Recompiling to CliffordAndRz

```
SetDirectory @ NotebookDirectory[];
Import["../Link/QuESTlink.m"];
```

```
testRecomp[gate_, showGates:(True|False):True] := Module[
    {recomp, error},
    recomp = RecompileCircuit[gate, "CliffordAndRz"];
    error = CalcCircuitMatrix[gate] - CalcCircuitMatrix[N@recomp] // Abs // Max;
    error = error // FullSimplify // Chop;
    If[showGates, Echo @ recomp];
    Echo @ DrawCircuit[{{gate}, recomp}];
    Echo[error, "error: "];
    If[error =!= 0, Style["ERRONEOUS DECOMPOSITION!", Red]]]
```

`? QuEST`Gate`*`

> **QuEST`Gate`**
>
> | | | | | | | |
> |---|---|---|---|---|---|---|
> | **Damp** | **G** | **KrausNonTP** | **Ph** | **Rz** | **U** | **Z** |
> | **Deph** | **H** | **M** | **R** | **S** | **UNonNorm** | |
> | **Depol** | **Id** | **Matr** | **Rx** | **SWAP** | **X** | |
> | **Fac** | **Kraus** | **P** | **Ry** | **T** | **Y** | |

Testing doc

`? RecompileCircuit`

> Symbol
>
> RecompileCircuit[circuit, method] returns an equivalent circuit,
>   transpiled to a differnet gate set. The input circuit can contain any unitary
>   gate, with any number of control qubits. Supported methods include:
> • "SingleQubitAndCNOT" decompiles the circuit into canonical single–qubit
>   gates (H, Ph, T, S, X, Y, Z, Rx, Ry, Rz), a global phase G, and two–qubit C[X] gates.
>   This method uses a combination of 23 analytic and numerical decompositions.
> • "CliffordAndRz" decompiles the circuit into Clifford gates
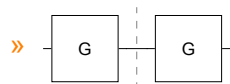>   (H, S, X, Y, Z, CX, CY, CZ, SWAP), a global phase G, and non–Clifford Rz.
> Note that the returned circuits are not necessarily
>   optimal/minimal, and may benefit from a subsequent call to SimplifyCircuit[].
>
> ⌄

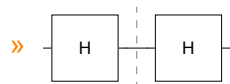# Testing decomp gates

G
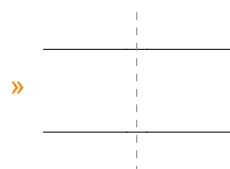
testRecomp @ G[x]

» {G[x]}

»

» error: 0

H

testRecomp @ H$_0$

» {H$_0$}

»

» error: 0

Id

testRecomp @ Id$_{0,1}$

» $\{Id_{0,1}\}$

»

» error: 0

S

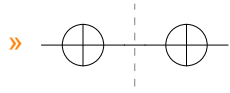**testRecomp @ S$_0$**

» {S$_0$}

»
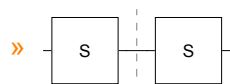
» error: 0
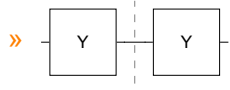
X, Y, Z

testRecomp @ X$_0$

» $\{X_0\}$

» error: 0
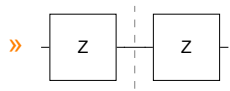
testRecomp @ Y_0

» $\{Y_0\}$

» error: 0

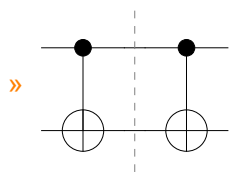**testRecomp @ Z_0**

» $\{Z_0\}$

» error: 0
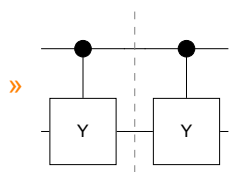
CX, CY, CZ

testRecomp @ $C_1@X_0$
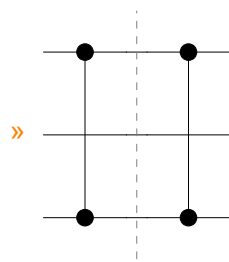
» $\{C_1[X_0]\}$

» error: 0

testRecomp @ $C_1@Y_0$

» $\{C_1[Y_0]\}$

» error: 0

testRecomp @ $C_2@Z_0$

» $\{C_2[Z_0]\}$

»

» error: 0

SWAP

testRecomp @ SWAP$_{0,1}$

» $\{SWAP_{0,1}\}$

»
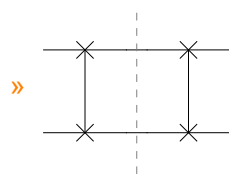
» error: 0

Rz

testRecomp @ Rz$_0$[x]
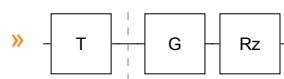
» $\{Rz_0[x]\}$

»
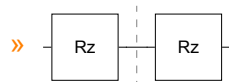
» error: 0

Testing canonical gates

Un-controlled

T

testRecomp @ T$_0$

» $\left\{G\left[\dfrac{\pi}{8}\right],\ Rz_0\left[\dfrac{\pi}{4}\right]\right\}$

»

» error: 0

Rx

testRecomp @ $Rx_0[\theta]$

» $\{H_0, Rz_0[\theta], H_0\}$

»
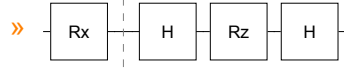
» error: 0

Ry

testRecomp @ $Ry_0[\theta]$

» $\{S_0, H_0, Rz_0[-\theta], H_0, S_0, S_0, S_0\}$

»
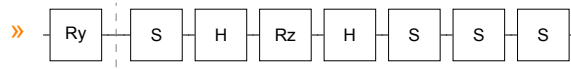
» error: 0

Ph

testRecomp @ $Ph_0[\theta]$
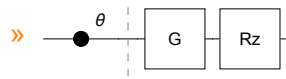
» $\left\{G\left[\dfrac{\theta}{2}\right], Rz_0[\theta]\right\}$

»

» error: 0

testRecomp @ $Ph_{0,1}[x]$

» $\left\{G\left[\dfrac{x}{4}\right], Rz_1\left[\dfrac{x}{2}\right], Rz_0\left[\dfrac{x}{2}\right], C_0[X_1], Rz_1\left[-\dfrac{x}{2}\right], C_0[X_1]\right\}$

»
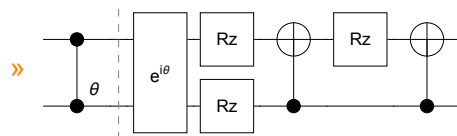
» error: 0

testRecomp @ $Ph_{0,1,2}[x]$

» $\left\{ G\left[\frac{\pi}{4} + \frac{x}{8}\right], Rz_1\left[\frac{x}{4}\right], Rz_2\left[\frac{x}{4}\right], C_2[X_1], Rz_1\left[-\frac{x}{4}\right], C_2[X_1], Rz_0\left[\frac{x}{4}\right], Rz_2\left[\frac{x}{4}\right], C_2[X_0], \right.$

$Rz_0\left[-\frac{x}{4}\right], C_2[X_0], H_1, C_0[X_1], Rz_1\left[-\frac{\pi}{4}\right], C_2[X_1], Rz_1\left[\frac{\pi}{4}\right], C_0[X_1], Rz_1\left[-\frac{\pi}{4}\right],$

$C_2[X_1], Rz_1\left[\frac{\pi}{4}\right], Rz_0\left[\frac{\pi}{4}\right], C_2[X_0], Rz_2\left[\frac{\pi}{4}\right], Rz_0\left[-\frac{\pi}{4}\right], C_2[X_0], H_1, Rz_1\left[-\frac{x}{4}\right],$

$Rz_2\left[-\frac{x}{4}\right], C_2[X_1], Rz_1\left[\frac{x}{4}\right], C_2[X_1], H_1, C_0[X_1], Rz_1\left[-\frac{\pi}{4}\right], C_2[X_1], Rz_1\left[\frac{\pi}{4}\right], C_0[X_1],$

$\left. Rz_1\left[-\frac{\pi}{4}\right], C_2[X_1], Rz_1\left[\frac{\pi}{4}\right], Rz_0\left[\frac{\pi}{4}\right], C_2[X_0], Rz_2\left[\frac{\pi}{4}\right], Rz_0\left[-\frac{\pi}{4}\right], C_2[X_0], H_1 \right\}$

»
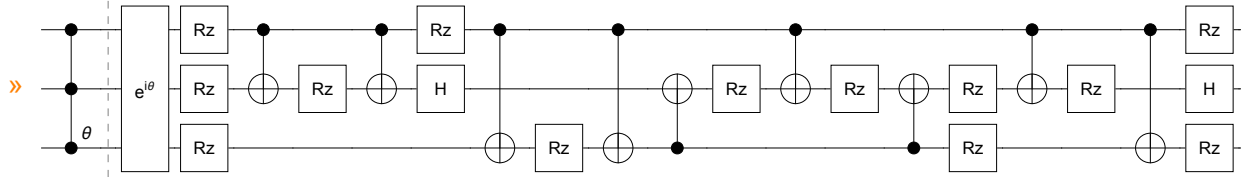
» error: 0

R

testRecomp @ R[x, X$_0$]

testRecomp @ R[x, Y$_0$]

» $\{H_0, Rz_0[x], H_0\}$

»
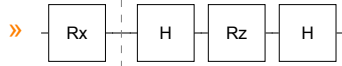
» error: 0

» $\{S_0, H_0, Rz_0[-x], H_0, S_0, S_0, S_0\}$

»
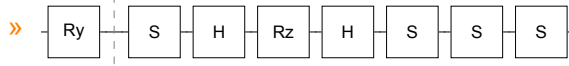
» error: 0

testRecomp @ R[x, X$_0$ Y$_1$]

» $\{H_0, C_0[X_1], S_1, H_1, Rz_1[-x], H_1, S_1, S_1, S_1, C_0[X_1], H_0\}$

»
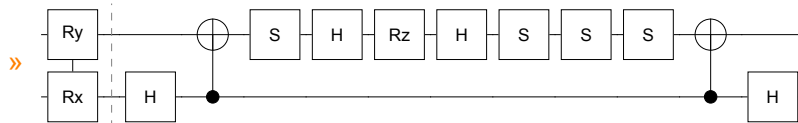
» error: 0

testRecomp @ R[x, Z$_0$ Y$_1$]

» $\{C_0[X_1], S_1, H_1, Rz_1[-x], H_1, S_1, S_1, S_1, C_0[X_1]\}$

»
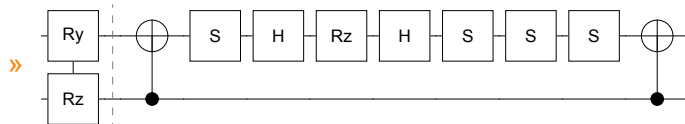
» error: 0

testRecomp @ R[x, Z$_0$ X$_1$]

» $\{H_1, C_1[X_0], Rz_0[x], C_1[X_0], H_1\}$
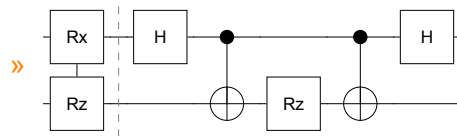
» error: 0

testRecomp @ R[x, $X_0 X_1 Y_2 Z_3 X_4$]

» $\{C_3[X_2], H_0, C_0[X_2], H_1, C_1[X_2], H_4, C_4[X_2], S_2, H_2,$
   $Rz_2[-x], H_2, S_2, S_2, S_2, C_4[X_2], H_4, C_1[X_2], H_1, C_0[X_2], H_0, C_3[X_2]\}$
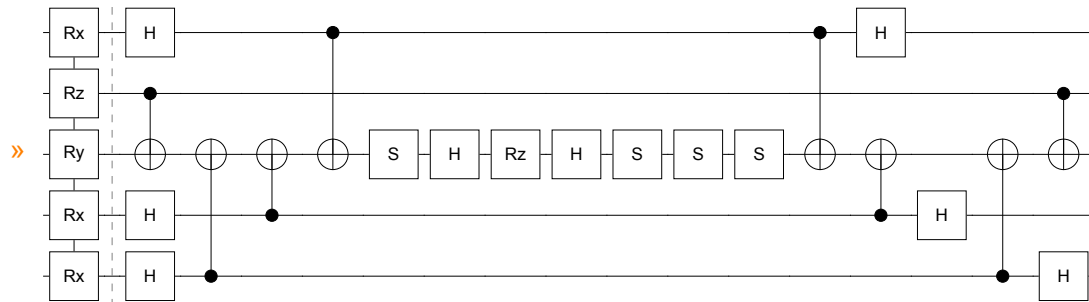
» error: 0

Rz^(n)

testRecomp @ $Rz_{0,1,2}[x]$

» $\{C_1[X_0], C_2[X_0], Rz_0[x], C_2[X_0], C_1[X_0]\}$

» error: 0

Singly-controlled

C[G]

```
(* cannot draw ill-formed input *)

RecompileCircuit[C₀@G[x], "CliffordAndRz"]
```

$\left\{G\left[\dfrac{x}{2}\right], Rz_0[x]\right\}$

C[H]

testRecomp @ $C_0[H_1]$

» $\left\{ G\left[-\dfrac{\pi}{4}\right],\ Rz_1\left[-\dfrac{\pi}{2}\right],\ H_1,\ Rz_1\left[-\dfrac{\pi}{4}\right],\ C_0[X_1],\ Rz_1\left[\dfrac{\pi}{4}\right],\ H_1,\ S_1 \right\}$

» error: 0

C[Ph]

testRecomp @ $C_0[Ph_1[x]]$

» $\left\{ G\left[\dfrac{x}{4}\right],\ Rz_1\left[\dfrac{x}{2}\right],\ Rz_0\left[\dfrac{x}{2}\right],\ C_0[X_1],\ Rz_1\left[-\dfrac{x}{2}\right],\ C_0[X_1] \right\}$
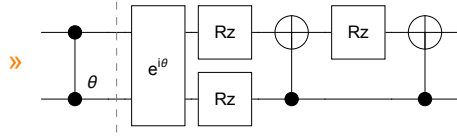
» error: 0

C[R]

testRecomp @ C_1@ $R[x,\ X_0]$
testRecomp @ C_1@ $R[x,\ Y_0]$
testRecomp @ C_1@ $R[x,\ Z_0]$
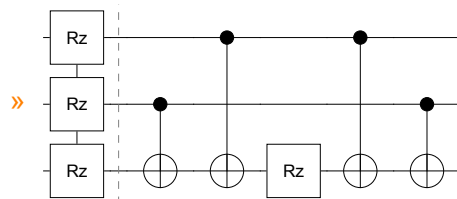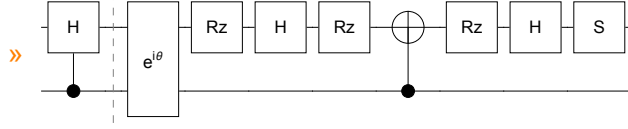
» $\left\{ Rz_0\left[\dfrac{\pi}{2}\right],\ S_0,\ H_0,\ Rz_0\left[-\dfrac{x}{2}\right],\ H_0,\ S_0,\ S_0,\ S_0, \right.$

$\left. C_1[X_0],\ S_0,\ H_0,\ Rz_0\left[\dfrac{x}{2}\right],\ H_0,\ S_0,\ S_0,\ S_0,\ C_1[X_0],\ Rz_0\left[-\dfrac{\pi}{2}\right] \right\}$
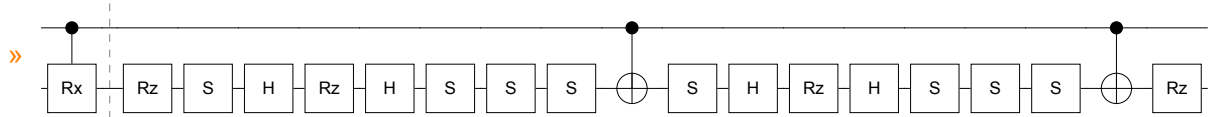
» error: 0

» $\left\{ S_0,\ H_0,\ Rz_0\left[-\dfrac{x}{2}\right],\ H_0,\ S_0,\ S_0,\ S_0,\ C_1[X_0],\ S_0,\ H_0,\ Rz_0\left[\dfrac{x}{2}\right],\ H_0,\ S_0,\ S_0,\ S_0,\ C_1[X_0] \right\}$
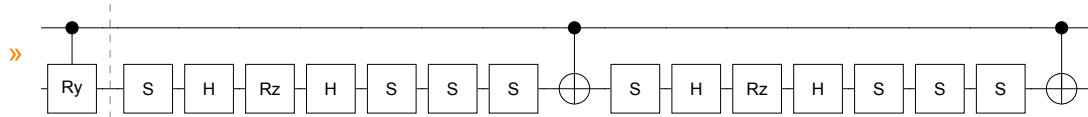
» error: 0

» $\left\{ Rz_0\left[\dfrac{x}{2}\right],\ C_1[X_0],\ Rz_0\left[-\dfrac{x}{2}\right],\ C_1[X_0] \right\}$
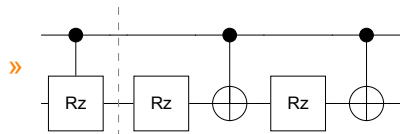
» error: 0

testRecomp @ C_2@$R[x,\ X_0\ Y_1\ X_4]$

» $\left\{H_0, C_0[X_1], H_4, C_4[X_1], S_1, H_1, Rz_1\left[-\dfrac{x}{2}\right], H_1, S_1, S_1, S_1,\right.$

$\left. C_2[X_1], S_1, H_1, Rz_1\left[\dfrac{x}{2}\right], H_1, S_1, S_1, S_1, C_2[X_1], C_4[X_1], H_4, C_0[X_1], H_0\right\}$

»
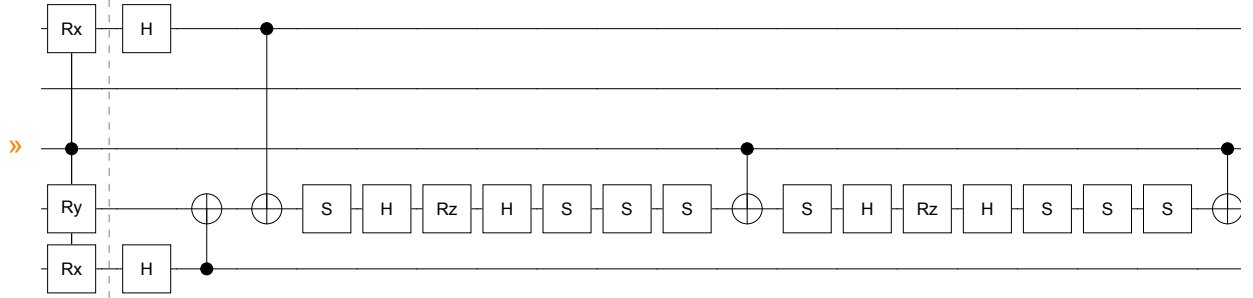
» error: 0

C[Rx]

testRecomp @ C_1@Rx_0[x]

» $\left\{Rz_0\left[\dfrac{\pi}{2}\right], S_0, H_0, Rz_0\left[-\dfrac{x}{2}\right], H_0, S_0, S_0, S_0,\right.$

$\left. C_1[X_0], S_0, H_0, Rz_0\left[\dfrac{x}{2}\right], H_0, S_0, S_0, S_0, C_1[X_0], Rz_0\left[-\dfrac{\pi}{2}\right]\right\}$

»
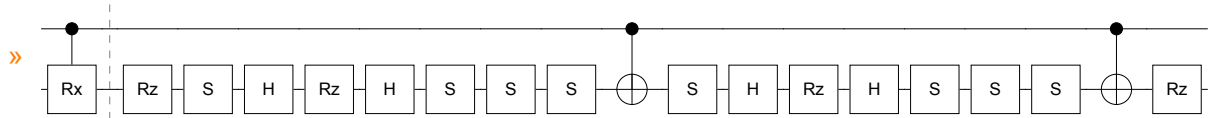
» error: 0

C[Ry], C[Rz]

testRecomp @ C_1@Ry_0[x]

» $\left\{S_0, H_0, Rz_0\left[-\dfrac{x}{2}\right], H_0, S_0, S_0, S_0, C_1[X_0], S_0, H_0, Rz_0\left[\dfrac{x}{2}\right], H_0, S_0, S_0, S_0, C_1[X_0]\right\}$

»
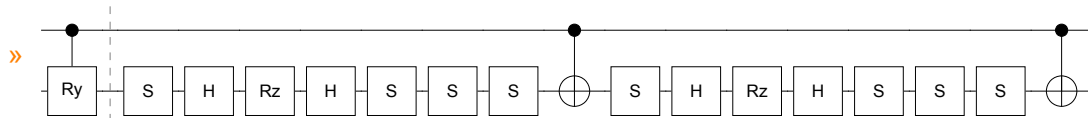
» error: 0

testRecomp @ C_1@Rz_0[x]

» $\left\{Rz_0\left[\dfrac{x}{2}\right], C_1[X_0], Rz_0\left[-\dfrac{x}{2}\right], C_1[X_0]\right\}$

»

» error: 0

C[S], C[T]

testRecomp @ $C_1 @ S_0$

» $\left\{ G\left[\frac{\pi}{8}\right], Rz_0\left[\frac{\pi}{4}\right], Rz_1\left[\frac{\pi}{4}\right], C_1[X_0], Rz_0\left[-\frac{\pi}{4}\right], C_1[X_0] \right\}$

»

» error: 0

testRecomp @ $C_1 @ T_0$

» $\left\{ G\left[\frac{\pi}{16}\right], Rz_0\left[\frac{\pi}{8}\right], Rz_1\left[\frac{\pi}{8}\right], C_1[X_0], Rz_0\left[-\frac{\pi}{8}\right], C_1[X_0] \right\}$

»

» error: 0

C[SWAP]

testRecomp @ $C_1 @ SWAP_{0,2}$

» $\left\{ G\left[\frac{\pi}{8}\right], C_0[X_2], H_0, C_1[X_2], Rz_0\left[\frac{\pi}{4}\right], Rz_2\left[-\frac{\pi}{4}\right], Rz_1\left[\frac{\pi}{4}\right], C_0[X_2], C_1[X_0], \right.$

$\left. Rz_2\left[\frac{\pi}{4}\right], C_1[X_2], Rz_0\left[-\frac{\pi}{4}\right], Rz_2\left[-\frac{\pi}{4}\right], C_1[X_0], C_0[X_2], H_0, Rz_2\left[\frac{\pi}{4}\right], C_0[X_2] \right\}$

»

» error: 0

Multi-controlled

C*[G]

(* cannot draw ill-formed input *)

DrawCircuit @ RecompileCircuit$\left[C_{0,1,2} @ G[x], \text{"CliffordAndRz"}\right]$
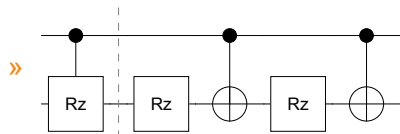
C*[H]

testRecomp$\left[\text{C}_{0,2}[\text{H}_1]\text{, False}\right]$
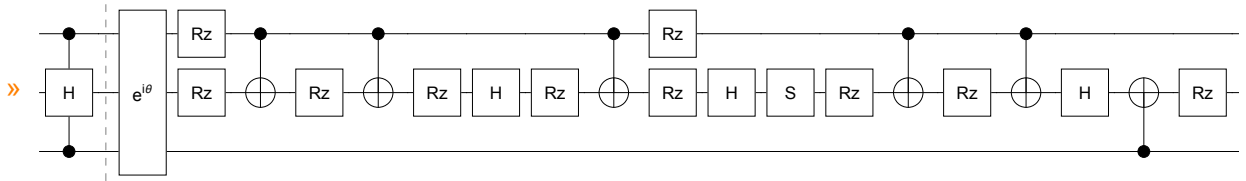
» error: 0

testRecomp$\left[\text{C}_{0,1,3}[\text{H}_2]\text{, False}\right]$
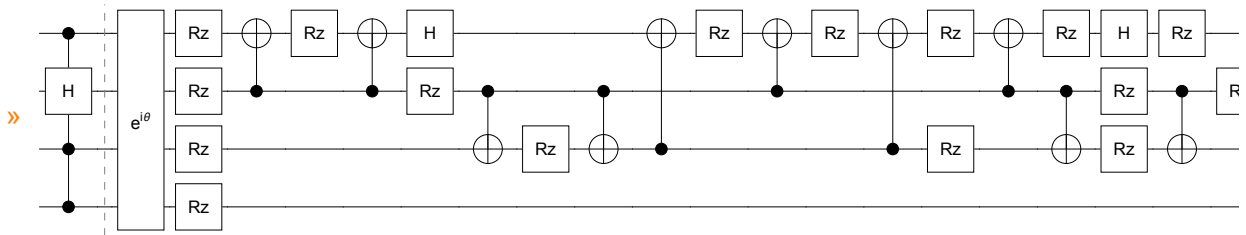
» error: 0

C*[Ph]

testRecomp$\left[\text{C}_{0,2}\left[\text{Ph}_{1,3}[.1]\right]\text{, False}\right]$
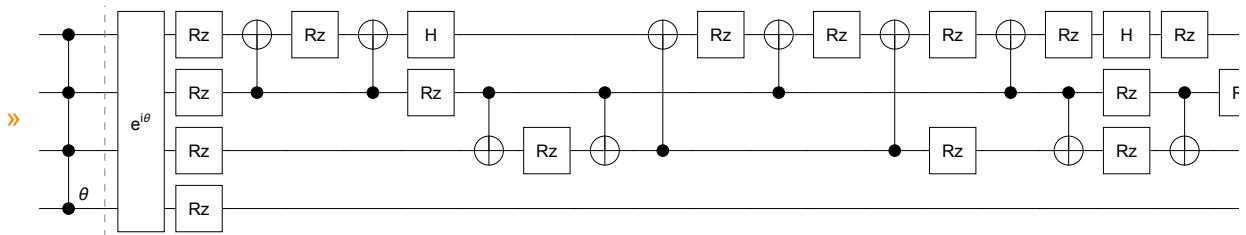
» error: 0

C*[R]

testRecomp @ $\text{C}_{1,2}$ @ R[-.1, X_0]

testRecomp @ $\text{C}_{1,2}$ @ R[x, Y_0]

testRecomp @ $\text{C}_{1,2}$ @ R[x, Z_0]

» $\left\{ Rz_0\left[\frac{\pi}{2}\right], S_0, H_0, Rz_0[0.025], H_0, S_0, S_0, S_0, C_1[X_0], S_0, H_0, Rz_0[-0.025], H_0, S_0, S_0, S_0,\right.$

$C_1[X_0], Rz_0\left[-\frac{\pi}{2}\right], C_2[X_1], Rz_0\left[\frac{\pi}{2}\right], S_0, H_0, Rz_0[-0.025], H_0, S_0, S_0, S_0, C_1[X_0], S_0,$

$H_0, Rz_0[0.025], H_0, S_0, S_0, S_0, C_1[X_0], Rz_0\left[-\frac{\pi}{2}\right], C_2[X_1], Rz_0\left[\frac{\pi}{2}\right], S_0, H_0, Rz_0[0.025],$

$\left. H_0, S_0, S_0, S_0, C_2[X_0], S_0, H_0, Rz_0[-0.025], H_0, S_0, S_0, S_0, C_2[X_0], Rz_0\left[-\frac{\pi}{2}\right]\right\}$

»

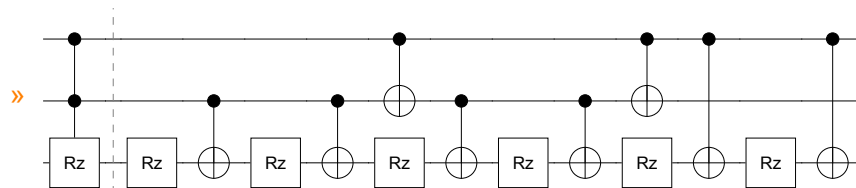| Rx | Rz | S | H | Rz | H | S | S | S | ⊕ | S | H | Rz | H | S | S | S | ⊕ | Rz | Rz |
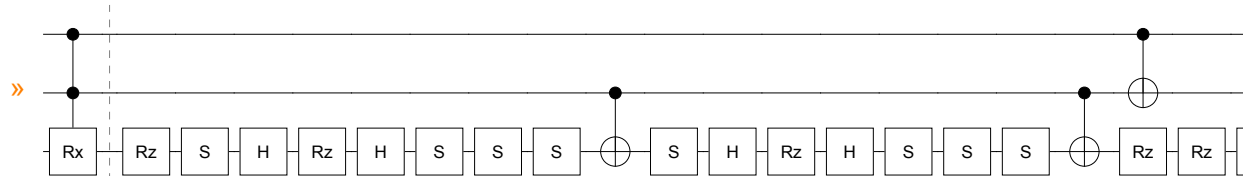
» error: 0

» $\left\{ S_0, H_0, Rz_0\left[-\frac{x}{4}\right], H_0, S_0, S_0, S_0, C_1[X_0], S_0, H_0, Rz_0\left[\frac{x}{4}\right], H_0, S_0, S_0, S_0, C_1[X_0], C_2[X_1], S_0,\right.$

$H_0, Rz_0\left[\frac{x}{4}\right], H_0, S_0, S_0, S_0, C_1[X_0], S_0, H_0, Rz_0\left[-\frac{x}{4}\right], H_0, S_0, S_0, S_0, C_1[X_0], C_2[X_1],$

$\left. S_0, H_0, Rz_0\left[-\frac{x}{4}\right], H_0, S_0, S_0, S_0, C_2[X_0], S_0, H_0, Rz_0\left[\frac{x}{4}\right], H_0, S_0, S_0, S_0, C_2[X_0]\right\}$

»

| Ry | S | H | Rz | H | S | S | S | ⊕ | S | H | Rz | H | S | S | S | ⊕ | S | H | Rz |
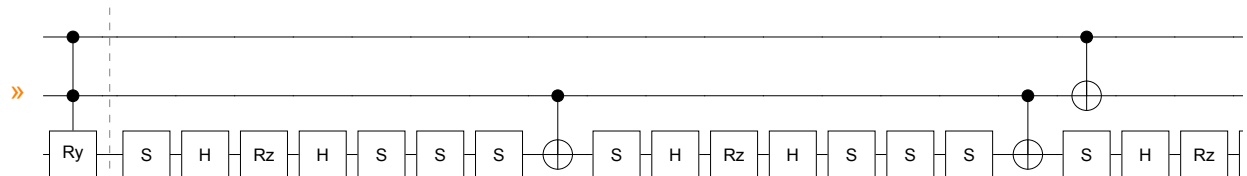
» error: 0

» $\left\{ Rz_0\left[\frac{x}{4}\right], C_1[X_0], Rz_0\left[-\frac{x}{4}\right], C_1[X_0], C_2[X_1], Rz_0\left[-\frac{x}{4}\right],\right.$

$\left. C_1[X_0], Rz_0\left[\frac{x}{4}\right], C_1[X_0], C_2[X_1], Rz_0\left[\frac{x}{4}\right], C_2[X_0], Rz_0\left[-\frac{x}{4}\right], C_2[X_0]\right\}$

»

| Rz | Rz | ⊕ | Rz | ⊕ | Rz | ⊕ | Rz | ⊕ | Rz | ⊕ | Rz | ⊕ |

» error: 0

testRecomp @ $C_{2,3}$@R[.4, $X_0 Y_1$]

» $\{H_0, C_0[X_1], S_1, H_1, Rz_1[-0.1], H_1, S_1, S_1, S_1, C_2[X_1], S_1, H_1, Rz_1[0.1],$
  $H_1, S_1, S_1, S_1, C_2[X_1], C_3[X_2], S_1, H_1, Rz_1[0.1], H_1, S_1, S_1, S_1, C_2[X_1],$
  $S_1, H_1, Rz_1[-0.1], H_1, S_1, S_1, S_1, C_2[X_1], C_3[X_2], S_1, H_1, Rz_1[-0.1], H_1,$
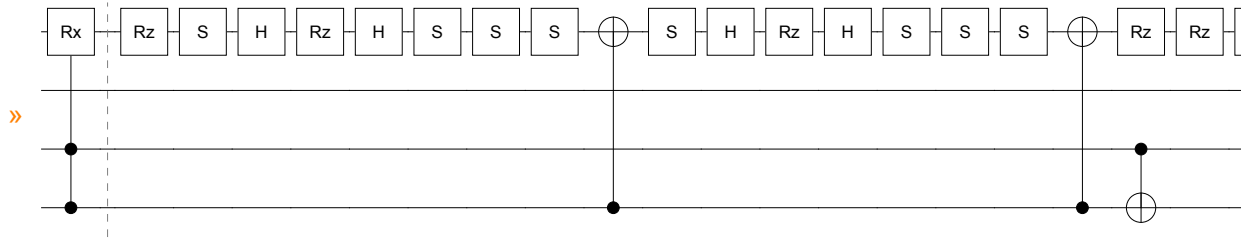 $S_1, S_1, S_1, C_3[X_1], S_1, H_1, Rz_1[0.1], H_1, S_1, S_1, S_1, C_3[X_1], C_0[X_1], H_0\}$
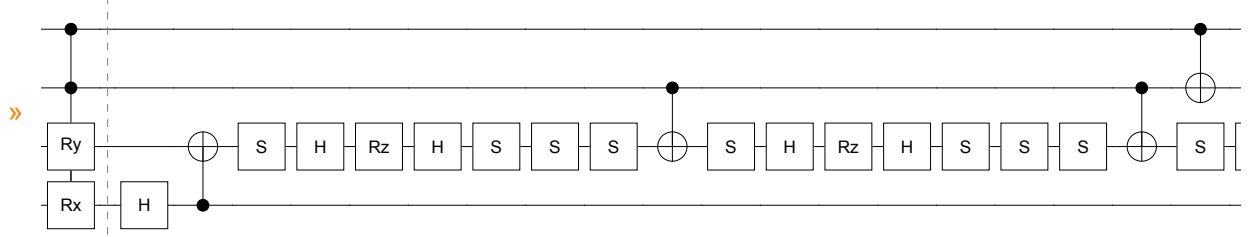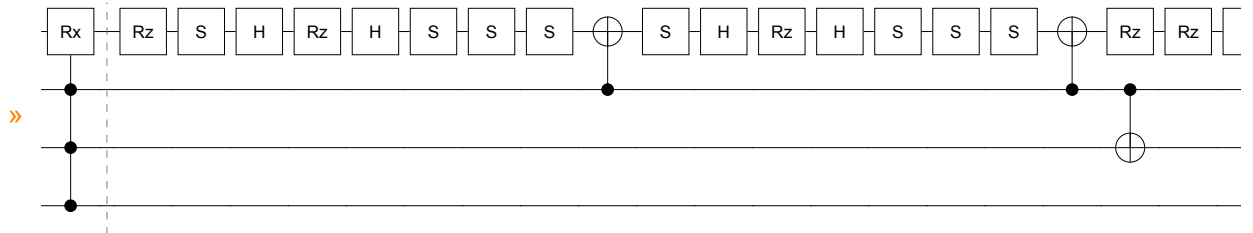

» error: 0

C*[Rx]

testRecomp @ $C_{0,1}[Rx_3[.1]]$

» $\left\{ Rz_3\left[\frac{\pi}{2}\right], S_3, H_3, Rz_3[-0.025], H_3, S_3, S_3, S_3, C_0[X_3], S_3, H_3, Rz_3[0.025], H_3, S_3, S_3, S_3, \right.$
  $C_0[X_3], Rz_3\left[-\frac{\pi}{2}\right], C_1[X_0], Rz_3\left[\frac{\pi}{2}\right], S_3, H_3, Rz_3[0.025], H_3, S_3, S_3, S_3, C_0[X_3], S_3, H_3,$
  $Rz_3[-0.025], H_3, S_3, S_3, S_3, C_0[X_3], Rz_3\left[-\frac{\pi}{2}\right], C_1[X_0], Rz_3\left[\frac{\pi}{2}\right], S_3, H_3, Rz_3[-0.025],$
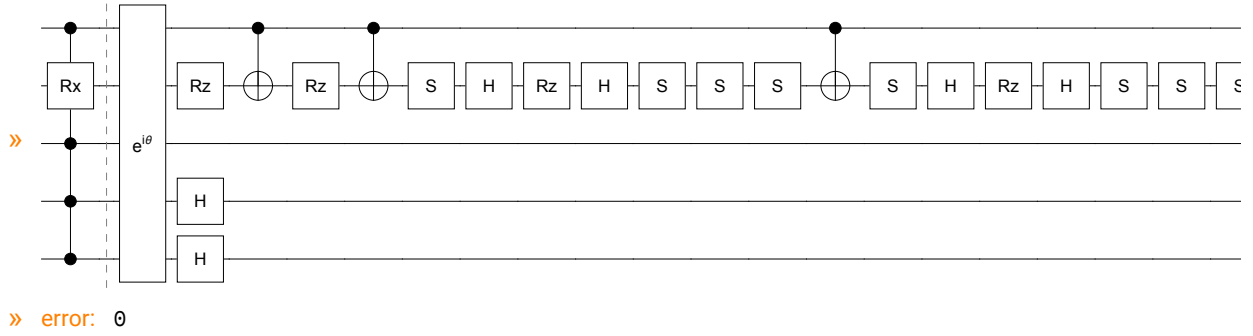 $\left. H_3, S_3, S_3, S_3, C_1[X_3], S_3, H_3, Rz_3[0.025], H_3, S_3, S_3, S_3, C_1[X_3], Rz_3\left[-\frac{\pi}{2}\right]\right\}$

» error: 0

testRecomp$\left[C_{0,1,2}[Rx_3[.1]], False\right]$

» error: 0

testRecomp$\left[C_{0,1,2,4}[Rx_3[.1]], False\right]$

» error: 0

C*[Ry]

testRecomp @ $C_{0,1}[Ry_3[-.3]]$

» $\{S_3, H_3, Rz_3[0.075], H_3, S_3, S_3, S_3, C_0[X_3], S_3, H_3, Rz_3[-0.075],$
$H_3, S_3, S_3, S_3, C_0[X_3], C_1[X_0], S_3, H_3, Rz_3[-0.075], H_3, S_3, S_3, S_3, C_0[X_3],$
$S_3, H_3, Rz_3[0.075], H_3, S_3, S_3, S_3, C_0[X_3], C_1[X_0], S_3, H_3, Rz_3[0.075],$
$H_3, S_3, S_3, S_3, C_1[X_3], S_3, H_3, Rz_3[-0.075], H_3, S_3, S_3, S_3, C_1[X_3]\}$

» error: 0

testRecomp @ $C_{0,1,2}[Ry_3[.1]]$

» $\{S_3, H_3, Rz_3[-0.0125], H_3, S_3, S_3, S_3, C_2[X_3], S_3, H_3, Rz_3[0.0125], H_3, S_3, S_3, S_3, C_2[X_3],$
$C_2[X_1], S_3, H_3, Rz_3[0.0125], H_3, S_3, S_3, S_3, C_1[X_3], S_3, H_3, Rz_3[-0.0125], H_3, S_3, S_3,$
$S_3, C_1[X_3], C_2[X_1], S_3, H_3, Rz_3[-0.0125], H_3, S_3, S_3, S_3, C_1[X_3], S_3, H_3, Rz_3[0.0125],$
$H_3, S_3, S_3, S_3, C_1[X_3], C_1[X_0], S_3, H_3, Rz_3[0.0125], H_3, S_3, S_3, S_3, C_0[X_3], S_3, H_3,$
$Rz_3[-0.0125], H_3, S_3, S_3, S_3, C_0[X_3], C_2[X_0], S_3, H_3, Rz_3[-0.0125], H_3, S_3, S_3, S_3,$
$C_0[X_3], S_3, H_3, Rz_3[0.0125], H_3, S_3, S_3, S_3, C_0[X_3], C_1[X_0], S_3, H_3, Rz_3[0.0125],$
$H_3, S_3, S_3, S_3, C_0[X_3], S_3, H_3, Rz_3[-0.0125], H_3, S_3, S_3, S_3, C_0[X_3], C_2[X_0], S_3, H_3,$
$Rz_3[-0.0125], H_3, S_3, S_3, S_3, C_0[X_3], S_3, H_3, Rz_3[0.0125], H_3, S_3, S_3, S_3, C_0[X_3]\}$

» error: 0

testRecomp$[C_{0,1,2,4}[Ry_3[.1]], False]$

» error: 0

C*[Rz]
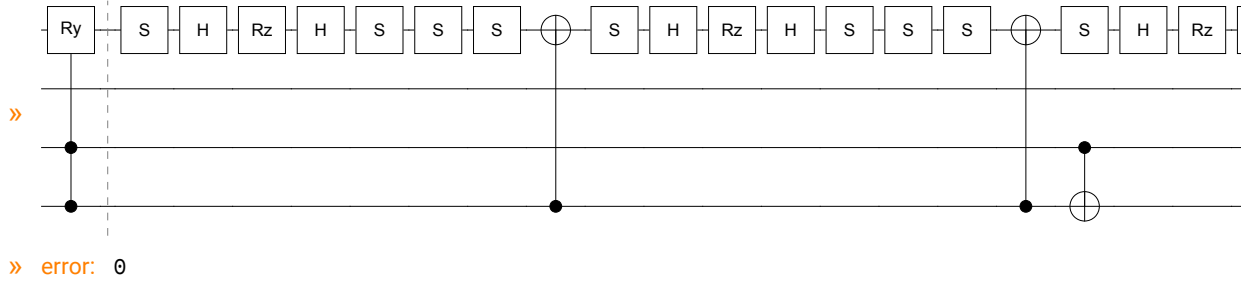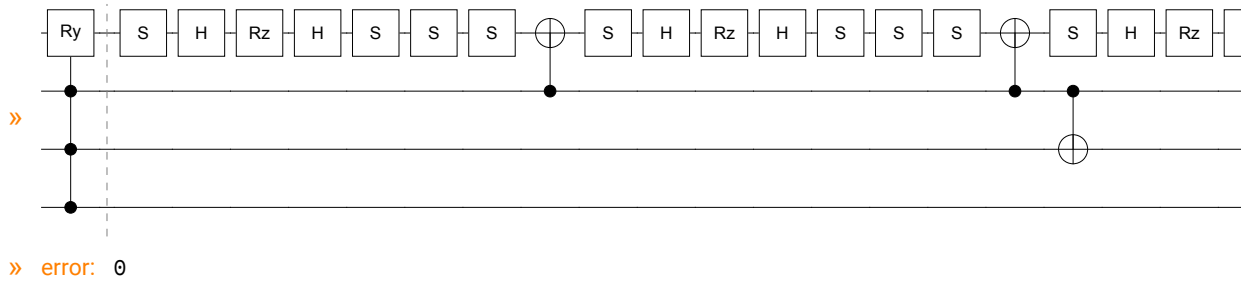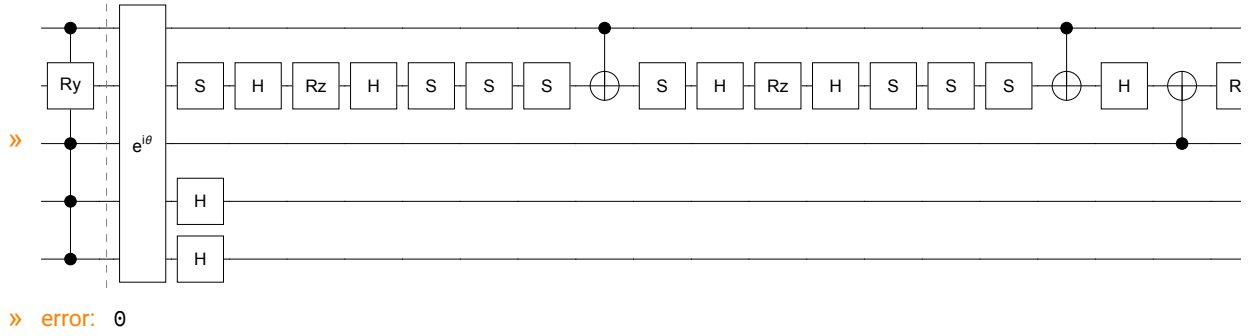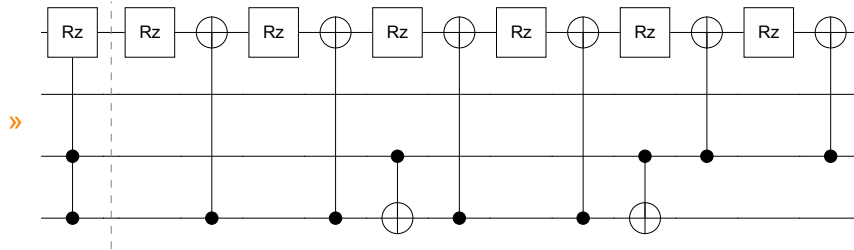
testRecomp @ $C_{0,1}[Rz_3[x]]$

» $\left\{ Rz_3\left[\frac{x}{4}\right], C_0[X_3], Rz_3\left[-\frac{x}{4}\right], C_0[X_3], C_1[X_0], Rz_3\left[-\frac{x}{4}\right], \right.$

$\left. C_0[X_3], Rz_3\left[\frac{x}{4}\right], C_0[X_3], C_1[X_0], Rz_3\left[\frac{x}{4}\right], C_1[X_3], Rz_3\left[-\frac{x}{4}\right], C_1[X_3] \right\}$

» error: 0

testRecomp @ $C_{0,1,2}[Rz_3[.1]]$

» $\{ Rz_3[0.0125], C_2[X_3], Rz_3[-0.0125], C_2[X_3], C_2[X_1], Rz_3[-0.0125],$
$C_1[X_3], Rz_3[0.0125], C_1[X_3], C_2[X_1], Rz_3[0.0125], C_1[X_3], Rz_3[-0.0125],$
$C_1[X_3], C_1[X_0], Rz_3[-0.0125], C_0[X_3], Rz_3[0.0125], C_0[X_3], C_2[X_0],$
$Rz_3[0.0125], C_0[X_3], Rz_3[-0.0125], C_0[X_3], C_1[X_0], Rz_3[-0.0125], C_0[X_3],$
$Rz_3[0.0125], C_0[X_3], C_2[X_0], Rz_3[0.0125], C_0[X_3], Rz_3[-0.0125], C_0[X_3] \}$
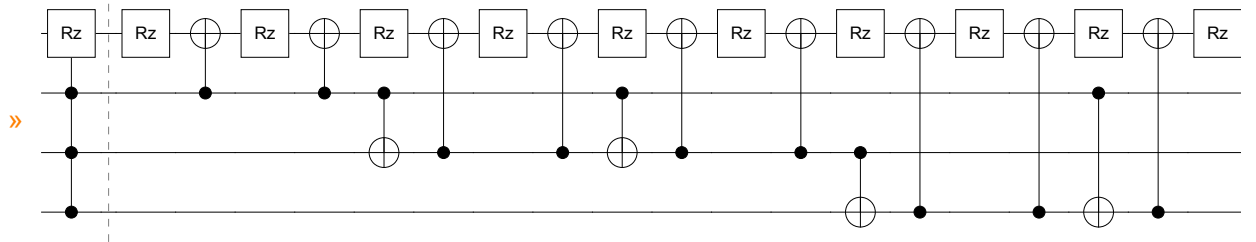
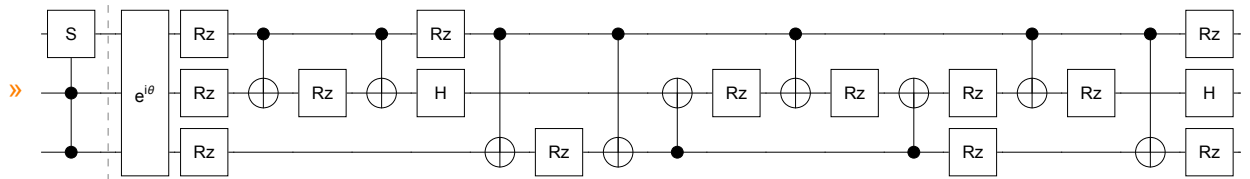» error: 0

testRecomp$\left[C_{0,1,2,4}[Rz_3[.1]], False \right]$
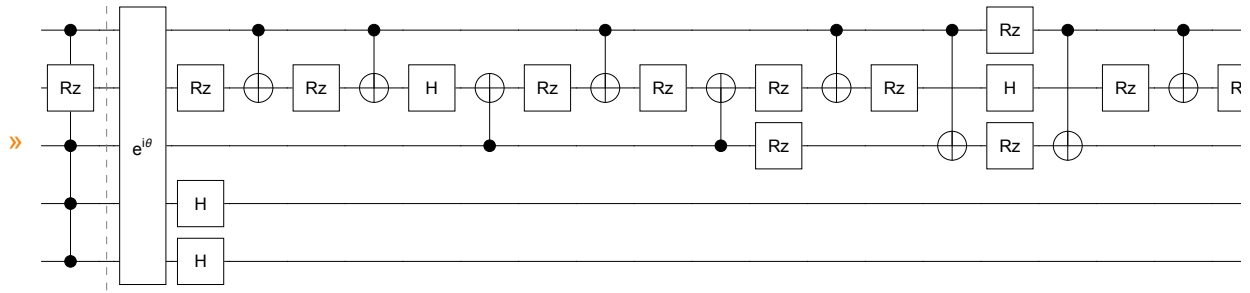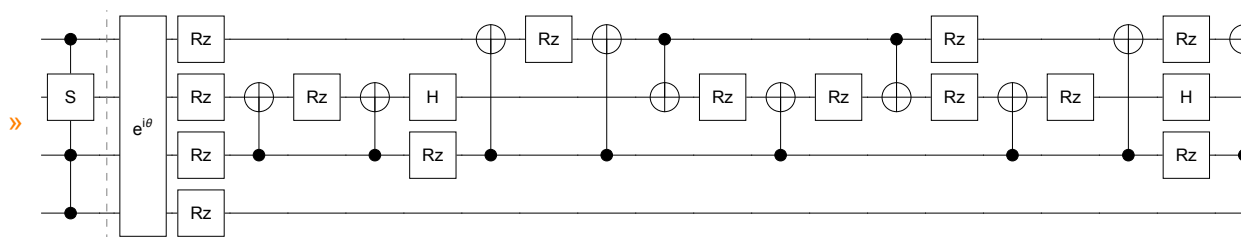
» error: 0

C*[S]

testRecomp @ $C_{0,1}[S_2]$

» $\left\{ G\left[\dfrac{5\pi}{16}\right], Rz_1\left[\dfrac{\pi}{8}\right], Rz_2\left[\dfrac{\pi}{8}\right], C_2[X_1], Rz_1\left[-\dfrac{\pi}{8}\right], C_2[X_1], Rz_0\left[\dfrac{\pi}{8}\right], Rz_2\left[\dfrac{\pi}{8}\right], C_2[X_0], \right.$

$Rz_0\left[-\dfrac{\pi}{8}\right], C_2[X_0], H_1, C_0[X_1], Rz_1\left[-\dfrac{\pi}{4}\right], C_2[X_1], Rz_1\left[\dfrac{\pi}{4}\right], C_0[X_1], Rz_1\left[-\dfrac{\pi}{4}\right],$

$C_2[X_1], Rz_1\left[\dfrac{\pi}{4}\right], Rz_0\left[\dfrac{\pi}{4}\right], C_2[X_0], Rz_2\left[\dfrac{\pi}{4}\right], Rz_0\left[-\dfrac{\pi}{4}\right], C_2[X_0], H_1, Rz_1\left[-\dfrac{\pi}{8}\right],$

$Rz_2\left[-\dfrac{\pi}{8}\right], C_2[X_1], Rz_1\left[\dfrac{\pi}{8}\right], C_2[X_1], H_1, C_0[X_1], Rz_1\left[-\dfrac{\pi}{4}\right], C_2[X_1], Rz_1\left[\dfrac{\pi}{4}\right], C_0[X_1],$

$\left. Rz_1\left[-\dfrac{\pi}{4}\right], C_2[X_1], Rz_1\left[\dfrac{\pi}{4}\right], Rz_0\left[\dfrac{\pi}{4}\right], C_2[X_0], Rz_2\left[\dfrac{\pi}{4}\right], Rz_0\left[-\dfrac{\pi}{4}\right], C_2[X_0], H_1 \right\}$

» error: 0

testRecomp$\left[C_{0,1,3}[S_2], \text{ False}\right]$

» error: 0

C*[T]

testRecomp @ $C_{0,1}[T_2]$

» $\left\{G\left[\dfrac{9\pi}{32}\right], Rz_1\left[\dfrac{\pi}{16}\right], Rz_2\left[\dfrac{\pi}{16}\right], C_2[X_1], Rz_1\left[-\dfrac{\pi}{16}\right], C_2[X_1], Rz_0\left[\dfrac{\pi}{16}\right], Rz_2\left[\dfrac{\pi}{16}\right],\right.$

$C_2[X_0], Rz_0\left[-\dfrac{\pi}{16}\right], C_2[X_0], H_1, C_0[X_1], Rz_1\left[-\dfrac{\pi}{4}\right], C_2[X_1], Rz_1\left[\dfrac{\pi}{4}\right], C_0[X_1], Rz_1\left[-\dfrac{\pi}{4}\right],$

$C_2[X_1], Rz_1\left[\dfrac{\pi}{4}\right], Rz_0\left[\dfrac{\pi}{4}\right], C_2[X_0], Rz_2\left[\dfrac{\pi}{4}\right], Rz_0\left[-\dfrac{\pi}{4}\right], C_2[X_0], H_1, Rz_1\left[-\dfrac{\pi}{16}\right],$

$Rz_2\left[-\dfrac{\pi}{16}\right], C_2[X_1], Rz_1\left[\dfrac{\pi}{16}\right], C_2[X_1], H_1, C_0[X_1], Rz_1\left[-\dfrac{\pi}{4}\right], C_2[X_1], Rz_1\left[\dfrac{\pi}{4}\right],$

$\left.C_0[X_1], Rz_1\left[-\dfrac{\pi}{4}\right], C_2[X_1], Rz_1\left[\dfrac{\pi}{4}\right], Rz_0\left[\dfrac{\pi}{4}\right], C_2[X_0], Rz_2\left[\dfrac{\pi}{4}\right], Rz_0\left[-\dfrac{\pi}{4}\right], C_2[X_0], H_1\right\}$

» error: 0

testRecomp$\left[C_{0,1,3}[T_2],$ False$\right]$

» error: 0

C*[X]

testRecomp @ $C_{0,1}[X_2]$

» $\left\{G\left[\dfrac{\pi}{8}\right], H_2, C_0[X_2], Rz_2\left[-\dfrac{\pi}{4}\right], C_1[X_2], Rz_2\left[\dfrac{\pi}{4}\right], C_0[X_2], Rz_2\left[-\dfrac{\pi}{4}\right],\right.$

$\left.C_1[X_2], Rz_2\left[\dfrac{\pi}{4}\right], Rz_0\left[\dfrac{\pi}{4}\right], C_1[X_0], Rz_1\left[\dfrac{\pi}{4}\right], Rz_0\left[-\dfrac{\pi}{4}\right], C_1[X_0], H_2\right\}$
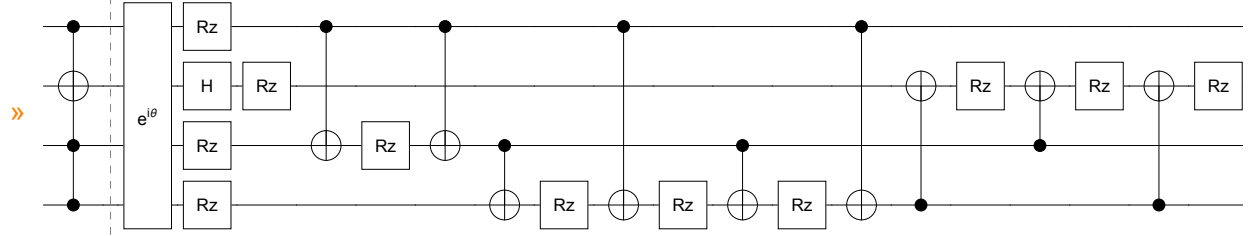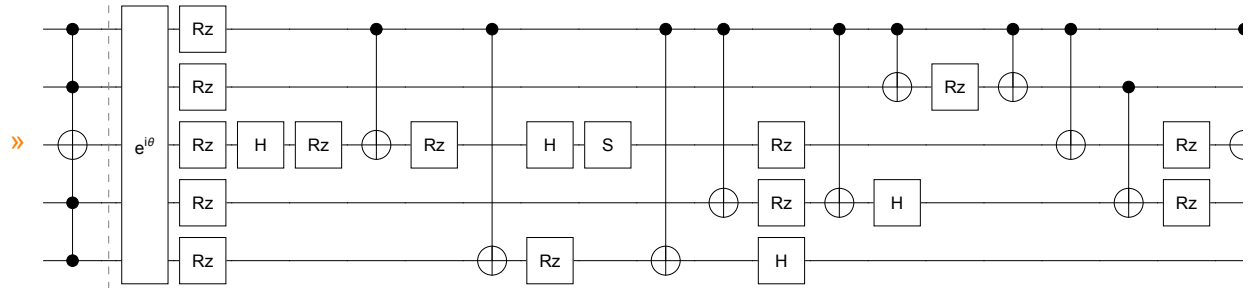

» error: 0

testRecomp @ $C_{0,1,3}[X_2]$

» $\left\{ G\left[\frac{\pi}{16}\right],\ H_2,\ Rz_0\left[\frac{\pi}{8}\right],\ Rz_1\left[\frac{\pi}{8}\right],\ Rz_3\left[\frac{\pi}{8}\right],\ Rz_2\left[\frac{\pi}{8}\right],\ C_3[X_1],\ Rz_1\left[-\frac{\pi}{8}\right],\ C_3[X_1],\ C_1[X_0],\ Rz_0\left[-\frac{\pi}{8}\right],\right.$

$C_3[X_0],\ Rz_0\left[\frac{\pi}{8}\right],\ C_1[X_0],\ Rz_0\left[-\frac{\pi}{8}\right],\ C_3[X_0],\ C_0[X_2],\ Rz_2\left[-\frac{\pi}{8}\right],\ C_1[X_2],\ Rz_2\left[\frac{\pi}{8}\right],\ C_0[X_2],$

$\left. Rz_2\left[-\frac{\pi}{8}\right],\ C_3[X_2],\ Rz_2\left[\frac{\pi}{8}\right],\ C_0[X_2],\ Rz_2\left[-\frac{\pi}{8}\right],\ C_1[X_2],\ Rz_2\left[\frac{\pi}{8}\right],\ C_0[X_2],\ Rz_2\left[-\frac{\pi}{8}\right],\ C_3[X_2],\ H_2\right\}$

»

» error: 0

testRecomp$\left[C_{0,1,3,4}[X_2],\ \text{False}\right]$

»

» error: 0

C*[Y]

testRecomp @ $C_{0,1}[Y_2]$

» $\left\{ G\left[\frac{\pi}{8}\right],\ Rz_2\left[-\frac{\pi}{4}\right],\ Rz_1\left[-\frac{\pi}{4}\right],\ C_1[X_2],\ Rz_2\left[\frac{\pi}{4}\right],\ C_1[X_2],\ H_2,\ C_0[X_2],\right.$

$Rz_2\left[-\frac{\pi}{4}\right],\ C_1[X_2],\ Rz_2\left[\frac{\pi}{4}\right],\ C_0[X_2],\ Rz_2\left[-\frac{\pi}{4}\right],\ C_1[X_2],\ Rz_2\left[\frac{\pi}{4}\right],\ Rz_0\left[\frac{\pi}{4}\right],\ C_1[X_0],$

$\left. Rz_1\left[\frac{\pi}{4}\right],\ Rz_0\left[-\frac{\pi}{4}\right],\ C_1[X_0],\ H_2,\ Rz_2\left[\frac{\pi}{4}\right],\ Rz_1\left[\frac{\pi}{4}\right],\ C_1[X_2],\ Rz_2\left[-\frac{\pi}{4}\right],\ C_1[X_2]\right\}$
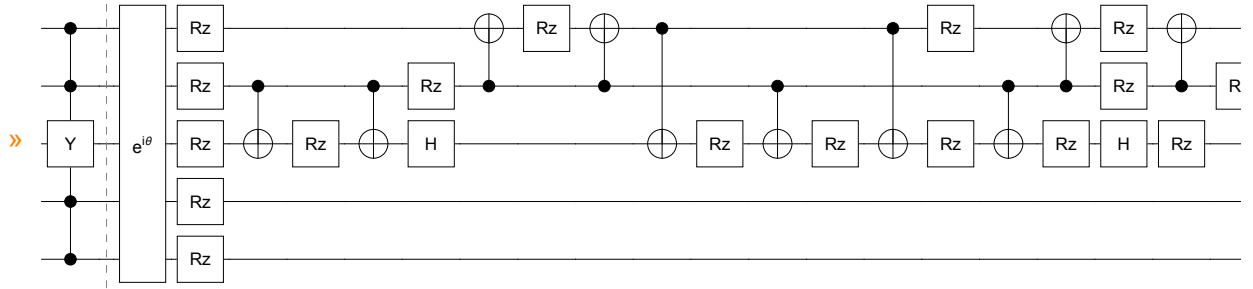
»
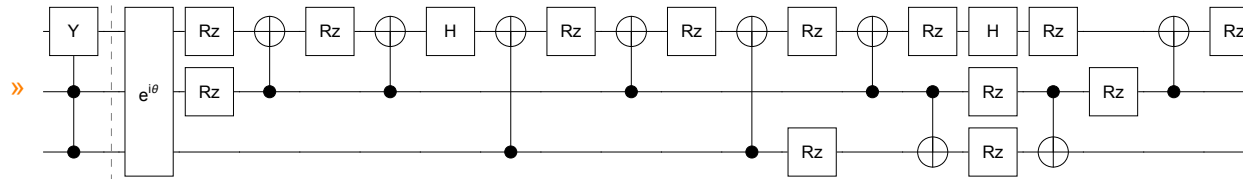
» error: 0

testRecomp$\left[C_{0,1,3}[Y_2],\ \text{False}\right]$
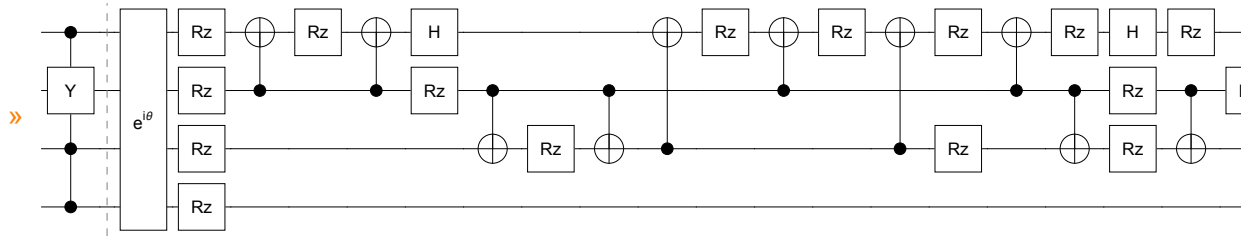
» error: 0

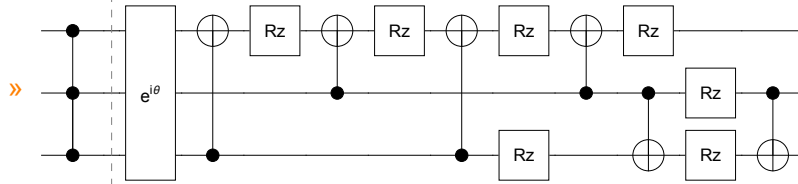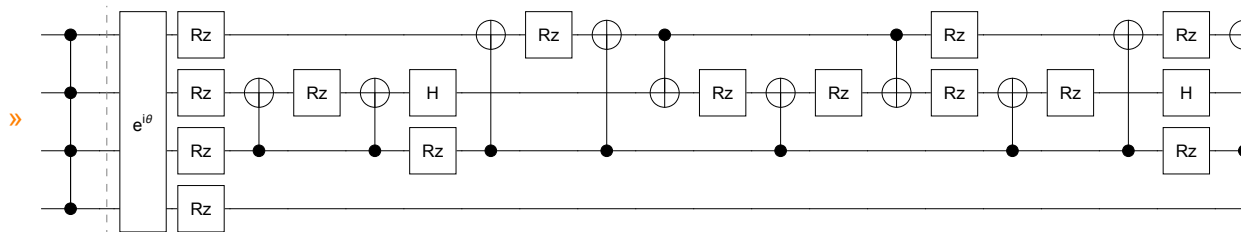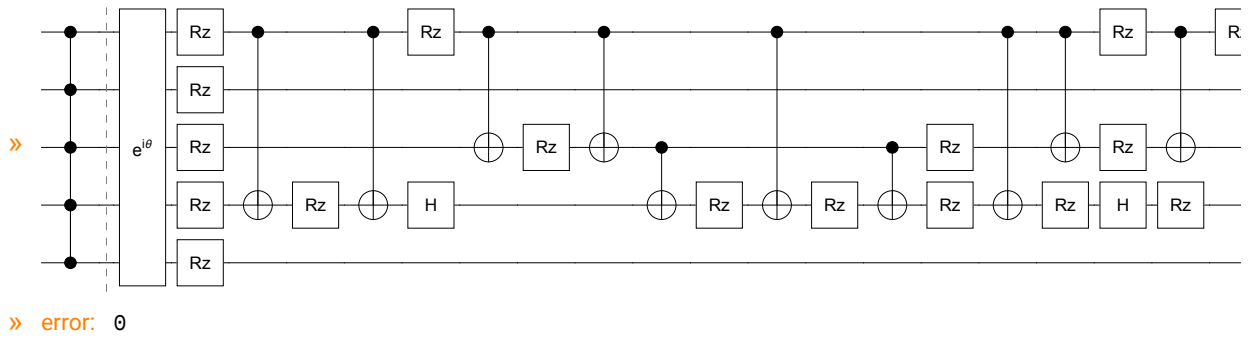**testRecomp[C$_{0,1,3,4}$[Y$_2$], False]**

» error: 0

C*[Z]

testRecomp @ C$_{0,1}$[Z$_2$]

» $\left\{ G\left[\dfrac{\pi}{8}\right],\ C_0[X_2],\ Rz_2\left[-\dfrac{\pi}{4}\right],\ C_1[X_2],\ Rz_2\left[\dfrac{\pi}{4}\right],\ C_0[X_2], \right.$

$\left. Rz_2\left[-\dfrac{\pi}{4}\right],\ C_1[X_2],\ Rz_2\left[\dfrac{\pi}{4}\right],\ Rz_0\left[\dfrac{\pi}{4}\right],\ C_1[X_0],\ Rz_1\left[\dfrac{\pi}{4}\right],\ Rz_0\left[-\dfrac{\pi}{4}\right],\ C_1[X_0] \right\}$

» error: 0

testRecomp[C$_{0,1,3}$[Z$_2$], False]

» error: 0

testRecomp[C$_{0,1,3,4}$[Z$_2$], False]

» error: 0

C*[SWAP]

testRecomp @ $C_{0,1}\left[SWAP_{2,3}\right]$

» $\left\{G\left[\dfrac{\pi}{16}\right], C_2[X_3], H_2, Rz_0\left[\dfrac{\pi}{8}\right], Rz_1\left[\dfrac{\pi}{8}\right], Rz_3\left[\dfrac{\pi}{8}\right], Rz_2\left[\dfrac{\pi}{8}\right], C_3[X_1],\right.$

$Rz_1\left[-\dfrac{\pi}{8}\right], C_3[X_1], C_1[X_0], Rz_0\left[-\dfrac{\pi}{8}\right], C_3[X_0], Rz_0\left[\dfrac{\pi}{8}\right], C_1[X_0], Rz_0\left[-\dfrac{\pi}{8}\right],$

$C_3[X_0], C_0[X_2], Rz_2\left[-\dfrac{\pi}{8}\right], C_1[X_2], Rz_2\left[\dfrac{\pi}{8}\right], C_0[X_2], Rz_2\left[-\dfrac{\pi}{8}\right], C_3[X_2], Rz_2\left[\dfrac{\pi}{8}\right],$

$\left.C_0[X_2], Rz_2\left[-\dfrac{\pi}{8}\right], C_1[X_2], Rz_2\left[\dfrac{\pi}{8}\right], C_0[X_2], Rz_2\left[-\dfrac{\pi}{8}\right], C_3[X_2], H_2, C_2[X_3]\right\}$
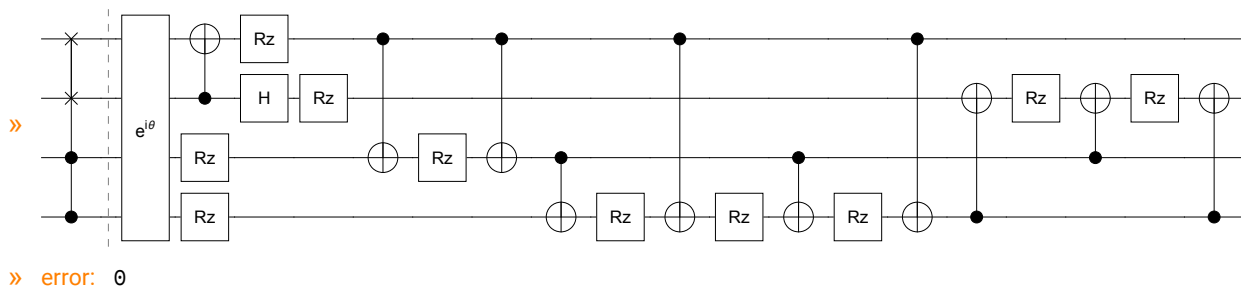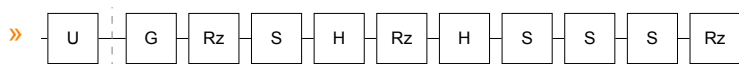

» error: 0

Testing U (matrix)

Un-controlled

U^(1)

testRecomp[U_0 @ RandomVariate @ CircularUnitaryMatrixDistribution @ 2, False]

» error: 0

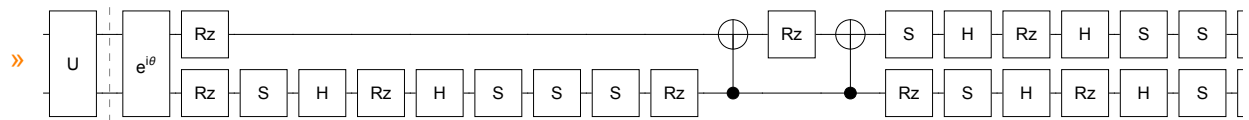**testRecomp[U_0 @ {{Exp[i .1], 0}, {0, Exp[$-i\pi/3$]}}, False]**

» error: 0

```
RecompileCircuit[ U₀@{{a, b}, {c, d}}, "SingleQubitAndCNOT"]
```

$$\Big\{\mathsf{G}\Big[\mathsf{ArcTan[Re[a], Im[a]]} +$$
$$\frac{1}{2} \left(-2\, \mathsf{ArcTan[Re[a], Im[a]]} + \mathsf{ArcTan[-Re[b], -Im[b]]} + \mathsf{ArcTan[Re[c], Im[c]]}\right)\Big],$$

$$\mathsf{Rz_0[-ArcTan[Re[a], Im[a]] + ArcTan[-Re[b], -Im[b]]]},\ \mathsf{Ry_0}\Big[2\, \mathsf{ArcTan}\Big[\frac{\mathsf{Abs[b]}}{\mathsf{Abs[a]}}\Big]\Big]\Big\},$$

$$\mathsf{Rz_0[-ArcTan[Re[a], Im[a]] + ArcTan[Re[c], Im[c]]]}\Big\}$$

U^(n)

```
testRecomp[ U₀,₁ @ RandomVariate @ CircularUnitaryMatrixDistribution[2²], False]
```


» error: 0

```
testRecomp[
    UNonNorm₀,₁,₂ @ RandomVariate @ CircularUnitaryMatrixDistribution[2³], False]
```
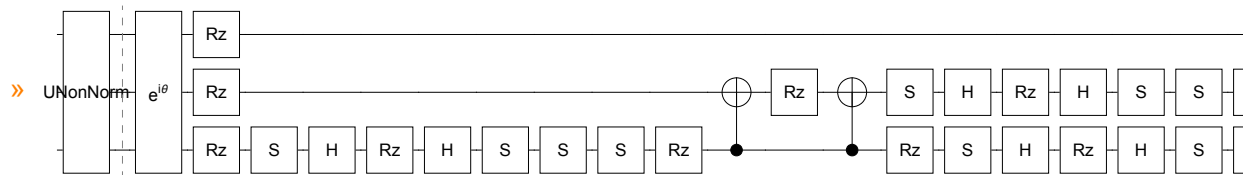

» error: 0

```
testRecomp[
    UNonNorm₀,₂,₁,₃ @ RandomVariate @ CircularUnitaryMatrixDistribution[2⁴], False]
```
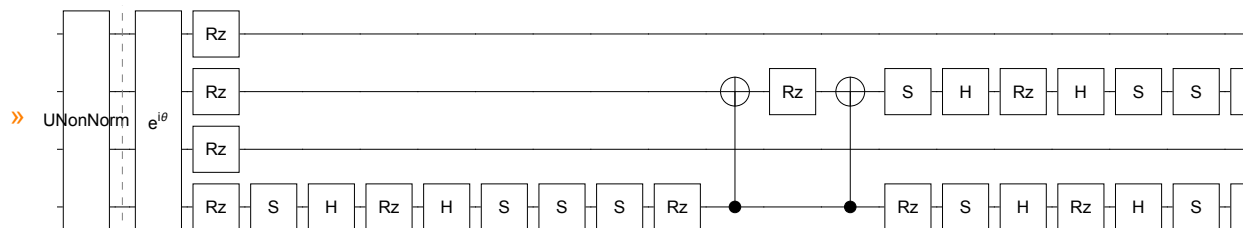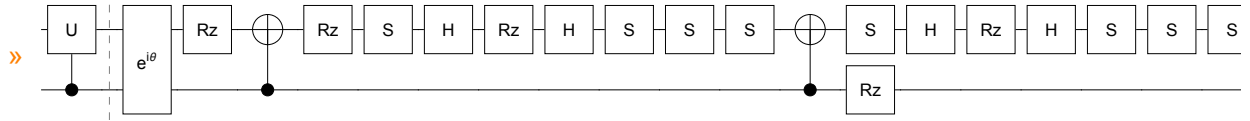

» error: 0

Singly-controlled

C[U^(1)]

```
testRecomp[C₀@U₁ @ RandomVariate @ CircularUnitaryMatrixDistribution @ 2]
```
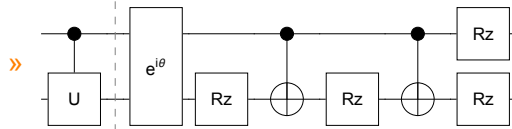
» $\{G[-0.145122], Rz_1[-1.71584], C_0[X_1], Rz_1[-0.373885], S_1, H_1, Rz_1[0.300537], H_1, S_1, S_1, S_1, C_0[X_1], S_1, H_1, Rz_1[-0.300537], H_1, S_1, S_1, S_1, Rz_1[2.08973], Rz_0[-0.290244]\}$

»

» error: 0

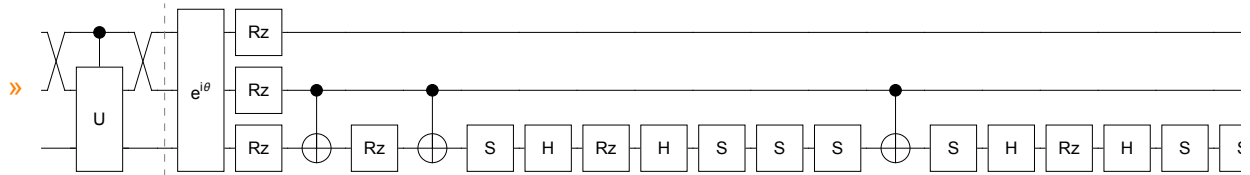**testRecomp[$C_1@U_0$ @ {{Exp[i .1], 0}, {0, Exp[-i π / 3]}}]**

» $\{G[-0.236799], Rz_0[0.573599], C_1[X_0],$
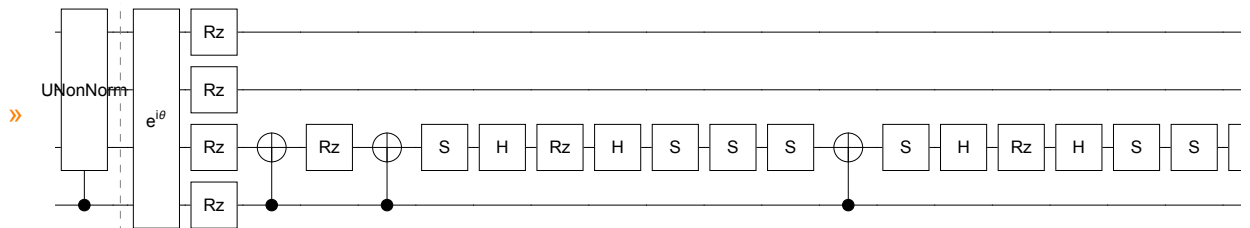$Rz_0[0.573599], C_1[X_0], Rz_0[-1.1472], Rz_1[-0.473599]\}$

»

» error: 0

C[U^(n)]

testRecomp[

$C_1@U_{0,2}$ @ RandomVariate @ CircularUnitaryMatrixDistribution[2^2], False]

»

» error: 0

testRecomp[C_0@

$UNonNorm_{1,2,3}$ @ RandomVariate @ CircularUnitaryMatrixDistribution[2^3], False]
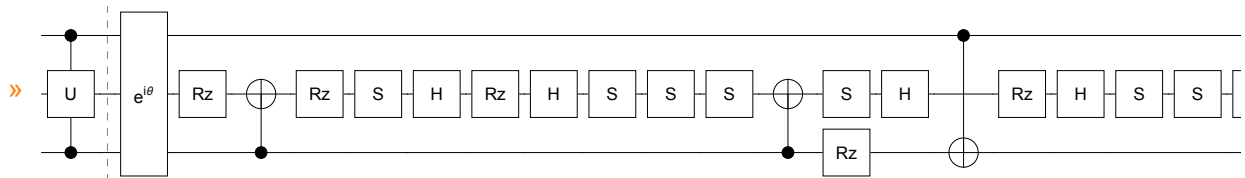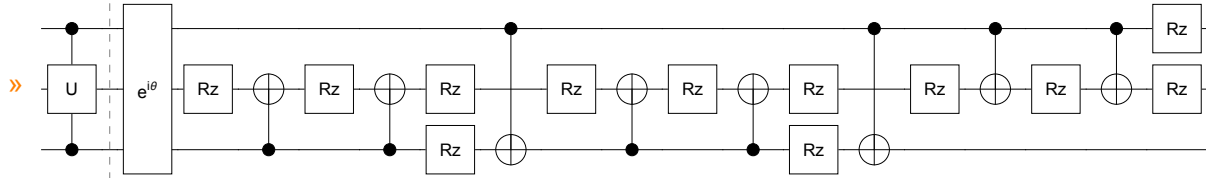
»

» error: 0

Multi-controlled

C*[U^(1)]

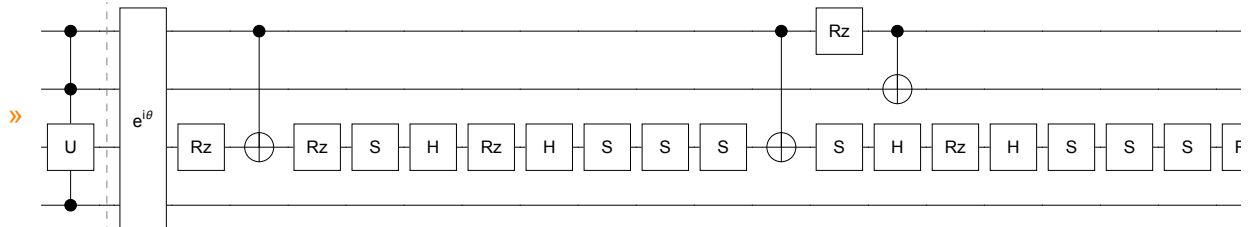**testRecomp[$C_{0,2}@U_1$ @ RandomVariate @ CircularUnitaryMatrixDistribution @ 2, False]**

» error: 0

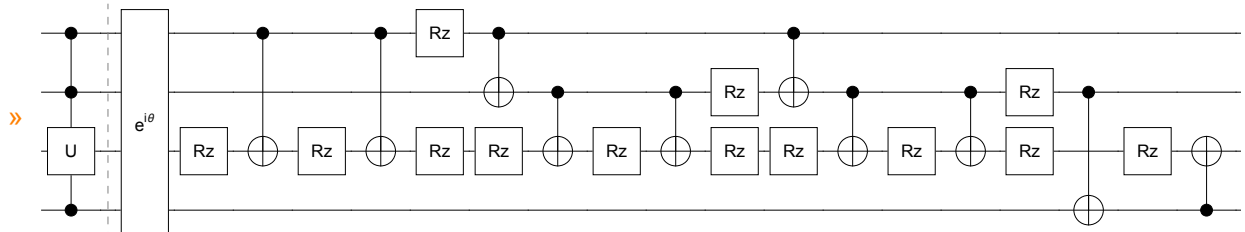testRecomp$[C_{0,2}@U_1 @ \{\{Exp[i\,.1], 0\}, \{0, Exp[-i\,\pi/3]\}\}, False]$

» error: 0

testRecomp$[$

   $C_{0,2,3}@U_1 @ RandomVariate @ CircularUnitaryMatrixDistribution @ 2, False]$

» error: 0

testRecomp$[C_{0,2,3}@U_1 @ \{\{Exp[i\,.1], 0\}, \{0, Exp[-i\,\pi/3]\}\}, False]$

» error: 0

testRecomp$[$

   $C_{0,2,3,4}@U_1 @ RandomVariate @ CircularUnitaryMatrixDistribution @ 2, False]$
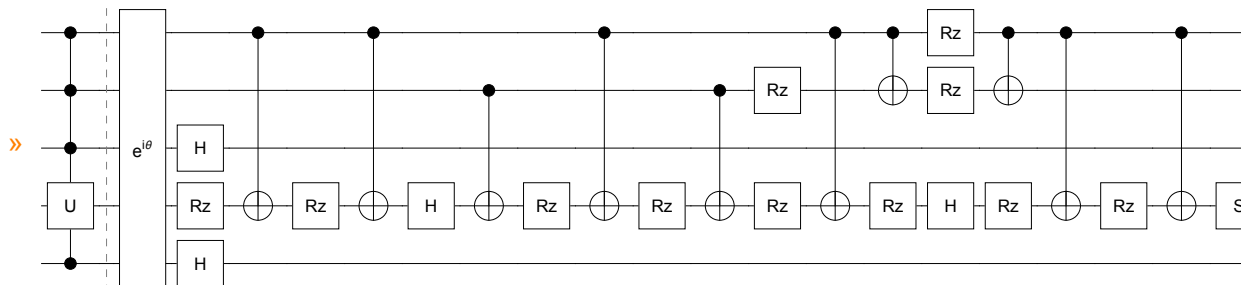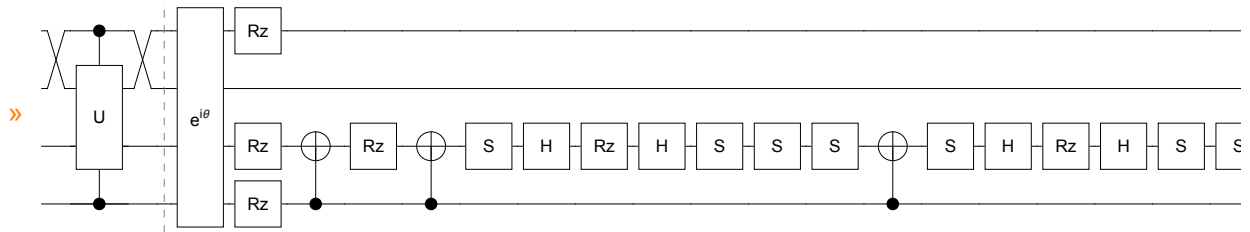

» error: 0

C*[U^(n)]

> **testRecomp[**
> $C_{0,2}@U_{1,3}$ @ RandomVariate @ CircularUnitaryMatrixDistribution[2^2], False**]**

» error: 0

> **testRecomp[**
> $C_{0,2,3}@U_{1,4}$ @ RandomVariate @ CircularUnitaryMatrixDistribution[2^2], False**]**
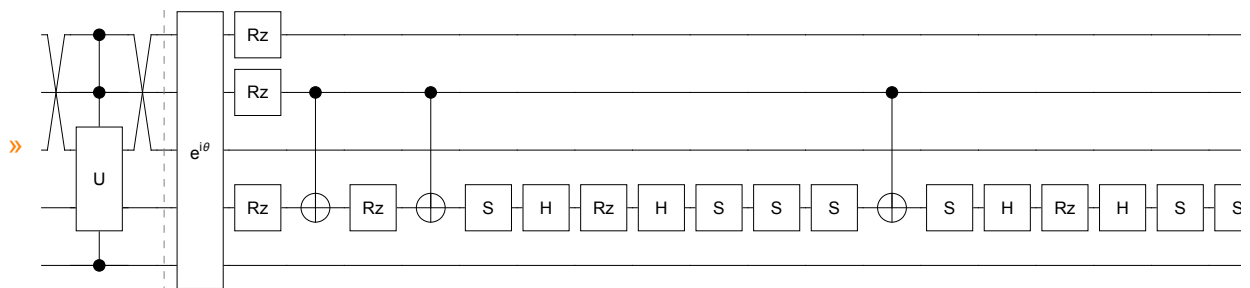
» error: 0

Testing U (vector)

Un-controlled

U^(1)

> **testRecomp @ U_0[{Exp[$\mathbb{i}$ .2], Exp[– $\mathbb{i}$ π / 3]}]**

» $\{G[-0.423599], Rz_0[-1.2472]\}$

» error: 0

U^(n)

not yet supported

Singly-controlled

C[U^(1)]

not yet supported

C[U^(n)]

not yet supported

Multi-controlled

C*[U^(1)]

not yet supported

C*[U^(n)]

not yet supported

Testing errors

invalid arguments

```
RecompileCircuit[bleh]
```

⚫⚫⚫ RecompileCircuit: Invalid arguments. See ?RecompileCircuit

```
$Failed
```

unrecognised method

```
RecompileCircuit[Damp₀[x], "eh"]
```

⚫⚫⚫ RecompileCircuit: Unrecognised method. See available methods via ?RecompileCircuit

unrecognised gates

```
RecompileCircuit[{Y₀, Poop₀, X₀, Blob₃}, "CliffordAndRz"]
```

⚫⚫⚫ RecompileCircuit: Recompilation failed. Could not recompile unrecognised gate: $Poop_0$

```
$Failed
```

unsupported gates

`RecompileCircuit[Damp`$_0$`[x], "SingleQubitAndCNOT"]`

⚫ RecompileCircuit: Recompilation failed. Could not recompile unrecognised gate: Damp$_0$[x]

`$Failed`

`RecompileCircuit`$\big[$`U`$_{0,1}$` @ {a, b, c, d}, "SingleQubitAndCNOT"`$\big]$

⚫ RecompileCircuit: Recompilation failed. Many–qubit diagonal gates are not yet supported by the recompiler.

`$Failed`

`RecompileCircuit`$\big[$`C`$_{1,2}$`@U`$_0$` @ {a, b}, "SingleQubitAndCNOT"`$\big]$

⚫ RecompileCircuit: Recompilation failed. Controlled diagonal gates are not yet supported by the recompiler.

`$Failed`

numerical issues

`RecompileCircuit`$\big[$
`    U`$_{0,1}$`[{{a, b}, {c, d}}],`
`    "SingleQubitAndCNOT"`$\big]$

⚫ RecompileCircuit: Recompilation failed. Encountered a non–numerical matrix in a two (or more) qubit U gate, which cannot be decomposed.

`$Failed`

`RecompileCircuit`$\big[$
`    U`$_{0,1}$` @ RandomComplex[{-`$\mathbb{i}$` - 1, `$\mathbb{i}$` + 1}, {2^2, 2^2}],`
`    "SingleQubitAndCNOT"`$\big]$

⚫ RecompileCircuit: Recompilation failed. Encountered a non–unitary U gate matrix which cannot be (spectrally) decomposed. Please use UNonNorm instead.

`$Failed`

`RecompileCircuit`$\big[$
`    U`$_{0,1}$` @ (2 IdentityMatrix @ 4),`
`    "SingleQubitAndCNOT"`$\big]$

⚫ RecompileCircuit: Recompilation failed. The cosine–sine decomposition involved in recompiling a U (or UNonNorm) gate failed.

`$Failed`