

## Analysis of continuous beam by force method

### Input data

Span lengths -  $\vec{l} = [4; 7; 3; 5] \cdot \text{m} = [4 \text{ m } 7 \text{ m } 3 \text{ m } 5 \text{ m}]$ Number of spans -  $n = \text{len}(\vec{l}) = 4$ Coordinates of supports -  $\vec{x} = [4 \text{ m } 11 \text{ m } 14 \text{ m } 19 \text{ m}]$ Total beam length -  $L = \vec{x}_4 = 19 \text{ m}$ Load -  $q = 10 \frac{\text{kN}}{\text{m}}$ 

### Material

Elastic modulus -  $E = 30 \text{ GPa}$ Poisson's ratio -  $\nu = 0.2$ Shear modulus -  $G = \frac{E}{2 \cdot (1 + \nu)} = \frac{30 \text{ GPa}}{2 \cdot (1 + 0.2)} = 12.5 \text{ GPa}$ 

### Cross section

Rectangular with dimensions:  $b = 250 \text{ mm}$ ,  $h = 500 \text{ mm}$ Area -  $A = b \cdot h = 250 \text{ mm} \cdot 500 \text{ mm} = 125000 \text{ mm}^2$ Moment of inertia -  $I = \frac{b \cdot h^3}{12} = \frac{250 \text{ mm} \cdot (500 \text{ mm})^3}{12} = 260416667 \text{ mm}^4$ Shear area -  $A_Q = \frac{5}{6} \cdot b \cdot h = \frac{5}{6} \cdot 250 \text{ mm} \cdot 500 \text{ mm} = 104167 \text{ mm}^2$ 

### Solution

The solution will be performed by the force method with a primary system - simply supported beam with internal supports removed and replaced by unknown forces  $X_i$

Bending moments

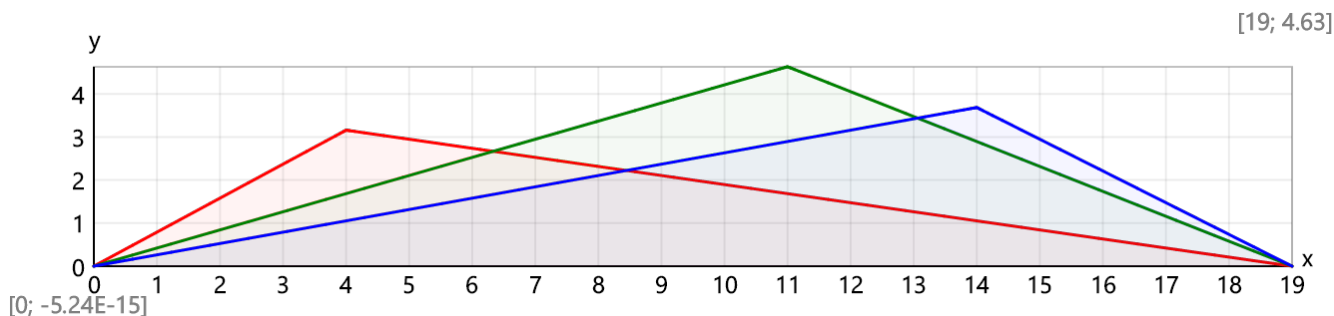
- in section  $a$  due to unit force at distance  $x$  from the beginning of the beam:

$$M_{1,\max}(x) = \left( \frac{x}{L} - 1 \right) \cdot x$$

$$M_{1,a}(a; x) = M_{1,\max}(x) \cdot \begin{cases} \text{if } a < x: \frac{a}{x} \\ \text{else: } \frac{L-a}{L-x} \end{cases}$$

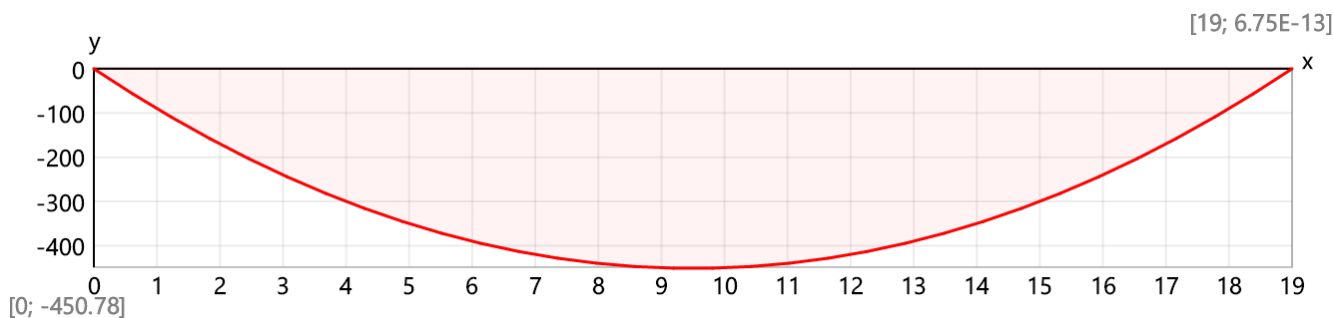
- in section  $a$ , due to unit force  $X_i$ :

$$M_1(a; i) = M_{1,a}(a; \vec{x}_i)$$



- due to external loads in primary system:

$$M_F(x) = \frac{q \cdot x}{2} \cdot (L - x)$$



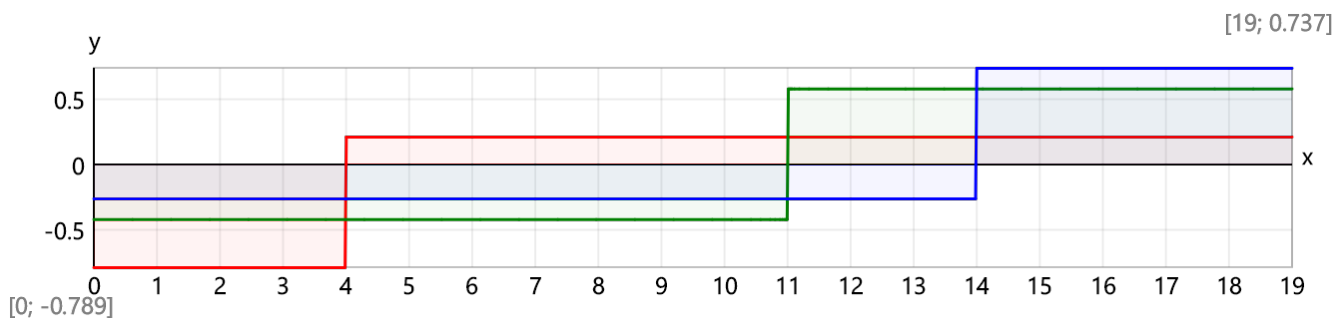
Shear forces

- in section  $a$  due to unit force at distance  $x$  from the beginning of the beam:

$$V_{1,a}(a; x) = \begin{cases} \text{if } a < x: \frac{x}{L} - 1 \\ \text{else: } \frac{x}{L} \end{cases}$$

- in section  $a$ , due to unit force  $X_i$ :

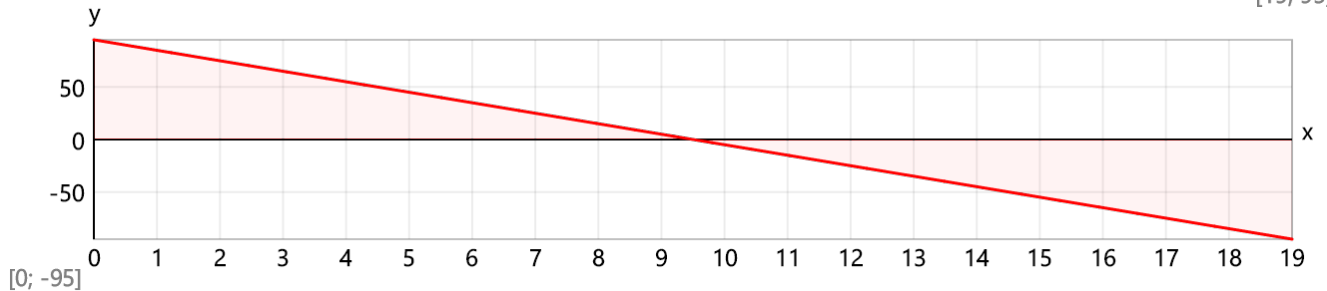
$$V_1(a; i) = V_{1,a}(a; \vec{x}_i)$$



- due to external loads in primary system:

$$V_F(x) = q \cdot \left( \frac{L}{2} - x \right)$$

[19; 95]

Number of unknowns by force method -  $n_1 = n - 1 = 4 - 1 = 3$ 

Flexibility coefficients

$$\delta(i; j) = \int_{0m}^L \frac{M_1(x; i) \cdot M_1(x; j)}{E \cdot I} dx + \int_{0m}^L \frac{V_1(x; i) \cdot V_1(x; j)}{G \cdot A_Q} dx$$

$$\Delta_F(i) = \int_{0m}^L \frac{M_1(x; i) \cdot M_F(x)}{E \cdot I} dx + \int_{0m}^L \frac{V_1(x; i) \cdot V_F(x)}{G \cdot A_Q} dx$$

$$\delta = \text{symmetric}(n_1) = \text{symmetric}(3) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \vec{\Delta}_F = \text{vector}(n_1) = \text{vector}(3) = [0 \ 0 \ 0]$$

$$\text{\$Repeat}\{\text{\$Repeat}\{\delta_{i,j} = \delta(i;j); i = 1 \dots n_1\}; j = 1 \dots n_1\} = 0.0011 \text{ m/kN}$$

$$\text{\$Repeat}\{\vec{\Delta}_{F,i} = \Delta_F(i); i = 1 \dots n_1\} = -0.161 \text{ m}$$

$$\delta = \begin{bmatrix} 0.000811 \text{ m/kN} & 0.00101 \text{ m/kN} & 0.000719 \text{ m/kN} \\ 0.00101 \text{ m/kN} & 0.00174 \text{ m/kN} & 0.00133 \text{ m/kN} \\ 0.000719 \text{ m/kN} & 0.00133 \text{ m/kN} & 0.0011 \text{ m/kN} \end{bmatrix}, \vec{\Delta}_F = [-0.135 \text{ m} \ -0.211 \text{ m} \ -0.161 \text{ m}]$$

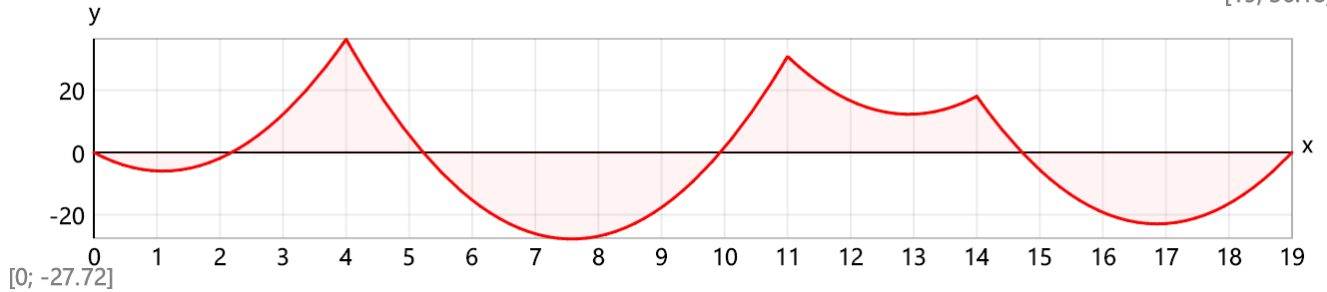
Calculation of the unknown forces  $X_i$ 

$$\vec{X} = -\text{csolve}(\delta; \vec{\Delta}_F) = [64.86 \text{ kN} \ 53.48 \text{ kN} \ 39.33 \text{ kN}]$$

## Results

$$\text{Bending moment diagram} - M(x) = M_F(x) + \sum_{i=1}^{n_1} M_1(x; i) \cdot X_i$$

[19; 36.18]



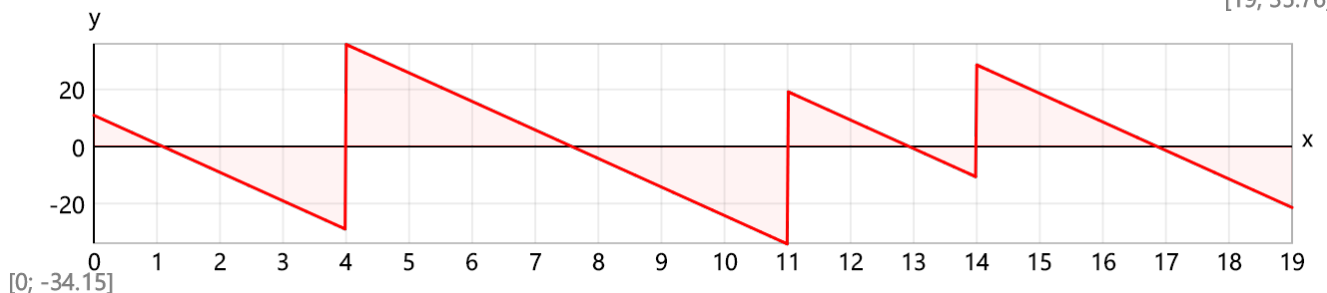
[0; -27.72]

$$\vec{M}_{\max} = [5.97 \text{ kNm} \quad 27.74 \text{ kNm} \quad -12.23 \text{ kNm} \quad 22.9 \text{ kNm}]$$

$$\vec{M}_{\min} = [-36.29 \text{ kNm} \quad -30.79 \text{ kNm} \quad -17.99 \text{ kNm}]$$

$$\text{Shear force diagram} - V(x) = V_F(x) + \sum_{i=1}^{n_1} V_1(\vec{x}; i) \cdot \vec{X}_i$$

[19; 35.76]



[0; -34.15]

$$\vec{V}_{\max} = [10.93 \text{ kN} \quad 35.79 \text{ kN} \quad 19.26 \text{ kN} \quad 28.6 \text{ kN}]$$

$$\vec{V}_{\min} = [-29.07 \text{ kN} \quad -34.21 \text{ kN} \quad -10.74 \text{ kN} \quad -21.4 \text{ kN}]$$

## Deflections

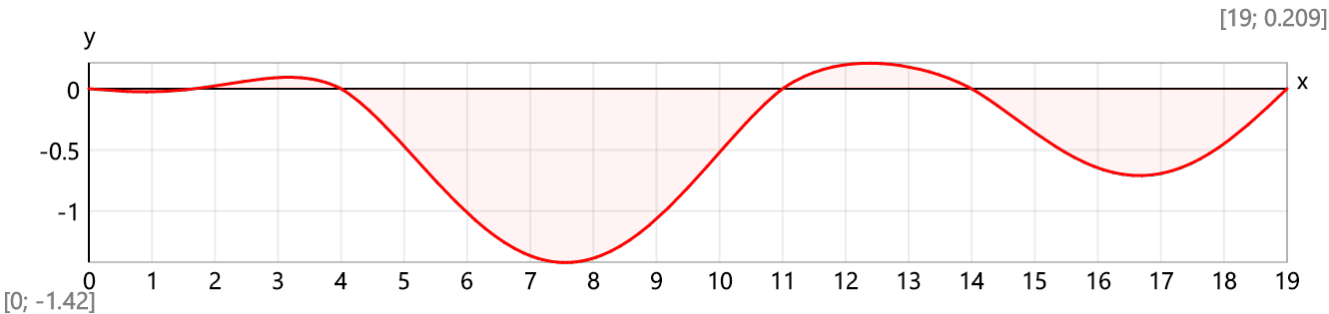
- in section  $a$ , due to unit force  $X_i$ :

$$d_1(a; i) = \int_{0m}^L \frac{M_{1,a}(x; a) \cdot M_1(x; i)}{E \cdot I} dx + \int_{0m}^L \frac{V_{1,a}(x; a) \cdot V_1(x; i)}{G \cdot A_Q} dx$$

- due to external loads in primary system:

$$d_F(a) = \int_{0m}^L \frac{M_{1,a}(x; a) \cdot M_F(x)}{E \cdot I} dx + \int_{0m}^L \frac{V_{1,a}(x; a) \cdot V_F(x)}{G \cdot A_Q} dx$$

$$d(x) = d_F(x) + \sum_{i=1}^{n_1} d_1(\vec{x}; i) \cdot \vec{X}_i$$



Maximum deflection -  $d_{\max} = \inf\{d(x); x \in [0\text{m}; L]\} = -1.42\text{ mm}$

At a distance from the origin -  $x_{\inf} = 7.56\text{ m}$