CalcPauliTransferMap

```
SetDirectory @ NotebookDirectory[];
Import["../Link/QuESTlink.m"];
```

Doc

```
? CalcPauliTransferMap
```

Symbol

CalcPauliTransferMap[ptm] produces a PTMap equivalent to the given PTM operator. See ?PTM.
CalcPauliTransferMap[circuit] produces a PTMap from a
given gate or circuit, and merely first invokes CalcPauliTransferMatrix.
By default, this returned map is from a Pauli-string index to a list of {index, coefficient} pairs,
encoding how the PTM would modify the indexed Pauli string. The indexing is the same
as used by GetIndexOfPauliString[] and GetPauliStringFromIndex[], where the subscripted
qubits of the PTM are treated as though given in order of increasing significance.
CalcPauliTransferMap accepts options "KroneckerForm"->True which replaces the Pauli-string indices
with their corresponding Pauli-string in the form produced by GetKroneckerOfPauliString[].
CalcPauliTransferMap also accepts option AssertValidChannels->False
to disable the automatic simplification of the map's coefficients through
the assertion of valid channel parameters. See ?AssertValidChannels.

```
? PTM
```

Symbol

PTM[matrix] is a Pauli-transfer matrix representation of an operator or
channel. The subscript indices specify which Paulis of a Pauli string are operated
upon. Such objects are produced by functions like CalcPauliTransferMatrix[].

```
? PTMap
```

Symbol

PTMap[map] is a representation of a Pauli transfer matrix as a map between Pauli tensors,
specified either as basis-state indices or in a Kronecker form. See ?CalcPauliTransferMap.

Correctness

PTM

```
iden = IdentityMatrix[4^2];
```

```
CalcPauliTransferMap[PTM_{0,1}[iden]]
CalcPauliTransferMap[PTM_{0,1}[iden], "KroneckerForm" → True]
```

```
PTMap_{0,1}[0 → {{0, 1}}, 1 → {{1, 1}}, 2 → {{2, 1}}, 3 → {{3, 1}}, 4 → {{4, 1}},
 5 → {{5, 1}}, 6 → {{6, 1}}, 7 → {{7, 1}}, 8 → {{8, 1}}, 9 → {{9, 1}}, 10 → {{10, 1}},
 11 → {{11, 1}}, 12 → {{12, 1}}, 13 → {{13, 1}}, 14 → {{14, 1}}, 15 → {{15, 1}}]
```

```
PTMap_{0,1}[Id⊗Id → {{Id⊗Id, 1}}, Id⊗X → {{Id⊗X, 1}}, Id⊗Y → {{Id⊗Y, 1}},
 Id⊗Z → {{Id⊗Z, 1}}, X⊗Id → {{X⊗Id, 1}}, X⊗X → {{X⊗X, 1}},
 X⊗Y → {{X⊗Y, 1}}, X⊗Z → {{X⊗Z, 1}}, Y⊗Id → {{Y⊗Id, 1}},
 Y⊗X → {{Y⊗X, 1}}, Y⊗Y → {{Y⊗Y, 1}}, Y⊗Z → {{Y⊗Z, 1}},
 Z⊗Id → {{Z⊗Id, 1}}, Z⊗X → {{Z⊗X, 1}}, Z⊗Y → {{Z⊗Y, 1}}, Z⊗Z → {{Z⊗Z, 1}}]
```

```
GetPauliStringFromIndex
GetIndexOfPauliString
```

```
GetKroneckerOfPauliString
```

Operator

```
CalcPauliTransferMap[Rz_0[x]]
```

```
PTMap_0[0 → {{0, 1}}, 1 → {{1, Cos[x]}, {2, Sin[x]}},
 2 → {{1, -Sin[x]}, {2, Cos[x]}}, 3 → {{3, 1}}]
```

```
CalcPauliTransferMap @ C_0[X_1]
Length /@ (List @@ %)[[All, 2]] // Max
```

```
PTMap_{1,0}[0 → {{0, 1}}, 1 → {{1, 1}}, 2 → {{14, 1}}, 3 → {{15, 1}},
 4 → {{5, 1}}, 5 → {{4, 1}}, 6 → {{11, 1}}, 7 → {{10, -1}},
 8 → {{9, 1}}, 9 → {{8, 1}}, 10 → {{7, -1}}, 11 → {{6, 1}},
 12 → {{12, 1}}, 13 → {{13, 1}}, 14 → {{2, 1}}, 15 → {{3, 1}}]
```

```
1
```

```
CalcPauliTransferMap @ R[x, X_0 Y_1 Z_4];
Length /@ (List @@ %)[[All, 2]] // Max
```

```
2
```

```
CalcPauliTransferMap @ H_1
```

```
PTMap_1[0 → {{0, 1}}, 1 → {{3, 1}}, 2 → {{2, -1}}, 3 → {{1, 1}}]
```

CalcPauliTransferMap @ $C_0[H_3]$

$$\text{PTMap}_{3,0}\left[0 \to \{\{0, 1\}\}, 1 \to \left\{\left\{1, \frac{1}{2}\right\}, \left\{3, \frac{1}{2}\right\}, \left\{13, \frac{1}{2}\right\}, \left\{15, -\frac{1}{2}\right\}\right\}, 2 \to \{\{14, 1\}\},\right.$$
$$3 \to \left\{\left\{1, \frac{1}{2}\right\}, \left\{3, \frac{1}{2}\right\}, \left\{13, -\frac{1}{2}\right\}, \left\{15, \frac{1}{2}\right\}\right\}, 4 \to \left\{\left\{5, \frac{1}{\sqrt{2}}\right\}, \left\{7, \frac{1}{\sqrt{2}}\right\}\right\},$$
$$5 \to \left\{\left\{4, \frac{1}{\sqrt{2}}\right\}, \left\{10, \frac{1}{\sqrt{2}}\right\}\right\}, 6 \to \left\{\left\{9, -\frac{1}{\sqrt{2}}\right\}, \left\{11, \frac{1}{\sqrt{2}}\right\}\right\},$$
$$7 \to \left\{\left\{4, \frac{1}{\sqrt{2}}\right\}, \left\{10, -\frac{1}{\sqrt{2}}\right\}\right\}, 8 \to \left\{\left\{9, \frac{1}{\sqrt{2}}\right\}, \left\{11, \frac{1}{\sqrt{2}}\right\}\right\},$$
$$9 \to \left\{\left\{6, -\frac{1}{\sqrt{2}}\right\}, \left\{8, \frac{1}{\sqrt{2}}\right\}\right\}, 10 \to \left\{\left\{5, \frac{1}{\sqrt{2}}\right\}, \left\{7, -\frac{1}{\sqrt{2}}\right\}\right\}, 11 \to \left\{\left\{6, \frac{1}{\sqrt{2}}\right\}, \left\{8, \frac{1}{\sqrt{2}}\right\}\right\},$$
$$12 \to \{\{12, 1\}\}, 13 \to \left\{\left\{1, \frac{1}{2}\right\}, \left\{3, -\frac{1}{2}\right\}, \left\{13, \frac{1}{2}\right\}, \left\{15, \frac{1}{2}\right\}\right\},$$
$$\left.14 \to \{\{2, 1\}\}, 15 \to \left\{\left\{1, -\frac{1}{2}\right\}, \left\{3, \frac{1}{2}\right\}, \left\{13, \frac{1}{2}\right\}, \left\{15, \frac{1}{2}\right\}\right\}\right]$$

CalcPauliTransferMap @ $\text{Damp}_0[x]$

$$\text{PTMap}_0\left[0 \to \{\{0, 1\}, \{3, x\}\}, 1 \to \left\{\left\{1, \sqrt{1-x}\right\}\right\}, 2 \to \left\{\left\{2, \sqrt{1-x}\right\}\right\}, 3 \to \{\{3, 1-x\}\}\right]$$

CalcPauliTransferMap @ $\text{Depol}_{0,2}[x]$

$$\text{PTMap}_{0,2}\left[0 \to \{\{0, 1\}\}, 1 \to \left\{\left\{1, 1-\frac{16\,x}{15}\right\}\right\}, 2 \to \left\{\left\{2, 1-\frac{16\,x}{15}\right\}\right\},\right.$$
$$3 \to \left\{\left\{3, 1-\frac{16\,x}{15}\right\}\right\}, 4 \to \left\{\left\{4, 1-\frac{16\,x}{15}\right\}\right\}, 5 \to \left\{\left\{5, 1-\frac{16\,x}{15}\right\}\right\},$$
$$6 \to \left\{\left\{6, 1-\frac{16\,x}{15}\right\}\right\}, 7 \to \left\{\left\{7, 1-\frac{16\,x}{15}\right\}\right\}, 8 \to \left\{\left\{8, 1-\frac{16\,x}{15}\right\}\right\}, 9 \to \left\{\left\{9, 1-\frac{16\,x}{15}\right\}\right\},$$
$$10 \to \left\{\left\{10, 1-\frac{16\,x}{15}\right\}\right\}, 11 \to \left\{\left\{11, 1-\frac{16\,x}{15}\right\}\right\}, 12 \to \left\{\left\{12, 1-\frac{16\,x}{15}\right\}\right\},$$
$$\left.13 \to \left\{\left\{13, 1-\frac{16\,x}{15}\right\}\right\}, 14 \to \left\{\left\{14, 1-\frac{16\,x}{15}\right\}\right\}, 15 \to \left\{\left\{15, 1-\frac{16\,x}{15}\right\}\right\}\right]$$

CalcPauliTransferMap @ Kraus_0 @ { a IdentityMatrix[2], b PauliMatrix[2]}

$$\text{PTMap}_0\left[0 \to \left\{\left\{0, \text{Abs}[a]^2 + \text{Abs}[b]^2\right\}\right\}, 1 \to \left\{\left\{1, \text{Abs}[a]^2 - b\,\text{Conjugate}[b]\right\}\right\},\right.$$
$$\left.2 \to \left\{\left\{2, \text{Abs}[a]^2 + \text{Abs}[b]^2\right\}\right\}, 3 \to \left\{\left\{3, \text{Abs}[a]^2 - b\,\text{Conjugate}[b]\right\}\right\}\right]$$

$\text{Kraus}_{2,4}$ @ Table[RandomComplex[{-1 - i, 1 + i}, $\{2^2, 2^2\}$], 6];
List @@ CalcPauliTransferMap[%, "KroneckerForm" → True] // Chop // Column

$\text{Id} \otimes \text{Id} \to$
{{Id ⊗ Id, 16.7312}, {Id ⊗ X, −1.41586}, {Id ⊗ Y, 0.196713}, {Id ⊗ Z, 0.154914},
{X ⊗ Id, 0.0542583}, {X ⊗ X, −1.45591}, {X ⊗ Y, −1.21574}, {X ⊗ Z, −1.58162},
{Y ⊗ Id, −1.02844}, {Y ⊗ X, −2.07866}, {Y ⊗ Y, 0.0907304}, {Y ⊗ Z, 0.311852},
{Z ⊗ Id, 0.443758}, {Z ⊗ X, 2.50728}, {Z ⊗ Y, −3.10869}, {Z ⊗ Z, 1.38539}}

$\text{Id} \otimes \text{X} \to$ {{Id ⊗ Id, 0.59228}, {Id ⊗ X, −1.44059}, {Id ⊗ Y, 1.34879}, {Id ⊗ Z, −1.70407},
{X ⊗ Id, −1.77884}, {X ⊗ X, −1.10503}, {X ⊗ Y, 0.596517}, {X ⊗ Z, 0.833307},
{Y ⊗ Id, 2.64471}, {Y ⊗ X, 0.322951}, {Y ⊗ Y, −0.893728}, {Y ⊗ Z, 1.67444},
{Z ⊗ Id, 0.0599756}, {Z ⊗ X, −1.91418}, {Z ⊗ Y, 3.21071}, {Z ⊗ Z, 0.138598}}

```
Id⊗Y → {{Id⊗Id, 1.75946}, {Id⊗X, -0.338879}, {Id⊗Y, -1.54278}, {Id⊗Z, 1.97007},
  {X⊗Id, 2.04071}, {X⊗X, -0.447619}, {X⊗Y, -0.0171636}, {X⊗Z, -1.48214},
  {Y⊗Id, 1.14163}, {Y⊗X, -2.43083}, {Y⊗Y, 0.0748366}, {Y⊗Z, 0.25137},
  {Z⊗Id, 0.034064}, {Z⊗X, -4.62242}, {Z⊗Y, -2.04677}, {Z⊗Z, 0.172367}}
Id⊗Z →
 {{Id⊗Id, -0.218347}, {Id⊗X, -0.301319}, {Id⊗Y, 0.343212}, {Id⊗Z, 2.68274},
  {X⊗Id, 0.843191}, {X⊗X, -1.27398}, {X⊗Y, 2.07347}, {X⊗Z, 1.51922},
  {Y⊗Id, -1.58945}, {Y⊗X, -2.7189}, {Y⊗Y, 3.52298}, {Y⊗Z, -0.882158},
  {Z⊗Id, -1.73229}, {Z⊗X, -0.30159}, {Z⊗Y, 1.17027}, {Z⊗Z, 0.833802}}
X⊗Id →
 {{Id⊗Id, -1.74265}, {Id⊗X, -2.83702}, {Id⊗Y, -4.69752}, {Id⊗Z, -1.06024},
  {X⊗Id, 0.983527}, {X⊗X, 1.37377}, {X⊗Y, -1.81577}, {X⊗Z, 0.108684},
  {Y⊗Id, 1.61577}, {Y⊗X, 1.07763}, {Y⊗Y, 1.56413}, {Y⊗Z, -1.46263},
  {Z⊗Id, 0.0469708}, {Z⊗X, -1.61487}, {Z⊗Y, -1.23402}, {Z⊗Z, 1.52635}}
X⊗X → {{Id⊗Id, -1.13401}, {Id⊗X, 0.17856}, {Id⊗Y, 2.00025}, {Id⊗Z, 0.176444},
  {X⊗Id, -0.178622}, {X⊗X, 1.44866}, {X⊗Y, 2.20307}, {X⊗Z, -2.91024},
  {Y⊗Id, 3.65132}, {Y⊗X, -1.04273}, {Y⊗Y, -0.193903}, {Y⊗Z, -3.03244},
  {Z⊗Id, -1.87032}, {Z⊗X, 0.335548}, {Z⊗Y, -0.921584}, {Z⊗Z, 0.330549}}
X⊗Y → {{Id⊗Id, 1.07247}, {Id⊗X, -1.64193}, {Id⊗Y, -1.6018}, {Id⊗Z, -1.30206},
  {X⊗Id, 0.250537}, {X⊗X, 2.38503}, {X⊗Y, -0.755889}, {X⊗Z, -2.09751},
  {Y⊗Id, 0.703808}, {Y⊗X, 0.72846}, {Y⊗Y, -2.39176}, {Y⊗Z, -3.16116},
  {Z⊗Id, 0.870071}, {Z⊗X, 2.41389}, {Z⊗Y, 2.99307}, {Z⊗Z, 0.308434}}
X⊗Z → {{Id⊗Id, -2.04953}, {Id⊗X, 1.23519}, {Id⊗Y, 2.76842}, {Id⊗Z, -2.56993},
  {X⊗Id, -0.135464}, {X⊗X, -1.64505}, {X⊗Y, -2.72144}, {X⊗Z, -0.745602},
  {Y⊗Id, -0.494476}, {Y⊗X, 0.722309}, {Y⊗Y, -0.426924}, {Y⊗Z, 3.29696},
  {Z⊗Id, -1.50126}, {Z⊗X, 0.209813}, {Z⊗Y, 0.856082}, {Z⊗Z, 1.45667}}
Y⊗Id → {{Id⊗Id, 2.57039}, {Id⊗X, -1.0105}, {Id⊗Y, -0.792324}, {Id⊗Z, 0.231285},
  {X⊗Id, 2.77091}, {X⊗X, -1.25154}, {X⊗Y, -0.213752}, {X⊗Z, -2.4503},
  {Y⊗Id, -0.490035}, {Y⊗X, -2.85679}, {Y⊗Y, 0.954808}, {Y⊗Z, -0.256108},
  {Z⊗Id, 1.54017}, {Z⊗X, -2.27416}, {Z⊗Y, -1.49655}, {Z⊗Z, -0.675587}}
Y⊗X → {{Id⊗Id, -1.64352}, {Id⊗X, -1.31141}, {Id⊗Y, 2.29969}, {Id⊗Z, -0.2515},
  {X⊗Id, 0.747353}, {X⊗X, 0.349126}, {X⊗Y, -0.247641}, {X⊗Z, 1.15239},
  {Y⊗Id, 1.58086}, {Y⊗X, -1.51398}, {Y⊗Y, 0.385115}, {Y⊗Z, -0.667356},
  {Z⊗Id, -0.085888}, {Z⊗X, -0.242368}, {Z⊗Y, 3.28483}, {Z⊗Z, -1.23265}}
Y⊗Y → {{Id⊗Id, 3.79457}, {Id⊗X, -1.09339}, {Id⊗Y, 0.631768}, {Id⊗Z, 1.09411},
  {X⊗Id, 0.954851}, {X⊗X, -2.96613}, {X⊗Y, 0.410646}, {X⊗Z, -2.42171},
  {Y⊗Id, 0.179661}, {Y⊗X, -3.16573}, {Y⊗Y, 1.63963}, {Y⊗Z, -2.65582},
  {Z⊗Id, 1.46622}, {Z⊗X, -2.61797}, {Z⊗Y, -1.84772}, {Z⊗Z, 3.39776}}
Y⊗Z → {{Id⊗Id, 4.58919}, {Id⊗X, 0.113049}, {Id⊗Y, 2.7307}, {Id⊗Z, 1.89674},
  {X⊗Id, 0.595218}, {X⊗X, 0.0333301}, {X⊗Y, -2.42914}, {X⊗Z, 0.868558},
  {Y⊗Id, -0.424232}, {Y⊗X, -1.25379}, {Y⊗Y, -0.413212}, {Y⊗Z, 2.21156},
  {Z⊗Id, -0.653779}, {Z⊗X, 1.69471}, {Z⊗Y, -2.94888}, {Z⊗Z, 2.57143}}
Z⊗Id → {{Id⊗Id, -0.683017}, {Id⊗X, 4.33221}, {Id⊗Y, 0.74947}, {Id⊗Z, 0.884625},
  {X⊗Id, 0.275214}, {X⊗X, 0.125808}, {X⊗Y, 0.669383}, {X⊗Z, -1.3845},
  {Y⊗Id, -1.36683}, {Y⊗X, 0.234427}, {Y⊗Y, 0.0627844}, {Y⊗Z, -0.19479},
  {Z⊗Id, 1.01639}, {Z⊗X, -1.44982}, {Z⊗Y, 0.315258}, {Z⊗Z, 0.658352}}
```

```
Z⊗X → {{Id⊗Id, -0.042419}, {Id⊗X, 1.22772}, {Id⊗Y, 2.28736}, {Id⊗Z, 0.356979},
  {X⊗Id, -1.30277}, {X⊗X, -3.51304}, {X⊗Y, 1.84021}, {X⊗Z, 0.431616},
  {Y⊗Id, 0.801144}, {Y⊗X, 2.62591}, {Y⊗Y, -0.309129}, {Y⊗Z, 1.349},
  {Z⊗Id, -2.81085}, {Z⊗X, -0.0965757}, {Z⊗Y, -1.67745}, {Z⊗Z, -0.323737}}
Z⊗Y → {{Id⊗Id, -1.47279}, {Id⊗X, -1.93399}, {Id⊗Y, 3.35757}, {Id⊗Z, -0.911516},
  {X⊗Id, 0.0771559}, {X⊗X, 1.56082}, {X⊗Y, -1.18551}, {X⊗Z, -2.0411},
  {Y⊗Id, 1.24232}, {Y⊗X, 1.36794}, {Y⊗Y, 0.319667}, {Y⊗Z, -2.16152},
  {Z⊗Id, -3.56871}, {Z⊗X, -2.47943}, {Z⊗Y, 2.53773}, {Z⊗Z, -0.35608}}
Z⊗Z →
 {{Id⊗Id, -1.24409}, {Id⊗X, 1.23402}, {Id⊗Y, -0.806405}, {Id⊗Z, -0.310091},
  {X⊗Id, -0.855996}, {X⊗X, -3.24621}, {X⊗Y, 0.682964}, {X⊗Z, 1.67708},
  {Y⊗Id, -2.47568}, {Y⊗X, -1.2844}, {Y⊗Y, -1.58042}, {Y⊗Z, 1.15069},
  {Z⊗Id, 1.6621}, {Z⊗X, -3.01658}, {Z⊗Y, 1.40091}, {Z⊗Z, 0.684578}}
```

Circuit

CalcPauliTransferMap @ Circuit[X_0 Y_1 H_2]

```
PTMap0,1,2[0 → {{0, 1}}, 1 → {{1, 1}}, 2 → {{2, -1}}, 3 → {{3, -1}}, 4 → {{4, -1}},
 5 → {{5, -1}}, 6 → {{6, 1}}, 7 → {{7, 1}}, 8 → {{8, 1}}, 9 → {{9, 1}},
 10 → {{10, -1}}, 11 → {{11, -1}}, 12 → {{12, -1}}, 13 → {{13, -1}}, 14 → {{14, 1}},
 15 → {{15, 1}}, 16 → {{48, 1}}, 17 → {{49, 1}}, 18 → {{50, -1}}, 19 → {{51, -1}},
 20 → {{52, -1}}, 21 → {{53, -1}}, 22 → {{54, 1}}, 23 → {{55, 1}}, 24 → {{56, 1}},
 25 → {{57, 1}}, 26 → {{58, -1}}, 27 → {{59, -1}}, 28 → {{60, -1}}, 29 → {{61, -1}},
 30 → {{62, 1}}, 31 → {{63, 1}}, 32 → {{32, -1}}, 33 → {{33, -1}}, 34 → {{34, 1}},
 35 → {{35, 1}}, 36 → {{36, 1}}, 37 → {{37, 1}}, 38 → {{38, -1}}, 39 → {{39, -1}},
 40 → {{40, -1}}, 41 → {{41, -1}}, 42 → {{42, 1}}, 43 → {{43, 1}}, 44 → {{44, 1}},
 45 → {{45, 1}}, 46 → {{46, -1}}, 47 → {{47, -1}}, 48 → {{16, 1}}, 49 → {{17, 1}},
 50 → {{18, -1}}, 51 → {{19, -1}}, 52 → {{20, -1}}, 53 → {{21, -1}},
 54 → {{22, 1}}, 55 → {{23, 1}}, 56 → {{24, 1}}, 57 → {{25, 1}}, 58 → {{26, -1}},
 59 → {{27, -1}}, 60 → {{28, -1}}, 61 → {{29, -1}}, 62 → {{30, 1}}, 63 → {{31, 1}}]
```

Options

CalcPauliTransferMap[X_0]

CalcPauliTransferMap[X_0, "KroneckerForm" → True]

```
PTMap0[0 → {{0, 1}}, 1 → {{1, 1}}, 2 → {{2, -1}}, 3 → {{3, -1}}]
```

```
PTMap0[⊗Id → {{⊗Id, 1}}, ⊗X → {{⊗X, 1}}, ⊗Y → {{⊗Y, -1}}, ⊗Z → {{⊗Z, -1}}]
```

```
CalcPauliTransferMap[Rz_{1,2}[x]]
CalcPauliTransferMap[Rz_{1,2}[x], "KroneckerForm" → True]
```

$$\text{PTMap}_{1,2}[0 \to \{\{0, 1\}\}, 1 \to \{\{1, \text{Cos}[x]\}, \{14, \text{Sin}[x]\}\},$$
$$2 \to \{\{2, \text{Cos}[x]\}, \{13, -\text{Sin}[x]\}\}, 3 \to \{\{3, 1\}\}, 4 \to \{\{4, \text{Cos}[x]\}, \{11, \text{Sin}[x]\}\},$$
$$5 \to \{\{5, 1\}\}, 6 \to \{\{6, 1\}\}, 7 \to \{\{7, \text{Cos}[x]\}, \{8, \text{Sin}[x]\}\},$$
$$8 \to \{\{7, -\text{Sin}[x]\}, \{8, \text{Cos}[x]\}\}, 9 \to \{\{9, 1\}\}, 10 \to \{\{10, 1\}\},$$
$$11 \to \{\{4, -\text{Sin}[x]\}, \{11, \text{Cos}[x]\}\}, 12 \to \{\{12, 1\}\},$$
$$13 \to \{\{2, \text{Sin}[x]\}, \{13, \text{Cos}[x]\}\}, 14 \to \{\{1, -\text{Sin}[x]\}, \{14, \text{Cos}[x]\}\}, 15 \to \{\{15, 1\}\}]$$

$$\text{PTMap}_{1,2}[\text{Id} \otimes \text{Id} \to \{\{\text{Id} \otimes \text{Id}, 1\}\}, \text{Id} \otimes \text{X} \to \{\{\text{Id} \otimes \text{X}, \text{Cos}[x]\}, \{\text{Z} \otimes \text{Y}, \text{Sin}[x]\}\},$$
$$\text{Id} \otimes \text{Y} \to \{\{\text{Id} \otimes \text{Y}, \text{Cos}[x]\}, \{\text{Z} \otimes \text{X}, -\text{Sin}[x]\}\}, \text{Id} \otimes \text{Z} \to \{\{\text{Id} \otimes \text{Z}, 1\}\},$$
$$\text{X} \otimes \text{Id} \to \{\{\text{X} \otimes \text{Id}, \text{Cos}[x]\}, \{\text{Y} \otimes \text{Z}, \text{Sin}[x]\}\}, \text{X} \otimes \text{X} \to \{\{\text{X} \otimes \text{X}, 1\}\},$$
$$\text{X} \otimes \text{Y} \to \{\{\text{X} \otimes \text{Y}, 1\}\}, \text{X} \otimes \text{Z} \to \{\{\text{X} \otimes \text{Z}, \text{Cos}[x]\}, \{\text{Y} \otimes \text{Id}, \text{Sin}[x]\}\},$$
$$\text{Y} \otimes \text{Id} \to \{\{\text{X} \otimes \text{Z}, -\text{Sin}[x]\}, \{\text{Y} \otimes \text{Id}, \text{Cos}[x]\}\}, \text{Y} \otimes \text{X} \to \{\{\text{Y} \otimes \text{X}, 1\}\},$$
$$\text{Y} \otimes \text{Y} \to \{\{\text{Y} \otimes \text{Y}, 1\}\}, \text{Y} \otimes \text{Z} \to \{\{\text{X} \otimes \text{Id}, -\text{Sin}[x]\}, \{\text{Y} \otimes \text{Z}, \text{Cos}[x]\}\},$$
$$\text{Z} \otimes \text{Id} \to \{\{\text{Z} \otimes \text{Id}, 1\}\}, \text{Z} \otimes \text{X} \to \{\{\text{Id} \otimes \text{Y}, \text{Sin}[x]\}, \{\text{Z} \otimes \text{X}, \text{Cos}[x]\}\},$$
$$\text{Z} \otimes \text{Y} \to \{\{\text{Id} \otimes \text{X}, -\text{Sin}[x]\}, \{\text{Z} \otimes \text{Y}, \text{Cos}[x]\}\}, \text{Z} \otimes \text{Z} \to \{\{\text{Z} \otimes \text{Z}, 1\}\}]$$

```
CalcPauliTransferMap[Depol_4[x]]
CalcPauliTransferMap[Depol_4[x], AssertValidChannels → False]
```

$$\text{PTMap}_4\left[0 \to \{\{0, 1\}\}, 1 \to \left\{\left\{1, 1 - \frac{4\,x}{3}\right\}\right\}, 2 \to \left\{\left\{2, 1 - \frac{4\,x}{3}\right\}\right\}, 3 \to \left\{\left\{3, 1 - \frac{4\,x}{3}\right\}\right\}\right]$$

$$\text{PTMap}_4\left[0 \to \left\{\left\{0, \frac{1}{2}\left(2\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right] + 2\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\},\right.$$
$$1 \to \left\{\left\{1, \frac{1}{2}\left(2\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right] - \frac{2}{3}\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\},$$
$$2 \to \left\{\left\{2, \frac{1}{2}\left(2\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right] - \frac{2}{3}\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\},$$
$$\left.3 \to \left\{\left\{3, \frac{1}{2}\left(2\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right] - \frac{2}{3}\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\}\right]$$

```
CalcPauliTransferMap[Damp_0[x]]
CalcPauliTransferMap[Damp_0[x], AssertValidChannels → False]
CalcPauliTransferMap[Damp_0[x],
 AssertValidChannels → False, "KroneckerForm" → True]
```

$$\text{PTMap}_0\left[0 \to \{\{0, 1\}, \{3, x\}\}, 1 \to \left\{\left\{1, \sqrt{1-x}\right\}\right\}, 2 \to \left\{\left\{2, \sqrt{1-x}\right\}\right\}, 3 \to \{\{3, 1-x\}\}\right]$$

$$\text{PTMap}_0\left[0 \to \left\{\left\{0, \frac{1}{2}\left(1+\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]+\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\},\right.\right.$$
$$\left.\left\{3, \frac{1}{2}\left(1-\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]+\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\},$$
$$1 \to \left\{\left\{1, \frac{1}{2}\left(\sqrt{1-x}+\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}, \left\{2, \frac{1}{2}\left(-i\sqrt{1-x}+i\,\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}\right\},$$
$$2 \to \left\{\left\{1, \frac{1}{2}\left(i\sqrt{1-x}-i\,\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}, \left\{2, \frac{1}{2}\left(\sqrt{1-x}+\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}\right\},$$
$$3 \to \left\{\left\{0, \frac{1}{2}\left(1-\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]-\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\},\right.$$
$$\left.\left.\left\{3, \frac{1}{2}\left(1+\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]-\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\}\right]$$

$$\text{PTMap}_0\left[\otimes\text{Id} \to \left\{\left\{\otimes\text{Id}, \frac{1}{2}\left(1+\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]+\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\},\right.\right.$$
$$\left.\left\{\otimes\text{Z}, \frac{1}{2}\left(1-\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]+\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\},$$
$$\otimes\text{X} \to \left\{\left\{\otimes\text{X}, \frac{1}{2}\left(\sqrt{1-x}+\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\},\right.$$
$$\left.\left\{\otimes\text{Y}, \frac{1}{2}\left(-i\sqrt{1-x}+i\,\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}\right\}, \otimes\text{Y} \to$$
$$\left\{\left\{\otimes\text{X}, \frac{1}{2}\left(i\sqrt{1-x}-i\,\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}, \left\{\otimes\text{Y}, \frac{1}{2}\left(\sqrt{1-x}+\text{Conjugate}\left[\sqrt{1-x}\right]\right)\right\}\right\},$$
$$\otimes\text{Z} \to \left\{\left\{\otimes\text{Id}, \frac{1}{2}\left(1-\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]-\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\},\right.$$
$$\left.\left.\left\{\otimes\text{Z}, \frac{1}{2}\left(1+\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right]-\sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right)\right\}\right\}\right]$$

Errors

```
CalcPauliTransferMap @ PTM_{1,1} @ IdentityMatrix[4^2]
```

CalcPauliTransferMap : The PTM target indices were not unique non-negative integers.

```
$Failed
```

```
CalcPauliTransferMap @ PTM_{-5,1} @ IdentityMatrix[4^2]
```

CalcPauliTransferMap : The PTM target indices were not unique non-negative integers.

```
$Failed
```

```
CalcPauliTransferMap @ PTM_{0,1} @ IdentityMatrix[3^2]
```

CalcPauliTransferMap : The PTM matrix was not a compatibly–sized square matrix.

```
$Failed
```

```
CalcPauliTransferMap @ PTM_{0,1} @ {1, 2, 3}
```

CalcPauliTransferMap : The PTM matrix was not a compatibly–sized square matrix.

```
$Failed
```

```
CalcPauliTransferMap @ X_{-1}
```

CalcPauliTransferMap : The PTM target indices were not unique non–negative integers.

```
$Failed
```

```
CalcPauliTransferMap @ Circuit[X_0 Y_1]
```

```
PTMap_{0,1}[0 → {{0, 1}}, 1 → {{1, 1}}, 2 → {{2, -1}}, 3 → {{3, -1}}, 4 → {{4, -1}},
 5 → {{5, -1}}, 6 → {{6, 1}}, 7 → {{7, 1}}, 8 → {{8, 1}}, 9 → {{9, 1}}, 10 → {{10, -1}},
 11 → {{11, -1}}, 12 → {{12, -1}}, 13 → {{13, -1}}, 14 → {{14, 1}}, 15 → {{15, 1}}]
```