

## RECURSION - WITH - ARRAYLISTS

### Get Subsequence

$abc \rightarrow \begin{array}{l} \underline{\underline{-}} \\ \underline{\underline{-\underline{c}}} \\ \underline{\underline{-b\underline{-}}} \\ \underline{\underline{-b\underline{c}}} \\ \underline{\underline{a\underline{-}}} \\ \underline{\underline{a\underline{-c}}} \\ \underline{\underline{a\underline{b\underline{-}}} } \\ \underline{\underline{a\underline{b\underline{c}}}} \end{array}$  This is  
subsequence  
of  
 $abc$   
 $\therefore [n \rightarrow 2^n]$

$abc \rightarrow \begin{array}{l} \underline{a} \\ \underline{ab} \\ \underline{abc} \\ \underline{b} \\ \underline{bc} \\ \underline{c} \end{array}$  This is  
sub-string  
 $\therefore [n \rightarrow \frac{n(n+1)}{2}]$

Hume input me  $\boxed{\text{getss}(abc)}$  → isko ek arraylist ~~bhar~~ bhar kर  
deni jisme saare subsequence padey ho!

### Expectation

$\text{getss}(abc) = \begin{array}{l} \underline{\underline{-}} \\ \underline{\underline{-\underline{c}}} \\ \underline{\underline{-b\underline{-}}} \\ \underline{\underline{-b\underline{c}}} \\ \underline{\underline{a\underline{-}}} \\ \underline{\underline{a\underline{-c}}} \\ \underline{\underline{a\underline{b\underline{-}}} } \\ \underline{\underline{a\underline{b\underline{c}}}} \end{array}$

### Faith

$\text{getss}(bc) = \begin{array}{l} \underline{\underline{-}} \\ \underline{\underline{-\underline{c}}} \\ \underline{\underline{b\underline{-}}} \\ \underline{\underline{bc}} \end{array}$

### Expectation meet Faith

$\text{getss}(abc) = \underbrace{\text{getss}(bc)}_{\{ \begin{array}{l} \underline{\underline{-}} \\ \underline{\underline{-\underline{c}}} \\ \underline{\underline{-b\underline{-}}} \\ \underline{\underline{-b\underline{c}}} \end{array} \}} + \underbrace{a \text{getss}(bc)}_{\{ \begin{array}{l} \underline{\underline{a\underline{-}}} \\ \underline{\underline{a\underline{-c}}} \\ \underline{\underline{a\underline{b\underline{-}}} } \\ \underline{\underline{a\underline{b\underline{c}}}} \end{array} \}}$

```
ps void m(s[] a){  
    Scanner s = new Scanner(System.in);  
    String str = s.next();  
    ArrayList<String> res = gss(str);  
    System.out.println(res);  
}
```

```
ps ArrayList<String> gss(String str){  
    if (str.length() == 0) { // Base Case  
        ArrayList<String> bres = new ArrayList<>();  
        bres.add("");  
        return bres;  
    }
```

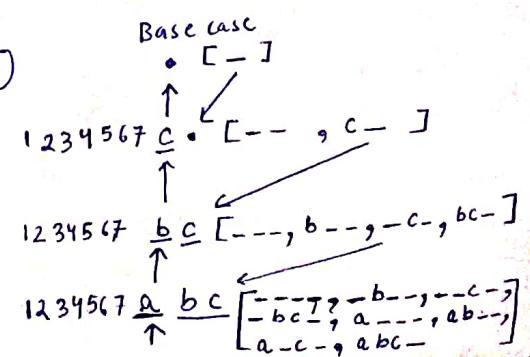
```
    char ch = str.charAt(0); ①  
    String ros = str.substring(1); ②  
    ArrayList<String> rres = gss(ros); ③  
    ArrayList<String> mres = new ArrayList<>(); ④  
    for (String val : rres){ ⑤  
        mres.add(ch + val);  
    }
```

```
    for (String val : mres){ ⑥  
        mres.add(ch + val);  
    }  
    return mres; ⑦
```

blank string kitni  
subsequence? → ①  
blank itself ← konki  
Ls why? → ② = 1

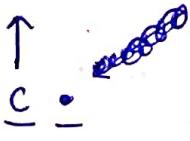
bc [-, -, c, b, -, bc]

↑  
abc [-, -, -, c, -, b, -, bc]  
[a, -, a, -, a, c, abc, abc]



= while going up

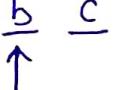
• [ ] (Base Case)



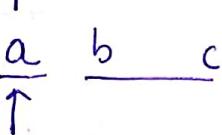
① ② ③



① ② ③



① ② ③



① ② ③ ④ ⑤ ⑥ ⑦

① ② ③ ④ ⑤ ⑥ ⑦

① ② ③ ④ ⑤ ⑥ ⑦

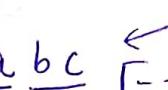
• [ ]



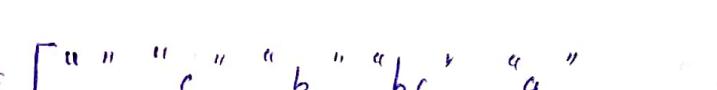
• [ --, c- ]



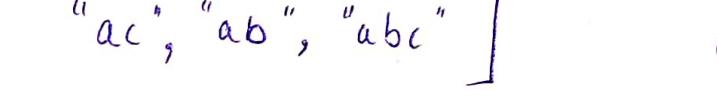
• [ ---, -c-, ]



• [ ---, --c-, -b-, -bc-, ]



• [ ---, -ac-, ab-, abc-, ]



• [ ---, a-c-, ab-, abc-, ]



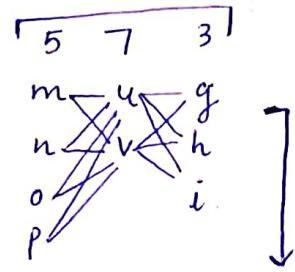
∴ FINAL OUTPUT = [ "", "c", "b", "bc", "a",  
"ac", "ab", "abc" ]

= TIME COMPLEXITY : O(n)

= SPACE COMPLEXITY : O(1)

# Gret KPC

1 → abc  
 2 → def  
 3 → ghi  
 4 → jkl  
 5 → mno  
 6 → pqrs  
 7 → tu  
 8 → vw  
 9 → yz  
 0 → ; ;



mug, muh, mui,  
 nug, nuh, nui,  
 oug, ouh, oui,  
 pug, puh, pui,  
 mvg, mvh, mvi,  
 nvg, nvh, nvi,  
 ovg, ovh, ovi,  
 prg, prh, pvi

73 [ ug uh ui ] (words)

573 [ mug, muh, mui, mvg, mvh, mvi, nug, nuh, nui, nvg, nvh, nvi, oug, ouh, oui, ovg, ovh, ovi, pug, puh, pui, prg, prh, pvi ] 24 words

## expectation

$$\begin{aligned}
 573 &\Rightarrow [ ] \\
 24 \text{ words} &\downarrow \\
 \text{milenay} & \\
 \downarrow & \\
 \text{why?} & \\
 \therefore 5 \rightarrow mnop &\therefore 4 ] 4 \times 2 \times 3 \\
 7 \rightarrow uv &\therefore 2 ] = 24 \\
 3 \rightarrow ghi &\therefore 3
 \end{aligned}$$

## Faith

$$\begin{aligned}
 73 &\Rightarrow [ ] \\
 6 \text{ words} &\downarrow \\
 \text{milenay} & \\
 \downarrow & \\
 \text{why} & \\
 7 \rightarrow \therefore 2 & ] 2 \times 3 \\
 3 \rightarrow \therefore 3 & ] = 6
 \end{aligned}$$

```

public static void main (String [] args) {
  Scanner s = new Scanner (System.in);
  String str = s.nextLine();
  ArrayList<String> words = getKPC (str);
  System.out.println (words);
}
  } to declare globally
  
```

**Static** String [] codes = {".;", "abc", "def", "ghi",  
 "jkl", "mno", "pqrs", "tu", "vw",  
 "yz"};

```

public static ArrayList<String> getKPC(String str) {
    if (str.length() == 0) {
        ArrayList<String> bres = new ArrayList<>();
        bres.add("");
        return bres;
    }
    // 678
    char ch = str.charAt(0); // 6 → ①
    String ros = str.substring(1); // 78 → ② → ③
    ArrayList<String> rres = getKPC(ros); // 6 words of 78
    ArrayList<String> mres = new ArrayList<>(); // 24 words of 678
    String codeforch = codes [ch - "0"];
    for (int i = 0; i < codeforch.length(); i++) {
        char chcode = codeforch.charAt(i);
        for (String val : rres) {
            mres.add(chcode + val);
        }
    }
    return mres;
}

```

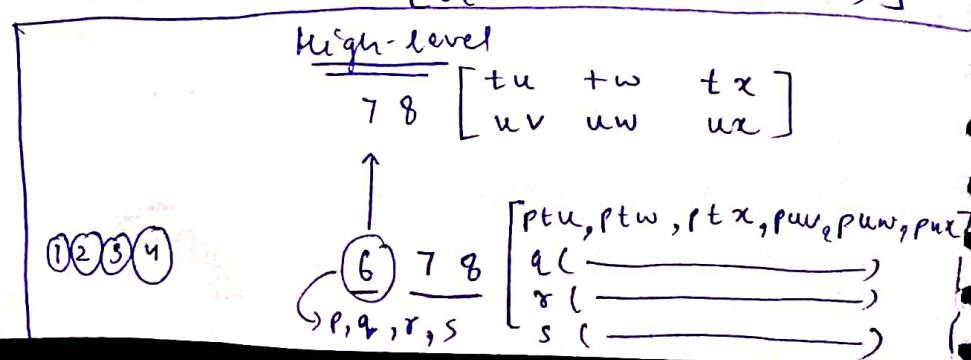
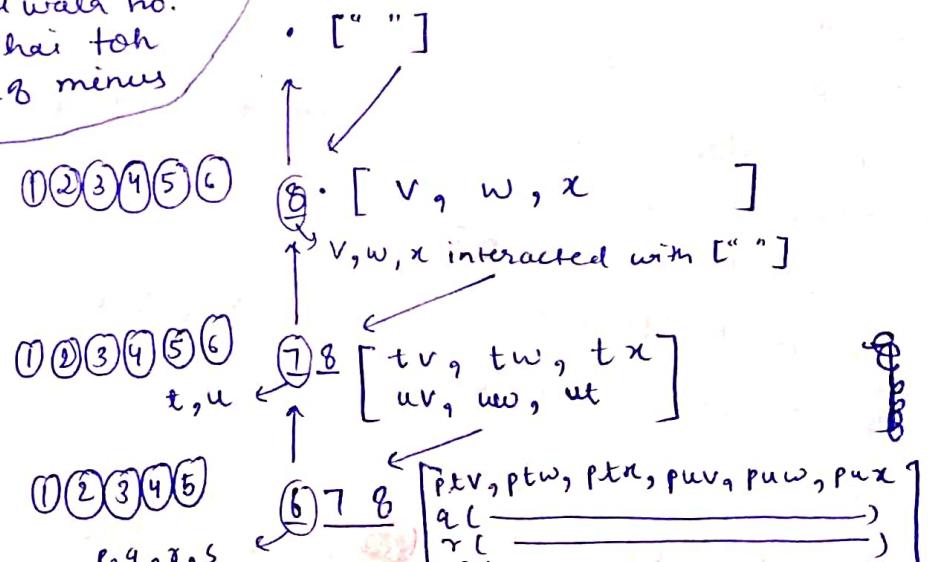
Base Case: Agar koi key nahi press hui  
toh ek word peda hoga vo word  
blank hoga! →  $2^0 = 1$

(Agar hum character waley  
no. ko asli wala no.  
bana chaty hai toh  
usme se 48 minus  
krna hoga)

6	7	8
p	t	v
q	u	w
r	x	
s		

'6' ≠ 6, no ka 6  
+  
character 6

'a' → 97	'3' → 51
'0' → 48	'4' → 52
'1' → 49	'5' → 53
'2' → 50	'6' → 54



Agar hum character waly number ko real waly number me convert karna chahiye toh character waly number se 48 minus karna hoga!

$$\therefore '6' \neq 6$$

∴ But,

$$'6' - '0' = 6$$

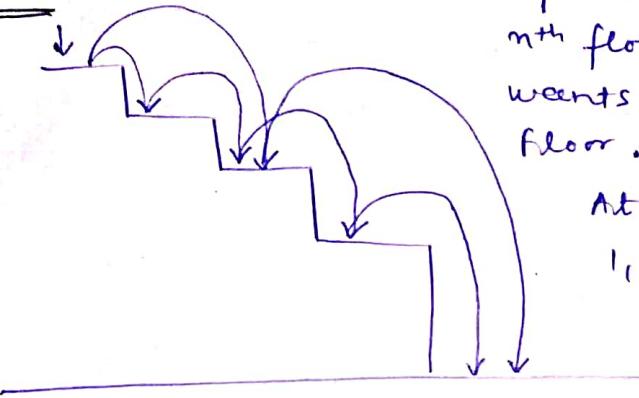
$\downarrow$        $\downarrow$

54 - 48

= Time Complexity :  $O(n)$

= Space Complexity :  $O(1)$

## Het Stair Path



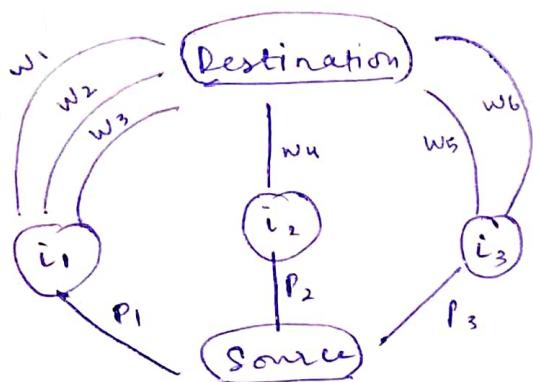
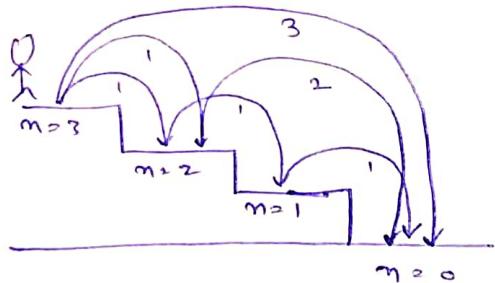
A person is standing on the  $n^{\text{th}}$  floor of staircase and wants to come to the  $0^{\text{th}}$  floor.

At a time he can go  
1, 2, or 3 step down

Home sare path  
point karne hai

For example

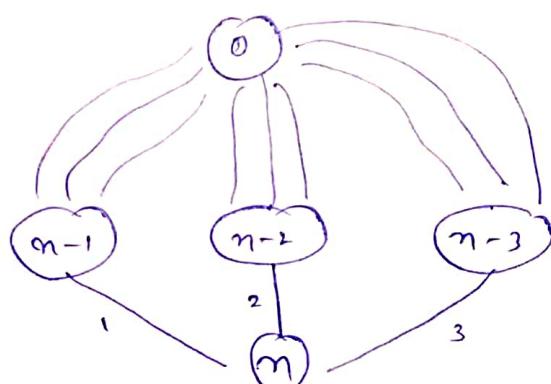
$$n = 3 \rightarrow \text{paths} \begin{array}{c} 111 \\ 12 \\ 21 \\ 3 \end{array}$$



$P_1, P_2, P_3 \rightarrow$  path

$w_1, w_2, w_3, \dots \rightarrow$  ways

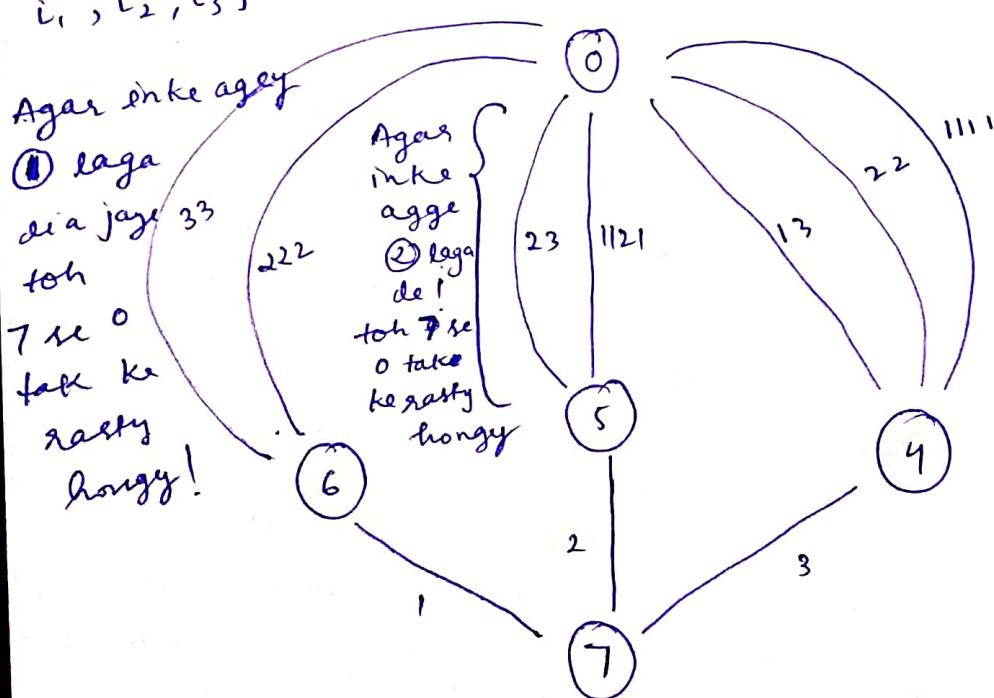
$i_1, i_2, i_3 \rightarrow$  intermediate



Agar ek ke aagey  
① laga  
dia jaye 33  
toh  
7 se 0  
tak ke  
raaste  
honge!

Agar  
in ke  
agge  
② laga  
de!  
toh 7 se  
0 tak  
ke raaste  
honge

Agar in sare  
sample raaste  
ke aage ③ laga  
dia jaye,  
toh yeh 7 se  
llke ④ tak ke  
raaste hongayengi!



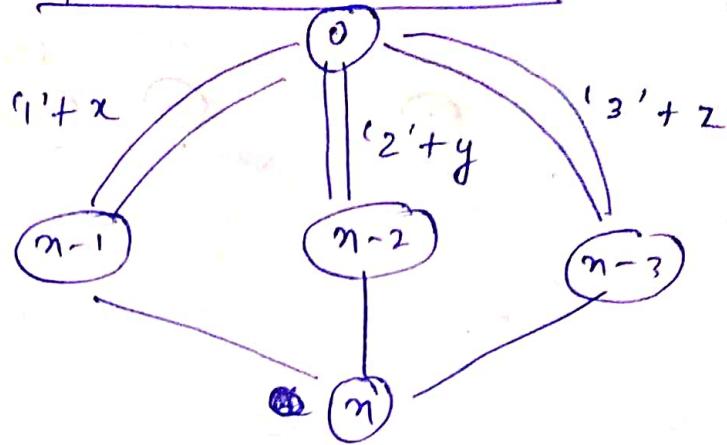
expectation :

(n) se 0 tak  
ke gasty  
chahiye  
[n, 0] paths

faith

(n-1) se 0,  
(n-2) se 0,  
(n-3) se 0,  
inn sabke gasty  
chahiye!  
[n-1, 0] paths  
[n-2, 0] paths  
[n-3, 0] paths

Expectation meets faith



ps v m (S [] a) {

Scanner s = new Scanner (System.in);

int n = s.nextInt();

ArrayList<String> paths = getStairPaths(n);

System.out.println(paths);

}

ps ArrayList<String> getStairPaths(int n) {

if (n == 0) {

ArrayList<String> bres = new ArrayList<>();

bres.add ("");

return bres;

} else if (n < 0) {

ArrayList<String> bres = new ArrayList<>();

return bres;

}

ArrayList<String> paths1 = getStairPaths (n-1); ①

paths2 = getStairPaths (n-2); ②

paths3 = getStairPaths (n-3); ③

ArrayList<String> paths = new ArrayList<>(); ④

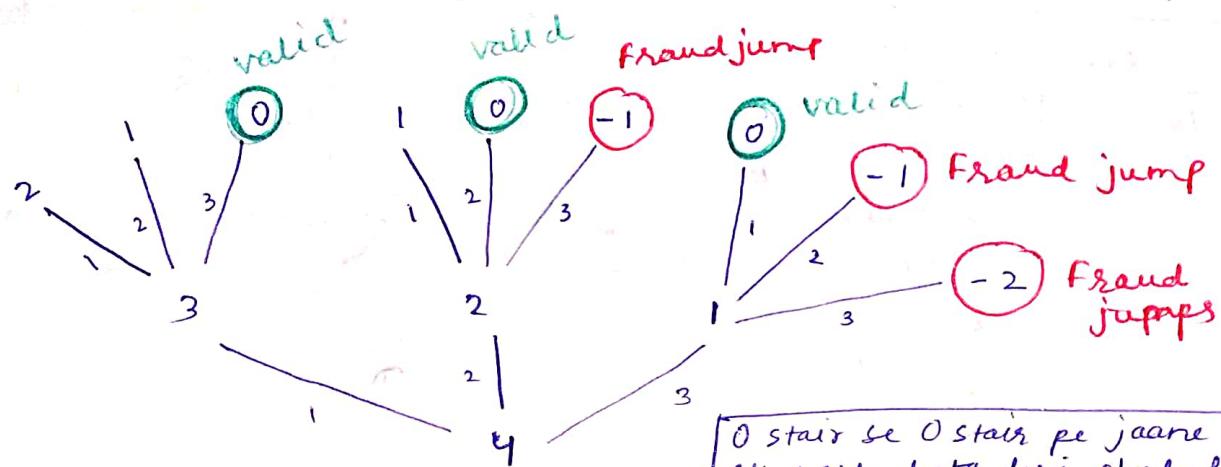
for (String path : paths1) {  
 paths.add ("1" + path); } ⑤

for (String path : paths2) {  
 paths.add ("2" + path); } ⑥

for (String path : paths3) {  
 paths.add ("3" + path); } ⑦

return paths; ⑧

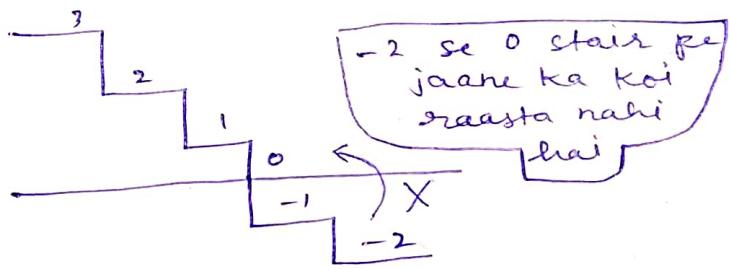
}



0 stair se 0 stair pe jaane ka ek rasta hota hai chalo hi mat

0 → Incorrect paths

0 → Correct path



`if (n == 0) {`

`ArrayList<String> bres = new ArrayList<String>();  
bres.add("");  
return bres;`

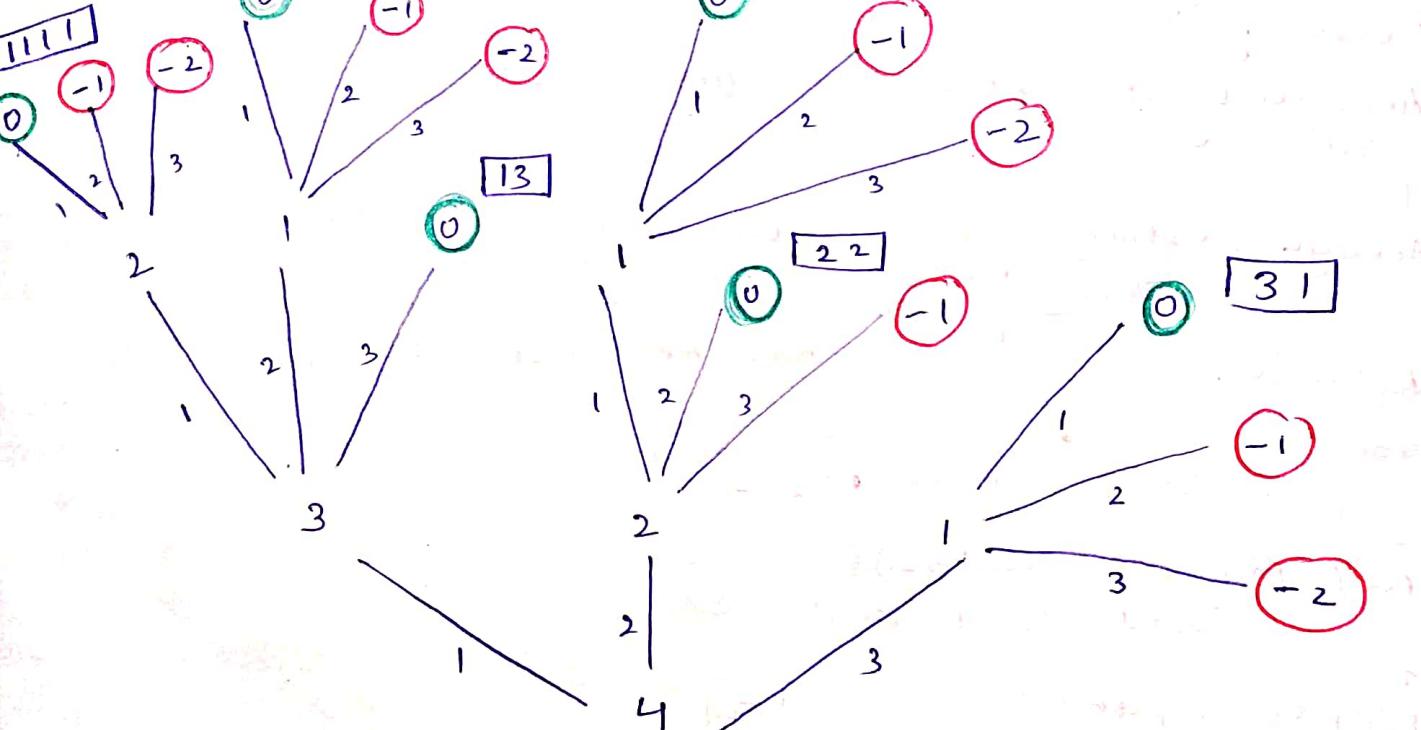
`} else if (n < 0) {`

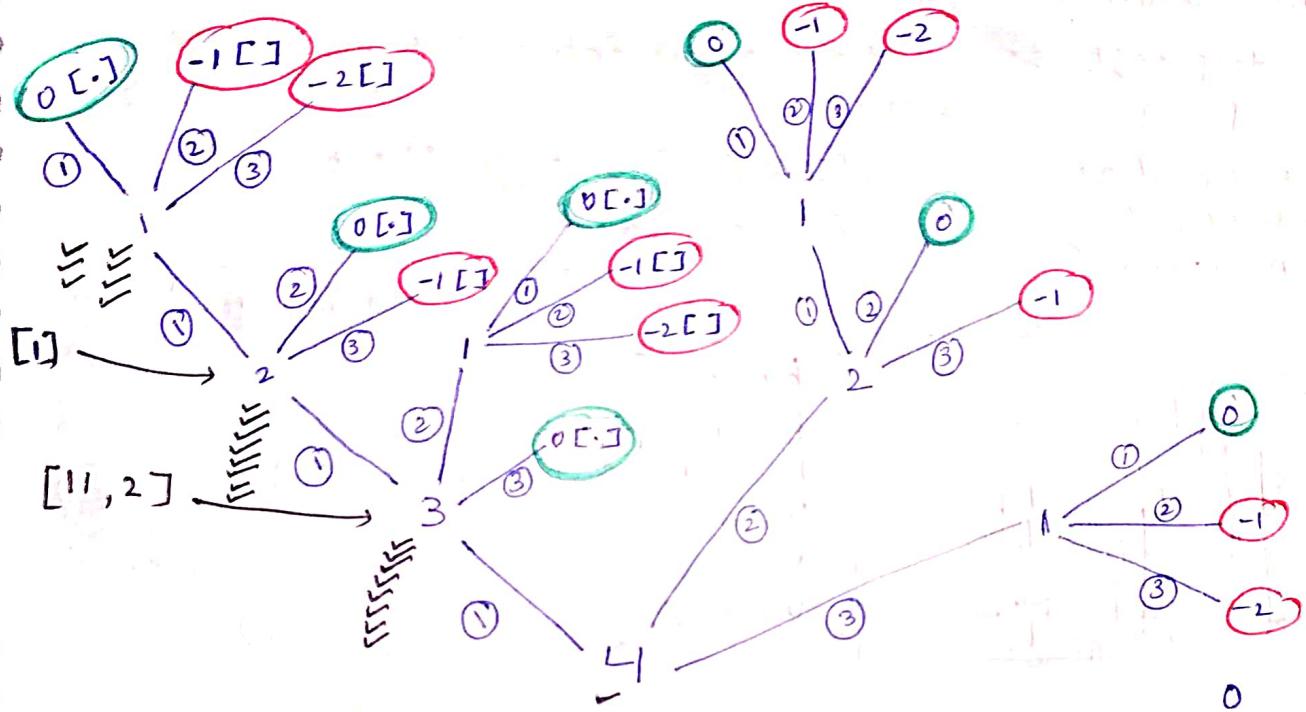
`ArrayList<String> bres = new ArrayList<String>();  
return bres;`

`}`

121

1211





Time Complexity

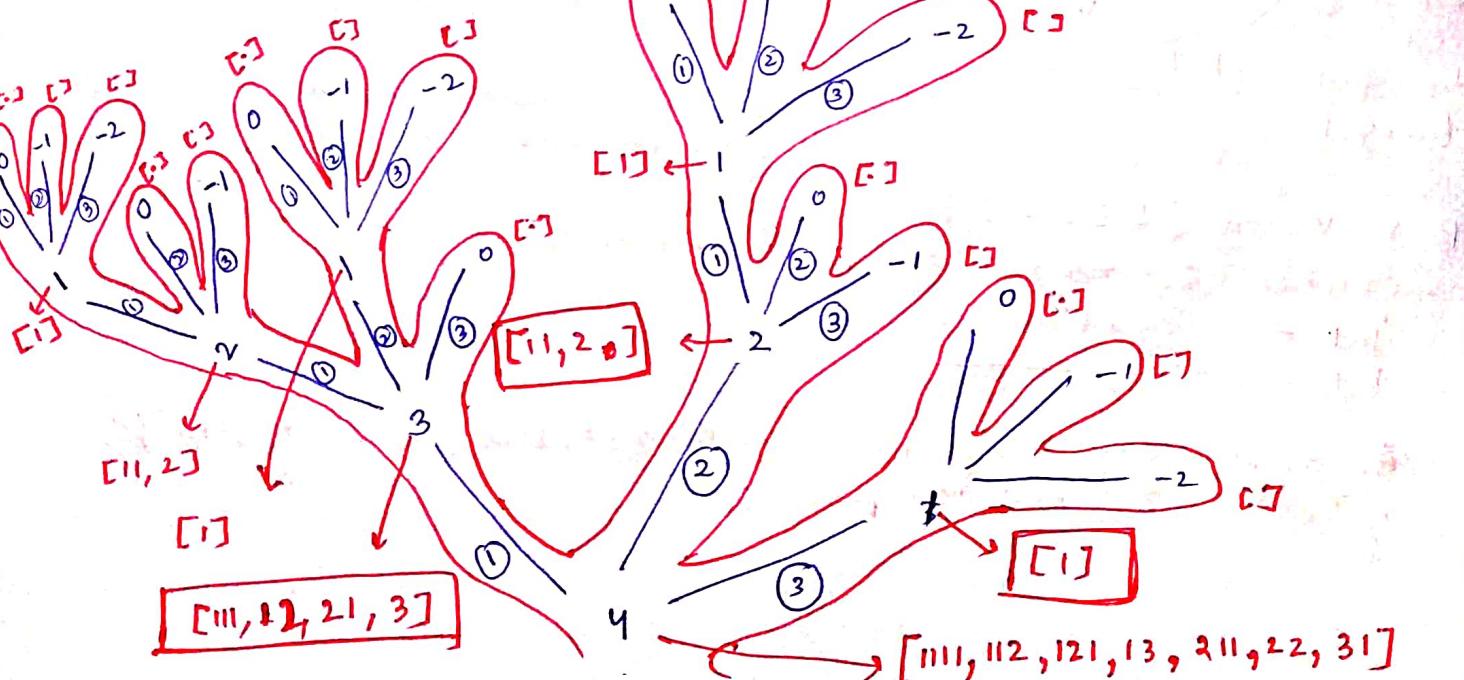
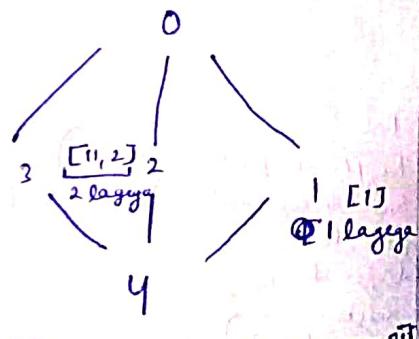
$$O(3^n)$$

↳ bcoz of each state,  
3 recursion calls are made

Space Complexity

$$O(1)$$

[111, 12, 21, 3]  
inke mith  
re ① lagega



## Q) het Maze Path

- Humne ek 2D maze dia hai

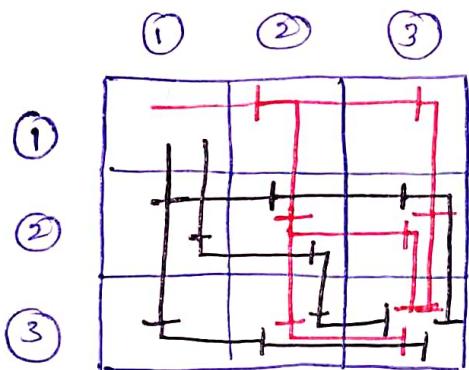
$n \rightarrow$  rows  
 $m \rightarrow$  columns

- Hum top left se bottom right corner tak jana hai

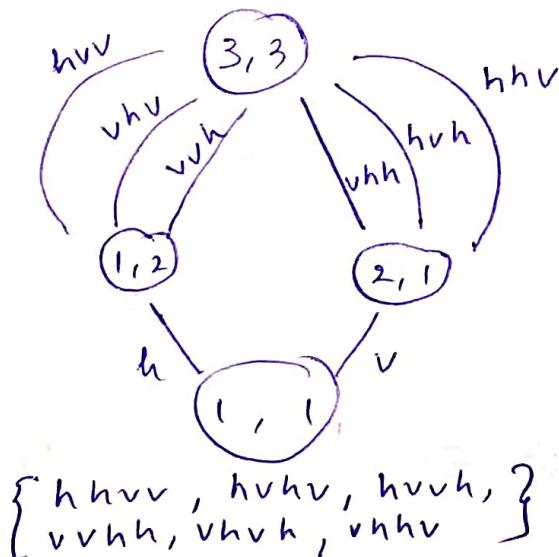
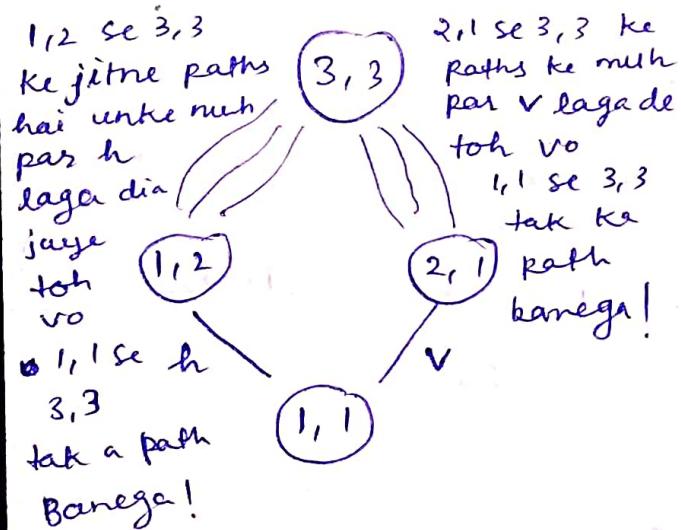
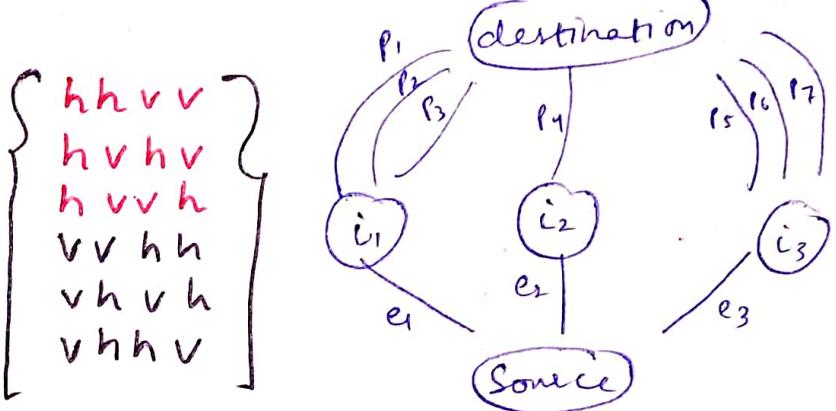
$$[1, 1] \rightarrow [n, m]$$

- Do tarikhe hai

↳ vertical  
↳ horizontal



Hume saare paths print krene hai!



P { s = new Scanner (System.in);

Scanner s = new Scanner (System.in);

int n = s.nextInt();

int m = \_\_\_\_\_;

ArrayList<String> paths = getMazePaths (1, 1, n, m);

System.out.println (paths);

}

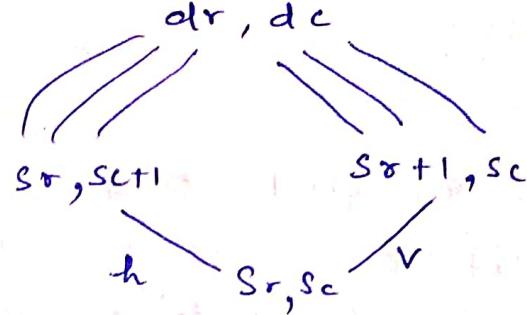
## Expectation

Humne  $(1,1)$  se  
 $(n, m)$  tak ke  
 Saare paths  
 chahiye

## Faith

$(1, 2) \rightarrow (n, m)$   
 $(2, 1) \rightarrow (m, n)$   
 Unke paths  
 humne milengy  
 humne faith hai

## Expectation meets Faith



p s ArrayList<String> getMazePaths (int sr, int sc, int dr, int dc)

ArrayList<String> hpaths = getMazePaths(sr, sc+1, dr, dc);  
 vpaths =

ArrayList<String> paths = new ArrayList<>();

for (String hpath : hpaths) { // horizontal paths pe loop  
 lagaya unke mukh per h lagaya!

paths.add ("h" + hpath);

for (String vpath : vpaths) { // vertical paths pe loop laga kr unke mukh par v lagadiya!  
 paths.add ("v" + vpath);

}

return paths;

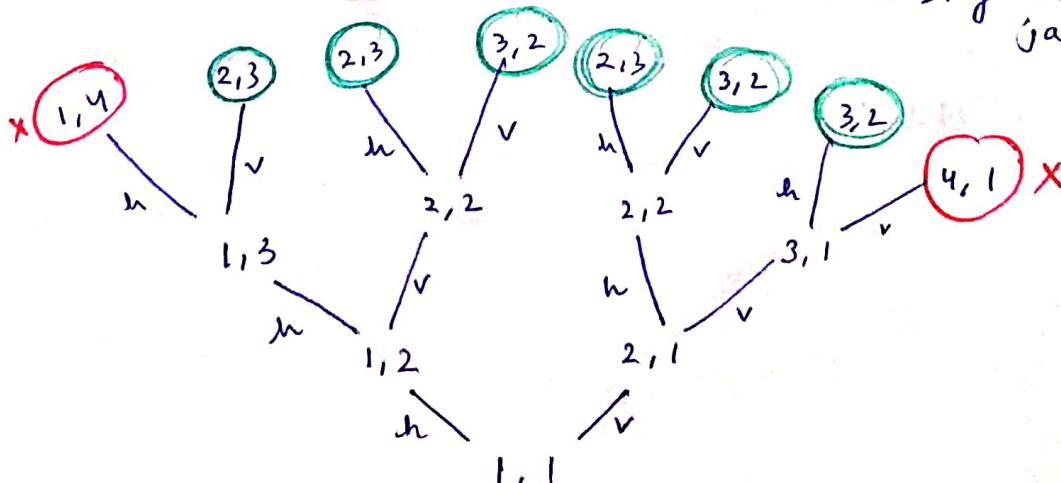
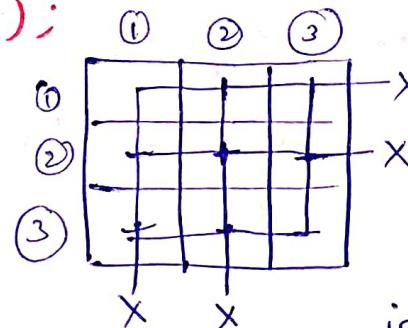
}

## DRY RUN

We conclude,

Jo Right-most wall hai, usse bahar nahi ja sakty! isse sirf vertical jaaty

Jo Bottom-most wall hai, usse bahar nahi ja sakty! isse sirf horizontal jaaty!



```

// sr = source row
// sc = source column
// dr = destination column
// dr = destination row

p s ArrayList<String> getMazePaths(int sr, int sc, int dr, int dc)
{
    if (sr == dr && sc == dc) {
        ArrayList<String> bres = new ArrayList<>();
        return bres;
    }

    ArrayList<String> hpaths = new ArrayList<>();
    ArrayList<String> vpaths = new ArrayList<>();

    if (sc < dc) {
        hpaths = getMazePaths(sr, sc+1, dr, dc);
    }

    if (sr < dr) {
        vpaths = getMazePaths(sr+1, sc, dr, dc);
    }

    ArrayList<String> paths = new ArrayList<>();

    for (String hpath : hpaths) {
        paths.add("h" + hpath);
    }

    for (String vpath : vpaths) {
        paths.add("v" + vpath);
    }

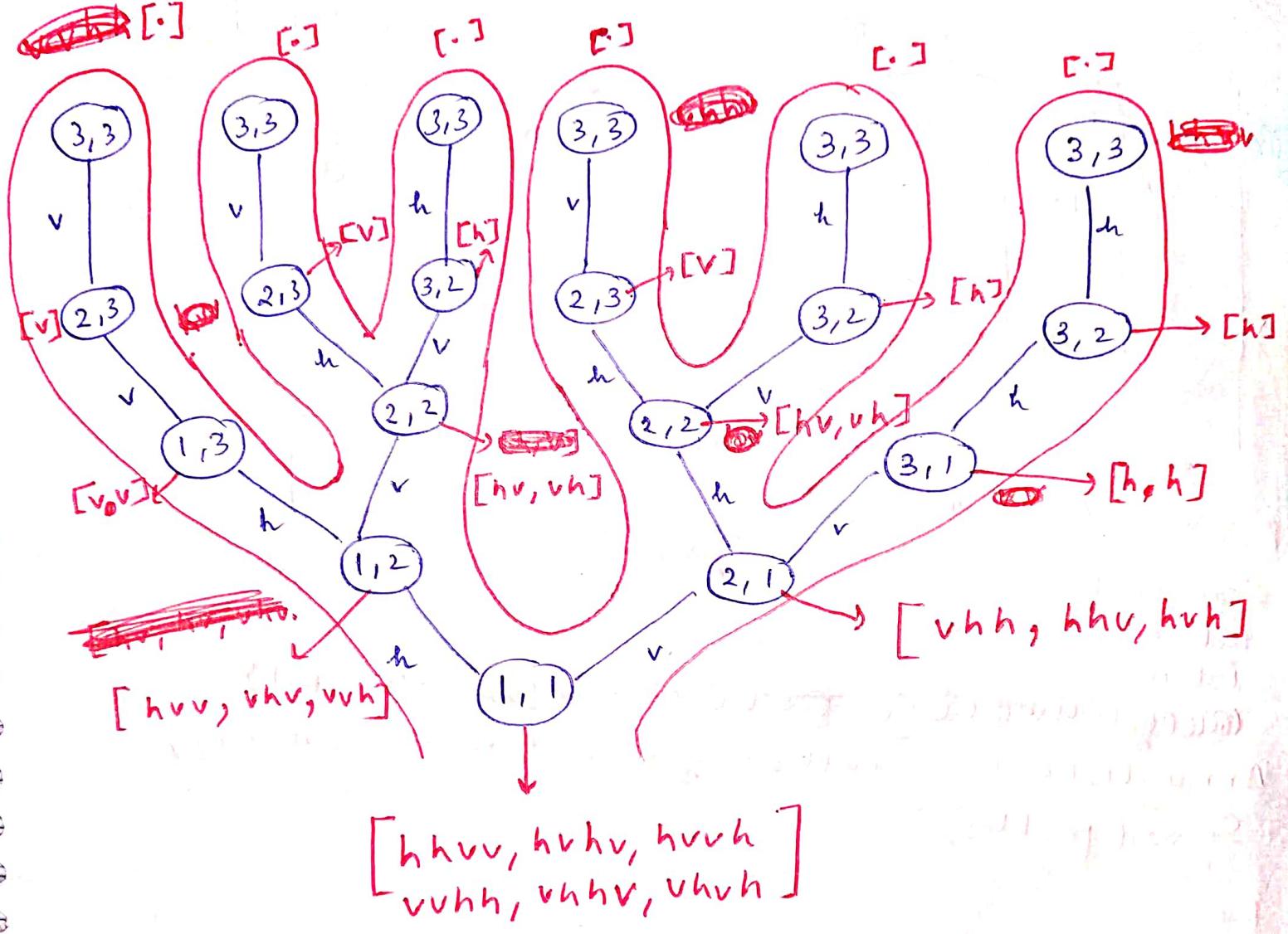
    return paths;
}

```

Base Case

To every path in hpaths, "h" is added.

To every path in vpaths, "v" is added.



Time Complexity:  $O(2^n)$

↳ Because of Each state,

2 recursion called.

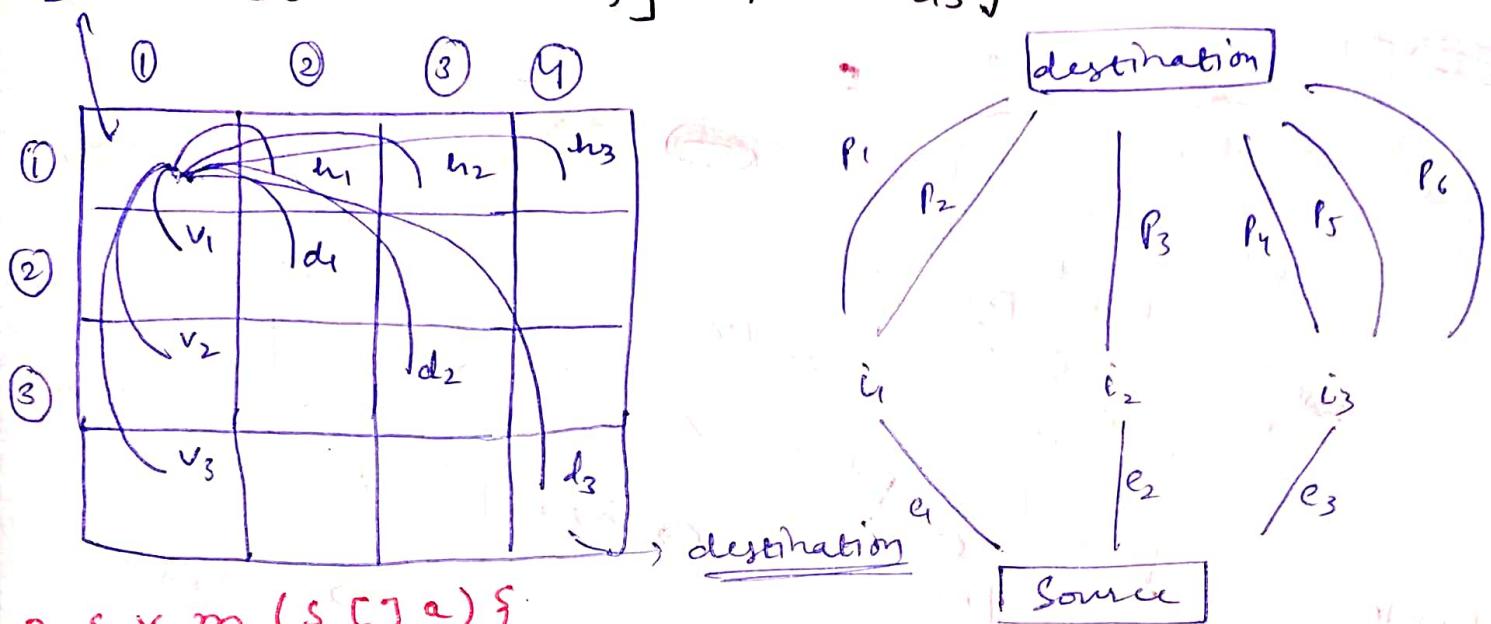
Space Complexity :

$O(1)$

## ④ het Maze Path With Jumps

Is baar heem, ek step me

$h_1$	horizontal	$v_1$	vertical	$d_1$	diagonal	Le Satty hai
$h_2$	steps	$v_2$	steps	$d_2$	steps	
$h_3$		$v_3$		$d_3$		



public static String getmazepaths(int sr, int sc, int dr, int dc) {

Scanner s = new Scanner(System.in);

int n = s.nextInt();

int m = \_\_\_\_\_;

~~ArrayList<String> paths = getmazepaths(sr, sc, dr, dc);~~

ArrayList<String> paths = getmazepaths(1, 1, n, m);

System.out.println(paths);

}

int sr = source row

int sc = source column

int dr = destination column

int dc = destination row

~~ArrayList<String> paths = getmazepaths(sr, sc, dr, dc);~~

④ HORIZONTAL MOVE

for (int ms = 1; m <= dc - sc; ms++)

{ ArrayList<String> hpaths = getmazepaths(sr, sc + ms, dr, dc);

for (String hpath : hpaths) {

paths.add("h" + ms + hpath);

}

}

## Vertical Move

```

for (int ms=1; ms <= dr - sr; ms++) {
    ArrayList<String> vpaths = getmazepaths(sr+ms, sc, dr, dc);
    for (String vpath : vpaths) {
        paths.add("v" + ms + vpath);
    }
}

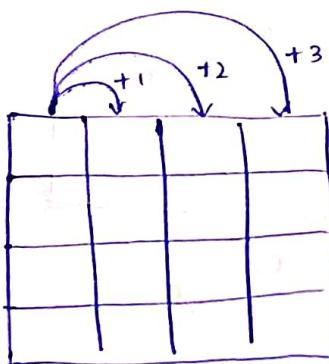
```

## Diagonal ~~Move~~ Move

```

for (int ms=1; ms <= dr - sr && m <= dc - sc; ms++) {
    ArrayList<String> dpaths = getmazepaths(sr+ms, sc+ms, dr, dc);
    for (String dpath : dpaths) {
        paths.add("d" + ms + dpath);
    }
}
return paths;
}

```



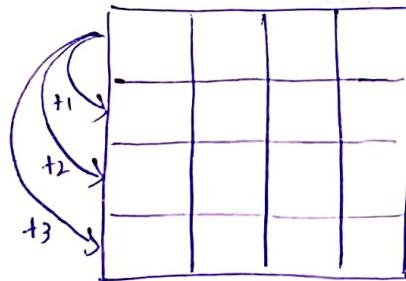
Max size of jump along a row

### BASE CASE

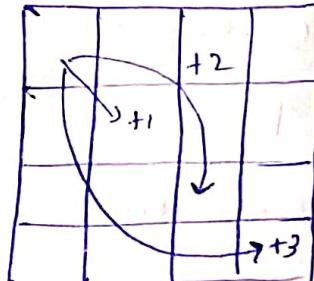
```

if (sr == dr && sc == dc) {
    ArrayList<String> bres = new ArrayList<>();
    bres.add("");
    return bres;
}
else if (sr > dr || sc > dc) {
    ArrayList<String> bres = new ArrayList<>();
    return bres;
}

```



Max size of jump along a column

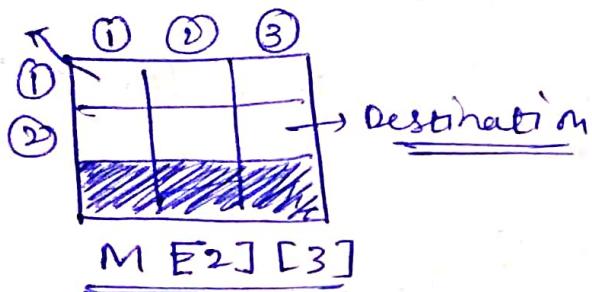


Max size of jump along row and columns

$$\left\{ \begin{array}{l} ms \leq dr - sr \& \& ms \leq dc - sc \end{array} \right.$$

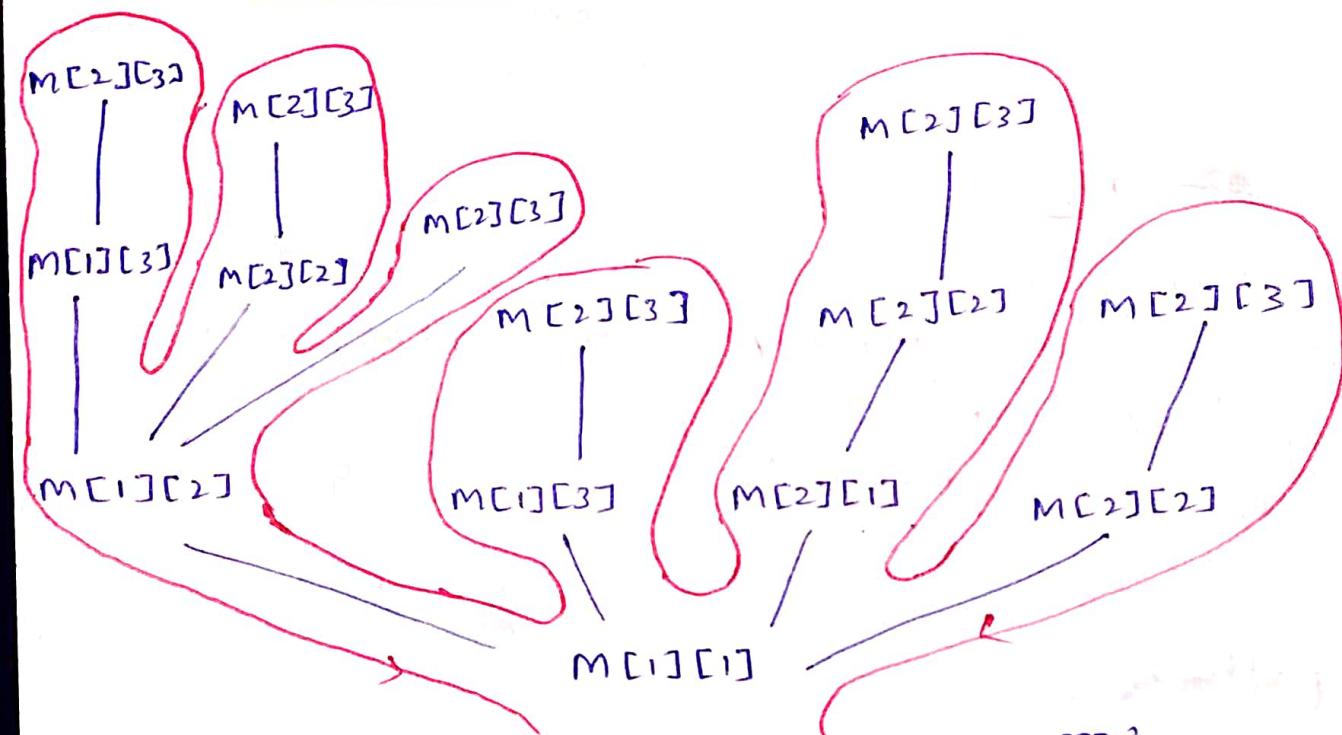
## = Low-level Thinking

Source



This is  $2 \times 3$  matrix

Enter Tree

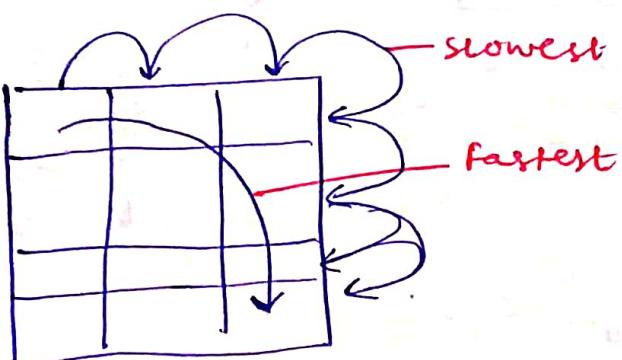


## Time Complexity

TC depends on 3 recursion calls

$$T(n, m) = T(n, m-1) + T(m-1, m) + T(n-1, m-1)$$

↓                  ↓                  ↓  
 Total Time      Row            Column      Diagonal  
 Complexity     call            call        cell



Fastest way → longest jump in diagonal

Slowest Way : One by one step (worst case)  
 Scenario : one by one step in columns.

we assume that the no. of rows is greater than the no. of columns.

$$T(n,m) = T(n,m-1) + \underbrace{T(n-1,m)}_{\substack{\rightarrow \text{maximum} \\ \text{impact on Time complexity}}} + T(n-1,m-1) \quad \therefore (n > m)$$

Replace  $T(n,m-1)$  with  $T(n-1,m)$   
 $T(n-1,m-1)$

∴ we get,

$$T(n,m) \leq 3T(n-1,m)$$

[TC: Time complexity]

- = As we assumed the no. of rows is the largest.
- = And traversal of columns depends upon the no. of rows.
- = so, they will be responsible for the overall TC the maximum.
- = If we replace TC function (other) with Column TC Function recursive call,
- = Equal sign changes as column complexity is the largest so its thrice will always be greater than or equal to the overall complexity.

### Calculations

$$\text{since, } T(n,m) \leq 3T(n-1,m) \quad \textcircled{1}$$

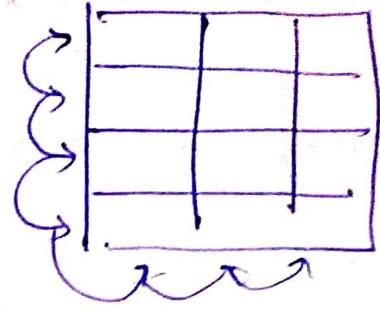
$$3T(n-1,m) \leq 3^2 T(n-2,m) \quad \textcircled{2}$$

$$3^2 T(n-2,m) \leq 3^3 T(n-3,m) \quad \textcircled{3}$$

$$3^{n-2} T(1,m) \leq 3^{n-1} T(0,m) \quad \textcircled{n}$$

$$T(n,m) \leq 3^n T(0,m)$$

We have moved along one column.  
Now, we will move along the row  
∴ TC will depend upon  $T(n, m-1)$   
We know that,



$$T(n, m) = T(n, m-1) + T(n-1, m) + T(n-1, m-1). \quad \text{Start}$$

Since  $T(n-1, m)$  has executed now  
TC will depend upon  $T(n, m-1)$ .

$$\therefore [T(n, m) \leq 3T(n, m-1)] \quad \textcircled{a}$$

Since,  
rows have been executed completely  
∴ put  $n=0$  in  $\textcircled{a}$

$$\therefore T(0, m) \leq 3T(0, m-1)$$

Now,

$$T(0, m) \leq 3\cancel{T(0, m-1)}$$

$$3\cancel{T(0, m-1)} \leq 3\cancel{3T(0, m-2)}$$

$$3^2\cancel{T(0, m-2)} \leq 3^2\cancel{3T(0, m-3)}$$

$$3^{m-2} \cancel{T(0, 1)} \leq \cancel{3^{m-1} 3T(0, 0)}$$

$$\underline{\underline{T(0, m) \leq 3^m T(0, 0)}}$$

Space complexity

$\Rightarrow O(1)$

$$\therefore T(n, m) = 3^n T(0, m)$$

$$T(0, m) = 3^m T(0, 0) - \{ \because T(0, 0) = 1 \}$$

$$\therefore T(n, m) = 3^n 3^m = \underline{\underline{3^{n+m}}}$$

Time Complexity  $\Rightarrow O(3^{n+m})$