Recompiling to SingleQubitAndCNOT

```
SetDirectory @ NotebookDirectory[];
Import["../Link/QuESTlink.m"];
```

```
testRecomp[gate_, showGates:(True|False):True] := Module[
    {recomp, error},
    recomp = RecompileCircuit[gate, "SingleQubitAndCNOT"];
    error = CalcCircuitMatrix[gate] - CalcCircuitMatrix[N@recomp] // Abs // Max;
    error = error // FullSimplify // Chop;
    If[showGates, Echo @ recomp];
    Echo @ DrawCircuit[{gate}, recomp}];
    Echo[error, "error: "];
    If[error =!= 0, Style["ERRONEOUS DECOMPOSITION!", Red]]]
```

```
? QuEST`Gate`*
```

QuEST`Gate`

Damp	G	KrausNonTP	Ph	Rz	U	Z
Deph	H	M	R	S	UNonNorm	
Depol	Id	Matr	Rx	SWAP	X	
Fac	Kraus	P	Ry	T	Y	

Testing doc

```
? RecompileCircuit
```

Symbol

RecompileCircuit[circuit, method] returns an equivalent circuit,
transpiled to a differnet gate set. The input circuit can contain any unitary
gate, with any number of control qubits. Supported methods include:

- "SingleQubitAndCNOT" decompiles the circuit into canonical single–qubit gates (H, Ph, T, S, X, Y, Z, Rx, Ry, Rz), a global phase G, and two–qubit C[X] gates. This method uses a combination of 23 analytic and numerical decompositions.
- "CliffordAndRz" decompiles the circuit into Clifford gates (H, S, X, Y, Z, CX, CY, CZ, SWAP), a global phase G, and non–Clifford Rz.

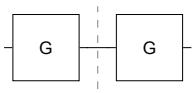
Note that the returned circuits are not necessarily optimal/minimal, and may benefit from a subsequent call to SimplifyCircuit[].

Testing decomp gates

G

```
testRecomp @ G[x]
```

» {G[x]}

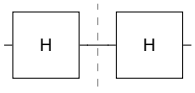
» 

» error: 0

H

```
testRecomp @ H0
```

» {H0}

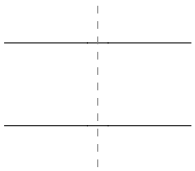
» 

» error: 0

Id

```
testRecomp @ Id0,1
```

» $\{\mathrm{Id}_{0,1}\}$

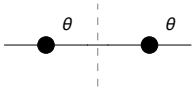


» error: 0

Ph

testRecomp @ $\mathrm{Ph}_0[\theta]$

» $\{\mathrm{Ph}_0[\theta]\}$

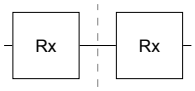


» error: 0

Rx, Ry, Rz

testRecomp @ $\mathrm{Rx}_0[\theta]$

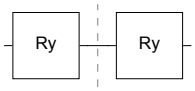
» $\{\mathrm{Rx}_0[\theta]\}$



» error: 0

testRecomp @ $\mathrm{Ry}_0[\theta]$

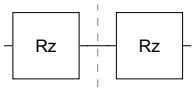
» $\{\mathrm{Ry}_0[\theta]\}$



» error: 0

testRecomp @ $\mathrm{Rz}_0[\theta]$

» $\{\mathrm{Rz}_0[\theta]\}$

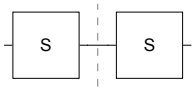


» error: 0

S

testRecomp @ S_0

» $\{\mathrm{S}_0\}$

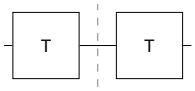


» error: 0

T

```
testRecomp @ T0
```

» {T0}

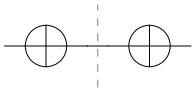
» 

» error: 0

X, Y, Z

```
testRecomp @ X0
```

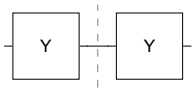
» {X0}

» 

» error: 0

```
testRecomp @ Y0
```

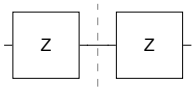
» {Y0}

» 

» error: 0

```
testRecomp @ Z0
```

» {Z0}

» 

» error: 0

Testing canonical gates

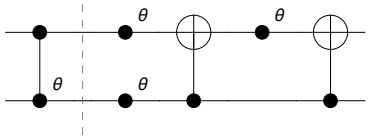
Un-controlled

Ph

```
testRecomp @ Ph0,1[x]
```

» $\left\{\text{Ph}_1\left[\frac{x}{2}\right], \text{Ph}_0\left[\frac{x}{2}\right], C_0[X_1], \text{Ph}_1\left[-\frac{x}{2}\right], C_0[X_1]\right\}$

»

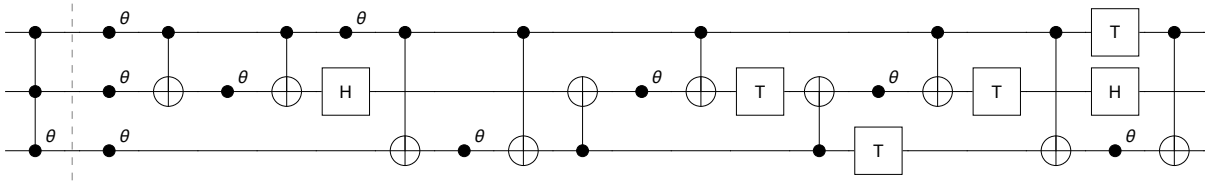


» error: 0

testRecomp @ $\text{Ph}_{0,1,2}[x]$

» $\left\{\text{Ph}_1\left[\frac{x}{4}\right], \text{Ph}_2\left[\frac{x}{4}\right], C_2[X_1], \text{Ph}_1\left[-\frac{x}{4}\right], C_2[X_1], \text{Ph}_0\left[\frac{x}{4}\right], \text{Ph}_2\left[\frac{x}{4}\right], C_2[X_0], \text{Ph}_0\left[-\frac{x}{4}\right],\right.$
$C_2[X_0], H_1, C_0[X_1], \text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, C_0[X_1], \text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, T_0, C_2[X_0],$
$T_2, \text{Ph}_0\left[-\frac{\pi}{4}\right], C_2[X_0], H_1, \text{Ph}_1\left[-\frac{x}{4}\right], \text{Ph}_2\left[-\frac{x}{4}\right], C_2[X_1], \text{Ph}_1\left[\frac{x}{4}\right], C_2[X_1], H_1, C_0[X_1],$
$\left.\text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, C_0[X_1], \text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, T_0, C_2[X_0], T_2, \text{Ph}_0\left[-\frac{\pi}{4}\right], C_2[X_0], H_1\right\}$

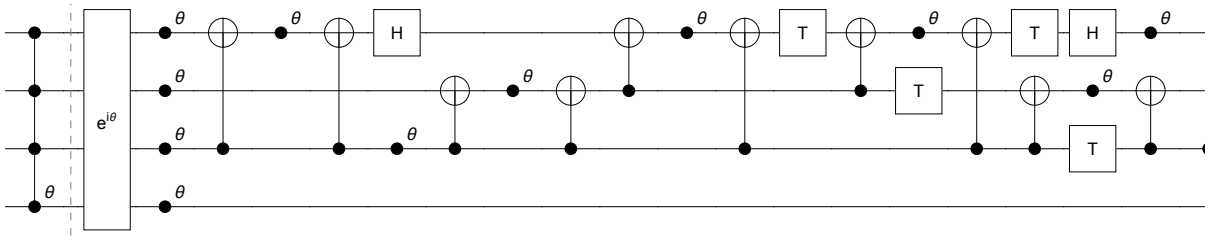
»



» error: 0

testRecomp$\left[\text{Ph}_{0,1,2,3}[-1.2], \text{False}\right]$

»

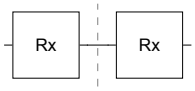


» error: 0

R

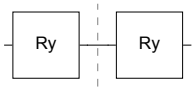
testRecomp @ R[x, X_0]
testRecomp @ R[x, Y_0]
testRecomp @ R[x, Z_0]

» {$Rx_0[x]$}

» 

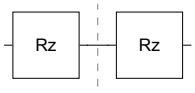
» error: 0

» {$Ry_0[x]$}

» 

» error: 0

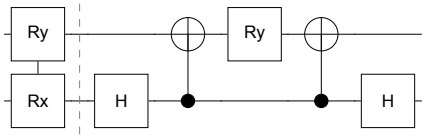
» {$Rz_0[x]$}

» 

» error: 0

testRecomp @ R[x, X_0 Y_1]

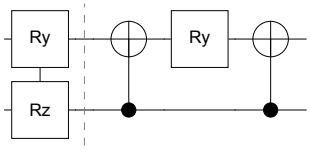
» {H_0, $C_0[X_1]$, $Ry_1[x]$, $C_0[X_1]$, H_0}

» 

» error: 0

testRecomp @ R[x, Z_0 Y_1]

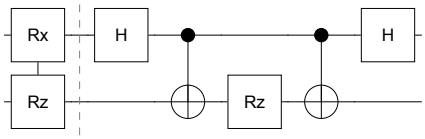
» {$C_0[X_1]$, $Ry_1[x]$, $C_0[X_1]$}

» 

» error: 0

testRecomp @ R[x, Z_0 X_1]

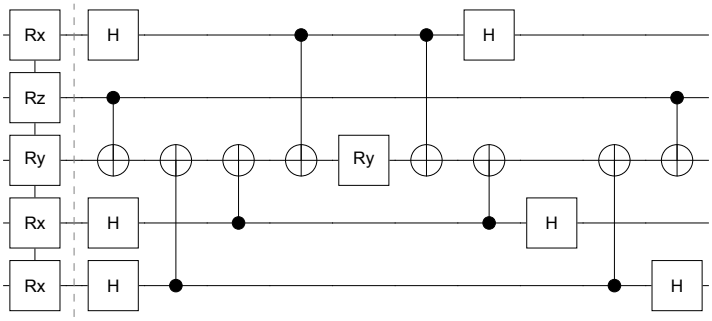
» {H_1, $C_1[X_0]$, $Rz_0[x]$, $C_1[X_0]$, H_1}

» 

» error: 0

testRecomp @ R[x, X_0 X_1 Y_2 Z_3 X_4]

» {C3[X2], H0, C0[X2], H1, C1[X2], H4, C4[X2], Ry2[x], C4[X2], H4, C1[X2], H1, C0[X2], H0, C3[X2]}

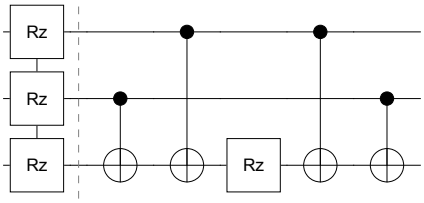


» error: 0

Rz^(n)

```
testRecomp @ Rz0,1,2[x]
```

» {C1[X0], C2[X0], Rz0[x], C2[X0], C1[X0]}

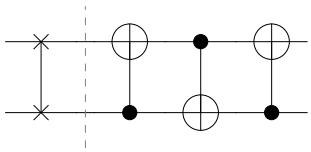


» error: 0

SWAP

```
testRecomp @ SWAP0,1
```

» {C0[X1], C1[X0], C0[X1]}



» error: 0

Singly-controlled

C[G]

```
(* cannot draw ill-formed input *)

DrawCircuit @ RecompileCircuit[C0@G[x], "SingleQubitAndCNOT"]
```

DrawCircuit: Invalid arguments. See ?DrawCircuit

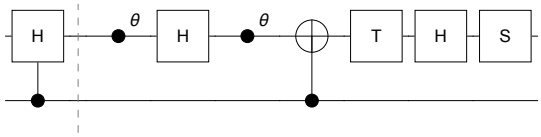
```
$Failed
```

C[H]

```
testRecomp @ C0[H1]
```

» $\left\{\text{Ph}_1\left[-\frac{\pi}{2}\right], \text{H}_1, \text{Ph}_1\left[-\frac{\pi}{4}\right], \text{C}_0[\text{X}_1], \text{T}_1, \text{H}_1, \text{S}_1\right\}$

»



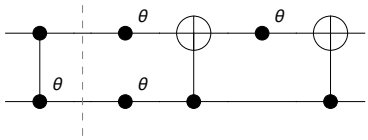
» error: 0

C[Ph]

```
testRecomp @ C0[Ph1[x]]
```

» $\left\{\text{Ph}_1\left[\frac{x}{2}\right], \text{Ph}_0\left[\frac{x}{2}\right], \text{C}_0[\text{X}_1], \text{Ph}_1\left[-\frac{x}{2}\right], \text{C}_0[\text{X}_1]\right\}$

»



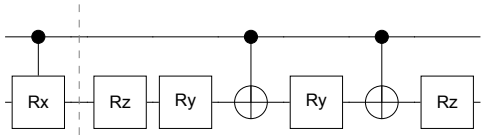
» error: 0

C[R]

```
testRecomp @ C1@ R[x, X0]
testRecomp @ C1@ R[x, Y0]
testRecomp @ C1@ R[x, Z0]
```

» $\left\{Rz_0\left[\frac{\pi}{2}\right], Ry_0\left[\frac{x}{2}\right], C_1[X_0], Ry_0\left[-\frac{x}{2}\right], C_1[X_0], Rz_0\left[-\frac{\pi}{2}\right]\right\}$

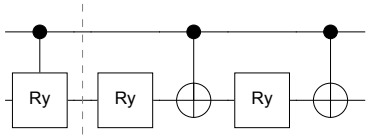
»



» error: 0

» $\left\{Ry_0\left[\frac{x}{2}\right], C_1[X_0], Ry_0\left[-\frac{x}{2}\right], C_1[X_0]\right\}$

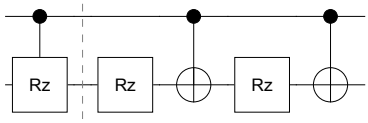
»



» error: 0

» $\left\{Rz_0\left[\frac{x}{2}\right], C_1[X_0], Rz_0\left[-\frac{x}{2}\right], C_1[X_0]\right\}$

»

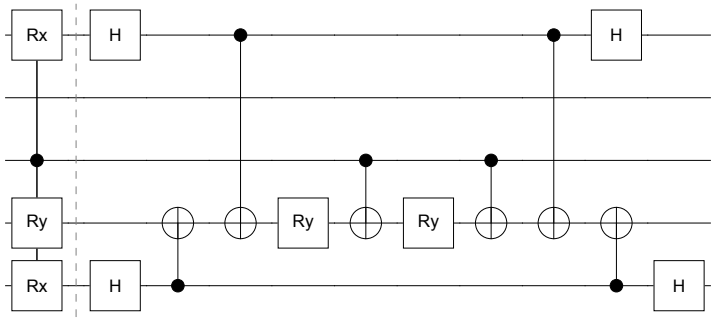


» error: 0

testRecomp @ C_2@R[x, X_0 Y_1 X_4]

» $\left\{H_0, C_0[X_1], H_4, C_4[X_1], Ry_1\left[\frac{x}{2}\right], C_2[X_1], Ry_1\left[-\frac{x}{2}\right], C_2[X_1], C_4[X_1], H_4, C_0[X_1], H_0\right\}$

»



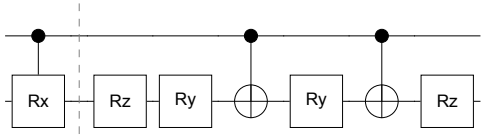
» error: 0

C[Rx]

testRecomp @ C_1@Rx_0[x]

» $\left\{Rz_0\left[\frac{\pi}{2}\right], Ry_0\left[\frac{x}{2}\right], C_1[X_0], Ry_0\left[-\frac{x}{2}\right], C_1[X_0], Rz_0\left[-\frac{\pi}{2}\right]\right\}$

»



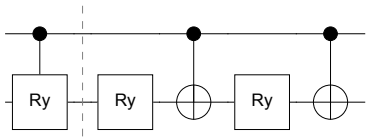
» error: 0

C[Ry], C[Rz]

testRecomp @ $C_1@Ry_0[x]$

» $\left\{Ry_0\left[\frac{x}{2}\right], C_1[X_0], Ry_0\left[-\frac{x}{2}\right], C_1[X_0]\right\}$

»

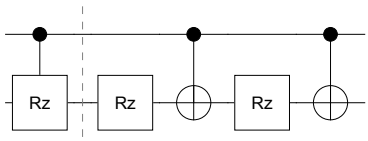


» error: 0

testRecomp @ $C_1@Rz_0[x]$

» $\left\{Rz_0\left[\frac{x}{2}\right], C_1[X_0], Rz_0\left[-\frac{x}{2}\right], C_1[X_0]\right\}$

»



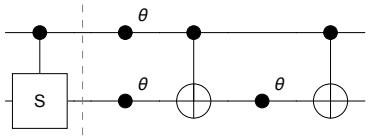
» error: 0

C[S], C[T]

testRecomp @ $C_1@S_0$

» $\left\{Ph_0\left[\frac{\pi}{4}\right], Ph_1\left[\frac{\pi}{4}\right], C_1[X_0], Ph_0\left[-\frac{\pi}{4}\right], C_1[X_0]\right\}$

»

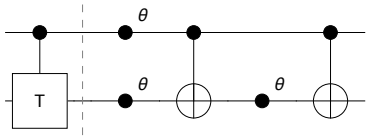


» error: 0

testRecomp @ $C_1@T_0$

» $\left\{Ph_0\left[\frac{\pi}{8}\right], Ph_1\left[\frac{\pi}{8}\right], C_1[X_0], Ph_0\left[-\frac{\pi}{8}\right], C_1[X_0]\right\}$

»



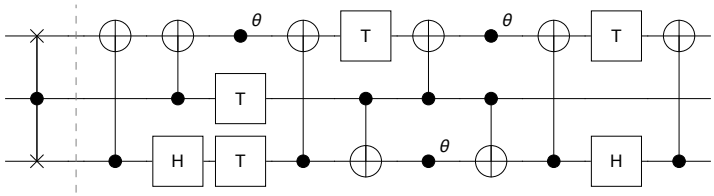
» error: 0

C[SWAP]

testRecomp @ $C_1@SWAP_{0,2}$

» $\left\{C_0[X_2], H_0, C_1[X_2], T_0, Ph_2\left[-\frac{\pi}{4}\right], T_1, C_0[X_2], C_1[X_0], T_2, C_1[X_2], Ph_0\left[-\frac{\pi}{4}\right], Ph_2\left[-\frac{\pi}{4}\right], C_1[X_0], C_0[X_2], H_0, T_2, C_0[X_2]\right\}$

»



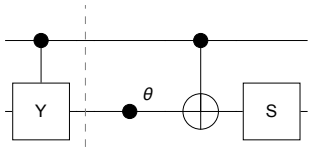
» error: 0

C[Y]

```
testRecomp @ C1@Y0
```

» $\left\{Ph_0\left[-\frac{\pi}{2}\right], C_1[X_0], S_0\right\}$

»



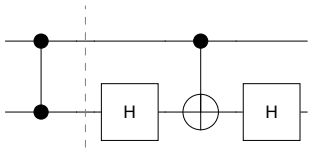
» error: 0

C[Z]

```
testRecomp @ C1@Z0
```

» {H0, C1[X0], H0}

»

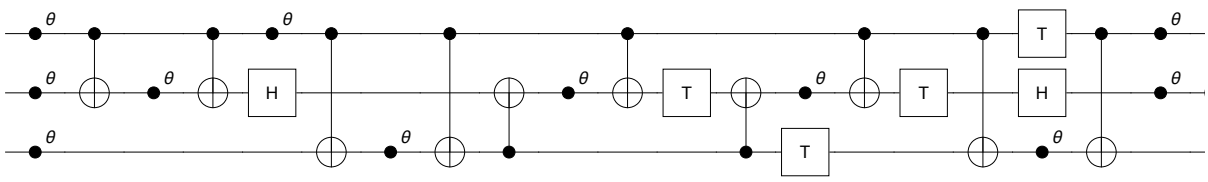


» error: 0

Multi-controlled

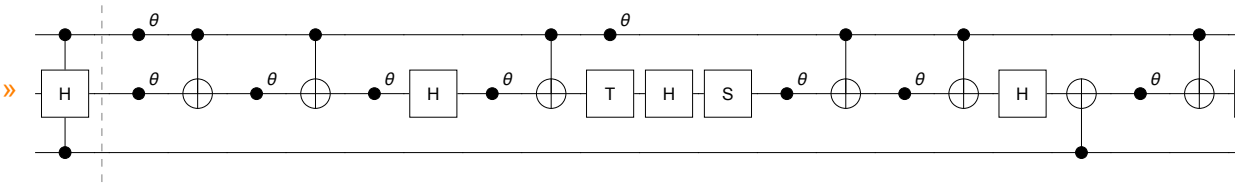
C*[G]

```
(* cannot draw ill-formed input *)
DrawCircuit @ RecompileCircuit[C0,1,2@G[x], "SingleQubitAndCNOT"]
```



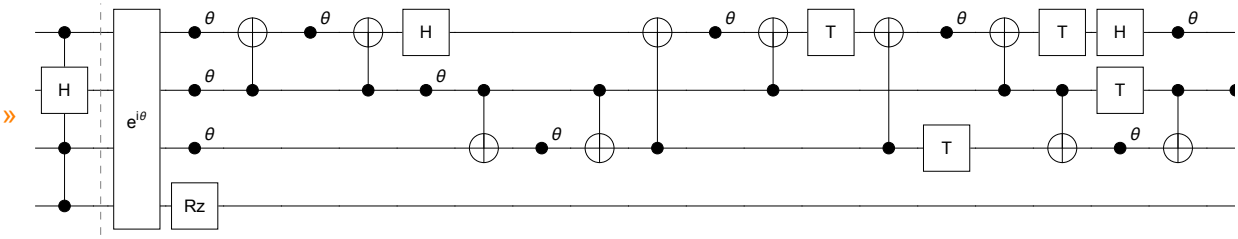
C*[H]

```
testRecomp[C0,2[H1], False]
```



» error: 0

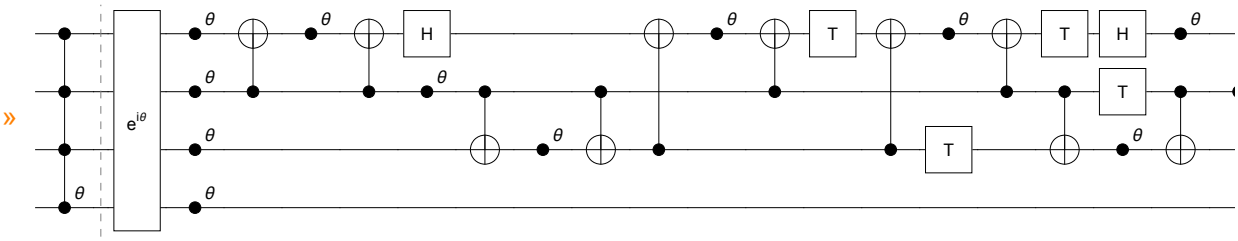
```
testRecomp[C0,1,3[H2], False]
```



» error: 0

C*[Ph]

```
testRecomp[C0,2[Ph1,3[.1]], False]
```



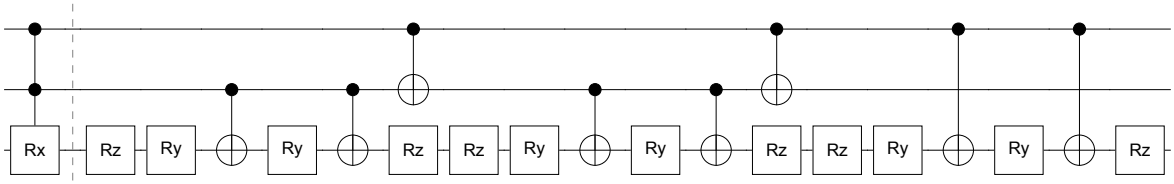
» error: 0

C*[R]

```
testRecomp @ C1,2@ R[-.1, X0]
testRecomp @ C1,2@ R[x, Y0]
testRecomp @ C1,2@ R[x, Z0]
```

» $\left\{Rz_0\left[\frac{\pi}{2}\right], Ry_0[-0.025], C_1[X_0], Ry_0[0.025], C_1[X_0], Rz_0\left[-\frac{\pi}{2}\right],\right.$
$C_2[X_1], Rz_0\left[\frac{\pi}{2}\right], Ry_0[0.025], C_1[X_0], Ry_0[-0.025], C_1[X_0], Rz_0\left[-\frac{\pi}{2}\right],$
$\left.C_2[X_1], Rz_0\left[\frac{\pi}{2}\right], Ry_0[-0.025], C_2[X_0], Ry_0[0.025], C_2[X_0], Rz_0\left[-\frac{\pi}{2}\right]\right\}$

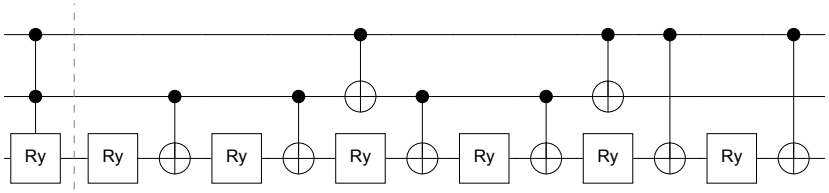
»



» error: 0

» $\left\{Ry_0\left[\frac{x}{4}\right], C_1[X_0], Ry_0\left[-\frac{x}{4}\right], C_1[X_0], C_2[X_1], Ry_0\left[-\frac{x}{4}\right],\right.$
$\left.C_1[X_0], Ry_0\left[\frac{x}{4}\right], C_1[X_0], C_2[X_1], Ry_0\left[\frac{x}{4}\right], C_2[X_0], Ry_0\left[-\frac{x}{4}\right], C_2[X_0]\right\}$

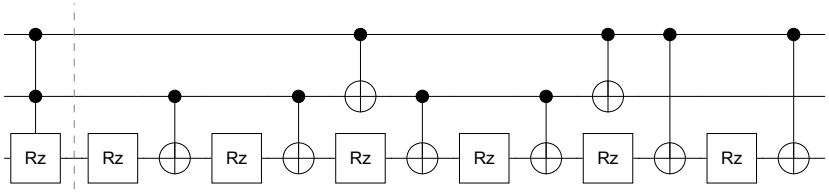
»



» error: 0

» $\left\{Rz_0\left[\frac{x}{4}\right], C_1[X_0], Rz_0\left[-\frac{x}{4}\right], C_1[X_0], C_2[X_1], Rz_0\left[-\frac{x}{4}\right],\right.$
$\left.C_1[X_0], Rz_0\left[\frac{x}{4}\right], C_1[X_0], C_2[X_1], Rz_0\left[\frac{x}{4}\right], C_2[X_0], Rz_0\left[-\frac{x}{4}\right], C_2[X_0]\right\}$

»

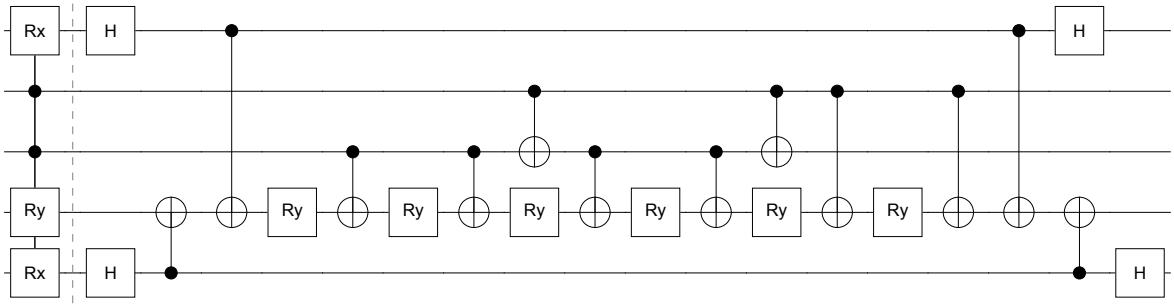


» error: 0

testRecomp @ $C_{2,3}$@R[x, X_0 Y_1 X_4]

» $\left\{H_0, C_0[X_1], H_4, C_4[X_1], Ry_1\left[\frac{x}{4}\right], C_2[X_1], Ry_1\left[-\frac{x}{4}\right], C_2[X_1], C_3[X_2], Ry_1\left[-\frac{x}{4}\right], C_2[X_1],\right.$
$Ry_1\left[\frac{x}{4}\right], C_2[X_1], C_3[X_2], Ry_1\left[\frac{x}{4}\right], C_3[X_1], Ry_1\left[-\frac{x}{4}\right], C_3[X_1], C_4[X_1], H_4, C_0[X_1], \left.H_0\right\}$

»

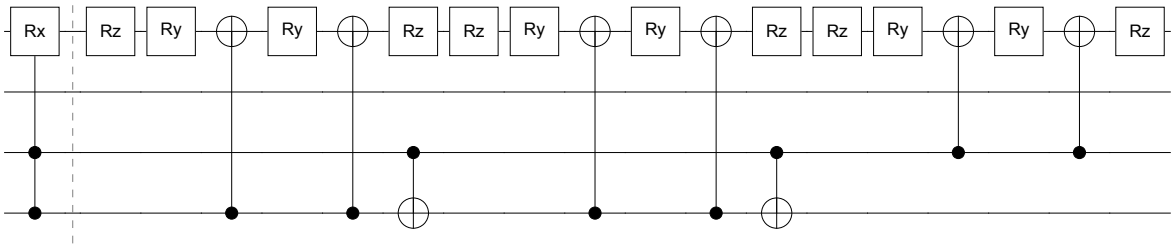


» error: 0

C*[Rx]

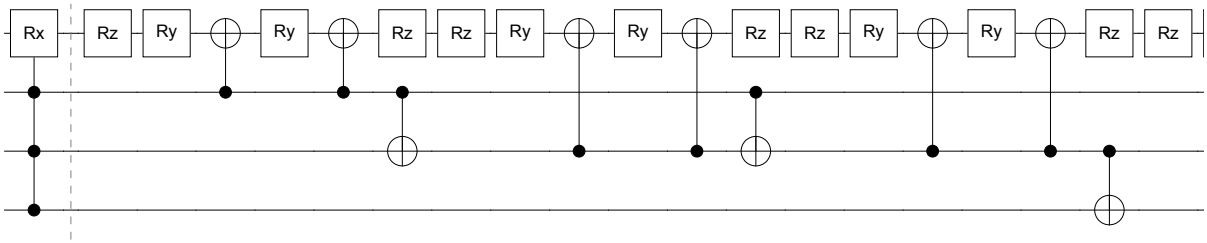
```
testRecomp @ C0,1[Rx3[.1]]
```

» $\left\{Rz_3\left[\frac{\pi}{2}\right], Ry_3[0.025], C_0[X_3], Ry_3[-0.025], C_0[X_3], Rz_3\left[-\frac{\pi}{2}\right],\right.$
$C_1[X_0], Rz_3\left[\frac{\pi}{2}\right], Ry_3[-0.025], C_0[X_3], Ry_3[0.025], C_0[X_3], Rz_3\left[-\frac{\pi}{2}\right],$
$\left.C_1[X_0], Rz_3\left[\frac{\pi}{2}\right], Ry_3[0.025], C_1[X_3], Ry_3[-0.025], C_1[X_3], Rz_3\left[-\frac{\pi}{2}\right]\right\}$



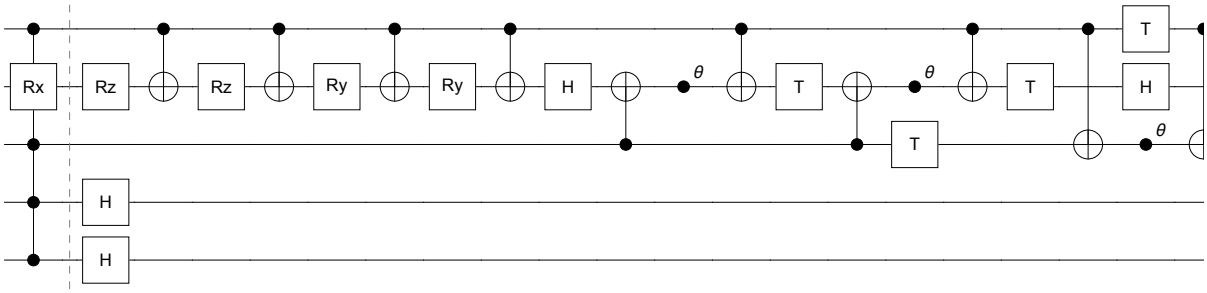
» error: 0

```
testRecomp[ C0,1,2[Rx3[.1]], False]
```



» error: 0

```
testRecomp[C0,1,2,4[Rx3[.1]], False]
```

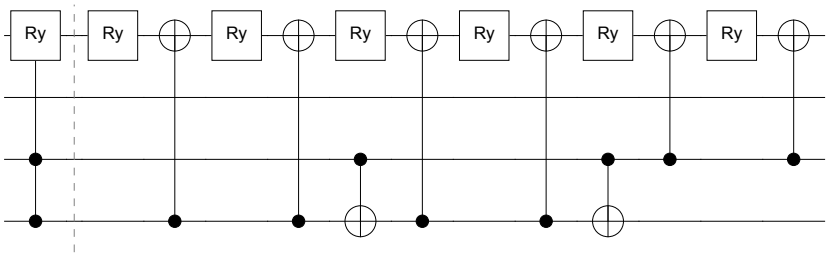


» error: 0

C*[Ry]

```
testRecomp @ C0,1[Ry3[x]]
```

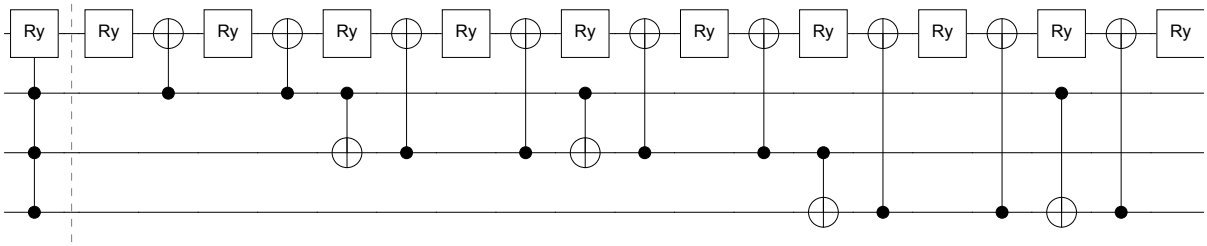
» $\left\{Ry_3\left[\frac{x}{4}\right], C_0[X_3], Ry_3\left[-\frac{x}{4}\right], C_0[X_3], C_1[X_0], Ry_3\left[-\frac{x}{4}\right],\right.$
$\left. C_0[X_3], Ry_3\left[\frac{x}{4}\right], C_0[X_3], C_1[X_0], Ry_3\left[\frac{x}{4}\right], C_1[X_3], Ry_3\left[-\frac{x}{4}\right], C_1[X_3]\right\}$



» error: 0

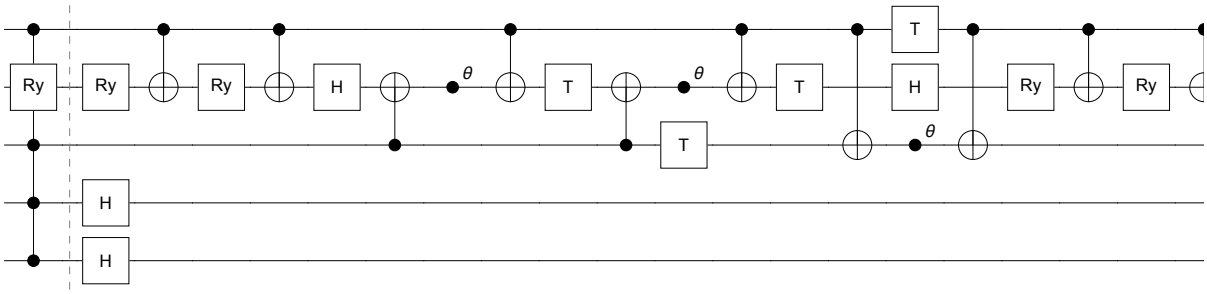
testRecomp @ $C_{0,1,2}[Ry_3[.1]]$

» {$Ry_3[0.0125], C_2[X_3], Ry_3[-0.0125], C_2[X_3], C_2[X_1], Ry_3[-0.0125],$
$C_1[X_3], Ry_3[0.0125], C_1[X_3], C_2[X_1], Ry_3[0.0125], C_1[X_3], Ry_3[-0.0125],$
$C_1[X_3], C_1[X_0], Ry_3[-0.0125], C_0[X_3], Ry_3[0.0125], C_0[X_3], C_2[X_0],$
$Ry_3[0.0125], C_0[X_3], Ry_3[-0.0125], C_0[X_3], C_1[X_0], Ry_3[-0.0125], C_0[X_3],$
$Ry_3[0.0125], C_0[X_3], C_2[X_0], Ry_3[0.0125], C_0[X_3], Ry_3[-0.0125], C_0[X_3]$}



» error: 0

testRecomp[$C_{0,1,2,4}[Ry_3[.1]]$, False]

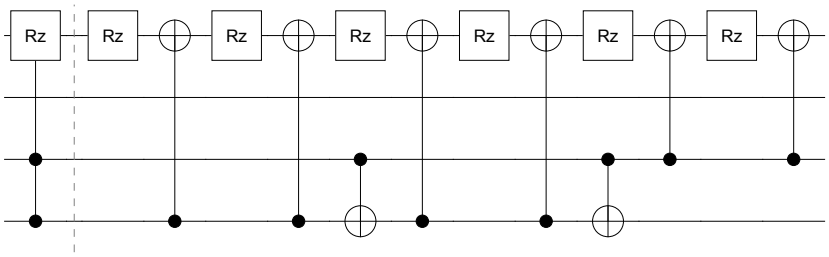


» error: 0

C*[Rz]

testRecomp @ $C_{0,1}[Rz_3[x]]$

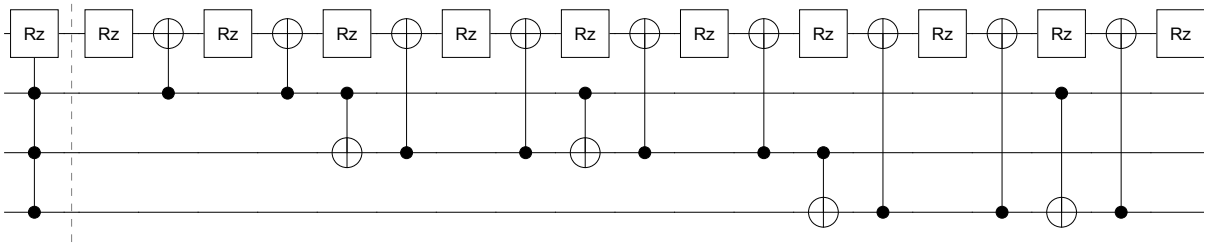
» $\left\{Rz_3\left[\frac{x}{4}\right], C_0[X_3], Rz_3\left[-\frac{x}{4}\right], C_0[X_3], C_1[X_0], Rz_3\left[-\frac{x}{4}\right],\right.$
$\left. C_0[X_3], Rz_3\left[\frac{x}{4}\right], C_0[X_3], C_1[X_0], Rz_3\left[\frac{x}{4}\right], C_1[X_3], Rz_3\left[-\frac{x}{4}\right], C_1[X_3]\right\}$



» error: 0

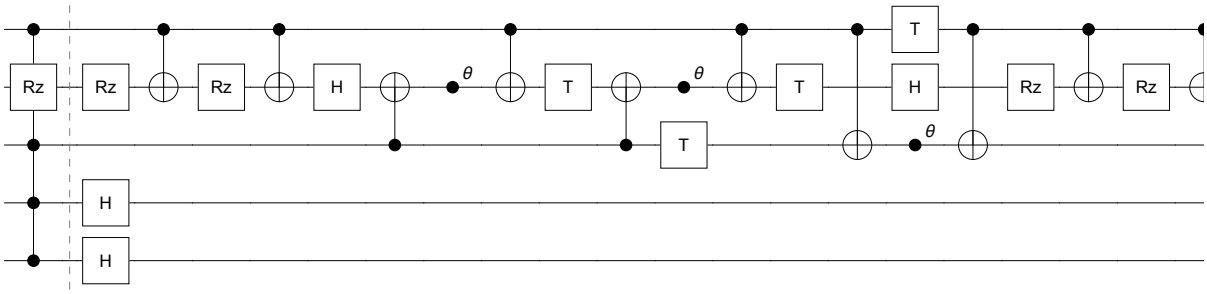
testRecomp @ $C_{0,1,2}[Rz_3[.1]]$

» {$Rz_3[0.0125], C_2[X_3], Rz_3[-0.0125], C_2[X_3], C_2[X_1], Rz_3[-0.0125],$
$C_1[X_3], Rz_3[0.0125], C_1[X_3], C_2[X_1], Rz_3[0.0125], C_1[X_3], Rz_3[-0.0125],$
$C_1[X_3], C_1[X_0], Rz_3[-0.0125], C_0[X_3], Rz_3[0.0125], C_0[X_3], C_2[X_0],$
$Rz_3[0.0125], C_0[X_3], Rz_3[-0.0125], C_0[X_3], C_1[X_0], Rz_3[-0.0125], C_0[X_3],$
$Rz_3[0.0125], C_0[X_3], C_2[X_0], Rz_3[0.0125], C_0[X_3], Rz_3[-0.0125], C_0[X_3]$}



» error: 0

testRecomp[$C_{0,1,2,4}[Rz_3[.1]]$, False]

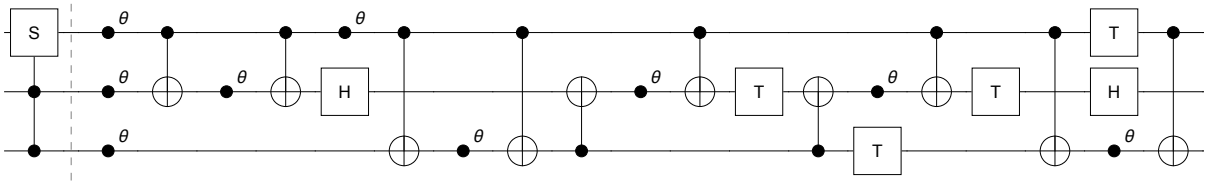


» error: 0

C*[S]

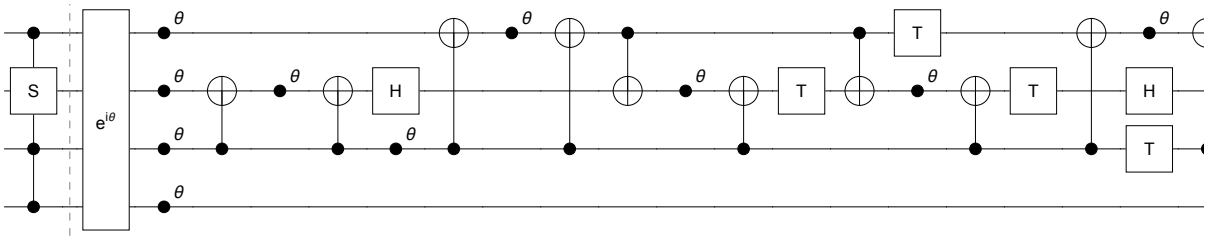
testRecomp @ $C_{0,1}[S_2]$

» $\left\{\text{Ph}_1\left[\frac{\pi}{8}\right], \text{Ph}_2\left[\frac{\pi}{8}\right], C_2[X_1], \text{Ph}_1\left[-\frac{\pi}{8}\right], C_2[X_1], \text{Ph}_0\left[\frac{\pi}{8}\right], \text{Ph}_2\left[\frac{\pi}{8}\right], C_2[X_0], \text{Ph}_0\left[-\frac{\pi}{8}\right],\right.$
$C_2[X_0], H_1, C_0[X_1], \text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, C_0[X_1], \text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, T_0, C_2[X_0],$
$T_2, \text{Ph}_0\left[-\frac{\pi}{4}\right], C_2[X_0], H_1, \text{Ph}_1\left[-\frac{\pi}{8}\right], \text{Ph}_2\left[-\frac{\pi}{8}\right], C_2[X_1], \text{Ph}_1\left[\frac{\pi}{8}\right], C_2[X_1], H_1, C_0[X_1],$
$\left.\text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, C_0[X_1], \text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, T_0, C_2[X_0], T_2, \text{Ph}_0\left[-\frac{\pi}{4}\right], C_2[X_0], H_1\right\}$

» 

» error: 0

testRecomp[$C_{0,1,3}[S_2]$, False]

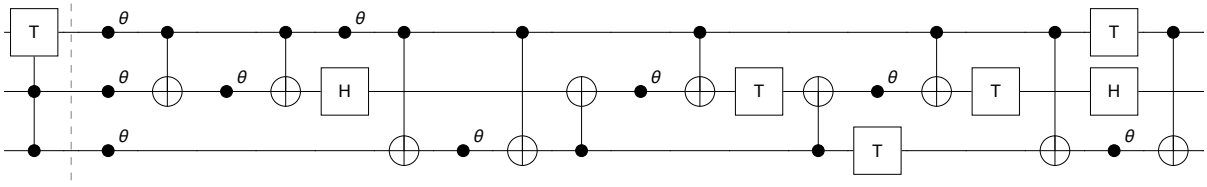
» 

» error: 0

C*[T]

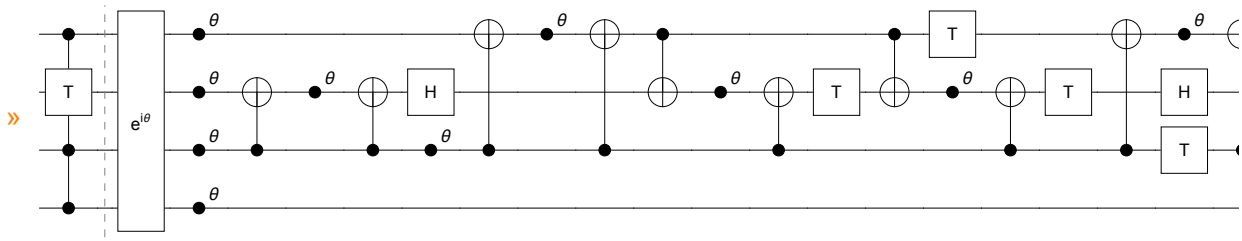
testRecomp @ $C_{0,1}[T_2]$

» $\left\{\text{Ph}_1\left[\frac{\pi}{16}\right], \text{Ph}_2\left[\frac{\pi}{16}\right], C_2[X_1], \text{Ph}_1\left[-\frac{\pi}{16}\right], C_2[X_1], \text{Ph}_0\left[\frac{\pi}{16}\right], \text{Ph}_2\left[\frac{\pi}{16}\right], C_2[X_0], \text{Ph}_0\left[-\frac{\pi}{16}\right],\right.$
$C_2[X_0], H_1, C_0[X_1], \text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, C_0[X_1], \text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, T_0, C_2[X_0],$
$T_2, \text{Ph}_0\left[-\frac{\pi}{4}\right], C_2[X_0], H_1, \text{Ph}_1\left[-\frac{\pi}{16}\right], \text{Ph}_2\left[-\frac{\pi}{16}\right], C_2[X_1], \text{Ph}_1\left[\frac{\pi}{16}\right], C_2[X_1], H_1, C_0[X_1],$
$\left.\text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, C_0[X_1], \text{Ph}_1\left[-\frac{\pi}{4}\right], C_2[X_1], T_1, T_0, C_2[X_0], T_2, \text{Ph}_0\left[-\frac{\pi}{4}\right], C_2[X_0], H_1\right\}$

» 

» error: 0

testRecomp[$C_{0,1,3}[T_2]$, False]

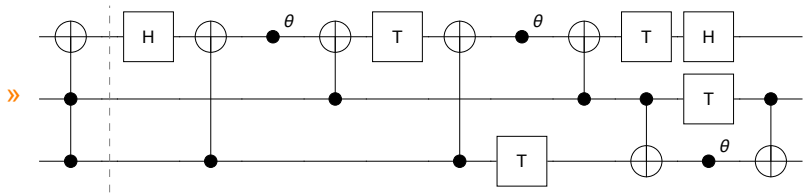


» error: 0

C*[X]

testRecomp @ $C_{0,1}[X_2]$

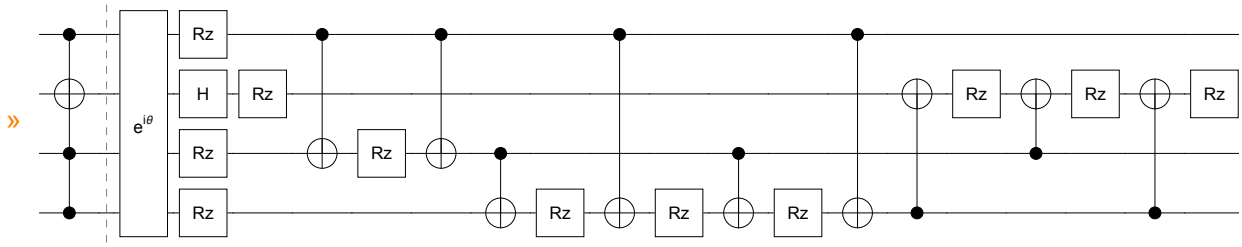
» $\{H_2, C_0[X_2], Ph_2[-\frac{\pi}{4}], C_1[X_2], T_2, C_0[X_2], Ph_2[-\frac{\pi}{4}], C_1[X_2], T_2, T_0, C_1[X_0], T_1, Ph_0[-\frac{\pi}{4}], C_1[X_0], H_2\}$



» error: 0

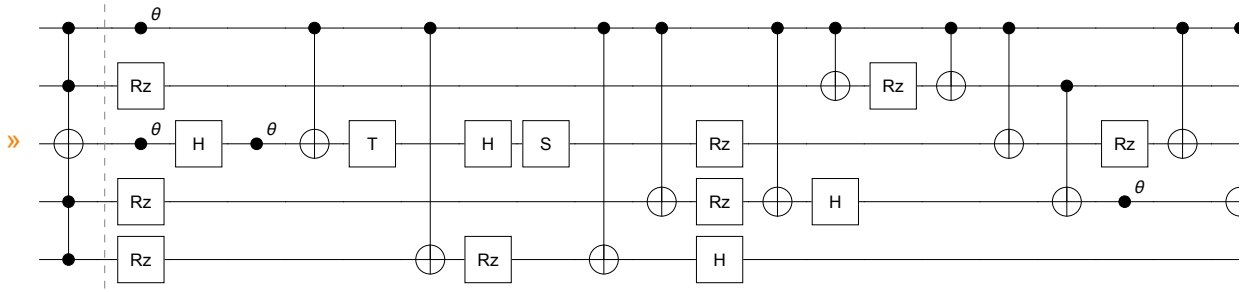
testRecomp @ $C_{0,1,3}[X_2]$

» $\{G[\frac{\pi}{16}], H_2, Rz_0[\frac{\pi}{8}], Rz_1[\frac{\pi}{8}], Rz_3[\frac{\pi}{8}], Rz_2[\frac{\pi}{8}], C_3[X_1], Rz_1[-\frac{\pi}{8}], C_3[X_1], C_1[X_0], Rz_0[-\frac{\pi}{8}], C_3[X_0], Rz_0[\frac{\pi}{8}], C_1[X_0], Rz_0[-\frac{\pi}{8}], C_3[X_0], C_0[X_2], Rz_2[-\frac{\pi}{8}], C_1[X_2], Rz_2[\frac{\pi}{8}], C_0[X_2], Rz_2[-\frac{\pi}{8}], C_3[X_2], Rz_2[\frac{\pi}{8}], C_0[X_2], Rz_2[-\frac{\pi}{8}], C_1[X_2], Rz_2[\frac{\pi}{8}], C_0[X_2], Rz_2[-\frac{\pi}{8}], C_3[X_2], H_2\}$



» error: 0

testRecomp[$C_{0,1,3,4}[X_2]$, False]

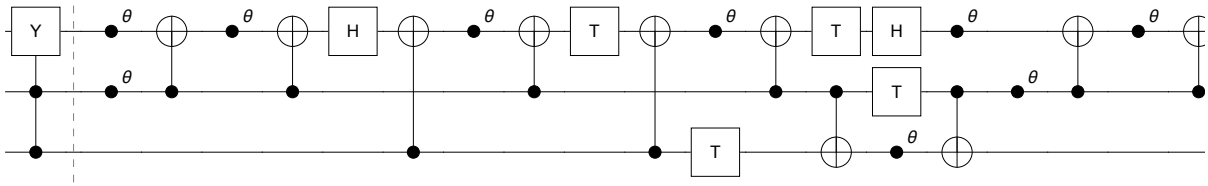


» error: 0

C*[Y]

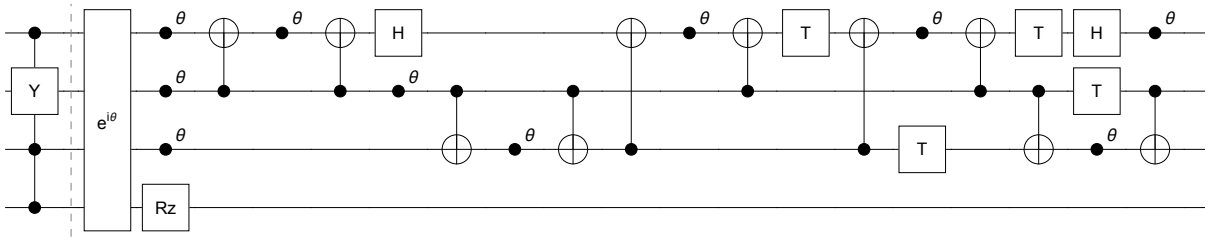
```
testRecomp @ C0,1[Y2]
```

» $\left\{\text{Ph}_2\left[-\frac{\pi}{4}\right], \text{Ph}_1\left[-\frac{\pi}{4}\right], C_1[X_2], \text{Ph}_2\left[\frac{\pi}{4}\right], C_1[X_2], H_2, C_0[X_2],\right.$
$\text{Ph}_2\left[-\frac{\pi}{4}\right], C_1[X_2], T_2, C_0[X_2], \text{Ph}_2\left[-\frac{\pi}{4}\right], C_1[X_2], T_2, T_0, C_1[X_0], T_1,$
$\left.\text{Ph}_0\left[-\frac{\pi}{4}\right], C_1[X_0], H_2, \text{Ph}_2\left[\frac{\pi}{4}\right], \text{Ph}_1\left[\frac{\pi}{4}\right], C_1[X_2], \text{Ph}_2\left[-\frac{\pi}{4}\right], C_1[X_2]\right\}$

» 

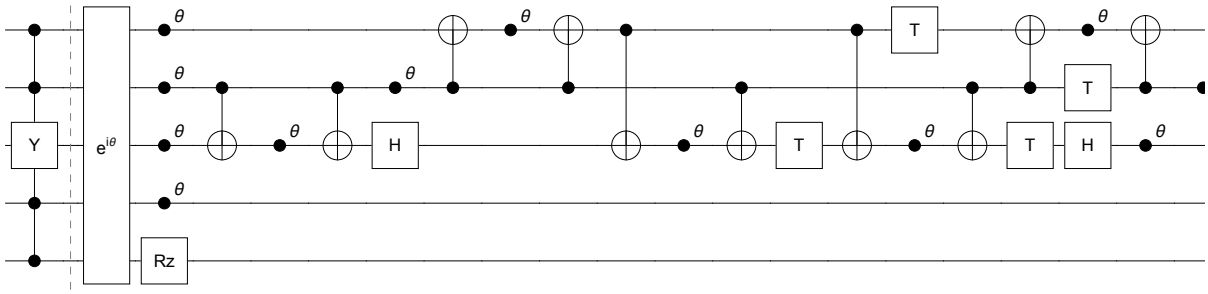
» error: 0

```
testRecomp[ C0,1,3[Y2], False]
```

» 

» error: 0

```
testRecomp[ C0,1,3,4[Y2], False]
```

» 

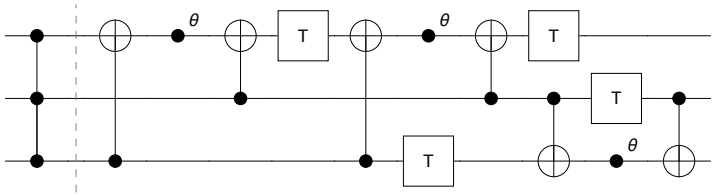
» error: 0

C*[Z]

```
testRecomp @ C0,1[Z2]
```

» $\left\{C_0[X_2], Ph_2\left[-\frac{\pi}{4}\right], C_1[X_2], T_2, C_0[X_2], Ph_2\left[-\frac{\pi}{4}\right], C_1[X_2], T_2, T_0, C_1[X_0], T_1, Ph_0\left[-\frac{\pi}{4}\right], C_1[X_0]\right\}$

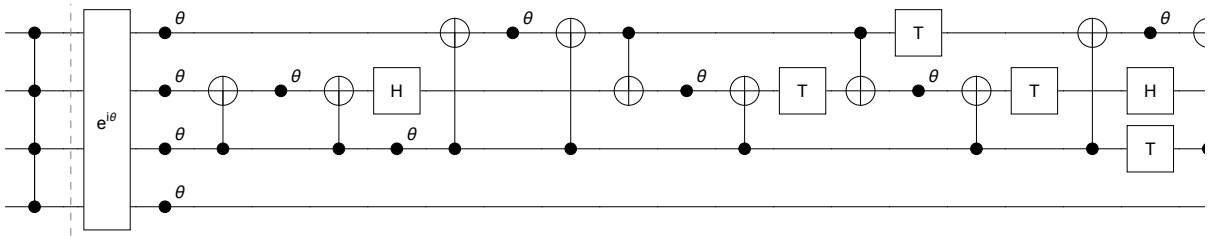
»



» error: 0

testRecomp[$C_{0,1,3}[Z_2]$, False]

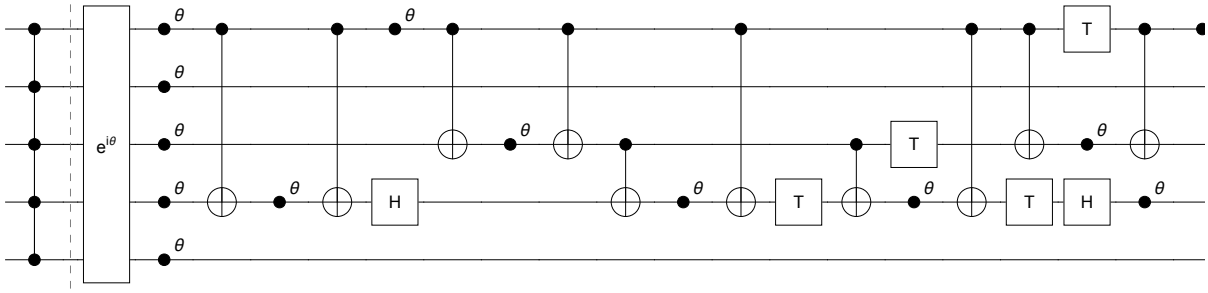
»



» error: 0

testRecomp[$C_{0,1,3,4}[Z_2]$, False]

»



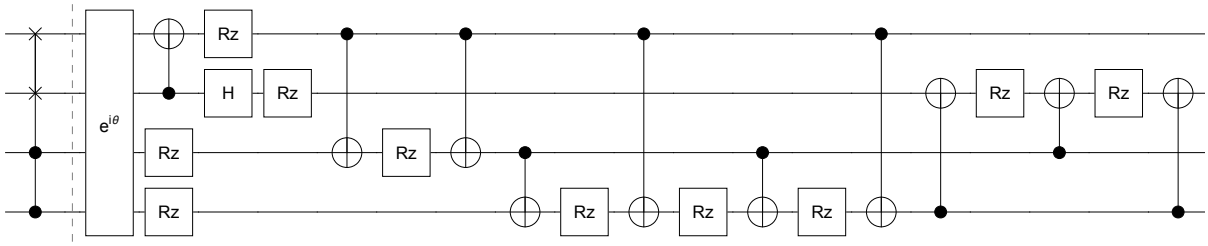
» error: 0

C*[SWAP]

testRecomp @ $C_{0,1}[SWAP_{2,3}]$

» $\left\{G\left[\frac{\pi}{16}\right], C_2[X_3], H_2, Rz_0\left[\frac{\pi}{8}\right], Rz_1\left[\frac{\pi}{8}\right], Rz_3\left[\frac{\pi}{8}\right], Rz_2\left[\frac{\pi}{8}\right], C_3[X_1],\right.$
$Rz_1\left[-\frac{\pi}{8}\right], C_3[X_1], C_1[X_0], Rz_0\left[-\frac{\pi}{8}\right], C_3[X_0], Rz_0\left[\frac{\pi}{8}\right], C_1[X_0], Rz_0\left[-\frac{\pi}{8}\right],$
$C_3[X_0], C_0[X_2], Rz_2\left[-\frac{\pi}{8}\right], C_1[X_2], Rz_2\left[\frac{\pi}{8}\right], C_0[X_2], Rz_2\left[-\frac{\pi}{8}\right], C_3[X_2], Rz_2\left[\frac{\pi}{8}\right],$
$\left.C_0[X_2], Rz_2\left[-\frac{\pi}{8}\right], C_1[X_2], Rz_2\left[\frac{\pi}{8}\right], C_0[X_2], Rz_2\left[-\frac{\pi}{8}\right], C_3[X_2], H_2, C_2[X_3]\right\}$

»



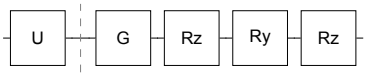
» error: 0

Testing U (matrix)

Un-controlled

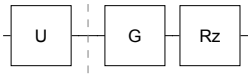
U^(1)

```
testRecomp[U₀ @ RandomVariate @ CircularUnitaryMatrixDistribution @ 2, False]
```

» 

» error: 0

```
testRecomp[U₀ @ {{Exp[i .1], 0}, {0, Exp[-i π / 3]}}, False]
```

» 

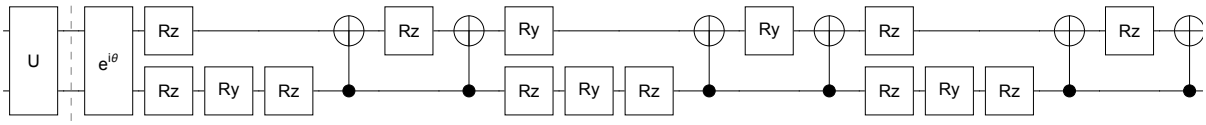
» error: 0

```
RecompileCircuit[ U₀@{{a, b}, {c, d}}, "SingleQubitAndCNOT"]
```

$$\left\{G\left[\text{ArcTan}[\text{Re}[a],\text{Im}[a]]+\frac{1}{2}\left(-2\,\text{ArcTan}[\text{Re}[a],\text{Im}[a]]+\text{ArcTan}[-\text{Re}[b],-\text{Im}[b]]+\text{ArcTan}[\text{Re}[c],\text{Im}[c]]\right)\right],\right.$$
$$\text{Rz}_0[-\text{ArcTan}[\text{Re}[a],\text{Im}[a]]+\text{ArcTan}[-\text{Re}[b],-\text{Im}[b]]],\ \text{Ry}_0\left[2\,\text{ArcTan}\left[\frac{\text{Abs}[b]}{\text{Abs}[a]}\right]\right],$$
$$\left.\text{Rz}_0[-\text{ArcTan}[\text{Re}[a],\text{Im}[a]]+\text{ArcTan}[\text{Re}[c],\text{Im}[c]]]\right\}$$

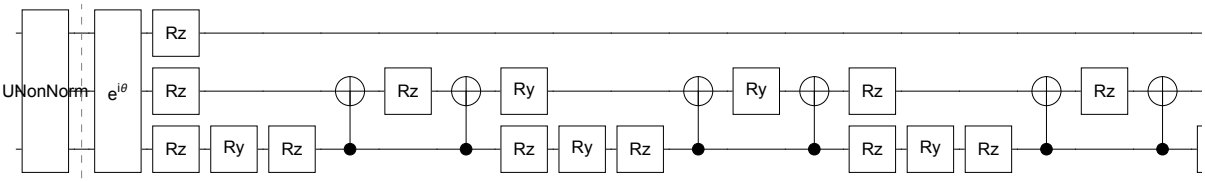
U^(n)

```
testRecomp[U₀,₁ @ RandomVariate @ CircularUnitaryMatrixDistribution[2²], False]
```

» 

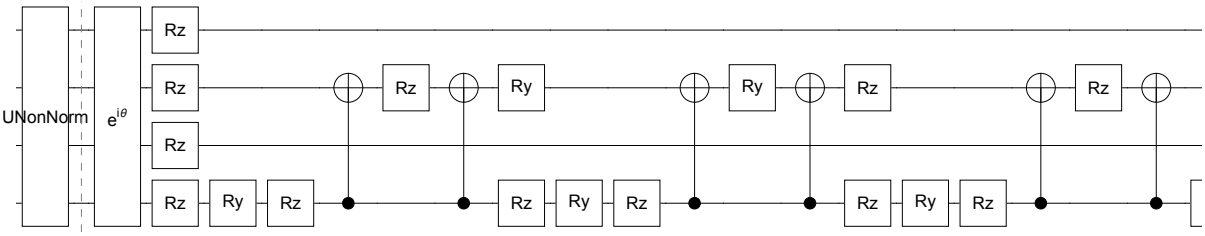
» error: 0

```
testRecomp[
 UNonNorm₀,₁,₂ @ RandomVariate @ CircularUnitaryMatrixDistribution[2³], False]
```

» 

» error: 0

```
testRecomp[
 UNonNorm₀,₂,₁,₃ @ RandomVariate @ CircularUnitaryMatrixDistribution[2⁴], False]
```

» 

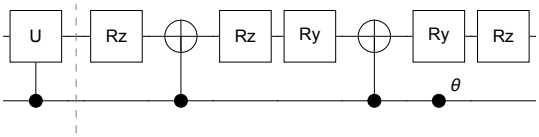
» error: 0

Singly-controlled

C[U^(1)]

```
testRecomp[C₀@U₁ @ RandomVariate @ CircularUnitaryMatrixDistribution @ 2]
```

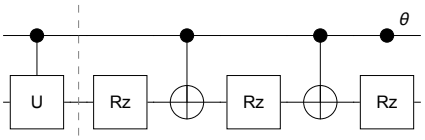
» {Rz₁[-1.56961], C₀[X₁], Rz₁[-1.62996], Ry₁[-0.645443], C₀[X₁], Ry₁[0.645443], Rz₁[3.19956], Ph₀[-0.693822]}

» 

» error: 0

```
testRecomp[C₁@U₀ @ {{Exp[i .1], 0}, {0, Exp[-i π / 3]}}]
```

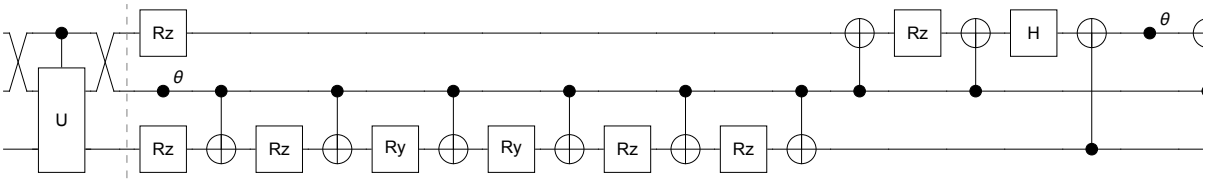
» {Rz₀[0.573599], C₁[X₀], Rz₀[0.573599], C₁[X₀], Rz₀[-1.1472], Ph₁[-0.473599]}

» 

» error: 0

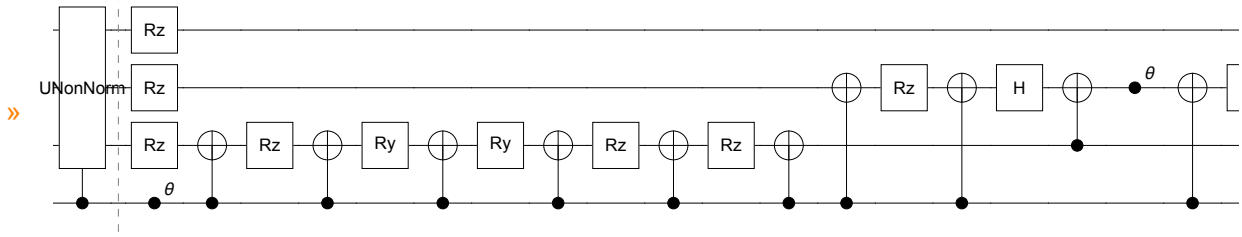
C[U^(n)]

```
testRecomp[
  C₁@U₀,₂ @ RandomVariate @ CircularUnitaryMatrixDistribution[2^2], False]
```

» 

» error: 0

```
testRecomp[C₀@
  UNonNorm₁,₂,₃ @ RandomVariate @ CircularUnitaryMatrixDistribution[2^3], False]
```

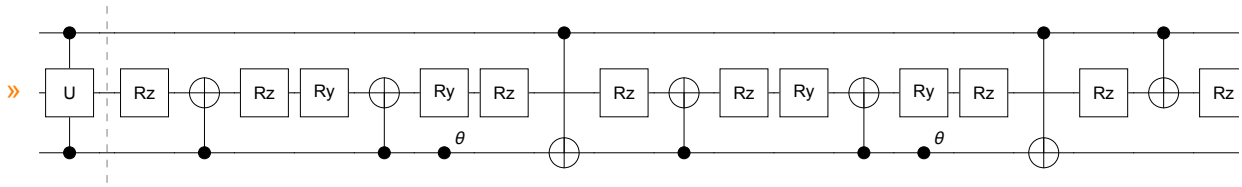


» error: 0

Multi-controlled

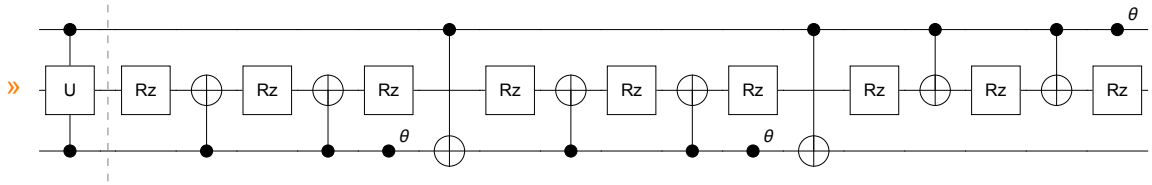
C*[U^(1)]

```
testRecomp[C0,2@U1 @ RandomVariate @ CircularUnitaryMatrixDistribution @ 2, False]
```



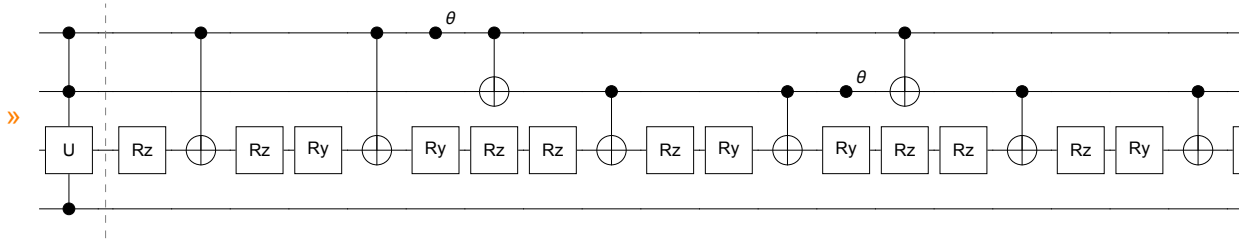
» error: 0

```
testRecomp[C0,2@U1 @ {{Exp[i .1], 0}, {0, Exp[-i π / 3]}}, False]
```



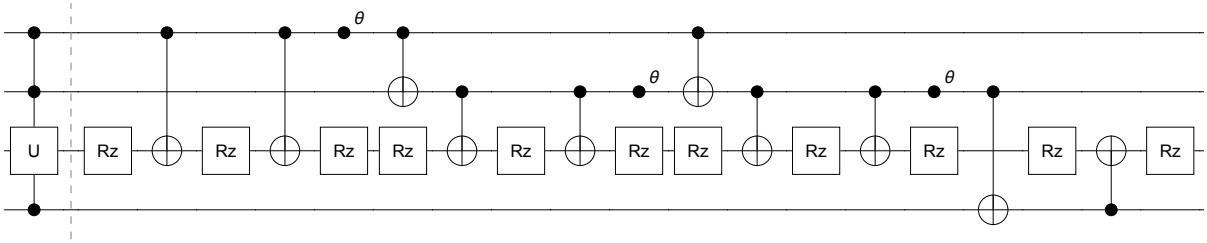
» error: 0

```
testRecomp[
 C0,2,3@U1 @ RandomVariate @ CircularUnitaryMatrixDistribution @ 2, False]
```



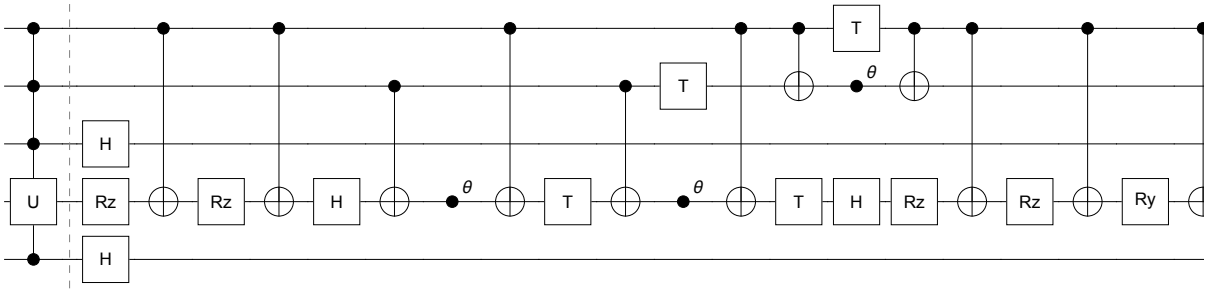
» error: 0

```
testRecomp[C0,2,3@U1 @ {{Exp[i .1], 0}, {0, Exp[-i π / 3]}}, False]
```



» error: 0

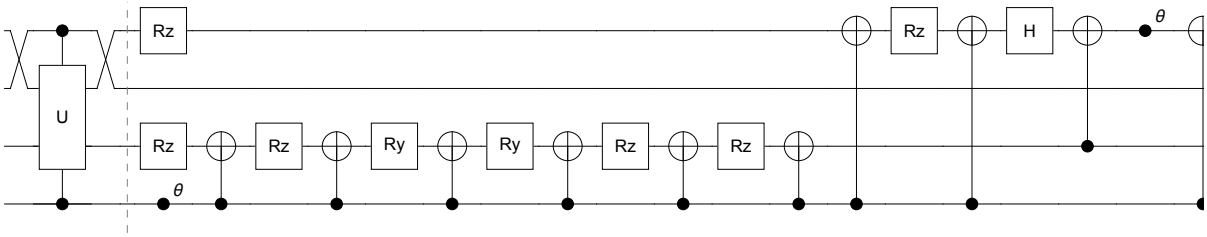
```
testRecomp[
 C0,2,3,4@U1 @ RandomVariate @ CircularUnitaryMatrixDistribution @ 2, False]
```



» error: 0

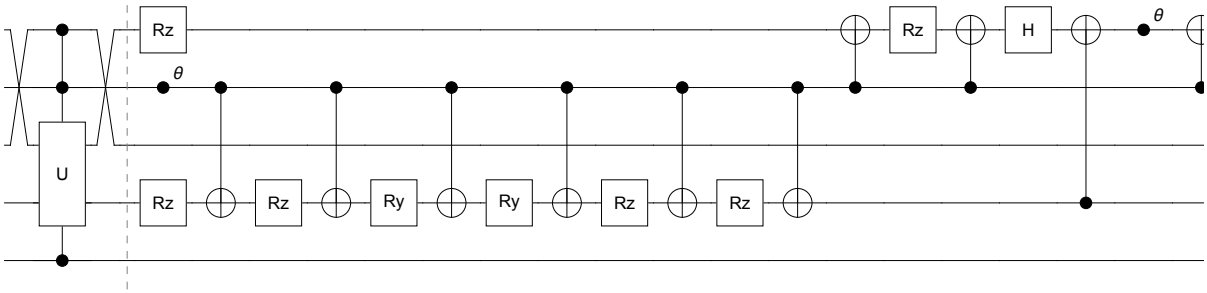
C*[U^(n)]

```
testRecomp[
 C0,2@U1,3 @ RandomVariate @ CircularUnitaryMatrixDistribution[2^2], False]
```



» error: 0

```
testRecomp[
 C0,2,3@U1,4 @ RandomVariate @ CircularUnitaryMatrixDistribution[2^2], False]
```



» error: 0

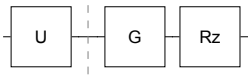
Testing U (vector)

Un-controlled

U^(1)

```
testRecomp @ U0[{Exp[i .2], Exp[ - i π / 3]}]
```

» {G[-0.423599], Rz0[-1.2472]}

» 

» error: 0

U^(n)

not yet supported

Singly-controlled

C[U^(1)]

not yet supported

C[U^(n)]

not yet supported

Multi-controlled

C*[U^(1)]

not yet supported

C*[U^(n)]

not yet supported

Testing errors

invalid arguments

```
RecompileCircuit[bleh]
```

RecompileCircuit: Invalid arguments. See ?RecompileCircuit

```
$Failed
```

unrecognised method

```
RecompileCircuit[X_0, "eh"]
```

RecompileCircuit: Unrecognised method. See available methods via ?RecompileCircuit

unrecognised gates

```
RecompileCircuit[{Y_0, Poop_0, X_0, Blob_3}, "SingleQubitAndCNOT"]
```

RecompileCircuit: Recompilation failed. Could not recompile unrecognised gate: $Poop_0$

```
$Failed
```

unsupported gates

```
RecompileCircuit[Damp_0[x], "SingleQubitAndCNOT"]
```

RecompileCircuit: Recompilation failed. Could not recompile unrecognised gate: $Damp_0[x]$

```
$Failed
```

```
RecompileCircuit[U_0,1 @ {a, b, c, d}, "SingleQubitAndCNOT"]
```

RecompileCircuit: Recompilation failed. Many−qubit diagonal gates are not yet supported by the recompiler.

```
$Failed
```

```
RecompileCircuit[C_1,2@U_0 @ {a, b}, "SingleQubitAndCNOT"]
```

RecompileCircuit: Recompilation failed. Controlled diagonal gates are not yet supported by the recompiler.

```
$Failed
```

numerical issues

```
RecompileCircuit[
    U_0,1[{{a, b}, {c, d}}],
    "SingleQubitAndCNOT"]
```

RecompileCircuit: Recompilation failed. Encountered a non−numerical matrix in a two (or more) qubit U gate, which cannot be decomposed.

```
$Failed
```

```
RecompileCircuit[
    U_{0,1} @ RandomComplex[{-i - 1, i + 1}, {2^2, 2^2}],
    "SingleQubitAndCNOT"]
```

RecompileCircuit: Recompilation failed. Encountered a non–unitary U gate matrix which cannot be (spectrally) decomposed. Please use UNonNorm instead.

```
$Failed
```

```
RecompileCircuit[
    U_{0,1} @ (2 IdentityMatrix @ 4),
    "SingleQubitAndCNOT"]
```

RecompileCircuit: Recompilation failed. The cosine–sine decomposition involved in recompiling a U (or UNonNorm) gate failed.

```
$Failed
```