
(ii) Find the transition matrix from S_1 to S_2.

(ii) Find the inverse of $A=\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ and verify that $AA^{-1}=I$. [5]

Section II

Q.4 (i) Find the basis for the null space of $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$. [5]

(ii) By using Cayley-Hamilton theorem, if $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, then find A^{-1} and express $A^5-4A^4-7A^3+11A^2-A-10I$ as a linear polynomial in A. [5]

(iii) Find the inverse of the following matrix by Gauss-Jordan elimination method $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 4 \\ 1 & 3 & 2 \end{bmatrix}$. [5]

OR

(iii) If $\nabla\phi = y^2 i + (2xy + z^3)j + 3yz^2 k$, determine ϕ. [5]

Q.5 (i) Show that T is a linear transformation from $V_3(R) \to V_3(R)$ defined by $T(a, b, c)= (a-b, a+b, b+c)$ for $a,b,c \in R$. [5]

(ii) Prove that $\vec{F} = (y^2 \cos x + z^3)i + (2y\, sinx - 4)j + 3xz^2 k$ is irrotational and find its scalar potential. [5]

OR

(i) Verify Green's theorem for the field F(x,y)= (x-y)i + xj and the region R bounded by unit circle C: r(t)= cos t i +sin t j , $0 \leq t \leq 2\pi$ [5]

(ii) Find the coordinate vector p relative to the basis S={p_1,p_2,p_3} where p=2-x+x^2, p_1=1+x, p_2=1+x^2, p_3=x+x^2. [5]

Q.6 (i) A map $T: R^3 \rightarrow R^3$ is defined by T(a,b,c) = (5a, a-b, 3a+b+c), a.b.c $\in$ R. Show that T is nonsingular and find T^{-1}. [5]

(ii) Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1\, x_3$ into canonical form. [5]

OR

(i) Evaluate $\iint_S A.dS$ where A= xi + (z^2 - zx)j – xy k and S is the surface of the triangle with vertices (2,0,0), (0,2,0), (0,0,4). [5]

(ii) Check whether following vectors are linearly independent or not: [5]

(4,-1,2), (-4,10,2), (4,0,1)

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Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

BE Semester–II (May 2013)

Mathematics –II (Linear Algebra and Vector Calculus)

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section I

Q.1 (i) Is the following system of equations consistent? If so solve $x+y+z=6$, $x-y+2z=5$, $2x-2y+3z=7$. [5]

(ii) Show that the set V of all pairs of real numbers of the form (1, x) with the operations defined as (1,y)+(1,y')= (1,y+y') and k(1,y) = (1, ky) is a vector space. [5]

(iii) $T:R^2 \to R^2$ is defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)=\begin{bmatrix} 2x_1+x_2 \\ x_1-3x_2 \end{bmatrix}$ and $S_1=\{w_1,w_2\}$ where $w_1=\begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $w_2=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ then find the matrix of T w. r. t the basis S_1. [5]

OR

(iii) A vector field is given by $\vec{F}=(x^2+xy^2)i+(y^2+yx^2)j$. Show that $\vec{F}$ is irrotational and find its scalar potential. [5]

Q.2 (i) Evaluate $\int_C [(x^2+xy)dx+(y^2+x^2)dy]$, where C is the square formed by the lines $x=\pm 1$ and $y=\pm 1$. [5]

(ii) By using Cayley-Hamilton theorem, if $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, then prove that [5]

$$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I=\begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

OR

(i) Evaluate $\oint_C \vec{F}.\vec{dr}$ by Stoke's theorem, where $\vec{F}=y^2i+$ [5]

$x^2j-(x+z)k$ and Cis the boundary of the triangle with vertices at (0,0,0), (1,0,0) and (1,1,0).

(ii) Find the algebraic multiplicity and geometric multiplicity of each Eigen value of the matrix A= $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ [5]

Q.3 (i) Let $T: R^2 \to P$ be a linear transformation defined by $T(a_1,a_2,a_3)=(-a_1+2a_2+a_3)+(-a_2+a_3)x$. Find which of the following vectors are in ker(T): (i) u= (6,2,2) (ii) u= (2,-1,1) [5]

(ii) Express the matrix A= $\begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & 2 & 13 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. [5]

OR

(i) For the basis $S=\{v_1,v_2,v_3\}$ of R^3, where $v_1=(1,1,1)$, $v_2=(1,1,0)$ and $v_3=(1,0,0)$. Let $T: R^3 \to R^3$ be a linear transformation such that $T(v_1)=(2,-1,4)$, $T(v_2)=(3,0,1)$, $T(v_3)=(-1,5,1)$. Find a formula for $T(x_1, x_2, x_3)$ and use it to find T(2,4,-1). [5]

(ii) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2,-1,2). [5]

Section II

Q.4 (i) Let C be the field of complex numbers and T be the transformation from C^3 to C^3 defined by $T(x_1,x_2,x_3)=(x_1-x_2+2x_3,\ 2x_1+x_2-x_3,\ -x_1-2x_2)$ Verify that T is a linear transformation. [5]

(ii) Diagonalize the matrix A= $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and find A^4. [5]

(iii) Find the basis for the Column space and Row space of [5]

$$A=\begin{bmatrix} 2 & -4 & 1 & 2 & -2 & -3 \\ -1 & 2 & 0 & 0 & 1 & -1 \\ 10 & -4 & -2 & 4 & -2 & 4 \end{bmatrix}$$

OR

(iii) The vectors $v_1=(1,-1,1)$, $v_2=(0,1,2)$, $v_3=(3,0,-1)$ form a basis of V. Let $S_1=\{v_1,v_2,v_3\}$ and $S_2=\{v_3,v_2,v_1\}$ are different orderings of these vectors. Determine the vector v in V having following coordinate vectors. [5]

(i) $v_{s_1}=(3,-1,8)$ (ii) $v_{s_2}=(3,-1,8)$

Q.5 (i) Prove that $\nabla^2 f(r) = f''(r)+\frac{2}{r}f'(r)$ [5]

(ii) Find the inverse of the following matrix by Gauss-Jordan elimination method $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$. [5]

OR

(i) Prove that $\vec{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is solenoidal and determine the constants a, b, c if $\vec{F}$ is irrotational. [5]

(ii) Find l, m, n and A^{-1} if $A=\begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal. [5]

Q.6 (i) Is the integral $\int_C [2xyz^2dx + (z\cos yz + x^2z^2)dy + (y\cos yz + 2x^2yz)dz]$ is independent of the path? If so, evaluate it from $(1,0,1)$ to $(0,\pi/2,1)$. [5]

(ii) Reduce the quadratic form $2x_1x_2+2x_2x_3+2x_2x_1$ into canonical form. Examine for definiteness. [5]

OR

(i) Use gauss' divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\, ds$ where $\vec{F} = 4xi - 2y^2j + z^2k$ and S is the surface bounding the region $x^2+y^2=4$; $z=0$ and $z=3$. [5]

(ii)(a) Which of the following are linearly independent : [3]

(i) $1, e^x, e^{2x}$ (ii) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$

(b) If u and v are two vectors are in R^n then show that $u.v = \frac{1}{4}\|u+v\|^2 - \frac{1}{4}\|u-v\|^2$ [2]

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Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,
Gandhinagar
December 2014
BE Semester–II Remedial (CC101B)
Mathematics –II (Linear Algebra and Vector Calculus)

Dt. 6.1.2015

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section I

Q.1 (i) Solve by Gauss-Jordan Elimination method -2y+3z=1, 3x+6y-3z = -2, 6x+6y+3z=5. [5]

(ii) Determine whether the set V of all pairs of real numbers (x, y) with the operations defined as $(x_1, y_1) + (x_2, y_2) = (x_1+x_2+1, y_1+y_2+1)$ and k(x,y) = (kx, ky) is a vector space. [5]

(iii) Determine whether the following function is linear transformation $T: P_2 \rightarrow P_3$ where T(p(x))=xp(x). [5]

OR

(iii) Show that $\vec{A} = 3y^4z^2\hat{\imath} + 4x^3z^2\hat{\jmath} - 3x^2y^2\hat{k}$ is solenoidal. [5]

Q.2 (i) Evaluate $\int_C x^2dx + xydy$ along the parabola $y^2 = x$ between the points (0, 0) and (1, 1). [5]

(ii) Verify Cayley-Hamilton theorem for A=$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$. [5]

OR

(i) Evaluate $\oint_C \vec{F}.\overrightarrow{dr}$ by Green's theorem, where $\vec{F} = (x-y)\hat{\imath} + 3xy\hat{\jmath}$ and C is the boundary of the region bounded by the parabolas $x^2=4y$ and $y^2=4x$. [5]

(ii) Find the algebraic multiplicity and geometric multiplicity of each Eigen value of the matrix A=$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. [5]

Q.3 (i) Let $T: R^2 \to R^2$ be a linear operator defined by T(x,y)=(2x-y, -8x+4y). Find a Basis for (a) ker(T) (b) R(T). [5]

(ii) Express the matrix $A=\begin{bmatrix} 2+3i & 2 & 3i \\ -2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{bmatrix}$ as the sum of a Hermition and a skew Hermition matrix. [5]

OR

(i) Prove that $\vec{F} = 2xyzi + (x^2z + 2y)j + x^2yk$ is irrotational and find its scalar potential. [5]

(ii) For the basis S={v_1,v_2,v_3} of R^3, where v_1=(1,1,1), v_2=(1,1,0) and v_3=(1,0,0). Let $T: R^3 \to R^2$ be a linear transformation such that T(v_1)=(1,0), T(v_2)=(2,-1), T(v_3)=(4, 3). Find a formula for T(x_1, x_2, x_3) and use it to find T(4, 3, -2). [5]

Section II

Q.4 (i) Find the basis for the null space of $A = \begin{bmatrix} 2 & -1 & -2 \\ -4 & 2 & 4 \\ -8 & 4 & 8 \end{bmatrix}$. [5]

(ii) Find the rank of the matrix $A=\begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$. [5]

(iii) Find the rank and nullity of transformation $T: R^3 \to R^2$, T(x_1, x_2, x_3)=(x_1+x_2+x_3, x_1+x_2). [5]

OR

(iii) Show that the subspace S={(1,1,1), (1,2,3), (2,-1,1)} of R^3 spans the entire vector space V. [5]

Q.5 (i) Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1. -2. 1). [5]

(ii) If $A=\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, find Eigen values for [5]

(i) A (ii) A^T (iii) A^{-1} (iv) A^2.

OR

(i) Find the inverse of the following matrix by Gauss-Jordan elimination method $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$. [5]

(ii) Verify $\nabla(\nabla \times \hat{A}) = \nabla(\nabla . \hat{A}) - \nabla^2 \hat{A}$ for $\hat{A} = x^2 y\hat{\imath} + x^3 y^2 \hat{\jmath} - 3x^2 z^2 \hat{k}$. [5]

Q.6 (i) If $\vec{F} = 2xyz\hat{\imath} + (x^2 z + 2y)\hat{\jmath} + x^2 y\hat{k}$, then [5]
(a) if $\vec{F}$ is conservative, find its scalar potential ϕ
(b) find the work done in moving a particle under this force field from (0, 1, 1) to (1, 2, 0).

(ii) Determine whether the following matrix is diagonalizable [5]
A=$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$.

OR

(i) Use Gauss' divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\, ds$ where $\vec{F} = 2xy\hat{\imath} + yz^2\hat{\jmath} + zx\hat{k}$ and S is the surface of the region bounded by x=0, y=0, z=0, y=3, x+2z=6. [5]

(ii) Which of the following sets are linearly independent : [5]

(a) $2-x+4x^2$, $3+6x+2x^2$, $2+10x-4x^2$

(b) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 6 \\ 4 & 6 \end{bmatrix}$

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Exam Seat No: ....

KADI SARVA VISHWAVIDYALAYA

B.E 2nd SEMESTER EXAMINATION (DECEMBER 2015)

SUBJECT: Linear algebra and vector calculus (Code: CC101B)

Date:23/12/2015 Time: 3 hour Marks: 70

Instruction:

1. Answer each section in separate Answer Sheet.
2. Use of scientific Calculator is permitted.
3. All questions are compulsory.
4. Indicate **clearly**,the option you attempted along with its respective question number.
5. Use the last page of main supplementary for rough work.

Section:1

Q.1 (a) Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ by Gauss-Jordan method. [05]

(b) Define Vector space and Hermitian matrix. [05]

(c) Find the directional derivative of $\phi(x, y, z) = x^3 - xy^2 - z$ at point $P(1, 1, 0)$ in the direction of the vector $\overrightarrow{v} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. [05]

OR

(c) Find unit normal to the surface $x^2y + 2xz^2 = 8$ at the point $(1, 0, 2)$ [05]

Q.2 (a) Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$ by normal form. [05]

(b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$. [05]

OR

Q.2 (a) Determine the value of following equations if they are consistent. [05]

$$\begin{aligned} 2x_1 - 2x_2 + x_3 &= 1 \\ x_1 + 2x_2 + 2x_3 &= 2 \\ 2x_1 + x_2 - 2x_3 &= 7 \end{aligned}$$

(b) Verify Cauchy-Schwarz inequality for the vectors $u = (-4, 2, 1)$ and $v = (8, -4, -2)$. [05]

Q.3 (a) Show that the set $W = \{(x, 0, 0) \in \mathbb{R}^3 |\ x \in \mathbb{R}\}$ is subspace of $\mathbb{R}^3$. Also state all possible subspace of $\mathbb{R}^3$. [05]

(b) Verify the matrix $A = \begin{pmatrix} \frac{1}{9} & \frac{\sqrt{8}}{9} \\ \frac{\sqrt{5}}{9} & \frac{2}{9} \end{pmatrix}$ is orthogonal and hence find their inverse. [05]

OR

Q.3 (a) Check whether the set $V = \{\ (x, y)\ /\ x, y \in \mathbb{R}$ and $y > 0\ \}$ is vector space or not under the addition and scalar multiplication $(a, b) + (c, d) = (0, b + d)$ and $k(a, b) = (ka, kb)$ respectively. [05]

(b) By Stokes'theorem evaluate $\oint_c \overline{F} \cdot d\overline{r}$ where $\overline{F} = (x + y)\ \hat{i} + (y + z)\ \hat{j} - x\ \hat{k}$ and S is the surface of the plane $2x + y + z = 2$ which is in the first octant. [05]

Section:2

Q.4 (a) Find the eigenvalue and eigenvector for the matrix $B = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. [05]

(b) Find the rank and nullity of the matrix $A = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$. [05]

(c) Find the basis and dimension for the solution space of the system [05]

$$\begin{aligned} x_1 + x_2 &= 0 \\ x_2 + x_3 &= 0 \\ x_3 + x_4 &= 0 . \end{aligned}$$

OR

(c) Show that the matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ is diagonalizable. [05]

Q.5 (a) Determine whether the following function are linear transformation [05]

(i) $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ where $T(x, y) = (x + 2y,\ 3x - y)$.
(ii) $F : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ where $F(x, y, z) = (2x - y + z,\ y - 4z)$.

(b) Let T be multiplication by the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ then find the rank and nullity of the linear transformation T. [05]

OR

Q.5 (a) Find the standard matrix of the stated composition of linear transformation is a rotation of $45°$ about the $Y-axis$, followed by dilation with factor $k = \sqrt{2}$ [05]

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation define by $T : (x, y) = (x - 2y, x + 2y)$ and let $S_1 = \{(1, -1),\ (0, 1)\}$be basis for standard basis for $\mathbb{R}^2$ then find matrix of T w.r.t S_1 and S_2 [05]

Q.6 (a) Evaluate $\int_C \overline{F} \cdot dr$, where $\overline{F} = (3x - 2y)\ \widehat{i} + (y + 2z)\ \widehat{j} - x^2\widehat{k}$ and C is the straight line joining the points (0,0,0) to (1,1,1). [05]

(b) Find the constants a, b, c so that $v = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational. [05]

OR

Q.6 (a) By Green's theorem evaluate the integral $\oint_c [\ (x - y)dx + 3xydy\ ]$, where C is the boundary of the region bounded by the parabolas $x^2 = 4y$ and $y^2 = 4x$. [05]

(b) Determine the nature(value class) of the quadratic form $6x^2 + 3y^2 + 3z^2 - 2yz + 4zx - 4xy$. [05]

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