Exam Seat no. .....

KADI SARVA VISHWAVIDHYALAYA,

BE Semester–III (November 2015)

Mathematics –III(CC301B) 27·11·15 **Max Marks: 70**

Discrete Mathematical structure **Duration: 3 hr**

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section- I

Q.1 (a) Define Binary Function. Let R be define on the set A= {1,2,3,4} as $R = \{(1,2),(2,3),(2,4)\}$ then compute R^2 and R^3. [5]

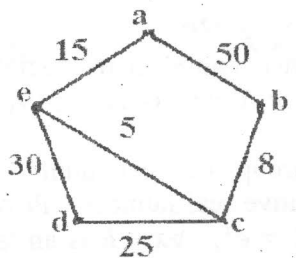
(b) Define Abelian Group. Show that cube roots of unity form a group under multiplication. [5]

(c) Define the following terms for graphs.
(i) Circuit (ii) Path (iii) Tree (iv) Multigraph (v) Pendant Vertex [5]

OR

(c) Define Boolean Algebra. For the set $A = \{a, b\}$, $P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Show that $(P(A), \wedge, \vee, \emptyset, A, C)$ is Boolean Algebra. [5]

Q.2 (a) Apply Dijkstra's algorithm to find the shortest path from a to d in the given weighted graph : [5]



(b) Use Karnaugh map representation to minimize the following [5]
(i) $f(x, y, z) = xy + x'y + x'y'$
(ii) $f(x, y, z) = xyz + xyz' + x'yz' + xy'z$

OR

(a) Define In-degree and out-degree of diagraph. From the following adjacency matrix, find the out-degree and in-degree of each node. Also verify your answer by drawing diagraph and its adjacency Matrix. [5]

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(b) Show that $\langle S_{30}, *, \oplus \rangle$ and $\langle p(A), \cap, \cup \rangle$ are isomorphic lattice for $A = \{a, b, c\}$. [5]

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[5]

Q.3 (a) Find all subgroup of cyclic group order 12 with generator 'a' and also find the order of generators of G.

(b) Draw the Hasse Diagram for the following posets: [5]

(i) $\langle S_{90}, D \rangle$

(ii) $\langle S_{72}, D \rangle$; where **aDb** means **a** divides **b**.

OR

(a) Define Right Coset and Left Coset. Consider group $(\mathbb{Z}, +)$ and its subgroup $H = (4\mathbb{Z}, +)$. Find all right cosets of H in G. [5]

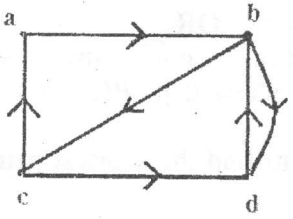
(b) Find the SOP and POS canonical expansions of given Boolean function $f(x, y, z) = x + (yz')$. [5]

Section –II

Q.4 (a) Show that $\langle \{1,2,3,6\}, \text{GCD}, \text{LCM} \rangle$ is a sublattice of the lattice $\langle S_{30}, \text{GCD}, \text{LCM} \rangle$. [5]

(b) Define Finite automata with output. Construct finite automata which accept all strings ending with 10 and also draw transition table for it where $\Sigma = \{0,1\}$. [5]

(c) Find all sub-graphs of following graph having three vertices a, b, and d [5]



OR

(c) Define Join-Irreducible Element and Meet-Irreducible Element. Find Meet-Irreducible elements and Atoms for lattice $\langle S_4 \times S_9, D \rangle$ and $\langle S_{30}, D \rangle$. [5]

Q.5 (a) Let R be the additive group of real numbers and R^+ be the multiplicative group of positive real numbers. Prove that the function $f: R \to R^+$ defined by $f(x) = e^x$, $\forall x \in R$ is an isomorphism from R onto R^+. [5]

(b) Let $M = (Q, \Sigma, \Gamma, q_0, \delta, \gamma)$, where $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$, $\Gamma = \{0,1\}$, q_0 is the initial state and the function δ *and* γ are defined as follows: [5]

	δ		γ	
	a	b	a	b
q_0	q_2	q_1	1	1
q_1	q_2	q_2	0	0
q_2	q_1	q_2	1	1

(i) Draw the transition diagram of M.

(ii) What is the output if the input strings are (1) **aabbb** (2) **abbbaba** (3) **bbbababa**

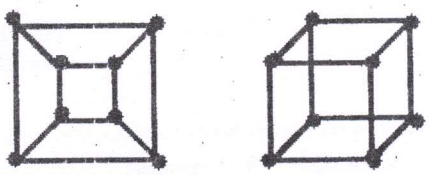
OR

(a) Let $G = (V_N, \Sigma, P, S)$ be a context-free grammer, where $V_N = \{S, A, B\}, \Sigma = \{a, b\}$ and $P = \{S \to aB, S \to bA, A \to a, A \to aS, A \to bAA, B \to b, B \to bS, B \to aBB\}$ [5]

(i) For the string **aabbab** find derivation.

(ii) Draw the derivation tree with **abbaba**.

(b) Show that following graphs are isomorphic given in following figures [5]



Q.6 (a) In a Lattice show that [5]

(i) $(a*b) \oplus (c*d) \leq (a \oplus c) * (b \oplus d)$

(ii) $(a*b) \oplus (b*c) \oplus (c*a) \leq (a \oplus b) * (b \oplus c) * (c \oplus a)$

(b) Define AND-Gate and NOR-Gate. [5]

Use K-map to minimize given Boolean expression and also draw circuit diagram

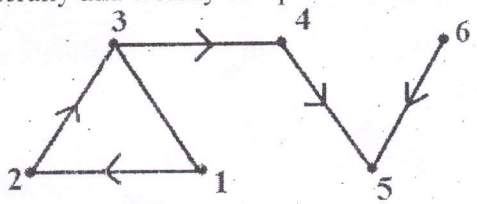
$f(x, y) = xy + x'y + x'y'$.

OR

(a) Using Quine – Mccluskey algorithm representation to minimize the following [5]

$f(x, y) = xyz + xy'$.

(b) Find strong, unilaterally and weakly component of following diagraph [5]



Best of Luck

Exam Seat no. .....

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

BE Semester–III (November 2013)

Discrete Mathematical Structure (CC 301 B)

Date: 23/11/2013 **Total Marks: 70**

Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section I

Q.1 A) Do as directed

1) Let A = {1, 4, 5} and R = {(1, 4), (1, 5), (4, 1), (4, 4), (5, 5)} be the relation defined on A. Then find relation matrix M_R also draw the graph of relation. [02]

2) Define the following a) Complimented lattice b) Equivalence relation. [02]

3) Every complete lattice is bounded. (True/False) [01]

B) Draw the Hasse diagram of the following posets. [05]

i) $< P(S), \subseteq >, S = \{a, b, c\}$

ii) $< S_{12}, R >$ where R is divides relation.

C) In a Lattice show that [05]

i) $(a*b) \oplus (c*d) \leq (a \oplus c) * (b \oplus d)$

ii) $(a*b) \oplus (b*c) \oplus (c*a) \leq (a \oplus b) * (b \oplus c) * (c \oplus a)$

OR

C) State and prove Distributive inequality. [05]

Q.2 A) Minimize $f(x, y, z) = xyz + xy'z + x'yz + x'y'z + x'y'z'$ using Quine-Mccluskey algorithm. [05]

B) In any Boolean algebra, show that [05]

$$a = b \Leftrightarrow ab' + a'b = 0$$

OR

Q.2 A) In any Boolean algebra Show that [05]

i) $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$

ii) $(a + b)(a' + c) = ac + a'b = ac + a'b + bc$

B) In any Boolean algebra, show that [05]

$$a = 0 \Leftrightarrow ab' + a'b = b$$

Q.3 A) For the Boolean function $f(x, y, z) = x + (yz')$ find the product of sum canonical and sum of products canonical form. [05]

B) Use Karnaugh map representation to minimize and also draw circuit diagram of the minimized Boolean expression [05]

$$f(x, y, z) = x'yz + x'yz' + xy'z + x'y'z + x'y'z'$$

OR

Q.3 A) Minimize Boolean function $f(x, y, z, t) = y't' + y'z't + x'y't + x'y'zt + yzt'$. [05]

B) Do as directed [05]

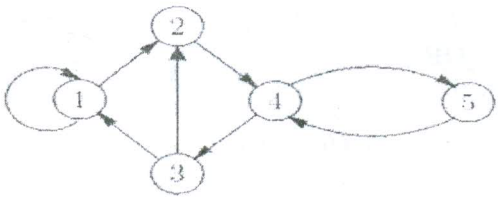
1) State stone's representation theorem
2) Define the following with example
   i) Direct product of two Boolean algebra
   ii) Join-irreducible element
   iii) Lattice as an algebraic structure

Section II

Q.4 A) Define Isomorphic graphs and verify that following pair of graphs G and H are isomorphic or not. [05]

G H

B) Consider the following graph G=<V,E> [05]



1) Define adjacency matrix and find it for above graph G.
2) How many paths are there of length 4(four) from vertex 1 to vertex 4.
3) Find path matrix.

C) Define the following terms for graphs with example [05]

i) Loop
ii) Null Graph
iii) Converse of a diagraph
iv) Elementary Path
v) Nodebase

OR

C) Draw digraph G = (V, E) with $V = \{1,2,3,4,5,6\}$ & [05]
$E = \{(1,2),(2,3),(3,4),(5,4),(6,5),(6,1),(4,1),(6,3)\}$.
Also find Strong & unilateral components for this graph.

Q.5 A) Define group. Prove that the set of fourth root of unity under multiplication $(G = \{\pm 1, \pm i\}, \times)$ is abelian group. Find the order of the group, and order of each element. [05]

B) Define subgroup. Prove that $(Z_6, +_6)$ is group. Find all Subgroups of this group. [05]

OR

Q.5 A) Define the cyclic group. Verify that $(Z_7 - \{0\}, \times_7)$ is cyclic group. [05]

B) Define Permutation Group. Prove that $(S_3, \circ)$ is group. Also find all subgroups of permutation group $(S_3, \circ)$. [05]

Q.6 A) Let $M =< Q, \Sigma, \Gamma, q0, \delta, \gamma >$, where $Q = \{q0, q1, q2, q3\}$, $\Sigma = \{0,1\}$, $\Gamma = \{a, b\}$, $q0$ is the initial state and the function δ and γ are defined as follows [05]

	δ		γ	
	0	1	0	1
q0	q2	q1	a	a
q1	q2	q2	a	b
q2	q3	q1	b	a
q3	q1	q3	b	b

a) Draw the transition diagram of M

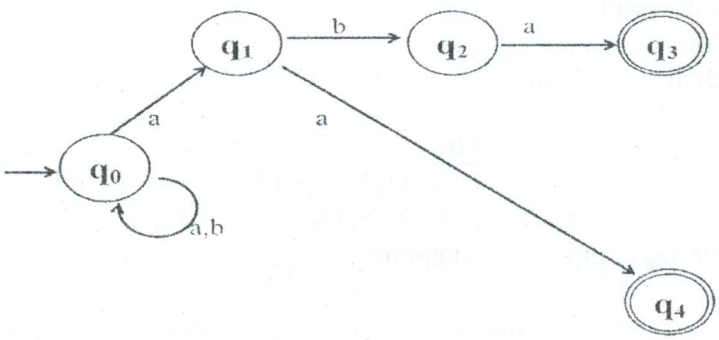
b) What is the output string and state if the input string is

i) 010001 ii) 101011 iii) 100111

B) Construct finite state machine which accept strings with even numbers of ones. [05]

OR

Q.6 A) Let M be the NDFA whose state diagram is given as [05]



a) Write the transition table for this NDFA

b) What are the final states?

c) Find $\delta^*(q0, baa)$.

d) Is $baaababa$ accepted by M?

B) Design a FSM, with input alphabet $\sum = \{a, b\}$, that outputs a 1 if the number of input symbols read so far is divisible by 3 [05]

BEST OF LUCK

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Exam Seat no. .....

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

BE Semester–III

Discrete Mathematical Structure

Date: 23/4/14 **Total Marks: 70**

Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
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5) Use the last page of main suppiementary for **rough work**.

Section I

Q.1 A) Do as directed

1) Let R be a Relation on a set N of positive integers defined by R={(a,b)/ a+b is even} Is R an Equivalence Relation ? [02]

2) Define the following with example i) lattice ii) Equivalence relation [02]

3) Define the composition relations and find the composition R_1 oR_2, (R_1 oR_2)oR_1 and (R_1 oR_2)oR_2 of the following relations [01]

R_1 = {(1,2),(1,6),(2,4),(3,4),(3,6),(3,8)}

R_2 = {(2,x)(4,y),(4,z),(6,z),(8,x)}

B) Draw the Hasse diagram of the following posets where D is divides relation i) $< S_{30}, D >$ ii) $< S_{90}, D >$ [05]

C) State and prove distributive inequality. [05]

OR

C) In a Lattice show that $(a*b) \oplus (c*d) \leq (a \oplus c) * (b \oplus d)$ [05]

Q.2 A) Find the sum of products canonical and product of sum canonical expansions of the following Boolean functions [05]

1) $f(x, y, z) = x + y + z$

B) In any Boolean algebra, show that [05]

$$a = 0 \Leftrightarrow ab' + a'b = b$$

OR

A) Use Quine-Mccluskey algorithm representation to minimize [05]

$$f(x,y) = xy + x'y + x'y'$$

z

B) Simplify the following Boolean expressions [05]

$$(a * b)' \oplus (a \oplus b)'$$

Q.3 A) In any Boolean algebra, show that [05]

i) $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$

ii) $a \leq b \Rightarrow a + bc = b(a + c)$

B) Use Karnaugh map representation to minimize [05]

$$f(x, y, z) = xyz + xyz' + x'yz' + xy'z$$

OR

A) Design a minimized logic circuit for the following Boolean function $F(A,B,C,D)=\sum(9,10,11,12,13,14,15)$ [05]

B) Define the following with example [05]

1) Boolean Algebra
2) Sub Boolean algebra
3) Direct product of two Boolean algebra
4) Boolean homomorphism

State Stone's Representation Theorem

Section II

Q.4 A) Do as directed

1) Check whether following statements are true or false. [02]
   i) Every Simple Circuit Elementary Circuit.
   ii) Every walk is path.
2) Define the following terms for graphs with their difference. [03]
   i) Elementary Path ii) Simple path

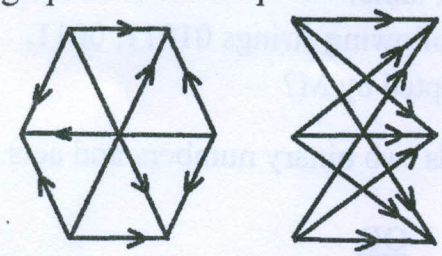
B) Find Strong & unilateral components from the following digraph G = (V, E) [05]
with
$V = \{1,2,3,4,5,6\}$ & $E =$
$\{(1,2), (2,3), (3,4), (5,4), (6,5), (6,1), (4,1), (6,3)\}$

C) Draw the digraph G = (V,E) & find Reachable set of each vertex(node) & node base; where V = $\{v_1, v_2, \ldots, v_7\}$ & [05]

E=$\{(v_2,v_3),(v_5,v_6),(v_1,v_6),(v_2,v_1),(v_6,v_5),(v_5,v_1),(v_2,v_3),(v_3,v_1),(v_2,v_7),(v_7,v_4),(v_3,v_7)\}$.

OR

C) Define Isomorphic graphs and verify that following pair of graphs are isomorphic or not. [05]



Q.5 A) Define group and order of group. Find the order of the group and order of each element for $(Z_5, +_5)$. [05]

B) Prove that the structure $(Z_5, +_5)$ is cyclic group and find all its generators. [05]

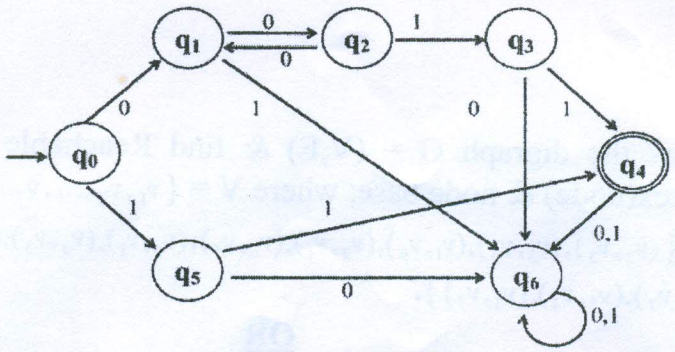
OR

A) Let $\alpha, \beta \in S_5$ such that $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 5 & 4 \end{pmatrix}$ & [05]

$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 5 & 2 \end{pmatrix}$, then find $\alpha^2 \circ \beta, o(\alpha)$ & α^2

B) Prove that Sub group H =$\{p_1, p_2, p_3\}$ is normal for the group $(S_3, \circ)$ .Check Whether Sub group H_1 =$\{p_1, p_4\}$ is normal for the group $(S_3, \circ)$? [05]

Q.6 A) Let M be Deterministic finite automaton whose transition diagram is as follows. [05]



i) What are the final states?
ii) Draw transition table.
iii) Which of the following strings 01011, 0011, 00111 are accepted by M?

B) Construct a FSM which adds two binary numbers and acts as a binary adder. [05]

OR

Q.6 A) For the FSM given below, draw the state diagram and find the output string given the input string as *aabbcc* [05]

$I=\{a,b,c\}$, $O=\{0,1\}$ $S=\{s_0,s_1,s_2\}$

	f			g		
Input/state	a	b	c	a	b	c
S_0	S_0	S_1	S_2	0	1	0
S_1	S_1	S_1	S_0	1	1	1
S_2	S_2	S_1	S_0	1	0	0

B) Construct a FSM that works as a language recognizer that gives an output 1 if and only if the last three bits received are all 1's. [05]

BEST OF LUCK

Exam Seat no. .....

KADI SARVA VISHWAVIDHYALAYA,

B.E. SEMESTER–III EXAMINATION (November 2014)

Subject Code: CC 301 B **Subject Name:Discrete Mathematical Structure**

Date: 11 /11/2014 **Duration: 3 hours** **Total Marks: 70**

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.
6) Make necessary assumption when value is not mention

Section I

Q.1 A) Do as directed

1) Consider the relation R on $A = \{4,5,6\}$ as $R = \{(4,4),(4,5),(5,5),(5,6),(6,6),(5,4)\}$. Check whether this relation is an equivalence relation? [03]

2) Define the following a) Loop b) Simple Graph [02]

B) In any Boolean algebra, show that $a = b \Leftrightarrow ab' + a'b = 0$ [05]

C) Let $< L \ , \leq >$ be a lattice and a, $b, c \in L$ then [05]
1) $b \leq c \Rightarrow a * b \leq a * c$
2) $b \leq c \Rightarrow a \oplus b \leq a \oplus c$

OR

C) State and prove distributive inequality [05]

Q.2 A) Define the following [05]
1) Join irreducible element
2) Boolean algebra
3) Bounded lattice
4) POSET
5) Lattice

B) Minimize following using Quine-Mccluskey algorithm [05]

$$f(x_1, x_2, x_3) = x_1x_2x_3 + x_1x_2x_3' + x_1'x_2x_3' + x_1'x_2'x_3' + x_1'x_2'x_3$$

OR

Q.2 A) Draw the Hasse diagram of the following posets. [05]
1) $< S_{15}, D >$ 2) $< S_{42}, D >$, where D is divides relation.

B) Write the following Boolean expression in an equivalent sum of products canonical form and product of sums canonical form [05]

$$f(x, y, z) = (x_1 + x_2) * (x'_3)$$

Q.3 A) In any Boolean algebra, verify that [05]

i) $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$

ii) $(a + b)(a' + c) = ac + a'b = ac + a'b + bc$

B) Use Karnaugh map representation to minimize [05]

$$f(x, y, z) = xyz + xyz' + xy'z + x'yz + x'y'z'$$

OR

Q.3 A) Find the value of following Boolean expressions for $a = 2$, $b = 10$ and $c = 15$ where Boolean algebra is S_{30} with divides relation. [05]

i) $f(a, b, c) = (a * c) \oplus c \oplus [(b \oplus a') * c]$

ii) $f(a, b, c) = (a' * b')' \oplus (a * b' * c) \oplus (a * b' * c')$

B) Define the following with example [05]

i) Complimented lattice
ii) Boolean homomorphism
iii) Equivalence relation
iv) Atom
v) Complete Lattice

Section II

Q.4 A) Define the following [05]
1) Graph 2) Isolated vertex 3) Parallel Edges 4) Cyclic graph 5) Tree

B) Consider the following graph G=<V,E> [05]

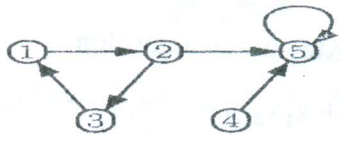
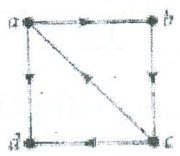


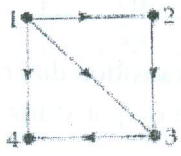
Figure 1

1) Define adjacency matrix and find it for above graph G.
2) How many paths are there of length 4(four) from vertex 1 to vertex 5.
3) Find path matrix.

C) Define Isomorphic graphs. Check whether the following graphs are isomorphic or not. [05]



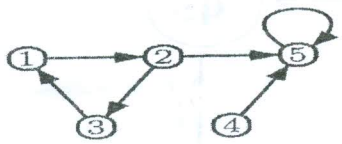
G_1



G_2

OR

C) Define Nodebase(Vertexbase). Also find it for the graph given by [05]



Q.5 A) Define subgroup and Normal subgroup. Find the order of the group and order of each element for $< Z_5 - \{0\}, X_5 >$. [05]

B) Define group. Prove that the set of cube root of unity under multiplication $(G = \{1, w, w^2\}, \times)$ is abelian group. Find the order of the group, and order of each element. [05]

OR

A) Define Permutation Group. Prove that $(S_3, \circ)$ is group. [05]

B) Verify that $< Z_7 - \{0\}, X_7 >$ is a group. [05]

Q.6 A) Define the following [05]

1) Finite state machine with output
2) Finite state Automata
3) Non-Deterministic finite state automaton

B) Construct finite state machine which accept strings with odd numbers of zeros (0s). [05]

OR

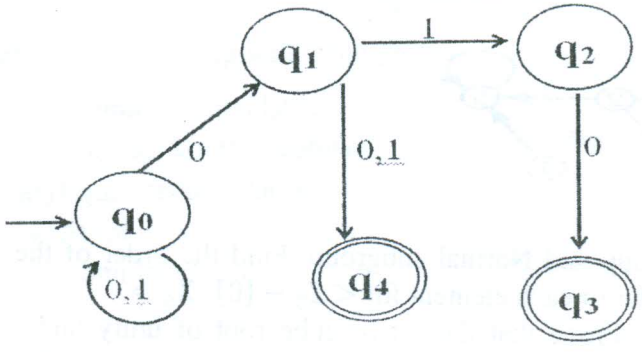
Q.6 A) Consider the following finite state machine [05]
$M =< Q, \Sigma, \Gamma, q0, \delta, \gamma >$, where $Q = \{q0, q1, q2, q3\}$, $\Sigma = \{0,1\}$, $\Gamma = \{0,1\}$, $q0$ is the initial state and the function δ and γ are defined as follows

	δ		γ	
	0	1	0	1
q0	q1	q0	1	0
q1	q2	q3	1	0
q2	q3	q2	0	0
q3	q2	q0	1	1

a) Draw the transition diagram of M

b) What is the output string if the input string is i) 001001 ii) 101010.

B) Consider the following non-deterministic finite automaton whose diagram is given as follows [05]



a) Write the transition table for this NDFA

b) What are the final states?

c) Find $\delta^*(q0, 010)$.

BEST OF LUCK