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Project: 24 Kensington Road Residential

Client: Dream Home Development Ltd

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Design by: ABC	Check by:

Analysis of continuous beam by force method

Input data

Span lengths -
$$\vec{l} = [4; 7; 3; 5] \cdot m = [4 \text{ m } 7 \text{ m } 3 \text{ m } 5 \text{ m}]$$

Number of spans -
$$n = \operatorname{len}(\vec{l}) = 4$$

Coordinates of supports -
$$\vec{x} = [4 \text{ m } 11 \text{ m } 14 \text{ m } 19 \text{ m}]$$

Total beam length -
$$\vec{L} = \vec{x}_4 = 19 \,\mathrm{m}$$

Load -
$$q = 10 \frac{\text{kN}}{\text{m}}$$

Material

Elastic modulus - E = 30GPa

Poisson's ratio -
$$v = 0.2$$

Shear modulus -
$$G = \frac{E}{2 \cdot (1 + v)} = \frac{30 \text{ GPa}}{2 \cdot (1 + 0.2)} = 12.5 \text{ GPa}$$

Cross section

Rectangular with dimensions: b = 250 mm, h = 500 mm

Area -
$$A = b \cdot h = 250 \,\text{mm} \cdot 500 \,\text{mm} = 125000 \,\text{mm}^2$$

Moment of inertia -
$$I = \frac{b \cdot h^3}{12} = \frac{250 \text{ mm} \cdot (500 \text{ mm})^3}{12} = 2604166667 \text{ mm}^4$$

Shear area -
$$A_Q = \frac{5}{6} \cdot b \cdot h = \frac{5}{6} \cdot 250 \text{ mm} \cdot 500 \text{ mm} = 104167 \text{ mm}^2$$

Solution

The solution will be performed by the force method with a primary system - simply supported beam with internal supports removed and replaced by unknown forces X_i

Bending moments

- in section a due to unit force at distance x from the beginning of the beam:

$$M_{1,\max}(x) = \left(\frac{x}{L} - 1\right) \cdot x$$

$$M_{1,a}(a; x) = M_{1,max}(x) \cdot \begin{cases} \text{if } a < x : \frac{a}{x} \\ \text{else: } \frac{L - a}{L - x} \end{cases}$$

- in section a, due to unit force X_i:

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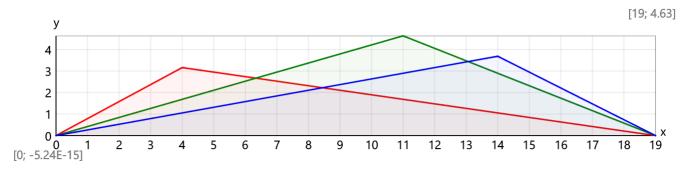
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$$M_1(a; i) = M_{1,a}(a; \vec{x_i})$$



- due to external loads in primary system:

$$M_{\mathsf{F}}(x) = \frac{q \cdot x}{2} \cdot (L - x)$$



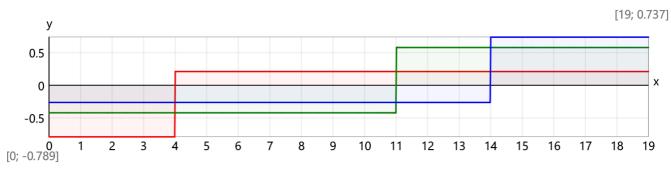
Shear forces

- in section a due to unit force at distance x from the beginning of the beam:

$$V_{1,a}(a; x) = \begin{cases} \text{if } a < x : \frac{x}{L} - 1 \\ \text{else: } \frac{x}{L} \end{cases}$$

- in section a, due to unit force X_i :

$$V_1(a; i) = V_{1,a}(a; \vec{x_i})$$



- due to external loads in primary system:

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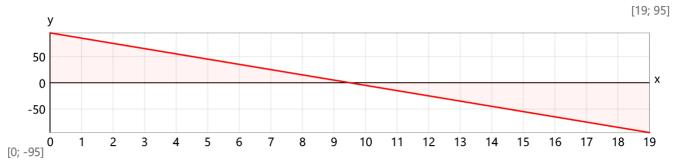
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$$V_{\mathsf{F}}(x) = q \cdot \left(\frac{L}{2} - x\right)$$



Number of unknowns by force method - $n_1 = n - 1 = 4 - 1 = 3$

Flexibility coefficients

$$\delta(i;j) = \int_{0 \text{ m}}^{L} \frac{M_1(x;i) \cdot M_1(x;j)}{E \cdot I} dx + \int_{0 \text{ m}}^{L} \frac{V_1(x;i) \cdot V_1(x;j)}{G \cdot A_Q} dx$$

$$\Delta_{\mathsf{F}}(i) = \int_{0\,\mathrm{m}}^{L} \frac{M_{\mathsf{1}}(x;\,i) \cdot M_{\mathsf{F}}(x)}{E \cdot I} \,\mathrm{d}x + \int_{0\,\mathrm{m}}^{L} \frac{V_{\mathsf{1}}(x;\,i) \cdot V_{\mathsf{F}}(x)}{G \cdot A_{\mathsf{Q}}} \,\mathrm{d}x$$

$$\delta = \mathbf{symmetric}(n_1) = \mathbf{symmetric}(3) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \vec{A}_F = \mathbf{vector}(n_1) = \mathbf{vector}(3) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Pext{Repeat} \{ \delta_{i,j} = \delta(i;j); i = 1...n_1 \}; j = 1...n_1 \} = 0.0011 \text{ m/kN}$$

$$Percent{\vec{A}_{F,i} = \Delta_F(i); i = 1...n_1} = -0.161 \text{ m}$$

$$\delta = \begin{bmatrix} 0.000811 \text{ m/kN } 0.00101 \text{ m/kN } 0.000719 \text{ m/kN} \\ 0.00101 \text{ m/kN } 0.00174 \text{ m/kN } 0.00133 \text{ m/kN} \\ 0.000719 \text{ m/kN } 0.00133 \text{ m/kN } 0.0011 \text{ m/kN} \end{bmatrix}, \vec{J}_{\mathsf{F}} = [-0.135 \text{ m } -0.211 \text{ m } -0.161 \text{ m}]$$

Calculation of the unknown forces X_i

$$\vec{X} = -\text{clsolve}(\delta; \vec{\Delta}_{\text{F}}) = [64.86 \,\text{kN} \, 53.48 \,\text{kN} \, 39.33 \,\text{kN}]$$

Results

Bending moment diagram -
$$M(x) = M_F(x) + \sum_{i=1}^{n_1} M_1(\vec{x}; i) \cdot X_i$$

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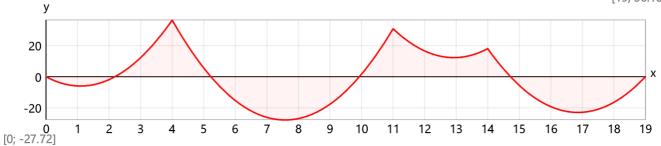
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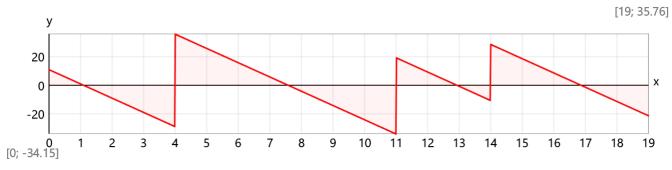
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$$\vec{M}_{\text{max}} = [5.97 \,\text{kNm} \, 27.74 \,\text{kNm} \, -12.23 \,\text{kNm} \, 22.9 \,\text{kNm}]$$

$$\vec{M}_{min} = [-36.29 \,\text{kNm} - 30.79 \,\text{kNm} - 17.99 \,\text{kNm}]$$

Shear force diagram -
$$V(x) = V_F(x) + \sum_{i=1}^{n_1} V_1(\vec{x}; i) \cdot \vec{X}_i$$



$$\vec{V}_{\text{max}} = [10.93 \,\text{kN} \, 35.79 \,\text{kN} \, 19.26 \,\text{kN} \, 28.6 \,\text{kN}]$$

$$\vec{V}_{min} = [-29.07 \,\text{kN} - 34.21 \,\text{kN} - 10.74 \,\text{kN} - 21.4 \,\text{kN}]$$

Deflections

- in section a, due to unit force X_i:

$$d_{1}(a; i) = \int_{0m}^{L} \frac{M_{1,a}(x; a) \cdot M_{1}(x; i)}{E \cdot I} dx + \int_{0m}^{L} \frac{V_{1,a}(x; a) \cdot V_{1}(x; i)}{G \cdot A_{Q}} dx$$

- due to external loads in primary system:

$$d_{\mathsf{F}}(a) = \int_{0,\mathrm{m}}^{L} \frac{M_{\mathsf{1,a}}(x;a) \cdot M_{\mathsf{F}}(x)}{E \cdot I} \, \mathrm{d}x + \int_{0,\mathrm{m}}^{L} \frac{V_{\mathsf{1,a}}(x;a) \cdot V_{\mathsf{F}}(x)}{G \cdot A_{\mathsf{Q}}} \, \mathrm{d}x$$

$$d(x) = d_{\mathsf{F}}(x) + \sum_{i=1}^{n_1} d_{\mathsf{1}}(\vec{x}; i) \cdot \vec{X}_i$$

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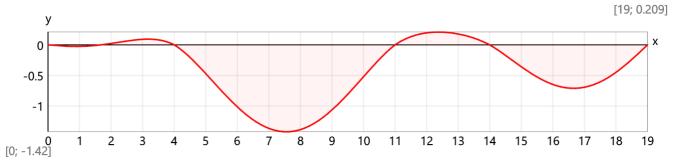
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Maximum deflection - $d_{\text{max}} = \frac{\inf\{d(x); x \in [0\text{m}; L]\}}{1 - 1.42 \text{mm}}$

At a distance from the origin - $x_{inf} = 7.56 \, m$