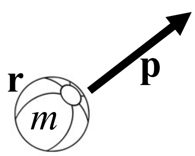
Matter
(Classical Mechanics)

- Objects have well-defined mass, position, and momentum.
- Any mass, position, and momentum is possible.

$$\mathbf{p} = m\,d\mathbf{r}/dt$$

$$m > 0; m \in \mathbb{R}$$

$$\mathbf{r}, \mathbf{p} \in \mathbb{R}^3$$



$$\mathbf{r}(t) = \mathbf{r}(0) + \tfrac{1}{m}\int_0^t \mathbf{p}(\tau)\,d\tau$$

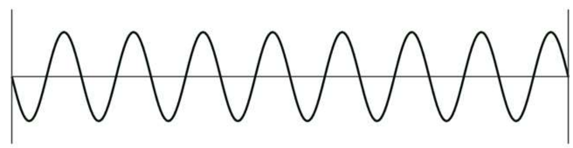
Radiation (light)
(Classical Electromagnetism)

- Waves have no position, mass, or momentum; they extend forever.
- The energy of a wave is proportional to its amplitude.
- Wavelength and frequency are arbitrary, linked by:

$$c = \lambda \nu$$

$$\lambda > 0; \lambda \in \mathbb{R}$$

$$\nu > 0; \nu \in \mathbb{R}$$



$$\Psi(\mathbf{r}, t) = Ae^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} \qquad |\mathbf{k}| = \tfrac{2\pi}{\lambda}; \omega = 2\pi\nu$$

It's a beautiful day in Hamilton Ontario and I hope you're doing well.

The purpose of this video is to introduce you to two key ideas in quantum mechanics. The first idea is wave-particle duality. Wave-particle duality means that matter sometimes behaves like waves, and that radiation sometimes behaves like particles. The second idea is quantization, which means that not every value for an observable is allowed. For example, if you measure the spin of an electron it is either "up" or "down", never "kinda up" or "mostly sideways."

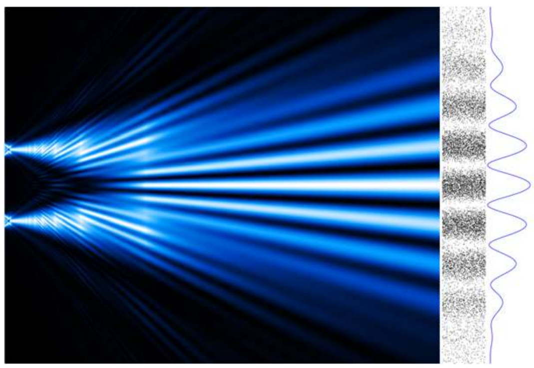
This is quite different from classical physics. In classical physics, matter is governed by the laws of classical mechanics. So particles have well-defined properties. Specifically, the mass, position, and momentum are well-defined. Also *any* mass, position, and momentum is possible. So if I want to make a glass bead with a mass of 50.21974 grams, place it exactly 2.54 cm from the edge of my desk, and then give it a velocity of 4.252009 meters/second, I can do that. You can increase the potential energy of the bead by raising the table, and increase its kinetic energy by increasing its velocity.

In classical physics, radiation (that is, light) is governed by the laws of classical electromagnetism. Light has no mass, but even the idea of light having mass is sort of absurd, since it's a wave that extends forever, with no position or momentum. Instead of position and momentum, light has a wavelength and a frequency. These parameters are related to each other because light is an oscillation in the electric and magnetic field with a specified frequency that propagates at a universal speed. You add energy to a wave by by increasing its amplitude.

Matter

(Davisson-Germer Experiment)

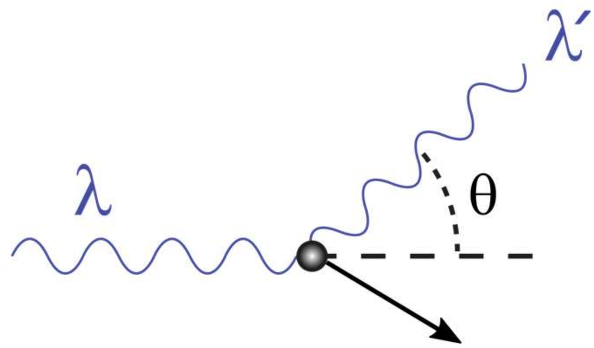
- Electrons show constructive and deconstructive interference as if it were a wave.



Radiation (light)

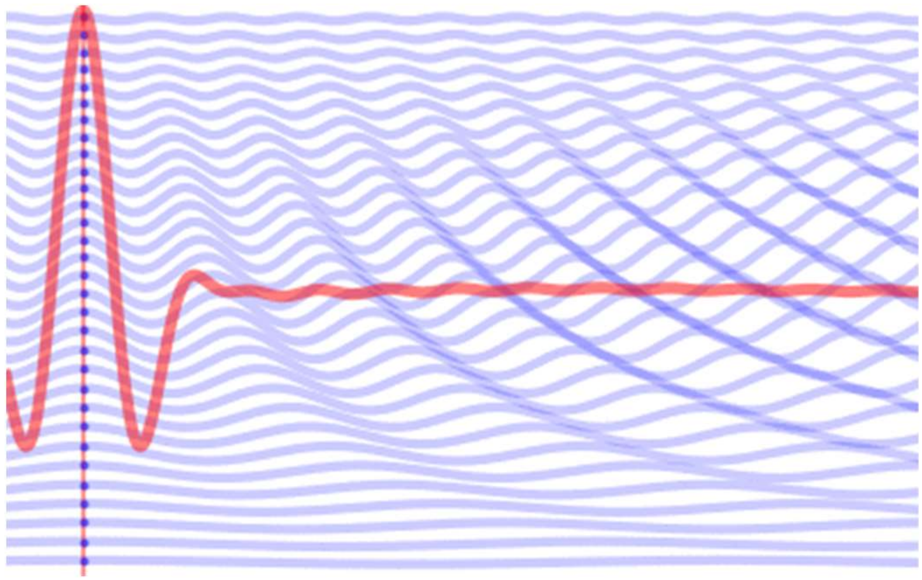
(Compton Experiment)

- Light transfers momentum to electrons as if it were composed of particles.



The genesis of quantum mechanics was some strange experiments that seemed to show radiation behaving like matter, and matter behaving like radiation. For example, the observed spectrum of blackbody radiation and the photoelectric effect suggested that radiation was "chunky", specifically, that just as an atom or molecule has an integer number of electrons, that radiation consists of an integer number of light "particles," called photons. The energy of these photons is proportional to their frequency, and Arthur Compton did an experiment that showed that they had momentum. Photons even have an effective, or relativistic mass, although not a real "rest mass" like matter. Around the same time, the Davisson-Germer electron diffraction experiment, that showed electrons diffracting through a material according to Bragg's law for radiation. So electrons were diffracting like X-rays in that experiment, and photons were knocking around electrons like classical particles in the Compton experiment.

Wave-Particle Duality



The conclusion was that there was not two distinct types of "stuff" in the university, matter and radiation. Rather, there is only one type of stuff in the universe. And that stuff is "chunky", as are its properties. The technical word for "chunkiness" is quantization. In classical physics, a quantity like energy could be any real number. But in quantum physics, the energy can only have certain discrete values.

Matter

(Quantum)

- Objects have well-defined mass.

$$m_{\text{total}} = n_{\text{particles}} m_{\text{particle}}$$

$$E_{\text{rest-mass}} = n_{\text{particles}} m_{\text{particle}} c^2$$

- Position and momentum cannot be observed simultaneously.
- Matter has a characteristic wavelength that is inversely proportional to its momentum.
- Energy comes in chunks. *E.g.*, there is no energy for the Sodium atom between the following values:

$$E\left(1s^2 2s^2 2p^6 3s^1\right); E\left(1s^2 2s^2 2p^6 3p^1\right)$$

Radiation (light)

(Quantum)

- Light is made up of discrete photons.
- The energy of a photon is proportional to its frequency.

$$E_{\text{photon}} = h \cdot \nu$$

$$E_{\text{light}} = n_{\text{photons}} E_{\text{photon}} = n_{\text{photons}} h\nu$$

- Photons have momentum.

$$p_{\text{photon}} = h \cdot \lambda^{-1}$$

- Photons have no rest mass but they have an effective “relativistic” mass.

$$m_{\text{effective;photon}} = p_{\text{photon}} / c = h \cdot \nu c^{-2}$$

- Planck’s constant is the fundamental proportionality constant.

$$h = 6.62607 \cdot 10^{-34} \text{ J} \cdot \text{s}$$

Light is chunky: the energy of a photon is proportional to its frequency, and the momentum of a photon is proportional to its wavenumber. The proportionality constant is called Planck's constant. To increase the energy, you can increase the number of photons.

Matter is also chunky. Obviously, the number of electrons in a system is an integer. But energy comes in chunks too. This is sort of intuitive for chemists; we write down an electron configuration, and understand that there are only certain orbitals that exist. You can put an electron in a 2p orbital or a 3p orbital but not a 2.71-p orbital. Similarly, you can increase the energy of an atom or molecule by exciting an electron from the HOMO to the LUMO, but there is not lower-energy way to excite the electron.

Let's make it even simpler and talk about a phenomena you have probably experienced, quantized dating. Back when I started teaching, I was notorious for using my unrequited crush on Uma Thurman as an analogy in my lectures. Suppose I ever found the courage to ask Miss Thurman out for a cup of coffee. She would probably look at me and say, "Based on my standards, you excite me to 1% of my threshold for a coffee date." But that's great! I get to share one sip of coffee with Uma Thurman! Best date ever!

Of course, sadly it doesn't work that way. In dating, as in chemistry, a minimum energy is required to excite the target, and anything less than that energy has no effect. Indeed, the first direct evidence for quantization in atoms and molecules came from the Franck-Hertz experiment. In that experiment, an electron collides with an atom but unless the electron is hot enough (has enough kinetic energy), the atom is not excited and the electron continues on its cold and lonely way.