Digital Mobile Communications - Coursework

October 31, 2017

Question 1

a) Thermal noise is given by:

$$P_n = kT_0B \quad (1)$$

Thus,

$$P_n = 1.38 \times 10^{-23} \times 290 \times 20 \times 10^6$$
$$\approx 8.004 \times 10^{-14} \text{ W}$$

Signal to noise ratio is given by:

$$SNR = \frac{P_r}{P_n} \quad (2)$$

Bandlimited capacity for AWGN channel is given by:

$$C = B\log_2(1 + SNR) \quad (3)$$

Thus,

$$C = 20 \times 10^6 \times \log_2\left(1 + \frac{P_r}{8.004\times10^{-14}}\right) = 50 \times 10^6$$
$$\therefore \frac{50}{20} = \log_2\left(1 + \frac{P_r}{8.004\times10^{-14}}\right)$$
$$\therefore P_r = \left(2^{\left(\frac{5}{2}\right)} - 1\right) \times 8.004 \times 10^{-14}$$
$$= 3.727 \times 10^{-13} \text{ W}$$
$$\approx -94.286 \text{ dBm}$$

Hence the required received signal power is roughly -94.286 dBm.

b) BPSK thus 1 bit per symbol and thus MPSK when M = 2. We can then state:

$$E_b = E_s = \frac{A^2T}{2} \quad (4)$$

Thus,

$$E_s = \frac{0.001^2\times0.01}{2} = 5 \times 10^{-9} \text{ J}$$

And we also know that:

$$SNR_{per\ bit} = \frac{E_s}{N_0} \quad (5)$$

Thus,

$$SNR_{per\ bit} = \frac{5\times10^{-9}}{10^{-9}} = 5 \approx 6.990 \text{ dB}$$

Hence the SNR per bit is roughly 6.990 dB.

c) The impulse response for the matched filter is given below by figure 1.1:

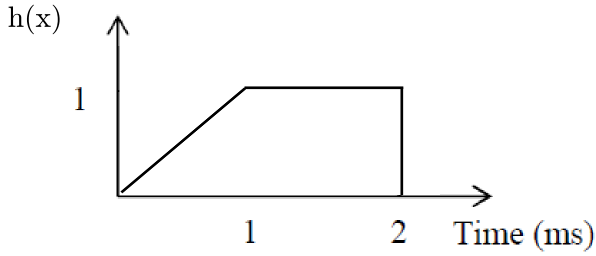

Figure 1.1 - Matched Filter

Hence matched filter is simply a time reversed and delayed version of the original pulse.

This is not a good pulse because, as we know from Fourier Series, sharp edges cause high frequency components. Such high frequency components imply a wider bandwidth which may cause interference with neighbouring symbols. This is known as ISI (Inter-Symbol Interference) which is why we need pulse shaping.

Question 2

a) The noise figure of the system is given by:

$$F_{sys(linear)} = F_{A(linear)} + F_{1(linear)} - 1 \quad (6)$$

Thus,

$$G_{sys} = 20\text{dB} = 100$$
$$F_A = 2\text{dB} \approx 1.585$$
$$F_1 = 8\text{dB} \approx 6.310$$
$$\therefore F_{sys} = 1.585 + 6.310 - 1 = 6.895$$

Noise power is given by:

$$P_n = F_{sys}\, G\, k\, T_0\, B \quad (7)$$

Thus,

$$P_n = 6.895 \times 1.38 \times 10^{-21} \times 290 \times 1.25 \times 10^6$$
$$\approx 3.449 \times 10^{-12} \text{ W} = -84.623 \text{ dBm}$$

Hence the noise power in the receiver is approximately -84.623 dBm.

b) The signal to noise ratio (SNR) is given by:

$$SNR = G + P_r - P_n \quad (8)$$

SNR must be larger than or equal to 15dB thus,

$$P_r = SNR + P_n - G$$
$$P_r \geq 45 \text{ dBm} - 84.623\text{dBm} - 50\text{dBm}$$
$$\therefore P_r \geq -89.623 \text{ dBm}$$

Hence minimum received signal power is -89.623 dBm.

c) Path loss is given by:

$$L_p = P_t + G_t + G_r + G_{sys} - SNR - P_n \tag{9}$$

From equation (9), maximum path loss occurs when SNR is minimum, thus:

$$L_p = 20 + 3 + 13 + 20 - 15 - (-114.623)$$
$$= 155.623 \text{ dBW}$$

Hence the maximum path loss is 155.623 dBW.

d) Path loss at distance d_0 is given by:

$$PL(d_0) = -10 \log \frac{\lambda^2}{(4\pi d)^2} \tag{10}$$

And since:

$$c = f\lambda \tag{11}$$

Thus,

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{1800\times10^6} = \frac{1}{6}$$
$$\therefore PL(d_0) = -10 \log \frac{\left(\frac{1}{6}\right)^2}{(4\pi 1)^2} \approx 37.547 \text{ dBW}$$

Since path loss at a distance d is given by:

$$PL(d) = PL(d_0) + 10n \log\left(\frac{d}{d_0}\right) \tag{12}$$

Thus, as we know from (9) that maximum $PL(d)$ is 155.623 dBW and $n = 4$, rearranging (12) for distance d gives:

$$d = 10^{\frac{PL(d) - PL(d_0)}{10\,n}} \times d_0 = 10^{\frac{155.623 - 37.547}{10\times4}} \times 1$$
$$= 895.159 \text{ m}$$

Hence maximum radius that this cell could cover is 895.159 meters.

Question 3

a) From Question3a.m RMS delay spread of the uniform power delay profile is 1.708×10^{-6} s.

b) From Question3b.m RMS delay spread vs decay factor is shown as follows:

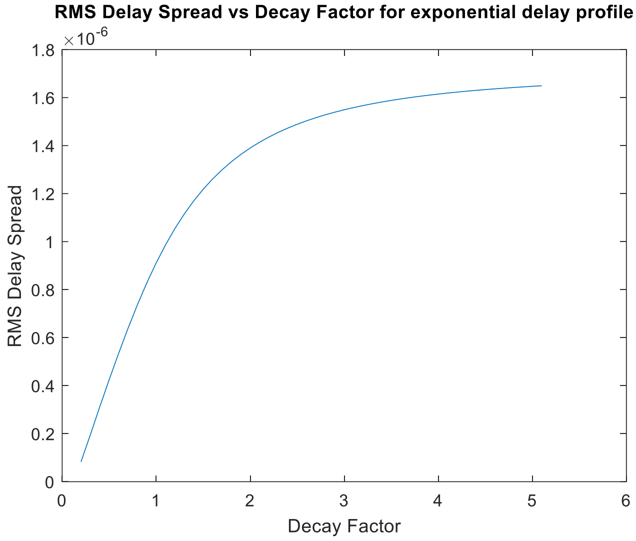

Figure 3.1 - RMS Delay Spread vs Decay Factor

c) Power delay profiles show us how many echoes there are and what the statistically significant average power of each echo is. Delay spread is a measure of difference in arrival times of the most and least significant multipath components (or echoes) of communication channels. Typically, delay spread is considered for its effect on ISI. Generally, if the symbol duration is very large with respect to delay spread, the channel is considered free from ISI. In the frequency domain, this is shown by the coherence bandwidth ($B_c \approx 1/D$). This is the bandwidth over which the channel is assumed to be flat. Indeed, we know that coherence bandwidth is inversely proportional to the delay spread (the shorter the delay spread, the larger the coherence bandwidth).

Figure 3.1 shows that as decay factor increases, RMS delay spread (standard deviation of excess delay) increases in a bounded exponential curve. The upper bound is approximately 1.7×10^{-6} s. We can notice that, as expected, the RMS delay of the exponential profile asymptotes toward the RMS delay of the uniform power profile for high values of decay factor.

This is because as the exponential factor increases, the power delay profile P approaches a linear decreasing power delay profile. This can be seen either by modifying the MATLAB code (shown in section 3d) or by inspecting the equation of exponential power delay from first principles:

$$P_k = c\, e^{\frac{-k/\Delta\tau}{c}} \tag{13}$$

Thus from (13), the standard deviation of an exponential delay profile can only be as high as its equivalent uniform profile. By taking the limit of (13) as c tends to infinity:

$$\lim_{c \to \infty} P_k = \lim_{c \to \infty} c\, e^{\frac{-k/\Delta\tau}{c}} \approx c \quad \therefore \quad P_{c \to \infty} \approx c \qquad (14)$$

From (14), P_k is demonstrated to reach a constant, uniform delay profile thus RMS delay spread approaches 1.7×10^{-6} s, as calculated in section 3a.

Again from (13), the smaller the decay factor will cause the most curved power delay profile thus the smallest RMS delay spread. In real terms, if the standard deviation of the excess delay is small, it means there aren't lots of echoes going on in the channel. Thus, the smaller the decay factor, the less power is spread to the echoes (multipath components).

The COST231 W-I model distinguishes between Line-Of-Sight (LOS) and Non-Line-Of-Sight (NLOS) cases. Often, objects like buildings, hills and high voltage power lines cause NLOS conditions. NLOS lowers the effective received power. LOS issues are usually addressed by using better antennas while NLOS usually requires alternative paths or multipath propagation techniques.

In this case, the uniform delay profile is effectively used to model an NLOS channel. This is because all instances of the signal will have no direct path between transmitter and receiver thus the echoes will have similar powers.

Furthermore, the exponential delay profile is used to model an LOS channel. This is because the first instance of the signal will always have a much higher power than subsequent instances as it has a direct path (which will be strongest) followed by weaker indirect echoes at later instances. The route taken by the indirect instances will have different gain, phase and arrival time due to effects such as longer distance, reflection, diffraction, scattering, interference, multipath fading and more.

Small RMS delay spreads imply a larger coherence bandwidth due to inversely proportional relation. If the coherence bandwidth is larger than the signal bandwidth (like in AMPS) flat fading will occur. This means no ISI will result and no equaliser will be needed which is significantly useful. However, deep fading could degrade system performance.

On the other hand, large RMS delay spreads imply a larger frequency correlation thus a smaller coherence bandwidth. Here, if the coherence bandwidth is smaller than the signal bandwidth (like in GSM) then frequency selective fading will occur. This is not desirable as ISI will occur and an equaliser will be needed.

d) `Question3a.m` is programmed as follows:

```
%Author: 9563426
%Subject: Digital Mobile Communications

%1. Set up global variables
channel_length = 6;
bin_width = 0.000001;

%2. Caculate RMS delay spread of uniform
power delay profile
tau = bin_width;
P = ones(1, channel_length);
mean_excess_delay = (P(1)*0*tau + P(2)*1*tau
+ P(3)*2*tau + P(4)*3*tau + P(5)*4*tau +
P(6)*5*tau)/(P(1)+P(2)+P(3)+P(4)+P(5)+P(6));
second_moment = (P(1)*(0*tau^2) +
P(2)*((1*tau)^2) + P(3)*((2*tau)^2) +
P(4)*((3*tau)^2) + P(5)*((4*tau)^2) +
P(6)*((5*tau)^2))/(P(1)+P(2)+P(3)+P(4)+P(5)+
P(6));
RMS_delay_spread = sqrt(second_moment -
(mean_excess_delay*mean_excess_delay));

%3. Return value to command window
disp(['RMS delay spread of uniform power
delay profile is: '
num2str(RMS_delay_spread) 's.'])
```

`Question3b.m` is programmed as follows:

```
%Author: 9563426
%Subject: Digital Mobile Communications

%1. Set up global variables
channel_length = 6;
bin_width = 0.000001;
exp_factor = 0.1;
xvals = [];
yvals = [];
i = 1;

%2. Caculate RMS delay spread of exponential
power delay profile (with increasing decay
factor)
while (exp_factor <= 5)
    tau = bin_width;
    exp_factor = exp_factor + 0.1;
    P = exp_factor*exp(-
[1:channel_length]/exp_factor);

    mean_excess_delay = (P(1)*0*tau +
P(2)*1*tau + P(3)*2*tau + P(4)*3*tau +
P(5)*4*tau +
P(6)*5*tau)/(P(1)+P(2)+P(3)+P(4)+P(5)+P(6));
    second_moment = (P(1)*(0*tau^2) +
P(2)*((1*tau)^2) + P(3)*((2*tau)^2) +
P(4)*((3*tau)^2) + P(5)*((4*tau)^2) +
P(6)*((5*tau)^2))/(P(1)+P(2)+P(3)+P(4)+P(5)+
P(6));
```

```
    RMS_delay_spread = sqrt(second_moment -
(mean_excess_delay*mean_excess_delay));

    xvals(i) = exp_factor;
    yvals(i) = RMS_delay_spread;
    i = i+1;
end

%3. Plot into a graph
plot(xvals, yvals);
title({'RMS Delay Spread vs Decay Factor for
exponential delay profile';''});
xlabel('Decay Factor')
ylabel('RMS Delay Spread')
```