ApplyPauliTransferMap

```
SetDirectory @ NotebookDirectory[];
Import["../Link/QuESTlink.m"];
```

Doc

```
? ApplyPauliTransferMap
```

Symbol

ApplyPauliTransferMap[pauliString, ptMap] returns the Pauli string produced by the given PTMap acting upon the given initial Pauli string.

ApplyPauliTransferMap[pauliString, circuit] automatically transforms the given circuit (composed of gates, channels, and PTMs, possibly intermixed) into PTMaps before applying them to the given Pauli string.

This method uses automatic caching to avoid needless re-computation of an operator's PTMap, agnostic to the targeted and controlled qubits, at the cost of additional memory usage. Caching behaviour can be controlled using option "CacheMaps":

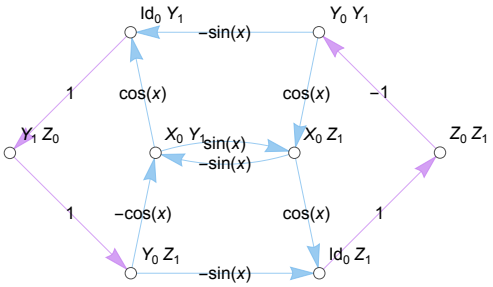
- "CacheMaps" -> "UntilCallEnd" (default) caches all computed PTMaps but clears the cache when ApplyPauliTransferMap[] returns.
- "CacheMaps" -> "Forever" maintains the cache even between multiple calls to ApplyPauliTransferMap[].
- "CacheMaps" -> "Never" disables caching (and clears the existing cache before computation), re-computing each operqtors' PTMap when encountered in the circuit.

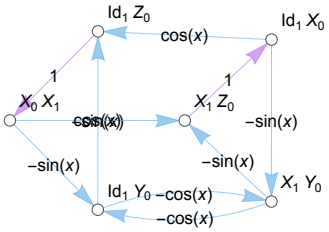
ApplyPauliTransferMap also accepts all options of CalcPauliTransferMap, like AssertValidChannels. See ?AssertValidChannels.

Correctness

Maps

```
map = CalcPauliTransferMap @ Circuit[H0 Rx0[x] C0[X1]];
DrawPauliTransferMap[map]
```






```
ApplyPauliTransferMap[X1 Z0, map]
```

X_0

```
ApplyPauliTransferMap[a X0 Z1 + b X1 + c X1 Z0, map]
```

$c\, X_0 + b\, X_1 - a\, \text{Sin}[x]\, X_0\, Y_1 + a\, \text{Cos}[x]\, Z_1$

```
ApplyPauliTransferMap[a X0 Z1 + b X1 + c X1 Z0, {map, map, map, map, map, map, map}]
```

$$-\frac{1}{2} c\, (1 + 3\, \text{Cos}[2\, x])\, \text{Sin}[x]^2\, X_0 + b\, X_1 + c\, \text{Cos}[x]^5\, X_0\, X_1 + c\, \text{Cos}[x]\, \text{Sin}[x]\, \left(\text{Cos}[x]^2 + \text{Cos}[x]^4 - 2\, \text{Sin}[x]^2\right)\, Y_0 + c\, \text{Cos}[x]^2\, (-1 + 2\, \text{Cos}[2\, x])\, \text{Sin}[x]\, X_1\, Y_0 + a\, \text{Sin}[x]\, \left(-\text{Cos}[x]^2 + \text{Sin}[x]^6 - \text{Sin}[2\, x]^2\right)\, X_0\, Y_1 - \frac{1}{8} a\, \text{Cos}[x]\, (3 - 12\, \text{Cos}[2\, x] + \text{Cos}[4\, x])\, Y_0\, Y_1 + \frac{1}{2} c\, \text{Sin}[x]\, \text{Sin}[4\, x]\, Z_0 + c\, \text{Cos}[x]^2\, \left(3 + \text{Cos}[x]^2\right)\, \text{Sin}[x]^2\, X_1\, Z_0 - \frac{1}{16} a\, (-1 - 16\, \text{Cos}[2\, x] + \text{Cos}[4\, x])\, \text{Sin}[2\, x]\, Y_1\, Z_0 - \frac{1}{8} a\, \text{Cos}[x]\, (3 - 20\, \text{Cos}[2\, x] + \text{Cos}[4\, x])\, \text{Sin}[x]^2\, Z_1$$

```
4 ^ 2
str = Z0 + Y1;
Table[
     Length @ ApplyPauliTransferMap[str, ConstantArray[map, reps]],
     {reps, 1, 10}]
```

16

{2, 3, 5, 8, 10, 10, 10, 10, 10, 10}

Circuits

```
ApplyPauliTransferMap[a X0 Z1 + b X1 + c X1 Z0,
 {map, C0[X1], PTM0@DiagonalMatrix@{a, b, c, d}}]
```

a b X_1 + b c X_0 X_1 − a c Sin[x] Y_0 Z_1 + a d Cos[x] Z_0 Z_1

```
n = 4;
u = GetKnownCircuit["QFT", n];
m = CalcCircuitMatrix[u];

ρin = RandomComplex[{-1 - i, 1 + i}, {2^n, 2^n}];
ρout = m . ρin . ConjugateTranspose[m];

σin = GetPauliString[ρin];
σout = ApplyPauliTransferMap[σin, u];

CalcPauliExpressionMatrix[σout] - ρout // Abs // Max
```

6.75322×10^{-16}

Fully-mixing channels

```
ApplyPauliTransferMap[X0, Deph0[.4]]
ApplyPauliTransferMap[X0, Deph0[.5]]
ApplyPauliTransferMap[X0, Deph0[.6], AssertValidChannels → False]
```

(0.2 + 0. i) X_0

0

(−0.2 + 0. i) X_0

```
ApplyPauliTransferMap[X0, Deph0,1[3 / 4]]
ApplyPauliTransferMap[X0, Depol0[3 / 4]]
ApplyPauliTransferMap[X0, Depol0,1[15 / 16]]
```

0

0

0

```
ApplyPauliTransferMap[X0, Damp0[1]]
```

```
0
```

Options

Caching

```
circ = Table[C0[X1], 500];
Timing @
 ApplyPauliTransferMap[a X0 Z1 + b X1 + c X1 Z0 + Y2, circ, "CacheMaps" → "Never"]
Timing @
 ApplyPauliTransferMap[a X0 Z1 + b X1 + c X1 Z0 + Y2, circ, "CacheMaps" → "Forever"]
```

```
{3.49939, b X1 + Y2 + c X1 Z0 + a X0 Z1}
```

```
{0.521339, b X1 + Y2 + c X1 Z0 + a X0 Z1}
```

```
ApplyPauliTransferMap[a X0 Z1 + b X1 + c X1 Z0 + Y2, circ, "CacheMaps" → "Forever"];
DownValues[QuEST`Private`obtainCachedPTMap] // First
```

```
ApplyPauliTransferMap[a X0 Z1 + b X1 + c X1 Z0 + Y2, circ, "CacheMaps" → "Never"];
DownValues[QuEST`Private`obtainCachedPTMap] // First
```

```
ApplyPauliTransferMap[a X0 Z1 + b X1 + c X1 Z0 + Y2,
  circ, "CacheMaps" → "UntilCallEnd"];
DownValues[QuEST`Private`obtainCachedPTMap] // First
```

```
HoldPattern[QuEST`Private`obtainCachedPTMap[{C1[X0]}, CacheMaps → Forever]] :>
 PTMap0,1[0 → {{0, 1}}, 1 → {{1, 1}}, 2 → {{14, 1}}, 3 → {{15, 1}},
  4 → {{5, 1}}, 5 → {{4, 1}}, 6 → {{11, 1}}, 7 → {{10, -1}},
  8 → {{9, 1}}, 9 → {{8, 1}}, 10 → {{7, -1}}, 11 → {{6, 1}},
  12 → {{12, 1}}, 13 → {{13, 1}}, 14 → {{2, 1}}, 15 → {{3, 1}}]
```

```
HoldPattern[QuEST`Private`obtainCachedPTMap[
   QuEST`Private`compGate_, QuEST`Private`opts___]] :>
 (QuEST`Private`obtainCachedPTMap[QuEST`Private`compGate, QuEST`Private`opts] =
   CalcPauliTransferMap[QuEST`Private`compGate, Sequence @@
     FilterRules[{QuEST`Private`opts}, Options[CalcPauliTransferMap]]])
```

```
HoldPattern[QuEST`Private`obtainCachedPTMap[
   QuEST`Private`compGate_, QuEST`Private`opts___]] :>
 (QuEST`Private`obtainCachedPTMap[QuEST`Private`compGate, QuEST`Private`opts] =
   CalcPauliTransferMap[QuEST`Private`compGate, Sequence @@
     FilterRules[{QuEST`Private`opts}, Options[CalcPauliTransferMap]]])
```

AssertValidChannels

```
ApplyPauliTransferMap[X_0 Y_1, Depol_{0,1}[x]]
ApplyPauliTransferMap[X_0 Y_1, Depol_{0,1}[x], AssertValidChannels → False]
```

$$\frac{1}{15}\,(15 - 16\,x)\,X_0\,Y_1$$

$$\frac{1}{15}\left(15\,\sqrt{1-x}\,\text{Conjugate}\left[\sqrt{1-x}\right] - \sqrt{x}\,\text{Conjugate}\left[\sqrt{x}\right]\right) X_0\,Y_1$$

Errors

```
ApplyPauliTransferMap[X_0, Deph_0[.6]]
```

ApplyPauliTransferMap : Could not pre–compute the Pauli transfer maps due to the below error:

CalcPauliTransferMatrix : The channels could not be asserted as completely positive trace–preserving maps and hence were not simplified. Hide this warning with AssertValidChannels –> False, or use Quiet[].

```
$Failed
```

```
ApplyPauliTransferMap[X_0, {blob_0}]
```

ApplyPauliTransferMap : Could not pre–compute the Pauli transfer maps due to the below error:

CalcPauliTransferMatrix : Circuit contained an unrecognised or unsupported gate: blob_0

```
$Failed
```

```
ApplyPauliTransferMap[X0, X0, "CacheMaps" → "Spaghetti"]
ApplyPauliTransferMap[X0, {X0}, "CacheMaps" → "Spaghetti"]
ApplyPauliTransferMap[X0, PTM0[x], "CacheMaps" → "Spaghetti"]
ApplyPauliTransferMap[X0, {PTM0[x]}, "CacheMaps" → "Spaghetti"]
ApplyPauliTransferMap[X0, PTMap0[x], "CacheMaps" → "Spaghetti"]
ApplyPauliTransferMap[X0, {PTMap0[x]}, "CacheMaps" → "Spaghetti"]
```

ApplyPauliTransferMap : Option "CacheMaps" must be one of "Forever", "UntilCallEnd" or "Never". See ?ApplyPauliTransferMap .

$Failed

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$Failed

```
ApplyPauliTransferMap[X0, {X0}, "BadOption" → True]
```

OptionValue : Unknown option BadOption for {ApplyPauliTransferMap , CalcPauliTransferMap}.

$Failed

```
ApplyPauliTransferMap[X0, {X0},
 "CombineStates" → "Only valid for CalcPauliTransferEval"]
```

OptionValue : Unknown option CombineStates for {ApplyPauliTransferMap , CalcPauliTransferMap}.

$Failed

```
ApplyPauliTransferMap[a, {H0}]
ApplyPauliTransferMap[X-1, {H0}]
ApplyPauliTransferMap[X0 X0, {H0}]
ApplyPauliTransferMap[X0 Y0, {H0}]
```

ApplyPauliTransferMap : Invalid arguments. See ?ApplyPauliTransferMap

```
$Failed
```

ApplyPauliTransferMap : Invalid arguments. See ?ApplyPauliTransferMap

```
$Failed
```

ApplyPauliTransferMap : Invalid arguments. See ?ApplyPauliTransferMap

```
$Failed
```

ApplyPauliTransferMap : Invalid arguments. See ?ApplyPauliTransferMap

```
$Failed
```