Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,

B.E. SEMESTER–I EXAMINATION (May-June 2015)

Subject Code: CC 101 A   Subject Name: Advance Calculus

Date: 03/06/2015   Duration: 3 hours   Total Marks: 70

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work.**
6) Make necessary assumption when value is not mention

Section I

Q.1 A) Trace the curve $y^2\,(2a-x) = x^3$. [05]

B) Evaluate $\int_0^{\pi/6} \cos^6 3\theta \ \sin^6 3\theta \ d\theta$. [05]

C) Evaluate $\iint_R e^{2x+3y}\, dA$, where R is the triangle bounded by $x = 0$, $y = 0$ and $x + y = 1$. [05]

OR

C) Test the convergence of following series. [05]

(i) $\sum_{n=2}^{\infty} \frac{1}{n\log n}$ (ii) $\sum_{n=1}^{\infty} \frac{2^n-1}{3^n}$

Q.2 A) State Euler's theorem for homogeneous function. Also show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 6u$ if $u = x^4y^2 \sin^{-1}\left({}^{y}/_{x}\right)$ [05]

B) If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. [05]

OR

Q.2 A) (i) find $\frac{dw}{dt}$ if $w = xy+z$, $x = \cos t$, $y = \sin t$, $z = t$ [05]

(ii) find $\frac{\partial z}{\partial s}$ if $Z = e^{2x} \sin 3y$, where $x = s\,t^2$ and $y = t\,s^2$

B) If $u = \tan^{-1}\left(\frac{y}{x}\right)$ verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ [05]

Q-3 A) Evaluate $\int_{\theta=0}^{\pi} \int_{r=0}^{a(1+\cos\theta)} r\, dr\, d\theta$ [05]

B) Evaluate $\iint_R xy\, dA$, where R is the region bounded by x-axis, ordinate x = 2a and the curve $x^2 = 4ay$. [05]

OR

Q.3 A) Change the order of integration and evaluate $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}\,dy\,dx$ [05]

B) By changing into polar coordinates, evaluate the integral $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2+y^2)\,dx\,dy$. [05]

Section II

Q.4 A) Evaluate $\lim_{x\to 0}(x)^{\sin x}$ [05]

B) Trace the curve r = a (1+cosθ) , a> 0 [05]

C) Find by double integration, the area lying between the parabola y = 4x - x^2 and the line y = x. [05]

OR

C) Find volume of the sphere $x^2+y^2+z^2 = a^2$ by triple integration. [05]

Q.5 A) Find the value of $\sqrt{18}$ using Taylor's series. [05]

B) Find maximum and minimum value of the function f (x, y) =$2x^4+y^2-x^2-2y$. [05]

OR

A) Expand $3x^3+8x^2+x-2$ in powers of (x-3). [05]

B) Show that the plane 3x+12y-6z-17=0 touches the surface $3x^2-6y^2+9z^2+17=0$. [05]

Q.6 A) Expand $e^x\, siny$ in powers of x and y up to second order terms. [05]

B) Find the minimum value of $x^2+y^2+z^2$, given that ax+by+cz = p. [05]

OR

Q.6 A) Determine the radius and interval of convergence for the series $x+2x^2+3x^3+4x^4+\ldots\ldots$ [05]

B) If u = x+y and v = $\frac{x}{x+y}$, find $\frac{\partial(u,v)}{\partial(x,y)}$. [05]

Exam Seat no. ………….

KADI SARVA VISHWAVIDHYALAYA,

B.E. SEMESTER–I EXAMINATION (DECEMBER 2015)

Subject Code: CC 101 A **Subject Name: Advance Calculus (Mathematics I)**
Date: 26/12/2015 **Duration: 3 hours** **Total Marks: 70**

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work.**

Section- I

Q.1 (a) Evaluate (i) $\lim_{x\to 0}\left(\frac{e^x+e^{2x}+e^{3x}}{3}\right)^{1/x}$ (ii) $\lim_{x\to \pi/2}\frac{\sin(x\cos x)}{\cos(x\sin x)}$ [5]

(b) Expand $\log_e x$ in power of $(x-1)$ up to fourth power and hence evaluate $\log_e 1.1$ correct to four decimal places. [5]

(c) Trace the curve $xy^2 = 4a^2(a-x)$. [5]

OR

(c) If $z = f(x,y)$ where $x = u\cos\alpha - v\sin\alpha$ and $y = u\sin\alpha + v\cos\alpha$, where α is constant, show that $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}$. [5]

Q.2 (a) Using the transformation $x+y=u$ and $y=uv$, Evaluate $\int_0^1\int_0^{1-x} e^{\frac{y}{x+y}}\,dxdy$. [5]

(b) Test the convergence of the series $\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+\cdots\infty$ [5]

OR

(a) Evaluate $\iint r^2 sin\theta drd\theta$ over the cardiod $r = a(1+cos\theta)$ above the initial line. [5]

(b) Determine absolute or conditional convergent of the series $\sum_{n=1}^{\infty}(-1)^n\frac{n^2}{n^3+1}$. [5]

Q.3 (a) If $ux+vy=0$ *and* $\frac{u}{x}+\frac{v}{y}=1$ then prove that $\left(\frac{\partial u}{\partial x}\right)_y-\left(\frac{\partial v}{\partial y}\right)_x=\frac{x^2+y^2}{y^2-x^2}$. [5]

(b) Verify $JJ^*=1$ for the following functions $x=u, y=utanv$ and $z=w$. [5]

OR

(a) Find the extreme value of $u(x,y)=x^3+y^3-63x-63y+12xy$. [5]

(b) If $z=\log(x^2+y^2)+\frac{x^2+y^2}{x+y}-2\log(x+y)$ then find the value of $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$. [5]

Section -II

Q. 4 (a) If $u = f(r)$ and $r^2 = x^2 + y^2 + z^2$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r} f'(r)$. [5]

(b) Test the convergence of the series(1) $\sum_{n=1}^{\infty} \frac{[(n+1)x]^n}{n^{n+1}}$, x>0 [5]

(2)$1 + \frac{2^p}{2!} + \frac{3^p}{3!} + \frac{4^p}{4!} + \cdots \ldots \ldots$ and $p > 0$.

(c) Evaluate $\int_0^1 \int_{4y}^4 e^{x^2}\, dxdy$ by changing the order of integration. [5]

OR

(c) Evaluate (1) $\int_{\pi/2}^{\pi} \int_1^2 x\cos xy\, dydx$ (2) $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\, dzdydx$ [5]

Q. 5 (a) Find the volume of the solid generated by revolving the curve $xy^2 = a^2(a - x)$ about the y-axis. [5]

(b) (1) Find the area bounded by the lemniscates $r^2 = a^2 \cos 2\theta$. [5]

(2)Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

OR

(a) Evaluate $\iiint \sqrt{x^2 + y^2}\, dv$, over the region bounded by the right circular cone $x^2 + y^2 = z^2, z > 0$ and the planes $z = 0$ and $z = 1$. [5]

(b) Evaluate (1) $\int_0^{\pi/4} \sin^7 2\theta\, d\theta$ (2) $\int_0^1 \frac{x^7}{\sqrt{1-x^4}}\, dx$ [5]

Q. 6 (a) Trace the curve $r = a\sin 3\theta$. [5]

(b) Test the convergence of the series $\frac{1}{1+\sqrt{2}} + \frac{2}{1+2\sqrt{3}} + \frac{3}{1+3\sqrt{4}} + \cdots \infty$ [5]

OR

(a) Evaluate (1) $\int_0^{\pi} x\sin^5 x\cos^5 x\, dx$ (2) $\int_0^{\infty} \frac{x^2}{(1+x^6)^{5/2}}\, dx$ [5]

(b) (I) Find the equations of the Tangent plane and normal line to the surface $x^2 + y^2 - 4z = 5$ at the point (3, 4, 5). [3]

(II)Write the relation between Cartesian and spherical coordinate in R^3. [1]

(III) State sandwich theorem. [1]

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Exam Seat no. …………

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

BE Semester–I (December 2013)

Mathematics –I(CC101A)

Advance Calculus

Dt. 21/12/13

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section- I

Q.1 (a) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{x} xy(x+y)dydx$. [5]

(b) Prove that if $x^2 = au + bv$ and $y^2 = au - bv$ then prove that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u$. [5]

(c) Test the convergence of the series $\frac{x}{1\cdot 2} + \frac{x^2}{3\cdot 4} + \frac{x^3}{5\cdot 6} + \cdots \infty, x > 0$ [5]

OR

(c) Expand $\frac{e^x}{e^x+1}$ in maclaurin's series. [5]

Q.2 (a) Trace the curve $x^2y^2 = a^2(y^2 - x^2)$. [5]

(b) Test the convergence of the series [5]
$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \cdots \infty$

OR

(a) Find the values of a and b such that $\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$. [5]

(b) Find the maximum value of $v(x,y,z) = xyz$ subjected to the constraint $2x + 2y + z = 108$. [5]

Q.3 (a) Determine the area bounded by the curves $xy = 2, 4y = x^2$ and $y = 4$. [5]

(b) If $x = e^v secu$ and $y = e^v tanu$ then verify JJ'=1. [5]

OR

(a) Evaluate $(1.99)^2(3.01)^3(0.98)^{\frac{1}{10}}$ using approximation. [5]

(b) State Sandwich theorem and find $\lim_{x \to 0} g(x)$ if $3 - x^2 \le g(x) \le 3secx$ for all x. [5]

Section –II

Q. 4 (a) By using the transformation $x + y = u, y = uv$, show that [5]

$$\int_0^1 \int_0^{1-x} e^{y/(x+y)}\, dxdy = \frac{1}{2}(e-1).$$

(b) If $u(x,y,z) = \log(tanx + tany + tanz)$ prove that [5]

$$sin2x\frac{\partial u}{\partial x} + sin2y\frac{\partial u}{\partial y} + sin2z\frac{\partial u}{\partial z} = 2.$$

(c) Expand $x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x-1)$ and hence find $f\left(\frac{11}{10}\right)$. [5]

OR

(c) Find the maximum and minimum values of [5]

$$sinx + siny + sinxy, 0 < x, y < \frac{\pi}{2}.$$

Q. 5 (a) Find the linearization of [5]
$(i) f(x,y,z) = x^3 - xy^2 + 8sinz$ at (2,1,0)
(ii) $f(x) = 4x^3 + 6x + 1$ at x=1.

(b) Find the area lying inside the circle $r = asin\theta$ and outside the cardioids $r = a(1 - cos\theta)$ where $a > 0$. [5]

OR

(a) Show that the plane $3x + 12y - 6z - 17 = 0$ touches the surface $3x^2 - 6y^2 + 9z^2 + 17 = 0$. Find also point of contact. [5]

(b) Find the volume of the solid generated by rotating the region bounded by $y = \sqrt{x}$ and the lines $y = 2, x = 0$ about the line $x = 0$. [5]

Q. 6 (a) Evaluate $\int_1^e \int_1^{logy} \int_1^{e^x} logz\, dzdxdy$. [5]

(b) Define Absolute and Conditional convergence. Examine absolute and conditional convergence for $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \cdots$ [5]

OR

(a) Evaluate (i) $\int_0^\infty \frac{x^2}{(1+x^6)^{\frac{7}{2}}} dx$ (ii) $\int_0^\pi sin^2x(1+cosx)^4\, dx$ [5]

(b) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. [5]

Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA

Gandhinagar

BE Semester–I (May 2013)

Mathematics –I (Advance Calculus)

4/6/13

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

		Section- I	
Q.1	**A**	Evaluate (i) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$ (ii) $\lim_{x\to 0}\left[\frac{1}{2x}-\frac{1}{x(e^{\pi/x}-1)}\right]$	[5]
	B	Prove that $tan^{-1}x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+ ....$	[5]
	C	Trace the curve $r = a(1+\cos\theta)$	[5]
		OR	
	C	If $u = \log\left[\frac{x^3+y^3}{xy}\right]$ show that $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}$	[5]
Q.2	**A**	Evaluate $\int_1^e\int_1^{\log y}\int_1^{e^x}\log z\ dxdydz$	[5]
	B	Test the convergence of the series $1+\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+\cdots$; x>0	[5]
		OR	
	A	Evaluate $\iiint\frac{z^2}{x^2+y^2+z^2}\,dxdydz$ over the volume of the sphere $x^2+y^2+z^2=2.$	[5]
	B	Test the convergence of the series $\sum(-1)^{n+1}\frac{x^n}{n^2}$; x>0	[5]
Q.3	**A**	If $x = r\cos\theta, y = r\sin\theta$, prove that (i) $\left(\frac{\partial x}{\partial r}\right)_\theta=\left(\frac{\partial r}{\partial x}\right)_y$ (ii) $\left(\frac{\partial x}{\partial\theta}\right)_r=r^2\left(\frac{\partial\theta}{\partial x}\right)_y$	[5]
	B	Define maxima of the function.Find the maximum and minimum values of $x^2+y^2+xy+x-4y+5$	[5]
		OR	
	A	Find Pressure P at any point (x,y,z) in space is P = 400 xyz^2. Find the highest pressure on the surface of the unit sphere $x^2+y^2+z^2=1.$	[5]

	B	If $x = a(u+v), y = b(u-v)$ & $u = r^2 \cos 2\theta$ $v = r^2 \sin 2\theta$, evaluate $J = \frac{\partial(x,y)}{\partial(u,v)}$ and $J' = \frac{\partial(u,v)}{\partial(x,y)}$ and hence verify that JJ'=1.	[5]
		Section -II	
Q. 4	A	If $u = \log(\tan x + \tan y + \tan z)$,then prove that $(\sin 2x)\, u_x + (\sin 2y)\, u_y + (\sin 2z)\, u_z = 2$	[5]
	B	Define Absolute convergence and Conditional convergence and Test the convergence of the series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \cdots$	[5]
	C	Evaluate by changing into polar co-ordinates of $\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\,dy\,dx$	[5]
		OR	
	C	Show that $\int_0^1 \left\{\int_0^1 \frac{x-y}{(x+y)^3}\,dy\right\} dx \neq \int_0^1 \left\{\int_0^1 \frac{x-y}{(x+y)^3}\,dx\right\} dy$	[5]
Q. 5	A	Evaluate $\iiint \frac{dx\,dy\,dz}{\sqrt{a^2-x^2-y^2-z^2}}$ over the region bdd by the sphere $x^2+y^2+z^2 = a^2$	[5]
	B	Evaluate $\iint (y-2x^2)\,dA$, where R is the region inside the square $\lvert x\rvert + \lvert y\rvert = 1$.	[5]
		OR	
	A	Find the volume of the region that lies under the paraboloid $z = x^2+y^2$ and above the triangle enclosed by the lines y=x, x=0 and x+y=2 in the xy-plane.	[5]
	B	Find the area common between curve $y = 4x - x^2$ & line $y = x$.	[5]
Q. 6	A	Trace the curve $x^3 + y^3 = 3axy$	[5]
	B	By Cauchy integral test, check the convergence of the following series: $\sum \frac{1}{n(\log n)^a}, for\ 0 \le a \le 1$.	[5]
		OR	
	A	Test the convergence of the series $\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \cdots$	[5]
	B	(i) Find the equations of the Tangent plane and normal line to the surface $x^2 + 2y^2 + 3z^2 = 12$ at the point (1,2,3).	[3]
		(ii) Find linearization of the function $f(x,y,z) = e^x + z + xy + 6z$ at (-1,0,1).	[2]

Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,

B.E. SEMESTER–I EXAMINATION (May-June 2015)

Subject Code: CC 101 A Subject Name: Advance Calculus

Date: 03/06/2015 Duration: 3 hours Total Marks: 70

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
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Section I

Q.1 A) Trace the curve $y^2\,(2a - x) = x^3$. [05]

B) Evaluate $\int_0^{\pi/6} \cos^6 3\theta \ \sin^6 3\theta \ d\theta$. [05]

C) Evaluate $\iint_R e^{2x+3y}\, dA$, where R is the triangle bounded by $x = 0, y = 0$ and $x + y = 1$. [05]

OR

C) Test the convergence of following series. [05]

(i) $\sum_{n=2}^{\infty} \frac{1}{n\log n}$ (ii) $\sum_{n=1}^{\infty} \frac{2^n - 1}{3^n}$

Q.2 A) State Euler's theorem for homogeneous function. Also show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 6u$ if $u = x^4 y^2 \sin^{-1}\left(y/_x\right)$ [05]

B) If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. [05]

OR

Q.2 A) (i) find $\frac{dw}{dt}$ if $w = xy+z$, $x = \cos t$, $y = \sin t$, $z = t$ [05]

(ii) find $\frac{\partial z}{\partial s}$ if $Z = e^{2x} \sin 3y$, where $x = s\,t^2$ and $y = t\,s^2$

B) If $u = \tan^{-1}\left(\frac{y}{x}\right)$ verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ [05]

Q-3 A) Evaluate $\int_{\theta=0}^{\pi} \int_{r=0}^{a(1+\cos\theta)} r\, dr\, d\theta$ [05]

B) Evaluate $\iint_R xy\,dA$, where R is the region bounded by x-axis, ordinate x = 2a and the curve $x^2 = 4ay$. [05]

OR

Q.3 A) Change the order of integration and evaluate $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}\,dy\,dx$ [05]

B) By changing into polar coordinates, evaluate the integral $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2+y^2)\,dx\,dy$. [05]

Section II

Q.4 A) Evaluate $\lim_{x\to 0}(x)^{\sin x}$ [05]

B) Trace the curve r = a (1+cosθ) , a> 0 [05]

C) Find by double integration, the area lying between the parabola y = 4x - x^2 and the line y = x. [05]

OR

C) Find volume of the sphere $x^2+y^2+z^2 = a^2$ by triple integration. [05]

Q.5 A) Find the value of $\sqrt{18}$ using Taylor's series. [05]

B) Find maximum and minimum value of the function f (x, y) =$2x^4+y^2-x^2-2y$. [05]

OR

A) Expand $3x^3+8x^2+x-2$ in powers of (x-3). [05]

B) Show that the plane 3x+12y-6z-17=0 touches the surface $3x^2-6y^2+9z^2+17=0$. [05]

Q.6 A) Expand $e^x\,\sin y$ in powers of x and y up to second order terms. [05]

B) Find the minimum value of $x^2+y^2+z^2$, given that ax+by+cz = p. [05]

OR

Q.6 A) Determine the radius and interval of convergence for the series $x+2x^2+3x^3+4x^4+\ldots\ldots$ [05]

B) If u = x+y and v = $\frac{x}{x+y}$, find $\frac{\partial(u,v)}{\partial(x,y)}$. [05]