EEEN30041 Control Systems II—Lab Exercise 1

Objectives

- Familiarise with the basics on control design and simulation using Matlab and Simulink
- Use Matlab for controller design
- Use Matlab for observer design
- Use Simulink for simulation & evaluation of closed-loop control system performance

Task 1

Simulate using Simulink the open-loop unit step response of the following system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned} \tag{1}$$

with the initial value $x_0 = [1, 1]'$.

Instruction

To complete Task 1, find the blocks needed to build the whole system first, and then connect them correctly, as shown in Fig.1.

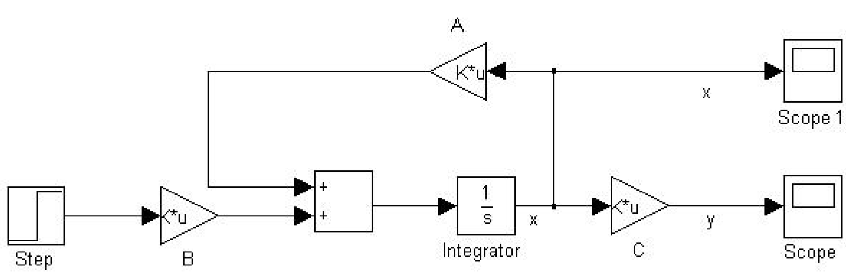

Figure 1: Diagram of Task 1

There are two ways to set values to gain blocks.

- To input matrix A, for example, double click the gain block named A, and type '[0, 1; 1, 1]' into the edit window popping up. Note that Matlab uses ';'(semicolon) to separate different rows of a matrix and ','(comma) or simply '␣' (space) to distinguish elements in a row. Therefore matrix B should be '[0; 1]'.

- Alternatively, you can input these matrices in Matlab command window (main window) first, and then recall them by referring to their names in Simulink. For example, input '$B = [0; 1]$' in Matlab command window, and then go to the diagram in Simulink, and type 'B'in the gain block B.

For multiplication option in gain blocks, always choose 'Matrix($K{*}u$)' from the dropdown list. To specify the initial conditions, double click the block 'Integrator' and type '[1; 1]'. If everything goes well, you should be able to see from both scopes that the states explode very quickly, as the open-loop system is unstable.

Task 2

Design a full-state feedback controller for the system shown in (1). The desired poles of the closed-loop system are at $[-1.5, -2.5]$. Simulate the closed-loop system with Simulink and observe the state responses.

Note: use the commands, like 'acker', to calculate the gain matrix K. Look up in Matlab the information on how to use those commands. Alternatively, you may use Ackermann's Formula, see page 34 in the lecture notes, to compute K.

Instruction

Use the command 'acker' to calculate the control gain matrix K: $K = acker(A, B, P_1)$, where P_1 is the column vector consisting of desired poles. For simulation refer to Fig.2.

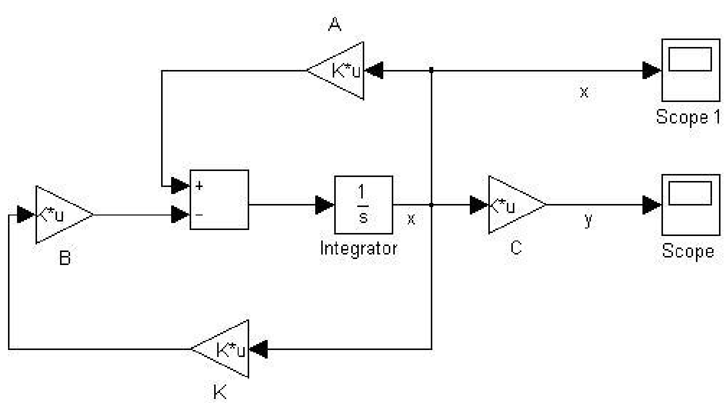

Figure 2: Diagram of Task 2

Note: It might be the case that 'acker' is not available when accessing the Matlab server due to license limit. Then $Ackermann's\ Formula$ can be used to calculate K(refer to page 34 in the lecture notes). For the n^{th} order system, if $P_1 = [s_1, s_2, \cdots, s_n]'$, $s_1, s_2, \cdots, s_n$ are the desired poles of the closed-loop system, type these codes in Matlab command window:

$P_b = [B, A*B, \cdots, \underbrace{A*A*\cdots*A}_{A^{n-1}}*B]$

$ds = poly(P_1)$

(With ds denoting the vector $[1, \alpha_1, \cdots, \alpha_n]$, then type)

$dA = \underbrace{A*A*\cdots*A}_{A^n} + \alpha_1 * \underbrace{A*A*\cdots*A}_{A^{n-1}} + \cdots + \alpha_n * eye(n)$

$K = [0, 0, \cdots, 1] * inv(P_b) * dA$

Task 3

Design an observer for the system shown in (1) with the poles of the observer error dynamics being $[-2.5, -3.5]$. Simulate the system and the observer. Show the observer error dynamics.

Instruction

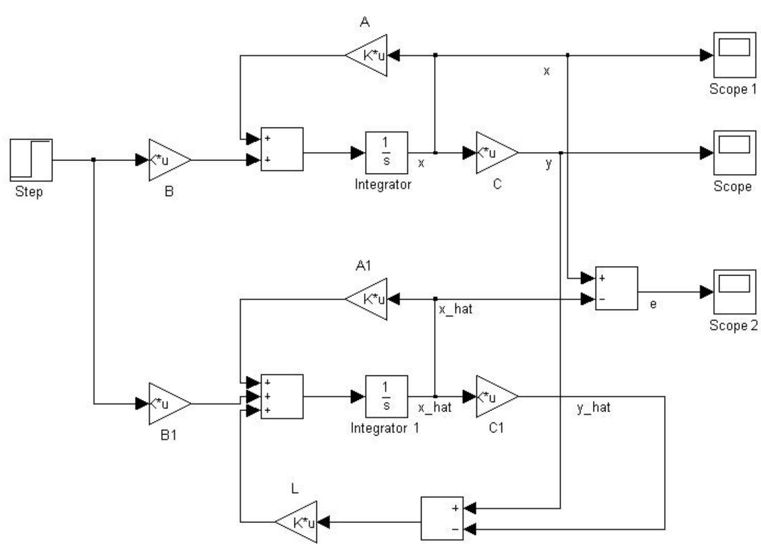

Figure 3: Diagram of Task 3

Use $L = acker(A', C', P_2)'$ to obtain the observer gain matrix L, where A', C' denote the transpose of matrices A and C respectively, P_2 is the column vector consisting of desired poles for the observer. Notice that the same control u goes to the original system as well as to the observer.

We do not need a controller for this task, so the controller designed in Task 2 would be removed from the diagram. Instead, we apply a unit step input to the system. For simulation purpose, the initial condition for the system is given as $x(0) = [1; 1]$, while the initial condition for the observer is usually set to zero. Note that the error $e(t)$ would eventually converge to zero, even if the original system is unstable itself. For simulation diagram refer to Fig.3, where $A = A1$,$B = B1$ and $C = C1$.

Task 4

Design a dynamic output compensator for system (1). The poles for the close-loop system and the observer error dynamics are the same as specified in Task 2 and Task 3, respectively. Simulate the overall system.

Instruction

See Fig.4 for simulation. Note that in the compensator design, the controller uses the estimate from the observer, that is, $u = -K\hat{x}$, not $u = -Kx$.

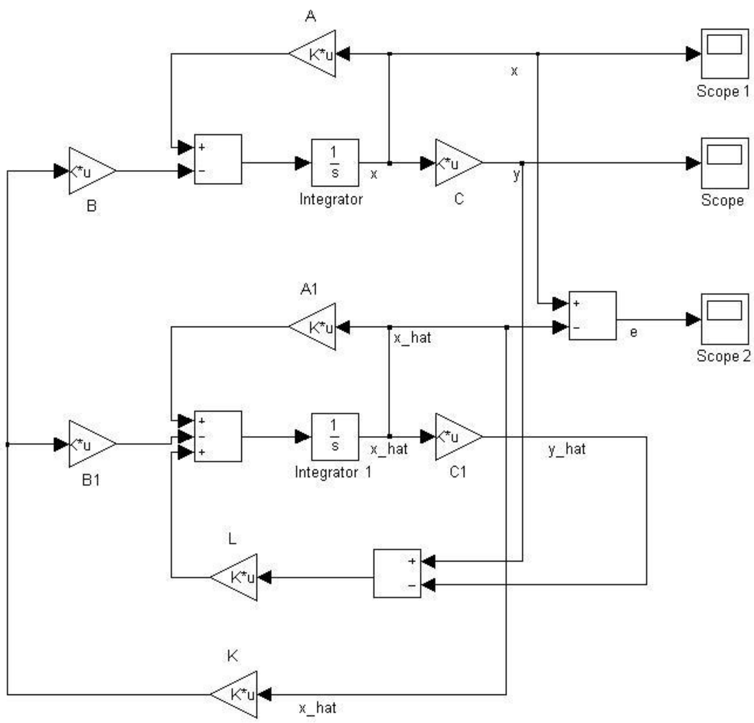

Figure 4: Diagram of Task 4

Task 5

A simplified linear model for the longitudinal dynamics of an aircraft is given by the following state-space equations (taken from Tewari, Atmospheric and space flight dynamics, p.427):

$$\begin{bmatrix} \dot{v} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -0.045 & 0.036 & -32 & -2 \\ -0.4 & -3 & -0.3 & 250 \\ 0 & 0 & 0 & 1 \\ 0.002 & -0.04 & 0.001 & -3.2 \end{bmatrix} \begin{bmatrix} v \\ \alpha \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ -30 \\ 0 \\ -10 \end{bmatrix} \delta$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ \alpha \\ \theta \\ q \end{bmatrix} \tag{2}$$

where the state variables denote longitudinal velocity (v), angle of attack (α), pitch angle (θ) and pitch rate (q), the control input is the elevator angle (δ) and the output is the pitch angle ($y = \theta$).

The task is to design a dynamic output compensator for the above system (2) and then use Simulink to verify your design. The selected pole configuration is $[-3, -1+1\jmath, -1-1\jmath, -2]$ for the close-loop system, and $[-2, -1.5+1.5\jmath, -1.5-1.5\jmath, -1]$ for the observer error dynamics. The initial values for simulation are $[0.001, 0.01, 0, 0]'$

Report

Write a report showing the results of Task 5 only, including the gain matrices K, L, and the simulated time response of the close-loop system. The report should not exceed two pages and should clearly state your full name and registration number. The report must be submitted to B21 by **2:00PM on the day one week after the lab**.