

# GetPauliStringFromMatrix

```
SetDirectory @ NotebookDirectory[];
Import["../Link/QuESTlink.m"];

test[in_] := Module[
  {out, check, error},
  out = GetPauliStringFromMatrix[in];
  check = Simplify @ Normal @ CalcPauliExpressionMatrix[out];
  error = check - in // N // Abs // Chop // Max;
  Echo[out, "output: "];
  Echo[error, "error: "];
  If[error != 0, Style["ERRONEOUS PAULI STRING!", Red]]]
```

## Doc

? GetPauliStringFromMatrix

Symbol

GetPauliStringFromMatrix[m] returns a complex-weighted sum of  
Pauli tensors equivalent to the given square, power-of-2 length matrix m.  
If the input matrix is Hermitian, the output can be passed  
to Chop[] in order to remove the negligible imaginary components.



## Floating-point

```
test @ RandomComplex[{1 + 1} {-10, 10}, {2, 2}]
```

» output:  $(-0.973073 + 0.741944 i) \text{Id}_0 - (2.97398 + 4.26202 i) X_0 +$   
 $(3.4839 - 1.23818 i) Y_0 + (7.76152 - 5.63547 i) Z_0$

» error: 0

```
test @ RandomComplex[{1 + 1} {-10, 10}, {8, 8}]
```

» **output:**  $(0.139814 - 1.54188 i) \text{Id}_2 + (1.73462 + 1.48615 i) X_0 +$   
 $(3.02282 + 2.05665 i) X_1 + (0.454681 - 1.87161 i) X_0 X_1 + (1.34732 + 2.02739 i) X_2 -$   
 $(1.86537 + 2.35614 i) X_0 X_2 + (2.65145 - 0.871312 i) X_1 X_2 + (0.326747 - 2.3702 i) X_0 X_1 X_2 -$   
 $(0.763019 - 0.169465 i) Y_0 + (3.48826 + 5.08657 i) X_1 Y_0 + (0.648935 - 0.403203 i) X_2 Y_0 +$   
 $(0.891398 + 1.90618 i) X_1 X_2 Y_0 - (0.164189 - 3.28807 i) Y_1 + (0.0127069 - 1.08187 i) X_0 Y_1 +$   
 $(1.72165 - 0.282772 i) X_2 Y_1 - (2.30141 - 0.24752 i) X_0 X_2 Y_1 + (1.3647 - 1.18187 i) Y_0 Y_1 +$   
 $(2.00364 - 0.882466 i) X_2 Y_0 Y_1 - (0.94013 - 1.09517 i) Y_2 + (1.78227 + 0.824745 i) X_0 Y_2 +$   
 $(0.101885 - 0.738079 i) X_1 Y_2 + (0.98813 + 5.06933 i) X_0 X_1 Y_2 + (1.54881 + 3.34238 i) Y_0 Y_2 -$   
 $(1.08937 - 3.08663 i) X_1 Y_0 Y_2 + (2.26801 + 1.45578 i) Y_1 Y_2 - (0.316869 + 3.18929 i) X_0 Y_1 Y_2 -$   
 $(3.74421 + 2.25177 i) Y_0 Y_1 Y_2 + (2.30453 - 1.37679 i) Z_0 + (0.0914269 - 0.760655 i) X_1 Z_0 +$   
 $(4.5844 - 0.492016 i) X_2 Z_0 - (1.29936 + 0.863364 i) X_1 X_2 Z_0 - (2.86909 + 0.758535 i) Y_1 Z_0 +$   
 $(0.843282 + 0.70642 i) X_2 Y_1 Z_0 - (0.24794 - 0.23487 i) Y_2 Z_0 - (3.04599 + 0.993896 i) X_1 Y_2 Z_0 -$   
 $(2.45986 - 2.84889 i) Y_1 Y_2 Z_0 - (3.58186 + 0.570081 i) Z_1 + (3.6915 + 2.96072 i) X_0 Z_1 +$   
 $(1.11257 + 3.25462 i) X_2 Z_1 + (3.26331 - 2.18514 i) X_0 X_2 Z_1 + (0.120717 - 0.0854912 i) Y_0 Z_1 -$   
 $(1.31298 + 0.759866 i) X_2 Y_0 Z_1 - (2.12883 - 1.45747 i) Y_2 Z_1 + (2.92552 + 0.810047 i) X_0 Y_2 Z_1 +$   
 $(2.29638 - 2.36478 i) Y_0 Y_2 Z_1 - (1.00239 - 0.251881 i) Z_0 Z_1 - (0.78218 - 1.65059 i) X_2 Z_0 Z_1 +$   
 $(1.40744 + 0.792362 i) Y_2 Z_0 Z_1 + (1.62985 + 2.38714 i) Z_2 + (3.27005 - 3.53969 i) X_0 Z_2 +$   
 $(1.86487 + 0.715973 i) X_1 Z_2 - (0.62627 - 0.902676 i) X_0 X_1 Z_2 + (1.89233 + 0.580809 i) Y_0 Z_2 +$   
 $(1.35633 + 1.71375 i) X_1 Y_0 Z_2 + (0.550727 - 2.01131 i) Y_1 Z_2 + (1.45508 - 0.858943 i) X_0 Y_1 Z_2 -$   
 $(0.144475 - 1.38886 i) Y_0 Y_1 Z_2 - (4.30294 + 2.655 i) Z_0 Z_2 + (2.09851 + 1.77015 i) X_1 Z_0 Z_2 +$   
 $(2.70976 + 0.358614 i) Y_1 Z_0 Z_2 - (2.19259 - 2.53062 i) Z_1 Z_2 - (2.44097 + 0.383743 i) X_0 Z_1 Z_2 -$   
 $(1.52336 + 0.754148 i) Y_0 Z_1 Z_2 + (0.382741 + 3.61576 i) Z_0 Z_1 Z_2$

» **error:** 0

**test @ RandomReal[{-10, 10}, {4, 4}]**

» **output:**  $-1.40379 \text{Id}_1 + 0.796864 X_0 + 0.492345 X_1 + 2.99213 X_0 X_1 - (0. + 1.9158 i) Y_0 +$   
 $(0. + 0.0733287 i) X_1 Y_0 - (0. + 2.16448 i) Y_1 + (0. + 2.06191 i) X_0 Y_1 +$   
 $0.101484 Y_0 Y_1 - 1.34646 Z_0 + 2.54854 X_1 Z_0 - (0. + 2.70442 i) Y_1 Z_0 +$   
 $0.518598 Z_1 + 2.25656 X_0 Z_1 + (0. + 4.69625 i) Y_0 Z_1 + 4.33439 Z_0 Z_1$

» **error:** 0

**hermitian = -i MatrixLog @ RandomVariate @ CircularUnitaryMatrixDistribution @ 4;**  
**test @ hermitian**

» **output:**  $(0.309257 - 6.57569 \times 10^{-17} i) \text{Id}_1 + (0.274875 + 1.38778 \times 10^{-17} i) X_0 +$   
 $(1.59668 - 2.56739 \times 10^{-16} i) X_1 - (0.0811653 - 2.77556 \times 10^{-16} i) X_0 X_1 -$   
 $(0.141415 + 2.22045 \times 10^{-16} i) Y_0 + (0.588161 + 2.77556 \times 10^{-17} i) X_1 Y_0 +$   
 $(0.0430516 - 5.55112 \times 10^{-17} i) Y_1 + (0.479953 - 2.498 \times 10^{-16} i) X_0 Y_1 -$   
 $(0.709693 - 2.77556 \times 10^{-16} i) Y_0 Y_1 + (0.0241915 + 1.96295 \times 10^{-16} i) Z_0 -$   
 $(0.292846 + 1.38778 \times 10^{-16} i) X_1 Z_0 + (0.275077 + 4.44089 \times 10^{-16} i) Y_1 Z_0 -$   
 $(0.166739 - 4.09774 \times 10^{-16} i) Z_1 - (0.747069 - 4.02456 \times 10^{-16} i) X_0 Z_1 -$   
 $(0.218264 - 1.11022 \times 10^{-16} i) Y_0 Z_1 - (0.171741 + 1.51734 \times 10^{-16} i) Z_0 Z_1$

» **error:** 0

**test @ Table[0., 2, 2]**

» **output:** 0. + 0. i

» **error:** 0

## Integer

**test @ RandomInteger[{-10, 10}, {2, 2}]**

» **output:**  $-7 \text{Id}_0 - 6 X_0 + 2 i Y_0 - Z_0$

» **error:** 0

**test @ RandomInteger[{-10, 10}, {4, 4}]**

» **output:** 
$$\frac{5 \text{Id}_1}{4} - \frac{X_0}{4} + \frac{5 X_1}{4} + X_0 X_1 - \frac{21 i Y_0}{4} - \frac{5}{2} i X_1 Y_0 + \frac{i Y_1}{4} -$$

$$3 i X_0 Y_1 + \frac{5 Y_0 Y_1}{2} + \frac{3 Z_0}{4} - \frac{25 X_1 Z_0}{4} - \frac{9}{4} i Y_1 Z_0 + \frac{Z_1}{4} + \frac{X_0 Z_1}{4} - \frac{3}{4} i Y_0 Z_1 + \frac{11 Z_0 Z_1}{4}$$

» **error:** 0

**test @ Table[1, {i, 8}, {i, 8}]**

» **output:**  $\text{Id}_2 + X_0 + X_1 + X_0 X_1 + X_2 + X_0 X_2 + X_1 X_2 + X_0 X_1 X_2$

» **error:** 0

**test @ Table[0, {i, 8}, {i, 8}]**

» **output:** 0

» **error:** 0

**test[ IdentityMatrix[8] ]**

» **output:**  $\text{Id}_2$

» **error:** 0

## Symbolic

**test @ {{a, b}, {c, d}}**

» **output:**  $\frac{1}{2} (a + d) \text{Id}_0 + \frac{1}{2} (b + c) X_0 + \frac{1}{2} (i b - i c) Y_0 + \frac{1}{2} (a - d) Z_0$

» **error:** 0

**test[ a IdentityMatrix[4] ]**

» **output:**  $a \text{Id}_1$

» **error:** 0

**test @ Table[a, {i, 8}, {i, 8}]**

» **output:**  $a \text{Id}_2 + a X_0 + a X_1 + a X_0 X_1 + a X_2 + a X_0 X_2 + a X_1 X_2 + a X_0 X_1 X_2$

» **error:** 0

**test @ RandomChoice[{a, b, c, d}, {4, 4}]**

» output:  $\frac{1}{4} (2a + c + d) \text{Id}_1 + \frac{1}{4} (b + c + 2d) X_0 + \frac{1}{4} (a + 2b + c) X_1 + \frac{1}{4} (a + 2b + c) X_0 X_1 +$   
 $\frac{1}{4} (ib + ic - 2id) Y_0 + \frac{1}{4} (ia - ic) X_1 Y_0 + \frac{1}{4} (ia - ic) Y_1 + \frac{1}{4} (-ia + ic) X_0 Y_1 +$   
 $\frac{1}{4} (a - 2b + c) Y_0 Y_1 + \frac{1}{4} (-c + d) Z_0 + \frac{1}{4} (-a + c) X_1 Z_0 + \frac{1}{4} (-ia + 2ib - ic) Y_1 Z_0 +$   
 $\frac{1}{4} (-c + d) Z_1 + \frac{1}{4} (b - c) X_0 Z_1 + \frac{1}{4} (ib - ic) Y_0 Z_1 + \frac{1}{4} (-2a + c + d) Z_0 Z_1$

» error: 0

test @ RandomChoice[{a, b, c, d}, {8, 8}]

» output:  $\frac{1}{8} (a + 5b + 2c) \text{Id}_2 + \frac{1}{8} (a + 4b + 2c + d) X_0 + \frac{1}{8} (a + b + c + 5d) X_1 +$   
 $\frac{1}{8} (3b + c + 4d) X_0 X_1 + \frac{1}{8} (5a + 2c + d) X_2 + \frac{1}{8} (2b + c + 5d) X_0 X_2 + \frac{1}{8} (3a + 3b + 2d) X_1 X_2 +$   
 $\frac{1}{8} (a + 2b + 3c + 2d) X_0 X_1 X_2 + \frac{1}{8} (-ia + 2ic - id) Y_0 + \frac{1}{8} (ib - ic) X_1 Y_0 +$   
 $\frac{1}{8} (-2ib - ic + 3id) X_2 Y_0 + \frac{1}{8} (-ia + ic) X_1 X_2 Y_0 + \frac{1}{8} (ia - ib - ic + id) Y_1 +$   
 $\frac{1}{8} (-ib - ic + 2id) X_0 Y_1 + \frac{1}{8} (ia - ib) X_2 Y_1 + \frac{1}{8} (ia - ic) X_0 X_2 Y_1 + \frac{1}{8} (-b - c + 2d) Y_0 Y_1 +$   
 $\frac{1}{8} (a - 2b + 3c - 2d) X_2 Y_0 Y_1 + \frac{1}{8} (-ia + id) Y_2 + \frac{1}{8} (ic - id) X_0 Y_2 + \frac{1}{8} (ia - ib) X_1 Y_2 +$   
 $\frac{1}{8} (-ia - 2ib + ic + 2id) X_0 X_1 Y_2 + \frac{1}{8} (c - d) Y_0 Y_2 + \frac{1}{8} (-a + c) X_1 Y_0 Y_2 + \frac{1}{8} (-3a + b + 2d) Y_1 Y_2 +$   
 $\frac{1}{8} (a - c) X_0 Y_1 Y_2 + \frac{1}{8} (-ia + 2ib + ic - 2id) Y_0 Y_1 Y_2 + \frac{1}{8} (a + b - 2c) Z_0 + \frac{1}{8} (a - b + c - d) X_1 Z_0 +$   
 $\frac{1}{8} (-a + d) X_2 Z_0 + \frac{1}{8} (a - b) X_1 X_2 Z_0 + \frac{1}{8} (ia + ib - ic - id) Y_1 Z_0 + \frac{1}{8} (-ia - ib + 2id) X_2 Y_1 Z_0 +$   
 $\frac{1}{8} (ia - 2ic + id) Y_2 Z_0 + \frac{1}{8} (-ia + 3ib - 2id) X_1 Y_2 Z_0 + \frac{1}{8} (-a + b) Y_1 Y_2 Z_0 +$   
 $\frac{1}{8} (a - b) Z_1 + \frac{1}{8} (-a - 2b + 2c + d) X_0 Z_1 + \frac{1}{8} (-a + d) X_2 Z_1 + \frac{1}{8} (2b - c - d) X_0 X_2 Z_1 +$   
 $\frac{1}{8} (ia - 2ib + 2ic - id) Y_0 Z_1 + \frac{1}{8} (-2ib + ic + id) X_2 Y_0 Z_1 + \frac{1}{8} (ia - 2ic + id) Y_2 Z_1 +$   
 $\frac{1}{8} (-ic + id) X_0 Y_2 Z_1 + \frac{1}{8} (-c + d) Y_0 Y_2 Z_1 + \frac{1}{8} (a - b) Z_0 Z_1 + \frac{1}{8} (-3a + 2c + d) X_2 Z_0 Z_1 +$   
 $\frac{1}{8} (-ia + id) Y_2 Z_0 Z_1 + \frac{1}{8} (a - 3b + 2c) Z_2 + \frac{1}{8} (-a + 2b - d) X_0 Z_2 + \frac{1}{8} (a + b + c - 3d) X_1 Z_2 +$   
 $\frac{1}{8} (-b - c + 2d) X_0 X_1 Z_2 + \frac{1}{8} (ia - 2ib + id) Y_0 Z_2 + \frac{1}{8} (-3ib + ic + 2id) X_1 Y_0 Z_2 +$   
 $\frac{1}{8} (ia - ib - ic + id) Y_1 Z_2 + \frac{1}{8} (-ib + ic) X_0 Y_1 Z_2 + \frac{1}{8} (-b + c) Y_0 Y_1 Z_2 +$   
 $\frac{1}{8} (a + b - 2c) Z_0 Z_2 + \frac{1}{8} (a - b + c - d) X_1 Z_0 Z_2 + \frac{1}{8} (ia + ib - ic - id) Y_1 Z_0 Z_2 +$   
 $\frac{1}{8} (a - b) Z_1 Z_2 + \frac{1}{8} (a - d) X_0 Z_1 Z_2 + \frac{1}{8} (-ia + id) Y_0 Z_1 Z_2 + \frac{1}{8} (a - b) Z_0 Z_1 Z_2$

» error: 0

# Errors

**GetPauliStringFromMatrix @ {1, 2, 3}**

... **GetPauliStringFromMatrix**: The input must be a square matrix with power-of-2 dimensions.

\$Failed

**GetPauliStringFromMatrix @ Table[1, {i, 7}, {j, 7}]**

... **GetPauliStringFromMatrix**: The input must be a square matrix with power-of-2 dimensions.

\$Failed

**GetPauliStringFromMatrix @ Table[1, {i, 2}, {j, 4}]**

... **GetPauliStringFromMatrix**: The input must be a square matrix with power-of-2 dimensions.

\$Failed

**GetPauliStringFromMatrix[a, b, c]**

... **GetPauliStringFromMatrix**: Invalid arguments. See ?GetPauliStringFromMatrix

\$Failed