Exam Seat no. ....

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Remedial Examination BE Semester–III April - 2014

DIFFERENTIAL EQUATION & INTEGRAL TRANSFORM

Mathematics –III (CC301A)

Date: 23/04/2014 **Max Marks: 70**

Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work.**

Section I

Q.1 (i) Solve following differential equations: [5]
(a)$\frac{dy}{dx} = xy + x + y + 1$
(b) $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$

(ii) Find Laplace Transform of (a)$f(t) = t\ sin2t$ [5]
(b)$f(t) = t^3e^{-4t}$

(iii) Solve the differential equation$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = x^2 + y^2$. [5]

OR

(iii) Find the General solution of $\frac{d^2y}{dx^2} - 4y = sin3x + e^{-3x}$. [5]

Q.2 (i) Solve by the method of undetermined coefficient [5]
$y''' - 5y'' + 6y' = x^2 + sinx$.

(ii) Obtain the Fourier series for $f(x) = 1 - x^2\ ;\ -1 \le x \le 1$ [5]

OR

(i) Solve by the method of variation of parameter [5]
$\frac{d^2y}{dx^2} + 4y = sec2x$

(ii) Find Z-transform of the function $f(k) = a^k, k \ge 0$ [5]
$= b^k, k < 0.$

Q.3 (i) State Convolution theorem and Using it, find $L^{-1}\left\{\frac{1}{s(s^2+a^2)}\right\}$. [5]

(ii) Solve Legendre's differential equation [5]
$(3x + 2)^2y'' + 3(3x + 2)y' - 36y = 3x^2 + 4x + 1$.

OR

(i) Find the Fourier transform of $f(x) = e^{-ax^2}$ [5]

(ii) Form PDE from $Z = f(ax + y) + g(ax - y)$. [5]

Section II

Q.4 (i) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = a^2$. [5]

(ii) Find Laplace transform of $f(t) = \left[\frac{e^{-bt} - e^{-at}}{t}\right] (a \neq b)$. [5]

(iii) Solve the Equation $(D^2 - 6D + 9)y = x^2 e^{3x}$. [5]

OR

(iii) If $Z\{f(k)\} = F(z)$, ROC: R then Prove that [5]

(i) $Z\{k\, f(k)\} = -z\frac{d}{dz}F(z)$ (ii) $Z\{f(-k)\} = F\left(\frac{1}{z}\right)$

Q.5 (i) Solve by Exactness method $ye^x dx + (2y + e^x)dy = 0$; Where $y(0) = -1$. [5]

(ii) Solve the equation $(D^5 - m^2 D^3)y = e^{ax}$. [5]

OR

(i) Define Laplace transform and find $L[1]$and $L[e^{-at}]$. [5]

(ii) Find the Fourier cosine and sine transform of $f(x) = e^{-ax}$, $a>0$. [5]

Q.6 (i) Solve by the general method $y''' - 3y'' + 9y' - 27y = cos3x$. [5]

(ii) Find the Fourier series of $(x) = |x|$; $-\pi < x < \pi$. [5]

OR

(i) By the method of separation of variables solve $\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t} + u$ Where $u(x, 0) = 4e^{-3x}$. [5]

(ii) Solve $L^{-1}\left[\frac{s+6}{s^2+6s+13}\right]$ [5]

BEST OF LUCK