Module 1

▶ **Step 1 :**

∵ States $Q_2$ and $Q_4$ are not reachable from start state they are removed.

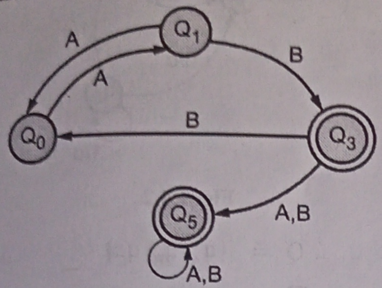


**Fig. Ex. 1.3.34(a)**

▶ **Step 2 and step 3 :**

| | | | |
|---|---|---|---|
| $Q_1$ | | | |
| $Q_3$ | 1 | 1 | |
| $Q_5$ | 1 | 1 | |
| | $Q_0$ | $Q_1$ | $Q_3$ |

▶ **Step 4 :**

Let $(Q_0, Q_1)$ be an unmarked cell

$$\delta(Q_0, A) = \frac{Q_1}{r} \qquad \delta(Q_1, A) = \frac{Q_0}{s}$$

List $(Q_1, Q_0) \rightarrow (Q_0, Q_1)$

$$\delta(Q_0, B) = \frac{Q_3}{r} \qquad \delta(Q_1, B) = \frac{Q_3}{r}$$

Then no decision can be taken

Let $(Q_3, Q_5)$ be an unmarked cell

$$\delta(Q_3, A) = \frac{Q_5}{r} \qquad \delta(Q_5, A) = \frac{Q_5}{r}$$

Then no decision can be taken

$$\delta(Q_3, B) = \frac{Q_5}{r} \qquad \delta(Q_5, B) = \frac{Q_5}{r}$$

Then no decision can be taken

▶ **Step 5 :**

δ :-

| $Q \setminus \Sigma$ | A | B |
|---|---|---|
| $\rightarrow Q_0$ | $Q_0$ | $Q_3$ |
| $\rightarrow Q_3^*$ | $Q_3$ | $Q_3$ |

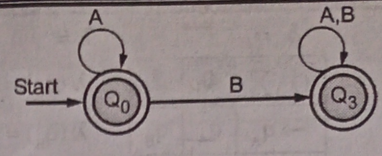


**Fig. Ex. 1.3.34(b)**

## ⏭ 1.4 MOORE MACHINE AND MEALY MACHINE

### 1.4.1 Moore Machine

**UQ.** Design Moore machine for binary adder.

**MU - Q. 1 (b) of May 15, Q. 2(b), Dec. 19, 10 Marks**

❑ **Definition :** It is a FA with no final state and it produces the o/p sequence for the given i/p sequence. In Moore m/c, the o/p symbol is associated with each state.

Moore machine is represented using six tupple representation and six tupple representation is given below :

$$M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$$

Where, $Q$ = finite set of states

$\Sigma$ = i/p alphabet

$\Delta$ = o/p alphabet

$\delta$ = transition function $\delta : Q \times \Sigma \rightarrow Q$

$\lambda$ = o/p mapping $\lambda : Q \rightarrow \Delta$

$q_0$ = start state $q_0$ E Q

e.g. :

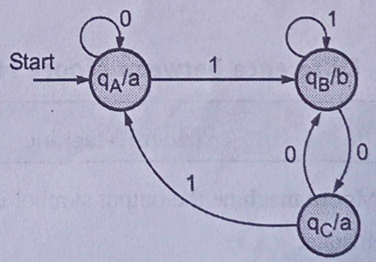


**Fig. 1.4.1**

$Q = \{q_A, q_B, q_C\}$ $\Delta = \{a, b\}$

$\Sigma = \{0, 1\}$ $q_0 = q_A$

$\delta$ =

| $Q \backslash \Sigma$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_A$ | $q_A$ | $q_B$ |
| $q_B$ | $q_C$ | $q_B$ |
| $q_C$ | $q_B$ | $q_A$ |

$\lambda$ =

$\lambda(q_A) = a$

$\lambda(q_B) = b$

$\lambda(q_C) = a$

**Working**

$\cap$      $\underline{n+1}$

$(q_A, 1101)$      abbaa

$+ (q_B, 101)$

$+ (q_B, 01)$

$+ (q_C, 1)$

$+ (q_A, \in)$

### 1.4.2 Mealy Machine

**Definition :** It is a FA with no final state and it produces the o/p sequence for the given i/p sequence. In mealy m/c, the o/p symbol is associated with each transition.

$$M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$$

Where,

$Q$ = finite set of states

$\Sigma$ = i/p alphabet

$\Delta$ = o/p alphabet

$\delta$ = transition function    $\delta : Q \times \Sigma \rightarrow Q$

$\lambda$ = o/p mapping    $\lambda : Q \times \Sigma \rightarrow \Delta$

$q_0$ = start state    $q_0$ E Q

e.g. :

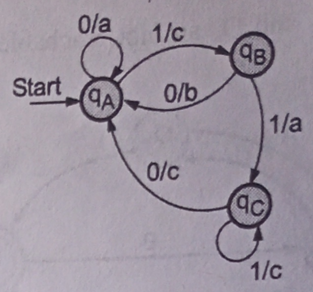


**Fig. 1.4.2**

$Q = \{q_A, q_B, q_C\}$    $\Delta = \{a, b, c\}$

$\Sigma = \{0, 1\}$    $q_0 = q_A$

$\delta$ :

| $Q \backslash \Sigma$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_A$ | $q_A$ | $q_B$ |
| $q_B$ | $q_A$ | $q_C$ |
| $q_C$ | $q_A$ | $q_C$ |

$\lambda$ =

| $Q / \Sigma$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_A$ | a | c |
| $q_B$ | b | a |
| $q_C$ | c | c |

**Working**

$\cap$      $\cap$

$(q_A, 1101)$      cacc

$(q_B, 101)$

$(q_C, 01)$

$(q_A, 1)$

$(q_B)$

### 1.4.3 Difference between Moore Machine and Mealy Machine

| Sr. No. | Moore Machine | Mealy Machine |
|---|---|---|
| 1. | In Moore machine the output symbol is associated with each state. | In Mealy machine the output symbol is associated with each transition. |
| 2. | In Moore M/c the output is dependent on the state. | In Mealy M/c the output is dependent on the state and input. |
| 3. | In Moore M/c. the output mapping is defined as $\lambda : Q \rightarrow \Delta$ | In Mealy M/c. the output mapping is defined as $\lambda : Q \times \Sigma \rightarrow \Delta$ |
| 4. | In Moore M/c if the length of input sequence is n then the length of the output sequence is n + 1 | In Mealy M/c if the length of input sequence is n then the length of the output sequence is also n. |

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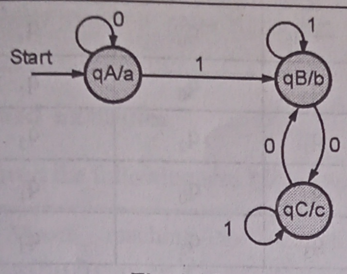
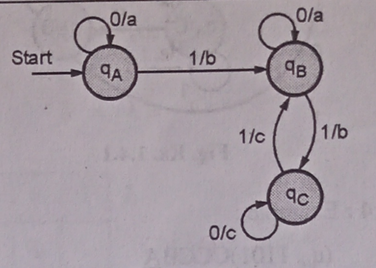
| Sr. No. | Moore Machine | Mealy Machine |
|---|---|---|
| 5. | In Moore M/c. we can get the output on $\in$. | In Mealy M/c. we cannot get the output on $\in$. |
| 6. | Example on Moore M/c. | Example on Mealy M/c. |
| |   Fig. 1.4.3 |   Fig. 1.4.4 |

## 1.4.4 Minimization of Moore Machine

States can be merged if

(1) All states have same transitions and

(2) Output symbols along with the states are also same.

## 1.4.4(A) Solved Examples on Moore Machine

**Examples : Convert the following into Moore**

**UEx. 1.4.1 MU - May 10, 5 Marks**

Design Moore machine to o/p. 'A' if i/p ends in "101", o/p 'B' if i/p ends in "110", otherwise o/p 'c' over $\Sigma = \{0, 1\}$

**Soln. :**

**Step 1 :** Theory

**Definition of Moore Machine**

*It is a FA with no final state and it produces the output sequence for the given input sequence.*

*In Moore machine, the output symbol is associated with each state.*

– Moore machine is represented using six tupple representation and it is defined as follows :

$$M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$$

where

$Q$ = Finite set of states

$\Sigma$ = Input alphabet

$\Delta$ = Output alphabet

$S$ = Transition function; $\delta : Q \times \Sigma \rightarrow Q$

$\lambda$ = Output mapping ; $\lambda : Q \rightarrow \Delta$

$q_0$ = Start state ; $q_0 \, E \, Q$

**Step 2 :** Logic

| | S\I | 0 | 1 | o/p |
|---|---|---|---|---|
| ends | $\rightarrow q_s$ | $q_0$ | $q_1$ | C |
| 0 | $q_0$ | $q_0$ | $q_1$ | C |
| 1 | $q_1$ | $q_2$ | $q_4$ | C |
| 10 | $q_2$ | $q_0$ | $q_3$ | C |
| 101 | $q_3$ | $q_2$ | $q_4$ | A |
| 11 | $q_4$ | $q_5$ | $q_4$ | C |
| 110 | $q_5$ | $q_0$ | $q_3$ | B |

$\rightarrow q_0$

**Step 3 :** Implementation

$$M = \{Q, \Sigma, \Delta, \delta, \lambda, q_0\}$$
$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$
$$\Sigma = \{0, 1\}$$
$$\Delta = \{A, B, C\}$$
$$q_0 = q_0$$

$\delta$ :

| S\I | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $q_0$ | $q_1$ |
| $q_1$ | $q_2$ | $q_4$ |
| $q_2$ | $q_0$ | $q_3$ |
| $q_3$ | $q_2$ | $q_4$ |
| $q_4$ | $q_5$ | $q_4$ |
| $q_5$ | $q_0$ | $q_3$ |

$\lambda$ :

$\lambda = (q_0) = C$

$\lambda = (q_1) = C$

$\lambda = (q_2) = C$

$\lambda = (q_3) = A$

$\lambda = (q_4) = C$

$\lambda = (q_5) = B$

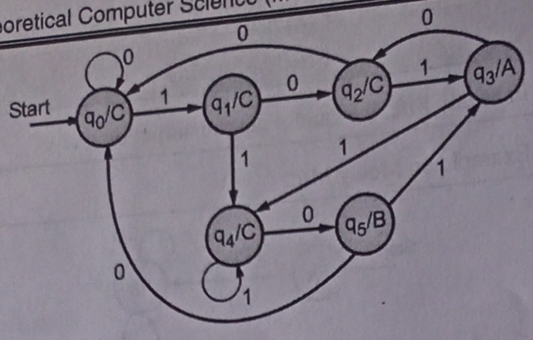



Fig. Ex. 1.4.1

▸ **Step 4 :** Example :

$(q_0, 1101)$CCCB$\underline{A}$

$\vdash (q_1, 101)$

$\vdash (q_4, 01)$

$\vdash (q_5, 1)$

$\vdash (q_3, \in)$

(Since last letter of the output is A sequence ends in 101)

**Ex. 1.4.2 :** Design Moore machine to print residue module 4 for binary numbers.

☑ **Soln. :**

▸ **Step 1 :** Define of Moore machine

**Definition of Moore Machine**

It is a FA with no final state and it produces the output sequence for the given input sequence.

In Moore machine, the output symbol is associated with each state.

– Moore machine is represented using six tupple representation and it is defined as follows :

$$M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$$

where

$Q$ = Finite set of states

$\Sigma$ = Input alphabet

$\Delta$ = Output alphabet

S = Transition function; $\delta : Q \times \Sigma \rightarrow Q$

$\lambda$ = Output mapping ; $\lambda : Q \rightarrow \Delta$

$q_0$ = Start state ; $q_0$ E Q

▸ **Step 2 :** Logic

| | | (2R + 0) mod 4 | (2R + 1) mod 4 | |
|---|---|---|---|---|
| | S\I | 0 | 1 | o/p |
| R | $\rightarrow q_s$ | $q_0$ | $q_1$ | 0 |
| 0 | $q_0$ | $q_0$ | $q_1$ | 0 |
| 1 | $q_1$ | $q_2$ | $q_3$ | 1 |
| 2 | $q_2$ | $q_0$ | $q_1$ | 2 |
| 3 | $q_3$ | $q_2$ | $q_3$ | 3 |

$q_0$

▸ **Step 3 :** Implementation

$$M = \{Q, \Sigma, \Delta, \delta, \lambda, q_0\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{0, 1\}$$

$$\Delta = \{0, 1, 2, 3\}$$

$$q_0 = q_0$$

$\delta$ =

| Q\Σ | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $q_0$ | $q_1$ |
| $q_1$ | $q_2$ | $q_3$ |
| $q_2$ | $q_0$ | $q_1$ |
| $q_3$ | $q_2$ | $q_3$ |

$\lambda$ =

$\lambda(q_0) = 0$

$\lambda(q_1) = 1$

$\lambda(q_2) = 2$

$\lambda(q_3) = 3$

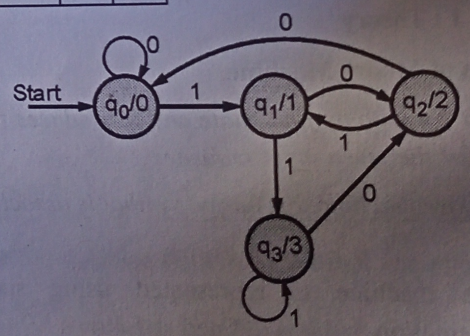



Fig. Ex. 1.4.2

▸ **Step 4 :** $(q_0, 1101)$ 0

$(q_1, 101)$ 1

$(q_3, 01)$ 3

$(q_2, 1)$ 2

$(q_1, \in)$ $\boxed{1}$