# DrawPauliTransferEval

SetDirectory @ NotebookDirectory[];
Import["../Link/QuESTlink.m"];

### Doc

#### ? DrawPauliTransferEval

#### Symbol

DrawPauliTransferEval [pauliString, circuit] renders and returns a graph of the evaluation of 'circuit' when converted to a series of Pauli transfer maps, acting upon the given initial Pauli string.

DrawPauliTransferEval [data] renders the pre-computed

evaluation graph 'data' as output by CalcPauliTransferEval[].

DrawPauliTransferEval accepts all options to Graph[], CalcPauliTransferEval[],

DrawPauliTransferMap [], and some additional options, which we summarise below.

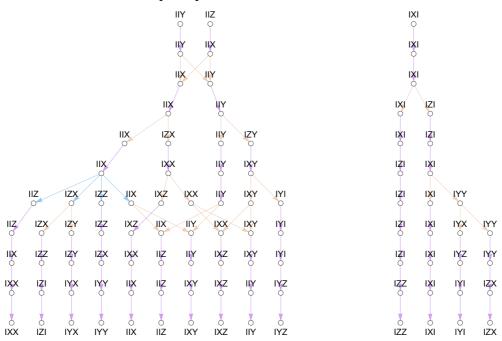
- "HighlightPathTo" -> pauliString (or a list of Pauli strings) highlights all edges ultimately contributing to the coefficient of the specified final pauliString(s). Symbolically weighted sums of Pauli strings are also accepted, in which case all edges to all non-orthogonal Pauli strings are highlighted.
- "CombineStrings" -> False disables combining incident
   Pauli strings so that the result is an (likely significantly larger) acyclic tree.
- "PauliStringForm" sets the vertex label format to one of "String", "Hidden" (these are the defaults depending on graph size), "Index", "Kronecker", or "Subscript". See ?GetPauliStringReformatted.
- "ShowCoefficients" -> True or False explicitly shows or hides the PTMap coefficient associated with each edge. The default is Automatic which auto-hides edge labels if there are too many.
- "EdgeDegreeStyles" specifies the style of edges from nodes of increasing outdegree. See ?DrawPauliTransferMap.
- "CacheMaps" controls the automatic caching of generated PTMaps. See ?ApplyPauliTransferMap.
- AssertValidChannels -> False disables the simplification of symbolic Pauli string coefficients, only noticeable when "ShowCoefficients"->True. See ?AssertValidChannels.
- Graph[] options override these settings. For example, specifying
   EdgeStyle -> Black will set all edges to Black regardless of their node's outdegree.

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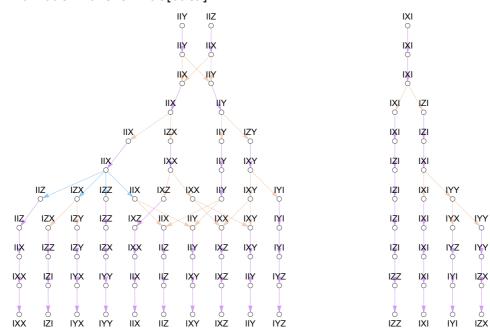
## Correctness

## Given evaluation data

```
u = Circuit[H_0 Rz_0[a] Ry_1[b] Damp_1[c] H_1 C_1[Ry_0[a]] Ph_0[x] H_0 C_0[X_1] Z_2];
in = Z_0 + Y_0 + X_1;
data = CalcPauliTransferEval[in, u];
DrawPauliTransferEval[data]
```

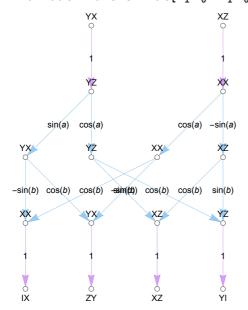


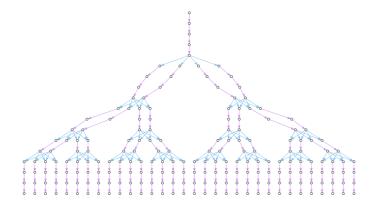
(\* detailed data should yield an identical graph \*) data = CalcPauliTransferEval[in, u, "OutputForm" → "Detailed"]; DrawPauliTransferEval[data]



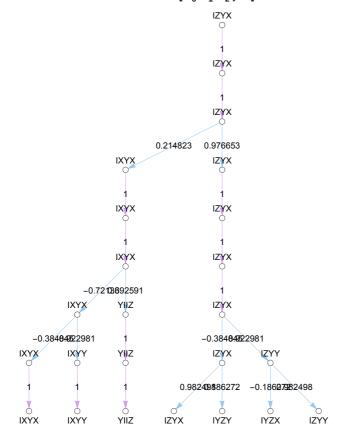
## Given circuit

 $u = Circuit[H_0 Ry_0[a] Rz_1[b] C_0[X_1]];$ DrawPauliTransferEval[ $X_1 Z_0 + Y_1 X_0$ , u]



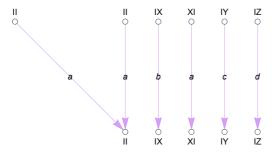


h = GetRandomPauliString[4, 8];
u = GetKnownCircuit["Trotter", h, 1, 1, π];
DrawPauliTransferEval[X<sub>0</sub> Y<sub>1</sub> Z<sub>2</sub>, u]

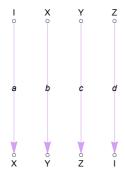


## Given PTMs and PTMaps

u = { PTM<sub>0</sub>@DiagonalMatrix@{a, b, c, d}}; DrawPauliTransferEval[ $Id_0 + X_0 + Y_0 + Z_0 + Id_1 + X_1$ , u]



```
u \; = \; \mathsf{PTMap}_{\theta} \, [\, 0 \, \to \, \{\{1,\, a\}\} \, , \, 1 \, \to \, \{\{2,\, b\}\} \, , \, 2 \, \to \, \{\{3,\, c\}\} \, , \, 3 \, \to \, \{\{0,\, d\}\}] \, ;
DrawPauliTransferEval[Id_0 + X_0 + Y_0 + Z_0, {u}]
```

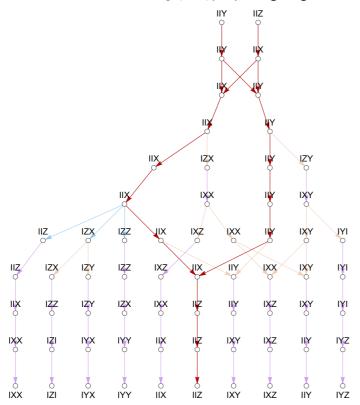


## **Options**

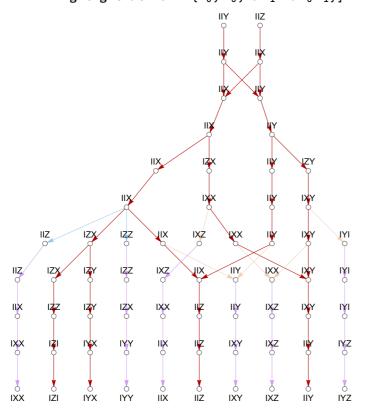
### "HighlightPathTo"

```
u = \mathsf{Circuit}[\mathsf{H}_0 \; \mathsf{Rz}_0[\mathsf{a}] \; \mathsf{Ry}_1[\mathsf{b}] \; \mathsf{Damp}_1[\mathsf{c}] \; \mathsf{H}_1 \; \mathsf{C}_1[\mathsf{Ry}_0[\mathsf{a}]] \; \mathsf{Ph}_0[\mathsf{x}] \; \mathsf{H}_0 \; \mathsf{C}_0[\mathsf{X}_1] \; \mathsf{Z}_2] \; ;
```

 $\label{eq:definition} DrawPauliTransferEval[Z_0+Y_0,\ u,\ "HighlightPathTo" \to Z_0]$ 

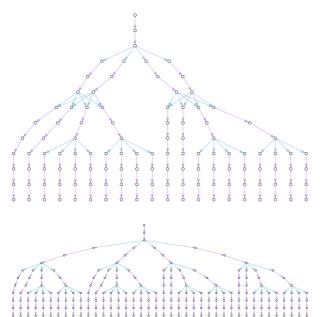


 $\label{eq:definition} DrawPauliTransferEval[Z_0+Y_0,\ u,$ "HighlightPathTo"  $\rightarrow$  {Z<sub>0</sub>, Y<sub>0</sub>, a Z<sub>1</sub> + b X<sub>0</sub> Y<sub>1</sub>}]



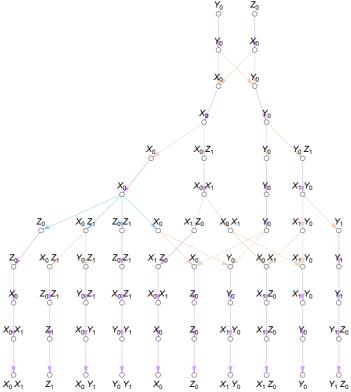
## "CombineStrings"

u = GetKnownCircuit["QFT", 4]; DrawPauliTransferEval[ $Z_0 X_1, u$ ]  $DrawPauliTransferEval[Z_0 \ X_1, \ u, \ "CombineStrings" \rightarrow False]$ 



### "PauliStringForm"

```
u = \mathsf{Circuit}[\mathsf{H}_0 \ \mathsf{Rz}_0[\mathsf{a}] \ \mathsf{Ry}_1[\mathsf{b}] \ \mathsf{Damp}_1[\mathsf{c}] \ \mathsf{H}_1 \ \mathsf{C}_1[\mathsf{Ry}_0[\mathsf{a}]] \ \mathsf{Ph}_0[\mathsf{x}] \ \mathsf{H}_0 \ \mathsf{C}_0[\mathsf{X}_1] \ \mathsf{Z}_2] \ ;
\label{eq:definition} DrawPauliTransferEval[Z_0 + Y_0, u, "PauliStringForm" \rightarrow "Subscript"]
```



```
u = Circuit[H_0 Rx_0[a] Damp_0[b] Depol_{0,1}[c] C_0[X_1]];
options = {"Hidden", "Subscript", "String", "Kronecker", "Index"};
```

```
Row @ Riffle[Table[
       Column@{form,
```

DrawPauliTransferEval[X₀, u, "PauliStringForm" → form, "ShowCoefficients" → False]},

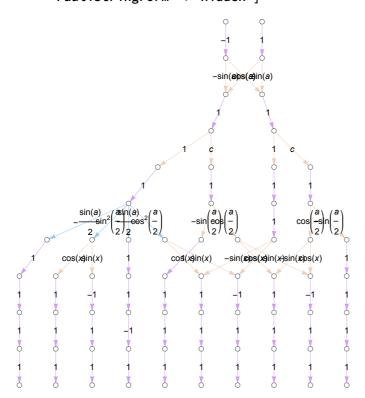
{form, options}], " "]

Hidden	Subscript	String	Kronecker	Index
j	$X_0$	IX	$\operatorname{Id} \otimes X$	1
	$\ddot{z_0}$	ΙŻ	ld⊗ Z	3
	$Y_0$ $Z_0$	IY IZ	ld⊗Mo⊗Z	2 3
	<b>Y</b> <sub>0</sub> <b>Z</b> <sub>0</sub>	iy iz	ld⊗M⊗Z	2 3
	$Y_0$ $Z_0$	IY IZ	ld⊗M⊗Z	2 3
	Ϋ́Υ	¥ ¥	Ϋ́Ϋ́	V V
Ĭ Ĭ	$X_1 Y_0 Z_0$	XY IZ	$X \otimes \mathbf{M} \otimes Z$	6 3

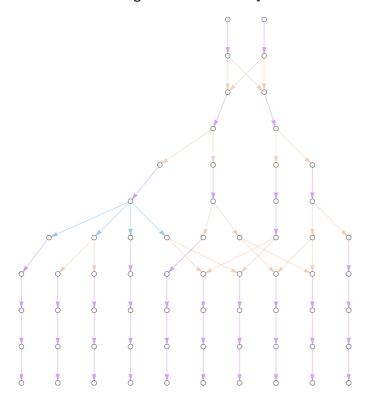
#### "ShowCoefficients"

```
u = \mathsf{Circuit}[\mathsf{H}_0 \ \mathsf{Rz}_0[\mathsf{a}] \ \mathsf{Ry}_1[\mathsf{b}] \ \mathsf{Damp}_1[\mathsf{c}] \ \mathsf{H}_1 \ \mathsf{C}_1[\mathsf{Ry}_0[\mathsf{a}]] \ \mathsf{Ph}_0[\mathsf{x}] \ \mathsf{H}_0 \ \mathsf{C}_0[\mathsf{X}_1] \ \mathsf{Z}_2] \ ;
```

DrawPauliTransferEval[ $Z_0 + Y_0$ , u, "ShowCoefficients"  $\rightarrow$  True, "PauliStringForm" → "Hidden"]



DrawPauliTransferEval[ $Z_0 + Y_0$ , u, "ShowCoefficients" → False, "PauliStringForm" → "Hidden"]

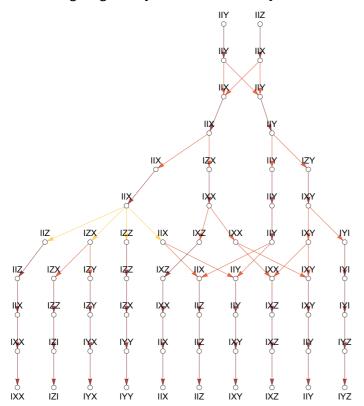


### "EdgeDegreeStyles"

 $u = \mathsf{Circuit}[\mathsf{H}_0 \; \mathsf{Rz}_0[\mathsf{a}] \; \mathsf{Ry}_1[\mathsf{b}] \; \mathsf{Damp}_1[\mathsf{c}] \; \mathsf{H}_1 \; \mathsf{C}_1[\mathsf{Ry}_0[\mathsf{a}]] \; \mathsf{Ph}_0[\mathsf{x}] \; \mathsf{H}_0 \; \mathsf{C}_0[\mathsf{X}_1] \; \mathsf{Z}_2] \; ;$ DrawPauliTransferEval[ $Z_0 + Y_0$ , u,  $\verb"EdgeDegreeStyles" \to \{Black, Lighter@Gray, White, LightGray\}]$ 

IIX IIX IIZ IYX IXX IXY IXZ

#### DrawPauliTransferEval[ $Z_0 + Y_0$ , u, "EdgeDegreeStyles" $\rightarrow$ ColorData["SolarColors"] /@ Range[0, 1, .3]]



### "CacheMaps"

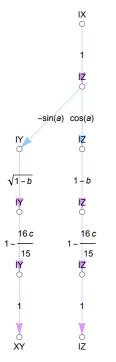
u = GetKnownCircuit["QFT", 5]; Timing @ DrawPauliTransferEval[ $X_3 Z_2 + X_2, u$ ] Timing @ DrawPauliTransferEval[ $X_3 Z_2 + X_2$ , u, "CacheMaps"  $\rightarrow$  "Never"]

1.56589,

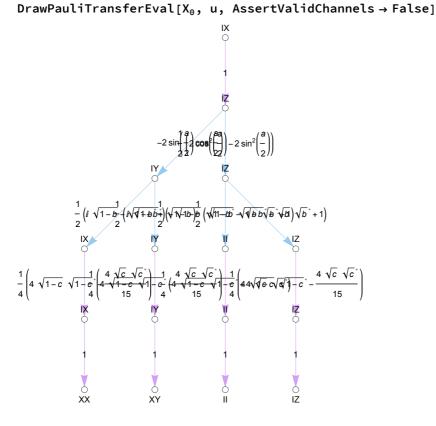
#### **AssertValidChannels**

 $\label{eq:u_signal} u \; = \; \mathsf{Circuit}\big[\mathsf{H}_0 \; \mathsf{Rx}_0[\mathsf{a}] \; \mathsf{Damp}_0[\mathsf{b}] \; \mathsf{Depol}_{0,1}[\mathsf{c}] \; \mathsf{C}_0[\mathsf{X}_1] \big] \, ;$ 

 $DrawPauliTransferEval[X_0, u]$ 



(∗ disabling simplification can cause zero weight branches to survive ∗)

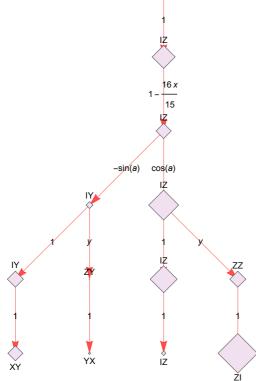


### $\label{eq:definition} DrawPauliTransferEval[X_0,\ Depol_0[3/4+.1],\ AssertValidChannels \rightarrow False]$

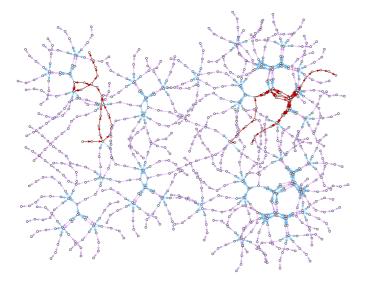


## Graph[] options

```
circ = Circuit[H_0 Depol_{0,1}[x] Rx_0[a] Damp_1[y] C_0[X_1]];
DrawPauliTransferEval[X₀, circ,
     EdgeStyle → Red,
     VertexShapeFunction → "Diamond",
     VertexSize ⇒ .5 RandomReal[],
     VertexStyle → LightPurple
]
```



```
u = GetKnownCircuit["QFT", 5];
DrawPauliTransferEval[X_3 Z_2 + X_2, u,
         "HighlightPathTo" \rightarrow {X<sub>2</sub>, Y<sub>1</sub> Y<sub>2</sub>, X<sub>4</sub> X<sub>3</sub> X<sub>2</sub> Y<sub>1</sub>},
        GraphLayout → "SpringEmbedding"
]
```



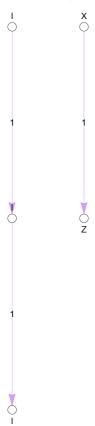
DrawPauliTransferEval[ $X_3 Z_2 + X_2$ , u, "HighlightPathTo"  $\rightarrow$  {X<sub>2</sub>, Y<sub>1</sub> Y<sub>2</sub>, X<sub>4</sub> X<sub>3</sub> X<sub>2</sub> Y<sub>1</sub>}, GraphHighlightStyle → "DehighlightGray" ]



# Warnings

#### $DrawPauliTransferEval[X_0 + Id_0, \{H_0, Depol_0[3/4]\}]$

••• DrawPauliTransferEval: Warning: the evaluation includes Pauli strings being mapped to null strings, as can occur from fully-mixing channels. The null strings and edges to them are not being rendered. Suppress this warning using Quiet[].



## **Errors**

#### DrawPauliTransferEval[ $X_0$ , Depol<sub>0</sub>[3/4 + .1]]

••• CalcPauliTransferEval: Could not pre-compute the Pauli transfer maps due to the below error:

... CalcPauliTransferMatrix: The channels could not be asserted as completely positive trace-preserving maps and hence were not simplified. Hide this warning with AssertValidChannels -> False, or use Quiet[].

••• DrawPauliTransferEval : Invalid arguments. See ?DrawPauliTransferEval

\$Failed

#### DrawPauliTransferEval[ $X_0$ , $X_0$ , "CombineStrings" $\rightarrow$ Eh]

••• DrawPauliTransferEval: Option "CombineStrings" must be True or False. See ?CalcPauliTransferEval.

\$Failed

```
DrawPauliTransferEval[X<sub>0</sub>, X<sub>0</sub>, "CacheMaps" → Eh]
w DrawPauliTransferEval: Option "CacheMaps" must be one of "Forever", "UntilCallEnd" or "Never". See
         ?ApplyPauliTransferMap.
$Failed
DrawPauliTransferEval[X₀, X₀, "ShowCoefficients" → Eh]
••• DrawPauliTransferEval: Option "ShowCoefficients" must be Automatic, True or False. See ?DrawPauliTransferEval.
$Failed
DrawPauliTransferEval[X<sub>0</sub>, X<sub>0</sub>, "PauliStringForm" → "Unknown"]
••• DrawPauliTransferEval: Invalid value for option "PauliStringForm". See ?DrawPauliTransferEval.
$Failed
DrawPauliTransferEval[X<sub>0</sub>, X<sub>0</sub>, "HighlightPathTo" → Eh]
DrawPauliTransferEval[X_0, X_0, "HighlightPathTo" \rightarrow X_{-1}]
••• DrawPauliTransferEval: Invalid value for option "HighlightPathTo". See ?DrawPauliTransferEval.
$Failed
••• DrawPauliTransferEval: Invalid value for option "HighlightPathTo". See ?DrawPauliTransferEval.
$Failed
DrawPauliTransferEval[X₀, X₀, "UnrecognisedOption" → Eh]
••• OptionValue: Unknown option UnrecognisedOption for
     {DrawPauliTransferEval, CalcPauliTransferEval, ApplyPauliTransferMap, CalcPauliTransferMap, Graph}.
$Failed
DrawPauliTransferEval[X<sub>-1</sub>, {X<sub>0</sub>}]
••• DrawPauliTransferEval: Invalid arguments. See ?DrawPauliTransferEval
$Failed
DrawPauliTransferEval[X<sub>0</sub> Y<sub>0</sub>, {X<sub>0</sub>}]
••• DrawPauliTransferEval: Invalid arguments. See ?DrawPauliTransferEval
$Failed
DrawPauliTransferEval[{X<sub>0</sub>}, {}]
DrawPauliTransferEval[{}, {X₀}]
DrawPauliTransferEval[]
••• DrawPauliTransferEval : Invalid arguments. See ?DrawPauliTransferEval
$Failed
••• DrawPauliTransferEval: Invalid arguments. See ?DrawPauliTransferEval
$Failed
```

••• DrawPauliTransferEval: Invalid arguments. See ?DrawPauliTransferEval

\$Failed