CalcPauliTransferMap

```
SetDirectory @ NotebookDirectory[];
Import["../Link/QuESTlink.m"];
```

Doc

```
? CalcPauliTransferMap
```

Symbol

CalcPauliTransferMap[ptm] produces a PTMap equivalent to the given PTM operator. See ?PTM.
CalcPauliTransferMap[circuit] produces a PTMap from the
given gate or circuit, by merely first invoking CalcPauliTransferMatrix[].
The returned map encodes how each basis Pauli-string (encoded by its integer index) is mapped
to a weighted sum of other strings (encoded as {index, coefficient} pairs) by the PTM.
The indexing convention is the same as used by GetPauliString[] where the subscripted
qubits of the PTM are treated as though given in order of increasing significance.
CalcPauliTransferMap also accepts option AssertValidChannels->False
to disable the automatic simplification of the map's coefficients through
the assertion of valid channel parameters. See ?AssertValidChannels.

```
? PTM
```

Symbol

PTM[matrix] is a Pauli-transfer matrix representation of an operator or
channel. The subscript indices specify which Paulis of a Pauli string are operated
upon. Such objects are produced by functions like CalcPauliTransferMatrix[].

```
? PTMap
```

Symbol

PTMap[map] is a representation of a Pauli transfer matrix as a map between Pauli tensors,
specified either as basis-state indices or in a Kronecker form. See ?CalcPauliTransferMap.

Correctness

PTM

```
iden = IdentityMatrix[4^2];

CalcPauliTransferMap[PTM0,1[iden]]
```

$$\text{PTMap}_{0,1}[0 \to \{\{0, 1\}\}, 1 \to \{\{1, 1\}\}, 2 \to \{\{2, 1\}\}, 3 \to \{\{3, 1\}\}, 4 \to \{\{4, 1\}\},$$
$$5 \to \{\{5, 1\}\}, 6 \to \{\{6, 1\}\}, 7 \to \{\{7, 1\}\}, 8 \to \{\{8, 1\}\}, 9 \to \{\{9, 1\}\}, 10 \to \{\{10, 1\}\},$$
$$11 \to \{\{11, 1\}\}, 12 \to \{\{12, 1\}\}, 13 \to \{\{13, 1\}\}, 14 \to \{\{14, 1\}\}, 15 \to \{\{15, 1\}\}]$$

Operator

```
CalcPauliTransferMap[Rz0[x]]
```

$$\text{PTMap}_{0}[0 \to \{\{0, 1\}\}, 1 \to \{\{1, \text{Cos}[x]\}, \{2, \text{Sin}[x]\}\},$$
$$2 \to \{\{1, -\text{Sin}[x]\}, \{2, \text{Cos}[x]\}\}, 3 \to \{\{3, 1\}\}]$$

```
CalcPauliTransferMap @ C0[X1]
Length /@ (List @@ %)[[All, 2]] // Max
```

$$\text{PTMap}_{1,0}[0 \to \{\{0, 1\}\}, 1 \to \{\{1, 1\}\}, 2 \to \{\{14, 1\}\}, 3 \to \{\{15, 1\}\},$$
$$4 \to \{\{5, 1\}\}, 5 \to \{\{4, 1\}\}, 6 \to \{\{11, 1\}\}, 7 \to \{\{10, -1\}\},$$
$$8 \to \{\{9, 1\}\}, 9 \to \{\{8, 1\}\}, 10 \to \{\{7, -1\}\}, 11 \to \{\{6, 1\}\},$$
$$12 \to \{\{12, 1\}\}, 13 \to \{\{13, 1\}\}, 14 \to \{\{2, 1\}\}, 15 \to \{\{3, 1\}\}]$$

```
1
```

```
CalcPauliTransferMap @ R[x, X0 Y1 Z4];
Length /@ (List @@ %)[[All, 2]] // Max
```

```
2
```

```
CalcPauliTransferMap @ H1
```

$$\text{PTMap}_{1}[0 \to \{\{0, 1\}\}, 1 \to \{\{3, 1\}\}, 2 \to \{\{2, -1\}\}, 3 \to \{\{1, 1\}\}]$$

CalcPauliTransferMap @ $C_0[H_3]$

$$\text{PTMap}_{3,0}\left[0 \to \{\{0, 1\}\}, 1 \to \left\{\left\{1, \frac{1}{2}\right\}, \left\{3, \frac{1}{2}\right\}, \left\{13, \frac{1}{2}\right\}, \left\{15, -\frac{1}{2}\right\}\right\}, 2 \to \{\{14, 1\}\},\right.$$
$$3 \to \left\{\left\{1, \frac{1}{2}\right\}, \left\{3, \frac{1}{2}\right\}, \left\{13, -\frac{1}{2}\right\}, \left\{15, \frac{1}{2}\right\}\right\}, 4 \to \left\{\left\{5, \frac{1}{\sqrt{2}}\right\}, \left\{7, \frac{1}{\sqrt{2}}\right\}\right\},$$
$$5 \to \left\{\left\{4, \frac{1}{\sqrt{2}}\right\}, \left\{10, \frac{1}{\sqrt{2}}\right\}\right\}, 6 \to \left\{\left\{9, -\frac{1}{\sqrt{2}}\right\}, \left\{11, \frac{1}{\sqrt{2}}\right\}\right\},$$
$$7 \to \left\{\left\{4, \frac{1}{\sqrt{2}}\right\}, \left\{10, -\frac{1}{\sqrt{2}}\right\}\right\}, 8 \to \left\{\left\{9, \frac{1}{\sqrt{2}}\right\}, \left\{11, \frac{1}{\sqrt{2}}\right\}\right\},$$
$$9 \to \left\{\left\{6, -\frac{1}{\sqrt{2}}\right\}, \left\{8, \frac{1}{\sqrt{2}}\right\}\right\}, 10 \to \left\{\left\{5, \frac{1}{\sqrt{2}}\right\}, \left\{7, -\frac{1}{\sqrt{2}}\right\}\right\}, 11 \to \left\{\left\{6, \frac{1}{\sqrt{2}}\right\}, \left\{8, \frac{1}{\sqrt{2}}\right\}\right\},$$
$$12 \to \{\{12, 1\}\}, 13 \to \left\{\left\{1, \frac{1}{2}\right\}, \left\{3, -\frac{1}{2}\right\}, \left\{13, \frac{1}{2}\right\}, \left\{15, \frac{1}{2}\right\}\right\},$$
$$\left.14 \to \{\{2, 1\}\}, 15 \to \left\{\left\{1, -\frac{1}{2}\right\}, \left\{3, \frac{1}{2}\right\}, \left\{13, \frac{1}{2}\right\}, \left\{15, \frac{1}{2}\right\}\right\}\right]$$

CalcPauliTransferMap @ $\text{Damp}_0[x]$

$$\text{PTMap}_0\left[0 \to \{\{0, 1\}, \{3, x\}\}, 1 \to \left\{\left\{1, \sqrt{1-x}\right\}\right\}, 2 \to \left\{\left\{2, \sqrt{1-x}\right\}\right\}, 3 \to \{\{3, 1-x\}\}\right]$$

CalcPauliTransferMap @ $\text{Depol}_{0,2}[x]$

$$\text{PTMap}_{0,2}\left[0 \to \{\{0, 1\}\}, 1 \to \left\{\left\{1, 1-\frac{16\,x}{15}\right\}\right\}, 2 \to \left\{\left\{2, 1-\frac{16\,x}{15}\right\}\right\},\right.$$
$$3 \to \left\{\left\{3, 1-\frac{16\,x}{15}\right\}\right\}, 4 \to \left\{\left\{4, 1-\frac{16\,x}{15}\right\}\right\}, 5 \to \left\{\left\{5, 1-\frac{16\,x}{15}\right\}\right\},$$
$$6 \to \left\{\left\{6, 1-\frac{16\,x}{15}\right\}\right\}, 7 \to \left\{\left\{7, 1-\frac{16\,x}{15}\right\}\right\}, 8 \to \left\{\left\{8, 1-\frac{16\,x}{15}\right\}\right\}, 9 \to \left\{\left\{9, 1-\frac{16\,x}{15}\right\}\right\},$$
$$10 \to \left\{\left\{10, 1-\frac{16\,x}{15}\right\}\right\}, 11 \to \left\{\left\{11, 1-\frac{16\,x}{15}\right\}\right\}, 12 \to \left\{\left\{12, 1-\frac{16\,x}{15}\right\}\right\},$$
$$\left.13 \to \left\{\left\{13, 1-\frac{16\,x}{15}\right\}\right\}, 14 \to \left\{\left\{14, 1-\frac{16\,x}{15}\right\}\right\}, 15 \to \left\{\left\{15, 1-\frac{16\,x}{15}\right\}\right\}\right]$$

CalcPauliTransferMap @ Kraus_0 @ { a IdentityMatrix[2], b PauliMatrix[2]}

$$\text{PTMap}_0\left[0 \to \left\{\left\{0, \text{Abs}[a]^2 + \text{Abs}[b]^2\right\}\right\}, 1 \to \left\{\left\{1, \text{Abs}[a]^2 - b\,\text{Conjugate}[b]\right\}\right\},\right.$$
$$\left.2 \to \left\{\left\{2, \text{Abs}[a]^2 + \text{Abs}[b]^2\right\}\right\}, 3 \to \left\{\left\{3, \text{Abs}[a]^2 - b\,\text{Conjugate}[b]\right\}\right\}\right]$$

$\text{Kraus}_{2,4}$ @ Table[RandomComplex[{-1 - i, 1 + i}, {2^2, 2^2}], 6];
List @@ CalcPauliTransferMap @ % // Chop // Column

```
0 → {{0, 16.143}, {1, 0.828285}, {2, 2.37628}, {3, -0.436849},
   {4, 0.920197}, {5, -2.40766}, {6, 1.83851}, {7, 1.82841},
   {8, 0.313788}, {9, 0.253652}, {10, -0.606174}, {11, 0.312817},
   {12, 0.836082}, {13, -4.01714}, {14, 0.867548}, {15, 0.241877}}
1 → {{0, 1.08021}, {1, -0.701388}, {2, -0.26064}, {3, 0.111841},
   {4, -0.546476}, {5, -2.57234}, {6, -0.94499}, {7, -4.0417},
   {8, -1.75636}, {9, 0.399539}, {10, -0.886122}, {11, -1.17487},
   {12, 1.44197}, {13, -1.30704}, {14, 0.918898}, {15, 0.36828}}
```

```
2 → {{0, 2.65716}, {1, -1.71901}, {2, 1.24961}, {3, 2.24631},
  {4, 3.4554}, {5, 0.108628}, {6, -0.578171}, {7, 1.85541},
  {8, -0.532512}, {9, 1.3179}, {10, -1.98812}, {11, 0.517915},
  {12, 3.8617}, {13, -1.24942}, {14, -0.742504}, {15, -1.26358}}
3 → {{0, 0.382334}, {1, -1.78957}, {2, 0.413454}, {3, -0.678119},
  {4, 1.64296}, {5, -0.75454}, {6, 1.15785}, {7, 1.25878},
  {8, -2.38834}, {9, 0.746835}, {10, -2.29613}, {11, 2.14672},
  {12, 0.644401}, {13, -1.18346}, {14, 0.187087}, {15, -0.124904}}
4 → {{0, 0.562046}, {1, -4.01189}, {2, 1.3458}, {3, -1.78997},
  {4, -0.0269316}, {5, -1.3861}, {6, 1.98814}, {7, 0.859748},
  {8, 0.373457}, {9, 0.514668}, {10, 2.01937}, {11, 1.25648},
  {12, 2.1317}, {13, -0.72674}, {14, 1.24101}, {15, 1.84862}}
5 → {{0, -0.648126}, {1, -1.90582}, {2, -0.294168}, {3, 1.68406},
  {4, 3.93092}, {5, 3.41811}, {6, 0.320825}, {7, -0.175675},
  {8, 0.700241}, {9, 1.65102}, {10, -1.00877}, {11, 2.37689},
  {12, -1.2336}, {13, -1.68802}, {14, -0.197871}, {15, 0.779338}}
6 → {{0, -0.228566}, {1, -1.79388}, {2, 2.35361}, {3, 1.73691},
  {4, 2.84352}, {5, -1.37628}, {6, 0.34009}, {7, -1.71708},
  {8, -1.41374}, {9, -1.01817}, {10, -1.85996}, {11, -0.752369},
  {12, 0.649584}, {13, -0.505122}, {14, 1.14723}, {15, 0.300438}}
7 → {{0, 0.0800607}, {1, -1.04788}, {2, -0.802117}, {3, -0.238219},
  {4, 0.640985}, {5, 1.36374}, {6, -0.33124}, {7, 0.306136},
  {8, -1.34994}, {9, -1.43486}, {10, 0.408579}, {11, -0.521505},
  {12, 1.55996}, {13, -2.17205}, {14, 1.24823}, {15, 0.349839}}
8 → {{0, -0.781548}, {1, -2.08569}, {2, -0.204932}, {3, -0.340131},
  {4, 0.148575}, {5, -0.692272}, {6, 1.68336}, {7, 0.985497},
  {8, 0.997196}, {9, 2.26409}, {10, -0.276514}, {11, 1.51312},
  {12, 0.0471851}, {13, -1.06711}, {14, -2.56593}, {15, -3.95103}}
9 → {{0, -3.39277}, {1, -1.58062}, {2, -0.770675}, {3, -1.63236},
  {4, -0.370995}, {5, 0.748404}, {6, -1.53827}, {7, 0.0360487},
  {8, -1.38793}, {9, -0.509538}, {10, 0.187069}, {11, -0.117591},
  {12, 0.470731}, {13, 0.0558733}, {14, 1.47123}, {15, -0.409816}}
10 → {{0, 1.16524}, {1, -0.293788}, {2, -2.72339}, {3, -3.06574},
  {4, -3.30728}, {5, -3.21998}, {6, -0.461746}, {7, 2.33459},
  {8, 0.342441}, {9, 0.598082}, {10, -1.7527}, {11, 0.35556},
  {12, -3.88784}, {13, -0.60323}, {14, -3.48391}, {15, -0.227843}}
11 → {{0, -1.4209}, {1, -0.433211}, {2, 1.77535}, {3, 1.55056},
  {4, 0.459927}, {5, -1.84665}, {6, -0.0717063}, {7, 1.39731},
  {8, -2.09944}, {9, -1.01098}, {10, -1.8876}, {11, -0.133866},
  {12, 1.73628}, {13, 0.00375726}, {14, -2.09281}, {15, 0.0363313}}
12 → {{0, -1.47497}, {1, -0.928566}, {2, -0.518169}, {3, -0.224772},
  {4, -0.600819}, {5, 0.478469}, {6, 0.680374}, {7, -0.0122904},
  {8, 2.90108}, {9, -1.46748}, {10, 0.886194}, {11, -1.71547},
  {12, -0.308425}, {13, 0.127799}, {14, -1.36318}, {15, 1.15867}}
```

```
13 → {{0, -3.09066}, {1, -0.0266088}, {2, -1.5786}, {3, 1.49811},
  {4, 0.782991}, {5, -0.397426}, {6, -0.220373}, {7, 1.14581},
  {8, 1.69728}, {9, 0.00619736}, {10, -0.159745}, {11, 1.16266},
  {12, 1.55277}, {13, 2.96617}, {14, -1.19769}, {15, 1.7439}}
14 → {{0, -1.71456}, {1, -0.851165}, {2, -1.18548}, {3, 0.313052},
  {4, -0.944925}, {5, -0.225451}, {6, 1.26152}, {7, 0.0228245},
  {8, -0.296356}, {9, -0.933703}, {10, 2.21206}, {11, -0.828187},
  {12, 1.8377}, {13, -0.188503}, {14, 1.07688}, {15, 4.02413}}
15 → {{0, -1.20968}, {1, 3.54816}, {2, -1.92136},
  {3, -0.55557}, {4, -2.26478}, {5, -0.185261}, {6, 0.553376},
  {7, 0.988695}, {8, 3.11115}, {9, 3.24555}, {10, 1.6314}, {11, 1.17491},
  {12, -0.610137}, {13, 0.874134}, {14, -0.137957}, {15, 0.627773}}
```

Circuit

```
CalcPauliTransferMap @ Circuit[X0 Y1 H2]
```

$$\text{PTMap}_{0,1,2}[0 \to \{\{0, 1\}\}, 1 \to \{\{1, 1\}\}, 2 \to \{\{2, -1\}\}, 3 \to \{\{3, -1\}\}, 4 \to \{\{4, -1\}\},$$
$$5 \to \{\{5, -1\}\}, 6 \to \{\{6, 1\}\}, 7 \to \{\{7, 1\}\}, 8 \to \{\{8, 1\}\}, 9 \to \{\{9, 1\}\},$$
$$10 \to \{\{10, -1\}\}, 11 \to \{\{11, -1\}\}, 12 \to \{\{12, -1\}\}, 13 \to \{\{13, -1\}\}, 14 \to \{\{14, 1\}\},$$
$$15 \to \{\{15, 1\}\}, 16 \to \{\{48, 1\}\}, 17 \to \{\{49, 1\}\}, 18 \to \{\{50, -1\}\}, 19 \to \{\{51, -1\}\},$$
$$20 \to \{\{52, -1\}\}, 21 \to \{\{53, -1\}\}, 22 \to \{\{54, 1\}\}, 23 \to \{\{55, 1\}\}, 24 \to \{\{56, 1\}\},$$
$$25 \to \{\{57, 1\}\}, 26 \to \{\{58, -1\}\}, 27 \to \{\{59, -1\}\}, 28 \to \{\{60, -1\}\}, 29 \to \{\{61, -1\}\},$$
$$30 \to \{\{62, 1\}\}, 31 \to \{\{63, 1\}\}, 32 \to \{\{32, -1\}\}, 33 \to \{\{33, -1\}\}, 34 \to \{\{34, 1\}\},$$
$$35 \to \{\{35, 1\}\}, 36 \to \{\{36, 1\}\}, 37 \to \{\{37, 1\}\}, 38 \to \{\{38, -1\}\}, 39 \to \{\{39, -1\}\},$$
$$40 \to \{\{40, -1\}\}, 41 \to \{\{41, -1\}\}, 42 \to \{\{42, 1\}\}, 43 \to \{\{43, 1\}\}, 44 \to \{\{44, 1\}\},$$
$$45 \to \{\{45, 1\}\}, 46 \to \{\{46, -1\}\}, 47 \to \{\{47, -1\}\}, 48 \to \{\{16, 1\}\}, 49 \to \{\{17, 1\}\},$$
$$50 \to \{\{18, -1\}\}, 51 \to \{\{19, -1\}\}, 52 \to \{\{20, -1\}\}, 53 \to \{\{21, -1\}\},$$
$$54 \to \{\{22, 1\}\}, 55 \to \{\{23, 1\}\}, 56 \to \{\{24, 1\}\}, 57 \to \{\{25, 1\}\}, 58 \to \{\{26, -1\}\},$$
$$59 \to \{\{27, -1\}\}, 60 \to \{\{28, -1\}\}, 61 \to \{\{29, -1\}\}, 62 \to \{\{30, 1\}\}, 63 \to \{\{31, 1\}\}]$$

Options

```
CalcPauliTransferMap[Depol4[x]]
CalcPauliTransferMap[Depol4[x], AssertValidChannels → False]
```

$$\text{PTMap}_4\left[0 \to \{\{0, 1\}\}, 1 \to \left\{\left\{1, 1 - \frac{4x}{3}\right\}\right\}, 2 \to \left\{\left\{2, 1 - \frac{4x}{3}\right\}\right\}, 3 \to \left\{\left\{3, 1 - \frac{4x}{3}\right\}\right\}\right]$$

$$\text{PTMap}_4\left[0 \to \left\{\left\{0, \frac{1}{2}\left(2\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right] + 2\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\},\right.$$
$$1 \to \left\{\left\{1, \frac{1}{2}\left(2\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right] - \frac{2}{3}\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\},$$
$$2 \to \left\{\left\{2, \frac{1}{2}\left(2\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right] - \frac{2}{3}\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\},$$
$$\left.3 \to \left\{\left\{3, \frac{1}{2}\left(2\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right] - \frac{2}{3}\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\}\right]$$

```
CalcPauliTransferMap[Damp_0[x]]
CalcPauliTransferMap[Damp_0[x], AssertValidChannels → False]
```

$$\text{PTMap}_0\left[0 \to \{\{0, 1\}, \{3, x\}\}, 1 \to \left\{\left\{1, \sqrt{1-x}\right\}\right\}, 2 \to \left\{\left\{2, \sqrt{1-x}\right\}\right\}, 3 \to \{\{3, 1-x\}\}\right]$$

$$\text{PTMap}_0\left[0 \to \left\{\left\{0, \frac{1}{2}\left(1+\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]+\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\},\right.\right.$$
$$\left.\left\{3, \frac{1}{2}\left(1-\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]+\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\},$$
$$1 \to \left\{\left\{1, \frac{1}{2}\left(\sqrt{1-x}+\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}, \left\{2, \frac{1}{2}\left(-i\sqrt{1-x}+i\,\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}\right\},$$
$$2 \to \left\{\left\{1, \frac{1}{2}\left(i\sqrt{1-x}-i\,\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}, \left\{2, \frac{1}{2}\left(\sqrt{1-x}+\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}\right\},$$
$$3 \to \left\{\left\{0, \frac{1}{2}\left(1-\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]-\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\},\right.$$
$$\left.\left.\left\{3, \frac{1}{2}\left(1+\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]-\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\}\right]$$

Errors

```
CalcPauliTransferMap @ PTM_{1,1} @ IdentityMatrix[4^2]
```

CalcPauliTransferMap : The PTM target indices were not unique non-negative integers.

```
$Failed
```

```
CalcPauliTransferMap @ PTM_{-5,1} @ IdentityMatrix[4^2]
```

CalcPauliTransferMap : The PTM target indices were not unique non-negative integers.

```
$Failed
```

```
CalcPauliTransferMap @ PTM_{0,1} @ IdentityMatrix[3^2]
```

CalcPauliTransferMap : The PTM matrix was not a compatibly-sized square matrix.

```
$Failed
```

```
CalcPauliTransferMap @ PTM_{0,1} @ {1, 2, 3}
```

CalcPauliTransferMap : The PTM matrix was not a compatibly-sized square matrix.

```
$Failed
```

```
CalcPauliTransferMap @ X_{-1}
```

CalcPauliTransferMap : The PTM target indices were not unique non-negative integers.

```
$Failed
```

```
CalcPauliTransferMap[Rz_{1,2}[x], "BadOtion" → True]
```

OptionValue : Unknown option BadOtion for CalcPauliTransferMap.

```
$Failed
```