CalcPauliTransferEval

```
SetDirectory @ NotebookDirectory[];
Import["../Link/QuESTlink.m"];
```

Doc

```
? CalcPauliTransferEval
```

Symbol

CalcPauliTransferEval[pauliString, ptMaps] returns the full evolution history
of the given Pauli string under the given list of PTMap operators. This is often
unnecessary to call directly – most users can call ApplyPauliTransferMap[] or
DrawPauliTransferEval[] instead – unless you wish to store or process the evaluation history.

CalcPauliTransferEval[pauliString, circuit] evolves the Pauli
string under the PTMaps automatically calculated from the given circuit.

There are two possible return formats, informed by option "OutputForm", which are respectively fast
and slow to evaluate, and both of which can be passed to functions like DrawPauliTransferEval[].

The "Simple" output is a list of sublists, each corresponding to a layer in the evaluation history (i.e.
the operation of a PTMap upon the current Pauli string) including the initial Pauli string.
Each item therein represents a Pauli product state and has form {prod,id,parents} where
'prod' is a Pauli basis state expressed in base−4 digits (see ?GetPauliStringReformatted),
'id' is a unique integer identifying the state, and 'parents' is a list of tuples of form
{parentId, factor}. These indicate the ancestor Pauli states from which the id'd state
was produced under the action of the previous PTMap, and the factor that the map
multiplies upon that parent state. The basis products of the initial state have parentId=0.

The "Detailed" output is an Association with the following items:

• "Ids" is a list of integers uniquely identifying each node
in the evaluation graph. Note these are not gauranteed to be contiguous
due to the merging of incident Pauli states (see "CombineStrings" below).

• "Layers" groups "Ids" into sublists according to their depth in the graph, i.e. which ptMap produced
the node. There are Length[ptMaps]+1 layers, including the initial pauliString layer.

• "States" is an Association of id –> pauliProduct,
identifying the Pauli basis state associated with the id'd node.

• "Parents" is an Association of id –> list of parent ids. A node's parents are the states of the
previous layer who were modified by the previous layer's PTMap to the node's state.

• "Children" is an Association of id –> list of child ids. A node's children are
the states the node is transformed to when modified by the next layer's PTMap.

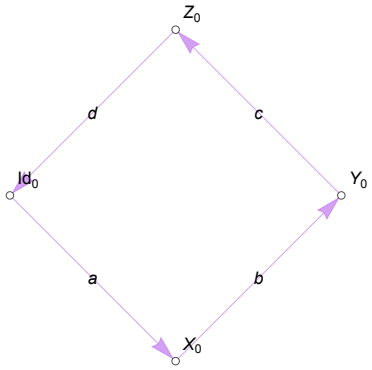
• "ParentFactors" is an Association of id –> Association, where the inner
Association is of parentId –> coefficient. The inner Association records the

factors multiplied upon the parents when the PTMap produced the node's state.
• "Weights" is an Association of id –> weight, where
weight is the number of non–identity Paulis in the id'd nodes state.
• "Indegree" is an Association of id –> degree, indicating the number of parents.
• "Outdegree" is an Association of id –> degree, indicating the number of children.
• "Coefficients" is an Association of id –> coeff, where coeff is the
coefficient of the id'd node's Pauli basis state at the node's layer of evaluation.
• "Strings" is a list of sums of weighted Pauli strings; one for each layer of evaluation. The resulting
Pauli string of the full circuit upon the initial Pauli string is the Last item of this list. The
strings are not automatically simplified, so each might be worth passing to SimplifyPaulis[].
• "NumQubits" is the number of qubits assumed
during the evaluation, informed by the initial Pauli string and PTMaps.
• "NumNodes" is the total number of nodes (or basis Pauli
states) processed during evaluation. This is merely the length of "Ids".
• "NumLeaves" is the number of nodes in the final
layer, equivalent to the number of Pauli products in the output string.
CalcPauliTransferEval accepts the below options:
• "OutputForm" –> "Simple" (default) or "Detailed", as explained above.
• "CombineStrings" –> False which disables combining incident Pauli
strings, so that the result is an acyclic tree, and each node has a single parent.
• "CacheMaps" which controls the automatic
caching of generated PTMaps (see ?ApplyPauliTransferMap).
• AssertValidChannels –> False which disables the
simplification of symbolic Pauli string coefficients (see ?AssertValidChannels).

Correctness

Maps

```
map = PTMap0[0 → {{1, a}}, 1 → {{2, b}}, 2 → {{3, c}}, 3 → {{0, d}}];
DrawPauliTransferMap[map]
```




```
CalcPauliTransferEval[X0, map];
Column @ MapAt[GetPauliString, %, {All, All, 1}]
```

```
{{X0, 1, {{0, 1}}}}
{{Y0, 2, {{1, b}}}}
```

```
CalcPauliTransferEval[X0, {map, map, map}];
Column @ MapAt[GetPauliString, %, {All, All, 1}]
```

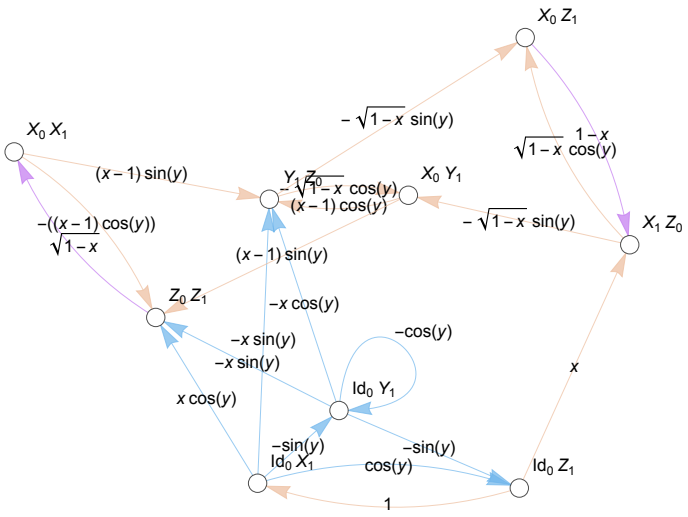
```
{{X0, 1, {{0, 1}}}}
{{Y0, 2, {{1, b}}}}
{{Z0, 3, {{2, c}}}}
{{Id0, 4, {{3, d}}}}
```

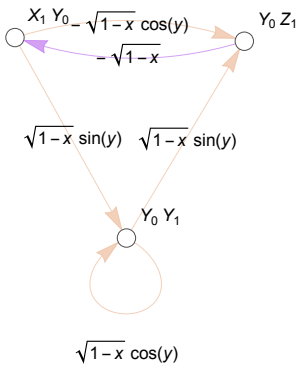
```
CalcPauliTransferEval[X5, {map, map, map, map, map, map}];
Column @ MapAt[GetPauliString, %, {All, All, 1}]
```

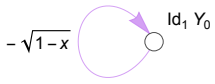
```
{{X5, 1, {{0, 1}}}}
{{X0 X5, 2, {{1, a}}}}
{{X5 Y0, 3, {{2, b}}}}
{{X5 Z0, 4, {{3, c}}}}
{{X5, 5, {{4, d}}}}
{{X0 X5, 6, {{5, a}}}}
{{X5 Y0, 7, {{6, b}}}}
```

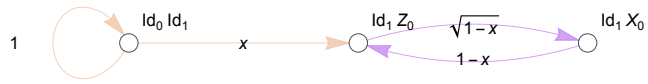
Circuits

```
circ = Circuit[H0 H1 Damp0[x] Rx1[y] ];
DrawPauliTransferMap[circ]
```










```
CalcPauliTransferEval[ Y0 Y1, circ]
GetPauliString /@ %[[-1, All, 1]]
```

```
{{{{2, 2}, 1, {{0, 1}}}}, {{{2, 2}, 2, {{1, -1}}}},
 {{{2, 2}, 3, {{2, -1}}}}, {{{2, 2}, 4, {{3, √(1 - x)}}}},
 {{{2, 2}, 5, {{4, Cos[y]}}}}, {{3, 2}, 6, {{4, Sin[y]}}}}}
```

```
{Y0 Y1, Y0 Z1}
```

```
CalcPauliTransferEval[ Y1, circ];
GetPauliString /@ %[[-1, All, 1]]
```

```
ApplyPauliTransferMap[Y1, circ]
```

```
{Y1, Z1, Y1 Z0, Z0 Z1}
```

```
-Cos[y] Y1 - x Cos[y] Y1 Z0 - Sin[y] Z1 - x Sin[y] Z0 Z1
```

```
repeated = Join @@ Table[circ, 10];

CalcPauliTransferEval[ Y1, repeated];
Plus @@ GetPauliString /@ %[[-1, All, 1]]

(ApplyPauliTransferMap[Y1, repeated] /. {x → .1, y → .15})[[All, 2 ;;]]
```

$X_1 + X_0\,X_1 + Y_1 + X_0\,Y_1 + X_1\,Z_0 + Y_1\,Z_0 + Z_1 + X_0\,Z_1 + Z_0\,Z_1$

$X_1 + X_0\,X_1 + Y_1 + X_0\,Y_1 + X_1\,Z_0 + Y_1\,Z_0 + Z_1 + X_0\,Z_1 + Z_0\,Z_1$

Options

OutputForm -> “Detailed”

```
circ = GetKnownCircuit["QFT", 3];
data = CalcPauliTransferEval[X0, circ, "OutputForm" → "Detailed"];

Column @ Keys @ data
```

```
Ids
Layers
NumQubits
NumNodes
NumLeaves
States
Parents
Children
ParentFactors
Weights
Indegree
Outdegree
Coefficients
Strings
```

```
data["Ids"]
```

```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
 19, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43}
```

```
data["Layers"]
```

```
{{1}, {2}, {3}, {4, 5, 6, 7}, {8, 9, 10, 11}, {12, 13, 14, 15, 16, 17, 18, 19},
 {28, 29, 30, 31, 32, 33, 34, 35}, {36, 37, 38, 39, 40, 41, 42, 43}}
```

```
data["States"]
```

$\langle|\, 1 \to X_0,\ 2 \to X_0,\ 3 \to X_0,\ 4 \to X_0,\ 5 \to X_0\,Z_2,\ 6 \to Y_0,\ 7 \to Y_0\,Z_2,\ 8 \to X_0,\ 9 \to X_0\,Z_2,$
$10 \to Y_0,\ 11 \to Y_0\,Z_2,\ 12 \to X_0,\ 13 \to X_0\,Z_1,\ 14 \to Y_0,\ 15 \to Y_0\,Z_1,\ 16 \to X_0\,Z_2,$
$17 \to X_0\,Z_1\,Z_2,\ 18 \to Y_0\,Z_2,\ 19 \to Y_0\,Z_1\,Z_2,\ 28 \to Z_0,\ 29 \to Z_0\,Z_1,\ 30 \to Y_0,\ 31 \to Y_0\,Z_1,$
$32 \to Z_0\,Z_2,\ 33 \to Z_0\,Z_1\,Z_2,\ 34 \to Y_0\,Z_2,\ 35 \to Y_0\,Z_1\,Z_2,\ 36 \to Z_2,\ 37 \to Z_1\,Z_2,$
$38 \to Y_2,\ 39 \to Y_2\,Z_1,\ 40 \to Z_0\,Z_2,\ 41 \to Z_0\,Z_1\,Z_2,\ 42 \to Y_2\,Z_0,\ 43 \to Y_2\,Z_0\,Z_1\,|\rangle$

```
data["Parents"]
```

```
⟨| 1 → {}, 2 → {1}, 3 → {2}, 4 → {3}, 5 → {3}, 6 → {3}, 7 → {3}, 8 → {4}, 9 → {5},
 10 → {6}, 11 → {7}, 12 → {8, 10}, 13 → {8, 10}, 14 → {8, 10}, 15 → {8, 10},
 16 → {9, 11}, 17 → {9, 11}, 18 → {9, 11}, 19 → {9, 11}, 28 → {12}, 29 → {13},
 30 → {14}, 31 → {15}, 32 → {16}, 33 → {17}, 34 → {18}, 35 → {19}, 36 → {28},
 37 → {29}, 38 → {30}, 39 → {31}, 40 → {32}, 41 → {33}, 42 → {34}, 43 → {35} |⟩
```

```
data["Children"]
```

```
⟨| 1 → {2}, 2 → {3}, 3 → {4, 5, 6, 7}, 4 → {8}, 5 → {9}, 6 → {10},
 7 → {11}, 8 → {12, 13, 14, 15}, 9 → {16, 17, 18, 19}, 10 → {12, 13, 14, 15},
 11 → {16, 17, 18, 19}, 12 → {28}, 13 → {29}, 14 → {30}, 15 → {31},
 16 → {32}, 17 → {33}, 18 → {34}, 19 → {35}, 28 → {36}, 29 → {37},
 30 → {38}, 31 → {39}, 32 → {40}, 33 → {41}, 34 → {42}, 35 → {43},
 36 → {}, 37 → {}, 38 → {}, 39 → {}, 40 → {}, 41 → {}, 42 → {}, 43 → {} |⟩
```

```
data["Weights"]
```

```
⟨| 1 → 1, 2 → 1, 3 → 1, 4 → 1, 5 → 2, 6 → 1, 7 → 2, 8 → 1,
 9 → 2, 10 → 1, 11 → 2, 12 → 1, 13 → 2, 14 → 1, 15 → 2, 16 → 2, 17 → 3,
 18 → 2, 19 → 3, 28 → 1, 29 → 2, 30 → 1, 31 → 2, 32 → 2, 33 → 3, 34 → 2,
 35 → 3, 36 → 1, 37 → 2, 38 → 1, 39 → 2, 40 → 2, 41 → 3, 42 → 2, 43 → 3 |⟩
```

```
data["Indegree"]
data["Outdegree"]
```

```
⟨| 1 → 0, 2 → 1, 3 → 1, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 1,
 9 → 1, 10 → 1, 11 → 1, 12 → 2, 13 → 2, 14 → 2, 15 → 2, 16 → 2, 17 → 2,
 18 → 2, 19 → 2, 28 → 1, 29 → 1, 30 → 1, 31 → 1, 32 → 1, 33 → 1, 34 → 1,
 35 → 1, 36 → 1, 37 → 1, 38 → 1, 39 → 1, 40 → 1, 41 → 1, 42 → 1, 43 → 1 |⟩
```

```
⟨| 1 → 1, 2 → 1, 3 → 4, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 4,
 9 → 4, 10 → 4, 11 → 4, 12 → 1, 13 → 1, 14 → 1, 15 → 1, 16 → 1, 17 → 1,
 18 → 1, 19 → 1, 28 → 1, 29 → 1, 30 → 1, 31 → 1, 32 → 1, 33 → 1, 34 → 1,
 35 → 1, 36 → 0, 37 → 0, 38 → 0, 39 → 0, 40 → 0, 41 → 0, 42 → 0, 43 → 0 |⟩
```

```
data["Coefficients"]
```

$$\left\langle \left| 1 \to 1,\ 2 \to 1,\ 3 \to 1,\ 4 \to \text{Cos}\left[\frac{\pi}{8}\right]^2,\ 5 \to \text{Sin}\left[\frac{\pi}{8}\right]^2,\ 6 \to \frac{1}{2\sqrt{2}},\right.\right.$$
$$7 \to -\frac{1}{2\sqrt{2}},\ 8 \to \text{Cos}\left[\frac{\pi}{8}\right]^2,\ 9 \to \text{Sin}\left[\frac{\pi}{8}\right]^2,\ 10 \to \frac{1}{2\sqrt{2}},\ 11 \to -\frac{1}{2\sqrt{2}},$$
$$12 \to -\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,\ 13 \to \frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,\ 14 \to \frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,$$
$$15 \to \frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,\ 16 \to \frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,\ 17 \to -\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,$$
$$18 \to -\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,\ 19 \to -\frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,\ 28 \to -\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,$$
$$29 \to \frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,\ 30 \to -\frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,\ 31 \to -\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,$$
$$32 \to \frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,\ 33 \to -\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,\ 34 \to \frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,$$
$$35 \to \frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,\ 36 \to -\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,\ 37 \to \frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,$$
$$38 \to -\frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,\ 39 \to -\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2,\ 40 \to \frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,$$
$$\left.\left.41 \to -\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,\ 42 \to \frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2,\ 43 \to \frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2 \right|\right\rangle$$

```
Column @ Normal @ data["Strings"]
```

X_0

X_0

X_0

$\text{Cos}\left[\frac{\pi}{8}\right]^2 X_0 + \frac{Y_0}{2\sqrt{2}} + \text{Sin}\left[\frac{\pi}{8}\right]^2 X_0 Z_2 - \frac{Y_0 Z_2}{2\sqrt{2}}$

$\text{Cos}\left[\frac{\pi}{8}\right]^2 X_0 + \frac{Y_0}{2\sqrt{2}} + \text{Sin}\left[\frac{\pi}{8}\right]^2 X_0 Z_2 - \frac{Y_0 Z_2}{2\sqrt{2}}$

$$\left(-\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) X_0 + \left(\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) Y_0 + \left(\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) X_0 Z_1 +$$
$$\left(\frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) Y_0 Z_1 + \left(\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) X_0 Z_2 + \left(-\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) Y_0 Z_2 +$$
$$\left(-\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) X_0 Z_1 Z_2 + \left(-\frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) Y_0 Z_1 Z_2$$

$$\left(-\frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) Y_0 + \left(-\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) Z_0 + \left(-\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) Y_0 Z_1 +$$
$$\left(\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) Z_0 Z_1 + \left(\frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) Y_0 Z_2 + \left(\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) Z_0 Z_2 +$$
$$\left(\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) Y_0 Z_1 Z_2 + \left(-\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) Z_0 Z_1 Z_2$$

$$\left(-\frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) Y_2 + \left(\frac{1}{4\sqrt{2}} - \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) Y_2 Z_0 +$$
$$\left(-\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) Y_2 Z_1 + \left(\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) Y_2 Z_0 Z_1 + \left(-\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) Z_2 +$$
$$\left(\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) Z_0 Z_2 + \left(\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Cos}\left[\frac{\pi}{8}\right]^2\right) Z_1 Z_2 + \left(-\frac{1}{4\sqrt{2}} + \frac{1}{2}\text{Sin}\left[\frac{\pi}{8}\right]^2\right) Z_0 Z_1 Z_2$$

```
SimplifyPaulis @ Last @ data @ "Strings"
```

$$-\frac{1}{4}\left(1+\sqrt{2}\right) Y_2+\frac{1}{4}\left(-1+\sqrt{2}\right) Y_2 Z_0+\frac{Y_2 Z_1}{4}+$$
$$\frac{1}{4} Y_2 Z_0 Z_1+\frac{Z_2}{4}+\frac{Z_0 Z_2}{4}+\frac{1}{4}\left(1+\sqrt{2}\right) Z_1 Z_2-\frac{1}{4}\left(-1+\sqrt{2}\right) Z_0 Z_1 Z_2$$

```
% // N
```

$$-0.603553\, Y_2+0.103553\, Y_2 Z_0+0.25\, Y_2 Z_1+0.25\, Y_2 Z_0 Z_1+$$
$$0.25\, Z_2+0.25\, Z_0 Z_2+0.603553\, Z_1 Z_2-0.103553\, Z_0 Z_1 Z_2$$

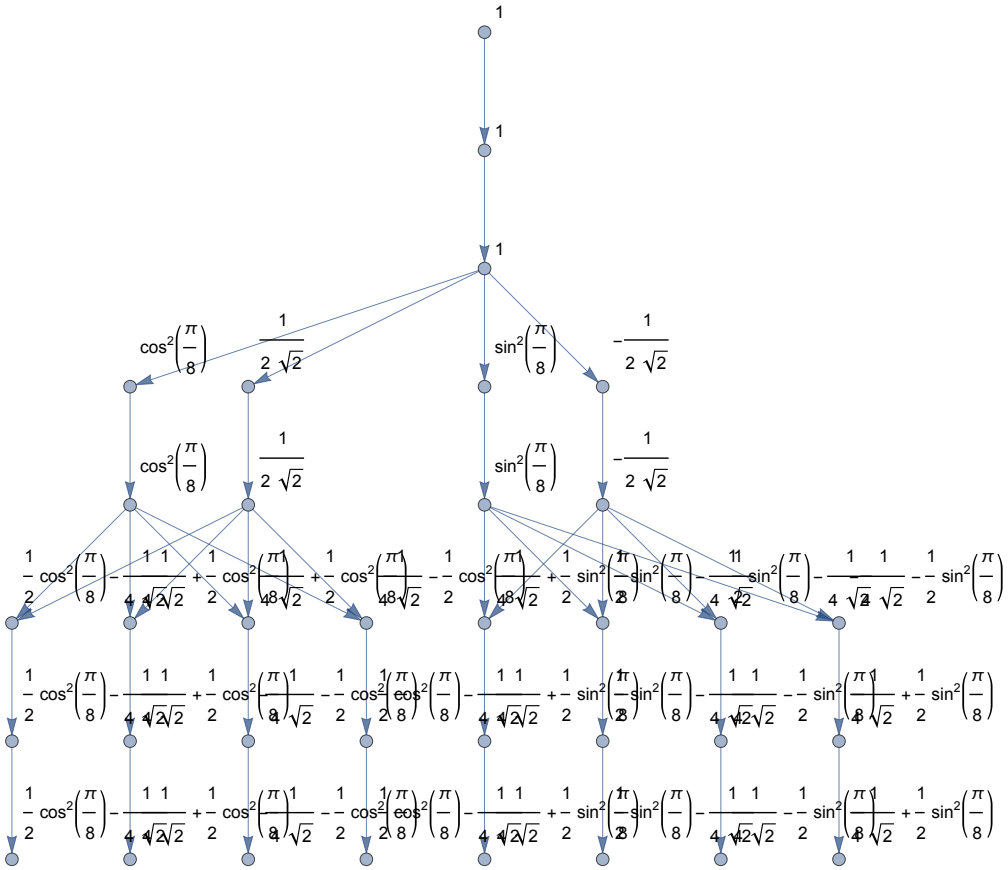
```
data["NumQubits"]
data["NumNodes"]
data["NumLeaves"]
```

```
3
```

```
35
```

```
8
```

```
Graph[
      Thread /@ Normal @ data @ "Children" // Flatten,
      VertexLabels → Normal @ data @ "Coefficients"
]
```



CombineStates

```
circ = Circuit[H0 H1 Damp0[x] Rx1[y] ];
circ = Join @@ Table[circ, 5];

CalcPauliTransferEval[ Y1, circ][[-1, All, 1]]
Length[%]

CalcPauliTransferEval[ Y1, circ, "CombineStrings" → False][[-1, All, 1]]
Length[%]
```

```
{{2, 0}, {3, 0}, {2, 3}, {3, 3}, {1, 0}, {1, 3}, {2, 1}, {3, 1}, {1, 1}}
```

```
9
```

```
{{2, 0}, {3, 0}, {2, 3}, {3, 3}, {1, 0}, {1, 3}, {2, 1}, {3, 1}, {1, 1}, {2, 0},
 {3, 0}, {2, 3}, {3, 3}, {2, 1}, {3, 1}, {2, 3}, {3, 3}, {1, 3}, {2, 3}, {3, 3},
 {2, 0}, {3, 0}, {2, 3}, {3, 3}, {1, 0}, {1, 3}, {2, 1}, {3, 1}, {1, 1}, {2, 3},
 {3, 3}, {1, 3}, {2, 1}, {3, 1}, {1, 1}, {2, 1}, {3, 1}, {2, 1}, {3, 1}, {1, 1},
 {2, 0}, {3, 0}, {2, 3}, {3, 3}, {1, 0}, {1, 3}, {2, 1}, {3, 1}, {1, 1}, {2, 0},
 {3, 0}, {2, 3}, {3, 3}, {2, 1}, {3, 1}, {2, 3}, {3, 3}, {1, 3}, {2, 3}, {3, 3},
 {2, 1}, {3, 1}, {1, 1}, {2, 1}, {3, 1}, {2, 3}, {3, 3}, {1, 3}, {2, 3},
 {3, 3}, {2, 3}, {3, 3}, {1, 3}, {2, 3}, {3, 3}, {1, 3}, {2, 3}, {3, 3}}
```

```
78
```

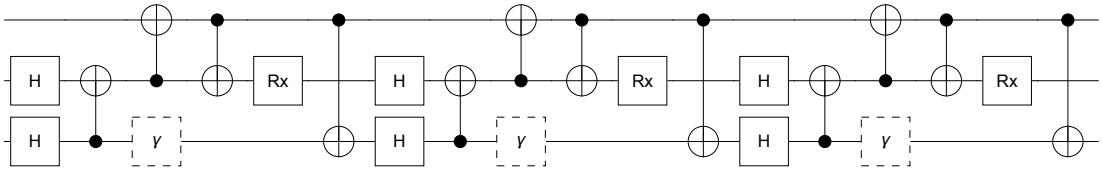
```
history = CalcPauliTransferEval[ Y1, circ];
ancestors = Flatten[history, 1][[All, 3]];
Length /@ ancestors // Max

history = CalcPauliTransferEval[ Y1, circ, "CombineStrings" → False];
ancestors = Flatten[history, 1][[All, 3]];
Length /@ ancestors // Max
```

```
2
```

```
1
```

CacheMaps

```
circ = Circuit[H0 H1 C0[X1] C1[X2] Damp0[x] C2[X1] Rx1[y] C2[X0] ];
circ = Join @@ Table[circ, 3];
DrawCircuit[circ]
```



```
First @ Timing @ CalcPauliTransferEval[ Y1, circ, "CacheMaps" → "Never"]
First @ Timing @ CalcPauliTransferEval[ Y1, circ, "CacheMaps" → "UntilCallEnd"]
```

```
0.093662
```

```
0.018925
```

```
First @ Timing @ CalcPauliTransferEval[ Y1, circ, "CacheMaps" → "Forever"]
First @ Timing @ CalcPauliTransferEval[ Y1, circ, "CacheMaps" → "Forever"]
```

```
0.021346
```

```
0.007441
```

AssertValidChannels

```
CalcPauliTransferEval[Y2 Y1 Z0, Damp0[x]]
CalcPauliTransferEval[Y2 Y1 Z0, Damp0[x], AssertValidChannels → False]
```

```
{{{{2, 2, 3}, 1, {{0, 1}}}}, {{{2, 2, 3}, 2, {{1, 1 - x}}}}}
```

$$\left\{\{\{\{2, 2, 3\}, 1, \{\{0, 1\}\}\}\},\ \left\{\left\{\{2, 2, 0\}, 2, \left\{\left\{1, \frac{1}{2}\left(1 - \sqrt{1-x}\ \text{Conjugate}\left[\sqrt{1-x}\right] - \sqrt{x}\ \text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\}\right\},\ \left\{\left\{\{2, 2, 3\}, 3, \left\{\left\{1, \frac{1}{2}\left(1 + \sqrt{1-x}\ \text{Conjugate}\left[\sqrt{1-x}\right] - \sqrt{x}\ \text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\}\right\}\right\}\right\}$$

```
CalcPauliTransferEval[Y2 Y1 Z0, Damp0[2], AssertValidChannels → False]
```

```
{{{{2, 2, 3}, 1, {{0, 1}}}}, {{{2, 2, 0}, 2, {{1, -1}}}}}
```

Errors

```
CalcPauliTransferEval[X0, Depol0[2]]
```

CalcPauliTransferEval : Could not pre–compute the Pauli transfer maps due to the below error:

CalcPauliTransferMatrix : The channels could not be asserted as completely positive trace–preserving maps and hence were not simplified. Hide this warning with AssertValidChannels –> False, or use Quiet[].

```
$Failed
```

```
CalcPauliTransferEval[Id1, PTMap2[{eh}]]
```

CalcPauliTransferEval : Could not pre–compute the Pauli transfer maps due to the below error:

CalcPauliTransferMatrix : Circuit contained an unrecognised or unsupported gate: PTMap_0[{eh}]

```
$Failed
```

```
CalcPauliTransferEval[Id1, Bop0]
```

CalcPauliTransferEval : Could not pre–compute the Pauli transfer maps due to the below error:

CalcPauliTransferMatrix : Circuit contained an unrecognised or unsupported gate: Bop_0

```
$Failed
```

```
CalcPauliTransferEval[Id_1, H_0]
```

CalcPauliTransferEval : Invalid arguments. See ?CalcPauliTransferEval

```
$Failed
```

```
CalcPauliTransferEval[1, H_0]
```

CalcPauliTransferEval : Invalid arguments. See ?CalcPauliTransferEval

```
$Failed
```

```
CalcPauliTransferEval[X_0, X_0, "CacheMaps" → "x"]
```

CalcPauliTransferEval : Option "CacheMaps" must be one of "Forever", "UntilCallEnd" or "Never".

```
$Failed
```

```
CalcPauliTransferEval[X_0, X_0, "CombineStrings" → "x"]
```

CalcPauliTransferEval : Option "CombineStrings" must be True or False. See ?CalcPauliTransferEval.

```
$Failed
```

```
CalcPauliTransferEval[X_0, X_0, "BadOption" → False]
```

OptionValue : Unknown option BadOption for {ApplyPauliTransferMap, CalcPauliTransferMap}.

```
$Failed
```

```
CalcPauliTransferEval[]
```

CalcPauliTransferEval : Invalid arguments. See ?CalcPauliTransferEval

```
$Failed
```