DrawPauliTransferMap

```
SetDirectory @ NotebookDirectory[];
Import["../Link/QuESTlink.m"];
```

Doc

? DrawPauliTransferMap

DrawPauliTransferMap[map] visualises the given PTMap as a graph where nodes
are basis Pauli strings, and edges indicate the transformative action of the map.

DrawPauliTransferMap also accepts PTM, circuit and gate
instances, for which the corresponding PTMap is automatically calculated.

DrawPauliTransferMap accepts options "PauliStringForm", "ShowCoefficients"
and "EdgeDegreeStyles", in addition to all options accepted by Graph[].

• "ShowCoefficients" –> False hides the map's
Pauli string coefficients which are otherwise shown as edge labels.

• "PauliStringForm" sets the vertex label format to one of
"Subscript" (default), "Index", "Kronecker", "String" or "Hidden". These (except
the latter) are the formats are supported by GetPauliStringReformatted[].

• "EdgeDegreeStyles" specifies a list of styles (default informed by ColorData["Pastel"]) to set upon
edges from nodes with increasing outdegree. For example, "EdgeDegreeStyles"–>{Red,Green,Blue}
sets edges from Pauli states which are mapped to a single other state to the colour
Red, but two–outdegree node out–edges become Green, and three–outdegree
become Blue. The list is assumed repeated for higher outdegree nodes than specified.

• Graph[] options override these settings, so specifying EdgeStyle
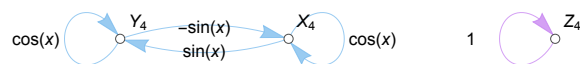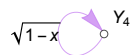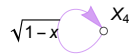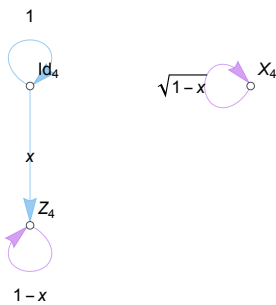–> Black will set all edges to Black regardless of their node's outdegree.

Correctness

PTMap

```
map = CalcPauliTransferMap @ Rz₄[x]
DrawPauliTransferMap[map]
```
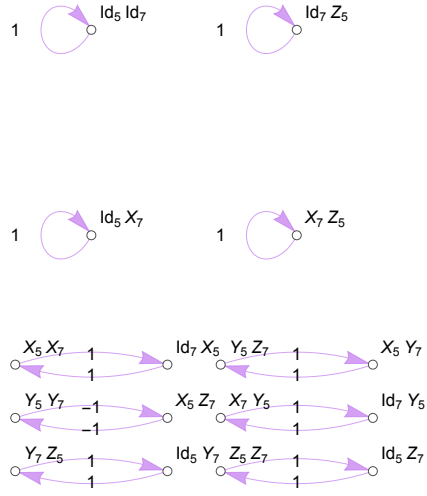
$\text{PTMap}_4[0 \to \{\{0, 1\}\}, 1 \to \{\{1, \text{Cos}[x]\}, \{2, \text{Sin}[x]\}\},$
$2 \to \{\{1, -\text{Sin}[x]\}, \{2, \text{Cos}[x]\}\}, 3 \to \{\{3, 1\}\}]$


```
DrawPauliTransferMap @ CalcPauliTransferMap @ Damp₄[x]
```

DrawPauliTransferMap @ CalcPauliTransferMap @ $C_5[X_7]$

1 $Id_5\, Id_7$ 1 $Id_7\, Z_5$

1 $Id_5\, X_7$ 1 $X_7\, Z_5$

$X_5\, X_7$ 1 $Id_7\, X_5$ $Y_5\, Z_7$ 1 $X_5\, Y_7$
     1 1

$Y_5\, Y_7$ -1 $X_5\, Z_7$ $X_7\, Y_5$ 1 $Id_7\, Y_5$
     -1 1

$Y_7\, Z_5$ 1 $Id_5\, Y_7$ $Z_5\, Z_7$ 1 $Id_5\, Z_7$
     1 1

DrawPauliTransferMap @ CalcPauliTransferMap @ $Depol_{3,9}[x]$

$1 - \dfrac{16\,x}{15}$ $Id_9\, Z_3$ $1 - \dfrac{16\,x}{15}$ $Id_9\, X_3$ $1 - \dfrac{16\,x}{15}$ $Y_3\, Y_9$ $1 - \dfrac{16\,x}{15}$ $X_3\, X_9$

$1 - \dfrac{16\,x}{15}$ $Id_9\, Y_3$ 1 $Id_3\, Id_9$ $1 - \dfrac{16\,x}{15}$ $Y_9\, Z_3$ $1 - \dfrac{16\,x}{15}$ $X_9\, Y_3$

$1 - \dfrac{16\,x}{15}$ $Id_3\, X_9$ $1 - \dfrac{16\,x}{15}$ $X_3\, Y_9$ $1 - \dfrac{16\,x}{15}$ $Id_3\, Z_9$ $1 - \dfrac{16\,x}{15}$ $X_9\, Z_3$

$1 - \dfrac{16\,x}{15}$ $X_3\, Z_9$ $1 - \dfrac{16\,x}{15}$ $Y_3\, Z_9$ $1 - \dfrac{16\,x}{15}$ $Z_3\, Z_9$ $1 - \dfrac{16\,x}{15}$ $Id_3\, Y_9$

DrawPauliTransferMap @ CalcPauliTransferMap @ C$_2$[H$_1$]

$\frac{1}{2}$ $\frac{1}{2}$

$Z_1 Z_2$ $\frac{1}{2}$ $X_1 Z_2$

$\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$ $\frac{2}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$Id_2 Z_1$ $\frac{1}{2}$ $Id_2 X_1$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$X_1 Y_2$ $\frac{1}{-\frac{\sqrt{2}}{\sqrt{2}}}$ $X_2 Y_1$

$\frac{1}{\sqrt{2}\sqrt{2}}$ $\frac{1}{\sqrt{2}\sqrt{2}}$

$Id_1 Y_2$ $\frac{1}{\frac{\sqrt{2}}{\sqrt{2}}}$ $Y_2 Z_1$

$X_1 X_2$ $\frac{1}{\frac{\sqrt{2}}{\sqrt{2}}}$ $Id_1 X_2$

$\frac{1}{\sqrt{2}\sqrt{2}}$ $\frac{1}{\sqrt{2}\sqrt{2}}$

$Y_1 Y_2$ $\frac{1}{-\frac{\sqrt{2}}{\sqrt{2}}}$ $X_2 Z_1$

1 $Id_1 Id_2$

1 $Id_1 Z_2$
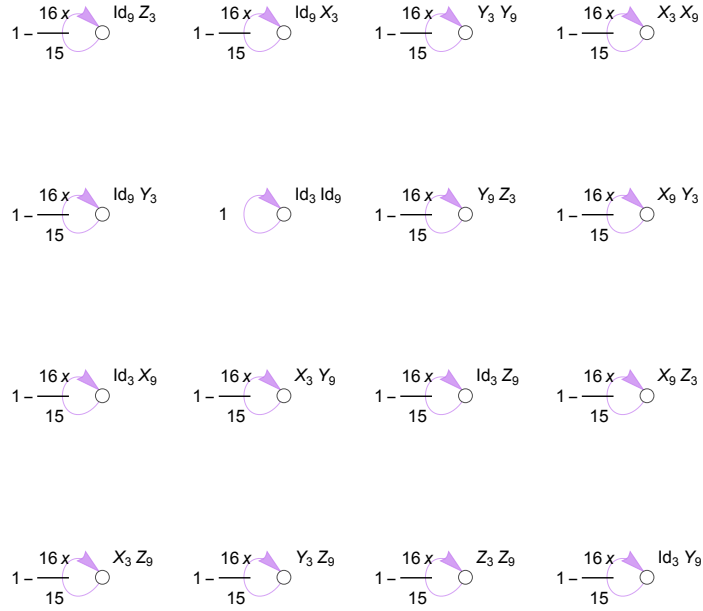
$Y_1 Z_2$ $\frac{1}{1}$ $Id_2 Y_1$
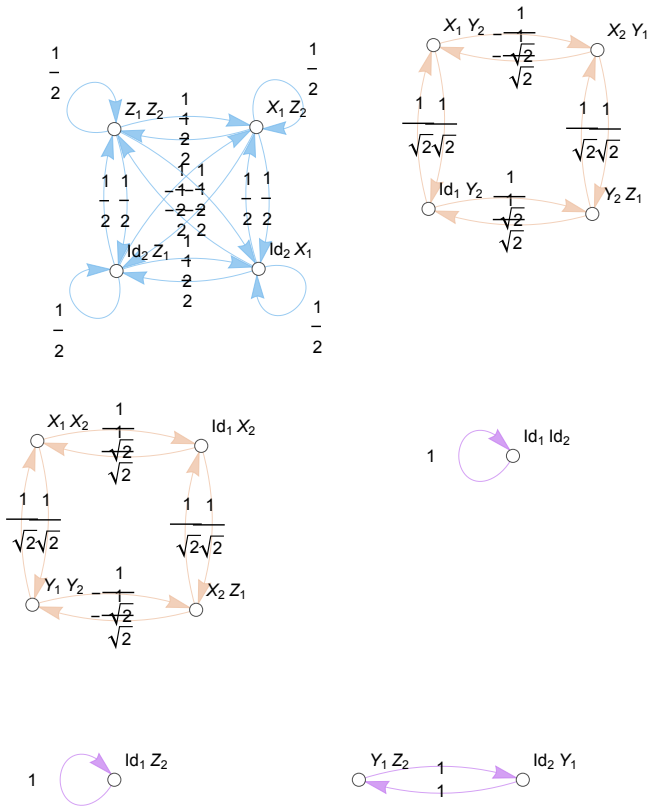
PTM, gates, circuits

DrawPauliTransferMap @ PTM$_0$ @ DiagonalMatrix[{a, b, c, d}]

c Y_0 d Z_0

b X_0 a Id_0
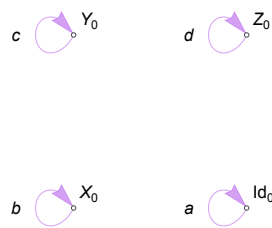
DrawPauliTransferMap @ PTM$_0$ @ Table[c$_{i,j}$, {i, 4}, {j, 4}]

$c_{4,4}$ $c_{3,3}$

Z_0 $c_{3,4}$ Y_0

$c_{4,3}$

$c_{3,2}$ $c_{1,4}$

$c_{4,2}$ $c_{2,4}$ $c_{4,3}$ $c_{2,3}$ $c_{3,1}$ $c_{1,3}$

X_0 $c_{1,2}$ Id_0

$c_{2,1}$

$c_{2,2}$ $c_{1,1}$
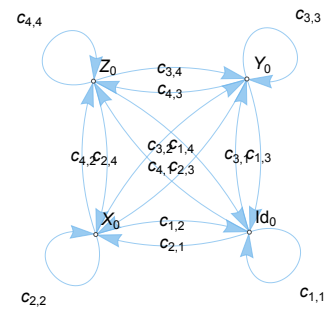
DrawPauliTransferMap @ Rz₀[x]

$\cos(x)$ $\qquad$ Y_0 $\qquad$ $-\sin(x)$ $\qquad$ X_0 $\quad \cos(x)$ $\qquad$ 1 $\qquad$ Z_0

$\sin(x)$

1 $\qquad$ Id_0

DrawPauliTransferMap @ Circuit[H₀ C₀[X₁]]

$X_0 Y_1$ $\qquad$ 1 $\qquad$ $\text{Id}_0 Y_1$ $\quad X_1 Z_0$ $\qquad$ 1 $\qquad$ $X_0 X_1$

-1 $\qquad$ 1 $\qquad$ 1 $\qquad$ 1 $\qquad$ 1 $\qquad$ $\text{Id}_0 X_1$

$Y_0 Z_1$ $\qquad$ 1 $\qquad$ $Y_1 Z_0$ $\quad \text{Id}_1 X_0$ $\qquad$ 1 $\qquad$ $\text{Id}_1 Z_0$

$Z_0 Z_1$ $\qquad$ 1 $\qquad$ $\text{Id}_0 Z_1$

-1 $\qquad$ 1 $\qquad$ 1 $\qquad$ $\text{Id}_0 \text{Id}_1$ $\qquad X_1 Y_0$ $\quad -1$ $\quad \text{Id}_1 Y_0$

$Y_0 Y_1$ $\qquad$ 1 $\qquad$ $X_0 Z_1$ $\qquad \qquad \qquad \qquad -1$
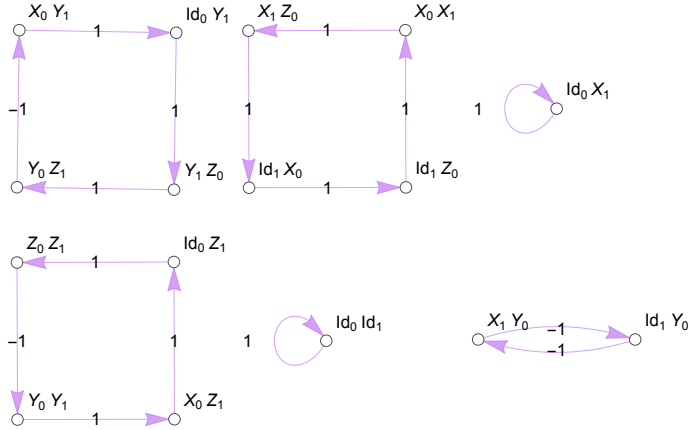
DrawPauliTransferMap @ Circuit[H₁ Damp₁[y] C₀[X₁]]

$X_1 Z_0$

1 $\qquad$ $\text{Id}_0 \text{Id}_1$ $\quad y \quad$ $Z_0 Z_1$ $\sqrt{1-y}$ $\quad 1-y \quad$ $\text{Id}_0 Z_1$ $\quad y \quad$ $\text{Id}_1 Z_0$ $\quad$ 1

$1-y$

$\text{Id}_0 X_1 \sqrt{1-y}$

$Y_0 Y_1$ $\qquad \qquad \qquad X_0 Y_1$

$X_0 Z_1 \sqrt{1-y}$ $\quad y-1 \quad$ $\qquad X_1 Y_0$ $1-y$ $\quad -\sqrt{1-y}$

$-y \qquad X_0 X_1$ $\qquad y \qquad Y_0 Z_1$

$\sqrt{1-y}$ $\quad \text{Id}_1 X_0$ $\quad 1$ $\qquad \qquad$ 1 $\quad \text{Id}_1 Y_0 \sqrt{1-y}$

$Y_1 Z_0 \sqrt{1-y}$ $\quad \text{Id}_0 Y_1$

$-\sqrt{1-y}$
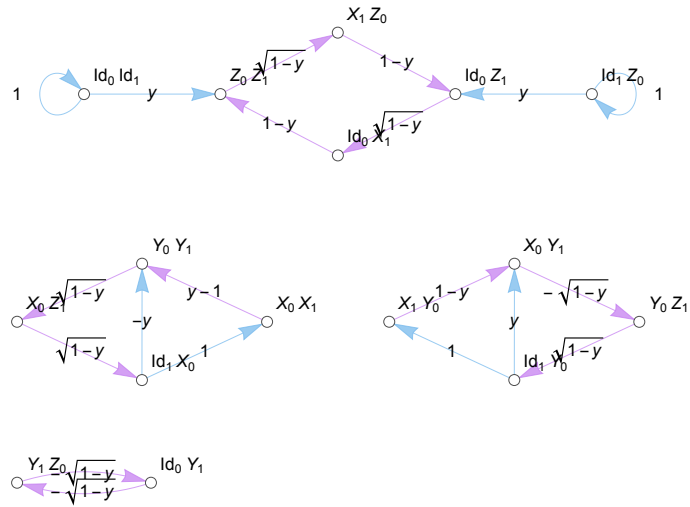
$$\text{DrawPauliTransferMap}\left[U_{0,1} @ \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right]$$
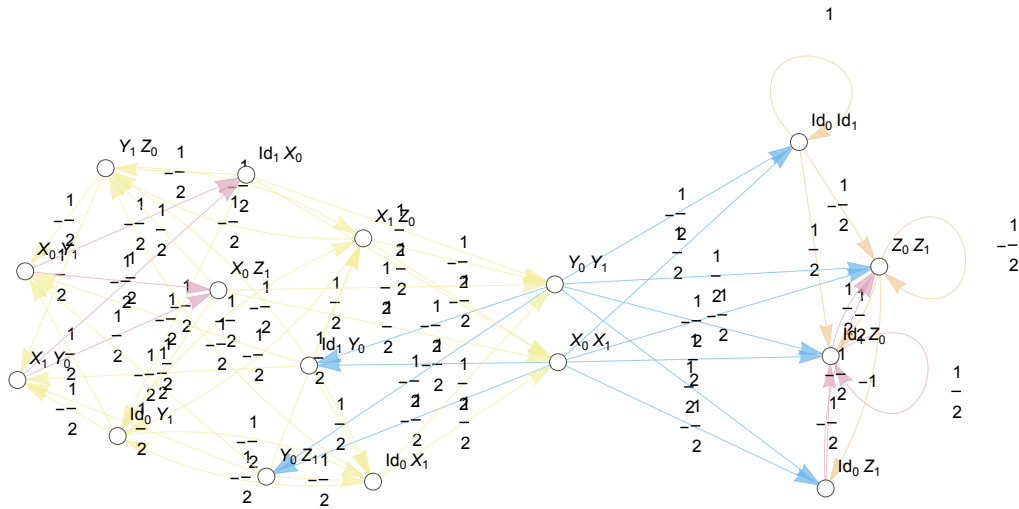
option "PauliStringForm"

```
circ = Circuit[H₀ C₀[X₁]];
```

```
DrawPauliTransferMap[circ, "PauliStringForm" → "String"]
```
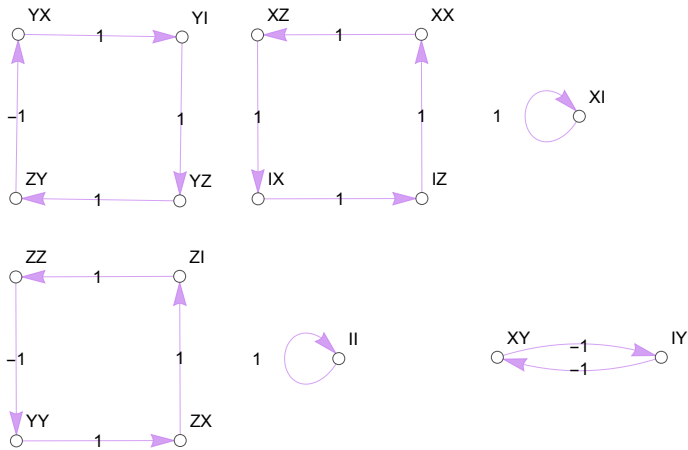
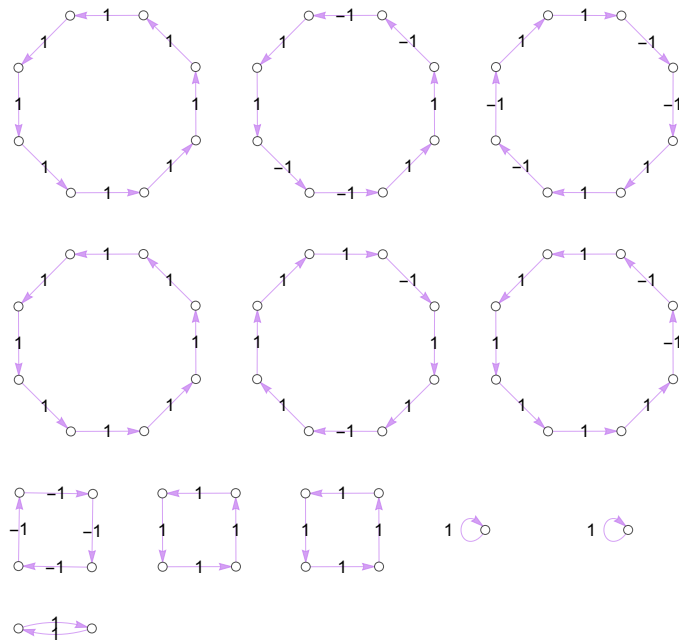**DrawPauliTransferMap[circ, "PauliStringForm" → "Kronecker"]**
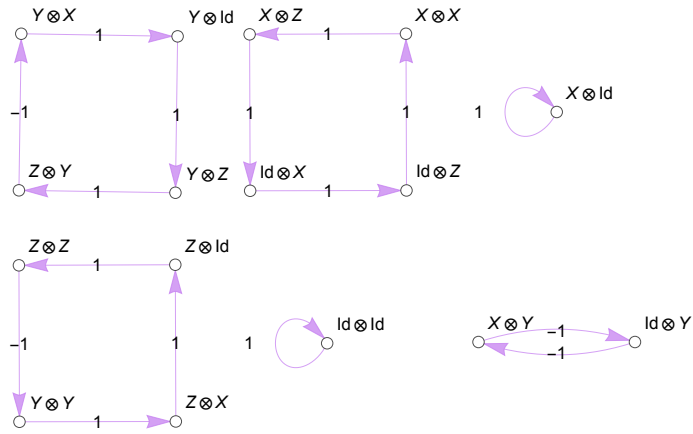
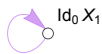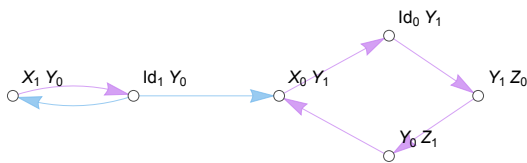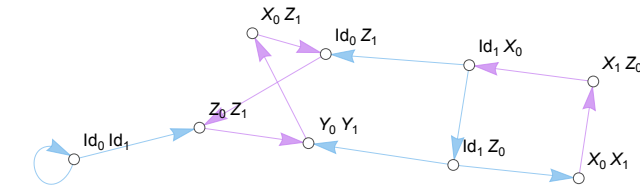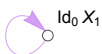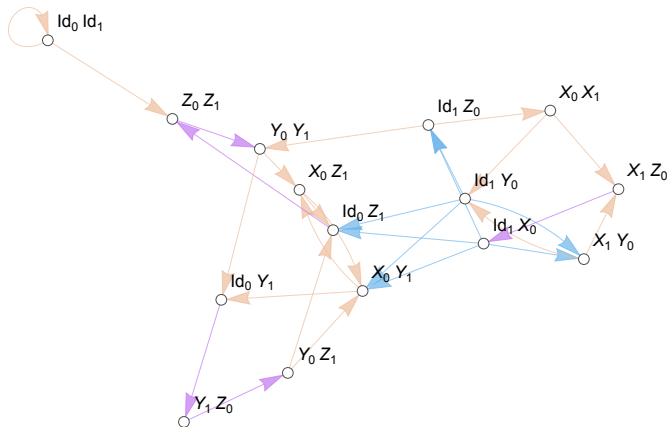**DrawPauliTransferMap[circ, "PauliStringForm" → "Index"]**
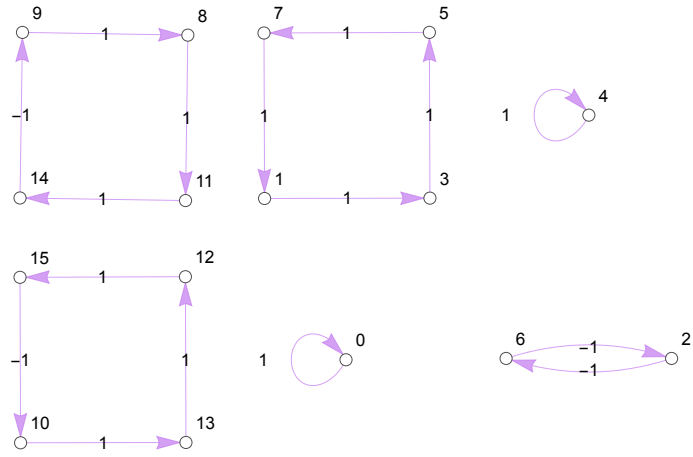
circ = Circuit[H_0 C_0[X_1] $SWAP_{1,2}$];

DrawPauliTransferMap[circ, "PauliStringForm" → "Hidden"]

option "ShowCoefficients"

```
circ = Circuit[H₀ Depol₀,₁[x] Damp₁[y] C₀[X₁]];
DrawPauliTransferMap[circ, "ShowCoefficients" → False]
```



```
circ = Circuit[H₁ Damp₀[x] Depol₀,₁[x] C₀[X₁] C₁[Rx₀[x]] R[x, X₀ Y₁]];
DrawPauliTransferMap[circ,
    "ShowCoefficients" → False,
    "PauliStringForm" → "String"]
```
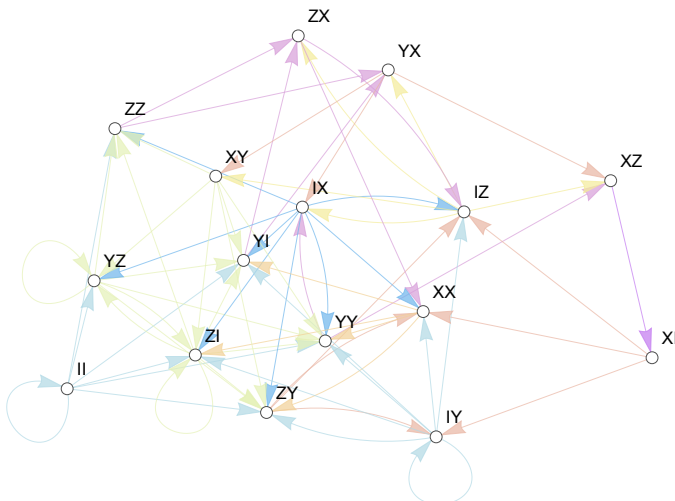
```
DrawPauliTransferMap[C₀,₂[H₁],
    "ShowCoefficients" → False,
    "PauliStringForm" → "Index"]
```



```
                       ⎛ 0 0 a 0 ⎞
                       ⎜ 0 1 0 0 ⎟
DrawPauliTransferMap[U₀,₁ @ ⎜ 1 0 0 1 ⎟ ,
                       ⎝ 0 0 0 0 ⎠

    "ShowCoefficients" → False, "PauliStringForm" → "Hidden"]
```

option "EdgeDegreeStyles"

```
circ = Circuit[H₀ Depol₀,₁[x] Rx₀[a] Damp₁[y] C₀[X₁]];
DrawPauliTransferMap[circ, "ShowCoefficients" → False]
```

```
colors = ColorData["SolarColors"] /@ Range[0, 1, .2]
```
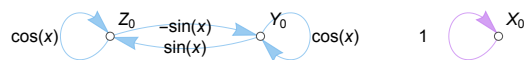
```
DrawPauliTransferMap[circ,
    "ShowCoefficients" → False,
    "EdgeDegreeStyles" → colors]
```

{ ■ , ■ , ■ , ■ , ■ , ■ }
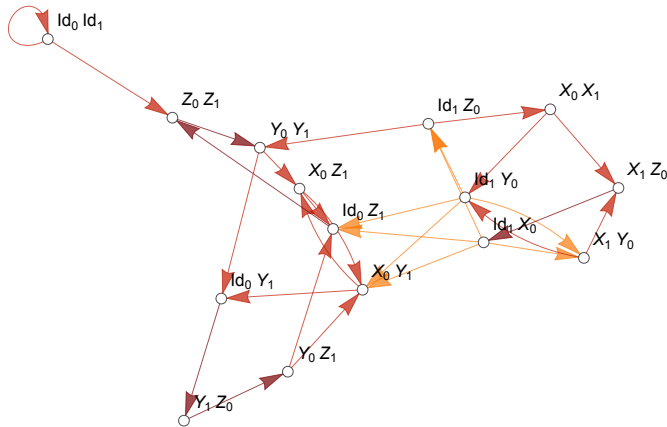
option AssertValidChannels

```
DrawPauliTransferMap[Rx₀[x], AssertValidChannels → True]
```

```
DrawPauliTransferMap[Rx₀[x], AssertValidChannels → False]
```

$$\frac{1}{2}\left(2\cos^2\left(\frac{x}{2}\right)-2\sin^2\left(\frac{x}{2}\right)\ \ \ldots\ \right)$$

$$\frac{1}{2}\left(2\sin^2\left(\frac{x}{2}\right)+2\cos^2\left(\frac{x}{2}\right)\right)$$
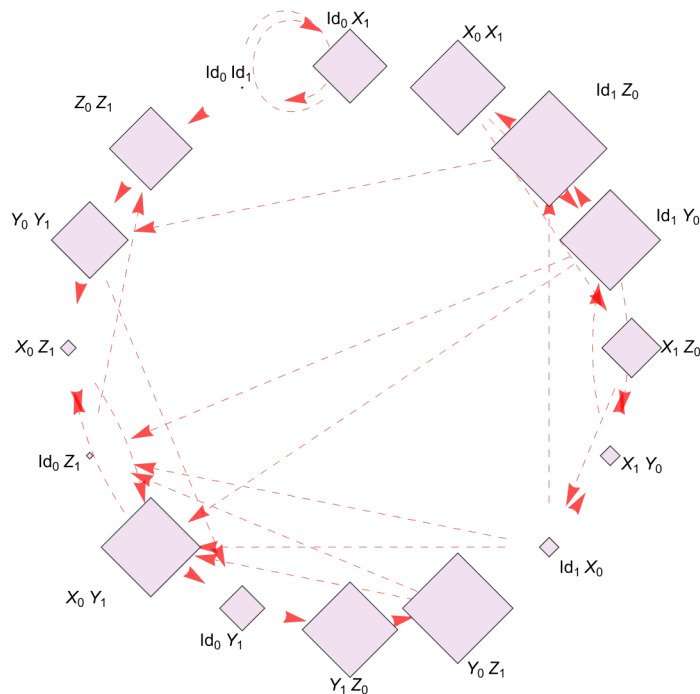
---

options of Graph

```
circ = Circuit[H₀ Depol₀,₁[x] Rx₀[a] Damp₁[y] C₀[X₁]];
DrawPauliTransferMap[circ, "ShowCoefficients" → False,

    EdgeStyle → Directive[Red, Dashed],
    VertexShapeFunction → "Diamond",
    VertexSize :→ RandomReal[],
    VertexStyle → LightPurple,
    GraphLayout → "CircularEmbedding"
]
```
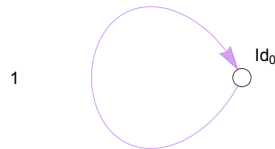
Warnings

`CalcPauliTransferMap[Depol₀[3/4]]`

$\mathrm{PTMap}_0[0 \to \{\{0, 1\}\}, 1 \to \{\}, 2 \to \{\}, 3 \to \{\}]$

`DrawPauliTransferMap[%]`

⬤⬤⬤ DrawPauliTransferMap : Warning: The Pauli transfer map produces no Pauli string from some initial strings. These edges (and null strings) are not being plotted. Hide this warning with Quiet[].

Errors

`DrawPauliTransferMap[CalcPauliTransferMap[X₀], "BadOption" → "bad"]`

⬤⬤⬤ OptionValue : Unknown option BadOption for {DrawPauliTransferMap , Graph}.

`$Failed`

`DrawPauliTransferMap[CalcPauliTransferMap[X₀], "PauliStringForm" → "bad"]`

⬤⬤⬤ DrawPauliTransferMap : Unrecognised value for option "PauliStringForm". See ?DrawPauliTransferMap

`$Failed`

`DrawPauliTransferMap[ PTMap₀,₋₁[x] ]`

⬤⬤⬤ DrawPauliTransferMap : Failed to automatically obtain the PTMap due to the below error:

⬤⬤⬤ CalcPauliTransferMatrix : Circuit contained an unrecognised or unsupported gate: PTMap$_{0,1}$[x]

`$Failed`

DrawPauliTransferMap[ ]

⬤⬤⬤ DrawPauliTransferMap : Invalid arguments. See ?DrawPauliTransferMap

$Failed

DrawPauliTransferMap @ Bad$_0$

⬤⬤⬤ DrawPauliTransferMap : Failed to automatically obtain the PTMap due to the below error:

⬤⬤⬤ CalcPauliTransferMatrix : Circuit contained an unrecognised or unsupported gate: Bad$_0$

$Failed