

# GetCircuitSuperoperator

```
SetDirectory @ NotebookDirectory[];  
Import["../Link/QuESTlink.m"];  
CreateLocalQuESTEnv["../quest_link"];
```

## Doc

### ? GetCircuitSuperoperator

#### Symbol

GetCircuitSuperoperator[circuit] returns the corresponding superoperator circuit upon doubly-many qubits as per the Choi-Jamiołkowski isomorphism. Decoherence channels become `Matr[]` superoperators.

GetCircuitSuperoperator[circuit, numQubits] forces the circuit to be assumed size `numQubits`, so that the output superoperator acts upon  $2*\text{numQubits}$ .

GetCircuitSuperoperator accepts optional argument `AssertValidChannels`.

### ? AssertValidChannels

#### Symbol

Optional argument to functions like `CalcCircuitMatrix`, `CalcPauliTransferMatrix`, `GetCircuitConjugated`, etc, specifying whether to simplify their outputs by asserting that all channels therein are completely-positive and trace-preserving (default `True`).

For example, this asserts that the symbolic argument to a damping channel is constrained between 0 and 1 (inclusive), and that the parameters of canonical parameterised gates (like `Rx`) are strictly real.

Specifying `AssertValidChannels->False` will not change the dimension of the outputs (i.e. the returned objects would be applied upon states in the same fashion), but will disable symbolic simplifications therein, and is necessary to obtain correct expressions when all symbolic parameters are permitted to be complex.

# Correctness

```
(* this method can only test when AssertValidChannels → True (default),
   and when all gate parameters are given numeric values *)
test[circ_, subs_:{}] := Module[
  {n, in, out, sup, ρ, err},
  n = 1 + Max @ GetCircuitQubits @ circ;
  in = RandomComplex[{-1-1i, 1+1i}, {2^n, 2^n}];
  ρ = SetQuregMatrix[CreateDensityQureg[n], in];
  ApplyCircuit[ρ, circ /. subs];
  sup = GetCircuitSuperoperator[circ];
  out = CalcCircuitMatrix[sup /. subs, 2 n] . Flatten @ Transpose @ in;
  err = Flatten @ Transpose @ GetQuregState[ρ] - out // N // Abs // Max // Chop;
  Echo[sup, "output: "];
  Echo[err, "error: "];
  If[err != 0 && err != 0., Style["ERRONEOUS SUPEROPERATOR!", Red]]]
```

## Gates

### Uncontrolled

**test @ H<sub>2</sub>**

» output: {H<sub>2</sub>, H<sub>5</sub>}

» error: 0

**test @ Id<sub>2</sub>**

» output: {Id<sub>2</sub>, Id<sub>5</sub>}

» error: 0

**test @ S<sub>2</sub>**

» output: {S<sub>2</sub>, Ph<sub>5</sub>[ $-\frac{\pi}{2}$ ]}

» error: 0

**test @ T<sub>1</sub>**

» output: {T<sub>1</sub>, Ph<sub>3</sub>[ $-\frac{\pi}{4}$ ]}

» error: 0

**test @ SWAP<sub>0,1</sub>**

» output: {SWAP<sub>0,1</sub>, SWAP<sub>2,3</sub>}

» error: 0

```

test @ X0
test @ Y1
test @ Z2
» output: {X0, X1}
» error: 0
» output: {Y1, U3[{{0, i}, {-i, 0}}]}
» error: 0
» output: {Z2, Z5}
» error: 0

test[Ph1,3,0[x], x → RandomReal[]]
» output: {Ph1,3,0[x], Ph5,7,4[-x]}
» error: 0

test[R[x, X0 X1 Z2], x → RandomReal[]]
test[R[x, Y0 Y1 Z2], x → RandomReal[]]
test[R[x, X0 Y1 Z2], x → RandomReal[]]
» output: {R[x, X0 X1 Z2], R[-x, X3 X4 Z5]}
» error: 0
» output: {R[x, Y0 Y1 Z2], R[-x, Y3 Y4 Z5]}
» error: 0
» output: {R[x, X0 Y1 Z2], R[x, X3 Y4 Z5]}
» error: 0

test[Rx0[x], x → RandomReal[]]
test[Rx0,2,3[x], x → RandomReal[]]
test[Ry0[x], x → RandomReal[]]
test[Ry0,2,3[x], x → RandomReal[]]
test[Rz0[x], x → RandomReal[]]
test[Rz0,2,3[x], x → RandomReal[]]
» output: {Rx0[x], Rx1[-x]}
» error: 0
» output: {Rx0,2,3[x], Rx4,6,7[-x]}
» error: 0
» output: {Ry0[x], Ry1[x]}
» error: 0
» output: {Ry0,2,3[x], Ry4,6,7[x]}
» error: 0
» output: {Rz0[x], Rz1[-x]}
» error: 0
» output: {Rz0,2,3[x], Rz4,6,7[-x]}
» error: 0

```

**test @  $U_{0,1}$  @ RandomVariate @ CircularUnitaryMatrixDistribution @ 4**

» output:  $\{U_{0,1}[\{\{0.181902 + 0.219331 i,$   
 $-0.508232 + 0.0095965 i, 0.205379 + 0.633037 i, -0.427538 - 0.186302 i\},$   
 $\{-0.345852 - 0.565724 i, -0.684023 - 0.0271223 i, -0.0370904 - 0.28199 i,$   
 $-0.0153565 - 0.10291 i\}, \{0.187056 + 0.504434 i, -0.433917 - 0.20901 i,$   
 $-0.233328 - 0.340244 i, -0.00833779 + 0.555257 i\}, \{-0.287874 - 0.326854 i,$   
 $0.107756 - 0.171478 i, -0.143546 + 0.534206 i, 0.00331872 + 0.680651 i\}\}],$   
 $U_{2,3}[\{\{0.181902 - 0.219331 i, -0.508232 - 0.0095965 i, 0.205379 - 0.633037 i,$   
 $-0.427538 + 0.186302 i\}, \{-0.345852 + 0.565724 i,$   
 $-0.684023 + 0.0271223 i, -0.0370904 + 0.28199 i, -0.0153565 + 0.10291 i\},$   
 $\{0.187056 - 0.504434 i, -0.433917 + 0.20901 i, -0.233328 + 0.340244 i,$   
 $-0.00833779 - 0.555257 i\}, \{-0.287874 + 0.326854 i,$   
 $0.107756 + 0.171478 i, -0.143546 - 0.534206 i, 0.00331872 - 0.680651 i\}\}]\}$

» error: 0

**test @  $U_{\text{NonNorm},1}$  @ RandomComplex[{-1 - i, 1 + i}, {4, 4}]**

» output:  $\{U_{\text{NonNorm},1}[\{\{-0.511282 + 0.248489 i, -0.633082 - 0.195985 i, -0.121728 + 0.180578 i,$   
 $0.976429 + 0.549827 i\}, \{-0.138027 - 0.106615 i, 0.128053 - 0.336605 i,$   
 $-0.0617404 - 0.856429 i, -0.13735 - 0.111372 i\},$   
 $\{0.203453 - 0.173808 i, 0.944329 + 0.29189 i, -0.906104 - 0.308604 i,$   
 $0.739776 - 0.960591 i\}, \{-0.641016 + 0.752223 i, 0.131106 - 0.156225 i,$   
 $-0.0873335 - 0.997022 i, 0.91434 - 0.810842 i\}\}],$   
 $U_{\text{NonNorm},2,3}[\{\{-0.511282 - 0.248489 i, -0.633082 + 0.195985 i,$   
 $-0.121728 - 0.180578 i, 0.976429 - 0.549827 i\}, \{-0.138027 + 0.106615 i,$   
 $0.128053 + 0.336605 i, -0.0617404 + 0.856429 i, -0.13735 + 0.111372 i\},$   
 $\{0.203453 + 0.173808 i, 0.944329 - 0.29189 i, -0.906104 + 0.308604 i,$   
 $0.739776 + 0.960591 i\}, \{-0.641016 - 0.752223 i, 0.131106 + 0.156225 i,$   
 $-0.0873335 + 0.997022 i, 0.91434 + 0.810842 i\}\}\}]\}$

» error: 0

## Controlled

**GetCircuitSuperoperator @  $C_1$  @  $H_2$**

$\{C_1[H_2], C_4[H_5]\}$

**test @  $C_0$  @  $\text{Id}_2$**

» output:  $\{C_0[\text{Id}_2], C_3[\text{Id}_5]\}$

» error: 0

**test @  $C_1$  @  $S_2$**

» output:  $\{C_1[S_2], C_4\left[\text{Ph}_5\left[-\frac{\pi}{2}\right]\right]\}$

» error: 0

**test @  $C_{0,2}$  @  $T_1$**

```

» output: {C0,2[T1], C3,5[Ph4[- $\frac{\pi}{4}$ ]]}

» error: 0

test @ C2,3@SWAP0,1

» output: {C2,3[SWAP0,1], C6,7[SWAP4,5]}

» error: 0

test @ C1,2,3 @ X0
test @ C2@Y1
test @ C3@Z2

» output: {C1,2,3[X0], C5,6,7[X4]}

» error: 0

» output: {C2[Y1], C5[U4[{{0, i}, {-i, 0}}]]]}

» error: 0

» output: {C3[Z2], C7[Z6]}

» error: 0

test[C2,4@Ph1,3,0[x], x → RandomReal[]]

» output: {C2,4[Ph1,3,0[x]], C7,9[Ph6,8,5[-x]]}

» error: 0

test[C3@R[x, X0 X1 Z2], x → RandomReal[]]
test[C3,4@R[x, Y0 Y1 Z2], x → RandomReal[]]
test[C3@R[x, X0 Y1 Z2], x → RandomReal[]]

» output: {C3[R[x, X0 X1 Z2]], C7[R[-x, X4 X5 Z6]]}

» error: 0

» output: {C3,4[R[x, Y0 Y1 Z2]], C8,9[R[-x, Y5 Y6 Z7]]}

» error: 0

» output: {C3[R[x, X0 Y1 Z2]], C7[R[x, X4 Y5 Z6]]}

» error: 0

test[C1,2@ Rx0[x], x → RandomReal[]]
test[C1@Rx0,2,3[x], x → RandomReal[]]
test[C1,2,3@ Ry0[x], x → RandomReal[]]
test[C1@ Ry0,2,3[x], x → RandomReal[]]
test[C1,2,3@Rz0[x], x → RandomReal[]]
test[C1,4@Rz0,2,3[x], x → RandomReal[]]

```

» output:  $\{C_{1,2}[Rx_0[x]], C_{4,5}[Rx_3[-x]]\}$

» error: 0

» output:  $\{C_1[Rx_{0,2,3}[x]], C_5[Rx_{4,6,7}[-x]]\}$

» error: 0

» output:  $\{C_{1,2,3}[Ry_0[x]], C_{5,6,7}[Ry_4[x]]\}$

» error: 0

» output:  $\{C_1[Ry_{0,2,3}[x]], C_5[Ry_{4,6,7}[x]]\}$

» error: 0

» output:  $\{C_{1,2,3}[Rz_0[x]], C_{5,6,7}[Rz_4[-x]]\}$

» error: 0

» output:  $\{C_{1,4}[Rz_{0,2,3}[x]], C_{6,9}[Rz_{5,7,8}[-x]]\}$

» error: 0

**test @ C<sub>2,3</sub>@ U<sub>0,1</sub>@ RandomVariate @ CircularUnitaryMatrixDistribution @ 4**

» output:  $\{C_{2,3}[$   
 $U_{0,1}[\{\{-0.0506663 + 0.299448 i, -0.178197 + 0.789933 i, 0.29792 + 0.0345961 i, 0.216312 +$   
 $0.339517 i\}, \{-0.37405 + 0.226039 i, 0.301695 - 0.150532 i, -0.517628 - 0.0356516 i,$   
 $0.105163 + 0.644239 i\}, \{0.00784152 - 0.482321 i, 0.300719 + 0.300631 i,$   
 $-0.167931 + 0.73856 i, -0.109945 + 0.0270907 i\}, \{0.637247 - 0.279245 i,$   
 $-0.0280165 - 0.221306 i, 0.253307 - 0.0547739 i, -0.0105128 + 0.631587 i\}\}],$   
 $C_{6,7}[U_{4,5}[\{\{-0.0506663 - 0.299448 i, -0.178197 - 0.789933 i, 0.29792 - 0.0345961 i,$   
 $0.216312 - 0.339517 i\}, \{-0.37405 - 0.226039 i, 0.301695 + 0.150532 i,$   
 $-0.517628 + 0.0356516 i, 0.105163 - 0.644239 i\},$   
 $\{0.00784152 + 0.482321 i, 0.300719 - 0.300631 i, -0.167931 - 0.73856 i,$   
 $-0.109945 - 0.0270907 i\}, \{0.637247 + 0.279245 i, -0.0280165 + 0.221306 i,$   
 $0.253307 + 0.0547739 i, -0.0105128 - 0.631587 i\}\}]\}$

» error: 0

**test @ C<sub>2,3</sub>@ UNonNorm<sub>0,1</sub>@ RandomComplex[{-1 - i, 1 + i}, {4, 4}]**

» output:  $\{C_{2,3}[$   
 $UNonNorm_{0,1}[\{\{0.0508448 + 0.720073 i, -0.585396 + 0.256124 i, 0.725307 - 0.446057 i,$   
 $-0.384357 - 0.363043 i\}, \{0.10545 + 0.582045 i,$   
 $0.16316 - 0.343721 i, 0.424913 + 0.637291 i, 0.765014 - 0.709477 i\},$   
 $\{0.50184 + 0.700342 i, 0.975469 - 0.318192 i, -0.932429 - 0.834725 i,$   
 $0.662282 - 0.238005 i\}, \{-0.366955 - 0.95043 i, 0.112315 + 0.844756 i,$   
 $0.0760064 + 0.00691771 i, -0.0920667 + 0.29749 i\}\}],$   
 $C_{6,7}[UNonNorm_{4,5}[\{\{0.0508448 - 0.720073 i, -0.585396 - 0.256124 i,$   
 $0.725307 + 0.446057 i, -0.384357 + 0.363043 i\}, \{0.10545 - 0.582045 i,$   
 $0.16316 + 0.343721 i, 0.424913 - 0.637291 i, 0.765014 + 0.709477 i\},$   
 $\{0.50184 - 0.700342 i, 0.975469 + 0.318192 i, -0.932429 + 0.834725 i,$   
 $0.662282 + 0.238005 i\}, \{-0.366955 + 0.95043 i, 0.112315 - 0.844756 i,$   
 $0.0760064 - 0.00691771 i, -0.0920667 - 0.29749 i\}\}]\}$

» error: 0

## Special

```
test[{G[x], Id0}, x → RandomReal[]]
» output: {Id0, Id1}
» error: 0

test[{Fac[x], Id0}, x → RandomReal[]]
» output: {Fac[x], Id0, Id1}
» error: 0

test[Matr0[{{1, 2}, {3, 4}}]]
» output: {Matr0[{{1, 2}, {3, 4}}]]
» error: 0
```

---

## Channels

```
test[Damp0[x], x → .2]
» output: {Matr0,1[{{1, 0, 0, x}, {0,  $\sqrt{1-x}$ , 0, 0}, {0, 0,  $\sqrt{1-x}$ , 0}, {0, 0, 0, 1-x}}]]}
» error: 0

test[Deph0[x], x → .1]
test[Deph1,2[x], x → .4]
```





» output:

$$\{\text{Matr}_{0,1}[\{\{1 - \frac{2x}{3}, 0, 0, \frac{2x}{3}\}, \{0, 1 - \frac{4x}{3}, 0, 0\}, \{0, 0, 1 - \frac{4x}{3}, 0\}, \{\frac{2x}{3}, 0, 0, 1 - \frac{2x}{3}\}\}\}\}]$$

» error: 0

» output:

$$\{\text{Matr}_{1,2,4,5}[\{\{1 - \frac{4x}{5}, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, \frac{4x}{15}\}, \{0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{\frac{4x}{15}, 0, 0, 0, 0, 1 - \frac{4x}{5}, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}\}, \{\frac{4x}{15}, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, 1 - \frac{4x}{5}, 0, 0, 0, 0, \frac{4x}{15}\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}\}, \{\frac{4x}{15}, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, 1 - \frac{4x}{5}\}\}\}\}]$$

» error: 0

```
m = Table[RandomVariate @ CircularUnitaryMatrixDistribution @ 4, 1];
test[Kraus0,1[m]]
```

» output:

$$\{\text{Matr}_{0,1,2,3}[\{\{0.276075 + 0. \text{i}, -0.107384 + 0.378888 \text{i}, -0.0833896 - 0.16882 \text{i}, -0.0666051 + 0.0698553 \text{i}, -0.107384 - 0.378888 \text{i}, 0.561759 + 0. \text{i}, -0.199255 + 0.18011 \text{i}, 0.121777 + 0.0642383 \text{i}, -0.0833896 + 0.16882 \text{i}, -0.199255 - 0.18011 \text{i}, 0.128422 + 0. \text{i}, -0.0225982 - 0.0618291 \text{i}, -0.0666051 - 0.0698553 \text{i}, 0.121777 - 0.0642383 \text{i}, -0.0225982 + 0.0618291 \text{i}, 0.0337444 + 0. \text{i}\}, \{0.113455 + 0.148148 \text{i}, 0.0882889 + 0.173687 \text{i}, 0.240396 + 0.2404 \text{i}, 0.263158 - 0.135861 \text{i}, 0.15919 - 0.213331 \text{i}, 0.204029 - 0.188727 \text{i}, 0.236422 - 0.42343 \text{i}, -0.288816 - 0.308316 \text{i}, -0.124862 + 0.0246289 \text{i}, -0.132878 + 0.00152585 \text{i}, -0.219618 + 0.0743885 \text{i}, 0.00359082 + 0.201959 \text{i}, 0.0101141 - 0.0644492 \text{i}, 0.0226477 - 0.064243 \text{i}, 0.00283116 - 0.118826 \text{i}, -0.0978657 - 0.0338095 \text{i}\}, \{0.0603701 + 0.0770828 \text{i}, 0.0851857 + 0.0347236 \text{i}, 0.0875352 + 0.275605 \text{i}, -0.408949 + 0.0846542 \text{i}, 0.0823074 - 0.112835 \text{i}, 0.0145208 - 0.130416 \text{i}, 0.344195 - 0.227335 \text{i}, 0.275248 + 0.528319 \text{i}\}\}\}]$$

$-0.0653712 + 0.0136332 i$ ,  $-0.0469642 + 0.0416027 i$ ,  $-0.194973 - 0.0297198 i$ ,  
 $0.0717587 - 0.275643 i$ ,  $0.00493953 - 0.0338722 i$ ,  $-0.0117655 - 0.0299318 i$ ,  
 $0.0486178 - 0.0886407 i$ ,  $0.120082 + 0.083053 i$ }, { $0.352123 - 0.177372 i$ ,  
 $-0.153346 - 0.225938 i$ ,  $-0.0989889 + 0.177817 i$ ,  $0.0590234 - 0.0340435 i$ ,  
 $-0.380391 - 0.414266 i$ ,  $-0.250434 + 0.298335 i$ ,  $0.282542 + 0.0666885 i$ ,  
 $-0.0696797 - 0.0677626 i$ ,  $0.00210264 + 0.268899 i$ ,  $0.18448 - 0.0255253 i$ ,  
 $-0.0788354 - 0.114242 i$ ,  $0.00298936 + 0.0463758 i$ ,  $-0.129833 - 0.0463055 i$ ,  
 $-0.0201734 + 0.0933103 i$ ,  $0.0688749 - 0.0178526 i$ ,  $-0.0228538 - 0.00672144 i$ }},  
{ $0.113455 - 0.148148 i$ ,  $0.15919 + 0.213331 i$ ,  $-0.124862 - 0.0246289 i$ ,  
 $0.0101141 + 0.0644492 i$ ,  $0.0882889 - 0.173687 i$ ,  $0.204029 + 0.188727 i$ ,  
 $-0.132878 - 0.00152585 i$ ,  $0.0226477 + 0.064243 i$ ,  $0.240396 - 0.2404 i$ ,  
 $0.236422 + 0.42343 i$ ,  $-0.219618 - 0.0743885 i$ ,  $0.00283116 + 0.118826 i$ ,  
 $0.263158 + 0.135861 i$ ,  $-0.288816 + 0.308316 i$ ,  $0.00359082 - 0.201959 i$ ,  
 $-0.0978657 + 0.0338095 i$ }, { $0.126124 + 0. i$ ,  $0.129487 + 0.024 i$ ,  
 $0.227796 - 0.0302079 i$ ,  $0.0352407 - 0.197049 i$ ,  $0.129487 - 0.024 i$ ,  $0.137507 + 0. i$ ,  
 $0.228122 - 0.0743602 i$ ,  $-0.00131583 - 0.209009 i$ ,  $0.227796 + 0.0302079 i$ ,  
 $0.228122 + 0.0743602 i$ ,  $0.418664 + 0. i$ ,  $0.110844 - 0.347455 i$ ,  $0.0352407 + 0.197049 i$ ,  
 $-0.00131583 + 0.209009 i$ ,  $0.110844 + 0.347455 i$ ,  $0.317705 + 0. i$ }},  
{ $0.0661738 - 0.000718267 i$ ,  $0.053641 - 0.0314426 i$ ,  $0.183869 + 0.0662883 i$ ,  
 $-0.122633 + 0.25424 i$ ,  $0.0678015 - 0.0133295 i$ ,  $0.0490881 - 0.0424882 i$ ,  
 $0.201385 + 0.0330676 i$ ,  $-0.0775237 + 0.284355 i$ ,  $0.11969 + 0.0145519 i$ ,  
 $0.104413 - 0.0439418 i$ ,  $0.316214 + 0.163763 i$ ,  $-0.282383 + 0.429818 i$ ,  
 $0.0196119 + 0.103185 i$ ,  $0.0641121 + 0.0750202 i$ ,  $-0.0521897 + 0.305788 i$ ,  
 $-0.431475 - 0.120557 i$ }, { $0.0495256 - 0.261849 i$ ,  $-0.184262 - 0.0105621 i$ ,  
 $0.0547407 + 0.126195 i$ ,  $0.00598753 - 0.0456636 i$ ,  $0.00101935 - 0.278255 i$ ,  
 $-0.191185 + 0.0242191 i$ ,  $0.0802135 + 0.119143 i$ ,  $-0.00254206 - 0.0480205 i$ ,  
 $0.152165 - 0.46107 i$ ,  $-0.33027 - 0.0632088 i$ ,  $0.0686438 + 0.241034 i$ ,  
 $0.0217511 - 0.0810402 i$ ,  $0.422936 + 0.00421209 i$ ,  $-0.0349834 - 0.290831 i$ ,  
 $-0.181864 + 0.120784 i$ ,  $0.0730152 - 0.0034044 i$ }, { $0.0603701 - 0.0770828 i$ ,  
 $0.0823074 + 0.112835 i$ ,  $-0.0653712 - 0.0136332 i$ ,  $0.00493953 + 0.0338722 i$ ,  
 $0.0851857 - 0.0347236 i$ ,  $0.0145208 + 0.130416 i$ ,  $-0.0469642 - 0.0416027 i$ ,  
 $-0.0117655 + 0.0299318 i$ ,  $0.0875352 - 0.275605 i$ ,  $0.344195 + 0.227335 i$ ,  
 $-0.194973 + 0.0297198 i$ ,  $0.0486178 + 0.0886407 i$ ,  $-0.408949 - 0.0846542 i$ ,  
 $0.275248 - 0.528319 i$ ,  $0.0717587 + 0.275643 i$ ,  $0.120082 - 0.083053 i$ }},  
{ $0.0661738 + 0.000718267 i$ ,  $0.0678015 + 0.0133295 i$ ,  $0.11969 - 0.0145519 i$ ,  
 $0.0196119 - 0.103185 i$ ,  $0.053641 + 0.0314426 i$ ,  $0.0490881 + 0.0424882 i$ ,  
 $0.104413 + 0.0439418 i$ ,  $0.0641121 - 0.0750202 i$ ,  $0.183869 - 0.0662883 i$ ,  
 $0.201385 - 0.0330676 i$ ,  $0.316214 - 0.163763 i$ ,  $-0.0521897 - 0.305788 i$ ,  
 $-0.122633 - 0.25424 i$ ,  $-0.0775237 - 0.284355 i$ ,  $-0.282383 - 0.429818 i$ ,  
 $-0.431475 + 0.120557 i$ }, { $0.0347236 + 0. i$ ,  $0.028323 - 0.0161916 i$ ,  
 $0.0960931 + 0.0358266 i$ ,  $-0.0657898 + 0.132694 i$ ,  $0.028323 + 0.0161916 i$ ,  
 $0.0306523 + 0. i$ ,  $0.0616743 + 0.0740308 i$ ,  $-0.115538 + 0.0775569 i$ ,  
 $0.0960931 - 0.0358266 i$ ,  $0.0616743 - 0.0740308 i$ ,  $0.302891 + 0. i$ ,  
 $-0.0451556 + 0.435094 i$ ,  $-0.0657898 - 0.132694 i$ ,  $-0.115538 - 0.0775569 i$ ,  
 $-0.0451556 - 0.435094 i$ ,  $0.631734 + 0. i$ }, { $0.0274759 - 0.137103 i$ ,  
 $-0.0966167 - 0.00659098 i$ ,  $0.0280022 + 0.0665225 i$ ,  $0.00340153 - 0.0239243 i$ ,  
 $0.086342 - 0.0990184 i$ ,  $-0.0757339 - 0.0504283 i$ ,  $-0.00817882 + 0.0673177 i$ ,  
 $0.0139304 - 0.0179282 i$ ,  $-0.0654219 - 0.407763 i$ ,  $-0.274175 + 0.0814462 i$ ,  
 $0.146128 + 0.155201 i$ ,  $-0.015271 - 0.0697171 i$ ,  $-0.575988 + 0.154767 i$ ,  
 $0.15787 + 0.381703 i$ ,  $0.201157 - 0.233047 i$ ,  $-0.0978701 + 0.0323299 i$ }},  
{ $0.352123 + 0.177372 i$ ,  $-0.380391 + 0.414266 i$ ,  $0.00210264 - 0.268899 i$ ,

```

-0.129833 + 0.0463055 i, -0.153346 + 0.225938 i, -0.250434 - 0.298335 i,
0.18448 + 0.0255253 i, -0.0201734 - 0.0933103 i, -0.0989889 - 0.177817 i,
0.282542 - 0.0666885 i, -0.0788354 + 0.114242 i, 0.0688749 + 0.0178526 i,
0.0590234 + 0.0340435 i, -0.0696797 + 0.0677626 i,
0.00298936 - 0.0463758 i, -0.0228538 + 0.00672144 i},
{0.0495256 + 0.261849 i, 0.00101935 + 0.278255 i, 0.152165 + 0.46107 i,
0.422936 - 0.00421209 i, -0.184262 + 0.0105621 i, -0.191185 - 0.0242191 i,
-0.33027 + 0.0632088 i, -0.0349834 + 0.290831 i, 0.0547407 - 0.126195 i,
0.0802135 - 0.119143 i, 0.0686438 - 0.241034 i, -0.181864 - 0.120784 i,
0.00598753 + 0.0456636 i, -0.00254206 + 0.0480205 i, 0.0217511 + 0.0810402 i,
0.0730152 + 0.0034044 i}, {0.0274759 + 0.137103 i, 0.086342 + 0.0990184 i,
-0.0654219 + 0.407763 i, -0.575988 - 0.154767 i, -0.0966167 + 0.00659098 i,
-0.0757339 + 0.0504283 i, -0.274175 - 0.0814462 i, 0.15787 - 0.381703 i,
0.0280022 - 0.0665225 i, -0.00817882 - 0.0673177 i, 0.146128 - 0.155201 i,
0.201157 + 0.233047 i, 0.00340153 + 0.0239243 i, 0.0139304 + 0.0179282 i,
-0.015271 + 0.0697171 i, -0.0978701 - 0.0323299 i}, {0.563077 + 0. i,
-0.0504264 - 0.386697 i, -0.2405 + 0.163201 i, 0.0971542 - 0.00550004 i,
-0.0504264 + 0.386697 i, 0.270082 + 0. i, -0.0905416 - 0.17978 i,
-0.00492347 + 0.0672139 i, -0.2405 - 0.163201 i, -0.0905416 + 0.17978 i,
0.150024 + 0. i, -0.0430904 - 0.0258099 i, 0.0971542 + 0.00550004 i,
-0.00492347 - 0.0672139 i, -0.0430904 + 0.0258099 i, 0.0168169 + 0. i}}}]
» error: 0

m = RandomComplex[{-1 - i, 1 + i}, {7, 4, 4}];
test[KrausNonTP0,1[m]]
» output: {Matr0,1,2,3[
{2.49289 + 0. i, -0.997707 - 1.44233 i, 1.17898 + 0.0431297 i, -0.0711321 + 1.4758 i,
-0.997707 + 1.44233 i, 3.46007 + 0. i, 0.890193 + 0.508589 i, -0.238343 - 1.04447 i,
1.17898 - 0.0431297 i, 0.890193 - 0.508589 i, 3.65023 + 0. i, -0.321455 - 0.0904113 i,
-0.0711321 - 1.4758 i, -0.238343 + 1.04447 i, -0.321455 + 0.0904113 i, 6.4003 + 0. i},
{-0.48417 - 1.10371 i, -0.781695 + 0.446238 i, 0.566639 - 1.04113 i,
0.1176 - 0.347512 i, 1.36196 + 0.536229 i, 1.87831 + 0.0685901 i, 1.71522 - 1.20387 i,
2.09015 + 0.432963 i, -0.805893 - 0.448595 i, -0.117784 + 0.668681 i,
1.80906 - 2.45177 i, 2.91595 - 0.273138 i, 1.02182 + 1.51325 i,
0.756708 - 0.431762 i, -0.65061 - 0.872411 i, -0.35737 + 0.188402 i},
{0.178585 + 0.463909 i, 0.139997 - 1.29744 i, 0.144713 - 1.70963 i,
-0.250428 - 1.16193 i, 1.12045 + 0.155689 i, 1.67458 + 1.07813 i, 0.246251 + 1.80658 i,
2.16059 + 0.865465 i, 3.23365 + 0.0713732 i, 1.47384 + 0.275105 i,
-0.45904 + 0.134044 i, 2.31387 + 1.47911 i, 0.455977 - 0.786856 i,
-2.62037 + 1.61997 i, -3.64869 - 0.44496 i, -0.836 + 1.96482 i},
{0.184798 - 0.326832 i, 0.197429 + 0.0563472 i, -0.570096 + 1.23257 i,
0.18935 + 0.187618 i, 0.380178 - 1.8411 i, -0.231103 - 0.056392 i,
-0.757504 - 1.61016 i, -0.527754 - 1.22736 i, 0.800536 - 2.89293 i,
0.679378 - 0.350116 i, -0.714096 - 0.678139 i, -1.31583 - 0.640512 i,
1.64447 - 0.888361 i, -3.17369 + 0.118918 i, 2.70308 + 0.134329 i, 2.25441 - 2.49014 i},
{-0.48417 + 1.10371 i, 1.36196 - 0.536229 i, -0.805893 + 0.448595 i,
1.02182 - 1.51325 i, -0.781695 - 0.446238 i, 1.87831 - 0.0685901 i,
-0.117784 - 0.668681 i, 0.756708 + 0.431762 i, 0.566639 + 1.04113 i,
1.71522 + 1.20387 i, 1.80906 + 2.45177 i, -0.65061 + 0.872411 i, 0.1176 + 0.347512 i,
2.09015 - 0.432963 i, 2.91595 + 0.273138 i, -0.35737 - 0.188402 i},

```

{4.96993 + 0. i, 2.33271 + 1.83897 i, 1.4335 - 0.0824692 i, -0.0528706 - 0.71527 i,  
 2.33271 - 1.83897 i, 6.479 + 0. i, 0.800278 - 3.49448 i, 0.313052 - 1.02489 i,  
 1.4335 + 0.0824692 i, 0.800278 + 3.49448 i, 5.72265 + 0. i, 2.40714 + 1.94948 i,  
 -0.0528706 + 0.71527 i, 0.313052 + 1.02489 i, 2.40714 - 1.94948 i, 3.83966 + 0. i},  
 {-0.110972 - 2.09594 i, 0.317239 + 0.0799497 i, 0.811696 - 0.0690812 i,  
 0.338765 + 0.305033 i, -0.112413 - 1.7492 i, -0.17621 + 0.203363 i,  
 -0.642015 - 0.828002 i, 0.346057 + 0.444142 i, 1.7943 + 1.48869 i,  
 -0.660655 + 0.218982 i, 1.0734 - 1.1303 i, -0.106032 + 2.04472 i, 3.68159 + 0.446919 i,  
 0.748491 + 0.421405 i, 0.343988 + 1.36986 i, 2.00761 + 1.77917 i},  
 {0.354423 - 0.0597601 i, -0.642735 + 2.27633 i, -0.570327 - 1.28483 i,  
 -1.42464 - 1.87055 i, -0.122057 - 0.845265 i, 1.008 - 0.191196 i,  
 -0.176703 - 0.423482 i, -1.87255 + 0.773001 i, 2.45498 - 1.18765 i,  
 0.617148 + 0.749458 i, 0.652035 - 1.20052 i, -1.44895 - 1.78769 i, 0.518931 - 2.75532 i,  
 0.142358 - 0.563047 i, -0.596319 - 0.756033 i, -1.3292 - 1.8802 i},  
 {0.178585 - 0.463909 i, 1.12045 - 0.155689 i, 3.23365 - 0.0713732 i,  
 0.455977 + 0.786856 i, 0.139997 + 1.29744 i, 1.67458 - 1.07813 i,  
 1.47384 - 0.275105 i, -2.62037 - 1.61997 i, 0.144713 + 1.70963 i,  
 0.246251 - 1.80658 i, -0.45904 - 0.134044 i, -3.64869 + 0.44496 i,  
 -0.250428 + 1.16193 i, 2.16059 - 0.865465 i, 2.31387 - 1.47911 i, -0.836 - 1.96482 i},  
 {-0.110972 + 2.09594 i, -0.112413 + 1.7492 i, 1.7943 - 1.48869 i, 3.68159 - 0.446919 i,  
 0.317239 - 0.0799497 i, -0.17621 - 0.203363 i, -0.660655 - 0.218982 i,  
 0.748491 - 0.421405 i, 0.811696 + 0.0690812 i, -0.642015 + 0.828002 i,  
 1.0734 + 1.1303 i, 0.343988 - 1.36986 i, 0.338765 - 0.305033 i,  
 0.346057 - 0.444142 i, -0.106032 - 2.04472 i, 2.00761 - 1.77917 i},  
 {5.47035 + 0. i, 1.43156 - 0.0948294 i, 0.117148 + 0.756608 i, 1.52203 + 2.29226 i,  
 1.43156 + 0.0948294 i, 5.51744 + 0. i, 1.00323 + 1.75548 i, 2.84959 - 0.804959 i,  
 0.117148 - 0.756608 i, 1.00323 - 1.75548 i, 4.73346 + 0. i, 0.305355 - 2.29736 i,  
 1.52203 - 2.29226 i, 2.84959 + 0.804959 i, 0.305355 + 2.29736 i, 4.73696 + 0. i},  
 {1.18928 - 2.34684 i, -0.448452 - 1.0005 i, 0.0646155 - 0.337814 i, -0.759659 -  
 2.08741 i, -0.193468 - 1.07464 i, 0.868041 + 0.27137 i, -2.19016 - 1.05128 i,  
 -0.582258 + 0.215905 i, -1.80933 + 0.895303 i, 0.263566 + 0.645783 i,  
 -0.383258 + 0.303869 i, -2.7026 + 0.408611 i, -0.946513 - 3.15003 i,  
 -0.180844 - 0.187503 i, -1.56097 - 0.871055 i, -1.62006 - 0.380897 i},  
 {0.184798 + 0.326832 i, 0.380178 + 1.8411 i, 0.800536 + 2.89293 i, 1.64447 + 0.888361 i,  
 0.197429 - 0.0563472 i, -0.231103 + 0.056392 i, 0.679378 + 0.350116 i,  
 -3.17369 - 0.118918 i, -0.570096 - 1.23257 i, -0.757504 + 1.61016 i,  
 -0.714096 + 0.678139 i, 2.70308 - 0.134329 i, 0.18935 - 0.187618 i,  
 -0.527754 + 1.22736 i, -1.31583 + 0.640512 i, 2.25441 + 2.49014 i},  
 {0.354423 + 0.0597601 i, -0.122057 + 0.845265 i, 2.45498 + 1.18765 i,  
 0.518931 + 2.75532 i, -0.642735 - 2.27633 i, 1.008 + 0.191196 i, 0.617148 - 0.749458 i,  
 0.142358 + 0.563047 i, -0.570327 + 1.28483 i, -0.176703 + 0.423482 i,  
 0.652035 + 1.20052 i, -0.596319 + 0.756033 i, -1.42464 + 1.87055 i,  
 -1.87255 - 0.773001 i, -1.44895 + 1.78769 i, -1.3292 + 1.8802 i},  
 {1.18928 + 2.34684 i, -0.193468 + 1.07464 i, -1.80933 - 0.895303 i, -0.946513 +  
 3.15003 i, -0.448452 + 1.0005 i, 0.868041 - 0.27137 i, 0.263566 - 0.645783 i,  
 -0.180844 + 0.187503 i, 0.0646155 + 0.337814 i, -2.19016 + 1.05128 i,  
 -0.383258 - 0.303869 i, -1.56097 + 0.871055 i, -0.759659 + 2.08741 i,  
 -0.582258 - 0.215905 i, -2.7026 - 0.408611 i, -1.62006 + 0.380897 i},  
 {3.92263 + 0. i, -0.105473 - 0.388833 i, 0.766349 - 0.132955 i, 1.92071 - 1.52407 i,  
 -0.105473 + 0.388833 i, 3.23068 + 0. i, -1.9985 - 0.541475 i, -1.28858 + 2.13755 i,  
 0.766349 + 0.132955 i, -1.9985 + 0.541475 i, 4.09897 + 0. i, -0.217755 - 1.58493 i,  
 1.92071 + 1.52407 i, -1.9985 - 0.541475 i, -0.217755 + 1.58493 i, 4.09897 - 0. i}

» error: 0

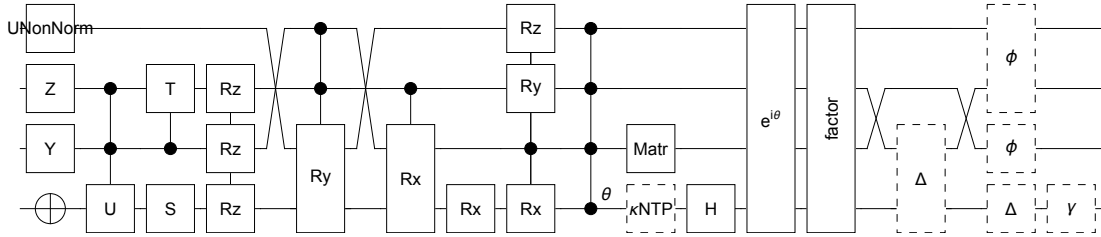
## Circuits

```

circ = Circuit[Damp0[.1] Deph1[.1] Deph2,3[.5] Depol0[.2] Depol0,2[.4]
  Fac[.3 + .3 i] × G[π] H0 Id1 KrausNonTP0@RandomComplex[{0, 1 + i}, {3, 2, 2}]
  Matr1@RandomComplex[{0, 1 + i}, {2, 2}] C0,1,2@Ph3[.6] C1@R[π, X0 Y2 Z3]
  Rx0[.1] C2@Rx1,0[.8] C1,2@Ry3,0[π / 3] Rz0,1,2[.9] S0 C1[T2]
  C1,2@U0@RandomVariate @ CircularUnitaryMatrixDistribution[2]
  UNonNorm3@RandomComplex[{0, 1 + i}, {2, 2}] X0 Y1 Z2] // Reverse;

```

```
DrawCircuit[%]
```



```
test @ circ
```

```

» output: {Z2, Z6, Y1, U5[{{0, i}, {-i, 0}}], X0,
  X4, UNonNorm3[{{0.939064 + 0.652941 i, 0.714666 + 0.48563 i},
    {0.930006 + 0.660488 i, 0.630213 + 0.453642 i}}],
  UNonNorm7[{{0.939064 - 0.652941 i, 0.714666 - 0.48563 i},
    {0.930006 - 0.660488 i, 0.630213 - 0.453642 i}}],
  C1,2[U0[{{0.238006 + 0.0751351 i, 0.43714 - 0.86407 i},
    {-0.59793 + 0.7617 i, -0.168787 - 0.183856 i}}]],
  C5,6[U4[{{0.238006 - 0.0751351 i, 0.43714 + 0.86407 i},
    {-0.59793 - 0.7617 i, -0.168787 + 0.183856 i}}]], C1[T2],
  C5[Ph6[- $\frac{\pi}{4}$ ]], S0, Ph4[- $\frac{\pi}{2}$ ], Rz0,1,2[0.9], Rz4,5,6[-0.9], C1,2[Ry3,0[ $\frac{\pi}{3}$ ]],
  C5,6[Ry7,4[- $\frac{\pi}{3}$ ]], C2[Rx1,0[0.8]], C6[Rx5,4[-0.8]], Rx0[0.1],
  Rx4[-0.1], C1[R[π, X0 Y2 Z3]], C5[R[π, X4 Y6 Z7]], C0,1,2[Ph3[0.6]],
  C4,5,6[Ph7[-0.6]], Matr1[{{0.0433638 + 0.037496 i, 0.747555 + 0.350197 i},
    {0.256458 + 0.556372 i, 0.314154 + 0.529126 i}}],
  Matr0,4[{{1.39652 + 0. i, 1.14074 - 0.943531 i, 1.14074 + 0.943531 i, 2.47912 + 0. i},
    {1.10516 + 0.638879 i, 1.61705 - 0.978374 i, 0.600817 + 1.5064 i, 2.66221 + 0.752945 i},
    {1.10516 - 0.638879 i, 0.600817 - 1.5064 i, 1.61705 + 0.978374 i, 2.66221 - 0.752945 i},
    {1.45759 + 0. i, 1.0415 - 1.36484 i, 1.0415 + 1.36484 i, 3.62453 + 0. i}}],
  Id1, Id5, H0, H4, Fac[0.3 + 0.3 i], Matr0,2,4,6[{{0.68 + 0. i, 0. + 0. i, 0. + 0. i,
    0. + 0. i, 0. + 0. i, 0.106667 + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i,
    0.106667 + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0.106667 + 0. i},
    {0. + 0. i, 0.573333 + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i,
    0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i},
    {0. + 0. i, 0. + 0. i, 0.573333 + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i,
    0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i},
    {0. + 0. i, 0. + 0. i, 0. + 0. i, 0.573333 + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i,
    0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i}]]

```



» error: 0

## AssertValidChannels -> False

```

Fac[x];
GetCircuitSuperoperator[%]
GetCircuitSuperoperator[%%, AssertValidChannels → False ]
{ Fac[x] }

{ Fac[x] }

GetCircuitSuperoperator[ G[x] ]
GetCircuitSuperoperator[ G[x], AssertValidChannels → False ]
{ }

{ G[2  $\pm$  Im[x]] }

Ph0[x];
GetCircuitSuperoperator[%]
GetCircuitSuperoperator[%%, AssertValidChannels → False ]
{ Ph0[x], Ph1[-x] }

{ Ph0[x], Ph1[-Conjugate[x]] }

C0,1 @ R[x, X2 Y3 Z4];
GetCircuitSuperoperator[%]
GetCircuitSuperoperator[%%, AssertValidChannels → False ]
{ C0,1[R[x, X2 Y3 Z4]], C5,6[R[x, X7 Y8 Z9]] }

{ C0,1[R[x, X2 Y3 Z4]], C5,6[R[Conjugate[x], X7 Y8 Z9]] }

Circuit[Rx0[x] Ry1[y] Rz2[z]];
GetCircuitSuperoperator[%]
GetCircuitSuperoperator[%%, AssertValidChannels → False ]
{ Rx0[x], Rx3[-x], Ry1[y], Ry4[y], Rz2[z], Rz5[-z] }

{ Rx0[x], Rx3[-Conjugate[x]], Ry1[y],
  Ry4[Conjugate[y]], Rz2[z], Rz5[-Conjugate[z]] }

Circuit[Ry0[x] Ry0,1[x] Ry0,1,2[x] Ry0,1,2,3[x]];
GetCircuitSuperoperator[%]
GetCircuitSuperoperator[%%, AssertValidChannels → False ]
{ Ry0[x], Ry4[x], Ry0,1[x], Ry4,5[-x], Ry0,1,2[x], Ry4,5,6[x], Ry0,1,2,3[x], Ry4,5,6,7[-x] }

{ Ry0[x], Ry4[Conjugate[x]], Ry0,1[x], Ry4,5[-Conjugate[x]],
  Ry0,1,2[x], Ry4,5,6[Conjugate[x]], Ry0,1,2,3[x], Ry4,5,6,7[-Conjugate[x]] }

```

```

Damp0[x];
GetCircuitSuperoperator[%]
GetCircuitSuperoperator[%%, AssertValidChannels → False ]
{Matr0,1[{{1, 0, 0, x}, {0,  $\sqrt{1-x}$ , 0, 0}, {0, 0,  $\sqrt{1-x}$ , 0}, {0, 0, 0, 1-x}}]}
{Matr0,1[{{1, 0, 0,  $\sqrt{x}$  Conjugate[ $\sqrt{x}$ ]}, {0,  $\sqrt{1-x}$ , 0, 0},
{0, 0, Conjugate[ $\sqrt{1-x}$ ], 0}, {0, 0, 0,  $\sqrt{1-x}$  Conjugate[ $\sqrt{1-x}$ ]}}]}

Deph1[x];
GetCircuitSuperoperator[%]
GetCircuitSuperoperator[%%, AssertValidChannels → False ]
{Matr1,3[{{1, 0, 0, 0}, {0, 1-2 x, 0, 0}, {0, 0, 1-2 x, 0}, {0, 0, 0, 1}}]}
{Matr1,3[{{ $\sqrt{1-x}$  Conjugate[ $\sqrt{1-x}$ ] +  $\sqrt{x}$  Conjugate[ $\sqrt{x}$ ], 0, 0, 0},
{0,  $\sqrt{1-x}$  Conjugate[ $\sqrt{1-x}$ ] -  $\sqrt{x}$  Conjugate[ $\sqrt{x}$ ], 0, 0},
{0, 0,  $\sqrt{1-x}$  Conjugate[ $\sqrt{1-x}$ ] -  $\sqrt{x}$  Conjugate[ $\sqrt{x}$ ], 0},
{0, 0, 0,  $\sqrt{1-x}$  Conjugate[ $\sqrt{1-x}$ ] +  $\sqrt{x}$  Conjugate[ $\sqrt{x}$ ]}}]}

Deph0,1[x];
GetCircuitSuperoperator[%]
GetCircuitSuperoperator[%%, AssertValidChannels → False ]

```



[illegible]

```

{Matr0,1,2,3[
  { {  $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] +  $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0 }, { 0,  $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ], 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0 }, { 0, 0,  $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ],
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 },
  { 0, 0, 0,  $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ],
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 }, { 0, 0, 0, 0,
     $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ], 0, 0, 0, 0, 0, 0, 0, 0, 0 },
  { 0, 0, 0, 0, 0,  $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] +  $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ],
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0 }, { 0, 0, 0, 0, 0, 0,
     $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ], 0, 0, 0, 0, 0, 0, 0, 0 },
  { 0, 0, 0, 0, 0, 0, 0,  $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ],
    0, 0, 0, 0, 0, 0, 0 }, { 0, 0, 0, 0, 0, 0, 0, 0,
     $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ], 0, 0, 0, 0, 0, 0 },
  { 0, 0, 0, 0, 0, 0, 0, 0, 0,  $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ],
    0, 0, 0, 0, 0 }, { 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
     $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] +  $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ], 0, 0, 0, 0, 0 },
  { 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,  $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ],
    0, 0, 0, 0 }, { 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
     $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ], 0, 0, 0 }, { 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0,  $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ], 0, 0 },
  { 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
     $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] -  $\frac{1}{3}$   $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ], 0 }, { 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0,  $\sqrt{1-x}$  Conjugate[  $\sqrt{1-x}$  ] +  $\sqrt{x}$  Conjugate[  $\sqrt{x}$  ] } } ] ] ]

```

Depol<sub>0,2</sub>[x];

GetCircuitSuperoperator[%]

GetCircuitSuperoperator[%%, AssertValidChannels → False]

$$\begin{aligned} & \left\{ \text{Matr}_{0,2,3,5} \left[ \left\{ \left\{ 1 - \frac{4x}{5}, 0, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, 0, \frac{4x}{15} \right\}, \right. \right. \\ & \quad \left\{ 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \quad \left\{ 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \quad \left\{ 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \quad \left\{ 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \quad \left\{ \frac{4x}{15}, 0, 0, 0, 0, 0, 1 - \frac{4x}{5}, 0, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, 0, \frac{4x}{15} \right\}, \\ & \quad \left\{ 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \quad \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \quad \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \quad \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \quad \left\{ \frac{4x}{15}, 0, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, 0, 1 - \frac{4x}{5}, 0, 0, 0, 0, 0, \frac{4x}{15} \right\}, \\ & \quad \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0, 0 \right\}, \\ & \quad \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0, 0 \right\}, \\ & \quad \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0, 0 \right\}, \\ & \quad \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 - \frac{16x}{15}, 0 \right\}, \\ & \quad \left. \left. \left. \left\{ \frac{4x}{15}, 0, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, 0, \frac{4x}{15}, 0, 0, 0, 0, 0, 1 - \frac{4x}{5} \right\} \right\} \right\} \right\} \\ & \left\{ \text{Matr}_{0,2,3,5} \left[ \left\{ \left\{ \sqrt{1-x} \text{ Conjugate}[\sqrt{1-x}] + \frac{1}{5} \sqrt{x} \text{ Conjugate}[\sqrt{x}], \right. \right. \right. \\ & \quad 0, 0, 0, 0, 0, \frac{4}{15} \sqrt{x} \text{ Conjugate}[\sqrt{x}], 0, 0, 0, 0, 0, \\ & \quad \frac{4}{15} \sqrt{x} \text{ Conjugate}[\sqrt{x}], 0, 0, 0, 0, 0, \frac{4}{15} \sqrt{x} \text{ Conjugate}[\sqrt{x}] \right\}, \\ & \quad \left\{ 0, \sqrt{1-x} \text{ Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{ Conjugate}[\sqrt{x}], 0, 0, 0, 0, 0, 0, 0, \right. \\ & \quad 0, 0, 0, 0, 0, 0, 0, 0 \left. \right\}, \left\{ 0, 0, \sqrt{1-x} \text{ Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{ Conjugate}[\sqrt{x}], \right. \\ & \quad 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \left. \right\}, \\ & \quad \left\{ 0, 0, 0, \sqrt{1-x} \text{ Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{ Conjugate}[\sqrt{x}], 0, 0, 0, 0, 0, 0, 0, \right. \\ & \quad 0, 0, 0, 0, 0, 0 \left. \right\}, \left\{ 0, 0, 0, 0, \sqrt{1-x} \text{ Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{ Conjugate}[\sqrt{x}], \right. \\ & \quad 0, 0, 0, 0, 0, 0 \left. \right\} \left. \right. \left. \right\} \end{aligned}$$

$$\begin{aligned}
&0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{4}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, \right. \\
&0, 0, \sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] + \frac{1}{5} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, 0, \\
&0, \frac{4}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, 0, 0, \frac{4}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}]\}, \\
&\left\{ 0, 0, 0, 0, 0, 0, 0, \sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], \right. \\
&0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \\
&\sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, 0, 0, 0, 0, 0\}, \\
&\left\{ 0, 0, 0, 0, 0, 0, 0, 0, \sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], \right. \\
&0, 0, 0, 0, 0, 0, 0\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \right. \\
&\sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, 0, 0, 0, 0\}, \\
&\left\{ \frac{4}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, 0, 0, \frac{4}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, \right. \\
&0, 0, \sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] + \frac{1}{5} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, \\
&0, 0, \frac{4}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}]\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \right. \\
&\sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, 0, 0\}, \left\{ 0, 0, 0, 0, 0, \right. \\
&0, 0, 0, 0, 0, 0, 0, \sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, 0\}, \\
&\left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \right. \\
&\sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0\}, \left\{ 0, 0, 0, 0, 0, 0, \right. \\
&0, 0, 0, 0, 0, 0, 0, \sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] - \frac{1}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0\}, \\
&\left\{ \frac{4}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, 0, 0, \frac{4}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], \right. \\
&0, 0, 0, 0, \frac{4}{15} \sqrt{x} \text{Conjugate}[\sqrt{x}], 0, 0, 0, 0, \\
&\left. \left. \left. \sqrt{1-x} \text{Conjugate}[\sqrt{1-x}] + \frac{1}{5} \sqrt{x} \text{Conjugate}[\sqrt{x}]\right\}\right\}\right\}
\end{aligned}$$

## Errors

**GetCircuitSuperoperator[e]**

GetCircuitSuperoperator: Invalid arguments. See ?GetCircuitSuperoperator

\$Failed

**GetCircuitSuperoperator[ $E_{h_0,1}$ ]**

... **GetCircuitSuperoperator**: Cannot obtain conjugate of unrecognised or unsupported operator:  $E_{h_0,1}$

\$Failed

**GetCircuitSuperoperator @  $M_0$**

... **GetCircuitSuperoperator**: Cannot obtain conjugate of unrecognised or unsupported operator:  $M_0$

\$Failed

**GetCircuitSuperoperator[  $\text{Damp}_0[-1]$  ]**

... **GetCircuitSuperoperator**: One or more channels could not be asserted as completely positive and trace-preserving (CPTP) and ergo could not be simplified. Prevent this error with `AssertValidChannels -> False`.

\$Failed

**GetCircuitSuperoperator[ $\text{Deph}_{0,1}[\mathbf{1}]$ ]**

... **GetCircuitSuperoperator**: One or more channels could not be asserted as completely positive and trace-preserving (CPTP) and ergo could not be simplified. Prevent this error with `AssertValidChannels -> False`.

\$Failed