

# GetCircuitParameterised

```
SetDirectory @ NotebookDirectory[];  
Import["../Link/QuESTlink.m"];
```

```
test[circ_, opts___] := Module[  
  {out, subs, check},  
  {out, subs} = GetCircuitParameterised[circ,  $\theta$ , opts];  
  Echo[out, "output circuit: "];  
  Echo[subs, "output params: "];  
  check = circ === (out /. subs);  
  If[Not @ check, Style["ERRONEOUS OUTPUT", Red]]]
```

## Doc

### ? GetCircuitParameterised

#### Symbol

GetCircuitParameterised[circuit, paramSymbol] returns {out, paramValues} where out is an equivalent circuit whereby each scalar gate parameter (like those to Rx, R, G, etc) has been substituted with paramSymbol[i]. The returned paramValues is a list of symbol substitutions, so that the original circuit is obtained with out /. paramValues.

This function is useful for tasks like obtaining gate strengths, finding repeated parameters, or transforming circuits into variational ansatze. Note that custom gates in the QuESTlink format are permitted, but will not be considered for parameterisation.

GetCircuitParameterised accepts the below options.

- "UniqueParameters" -> True will force every substituted gate to receive a unique symbol (i.e. paramSymbol[i] for unique i), even if multiple gates have the same scalar parameter. Otherwise, gates with the same scalar parameter will automatically use the same repeated symbol, shrinking the length of paramValues.
- "ExcludeChannels" -> False will permit scalar-parameterised decoherence channels (like Damp, Depol, etc) to be parameterised in the output.
- "ExcludeGates" -> gatePattern(s) will prevent gates matching the given pattern (or list of patterns) from being parameterised. Note that gates and their controlled variants are treated separately.
- "ExcludeParameters" -> paramPattern(s) will prevent any gate parameter matching the given pattern (or list of patterns) from being substituted.

# Correctness

```

test @ {X0}
» output circuit: {X0}
» output params: {}

test @ {Ignored0[x], Rx0[x]}
» output circuit: {Ignored0[x], Rx0[ $\theta[1]$ ]}
» output params: { $\theta[1] \rightarrow x$ }

test @ Circuit[ Rz0[3 / 2] C0[Rx1[a]] R[b, X0 Y1]
               C1,2[Ph0[a + c]] G[d] × Fac[e] Damp0[1] Depol1,2[2] C2,3@R[b, X0 Y1] ]
» output circuit: {Rz0[ $\theta[1]$ ], C0[Rx1[ $\theta[2]$ ]], R[ $\theta[3]$ , X0 Y1], C1,2[Ph0[ $\theta[4]$ ]],
                  G[ $\theta[5]$ ], Fac[ $\theta[6]$ ], Damp0[1], Depol1,2[2], C2,3[R[ $\theta[3]$ , X0 Y1]]}
» output params: { $\theta[1] \rightarrow \frac{3}{2}$ ,  $\theta[2] \rightarrow a$ ,  $\theta[3] \rightarrow b$ ,  $\theta[4] \rightarrow a + c$ ,  $\theta[5] \rightarrow d$ ,  $\theta[6] \rightarrow e$ }

test @ GetKnownCircuit["QFT", 5]
» output circuit:
  {H4, Ph4,3[ $\theta[1]$ ], Ph4,2[ $\theta[2]$ ], Ph4,1[ $\theta[3]$ ], Ph4,0[ $\theta[4]$ ], H3, Ph3,2[ $\theta[1]$ ], Ph3,1[ $\theta[2]$ ],
   Ph3,0[ $\theta[3]$ ], H2, Ph2,1[ $\theta[1]$ ], Ph2,0[ $\theta[2]$ ], H1, Ph1,0[ $\theta[1]$ ], H0, SWAP0,4, SWAP1,3}
» output params: { $\theta[1] \rightarrow \frac{\pi}{2}$ ,  $\theta[2] \rightarrow \frac{\pi}{4}$ ,  $\theta[3] \rightarrow \frac{\pi}{8}$ ,  $\theta[4] \rightarrow \frac{\pi}{16}$ }

test[ C5,6 /@ GetKnownCircuit["QFT", 5] ]
» output circuit: {C5,6[H4], C5,6[Ph4,3[ $\theta[1]$ ]], C5,6[Ph4,2[ $\theta[2]$ ]],
                  C5,6[Ph4,1[ $\theta[3]$ ]], C5,6[Ph4,0[ $\theta[4]$ ]], C5,6[H3], C5,6[Ph3,2[ $\theta[1]$ ]],
                  C5,6[Ph3,1[ $\theta[2]$ ]], C5,6[Ph3,0[ $\theta[3]$ ]], C5,6[H2], C5,6[Ph2,1[ $\theta[1]$ ]], C5,6[Ph2,0[ $\theta[2]$ ]],
                  C5,6[H1], C5,6[Ph1,0[ $\theta[1]$ ]], C5,6[H0], C5,6[SWAP0,4], C5,6[SWAP1,3]}
» output params: { $\theta[1] \rightarrow \frac{\pi}{2}$ ,  $\theta[2] \rightarrow \frac{\pi}{4}$ ,  $\theta[3] \rightarrow \frac{\pi}{8}$ ,  $\theta[4] \rightarrow \frac{\pi}{16}$ }

```

## Options

### “UniqueParameters”

```
test[ C5,6 /@ GetKnownCircuit["QFT", 5], "UniqueParameters" → True]
```

» **output circuit:**  $\{C_{5,6}[H_4], C_{5,6}[Ph_{4,3}[\theta[1]]], C_{5,6}[Ph_{4,2}[\theta[2]]],$   
 $C_{5,6}[Ph_{4,1}[\theta[3]]], C_{5,6}[Ph_{4,0}[\theta[4]]], C_{5,6}[H_3], C_{5,6}[Ph_{3,2}[\theta[5]]],$   
 $C_{5,6}[Ph_{3,1}[\theta[6]]], C_{5,6}[Ph_{3,0}[\theta[7]]], C_{5,6}[H_2], C_{5,6}[Ph_{2,1}[\theta[8]]], C_{5,6}[Ph_{2,0}[\theta[9]]],$   
 $C_{5,6}[H_1], C_{5,6}[Ph_{1,0}[\theta[10]]], C_{5,6}[H_0], C_{5,6}[SWAP_{0,4}], C_{5,6}[SWAP_{1,3}]\}$

» **output params:**  $\{\theta[1] \rightarrow \frac{\pi}{2}, \theta[2] \rightarrow \frac{\pi}{4}, \theta[3] \rightarrow \frac{\pi}{8}, \theta[4] \rightarrow \frac{\pi}{16},$   
 $\theta[5] \rightarrow \frac{\pi}{2}, \theta[6] \rightarrow \frac{\pi}{4}, \theta[7] \rightarrow \frac{\pi}{8}, \theta[8] \rightarrow \frac{\pi}{2}, \theta[9] \rightarrow \frac{\pi}{4}, \theta[10] \rightarrow \frac{\pi}{2}\}$

```
h = GetRandomPauliString[5, 4];
u = GetKnownCircuit["Trotter", h, 1, 5, t];
```

```
test[u, "UniqueParameters" → True]
test[u, "UniqueParameters" → False]
```

» **output circuit:**  
 $\{R[\theta[1], X_4 Y_1 Y_2 Z_0], R[\theta[2], X_4 Y_3 Z_1 Z_2], R[\theta[3], X_1 X_4 Z_0 Z_2 Z_3], R[\theta[4], X_0 X_1 Z_3 Z_4],$   
 $R[\theta[5], X_4 Y_1 Y_2 Z_0], R[\theta[6], X_4 Y_3 Z_1 Z_2], R[\theta[7], X_1 X_4 Z_0 Z_2 Z_3], R[\theta[8], X_0 X_1 Z_3 Z_4],$   
 $R[\theta[9], X_4 Y_1 Y_2 Z_0], R[\theta[10], X_4 Y_3 Z_1 Z_2], R[\theta[11], X_1 X_4 Z_0 Z_2 Z_3], R[\theta[12], X_0 X_1 Z_3 Z_4],$   
 $R[\theta[13], X_4 Y_1 Y_2 Z_0], R[\theta[14], X_4 Y_3 Z_1 Z_2], R[\theta[15], X_1 X_4 Z_0 Z_2 Z_3], R[\theta[16], X_0 X_1 Z_3 Z_4],$   
 $R[\theta[17], X_4 Y_1 Y_2 Z_0], R[\theta[18], X_4 Y_3 Z_1 Z_2], R[\theta[19], X_1 X_4 Z_0 Z_2 Z_3], R[\theta[20], X_0 X_1 Z_3 Z_4]\}$

» **output params:**  
 $\{\theta[1] \rightarrow -0.180761 t, \theta[2] \rightarrow -0.306008 t, \theta[3] \rightarrow -0.190264 t, \theta[4] \rightarrow 0.193326 t,$   
 $\theta[5] \rightarrow -0.180761 t, \theta[6] \rightarrow -0.306008 t, \theta[7] \rightarrow -0.190264 t, \theta[8] \rightarrow 0.193326 t,$   
 $\theta[9] \rightarrow -0.180761 t, \theta[10] \rightarrow -0.306008 t, \theta[11] \rightarrow -0.190264 t, \theta[12] \rightarrow 0.193326 t,$   
 $\theta[13] \rightarrow -0.180761 t, \theta[14] \rightarrow -0.306008 t, \theta[15] \rightarrow -0.190264 t, \theta[16] \rightarrow 0.193326 t,$   
 $\theta[17] \rightarrow -0.180761 t, \theta[18] \rightarrow -0.306008 t, \theta[19] \rightarrow -0.190264 t, \theta[20] \rightarrow 0.193326 t\}$

» **output circuit:**  
 $\{R[\theta[1], X_4 Y_1 Y_2 Z_0], R[\theta[2], X_4 Y_3 Z_1 Z_2], R[\theta[3], X_1 X_4 Z_0 Z_2 Z_3], R[\theta[4], X_0 X_1 Z_3 Z_4],$   
 $R[\theta[1], X_4 Y_1 Y_2 Z_0], R[\theta[2], X_4 Y_3 Z_1 Z_2], R[\theta[3], X_1 X_4 Z_0 Z_2 Z_3], R[\theta[4], X_0 X_1 Z_3 Z_4],$   
 $R[\theta[1], X_4 Y_1 Y_2 Z_0], R[\theta[2], X_4 Y_3 Z_1 Z_2], R[\theta[3], X_1 X_4 Z_0 Z_2 Z_3], R[\theta[4], X_0 X_1 Z_3 Z_4],$   
 $R[\theta[1], X_4 Y_1 Y_2 Z_0], R[\theta[2], X_4 Y_3 Z_1 Z_2], R[\theta[3], X_1 X_4 Z_0 Z_2 Z_3], R[\theta[4], X_0 X_1 Z_3 Z_4]\}$

» **output params:**  
 $\{\theta[1] \rightarrow -0.180761 t, \theta[2] \rightarrow -0.306008 t, \theta[3] \rightarrow -0.190264 t, \theta[4] \rightarrow 0.193326 t\}$

## “ExcludeChannels”

```
u = Circuit[ Damp0[a] Deph0[b] Depol1,2[b] Depol5[b] Deph2,1[a] ];
```

```
test[u]
```

» **output circuit:**  $\{Damp_0[a], Deph_0[b], Depol_{1,2}[b], Depol_5[b], Deph_{2,1}[a]\}$

» **output params:**  $\{\}$

```
test[u, "ExcludeChannels" → False]
```

» **output circuit:**  $\{Damp_0[\theta[1]], Deph_0[\theta[2]], Depol_{1,2}[\theta[2]], Depol_5[\theta[2]], Deph_{2,1}[\theta[1]]\}$

» **output params:**  $\{\theta[1] \rightarrow a, \theta[2] \rightarrow b\}$

```
test[u, "ExcludeChannels" → False, "UniqueParameters" → True]
```

» **output circuit:**  $\{ \text{Damp}_0[\theta[1]], \text{Deph}_0[\theta[2]], \text{Depol}_{1,2}[\theta[3]], \text{Depol}_5[\theta[4]], \text{Deph}_{2,1}[\theta[5]] \}$

» **output params:**  $\{ \theta[1] \rightarrow a, \theta[2] \rightarrow b, \theta[3] \rightarrow b, \theta[4] \rightarrow b, \theta[5] \rightarrow a \}$

## “ExcludeGates”

### Gates

```
u = Circuit[ Rz0[3/2] Rz1[4] C0[Rx1[a]] R[b, X0 Y1]
             C1,2[Ph0[a + c]] G[d] × Fac[e] Damp0[1] Depol1,2[2] C2,3@R[b, X0 Y1] ];
```

```
test[u]
```

» **output circuit:**  $\{ \text{Rz}_0[\theta[1]], \text{Rz}_1[\theta[2]], \text{C}_0[\text{Rx}_1[\theta[3]]], \text{R}[\theta[4], \text{X}_0 \text{Y}_1], \text{C}_{1,2}[\text{Ph}_0[\theta[5]]], \text{G}[\theta[6]], \text{Fac}[\theta[7]], \text{Damp}_0[1], \text{Depol}_{1,2}[2], \text{C}_{2,3}[\text{R}[\theta[4], \text{X}_0 \text{Y}_1]] \}$

» **output params:**  $\{ \theta[1] \rightarrow \frac{3}{2}, \theta[2] \rightarrow 4, \theta[3] \rightarrow a, \theta[4] \rightarrow b, \theta[5] \rightarrow a + c, \theta[6] \rightarrow d, \theta[7] \rightarrow e \}$

```
test[u, "ExcludeGates" → Rz[_]]
```

» **output circuit:**  $\{ \text{Rz}_0[\frac{3}{2}], \text{Rz}_1[4], \text{C}_0[\text{Rx}_1[\theta[1]]], \text{R}[\theta[2], \text{X}_0 \text{Y}_1], \text{C}_{1,2}[\text{Ph}_0[\theta[3]]], \text{G}[\theta[4]], \text{Fac}[\theta[5]], \text{Damp}_0[1], \text{Depol}_{1,2}[2], \text{C}_{2,3}[\text{R}[\theta[2], \text{X}_0 \text{Y}_1]] \}$

» **output params:**  $\{ \theta[1] \rightarrow a, \theta[2] \rightarrow b, \theta[3] \rightarrow a + c, \theta[4] \rightarrow d, \theta[5] \rightarrow e \}$

```
test[u, "ExcludeGates" → {Rz[_]}]
```

» **output circuit:**  $\{ \text{Rz}_0[\frac{3}{2}], \text{Rz}_1[\theta[1]], \text{C}_0[\text{Rx}_1[\theta[2]]], \text{R}[\theta[3], \text{X}_0 \text{Y}_1], \text{C}_{1,2}[\text{Ph}_0[\theta[4]]], \text{G}[\theta[5]], \text{Fac}[\theta[6]], \text{Damp}_0[1], \text{Depol}_{1,2}[2], \text{C}_{2,3}[\text{R}[\theta[3], \text{X}_0 \text{Y}_1]] \}$

» **output params:**  $\{ \theta[1] \rightarrow 4, \theta[2] \rightarrow a, \theta[3] \rightarrow b, \theta[4] \rightarrow a + c, \theta[5] \rightarrow d, \theta[6] \rightarrow e \}$

```
test[u, "ExcludeGates" → {Rz[4]}]
```

» **output circuit:**  $\{ \text{Rz}_0[\theta[1]], \text{Rz}_1[4], \text{C}_0[\text{Rx}_1[\theta[2]]], \text{R}[\theta[3], \text{X}_0 \text{Y}_1], \text{C}_{1,2}[\text{Ph}_0[\theta[4]]], \text{G}[\theta[5]], \text{Fac}[\theta[6]], \text{Damp}_0[1], \text{Depol}_{1,2}[2], \text{C}_{2,3}[\text{R}[\theta[3], \text{X}_0 \text{Y}_1]] \}$

» **output params:**  $\{ \theta[1] \rightarrow \frac{3}{2}, \theta[2] \rightarrow a, \theta[3] \rightarrow b, \theta[4] \rightarrow a + c, \theta[5] \rightarrow d, \theta[6] \rightarrow e \}$

```
test[u, "ExcludeGates" → C2,3[_]]
```

» **output circuit:**  $\{ \text{Rz}_0[\theta[1]], \text{Rz}_1[\theta[2]], \text{C}_0[\text{Rx}_1[\theta[3]]], \text{R}[\theta[4], \text{X}_0 \text{Y}_1], \text{C}_{1,2}[\text{Ph}_0[\theta[5]]], \text{G}[\theta[6]], \text{Fac}[\theta[7]], \text{Damp}_0[1], \text{Depol}_{1,2}[2], \text{C}_{2,3}[\text{R}[\text{b}, \text{X}_0 \text{Y}_1]] \}$

» **output params:**  $\{ \theta[1] \rightarrow \frac{3}{2}, \theta[2] \rightarrow 4, \theta[3] \rightarrow a, \theta[4] \rightarrow b, \theta[5] \rightarrow a + c, \theta[6] \rightarrow d, \theta[7] \rightarrow e \}$

```
test[u, "ExcludeGates" → C__[_]]
```

» **output circuit:**  $\{ \text{Rz}_0[\theta[1]], \text{Rz}_1[\theta[2]], \text{C}_0[\text{Rx}_1[a]], \text{R}[\theta[3], \text{X}_0 \text{Y}_1], \text{C}_{1,2}[\text{Ph}_0[a + c]], \text{G}[\theta[4]], \text{Fac}[\theta[5]], \text{Damp}_0[1], \text{Depol}_{1,2}[2], \text{C}_{2,3}[\text{R}[\text{b}, \text{X}_0 \text{Y}_1]] \}$

» **output params:**  $\{ \theta[1] \rightarrow \frac{3}{2}, \theta[2] \rightarrow 4, \theta[3] \rightarrow b, \theta[4] \rightarrow d, \theta[5] \rightarrow e \}$

```
test[u, "ExcludeGates" → {R[_], Rz[_], C[_], G[_]}]
```

- » output circuit:  $\left\{ R_z\left[\frac{3}{2}\right], R_z[4], C_0[Rx_1[a]], R[b, X_0 Y_1], \right.$   
 $C_{1,2}[Ph_0[a + c]], G[d], Fac[\theta[1]], Damp_0[1], Depol_{1,2}[2], C_{2,3}[R[b, X_0 Y_1]] \left. \right\}$
- » output params:  $\{\theta[1] \rightarrow e\}$

## Channels

```
u = Circuit[ Damp_0[a] Deph_0[b] Depol_{1,2}[b] Depol_5[b] Deph_{2,1}[a] ];
```

```
test[u, "ExcludeChannels" → False, "ExcludeGates" → {Deh_0[_]}]
```

- » output circuit:  $\{Damp_0[\theta[1]], Deph_0[b], Depol_{1,2}[\theta[2]], Depol_5[\theta[2]], Deph_{2,1}[\theta[1]]\}$
- » output params:  $\{\theta[1] \rightarrow a, \theta[2] \rightarrow b\}$
- ```
test[u, "ExcludeChannels" → False, "ExcludeGates" → (Damp | Deph) _[_]]
```
- » output circuit:  $\{Damp_0[a], Deph_0[b], Depol_{1,2}[\theta[1]], Depol_5[\theta[1]], Deph_{2,1}[\theta[2]]\}$
- » output params:  $\{\theta[1] \rightarrow b, \theta[2] \rightarrow a\}$

## “ExcludeParameters”

### Gates

```
u = Circuit[ Rz_0[3 / 2] Rz_1[4] C_0[Rx_1[a]] R[b, X_0 Y_1]
```

```
  C_{1,2}[Ph_0[a + c]] G[d] × Fac[e] Damp_0[1] Depol_{1,2}[2] C_{2,3}@R[b, X_0 Y_1] ];
```

```
test[u, "ExcludeParameters" → 3 / 2]
```

- » output circuit:  $\left\{ R_z\left[\frac{3}{2}\right], R_z[\theta[1]], C_0[Rx_1[\theta[2]]], R[\theta[3], X_0 Y_1], \right.$   
 $C_{1,2}[Ph_0[\theta[4]]], G[\theta[5]], Fac[\theta[6]], Damp_0[1], Depol_{1,2}[2], C_{2,3}[R[\theta[3], X_0 Y_1]] \left. \right\}$
- » output params:  $\{\theta[1] \rightarrow 4, \theta[2] \rightarrow a, \theta[3] \rightarrow b, \theta[4] \rightarrow a + c, \theta[5] \rightarrow d, \theta[6] \rightarrow e\}$

```
test[u, "ExcludeParameters" → {a, b, a + c}]
```

- » output circuit:  $\{R_z[\theta[1]], R_z[\theta[2]], C_0[Rx_1[a]], R[b, X_0 Y_1],$   
 $C_{1,2}[Ph_0[a + c]], G[\theta[3]], Fac[\theta[4]], Damp_0[1], Depol_{1,2}[2], C_{2,3}[R[b, X_0 Y_1]] \left. \right\}$
- » output params:  $\left\{ \theta[1] \rightarrow \frac{3}{2}, \theta[2] \rightarrow 4, \theta[3] \rightarrow d, \theta[4] \rightarrow e \right\}$

```
test[u, "ExcludeParameters" → _]
```

- » output circuit:  $\left\{ R_z\left[\frac{3}{2}\right], R_z[4], C_0[Rx_1[a]], R[b, X_0 Y_1], \right.$   
 $C_{1,2}[Ph_0[a + c]], G[d], Fac[e], Damp_0[1], Depol_{1,2}[2], C_{2,3}[R[b, X_0 Y_1]] \left. \right\}$
- » output params:  $\{\}$

```
test[u, "ExcludeParameters" → _Plus]
```

» **output circuit:**  $\{Rz_0[\theta[1]], Rz_1[\theta[2]], C_0[Rx_1[\theta[3]]], R[\theta[4], X_0 Y_1], C_{1,2}[Ph_0[a + c]], G[\theta[5]], Fac[\theta[6]], Damp_0[1], Depol_{1,2}[2], C_{2,3}[R[\theta[4], X_0 Y_1]]\}$

» **output params:**  $\{\theta[1] \rightarrow \frac{3}{2}, \theta[2] \rightarrow 4, \theta[3] \rightarrow a, \theta[4] \rightarrow b, \theta[5] \rightarrow d, \theta[6] \rightarrow e\}$

**test[u, "ExcludeParameters" → (\_Plus | \_Rational), "UniqueParameters" → True]**

» **output circuit:**  $\{Rz_0[\frac{3}{2}], Rz_1[\theta[1]], C_0[Rx_1[\theta[2]]], R[\theta[3], X_0 Y_1], C_{1,2}[Ph_0[a + c]], G[\theta[4]], Fac[\theta[5]], Damp_0[1], Depol_{1,2}[2], C_{2,3}[R[\theta[6], X_0 Y_1]]\}$

» **output params:**  $\{\theta[1] \rightarrow 4, \theta[2] \rightarrow a, \theta[3] \rightarrow b, \theta[4] \rightarrow d, \theta[5] \rightarrow e, \theta[6] \rightarrow b\}$

## Channels

**u = Circuit[ Damp<sub>0</sub>[a] Deph<sub>0</sub>[b] Depol<sub>1,2</sub>[b] Depol<sub>5</sub>[b] Deph<sub>2,1</sub>[c] ];**

**test[u, "ExcludeChannels" → False, "ExcludeParameters" → a]**

» **output circuit:**  $\{Damp_0[a], Deph_0[\theta[1]], Depol_{1,2}[\theta[1]], Depol_5[\theta[1]], Deph_{2,1}[\theta[2]]\}$

» **output params:**  $\{\theta[1] \rightarrow b, \theta[2] \rightarrow c\}$

**test[u, "ExcludeChannels" → False, "ExcludeParameters" → {a, b}]**

» **output circuit:**  $\{Damp_0[a], Deph_0[b], Depol_{1,2}[b], Depol_5[b], Deph_{2,1}[\theta[1]]\}$

» **output params:**  $\{\theta[1] \rightarrow c\}$

## Errors

**GetCircuitParameterised[X<sub>0</sub>, x, "UniqueParameters" → bad]**  
**GetCircuitParameterised[X<sub>0</sub>, x, "ExcludeChannels" → bad]**

... **GetCircuitParameterised:** Option "UniqueParameters" must be True or False

\$Failed

... **GetCircuitParameterised:** Option "ExcludeChannels" must be True or False

\$Failed

**GetCircuitParameterised[bad, x]**

... **GetCircuitParameterised:** Invalid arguments. See ?GetCircuitParameterised

\$Failed