Control Systems II - Laboratory 2

March 22, 2018

1. Task 2

1.1 System description & control design

Figures 1.2.1 and 1.2.2 show the simulation in Simulink and MATLAB of a closed loop system with a Tracking and Internal Model Design. The system was first built in Simulink with the correct system state space matrices A, B and C. Then the 3 gain matrices *k1, k2* and *k3* were calculated in MATLAB using the `acker()` command which uses the Ackermann formula to compute the matrices. This formula runs for the following poles: [-3+2.5i, -3-2.5i, -4+3.5i, -4-3.5i] (closed-loop system poles). The gain matrices are shown below.

1.2 Gain Matrix *K* & simulated time response discussion

K = [430.8125 291.5000 88.5000 10.0000]

k1 = 430.8125

k2 = 291.5000

k3 = [88.5000 10.0000]

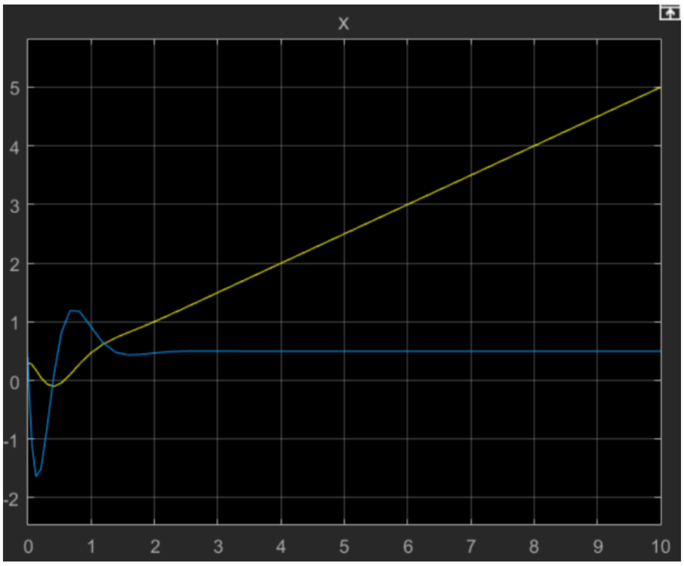

Figure 1.2.1 - Scope 1 (system input)

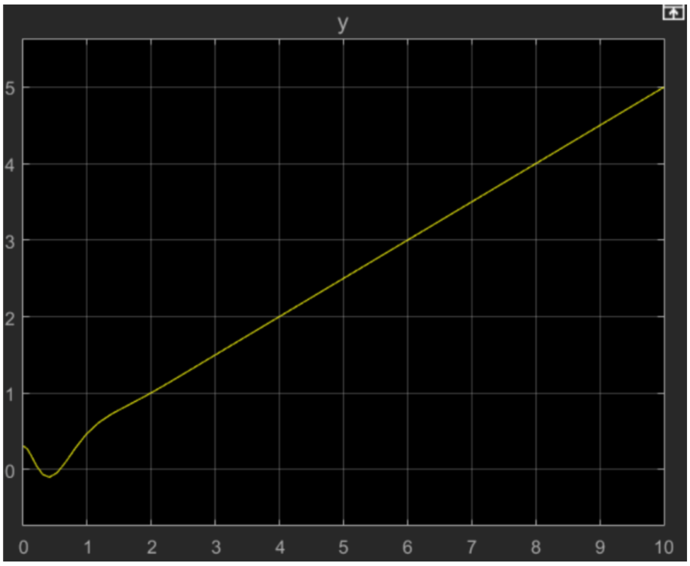

Figure 1.2.2 - Scope (system output)

Here these simulations are in line with the expected theoretical values as the system is clearly tracking the 0.5t value. Indeed, for each incrementing value of t, the output increases by half.

As expected there is a small oscillatory period at the beginning of the simulation which is expected for a tracking and internal model design. This is normal as the tracking error is non-zero and thus state feedback ensures stability and implies an asymptotic tracking with error eventually converges to zero.

2. Task 3

2.1 System description & control design

Figures 2.2.1, 2.2.2, 2.2.3 and 2.2.4 show the simulation in Simulink and MATLAB of a closed loop system with an Optimal Control Design (more specifically, a Linear Quadratic Regulator). The system was first built in Simulink with the correct system state space matrices A, B and C. Then the optimal gain matrix *K* was calculated in MATLAB, by solving the Riccati equation via the command `lqr()`, which is short for Linear Quadratic Regulator. This was done the minimise the cost function J (also known as a performance index), for both pairs of values for Q and R. The gain matrices are shown on the following page along with a discussion of the various performances.

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2.2 Gain matrices & simulated time response discussion

For R = 3260, $\mathbf{K = [-0.0004 \quad 0.5560 \quad -0.2485 \quad 0.0595]}$

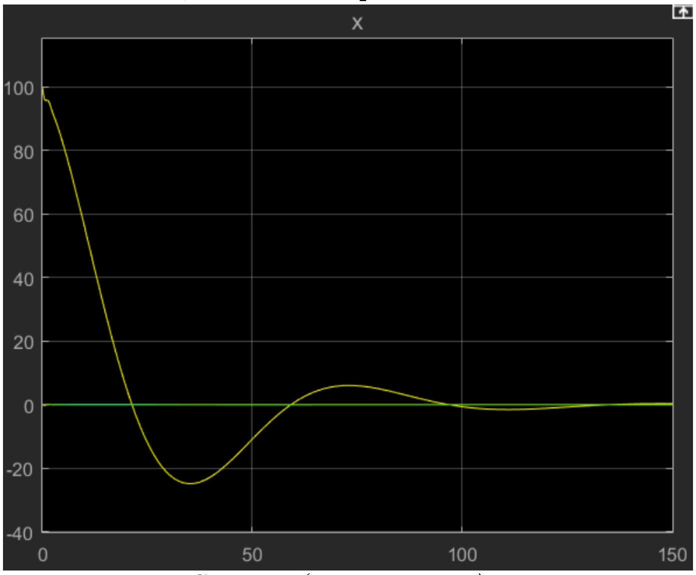

Figure 2.2.1 - Scope 1 (system input)

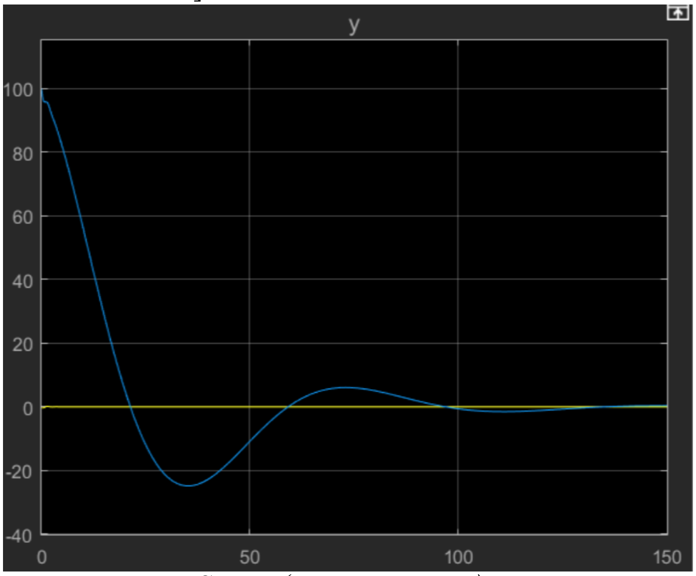

Figure 2.2.2 - Scope (system output)

For R = 800, $\mathbf{K = [-0.0016 \quad 1.0663 \quad -1.0689 \quad 0.1068]}$

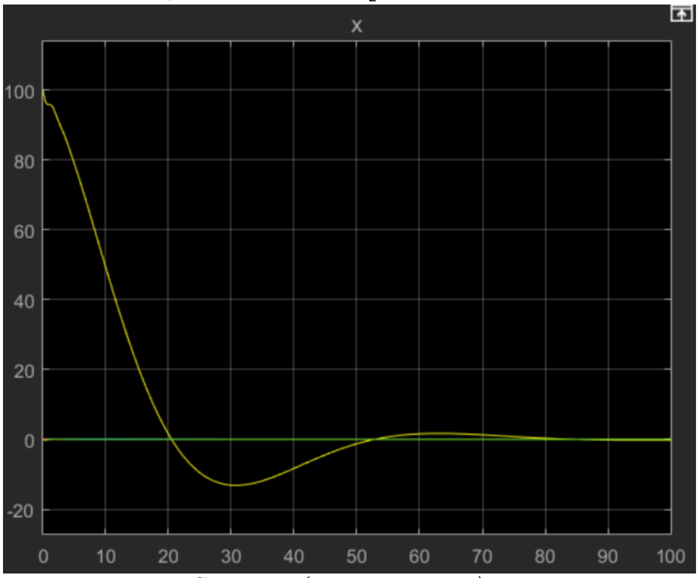

Figure 2.2.3 - Scope 1 (system input)

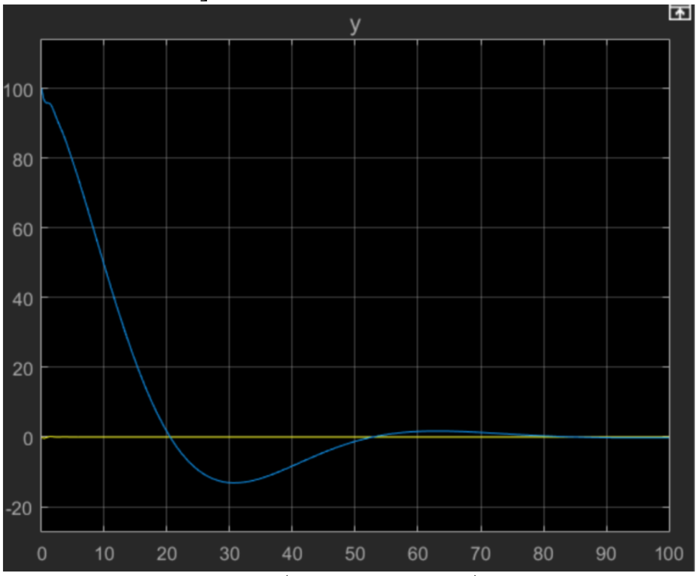

Figure 2.2.4 - Scope (system output)

Here the second controller clearly outperforms the first controller. This can evidently be seen by two differences in control response. Firstly, the second controller converges around 0 faster than the first, in around t = 100 rather than t = 150. Secondly, the second controller has less overshoot and subsequently smaller control swings. This can be seen quantitatively as, in the first case, the maximum overshoot is around -25 while in the second it is smaller at around -15.

Indeed, these graphs confirm the theoretical expected dynamics of a linear quadratic regulator. Here, we can think of minimising the cost function as an attempt to minimise the control energy. The Q and R parameters are the design parameters used to penalise the state variables and control signals. Hence since R is smaller in the second set of tuning parameters, the controller penalises with less energy thus performs better as seen in the figures above. This is because in the first case the controller is over energetic hence overshoots. This can be confirmed mathematically of as since R is larger, its relative impact on the input (u) will be larger as shown in the cost function below:

$$J = \int_0^\infty x^T Q x + R u^2 \ dt$$

On the other hand, increasing Q would increase its control impact on x (as shown in the equation above). As expected, the converse is also true as decreasing Q will diminish its impact on x.

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