

Analysis of multi-story multi-bay RC plane frame

Number of stories - $n_{st} = 5$, Number of bays - $n_b = 3$

Story height - $h_{st} = 2.85$ m, Bay length- $l_b = 4$ m

Slab thickness - $h_{pl} = 18$ cm

Cross section of columns - $b_c = 25$ cm , $h_c = 60$ cm

Cross section of beams - $b_b = 25$ cm , $h_b = 40$ cm

Joint coordinates - $n_j = 24$

$\vec{x}_j = [0 \ 4 \ 8 \ 12 \ 0 \ 4 \ 8 \ 12 \ 0 \ 4 \ \dots 12]$ m, $\vec{y}_j = [0 \ 0 \ 0 \ 0 \ 2.85 \ 2.85 \ 2.85 \ 2.85 \ 5.7 \ 5.7 \ \dots 14.25]$ m

Elements - [J1; J2] - $n_E = 35$

$\text{transp}(e_j) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & \dots & 23 \\ 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & \dots & 24 \end{bmatrix}$

Element endpoint coordinates

$x_1(e) = \vec{x}_{j_{e_j,1}}$, $y_1(e) = \vec{y}_{j_{e_j,1}}$, $x_2(e) = \vec{x}_{j_{e_j,2}}$, $y_2(e) = \vec{y}_{j_{e_j,2}}$

Element length - $l(e) = \sqrt{(x_2(e) - x_1(e))^2 + (y_2(e) - y_1(e))^2}$

Element direction - $c(e) = \frac{x_2(e) - x_1(e)}{l(e)}$, $s(e) = \frac{y_2(e) - y_1(e)}{l(e)}$

Transformation matrix

Diagonal 3x3 block - $t(e) = \text{hp}([c(e); s(e); 0 \mid -s(e); c(e); 0 \mid 0; 0; 1])$

Generation of the full transformation matrix

$T(e) = \text{add}(t(e); \text{add}(t(e); \text{matrix}(6; 6); 1; 1); 4; 4)$

Supports - $n_c = 4$

$c = \begin{bmatrix} 1 & 10^{20} \text{ kN/m} & 10^{20} \text{ kN/m} & 0 \text{ kNm} \\ 2 & 10^{20} \text{ kN/m} & 10^{20} \text{ kN/m} & 0 \text{ kNm} \\ 3 & 10^{20} \text{ kN/m} & 10^{20} \text{ kN/m} & 0 \text{ kNm} \\ 4 & 10^{20} \text{ kN/m} & 10^{20} \text{ kN/m} & 0 \text{ kNm} \end{bmatrix}$

Unit weights of building materials

- concrete - $\gamma_c = 25 \text{ kN/m}^3$

- screed - $\gamma_{scr} = 21 \text{ kN/m}^3$

- finishes - $\gamma_{fin} = 18 \text{ kN/m}^3$

- brickwork - $\gamma_{bw} = 16 \text{ kN/m}^3$

- plaster/render - $\gamma_{pla} = 16 \text{ kN/m}^3$

- insulation - $\gamma_{ins} = 0.5 \text{ kN/m}^3$

Loads

Total halfwidth of adjacent plate spans - $a = \frac{5\text{ m}}{2} = 2.5\text{ m}$

Self weight

$$\text{Plate} - g_{pl} = h_{pl} \cdot a \cdot \gamma_c = 11.25\text{ kN/m}$$

$$\text{Beam} - g_b = b_b \cdot (h_b - h_{pl}) \cdot \gamma_c = 1.38\text{ kN/m}$$

$$\text{Total for beam} - sw = g_{pl} + g_b = 12.62\text{ kN/m}$$

$$\text{Column} - g_c = b_c \cdot h_c \cdot \gamma_c = 3.75\text{ kN/m}$$

Dead loads

$$\text{Screed} - g_{scr} = 8\text{ cm} \cdot a \cdot \gamma_{scr} = 4.2\text{ kN/m}$$

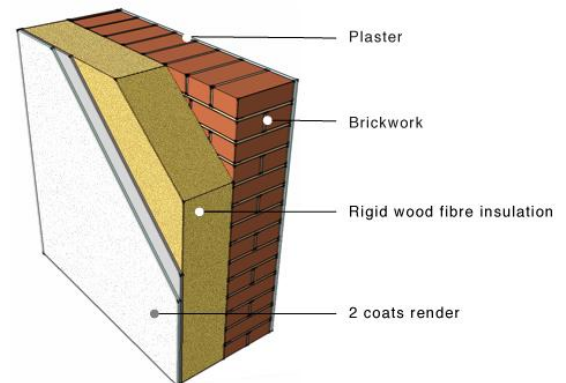
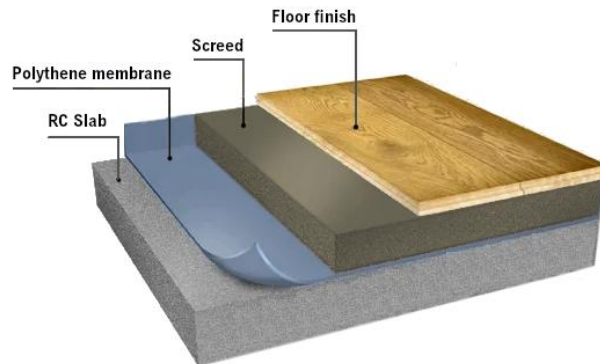
$$\text{Finishes} - g_{fin} = 2\text{ cm} \cdot a \cdot \gamma_{fin} = 0.9\text{ kN/m}$$

$$\text{Plaster ceiling} - g_{pls} = 2\text{ cm} \cdot a \cdot \gamma_{pla} = 0.8\text{ kN/m}$$

$$\text{Brick wall} - g_{bw} = 25\text{ cm} \cdot (h_{st} - h_b) \cdot \gamma_{bw} = 9.8\text{ kN/m}$$

$$\text{Wall insulation} - g_{ins} = 15\text{ cm} \cdot h_{st} \cdot \gamma_{ins} = 0.214\text{ kN/m}$$

$$\text{Wall plaster/render} - g_{plw} = 2 \cdot 2\text{ cm} \cdot h_{st} \cdot \gamma_{pla} = 1.82\text{ kN/m}$$



Total dead load

$$dl = g_{scr} + g_{fin} + g_{pls} + g_{bw} + g_{ins} + g_{plw} = 17.74\text{ kN/m}$$

$$\text{Live load} - ll = a \cdot 2 \frac{\text{kN}}{\text{m}^2} = 5\text{ kN/m}$$

Total load

$$\text{On beams} - p_b = (sw + dl) \cdot 1.35 + ll \cdot 1.5 = 48.49\text{ kN/m}$$

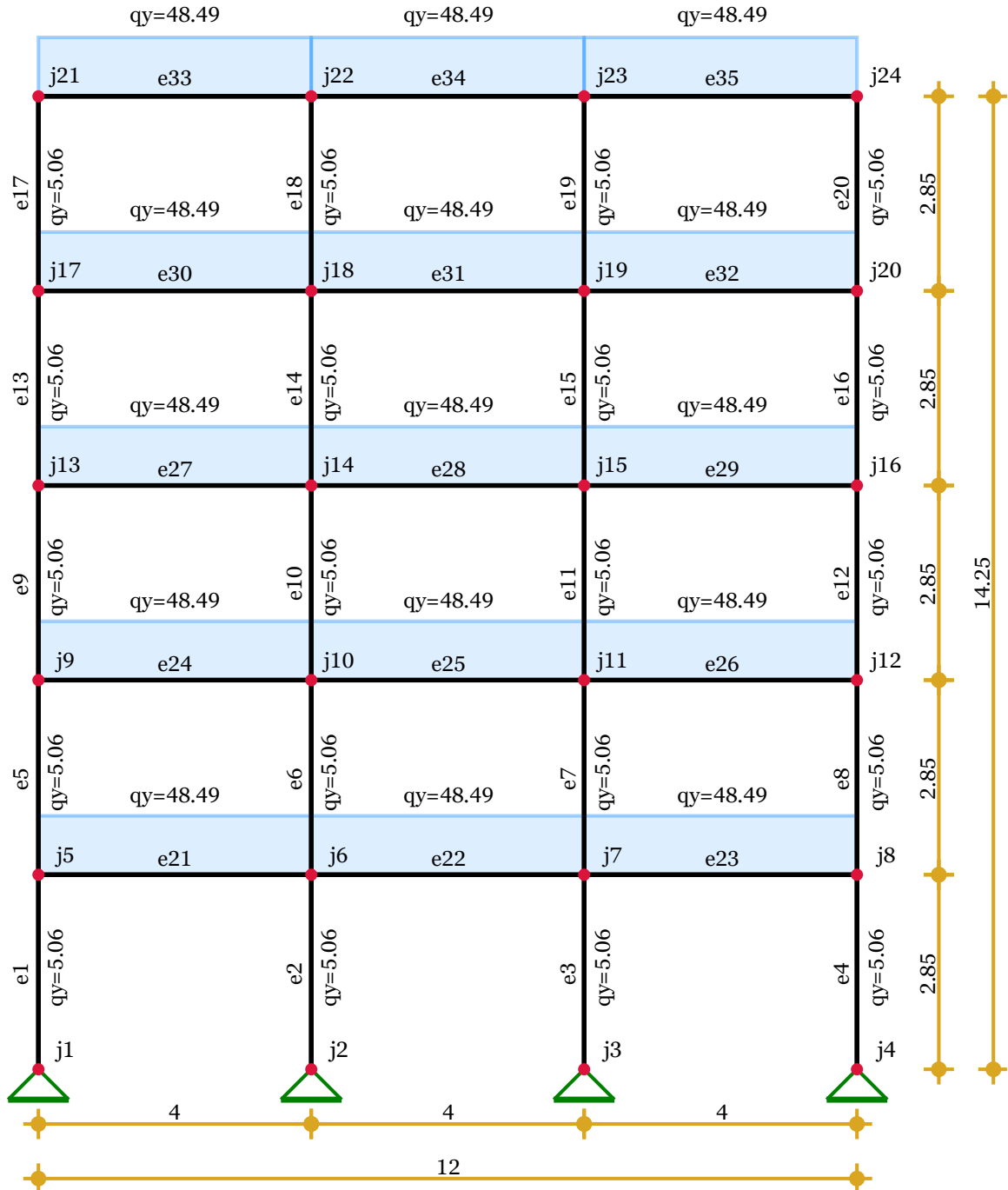
$$\text{On columns} - p_c = g_c \cdot 1.35 = 5.06\text{ kN/m}$$

Load values on elements

$$\vec{q}_x = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 0]\text{ kN/m}$$

$$\vec{q}_y = [-5.06 \ -5.06 \ -5.06 \ -5.06 \ -5.06 \ -5.06 \ -5.06 \ -5.06 \ -5.06 \ -5.06 \ \dots \ -48.49]\text{ kN/m}$$

Scheme of the structure



Materials

Modules of elasticity - $\vec{E} = \mathbf{hp}([35 \text{ GPa}]) = [35] \text{ GPa}$

Poisson coefficients - $\vec{\nu} = \mathbf{hp}([0.2]) = [0.2]$

Shear modules - $\vec{G} = \frac{\vec{E}}{2 \cdot (1 + \vec{\nu})} = [14.58] \text{ GPa}$

Assignments on elements

$$\vec{e}_M = \text{fill}(\text{vector}_{hp}(n_E); 1) = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ \dots \ 1]$$

Cross sections

Calculation of effective flange width

$$l_0 = 0.85 \cdot l_b = 3.4 \text{ m}$$

$$b_{eff} = b_b + \min(0.2 \cdot a + 0.1 \cdot l_0; 0.2 \cdot l_0) = 93 \text{ cm}$$

Section 1 - $\vec{b}_1 = b_c = 25 \text{ cm}$, $\vec{h}_1 = h_c = 60 \text{ cm}$ - columns

Section 2 - $\vec{b}_2 = b_b = 25 \text{ cm}$, $\vec{h}_2 = h_b = 40 \text{ cm}$ - beams

$$\vec{b}_f = b_{eff} = 93 \text{ cm}, \vec{h}_f = h_{pl} = 18 \text{ cm}$$

Cross section properties

Area

$$\text{Web} - \vec{A}_w = \vec{b} \odot \vec{h} = [1500 \ 1000] \text{ cm}^2$$

$$\text{Flange} - \vec{A}_f = (\vec{b}_f - \vec{b}) \odot \vec{h}_f = [0 \ 1224] \text{ cm}^2$$

$$\text{Total} - \vec{A} = \vec{A}_w + \vec{A}_f = [1500 \ 2224] \text{ cm}^2$$

$$\text{First moment of area} - \vec{S} = \frac{\vec{A}_w \odot \vec{h}}{2} + \vec{A}_f \odot \left(\vec{h} - \frac{\vec{h}_f}{2} \right) = [45000 \ 57944] \text{ cm}^3$$

$$\text{Geometrical center} - \vec{z}_c = \frac{\vec{S}}{\vec{A}} = [300 \ 260.54] \text{ mm}$$

Second moment of area

$$\text{Web} - \vec{I}_w = \vec{A}_w \odot \left(\frac{\vec{h} \odot^2}{12} + \left(\vec{z}_c - \frac{\vec{h}}{2} \right) \odot^2 \right) = [450000 \ 169984] \text{ cm}^4$$

$$\text{Flange} - \vec{I}_f = \vec{A}_f \odot \left(\frac{\vec{h}_f \odot^2}{12} + \left(\vec{h} - \vec{z}_c - \frac{\vec{h}_f}{2} \right) \odot^2 \right) = [0 \ 62991.1] \text{ cm}^4$$

$$\text{Total} - \vec{I} = \vec{I}_w + \vec{I}_f = [450000 \ 232975] \text{ cm}^4$$

$$\text{Shear area} - \vec{A}_s = \frac{\vec{A}}{1.2} = [1250 \ 1853.33] \text{ cm}^2$$

Assignment on elements

$$\text{Columns} - \vec{e}_{SC} = \text{fill}(\text{vector}_{hp}(n_{CE}); 1) = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ \dots \ 1]$$

$$\text{Beams} - \vec{e}_{SB} = \text{fill}(\text{vector}_{hp}(n_{BE}); 2) = [2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ \dots \ 2]$$

$$\text{All} - \vec{e}_S = \text{hp}([\vec{e}_{SC}; \vec{e}_{SB}]) = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ \dots \ 2]$$

Element stiffness matrix

Elastic properties for element "e"

$$EA(e) = \vec{E}_{e_{M,e}} \cdot \vec{A}_{e_{S,e}}, \quad EI(e) = \vec{E}_{e_{M,e}} \cdot \vec{I}_{e_{S,e}}, \quad GA_s(e) = \vec{G}_{e_{M,e}} \cdot \vec{A}_{e_{S,e}}$$

$$k_s(e) = \frac{12 \cdot EI(e)}{GA_s(e) \cdot l(e)^2}, \quad \alpha(e) = \frac{EA(e)}{l(e)}, \quad \beta(e) = \frac{EI(e)}{l(e)^3 \cdot (1 + k_s(e))}$$

Stiffness matrix coefficients for element "e"

$$k_{11}(e) = \alpha(e) \cdot \frac{m}{kN}, \quad k_{22}(e) = 12 \cdot \beta(e) \cdot \frac{m}{kN}, \quad k_{23}(e) = 6 \cdot \beta(e) \cdot l(e) \cdot \frac{1}{kN}$$

$$k_{33}(e) = (4 + k_s(e)) \cdot \beta(e) \cdot l(e)^2 \cdot \frac{1}{kNm}, \quad k_{36}(e) = (2 - k_s(e)) \cdot \beta(e) \cdot l(e)^2 \cdot \frac{1}{kNm}$$

Assembling the 3x3 stiffness matrix blocks for element "e"

$$k_{ii}(e) = \mathbf{hp}([k_{11}(e) \mid 0; k_{22}(e); k_{23}(e) \mid 0; k_{23}(e); k_{33}(e)])$$

$$k_{ij}(e) = \mathbf{hp}([-k_{11}(e) \mid 0; -k_{22}(e); k_{23}(e) \mid 0; -k_{23}(e); k_{36}(e)])$$

$$k_{ji}(e) = \mathbf{transp}(k_{ij}(e))$$

$$k_{jj}(e) = \mathbf{hp}([k_{11}(e) \mid 0; k_{22}(e); -k_{23}(e) \mid 0; -k_{23}(e); k_{33}(e)])$$

Full element stiffness matrix

$$k_E(e) = \mathbf{stack}\left(\mathbf{augment}(k_{ii}(e); k_{ij}(e)); \mathbf{augment}(k_{ji}(e); k_{jj}(e))\right)$$

Stiffness matrices obtained in local coordinates

$$k_E(1) = \begin{bmatrix} 1842105 & 0 & 0 & -1842105 & 0 & 0 \\ 0 & 72402.7 & 103174 & 0 & -72402.7 & 103174 \\ 0 & 103174 & 202286 & 0 & -103174 & 91759.5 \\ -1842105 & 0 & 0 & 1842105 & 0 & 0 \\ 0 & -72402.7 & -103174 & 0 & 72402.7 & -103174 \\ 0 & 103174 & 91759.5 & 0 & -103174 & 202286 \end{bmatrix}$$

$$n_{b1} = n_{CE} + 1 = 21$$

$$k_E(n_{b1}) = \begin{bmatrix} 1946000 & 0 & 0 & -1946000 & 0 & 0 \\ 0 & 14950.7 & 29901.4 & 0 & -14950.7 & 29901.4 \\ 0 & 29901.4 & 80188 & 0 & -29901.4 & 39417.4 \\ -1946000 & 0 & 0 & 1946000 & 0 & 0 \\ 0 & -14950.7 & -29901.4 & 0 & 14950.7 & -29901.4 \\ 0 & 29901.4 & 39417.4 & 0 & -29901.4 & 80188 \end{bmatrix}$$

Stiffness matrices obtained in global coordinates

$$\mathbf{transp}(T(1)) \cdot k_E(1) \cdot T(1) = \begin{bmatrix} 72402.7 & 0 & -103174 & -72402.7 & 0 & -103174 \\ 0 & 1842105 & 0 & 0 & -1842105 & 0 \\ -103174 & 0 & 202286 & 103174 & 0 & 91759.5 \\ -72402.7 & 0 & 103174 & 72402.7 & 0 & 103174 \\ 0 & -1842105 & 0 & 0 & 1842105 & 0 \\ -103174 & 0 & 91759.5 & 103174 & 0 & 202286 \end{bmatrix}$$

$$\text{transp}(T(n_{b1})) \cdot k_E(n_{b1}) \cdot T(n_{b1}) = \begin{bmatrix} 1946000 & 0 & 0 & -1946000 & 0 & 0 \\ 0 & 14950.7 & 29901.4 & 0 & -14950.7 & 29901.4 \\ 0 & 29901.4 & 80188 & 0 & -29901.4 & 39417.4 \\ -1946000 & 0 & 0 & 1946000 & 0 & 0 \\ 0 & -14950.7 & -29901.4 & 0 & 14950.7 & -29901.4 \\ 0 & 29901.4 & 39417.4 & 0 & -29901.4 & 80188 \end{bmatrix}$$

Global stiffness matrix

$$K = \begin{bmatrix} 10^{20} & 0 & -103174 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 10^{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ -103174 & 0 & 202286 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 10^{20} & 0 & -103174 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 10^{20} & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & -103174 & 0 & 202286 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10^{20} & 0 & -103174 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^{20} & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -103174 & 0 & 202286 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^{20} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 282474 \end{bmatrix}$$

Element load vector

Lateral load in local CS - $q_E(e) = -\vec{q}_{x,e} \cdot s(e) + \vec{q}_{y,e} \cdot c(e)$

Axial load in local CS - $n_E(e) = \vec{q}_{x,e} \cdot c(e) + \vec{q}_{y,e} \cdot s(e)$

Equivalent loads at element endpoints

$$F_{Ex}(e) = \frac{\vec{q}_{x,e} \cdot l(e)}{2} \cdot \frac{1}{kN}, F_{Ey}(e) = \frac{\vec{q}_{y,e} \cdot l(e)}{2} \cdot \frac{1}{kN}, M_E(e) = \frac{q_E(e) \cdot l(e)^2}{12} \cdot \frac{1}{kNm}$$

Load vector - $F_E(e) = \text{hp}([F_{Ex}(e); F_{Ey}(e); M_E(e); F_{Ex}(e); F_{Ey}(e); -M_E(e)])$

Global load vector

$$\vec{F} = [0 \ -7.21 \ 0 \ 0 \ -7.21 \ 0 \ 0 \ -7.21 \ 0 \ 0 \ \dots \ 64.65]$$

Results

Solution of the system of equations by PCG method

$$\vec{Z} = \text{solve}(K; \vec{F})$$

$$= [0 \ -5.72 \times 10^{-18} \ 7.06 \times 10^{-5} \ 0 \ -1.03 \times 10^{-17} \ 3.03 \times 10^{-6} \ 0 \ -1.03 \times 10^{-17} \ -3.03 \times 10^{-6} \ 0 \ \dots \ 0.000283]$$

Joint displacements

$$z_j(j) = \text{slice}(\vec{Z}; 3 \cdot j - 2; 3 \cdot j) \quad z(j) = \text{round}\left(\frac{z_j(j)}{\delta z}\right) \cdot \delta z \cdot 1000 \cdot [mm; mm; 1]$$

Support reactions

$$r(i) = \text{row}(c; i), j(i) = \text{take}(1; r(i))$$

$$R(i) = -z_j(j(i)) \cdot [m; m; 1] \cdot \text{last}(r(i); 3)$$

Joint **J1** - [8.19 kN 571.77 kN 0 kNm]

Joint **J2** - [0.174 kN 1027.2 kN 0 kNm]

Joint **J3** - [-0.174 kN 1027.2 kN 0 kNm]

Joint **J4** - [-8.19 kN 571.77 kN 0 kNm]

Element end forces

$$z_E(e) = \text{hp} \left(\left[z_J(e_{J_{e,1}}); z_J(e_{J_{e,2}}) \right] \right)$$

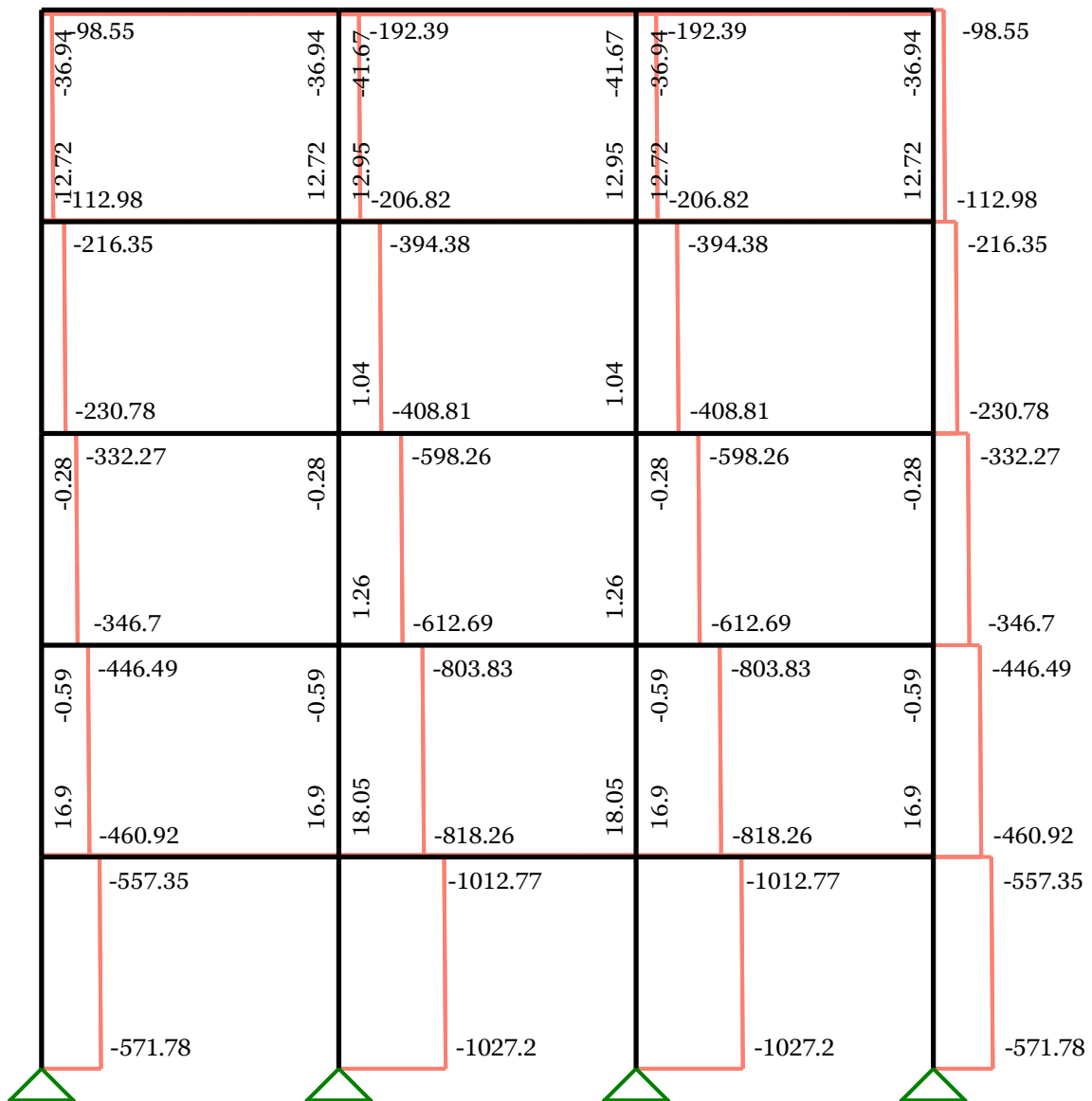
$$R_E(e) = \text{col}(k_E(e) \cdot T(e) \cdot z_E(e) - T(e) \cdot F_E(e); 1) \cdot [1; 1; m; 1; 1; m] \cdot kN$$

Element internal forces

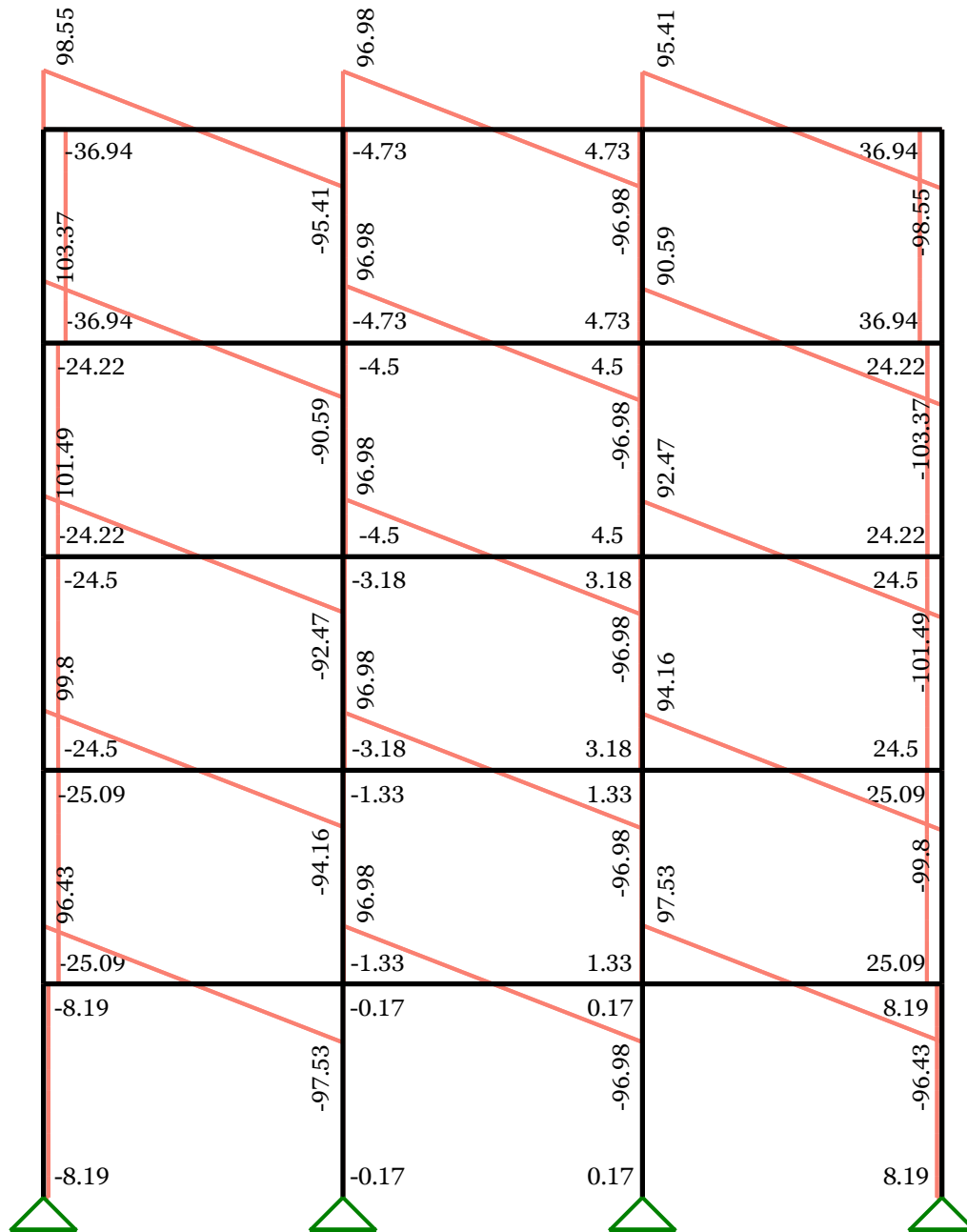
$$N(e; x) = -\text{take}(1; R_E(e)) - n_E(e) \cdot x, \quad Q(e; x) = \text{take}(2; R_E(e)) + q_E(e) \cdot x$$

$$M(e; x) = -\text{take}(3; R_E(e)) + \text{take}(2; R_E(e)) \cdot x + \frac{q_E(e) \cdot x^2}{2}$$

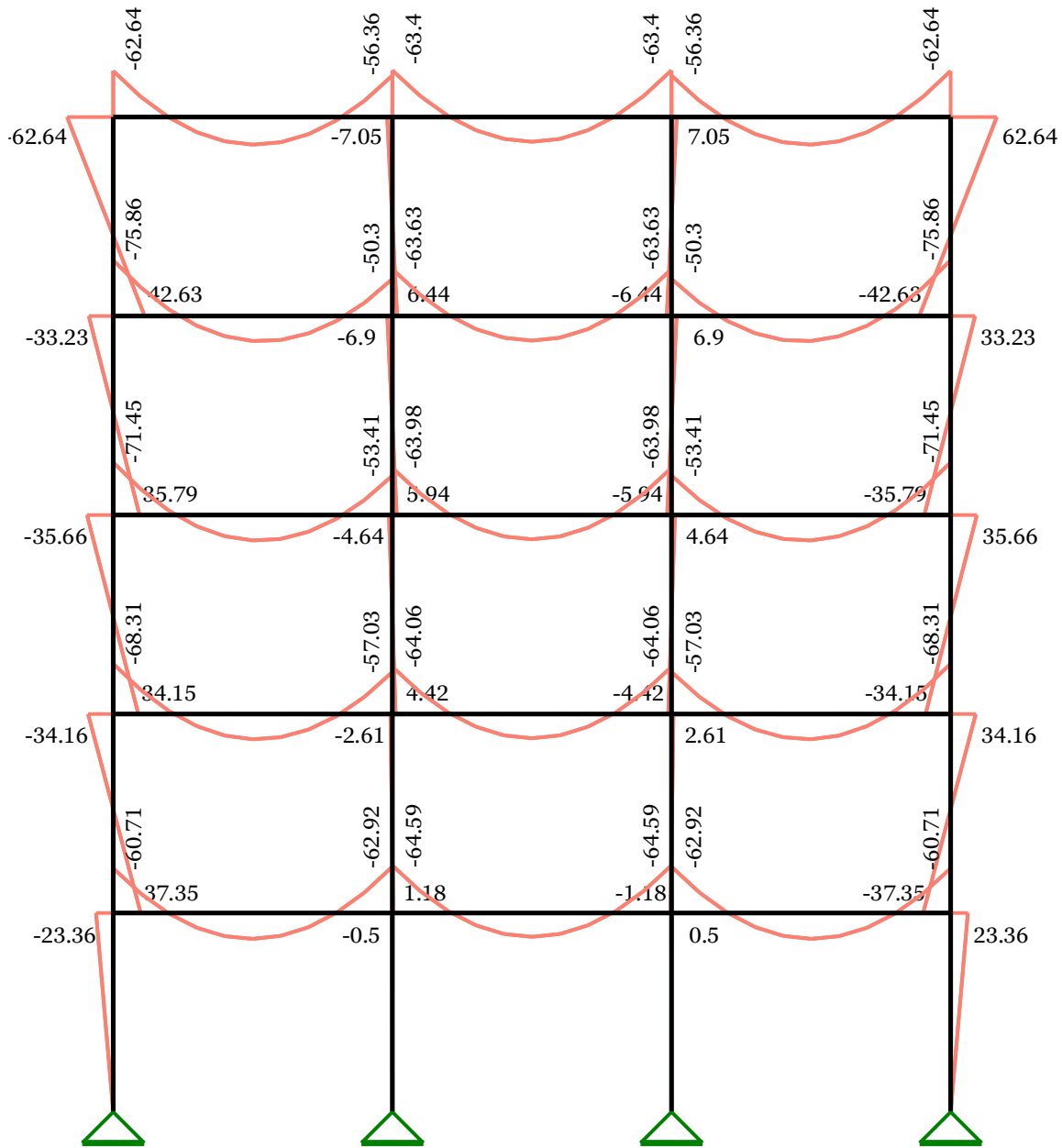
Axial forces diagram, kN



Shear forces diagram, kN



Bending moments diagram, kNm



Deformed shape

Shape function in relative coordinates $\xi = x/l$ (with account to shear deflections)

$$\Phi_1(e; \xi) = \frac{1}{1 + k_s(e)} \cdot (1 + k_s(e) - k_s(e) \cdot \xi - 3 \cdot \xi^2 + 2 \cdot \xi^3)$$

$$\Phi_2(e; \xi) = \frac{\xi \cdot l(e) \cdot m^{-1}}{1 + k_s(e)} \cdot \left(1 + \frac{k_s(e)}{2} - \left(2 + \frac{k_s(e)}{2} \right) \cdot \xi + \xi^2 \right)$$

$$\Phi_3(e; \xi) = \frac{\xi}{1 + k_s(e)} \cdot (k_s(e) + 3 \cdot \xi - 2 \cdot \xi^2)$$

$$\Phi_4(e; \xi) = \frac{\xi \cdot l(e) \cdot m^{-1}}{1 + k_s(e)} \cdot \left(\frac{-k_s(e)}{2} - \left(1 - \frac{k_s(e)}{2} \right) \cdot \xi + \xi^2 \right)$$

Element endpoint displacements and rotations

$$\mathbf{z}_{E,loc}(e) = \mathbf{T}(e) \cdot \mathbf{z}_E(e)$$

$$u_1(e) = \text{take}\left(1; \mathbf{z}_{E,loc}(e)\right), v_1(e) = \text{take}\left(2; \mathbf{z}_{E,loc}(e)\right), \varphi_1(e) = \text{take}\left(3; \mathbf{z}_{E,loc}(e)\right)$$

$$u_2(e) = \text{take}\left(4; \mathbf{z}_{E,loc}(e)\right), v_2(e) = \text{take}\left(5; \mathbf{z}_{E,loc}(e)\right), \varphi_2(e) = \text{take}\left(6; \mathbf{z}_{E,loc}(e)\right)$$

Displacement functions

$$u(e; \xi) = u_1(e) \cdot (1 - \xi) + u_2(e) \cdot \xi$$

$$v(e; \xi) = v_1(e) \cdot \Phi_1(e; \xi) + \varphi_1(e) \cdot \Phi_2(e; \xi) + v_2(e) \cdot \Phi_3(e; \xi) + \varphi_2(e) \cdot \Phi_4(e; \xi)$$

Deformed shape, mm

