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**Subject : Analysis of Algorithms (AOA)
Assignment-04**

Assignment No-04

Q.1)

Find the Longest Common Subsequence (LCS) for given strings

 $X = ABACABB$: $Y = BABACAB$.

Soln:-

We have the following recursive formula to compute the length of an LCS of $A[1..i]$ & $B[1..j]$: Hence $c(i, j)$ gives the length of LCS of sub-problem $A[1..i]$ & $B[1..j]$.

$$c(i, j) = \begin{cases} 0 & \text{if } i=0 \text{ or } j=0 \\ c(i-1, j-1) + 1 & \text{if } i > 0 \text{ and } j > 0 \text{ and } a_i = b_j \\ \max\{c(i, j-1), c(i-1, j)\} & \text{if } i > 0 \text{ and } j > 0 \text{ and } a_i \neq b_j \end{cases}$$

- The given string $X = ABACABB$ and $Y = BABACAB$.

| | J | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|----|---|----------------|----------------|----------------|----------------|----------------|----------------|
| i | Yj | B | A | B | C | A | B | |
| 0 | X | 0 | 0 | 0 | 0 | 0 | 0 | |
| 1 | A | 0 | 0 [↑] | 1 [↑] | 1 | 1 | 1 [↑] | 1 |
| 2 | B | 0 | 1 [↑] | 1 [↑] | 2 [↑] | 2 | 2 | 2 [↑] |
| 3 | A | 0 | 1 [↑] | 1 [↑] | 2 [↑] | 2 [↑] | 3 [↑] | 2 |
| 4 | C | 0 | 1 [↑] | 1 [↑] | 2 [↑] | 3 [↑] | 3 [↑] | 3 |
| 5 | A | 0 | 1 [↑] | 1 [↑] | 2 [↑] | 3 [↑] | 4 [↑] | 4 [↑] |
| 6 | B | 0 | 1 [↑] | 2 [↑] | 2 [↑] | 3 [↑] | 4 [↑] | 5 [↑] |
| 7 | B | 0 | 1 [↑] | 2 [↑] | 3 [↑] | 3 [↑] | 4 [↑] | 5 [↑] |

$$c(0,0) = c(1,0) = c(2,0) = c(3,0) = c(4,0) = c(5,0) = c(6,0) = c(7,0) = 0$$

$$c(1,1) = \max\{c(1,0), c(0,1)\} = \max(0,0) = 0; s(1,1) = "\uparrow"$$

$$c(2,1) = c(1,0) + 1 = 1; s(2,1) = "\nwarrow"$$

$$c(3,1) = \max\{c(3,0), c(2,1)\} = \max(0,1) = 1; s(3,1) = "\uparrow"$$

$$c(4,1) = \max\{c(4,0), c(3,1)\} = \max(0,1) = 1; s(4,1) = "\uparrow"$$

$$c(5,1) = \max\{c(5,0), c(4,1)\} = \max(0,1) = 1; s(5,1) = "\uparrow"$$

$$c(6,1) = c(5,0) + 1 = 0 + 1 = 1; s(6,1) = "\nwarrow"$$

$$c(7,1) = c(6,0) + 1 = 0 + 1 = 1; s(7,1) = "\nwarrow"$$

$$c(1,2) = c(0,1) + 1 = 0 + 1 = 1; s(1,2) = "\nwarrow";$$

$$c(1,3) = \max\{c(1,2), c(0,3)\} = \max(1,0) = 1; s(1,3) = "\nwarrow"$$

$$c(1,4) = \max\{c(1,3), c(0,4)\} = \max(1,0) = 1; s(1,4) = "\nwarrow"$$

$$c(1,5) = c(0,4) + 1 = 0 + 1 = 1; s(1,5) = "\nwarrow"$$

$$c(1,6) = \max\{c(1,5), c(0,6)\} = \max(1,0) = 1; s(1,6) = "\nwarrow"$$

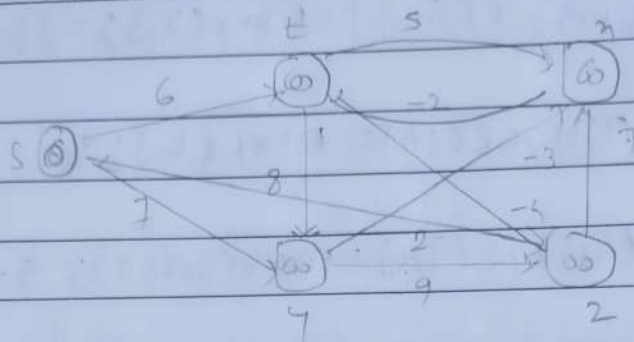
$$c(2,2) = c(1,1) + 1 = 0 + 1 = 1; s(2,2) = "\nwarrow"$$

$$c(3,2) = \max\{c(3,1), c(2,2)\} = \max(1,1) = 1; s(3,2) = "\uparrow"$$

$$c(4,2) = \max\{c(4,1), c(3,2)\} = \max(1,1) = 1; s(4,2) = "\uparrow"$$

Q.2)

find the single source shortest path using Bellman Ford algorithm for the given graph. Consider S as source vertex



→ (S, T), (S, X), (X, Y), (Y, Z), (T, Z)

(X, T) = -2, (T, Z) = -3, (Z, Y) = 2, (Y, X) = 7, (Z, S) = 7

1st iteration :

| distance | S | T | X | Y | Z |
|----------|---|---|---|---|---|
| path | 0 | 6 | ∞ | 7 | ∞ |
| | | 5 | | 5 | |

2nd iteration :-

| distance | S | T | X | Y | Z |
|----------|---|---|---|---|---|
| path | 0 | 2 | 4 | 7 | 2 |
| | | X | Y | Z | |

3rd iteration

| distance | S | T | X | Y | Z |
|----------|---|---|---|---|---|
| path | 0 | 6 | 4 | 7 | 2 |
| | | 3 | Y | S | T |

4th iteration

value of 2 is update until 4th iteration completely find to every node from so an code. Now we have do one more iteration to find whether there exist edge upon or not

Hence shortest path is

$(s, y) ; (y, x) ; (x, t) ; (t, z)$

Q.3) Let $n=4$

profit = $\{1, 2, 5, 6\}$

weight = $\{2, 3, 4, 5\}$

find the solution to 0/1 knapsack problem using Dynamic programming

→ $p_i = 1 \quad 2 \quad 5 \quad 6$
 $w_i = 2 \quad 3 \quad 4 \quad 5$

W →

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 1 | 2 | 2 | 3 | 3 | 3 | 3 |
| 3 | 0 | 0 | 1 | 2 | 5 | 5 | 6 | 7 | 7 |
| 4 | 0 | 0 | 1 | 2 | 0 | 6 | 6 | 7 | 8 |

$$\begin{aligned} \max(2+0, 1) \\ \max(2, 1) \\ = (2) \end{aligned}$$

$$\begin{aligned} \max(2+0, 1) \\ (2-1) \\ = 2 \end{aligned}$$

$$\begin{aligned} \max(2+1, 1) \\ (3, 1) \\ = 3 \end{aligned}$$

$$\max(6+0, 5)$$

$$r(6, 5)$$

$$= 5$$

$$\max(6+0, 5)$$

$$(6, 6)$$

$$= 6$$

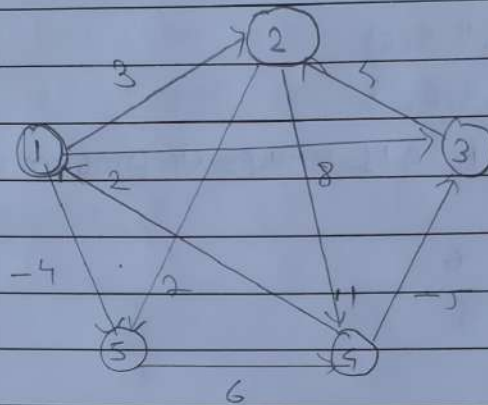
$$\max(6+1, 7)$$

$$(7)$$

Hence the max value for given solⁿ is

$$x_i(0, 0, 1).$$

Q.4) Find the All Shortest path for following Graph.



DD =

| | 1 | 2 | 3 | 4 | 5 |
|---|----------|----------|----------|----------|----------|
| 1 | 0 | 3 | 8 | ∞ | -4 |
| 2 | ∞ | 0 | ∞ | 1 | 7 |
| 3 | ∞ | 5 | 6 | ∞ | ∞ |
| 4 | ∞ | ∞ | -5 | 0 | ∞ |
| 5 | ∞ | ∞ | ∞ | 6 | 0 |

$\pi_0 =$

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | n | 1 | 1 | n | 1 |
| 2 | n | n | n | 2 | 2 |
| 3 | n | 3 | 3 | n | n |
| 4 | 4 | n | 4 | n | n |
| 5 | n | n | n | 5 | n |

D1

| | 1 | 2 | 3 | 4 | 5 |
|---|----------|----------|----------|----------|----------|
| 1 | 0 | 3 | ∞ | ∞ | -4 |
| 2 | ∞ | 0 | ∞ | 1 | 7 |
| 3 | ∞ | 5 | 0 | ∞ | ∞ |
| 4 | 2 | 5 | -5 | 0 | -2 |
| 5 | ∞ | ∞ | ∞ | 6 | 0 |

$$2,3 = \min(2,3 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow \infty, \infty + \infty))$$

$$(4,2) = \min \infty$$

II

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | N | 1 | 1 | N | 1 |
| 2 | N | N | N | 2 | 2 |
| 3 | N | 3 | 1 | N | N |
| 4 | 4 | 1 | 4 | N | 1 |
| 5 | N | N | N | 5 | N |

D2

| | 1 | 2 | 3 | 4 | 5 |
|---|----------|----------|----------|-----|------|
| 1 | 0 | 3 | ∞ | (3) | -4 |
| 2 | ∞ | 0 | ∞ | 1 | 7 |
| 3 | ∞ | 5 | 0 | (5) | (11) |
| 4 | 2 | 5 | -5 | 0 | -2 |
| 5 | ∞ | ∞ | ∞ | 6 | 0 |

Q5) π_2

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | h | 1 | 1 | 2 | 1 |
| 2 | n | n | n | 2 | 2 |
| 3 | n | 3 | h | 2 | 2 |
| 4 | h | 1 | h | n | 1 |
| 5 | n | h | n | 5 | n |

D_3

| | 1 | 2 | 3 | 4 | 5 |
|---|------------|----------|----------|---|----|
| 1 | 0 | 3 | 8 | h | -4 |
| 2 | ∞ | 0 | ∞ | 6 | 7 |
| 3 | ∞ | h | 0 | 5 | 11 |
| 4 | ∞ 2 | (-1) | 65 | 0 | -2 |
| 5 | ∞ | ∞ | ∞ | 6 | 0 |

π_3

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | n | 1 | 1 | 2 | 1 |
| 2 | n | n | n | 2 | 2 |
| 3 | h | 2 | n | 2 | 2 |
| 4 | h | 3 | n | 7 | 1 |
| 5 | n | n | n | 5 | n |

D_4

| | 1 | 2 | 3 | 4 | 5 |
|---|-----|-----|------|---|----|
| 1 | 0 | 3 | (-1) | h | -4 |
| 2 | (3) | 0 | (-3) | 1 | -1 |
| 3 | (7) | h | 0 | 5 | 3 |
| 4 | 2 | -1 | -5 | 0 | -2 |
| 5 | (8) | (5) | 1 | 6 | 0 |

3, 1, 2, 3, 1, 3

π_4

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | n | 1 | 4 | 2 | 1 |
| 2 | 4 | n | n | 2 | 2 |
| 3 | 4 | 3 | n | 2 | 2 |
| 4 | 4 | 3 | n | 7 | 1 |
| 5 | 4 | n | n | 5 | n |

 D_4

| | 1 | 2 | 3 | 4 | 5 |
|---|-------------|-------------|-------------|---|----|
| 1 | 0 | 3 | $\ominus 1$ | 4 | -4 |
| 2 | $\ominus 3$ | 0 | $\ominus 3$ | 1 | -1 |
| 3 | $\ominus 7$ | 4 | $\ominus 1$ | 5 | 3 |
| 4 | 2 | -1 | -5 | 0 | -2 |
| 5 | $\ominus 8$ | $\ominus 5$ | 1 | 6 | 0 |

 $\{1, 2, 3, 4\}$ π_4

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | n | 1 | 4 | 2 | 1 |
| 2 | 4 | n | 4 | 2 | 1 |
| 3 | 4 | 3 | n | 2 | 1 |
| 4 | 4 | 3 | 4 | n | 1 |
| 5 | 4 | 3 | 4 | 5 | n |

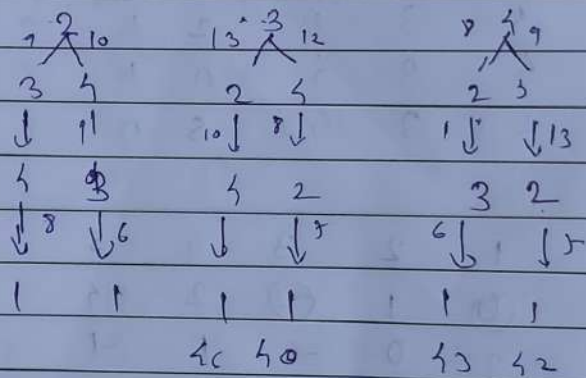
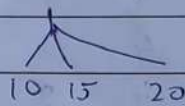
 D_5

| | 1 | 2 | 3 | 4 | 5 |
|---|-------------|----|-------------|---|----|
| 1 | $\ominus 1$ | 1 | $\ominus 2$ | 2 | -4 |
| 2 | 3 | 0 | -4 | 1 | -1 |
| 3 | 7 | 4 | 0 | 5 | 3 |
| 4 | 2 | -1 | -5 | 0 | -2 |
| 5 | 6 | 5 | 1 | 6 | 0 |

| π_s | 1 | 2 | 3 | 4 | 5 |
|---------|---|---|---|---|---|
| 1 | M | 3 | 4 | 5 | 1 |
| 2 | 4 | 1 | 5 | 2 | 1 |
| 3 | 4 | 3 | 3 | 2 | 1 |
| 4 | 4 | 3 | 4 | 1 | 1 |
| 5 | 4 | 3 | 5 | 5 | M |

Q-5) Find the shortest distance using traveling salesman problem for the given graph consider source vertex is 1

| | 2 | 2 | 3 | 4 |
|-----|---|----|----|----|
| → 1 | 0 | 10 | 15 | 20 |
| 2 | 5 | 0 | 9 | 10 |
| 3 | 6 | 13 | 0 | 12 |
| 4 | 8 | 8 | 9 | 0 |



Shortest distance is

