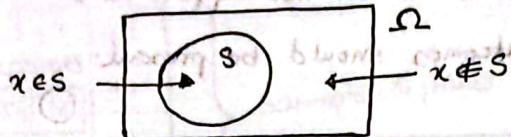


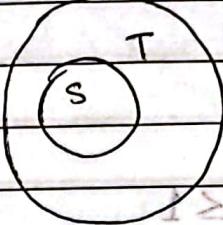
$\{a, b, c, d\}$ $\{x \mid x \in \mathbb{R} \wedge \cos(x) > \frac{1}{2}\}$

exact/10/11



Date: 12/01/2022

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 $S^c \text{ or } S' = \{x \mid x \notin S \wedge x \in \Omega\}$  $\Omega^c = \emptyset = \{\}$ = Null set(note: $S \cap T : x \in S \rightarrow x \in T$)If ~~not~~ $\bigcap_{i=1}^n A_i = \emptyset$ then $(A \cap (A)^c) \geq (A \cap A)^c - (A)^c + (A)^c = (A \cup A)^c$

A_i's are disjoint.

 $\Omega \supseteq A \mid A \neq \emptyset$ If $S \cup T = \Omega$ and $S \cap T = \emptyset$ then S and T are partition of Ω .

$$(S \cup T)^c = (S \cap T)^c = (S \cap T)^c = (S \cup T)^c$$

DeMorgan's Law

 $S \cup T = \{x \mid x \in S \vee x \in T\}$

$$(\bigcap_n S_n)^c = \bigcup_n S_n^c = (S \cup T)^c$$

 $S \cap T = \{x \mid x \in S \wedge x \in T\}$

$$(\bigcup_n S_n)^c = \bigcap_n S_n^c = (S \cap T)^c$$

$$\bigcap_n S_n = \{x \mid x \in S_n \wedge x \in S_m\}$$

$$(S \cup T)^c = (S \cap T)^c = (S \cap T)^c = (S \cup T)^c$$

$$(\emptyset)^c - x \in (A \cap B)^c = (\Omega)^c = (\Omega)^c$$

$$x \notin A \cap B = (\emptyset)^c - x = x$$

$$x \notin A \wedge x \notin B = (\emptyset)^c = (\emptyset)^c$$

$$x \in A^c \vee x \in B^c$$

$$x \in A^c \cup B^c$$

Sample space: set of all possible outcomes.

Sample space must be

- Mutually exclusive \leftrightarrow two outcomes can not happen at once.
- Collectively exhaustive \leftrightarrow all outcomes should be present. Date: 17/01/2022
- At "right" granularity.

(M)	T	W	T	F	S	S
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Axioms:

$$0 \leq P(A) \leq 1$$

$$P(\Omega) = 1 \quad (\text{Normalization})$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \leq P(A) + P(B)$$

Event: $\{A \mid A \subseteq \Omega\}$

$$P\left(\bigcup_n A_n\right) = \sum_n P(A_n) \quad \text{iff} \quad \bigcap_n A_n = \emptyset$$

we know that,

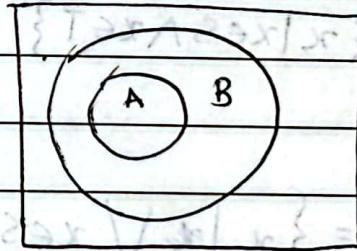
$$A \cup A^c = \Omega \wedge A \cap A^c = \emptyset$$

$$P(A \cup A^c) = P(\Omega)$$

$$P(A) + P(A^c) = 1$$

$$P(A^c) = 1 - P(A) \leq 1$$

$$A \subseteq B \iff P(A) \leq P(B)$$



$$B = A \cup (A^c \cap B)$$

$$P(A \cup A^c) = P(A) + P(A^c) - P(A \cap A^c)$$

$$P(\Omega) = P(A) + 1 - P(A) - P(\emptyset)$$

$$Y = X - P(\emptyset)$$

$$P(\emptyset) = 0$$

$$\cancel{P(B) = P(A)}$$

$$P(B) = P(A \cup (A^c \cap B))$$

$$P(B) = P(A) + P(A^c \cap B) \geq P(A)$$

Two Tetrahedral dies are rolled.

4	14	24	34	44	}
3	13	23	33	43	
2	12	22	32	42	
1	11	21	31	41	
	1	2	3	4	

$X = \text{second}$

$Y = \text{first roll}$

Roll

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Each event is equi-likely

$$P(X=1) = P(\{11, 12, 13, 14\})$$

$$= P(\{11\} \cup \{12\} \cup \{13\} \cup \{14\})$$

$$= P(\{11\}) + P(\{12\}) + P(\{13\}) + P(\{14\}) = 4$$

$$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$$

$$P(X=1) = \frac{1}{4}$$

$$P(Z=2) = P(\{22, 32, 42, 23, 33, 43, 24, 34\})$$

$$= P(\{22\} \cup \{32\} \cup \{42\})$$

$$\cup \{23\} \cup \{33\} \cup \{43\}$$

$$= P(\{22\}) + P(\{32\}) + P(\{42\})$$

$$+ P(\{23\}) + P(\{33\}) + P(\{43\})$$

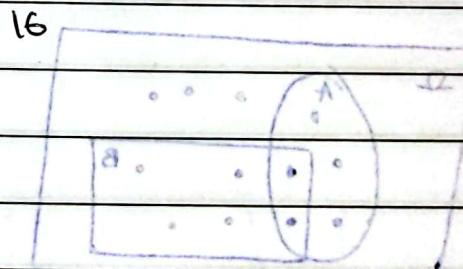
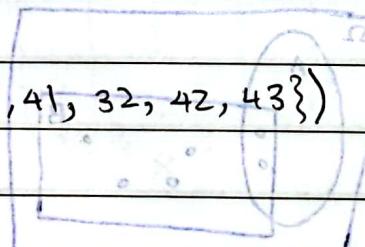
$$+ P(\{24\}) + P(\{34\})$$

$$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$$

$$P(\min(X, Y) = 2) = \frac{5}{16}$$

$$P(X > Y) = P(\{21, 31, 41, 32, 42, 43\})$$

$$= \frac{3}{8}$$



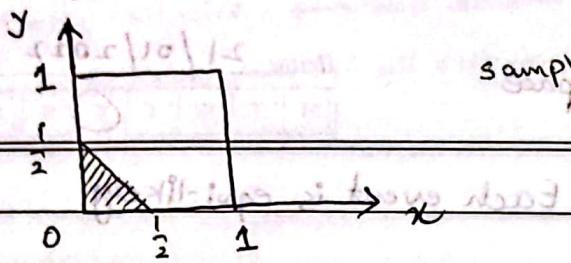
$$P(X+Y=0 \pmod{2}) = P(\{11, 31, 22, 42, 13, 33, 24, 44\})$$

$$= \frac{8}{16} = \frac{1}{2}$$

$$\underline{1} = (8/16) = \underline{1/2}$$

$$0 < (8/16) = (8/16) = (8/16)$$

~~Sample Space~~ $A = \{(x, y) \mid x, y \in [0, 1] \wedge x+y \leq \frac{1}{2}\}$



sample space: $x, y \in [0, 1]$

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$$P(A) = \frac{1}{2} \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}} dx dy = \frac{1}{8}$$

Discrete but infinite sample space.

$$(\{P1, E1, S1, I1\})9 = (E=X)9$$

$$(\{P1\} \cup \{E1\} \cup \{S1\} \cup \{I1\})9 =$$

$$\text{Sample space} = \mathbb{N} = \{P1\}9 + \{E1\}9 + \{S1\}9 + \{I1\}9 =$$

$$\text{Probability Law: } P(n) = \frac{1}{2^n} = \frac{1}{21} + \frac{1}{21} + \frac{1}{21} + \frac{1}{21} =$$

$$P(\Omega) = \sum_{n=1}^{\infty} \frac{1}{2^n} = 1 \quad (S=S)9$$

$$\{SP1\} \cup \{SE1\} \cup \{SS1\}9 =$$

Countable Additivity Axiom

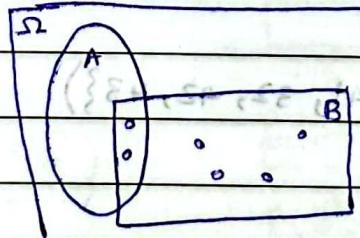
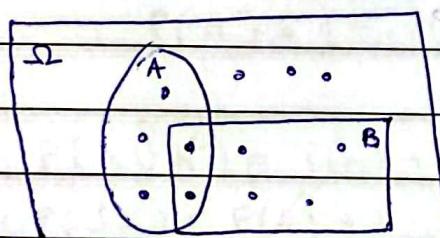
$$(\{PP\})9 = (P=(X, X) \text{ min})9$$

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n) \quad \frac{1}{3} =$$

as $N + N + N + N + N =$

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Conditional Probability. $(Y, X) \text{ min})9$



$$P(A) = \frac{5}{12}, P(B) = \frac{6}{12}$$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ Probability of A given B has occurred.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad P(B) > 0$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{(A \cap (B \cup A))}{(A)} = (A \cap A) = 0$$

$$P(A|B)P(B) = P(A \cap B) \rightarrow \text{Multiplication Rule}$$

$$\frac{((A \cap B) \cup (A \cap A))}{(A)} = (A \cap B) + (A \cap A) = 1$$

$$P(A|B)P(B) = P(B \cap A)$$

$$(B \cap A) = (A \cap B) + (B \cap A) = 1$$

$$P(A|B)P(B) = P(B \cap A) \times P(A) \quad \because P(A) > 0$$

$$(A \cap (B \cap A)) = (A \cap B) + (A \cap A) = 1$$

$$P(A|B)P(B) = P(B|A)P(A)$$

$$\text{Bayes' Theorem} \quad (A \cap (B \cap A)) = (A \cap B) + (B \cap A) = 1$$

If Ω is a sample space $|\Omega| \in \mathbb{N}$ and B is an event $B \neq \emptyset$.

The discrete uniform law for Ω does not hold after B occurs.

$$\Omega \cap (A \cap B) = (A \cap B) = 1$$

$$(A \cap (A \cap B)) = (A \cap B) = 1$$

$$\text{After } B \text{ occurs, } P(\{x\}) = 0$$

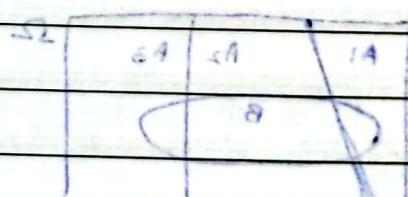
$$y = \text{Second Roll} \quad 4 \quad | \quad (B \cap A) \cup (B \cap A) = 0$$

$$x = \text{First Roll} \quad 3 \quad | \quad (B \cap A) \cup (B \cap A) = 0$$

$$0 = (A \cap B) + (A \cap B) = 0$$

$$(A \cap B) + (A \cap B) = 0$$

$$P(M=1|B) = 0$$



$$\Omega = \{A \cap A, A \cap B, B \cap A, B \cap B\}$$

$$P(A|B) \geq 1$$

$$P(\Omega|B) = 1$$

$$P(A|A) = 1$$

$$\text{If } A \cap C = \emptyset \Rightarrow P(A \cup C|B) = P(A|B) + P(C|B) = 0$$

$$(A \cap A) \cup (B \cap A) \cup (B \cap A) = 0$$

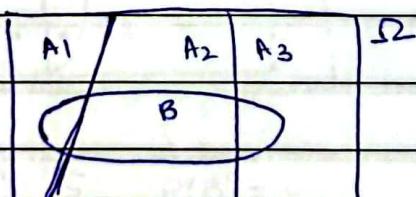
$$\begin{aligned}
 P(A \cup C | B) &= \frac{P((A \cup C) \cap B)}{P(B)} \\
 &= \frac{P(A \cap B) \cup (C \cap B)}{P(B)} \\
 &= \frac{P(A \cap B) + P(C \cap B) - P(A \cap B \cap C)}{P(B)} \\
 &= \frac{P(A \cap B)}{P(B)} + \frac{P(C \cap B)}{P(B)} - \frac{P((A \cap C) \cap B)}{P(B)} \\
 P(A \cup C | B) &= P(A | B) + P(C | B) - P(A \cap C | B)
 \end{aligned}$$

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Q: 8 दोनों घटनाएँ A तथा B का अन्तर्गत हैं। $P(A) = 0.2$, $P(B) = 0.3$, $P(A \cap B) = 0.1$ तथा $P(A \cup B) = 0.5$ हैं। घटना C का अन्तर्गत है, जिसका प्रायिकता $P(C | A \cup B) = 0.4$ है। घटना C का प्रायिकता क्या है?

परिवर्तन का नियम:

$$\begin{aligned}
 P(A^c \cap B \cap C^c) &= P(A^c \cap B) P(C^c | A^c \cap B) \\
 &= P(A^c) P(B | A^c) P(C^c | A^c \cap B) \\
 P\left(\bigcap_{i=1}^n A_i\right) &= P(A) \quad \text{जब } i=2 \\
 &= P(A_i | \bigcap_{j=i+1}^{i-1} A_j) \\
 (A \cap B) \cup (A^c \cap B) &= P(A) P(B | A) + P(A^c) P(B | A^c) = 0 \\
 P(B) &= P(A) P(B | A) + P(A^c) P(B | A^c) \\
 \text{Total Probability Theorem.}
 \end{aligned}$$



$$A_1 \cup A_2 \cup A_3 = \Omega \wedge A_1 \cap A_2 \cap A_3 = \emptyset$$

A_1, A_2 and A_3 are partitions of Ω

$$P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$$

~~$B = \Omega \cap B$~~

$$P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$$

$$\begin{aligned}
 B &= (A_1 \cup A_2 \cup A_3) \cap B \\
 &= (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B)
 \end{aligned}$$

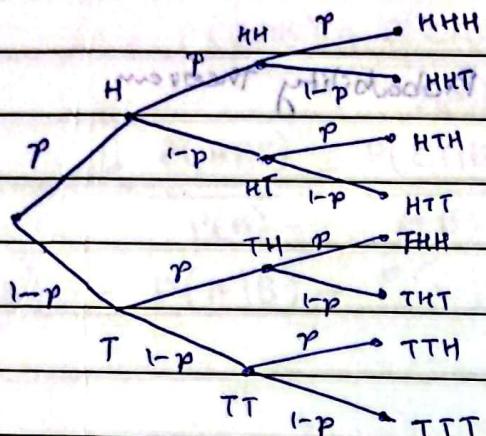
$$P(H) = p \wedge P(T) = 1-p$$

$$(A \cap A)9 + (B \cap A)9 + (A \cap A)9 = (A)9$$

$$(A|B)9 (A|A)9 + (A|B)9 (A|A)9 + (A|A)9 (A|A)9 = (A)9$$

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$$P(T|H) = P(T)$$

$$P(T|T \cap H) = P(T)$$

$$P(T|T \cap H) = P(T)$$

$$P(1 \text{ head}) = P(\{HTT, THT, TTH\}) = P(\{HTT\}) + P(\{THT\}) + P(\{TTH\})$$

$$= p(1-p)^2 + p(1-p)^2 + p(1-p)^2 = (3p(1-p)^2)$$

$$= 3p(1-p)^2$$

$$(0.5 \times 0.5)9 (0.5 \times 0.5)9 + (0.5 \times 0.5)9 (0.5 \times 0.5)9 = (1.5)9$$

$$P(1 \text{ head}) =$$

$$P(\text{First is head} | 1 \text{ Head}) = P(\text{First head} | 1 \text{ Head})$$

$$= \frac{P(1 \text{ Head})}{P(1 \text{ Head})} = \frac{2p \cdot 0.5}{2p \cdot 0.5} = (1)9$$

$$= \frac{P(1-p)^2}{3(1-p)^2}$$

$$= \frac{1}{3}$$

$$P(\text{First is head} | 1 \text{ Head}) = \frac{1}{3}$$

$$(0.5 \times 0.5)9 (0.5 \times 0.5)9 = (1.5)9$$

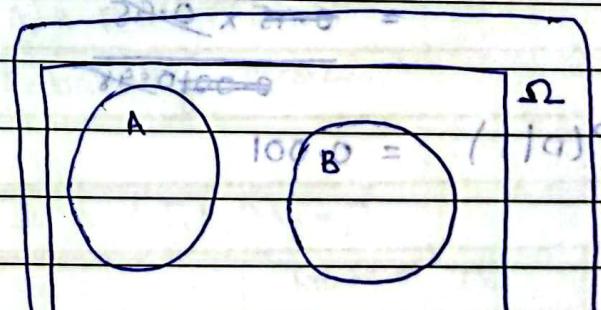
$$(1.5)9$$

Independence of 2 ~~set~~ events.

$$\text{#} P(A \cap B) = P(A) P(B)$$

iff

$$P(A|B) = P(A) \wedge P(B|A) = P(B)$$



$$P(A) > 0, P(B) > 0$$

$P(A \cap B) = 0 \Rightarrow P(A)P(B) = 0$ but
 $P(A) > 0$ and $P(B) > 0$ this contradicts
 Therefore A and B are dependent.

If A be an event, a subset of the sample space Ω . Are A and Ω independent?

$$A \subseteq \Omega$$

$$(5/8)^9 (5/1)^9 = (5/8 \cap A)^9$$

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$$P(A|\Omega) = P(A)$$

$$P(\Omega|A) = P(\Omega) = 1$$

$$P(\Omega \cap A) = P(A) P(\Omega) = P(A)$$

They are independent. $(5/8)^9 (5/1)^9 = (5/8 \cap A)^9 \Leftrightarrow (5/8)^9 (5/1)^9 = (5/8 \cap A)^9$

$$0 = (5/8 \cap A)^9 \therefore$$

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$$A = (A \cap B^c) \cup (A \cap B)$$

$$P(A) = P((A \cap B^c) \cup (A \cap B))$$

$$= P(A \cap B^c) + P(A \cap B)$$

$$P(A) = P(A \cap B^c) + P(A) P(B)$$

$$P(A \cap B^c) \leq P(A) (1 - P(B))$$

$$P(A \cap B^c) = P(A) P(B^c) \text{ (iff) } P(A \cap B) = P(A) P(B)$$

$$P \cdot 0 = (A \cap B) \cap H^9$$

$$1 \cdot 0 = (B \cap H)^9$$

$$(1 - P(A)) (1 - P(B)) = 1 - (P(A) + P(B)) + P(A) P(B)$$

$$= 1 - (P(A \cup B) + P(A \cap B)) + P(A) P(B)$$

$$= 1 - P(A \cup B) - P(A \cap B) + P(A) P(B)$$

$$= 1 - P(A \cup B) - P(A) P(B) + P(A) P(B)$$

$$= 1 - P(A \cup B)$$

$$= P(A^c \cap B^c)$$

If A and B are independent

then A^c and B^c are also

independent.

$$(A \cap B)^9 \cap (H = 11 \text{ and } 9) (A \cap B)^9 = (H = 11 \text{ and } 9)$$

$$(B \cap H)^9 \cap (H = 11 \text{ and } 9) (B \cap H)^9 +$$

$$\left(1 \cdot 0\right) \left(\frac{1}{2}\right) + \left(P \cdot 0\right) \left(\frac{1}{2}\right) =$$

$$\frac{1}{2} =$$

Given three events A, B, and C.

Set to be Ω , then we get A \cap Ω

$$P(A \cap B | C) = P(A | C) P(B | C)$$

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if this equation holds, then

A and B are independent

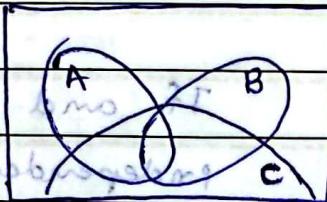
given C.

$$(A) \Omega = (\Omega | A) \Omega$$

$$I = (\Omega) \Omega = (A | \Omega) \Omega$$

$$P(A \cap B | C) = P(A | C) P(B | C) \Rightarrow P(A \cap B | C^c) = P(A | C^c) P(B | C^c)$$

$\therefore P(A \cap B | C^c) = 0$



Ω

$$(A \cap A) \cup (\Omega \cap A) = A$$

$$((A \cap A) \cup (\Omega \cap A)) \Omega = (A) \Omega$$

$$(A \cap A) \Omega + (\Omega \cap A) \Omega =$$

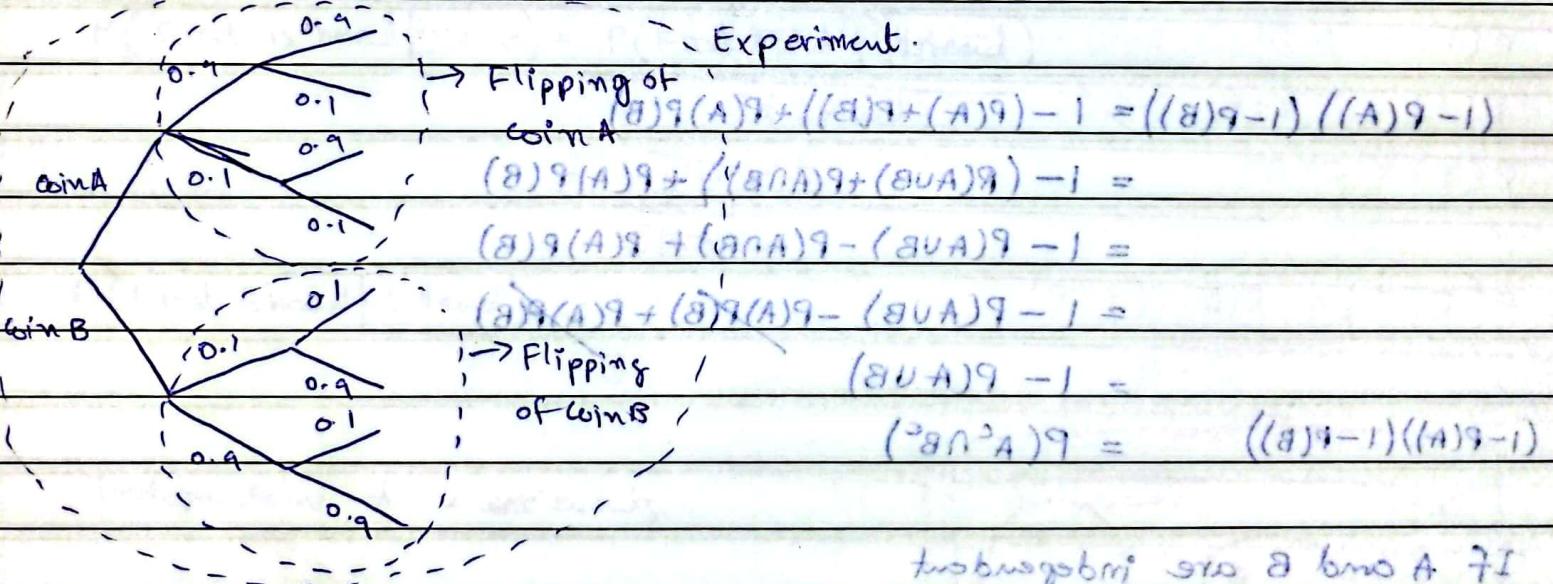
$$(A) \Omega (A) \Omega + (\Omega \cap A) \Omega = (A) \Omega$$

$$(A) \Omega - 1 / 2 = (A) \Omega$$

$$P(H | \text{coin } A) = 0.9$$

$$(A) \Omega (A) \Omega P(\text{coin } A) + P(\text{coin } B) = 1 / 2 = (A) \Omega$$

$$P(H | \text{coin } B) = 0.1$$



$$P(\text{toss } 11 = H) = P(\text{coin } A) P(\text{toss } 11 = H) + P(\text{coin } B)$$

$$+ P(\text{coin } B) P(\text{toss } 11 = H) \text{ coin } B$$

$$= \left(\frac{1}{2}\right)(0.9) + \left(\frac{1}{2}\right)(0.1)$$

$$= \frac{1}{2}$$

$$P(\text{toss } 11 = H \mid \text{first 10 tosses are } H) = \frac{P(\text{toss } 11 = H \cap \text{first 10 tosses are } H)}{P(\text{first 10 tosses are } H)}$$

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$$P(\text{first 10 tosses are } H) = P(\text{coin A}) P(\text{first 10 tosses are } H \mid \text{coin A})$$

$$P(\text{first 10 tosses are } H) = P(\text{coin B}) P(\text{first 10 tosses are } H \mid \text{coin B})$$

$$= \frac{1}{2} (0.9)^{10} + \frac{1}{2} (0.1)^{10}$$

$$P(\text{first 10 tosses are } H) = 0.1743$$

$$\binom{10}{5} = 120 \text{ ways, } A \in \mathbb{C}$$

$$P(\text{toss } 11 = H \cap \text{first 10 tosses are } H) = P(\text{11 tosses are Heads})$$

$$= P(\text{coin A}) P(\text{11 tosses are heads} \mid \text{coin A}) + P(\text{coin B}) P(\text{11 tosses are heads} \mid \text{coin B})$$

$$= \frac{1}{2} (0.9)^{11} + \frac{1}{2} (0.1)^{11}$$

$$P(\text{toss } 11 = H \cap \text{first 10 tosses are } H) = 0.1569$$

$$P(\text{toss } 11 = H \mid \text{first 10 tosses are } H) = 0.899$$

Independence of collection of sets, $(q-1)^{q-1} = (q-1)^{q-1} = (q-1)^{q-1}$

$A_1, A_2, A_3, \dots, A_n$

$$P\left(\bigcap_i A_i\right) = \prod_i P(A_i) = (q-1)^{q-1}$$

~~$$P(A_1 \cap A_2 \cap \dots \cap A_n) = \prod_{k=1}^n P(A_k)$$~~

check

$$1 \times 2 \times \dots \times n = n!$$

$$\frac{1 \times 2 \times \dots \times n}{1 \times 2 \times \dots \times n \times 1} = \binom{n}{n}$$

For $|\Omega| = n$, if we have to arrange k elements such that $k \leq n$
 in such a way that order matters, then
 if repetition is allowed: $= n^k$

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if repetition not allowed: $= \frac{n^k}{(n-k)!} = {}^n P_k$

for any set $A \Rightarrow |A|=n$ if $C \subseteq A \Rightarrow |C|=k$ and
 $C \subseteq A$, then $|B|= \binom{n}{k}$.

$$E(HF) = 0 = (H \text{ and } F \text{ both not})$$

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$

Let $P(H) = p$ and $P(T) = 1-p$ Independent tosses.

$$P(HTTHHH) = p^4(1-p)^2$$

$$P(\text{particular seq. of } H \text{ and } T) = p^{\# \text{ of } H} (1-p)^{\# \text{ of } T}$$

$$P(\text{a particular } k\text{-head seq.}) = p^k (1-p)^{n-k}$$

$$P(k \text{ heads in } n \text{ flip}) = \binom{n}{k} p^k (1-p)^{n-k}$$

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Number of partition of a set of size n into n_1, n_2, \dots, n_r sized sets such that, $n_1 + n_2 + \dots + n_r = n$, will be,

$$\frac{n!}{n_1! n_2! \dots n_r!}$$

$$\binom{n}{n_1, n_2, \dots, n_r} = \frac{n!}{n_1! n_2! \dots n_r!}$$

with replacement

n^r

Order matters

Order doesn't matter

$$\binom{n+r-1}{r}$$

without replacement

PP	PE	EE	EE	P	S
EP	EE	EE	EE	E	S
EE	EE	EE	EE	E	S

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${}^n C_r$ (M, T, W, T, F, S, S)

P Non Repetition = X

10.1 Discrete Random Variables

Example Random Variable

$$X: \Omega \rightarrow \mathbb{R}$$

ii. Sample Space

ii. $\Omega = \{0, 1, 2, \dots, n-1\}$ $\text{Range} = \{0, 1, 2, \dots, n-1\}$

q = 75% chance

$$\omega \in \Omega, X(\omega) = x$$

Probability measure function: \mathbb{P}

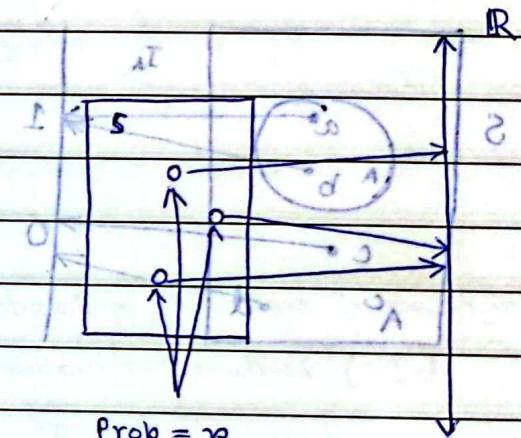
There can be more than one random variables on a sample space. Any function of those random variables is also a random variables.

$$\mathbb{I} = (\omega)_{\mathbb{A}}$$

$$\mathbb{I} = (\omega)_{\mathbb{A}}$$

$$\mathbb{O} = (\omega)_{\mathbb{A}}$$

$$\mathbb{O} = (\omega)_{\mathbb{A}}$$

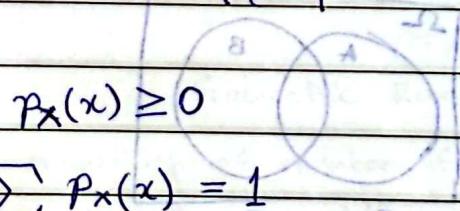


Prob = p

Probability Mass Function (PMF)

$$p_X(x) := P(X=x)$$

$$= P(\{\omega \mid \omega \in \Omega \wedge X(\omega) = x\})$$



$$p_X(x) \geq 0$$

$$\sum_x p_X(x) = 1$$

$$(A)q = (\mathbb{I} = \mathbb{A})q = (\mathbb{I})_{\mathbb{A}}q$$

smallest set
easier to find
A

subset of A, becomes
subset of A, in A
subset of A, in A
subset of A, in A

A is in A.

$$(B \cap A)q = (\mathbb{I} = \mathbb{B} \cap \mathbb{A})q = (\mathbb{I})_{\mathbb{B} \cap \mathbb{A}}q$$

4	14	24	34	44
3	13	23	33	43
2	12	22	32	42
1	11	21	31	41

$X = \text{First Roll}$

$(1-3+1)$

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Bernoulli Random Variable.

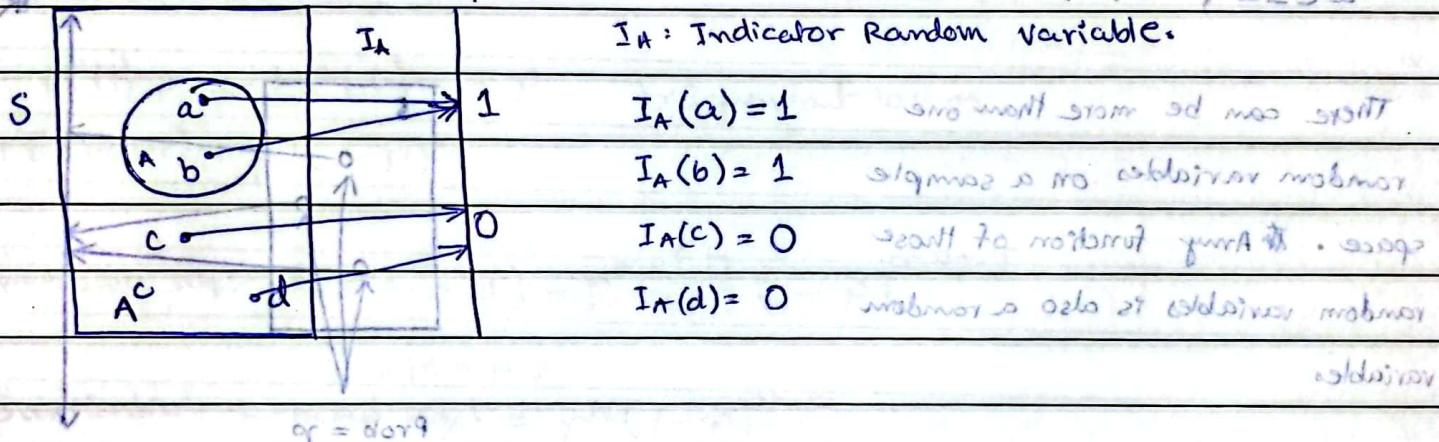
parameter: = p

~~$\{0, 1\}$~~ $\rightarrow \{B\}$

$X := \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases}$

Used when there is one trial with two possibilities.

$x = (\omega)X, \Omega \ni \omega$



$$P_{I_A}(1) = P(I_A = 1) = P(A)$$

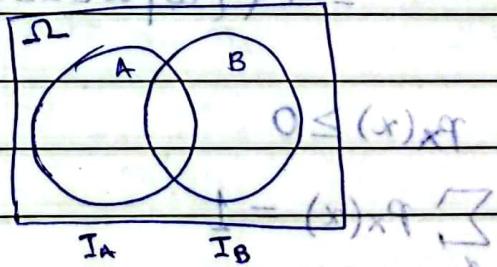
The outcome is from event A.

without loss of generality (T.M.Q)

$$(x = X) \Omega = \{x | x \in \Omega\}$$

$$(\{x = (\omega)X \wedge \Omega \ni \omega | \omega\}) \Omega =$$

$I_A + I_B$ is not an IRV, because in $A \cap B$ ~~IRV~~ $I_A + I_B = 2$, which is absurd.



$I_A I_B$ is an IRV.

$$P_{I_A I_B}(1) = P(I_A I_B = 1) = P(A \cap B)$$

Discrete Uniform random variable.

parameter: $a, b \rightarrow$ ending number.
starting number

$$\Omega := \{a, a+1, a+2, \dots, b-1, b\}$$

Model of complete ignorance

$$q^{\Omega} = q^{b-a} = q^{(q-1)}$$

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$$\text{Random Variable: } X(\omega) = \omega + \frac{0}{q-1} + \frac{1}{q-1} + \dots + \frac{q-1}{q-1} = \omega + \frac{q-1}{q-1} = \omega + 1$$

$$p_X(\omega) = P(X=\omega) = \frac{1}{q-1} = \frac{1}{q-1}$$

special case:-

$$a=b \quad p_X(\omega) = P(X=\omega) = 1 = (0 \leq X)q$$

X is called constant / Deterministic Random variable.

$$(0 \leq X)q-1 = (0 \leq X)q$$

Binomial Random Variable $X = \sum_{i=1}^n I_{\{X_i=1\}}$

Parameters: $n :=$ number of independent trials.

$$p \in [0, 1]$$

$$q^{n-q} =$$

$$q^{q-1} =$$

conditions: n trials are independent, and the probability of success in each trial remain same. we call it (Independent) ~~and~~ Identical (IID)

Random Variable X : # of success observed.

$$p^{q-1} = (0 \leq X)q$$

$$p_X(k) = P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

↳ ~~more trials~~ more probability of success

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Geometric Random Variable.

$$(0 \leq X)q = [X]q$$

Tells the probability of number of trials took until first success.

number of failures until success.

$$p(\text{success}) = p$$

$$p_X(k) = P(X=k) = (1-p)^{k-1} p$$

X is a G.R.V.

$$P(X \geq 10) = \sum_{i=10}^{\infty} P(X=i)$$
$$= \sum_{i=10}^{\infty} (1-p)^{i-1} p$$

$$= p \left((1-p)^9 + (1-p)^{10} + \dots \right) = (w) X = \text{Expected value}$$
$$= p \frac{(1-p)^9}{1 - (1-p)} = (w=X) 9 = (w) \times q$$
$$= p (1-p)^9$$

$$P(X \geq 10) = (1-p)^9 = (w) \times q \quad d=0$$

$$P(X \geq 10) = 1 - P(X < 10)$$

$$= 1 - \sum_{i=1}^9 P(X=i)$$
$$= 1 - \sum_{i=1}^9 (1-p)^i p$$
$$= 1 - p \frac{1 - (1-p)^9}{1 - (1-p)}$$

$$(b) \text{ Let } P(X \geq 10) = p \quad \text{then } 1-p = q$$

$$= 1 - p \frac{1 - (1-p)^9}{1 - (1-p)}$$

$$P(X \geq 10) = (1-p)^9$$

$$\text{Expectation: } \sum_{i=1}^{\infty} i p_i = (w=X) q = (w) \times q$$

also called population average,

$$E[X] = \sum x p_x(x)$$

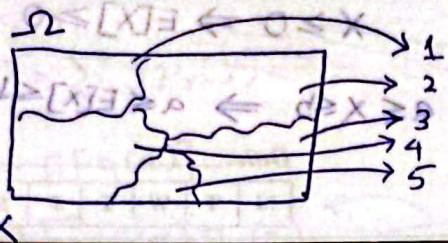
also called population average,

• x success if not x fails to minimum to maximum not else

• x success if x equals to minimum

$$q = (w) \times q$$

$$q^{1-q} (q-1) = (w=X) q = (w) \times q$$



Probabilistic Model

Sample Space

Probability Date: 25/02/2022

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Random Variable - maps some subset of sample space to a unique number in order to identify events.

$$\varepsilon < r < s$$

$$\frac{(q-1)q}{1m} = \frac{1m}{(1-m)(1-s)}$$

Variance

Tells how much data is spreaded.

$$\text{Var}(X) = E[X^2] - (E[X])^2 = E[(X-\mu)^2] = \sigma^2 (1-m) \left(\frac{1}{1-s} \right) =$$

$$\text{Poisson Random Variable} = \sum_{k=0}^m \frac{(q-1)q}{(1-m)} = E[X]$$

$$p_X(x) = P(X=x) = e^{-\lambda} \lambda^x$$

$$= \frac{(q-1)q}{(1-m)} \left(\frac{m}{1} \right) + x! + \frac{(q-1)q}{(1-m)} \left(\frac{m}{1} \right) (1-m) = 25/02/2022$$

$$\text{Expectation: } E[X] = \sum_{k=0}^m k p_X(k) = \sum_{k=0}^m k \left(\frac{(q-1)q}{(1-m)} \right) (1-m) =$$

Bernoulli Random Variable

$$E[X] = (0)p_X(0) + (1)p_X(1)$$

$$E[X] = p$$

$$\left(\frac{q}{q-1} \right) \times (1) = E[X] =$$

Uniform Random Variable

$$E[X] = (0)p_X(0) + (1)p_X(1) + \dots + (m)p_X(m) =$$

$$= \frac{0}{m+1} + \frac{1}{m+1} + \dots + \frac{m}{m+1}$$

$$= \frac{1}{m+1} \times (1+2+3+\dots+m)$$

$$= \frac{1}{m+1} \times \frac{m(m+1)}{2}$$

$$\text{Page No. } E[X] = m/2$$

RC

Signature

Binomial Random Variable

$$X \geq 0 \Rightarrow E[X] \geq 0$$

$$E[X] = \sum_{k=0}^n k \binom{n}{k} p^k (1-p)^{n-k} \quad (i)$$

$$a \leq X \leq b \Rightarrow a \leq E[X] \leq b$$

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$$\begin{aligned}
 E[X] &= \sum_{k=0}^n k \binom{n}{k} p^k (1-p)^{n-k} \\
 &= \sum_{k=0}^n \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} \\
 &= \sum_{k=0}^n \frac{n!}{(k-1)!(n-k)!} p^k (1-p)^{n-k} \\
 &= \sum_{k=0}^n \binom{n}{k-1} (n-k+1) p^k (1-p)^{n-k} \quad E[X] = ((x-4) \leq x \leq 7) = (x) \text{ for } -3 \leq -y \leq 2
 \end{aligned}$$

$$E[X] = \sum_{k=0}^n (n-k) \binom{n}{n-k} p^{n-k} (1-p)^k \quad (ii)$$

$$\begin{aligned}
 2E[X] &= \sum_{k=0}^n (n-k) \binom{n}{n-k} p^{n-k} (1-p)^k + \sum_{k=0}^n k \binom{n}{k} p^k (1-p)^{n-k} \\
 &= \sum_{k=0}^n \left\{ (n-k) \binom{n}{k} p^n \left(\frac{1-p}{p}\right)^k + \sum_{k=0}^n \left\{ k \binom{n}{k} (1-p)^n \left(\frac{p}{1-p}\right)^k \right\} \right\} \\
 &= \sum_{k=0}^n \binom{n}{k} \left\{ (n-k) p^n \left(\frac{1-p}{p}\right)^k + k (1-p)^n \left(\frac{p}{1-p}\right)^k \right\} \\
 &= E[X] = \sum_{k=0}^n \binom{n}{k} p^k (1-p)^k \quad E[X] = E[X]
 \end{aligned}$$

$$E[X] = p(1-p)^{n+1} \sum_{k=0}^n \binom{n}{k} p^k (1-p)^k = p(1-p)^{n+1} (1) + (0) = E[X]$$

$$\frac{1}{1+x} + \dots + \frac{1}{1+x} + \frac{1}{1+x} =$$

$$(1+x) + \dots + 1 + \frac{1}{1+x} =$$

$$\frac{(1+x)^n}{1+x} =$$

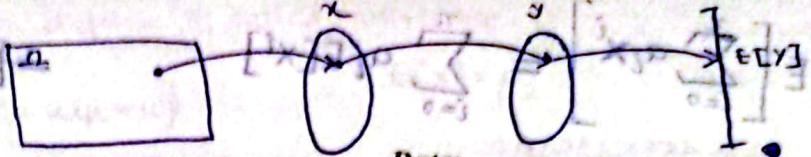
$$Y = g(X)$$

$$E[Y] = \sum_y y p_Y(y)$$

$$E[Y] = E[g(X)]$$

$$= \sum_x g(x) p_X(x)$$

Expected value rule



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$$(x) \underset{x \in \Omega}{\mathbb{E}[x]} = [x] \mathbb{E}$$

$$(x) \underset{x \in \Omega}{\mathbb{E}[x]} = \text{Date: } 02/03/2022$$

$$\sum_y \sum_{x: g(x)=y} g(x) p_X(x)$$

$$[x] \mathbb{E} =$$

$$E[x^2] \neq (E[x])^2$$

$$F = [x] \mathbb{E} \text{ bmo } S = [x] \mathbb{E} \text{ IT}$$

$$= \sum_y \sum_{x: g(x)=y} y p_X(x)$$

$$[x] \mathbb{E} - 8 = [x-8] \mathbb{E}$$

$$5 - 8 =$$

$$= \sum_y y \sum_{x: g(x)=y} p_X(x)$$

Summing over

$$\delta =$$

$$= \sum_y y p_Y(y)$$

particular value of Y

$$[p] \mathbb{E} = E[(x+x)(x-x)] \mathbb{E}$$

$$= E[Y]$$

$$P-F \mathbb{E} =$$

uniform Random variable =

$$\exists \Omega = \{-1, 0, 1, 2\} \ni \forall \exists X(\Omega) \rightarrow \mathbb{R}, E[X^4] = ?$$

following problem more; 09

$$E[X^4] = \sum_{x \in \{-1, 0, 1, 2\}} x^4 p_X(x) = \sum_{x \in \{-1, 0, 1, 2\}} x^4 \left(\frac{1}{4}\right) = \frac{1}{4} \underset{!x}{\sum_{x=-1}^2} x^4 = (x) \underset{!x}{\mathbb{E}}$$

$$= \frac{1}{4} \left((-1)^4 + (0)^4 + (1)^4 + (2)^4 \right) = \frac{1}{4} \underset{!x}{\sum_{x=-1}^2} x^4 = \frac{1}{4} \underset{!x}{\sum_{x=-1}^2} x^4 = [x] \mathbb{E}$$

$$E[X^4] = 4 \cdot 5$$

$$\frac{1}{4} \underset{!x}{\sum_{x=-1}^2} x^4 =$$

$$\frac{x^4}{!x} \underset{!x}{\sum_{x=-1}^2} =$$

$$x^4 \underset{!x}{\sum_{x=-1}^2} =$$

$$x = [x] \mathbb{E}$$

$$E\left[\sum_{i=0}^n a_i x^i\right] = \sum_{i=0}^n a_i E[x^i]$$

~~$E[g(x)] = g(E[x])$~~

$(x)_E = x$

$E[Y] = E$

Proof:-

↓ Einstein Notation.

$$E[a_i x^i] = \sum_x a_i x^i p_x(x)$$

$$= a_i \sum_x x^i p_x(x)$$

$$= a_i E[x^i]$$

$$(x)_E \neq [x]_E$$

$$\text{If } E[x] = 2 \text{ and } E[x^2] = 7$$

$$E[8-x] = 8 - E[x]$$

$$= 8 - 2$$

$$= 6$$

$$E[(x-3)(x+3)] = E[x^2 - 9]$$

$$= E[x^2] - 9E[1]$$

$$= 7 - 9$$

$$= -2$$

Poisson Random Variable

$$p_x(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$E[x] = \sum_{x=0}^{\infty} x \frac{e^{-\lambda} \lambda^x}{x!} = \sum_{x=1}^{\infty} \frac{e^{-\lambda} \lambda^x}{x!} =$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^{x+1}}{x!}$$

$$= \frac{e^{-\lambda} \lambda}{x!} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda} \lambda e^{\lambda}$$

$$E[x] = \lambda$$

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Variance :- The amount of data disperse is called variance. $E[X] = \mu$

$$\text{var}(x) = E[(x-\mu)^2] = E[X^2 - 2x\mu + \mu^2]$$

$$= E[X^2] - 2\mu E[X] + \mu^2$$

$$= E[X^2] - 2\mu\mu + \mu^2$$

$$= E[X^2] - 2\mu^2$$

$$\text{var}(x) = E[X^2] - (E[X])^2$$

$$\text{SD}(x) = \sqrt{\text{var}(x)} = \sigma(x)$$

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Variance is conserved under linear transformation of PMFs along x-axis

$$\text{var}(x+b) = \text{var}(x)$$

Proof :-

$$E[X+b] = \mu + b$$

$$\begin{aligned} \text{var}(x+b) &= E[(x+b) - (\mu+b)]^2 \\ &= E[(x-\mu)^2] \\ &= \text{var}(x) \end{aligned}$$

$$\text{var}(ax) = a^2 \text{var}(x)$$

Proof :-

$$E[ax] = a\mu$$

$$\begin{aligned} \text{var}(ax) &= E[(ax - a\mu)^2] \\ &= a^2 E[(x-\mu)^2] \end{aligned}$$

$$\text{var}(ax) = a^2 \text{var}(x)$$

Bernoulli Random Variable :-

$$\text{var}(x) = E[(x-\mu)^2]$$

$$= E[(x-p)^2]$$

$$= (0-p)^2(1-p) + (1-p)^2(p)$$

$$= p(1-p)(p+1-p)$$

$$\text{var}(x) = p(1-p)$$

$$\left(\frac{m}{2} - \frac{1+m}{8} \right) \frac{m}{2} =$$

$$\left(\frac{m}{2} - \frac{1+m}{8} \right) \frac{m}{2} =$$

$$\left(\frac{m}{2} - \frac{1+m}{8} \right) \frac{m}{2} = (X)_{\text{var}}$$

$$(s+p-d) \frac{(d-d)}{s} = (Y)_{\text{var}}$$

For $p=1$ or 0 , in $X \sim \text{Ber}(p)$, $\text{Var}(X)=0$, means the experiment is bias.

$$\mu = [x] \exists$$

$$[24 + 14x_2 - x_3] \exists = [^2(\mu) - x] \exists = (x) \text{Var}$$

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Uniform Random Variable,

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$$X: \{0, 1, 2, \dots, n\} \rightarrow \mathbb{R} \quad [x] \exists =$$

$$\mu = \frac{n}{2}$$

$$[x] \exists - [x] \exists = (x) \text{Var}$$

$$(x) \text{Var} = \sqrt{(x) \text{Var}} = (x) \text{SD}$$

~~$$\text{Var}(x) = E[x^2] - (E[x])^2$$~~

~~$$= \sum_{i=0}^n i^2 \frac{1}{n} - \left(\frac{n}{2}\right)^2$$~~

Advantages of calculating variance

~~$$= \frac{1}{n+1} \sum_{i=0}^n i^2 - \frac{n^2}{4}$$~~

~~$$E[x] = [x] \exists$$~~

~~$$= \frac{1}{2n+1} \frac{2n(2n+1)(2n+1) - (n^3)}{6} = (d+x) \text{Var}$$~~

~~$$[x] \exists =$$~~

~~$$(x) \text{Var} =$$~~

~~$$E[x] = \sum_{i=0}^n i \frac{1}{n+1} = \frac{n}{2} = \mu$$~~

~~$$(x) \text{Var} = (x) \text{Var}$$~~

- 2022

$$\text{Var}(x) = E[x^2] - (E[x])^2$$

$$E[x] = [x] \exists$$

$$= \sum_{i=0}^n i^2 \frac{1}{n+1} - \frac{n^2}{4} [x] \exists = (x) \text{Var}$$

$$[x] \exists =$$

$$= \frac{1}{n+1} \times \frac{n(n+1)(2n+1)}{6} - \frac{n^3}{4} = (x) \text{Var}$$

$$= \frac{n}{2} \left(\frac{2n+1}{3} - \frac{n}{2} \right)$$

Advantages of variance

$$= \frac{n}{2} \left(\frac{4n+2 - 3n}{6} \right)$$

$$[x] \exists = (x) \text{Var}$$

$$[x] \exists =$$

$$\text{Var}(x) = \frac{n}{12} (n+2)$$

$$(q) [x] \exists + (q-1) [x] \exists =$$

$$(q-1+q) (q-1) q =$$

$$\text{If } X: \{a, a+1, a+2, \dots, b\} \rightarrow \mathbb{R}$$

$$(q-1) q = (x) \text{Var}$$

$$\text{Var}(x) = \frac{(b-a)}{12} (b-a+2)$$

conditional PMFs and Expectations-

7M9 2019/2020

$$p_x(x) = P(X=x) \quad \text{unconditional PMF} \quad \text{Date: } \boxed{\text{M T W T F S S}}$$

$$p_{x|A}(x) = P(X=x|A)$$

$$p_{x|A}(x) = P(X=x|A) = P(\{\omega : X(\omega) = x\} \cap \{\omega' : \omega' \in A\})$$

$$\{\bar{\omega} = (\omega) \text{ s.t. } \omega \in A \text{ & } \omega \in \Omega \text{ s.t. } \omega \in A\} \cap \{\omega' : \omega' \in A\} = \{\bar{\omega} = \bar{\omega} \text{ b.s.t. } \bar{\omega} \in A\} = (\bar{\omega}, \bar{\omega}) \in A$$

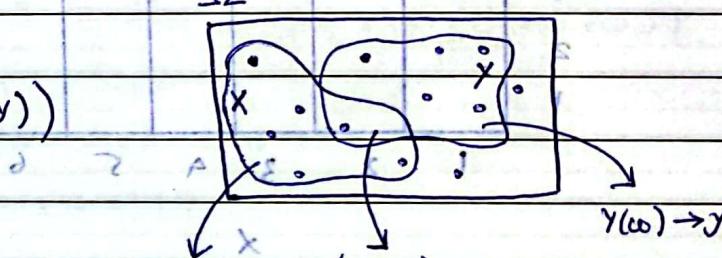
$$\sum_x p_{x|A}(x) = 1 \quad \underline{\Sigma} = E[g(x)|A] = \sum_x g(x) p_{x|A}(x)$$

$$E[X|A] = \sum_x x p_{x|A}(x)$$

09/03/2022

Joint PMFs.

$$p_{x,y}(x,y) = P(X=x \wedge Y=y)$$



$$p_{x,y}(x,y) = P(\{\omega : X(\omega) = x \wedge Y(\omega) = y\}) \quad X(\omega) \rightarrow x \quad Y(\omega) \rightarrow y$$

4	$1/20$	$2/20$	$2/20$		$\sum_{i=1}^4 \sum_{j=1}^5 p_{x,y}(x,y) = 1$
3	$2/20$	$4/20$	$3/20$	$2/20$	$\sum_{i=1}^4 \sum_{j=1}^5 p_{x,y}(x,y) = 1$
2		$1/20$	$3/20$	$Y/20$	$\sum_{i=1}^4 \sum_{j=1}^5 p_{x,y}(x,y) = 1$
1		$1/20$			$\sum_{i=1}^4 \sum_{j=1}^5 p_{x,y}(x,y) = 1$
	1	2	3	4	X

$$p_{x,y}(1,3) = \frac{2}{20} = \frac{1}{10}$$

$$p_x(4) = \sum_{y=1}^5 p_{x,y}(4,y) = \frac{3}{20}$$

$$p_Y(y) = \sum_x p_{x,y}(x,y)$$

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For 2 rolls of D_6 ,

$$\{x = (\omega)X : \omega\}^q = (x = X)^q = (x)^q$$

$$\{A \ni \omega : \omega \in \omega\} \cap \{x = (\omega)X : \omega\}^q = (A | x = X)^q = (A)^q$$

$$p_{x,y}(3,5) = p(x=3 \text{ and } y=5) = p\{\omega : \omega \in \Omega \wedge \min(\omega) = 3 \wedge \max(\omega) = 5\}$$

$$(x)_{A|x}^q \text{ (28)} \quad \boxed{3} = [A | (x)_R]_3 = \frac{2}{36} \quad 1 = (x)_{A|x}^q, \boxed{3}$$

$$6 \quad | \quad | \quad | \quad | \quad | \quad | \quad (x)_{A|x}^q \times \boxed{3} = [A | X]_3$$

$$500 \times 100 \times 100 \quad \checkmark$$

4

3

2

1

0

✓

27M9 2010

$$(x = X) \wedge (x = x) \quad (x = X)_{x,x}^q = (x,x)_{x,x}^q$$

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

$$v \leftarrow (\omega)X$$

$$v, x \leftarrow (\omega)X \cap X$$

$$x \leftarrow (\omega)X$$

$$\{x = (\omega)X \wedge x = (\omega)X : \omega\}^q = (v, x)_{v,x}^q$$

$$p_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) = p\left(\bigwedge_{i=1}^n (x_i = x_i)\right)$$

$$1 = (x, x)_{v,x}^q \quad \boxed{3} \quad \boxed{3}$$

$$= p\left(\{\omega : \bigwedge_{i=1}^n (x_i(\omega) = x_i)\}\right)$$

n	0.15	0.15	0.11	p
0.15	0.15	0.11	0.11	1
0.15	0.15	0.11	0.11	Y

$$\sum_{x_1} \sum_{x_2} \dots \sum_{x_n} p_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) = 1$$

$$1 \quad \varepsilon \quad \varepsilon \quad 1$$

x

$$\sum_{x_1} \sum_{x_2} \dots \sum_{x_{i-1}} \sum_{x_{i+1}} \dots \sum_{x_n} p_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) = p_{x_i, x_{i+1}, \dots, x_{j-1}, x_j, x_{i+1}, \dots, x_{j-1}, x_j}^q$$

$$\frac{\varepsilon}{0.15} \times (E, A)_{v,x}^q + (E, B)_{v,x}^q = (A)_{v,x}^q$$

Functions of multi random variables $= (c = Y/x = X)q = (Y/x)_{x/x}q$

$$y = g(x_1, x_2, \dots, x_n)$$

$$P_Y(y) = P(Y=y) = P(g(x_1, x_2, \dots, x_n) = y)$$

Date: 01-01-2022

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$$= \sum_{(u, x_2, \dots, x_n) : g(x_1, u, \dots, x_n) = y} p_{x_1, x_2, \dots, x_n} \frac{1}{x_1} = (x_1, x_2, \dots, x_n)_{x_1/x} q$$

$$(x/x)_{x/x} q = (c)_{x} q = (c/x)_{x/x} q$$

$$E[Y] = \sum_{x_1} \sum_{x_2} \dots \sum_{x_n} g(x_1, x_2, \dots, x_n) p_{x_1, x_2, \dots, x_n} (x_1, x_2, \dots, x_n)$$

$$(x/x)_{x/x} q = (c/x)_{x/x} q$$

11/03/2022

$$(x/x)_{x/x} q = (x/x)_{x/x} q$$

$$\sum_{v \in \{0, 1, 3\}} \sum_{w \in \{0, 1, 2\}} c(v+w) = 1$$

$$(S = \Sigma + \tau = \sum_{i=1}^n x_i) q = \sum_i E[x_i] q$$

$$c \sum_{v \in \{0, 1, 3\}} (3v + 3) = 1$$

$$(S = \Sigma, v = Y, x = X) q = (S = \Sigma, x = X) q =$$

$$c(3+6) = (1 = \Sigma, x = X) q = (S = \Sigma) q$$

$$c = \frac{1}{9}$$

$$(S, x/x)_{S, x/x} q = (S/x)_{S/x} q =$$

$$p_{v}(1) = \sum_{w \in \{0, 1, 2\}} \frac{1}{9} (1+w)$$

moratorium

$$(x/x)_{x/x} q = \frac{1}{9} (6) = [x = Y / (x)_{x}] q = [x = Y / x] q$$

$$2 \frac{2}{3}$$

$$(S/x)_{x/x} q = (x/x)_{x/x} q = (x/x)_{x/x} q$$

~~$$\sum_i E[x_i]$$~~

$$(x/x)_{x/x} q = (x/x)_{x/x} q = (x/x)_{x/x} q$$

$$(x/x)_{x/x} q = (x/x)_{x/x} q = (x/x)_{x/x} q$$

$$[x = Y / x] q = (x/x)_{x/x} q = (x/x)_{x/x} q$$

$$p_{x|y}(x|y) = P(X=x|Y=y) = \frac{P(X=x, Y=y)}{P(Y=y)} = \frac{p_{x,y}(x,y)}{p_y(y)}$$

$$p_{x|y}(1,1) = 0$$

Date: 14/03/2022

M T W T = F C S S

$$P_{x|y}(2,2) = \frac{1}{6}$$

3

C = (m1, ..., m, n) & D = (m1, ..., m, n)

$$p_{x,y}(x,y) = p_y(y) = p_{x,y}(x|y)$$

$$(m1, \dots, m, n) \quad p_{x,y}(x,y) = p_x(x) p_{x,y}(y|x)$$

so on

$$p_x(x) p_{x|y}(y|x) = p_y(y) p_{x,y}(x|y)$$

$$p_{x,y|z}(x,y|z) = \frac{P(X=x, Y=y, Z=z)}{P(Z=z)}$$

$1 = (\omega + 1) \quad 3, 3$

$$= \frac{P(X=x, Z=z)}{P(Z=z)} \quad \frac{P(X=x, Y=y, Z=z)}{P(X=x, Z=z)} = (1 + \epsilon) \quad 1 = 0$$

$$p_{x|z}(x|z) p_{y|x,z}(y|x,z)$$

$$\frac{1}{P}$$

conditional Expectation.

$$(\omega + 1) \frac{1}{P} = (1 + \epsilon)$$

$$E[g(x)|Y=y] = \sum_x g(x) p_{x|y}(x|y)$$

~~$$p_x(x) = p_{x|y}(x|y) p_y(y)$$~~

$$p_x(x) = \sum_y p_y(y) p_{x|y}(x|y)$$

$$\sum_n x p_x(x) = \sum_n x \sum_y p_y(y) p_{x|y}(x|y)$$

$$E[X] = \sum_y p_y(y) \sum_n x p_{x|y}(x|y)$$

$$E[X] = \sum_y p_y(y) = E[X|Y=y]$$

D.R.Vs Independence.

$$p_{x,y}(x,y) = p_x(x) p_y(y)$$

$$p_{x,y}(x|y) = p_x(x) p_{y|x}(y|x) = p_x(x) \quad \text{for } x \geq 0$$

Date: 16/03/2022

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$$p_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) = \prod_{i=1}^n p_{x_i}(x_i)$$

if $x_i \geq 0$

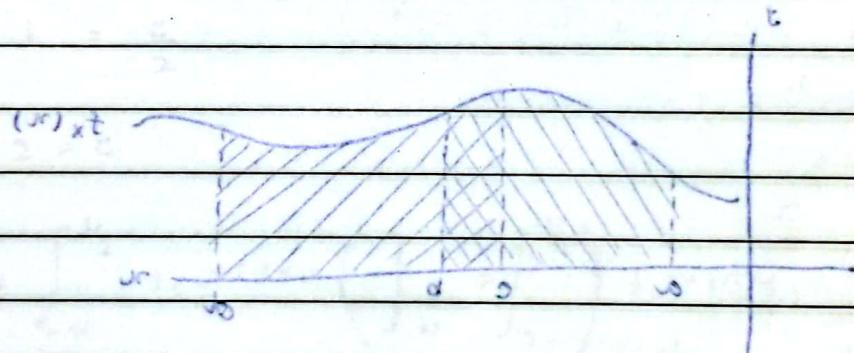
$\left. \begin{array}{l} \text{if } x_i \geq 0 \\ \text{if } x_i < 0 \end{array} \right\} \text{for } i = 1, 2, \dots, n$

25/03/2022

If X and Y are independent

$$\text{cov}(XY) \neq E[X]E[Y] \quad p_{x,y}(x,y) = p_x(x) p_y(y)$$

$$\begin{aligned} \text{var}(x+y) &= E((x+y)^2) = E(x^2 + y^2 + 2xy) \quad \because E[x] = 0 \wedge E[y] = 0 \text{ b/c } 0 \leq x, y \\ &= E(x^2) + E(y^2) + 2E(x)E(y) \quad 0 \leq x, y \\ &= E(x^2) - (E(x))^2 + E(y^2) - (E(y))^2 = \text{var}(x) + \text{var}(y) \end{aligned}$$



$$(d \geq x \geq 0) \neq (H \geq x \geq 0) + (d \geq x \geq 0) = ((b \geq x \geq 0) \cup (d \geq x \geq 0))$$



$$(d = x) \cup (d > x > 0) \cup (d = x) = (d \geq x \geq 0)$$

$$(d = x) + (d > x > 0) + (d = x) =$$

$$0 + (d > x > 0) + 0 =$$

$$(d > x > 0) = (d \geq x \geq 0)$$

Continuous Random Variable.

- Outcomes will be taken out from some intervals,

$$P(a \leq X \leq b)$$

- Instead of PMF (Probability Mass Function): $p_x(x)$

Date: 28/03/2022

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we have PDF (Probability Density Function): $f_x(x)$

$$P(a \leq X \leq b) :=$$

$$\sum_{x \in \{a, b\}} p_x(x) \quad \text{if } X \text{ is discrete.}$$

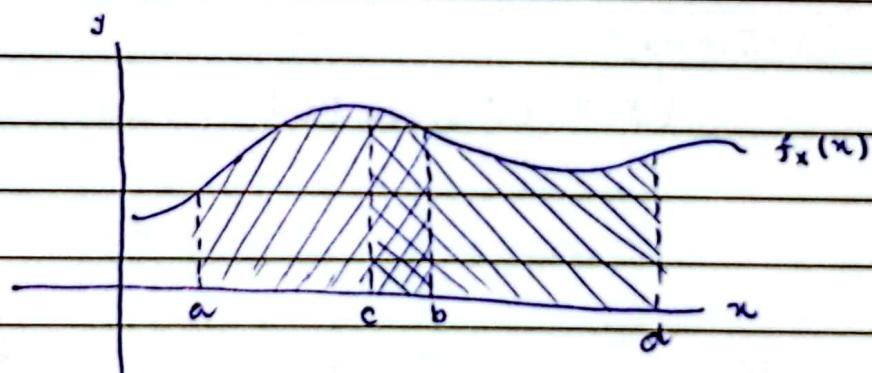
$$\int_a^b f_x(x) dx \quad \text{if } X \text{ is continuous.}$$

Valid PDF:

- $f_x(x) \geq 0$

$f_x(x)$ could be continuous or not.

- $\int_{-\infty}^{+\infty} f_x(x) dx = 1$



$$P((a \leq X \leq b) \cup (c \leq X \leq d)) = P(a \leq X \leq b) + P(c \leq X \leq d) - P(c \leq X \leq b)$$



$$P(a \leq X \leq b) = P((x=a) \cup (a < x < b) \cup (x=b))$$

$$= P(x=a) + P(a < x < b) + P(x=b)$$

$$= 0 + P(a < x < b) + 0$$

$$P(a \leq X \leq b) = P(a < X < b)$$

$f_x(x)$ $\delta \rightarrow 0$

$P(a \leq x \leq a+\delta) \approx f_x(a)\delta$

$f_x(a) \approx \frac{P(a \leq x \leq a+\delta)}{\delta}$

 $f_x(a)$ is density of the probability, that's why

it is called Probability Density Function.

example:-

Ex

$$f_x(x) := \begin{cases} c(1-x) & \text{if } x \in [0, 1] \\ 0 & \text{else.} \end{cases}$$

$L = (n-d)(x)_x$

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$$\int_{-\infty}^{+\infty} f_x(x) dx = \int_{-\infty}^0 f_x(x) dx + \int_0^1 f_x(x) dx + \int_1^{+\infty} f_x(x) dx$$

$$= \int_0^1 c(1-x) dx = c \left[x - \frac{x^2}{2} \right] \Big|_{x=0}^{x=1} = c \left[1 - \frac{1}{2} \right] = c \left[\frac{1}{2} \right]$$

$$= c \left(1 - \frac{1}{2} \right) - c(0) = c \left(\frac{1}{2} \right) = \frac{c}{2}$$

$$1 = \frac{c}{2} \quad (1-\delta) \circ + (1) \circ \delta = 1$$

$0\delta + 1\delta = 1$

$2 = c$

$L = 0$

$$P(x \leq \frac{1}{2}) = \int_{-\infty}^{1/2} f_x(x) dx = \left(\int_{-\infty}^0 + \int_0^{1/2} \right) f_x(x) dx = (n \geq x \geq 0) \%$$

$$= \int_0^{1/2} 2(1-x) dx$$

$$= 2 \left(x - \frac{x^2}{2} \right) \Big|_{x=0}^{x=1/2}$$

$$= 2 \left(\frac{1}{2} - \frac{1}{8} \right)$$

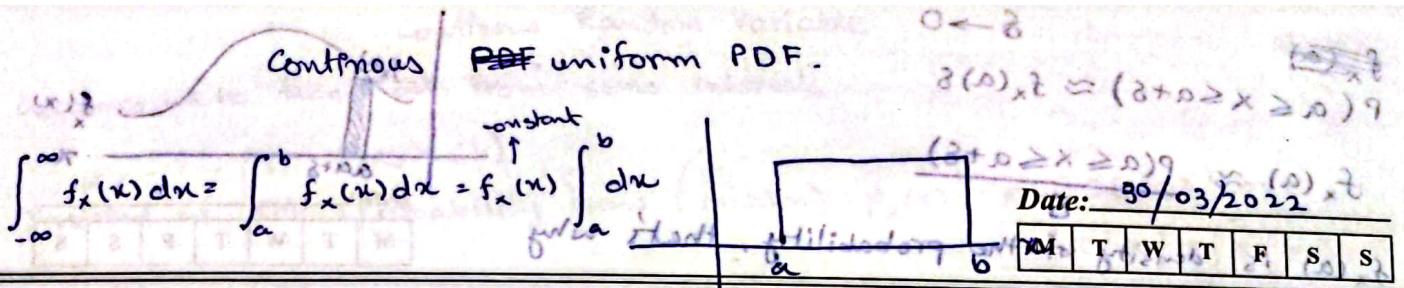
$$P(x \leq \frac{1}{2}) = \frac{3}{4}$$

$$= 2 \left(\frac{1}{2} - \frac{1}{8} \right) = \frac{3}{4}$$

$$= \left(1 - \frac{1}{2} \right) \frac{1}{2} + \left(\frac{1}{2} - 1 \right) \frac{1}{2} =$$

$$= \left(\frac{1}{2} \right) \frac{1}{2} + \left(\frac{1}{2} \right) \frac{1}{2} =$$

$$= \left(\frac{1}{2} + \frac{1}{2} \right) \frac{1}{2} =$$



$$0 \leftarrow \delta$$

$$\delta(x)_{x \geq 0} \approx (\delta + \delta \geq x \geq 0) \approx$$

$$(\delta + \delta \geq x \geq 0) \approx$$

Date: 30/03/2022

MO T W T F S S

$$f_x(x)(b-a) = 1$$

$$f_x(x) = \frac{1}{b-a}$$

$$[1, 0] \rightarrow x \geq 1$$

$$(x-1) \geq 0$$

sigmas

$$\therefore \text{se } 0 \leq x \leq 1 \quad \left\{ \begin{array}{ll} 2c & \text{if } 0 \leq x \leq 1 \\ 0 & \text{else} \end{array} \right\} =: (x)_{x \geq 0}$$

$$f_x(x) = \left\{ \begin{array}{ll} c & \text{if } 1 \leq x \leq 3 \\ 0 & \text{else} \end{array} \right\} =: (x)_{x \geq 0}$$

$$\int_{-\infty}^{\infty} f_x(x) dx = \left(\int_{-\infty}^0 + \int_0^1 + \int_1^3 + \int_3^{\infty} \right) f_x(x) dx$$

$$1 = \int_0^1 2c dx + \int_3^{\infty} c dx$$

$$1 = 2c(1) + c(3-1)$$

$$1 = 2c + 2c$$

$$c = \frac{1}{4}$$

$$\omega = \frac{\pi}{4}$$

$$P(1/2 \leq x \leq 3/2) = \int_{1/2}^{3/2} f_x(x) dx = \int_{1/2}^{3/2} \left(\frac{1}{2} + \frac{1}{4} \right) dx = \frac{1}{2} (x-1) \Big|_{1/2}^{3/2} = \left(\frac{1}{2} \geq x \right) \approx$$

$$= \left(\int_{1/2}^1 + \int_1^{3/2} \right) f_x(x) dx$$

$$\frac{1}{2} (x-1) \Big|_{1/2}^{3/2} =$$

$$= \int_{1/2}^1 \frac{1}{2} dx + \int_1^{3/2} \frac{1}{4} dx$$

$$\frac{1}{2} (x-1) \Big|_{1/2}^{3/2} =$$

$$= \frac{1}{2} \left(1 - \frac{1}{2} \right) + \frac{1}{4} \left(\frac{3}{2} - 1 \right)$$

$$\left(\frac{1}{2} - 1 \right) \left(\frac{1}{4} \right) =$$

$$\frac{1}{2} = \left(\frac{1}{2} \geq x \right) \approx$$

$$= \frac{1}{2} \left(\frac{1}{2} + \frac{1}{4} \right)$$

Page No. $P\left(\frac{1}{2} \leq x \leq \frac{3}{2}\right) = \frac{3}{8}$

RC

Signature _____

$$E[X] = \int_{-\infty}^{+\infty} x f_X(x) dx \quad E[g(x)] = \int_{-\infty}^{+\infty} g(x) f_X(x) dx = (x)_{\text{var}}$$

continuous random variable.

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$$f_X(x) = \begin{cases} \frac{1}{26}(4x+1) & \text{if } 2 \leq x \leq 4 \\ 0 & \text{else} \end{cases} \quad = \text{prob}(x)_{\text{var}} \quad = E[X]$$

$$E[X] = \int_{-\infty}^{+\infty} x f_X(x) dx = \int_2^4 x \frac{1}{26}(4x+1) dx \quad \frac{(x-d)}{s} = (x)_{\text{var}}$$

no bias, no variance

$$= \frac{1}{26} \int_2^4 (4x^2 + x) dx \quad \begin{matrix} (x)_{\text{var}} = (x \geq x) \frac{9}{10} = (x)_{\text{var}} \\ \Leftrightarrow \text{prob}(x)_{\text{var}} = (x \geq x) 9 = (x)_{\text{var}} \end{matrix}$$

$$= \frac{1}{26} \left(\frac{4x^3}{3} + \frac{x^2}{2} \right) \Big|_{x=2}^{x=4}$$

$$\frac{1}{2} \left(\frac{1}{26} \left(\frac{4}{3}(4)^3 + \frac{4}{2} \right) - \frac{1}{26} \left(\frac{4}{3}(2)^3 + \frac{4}{2} \right) \right) = (x)_{\text{var}}$$

$$= \frac{140}{39} \quad (1 - \frac{19}{39}) = (x)_{\text{var}} \quad (x)_{\text{var}} \leq (x)_{\text{var}} \Leftrightarrow x \leq x$$

$$= \frac{121}{39} \quad 0 \leq (x)_{\text{var}} \Leftrightarrow \infty \leq x$$

$$E[X] = 3.10256 \dots$$

$$\text{var}(X) = E[(X - \mu)^2]$$

$$= \int_{-\infty}^{+\infty} (x - \mu)^2 f_X(x) dx$$

$$x \geq 0 \Rightarrow E[X] \geq 0 \quad \sigma_X = \sqrt{\text{var}(X)}$$

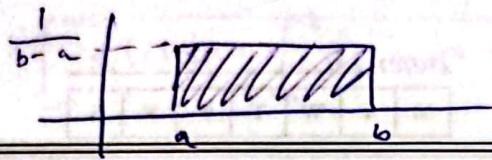
$$a \leq x \leq b \Rightarrow a \leq E[X] \leq b \quad \begin{matrix} \mu \geq x \geq \mu \\ \text{var}(ax + b) = a^2 \text{var}(x) \end{matrix} \quad \left. \begin{matrix} (1) x \geq (x)_{\text{var}} \\ (2) x \leq (x)_{\text{var}} \end{matrix} \right\} = (x)_{\text{var}}$$

$$\text{var}(X) = \int_{-\infty}^{\infty} (x - \frac{121}{39})^2 f_X(x) dx$$

$$= \int_2^4 (x - \frac{121}{39})^2 \frac{1}{26}(4x+1) dx \quad (1) x \geq (x)_{\text{var}} \Leftrightarrow (x \geq x) 9$$

$$= \frac{1}{26} \int_2^4 \left(x^2 - \frac{242}{39}x + \frac{14641}{1521} \right) (4x+1) dx \quad \frac{1}{26} =$$

$$\text{var}(x) = \frac{491}{1521} = 0.3228 \dots \quad \boxed{[(x)_{\text{P}}] \exists} \quad \boxed{\text{var}(x)_{\text{P}} x} \quad \boxed{[x] \exists} \quad \uparrow$$



Date: 04/04/2022

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$$E[x] = \int_{-\infty}^{\infty} x f_x(x) dx = \int_a^b x \frac{1}{b-a} dx = \frac{b+a}{2} \quad \boxed{[(x)_{\text{P}}] \frac{1}{2x}} = (x)_{\text{P}}$$

$$\text{var}(x) = \frac{(b-a)^2}{12}$$

~~cumulative~~ \Rightarrow Distribution Function.

$$F_x(x) = P(X \leq x) = \int_{-\infty}^x f_x(t) dt \quad \Leftrightarrow \quad \frac{dF_x(x)}{dx} = \frac{dP(X \leq x)}{dx} = f_x(x)$$

$$b \geq a \Rightarrow P(X \leq b) = P(X \leq a) + P(a < X \leq b)$$

$$y \geq x \Rightarrow F_x(y) \geq F_x(x) \quad p_x(k) = F_x(k) - F(k-1)$$

$$x \rightarrow \infty \Rightarrow F_x(x) \rightarrow 1$$

$$x \rightarrow -\infty \Rightarrow F_x(x) \rightarrow 0$$

$$f_x(x) = \begin{cases} \frac{1}{26}(4x-1) & 2 \leq x \leq 4 \\ \frac{1}{26} & \text{else.} \end{cases} \quad \boxed{[(x)_{\text{P}}] = x0} \quad \boxed{0 \leq [x] \leq 0 \leq x}$$

$$F_x(x) = \begin{cases} \frac{1}{26}(2x^2 + x) & 2 \leq x \leq 4 \\ 0 & \text{else.} \end{cases} \quad \boxed{d \geq [x] \geq 0 \Leftrightarrow d \geq x \geq 0}$$

$$\begin{aligned} P(1 \leq x \leq 3) &= F_x(3) - F_x(1) \\ &= \frac{1}{26}(2(3)^2 + 3) - 5 \\ &= \frac{121}{26} \end{aligned}$$

Exponential Random Function -

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\left. \begin{array}{l} 0 \leq x \\ x > 0 \end{array} \right\} = (x)_x^t$$

Date: 06/04/2022

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$$E[X] = \int_{-\infty}^{\infty} x \lambda e^{-\lambda x} dx = \lambda \int_{0}^{\infty} x e^{-\lambda x} dx = (x)_x^t$$

$$= \lambda \int_{0}^{\infty} x e^{-\lambda x} dx = \frac{\lambda}{\lambda} \int_{0}^{\infty} (x\lambda) e^{-\lambda x} d(x\lambda) =$$

$$= \lambda \int_{0}^{\infty} x e^{-\lambda x} dx = \frac{\Gamma(2)}{\lambda} \frac{x e^{-\lambda x} - 1}{\lambda} = (x)_x^t$$

$$= \lambda \int_{0}^{\infty} x e^{-\lambda x} dx = \frac{1!}{\lambda} = (x)_x^t$$

$$= \frac{1}{\lambda}$$

$$[1 + x\lambda + x^2 \lambda^2] \mathbb{E} = [e(1 + x\lambda)] \mathbb{E}$$

$$1 + [x]\mathbb{E} + [x^2]\mathbb{E}^2 =$$

$$1 + \left(\frac{1}{\lambda}\right) \mathbb{E} + \left(\frac{1}{\lambda^2}\right) \mathbb{E}^2 =$$

$$\text{var}(x) = E[x^2] - (E[x])^2$$

$$2.8 = [e(1 + x\lambda)] \mathbb{E}$$

$$= \int_0^{\infty} x^2 \lambda e^{-\lambda x} dx - \frac{1}{\lambda^2}$$

Normal distribution formula

$$= \frac{1}{\lambda^2} \int_0^{\infty} (\lambda x)^2 e^{-\lambda x} d(\lambda x) - \frac{1}{\lambda^2}$$

$$= \frac{1}{\lambda^2} \left(\frac{(\lambda x)^2}{2} \right) \Big|_0^{\infty} = \frac{1}{\lambda^2} = (0.4/x)_x^t$$

$$\text{Var}(x) = \frac{1}{\lambda^2} \text{Normal distribution formula: } \left(\frac{(\lambda x)^2}{2} \right) \Big|_0^{\infty} = \frac{1}{\lambda^2} = (0.4/x)_x^t$$

Normal distribution formula

$$f_X(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

ssac (10/20)

→ not same without logarithm
Seeing Theory
Date: _____

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$$F_X(x) = \int_{-\infty}^x \int_{-\infty}^t e^{-2t} f_X(t) dt dt = \int_0^x 2e^{-2t} dt = [x] \rightarrow = [x] \rightarrow$$

$$= \left. \frac{2}{-2} e^{-2t} \right|_0^x = \left. -e^{-2t} \right|_0^{\infty} =$$

$$F_X(x) = 1 - e^{-2x} \quad (s) \rightarrow =$$

$$F_X(x) = \begin{cases} 1 - e^{-2x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$E[(3x+1)^2] = E[9x^2 + 6x + 1]$$

$$= 9E[x^2] + 6E[x] + 1$$

$$= 9 \left(\frac{2}{2^2} \right) + 6 \left(\frac{1}{2} \right) + 1$$

$$E[(3x+1)^2] = 8.5$$

$$E[(x) \rightarrow] - E[x] \rightarrow = (x) \rightarrow$$

$$1 - \int_0^{\infty} e^{-2t} dt =$$

Normal Random variable

$$f_X(x | \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left(\frac{1 - (x-\mu) \sigma}{\sigma^2} \right) \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma}$$

$$f_X(x | 0, 1) = \frac{1}{\sqrt{2\pi}} \exp(-x^2) ; \text{ Standard Normal Random variable } \rightarrow$$

$$\mathcal{N}(\mu, \sigma^2) \rightarrow \text{NRV.}$$

$$X \sim N(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2 - 2x\mu + \mu^2}{\sigma^2}\right)$$

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$$= \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{\sigma^2}\right) \exp\left(\frac{2x\mu}{\sigma^2}\right) \exp\left(-\frac{\mu^2}{\sigma^2}\right)$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \left(\exp(-x^2)\right)^{1/\sigma^2} \exp\left(\frac{2x\mu - \mu^2}{\sigma^2}\right)$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \left(\frac{\sqrt{2\pi}}{\sqrt{2\pi}} \exp(-x^2)\right)^{1/\sigma^2} \exp\left(\frac{2x\mu - \mu^2}{\sigma^2}\right)$$

$$= \frac{1}{\sigma\sqrt{2\pi}} (2\pi)^{2/\sigma^2} \left(\frac{1}{\sqrt{2\pi}} e^{-x^2}\right)^{1/\sigma^2} \exp\left(\frac{2x\mu - \mu^2}{\sigma^2}\right)$$

$$N(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} (2\pi)^{2/\sigma^2-1} (N(0, 1))^{1/\sigma^2} \exp\left(\frac{2x\mu - \mu^2}{\sigma^2}\right)$$

$$X \sim N(\mu, \sigma^2)$$

$$\text{Let } Y = \frac{X-\mu}{\sigma}$$

$$\frac{Y}{\sigma} = (X)_{AIXT}$$

$$E[Y] = \frac{1}{\sigma} (E[X] - \mu) = 0$$

$$\frac{dE[Y]}{d\sigma} = \frac{d}{d\sigma} \left(\frac{1}{\sigma} (E[X] - \mu) \right) = \frac{1}{\sigma^2} (E[X] - \mu) \quad \boxed{= [AIX]E}$$

$$\text{var}(Y) = \text{var}\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2} \text{var}(X) = 1$$

$$\frac{d^2E[Y]}{d\sigma^2} = \frac{d}{d\sigma^2} \left(\frac{1}{\sigma^2} (E[X] - \mu) \right) = \frac{2}{\sigma^4} (E[X] - \mu) \quad \boxed{= [AIX]E}$$

Conditional PDFs.

~~$$f_{x|y}(x, y) = \frac{f_{x,y}(x, y)}{f_y(y)}$$~~

$$\int_B f_{x|y}(x, y) dy = \int_B \frac{f_{x,y}(x, y)}{f_y(y)} dy = f_{x|B}(x) =$$

$$P(X \in A) = \int_{-\infty}^{\infty} f_{X|A}(x) dx \leq \int_{-\infty}^{\infty} \left(\frac{1}{\pi \sqrt{b-a}} \right) dx = \left(\frac{1}{\pi \sqrt{b-a}} \right) (b-a) = \frac{1}{\pi \sqrt{b-a}}$$

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$$E[g(x)|A] = \int x f_{X|A}(x) dx = \int x \left(\frac{1}{\pi \sqrt{b-a}} \right) dx$$

$f_X(x)$ is not defined, we only know the intervals.

$$A: a+b \leq x \leq b$$

$$f_{X|A}(x) = \frac{1}{\pi \sqrt{b-a}} \quad \text{for } a+b \leq x \leq b$$

$$\left(\frac{1}{\pi \sqrt{b-a}} \right) \int_{a+b}^b \left(\frac{1}{\pi \sqrt{b-a}} \right) dx = \frac{1}{\pi \sqrt{b-a}} \quad \text{for } a+b \leq x \leq b$$

The area of shaded rectangle must be

1. therefore,

$$f_{X|A}(x) = \frac{2}{b-a}$$

$$\frac{4-x}{2} = Y \quad \text{for } a+b \leq x \leq b$$

$$E[X|A] = \int_{a+b}^b x \frac{2}{b-a} dx = \frac{a+3b}{4}$$

$0 = (4 - E[X]) \frac{1}{2} = [Y] \exists$

$$E[X^2|A] = \int_{a+b}^b x^2 \frac{2}{b-a} dx = \frac{7b^2 + 4ab + a^2}{12}$$

$1 = (Y)^2 \text{ for } \frac{1}{2} = \left(\frac{4-Y}{2} \right)^2 = (Y)^2$

$$\frac{(x, x) \cdot t}{(x, x)} = (x, x) \cdot \frac{t}{(x, x)}$$

$$(x, x) \cdot \frac{t}{(x, x)} = t \cdot \frac{(x, x) \cdot t}{(x, x)} = t \cdot (x, x) \cdot \frac{t}{(x, x)} = t^2$$

Memorylessness of exponential PDF

Light-bulb follows exponential random variable.

T is the lifetime of Bulb.

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$$P(T > t) = e^{-\lambda t}$$

The remaining life time of bulb is $T - t$. Now select an arbitrary point from interval $[T-t, T]$. What is the probability of lifetime (burning out of the bulb at x , given it was open for time t).

$$\begin{aligned} P(T-t > x | T > t) &= P(T > x+t | T > t) \\ &= \frac{P((T > x+t) \cap (T > t))}{P(T > t)} = (3 \times x)^{-\lambda} \\ &= \frac{P(T > x+t)}{P(T > t)} \\ &= \frac{e^{-\lambda(x+t)}}{e^{-\lambda t}} \\ &= e^{-\lambda x} \end{aligned}$$

It's like bulb ~~doesn't~~ doesn't know it was turned on for some time t .

If $\{A_1, A_2, A_3, \dots, A_n\}$ are partition sets of Ω , and $B \subseteq \Omega$, then,

$$P(B) = \sum_{i=1}^n P(A_i) P(B | A_i)$$

$$P_X(x) = \sum_{i=1}^n P(A_i) P_{X|A_i}(x)$$

$$F_X(x) = P(X \leq x) = \sum_{i=1}^n P(A_i) P(X \leq x | A_i)$$

$$F_X(x) = \sum_{i=1}^n P(A_i) F_{X|A_i}(x)$$

$$\frac{d}{dx}(F_x(x)) = \sum_{i=1}^n P(A_i) \frac{d}{dx}(F_{x|A_i}(x))$$

Lithographs to extract gamma
Also many lithographs available online

$$f_x(x) = \sum_{i=1}^n P(A_i) f_{x|A_i}(x)$$

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$$\int x f_x(x) dx = \sum_{i=1}^n P(A_i) \int x f_{x|A_i}(x) dx$$

$$x - s = (x - T)q$$

$$E[X] = \sum_{i=1}^n P(A_i) E[X|A_i]$$

allied to emit still premium SMT

emit to utilizing existing facilities. $[T, t-T] \rightarrow \boxed{[T, t]}$ lowest cost twice
+ emit not mega now + i want x to emit SMT (04/04/2022 primed)

Joint PDFs,

$$B \subseteq \mathbb{R} \quad (t < T \mid t+x < T)q = (t < T \mid s < t-T)q$$

$$P((x, y) \in B) = \int_B (f_{x,y}(x, y) dx dy) q =$$

$$(t < T)q =$$

$$B \subseteq \mathbb{R}^n, \forall n \in \mathbb{N} \quad (t < T)q$$

$$(t+x)q =$$

$$\text{Proof: } P(B) = \int_B f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) d^n x$$

no benefit now + i want

x_1, x_2, \dots, x_n are jointly continuous, if they fulfill below requirements

$$1. f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) \geq 0, \forall x_i \in \mathbb{R}, \text{ s.t. } \{A_1, \dots, A_n, A\} \in \mathcal{F}$$

$$2. \int_{\mathbb{R}^n} f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) d^n x = 1 \quad (\text{A})q \quad \boxed{\int_{\mathbb{R}^n} f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) d^n x = 1} = 1 \quad (\text{A})q$$

$$(x)_{\text{A}} q = (\text{A})q \quad \boxed{\int_{\mathbb{R}^n} f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) d^n x = 1} = 1 \quad (\text{A})q$$

$$(\text{A} / x \geq x)q = (\text{A})q \quad \boxed{\int_{\mathbb{R}^n} f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) d^n x = 1} = 1 \quad (\text{A})q$$

$$(x)_{\text{A}} q = (\text{A})q \quad \boxed{\int_{\mathbb{R}^n} f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) d^n x = 1} = 1 \quad (\text{A})q$$

From joint to the marginal

$$f_x(x) = \int_{-\infty}^{\infty} f_{x,y}(x,y) dy$$

$$F_x(x) = P(X \leq x) = \int_{-\infty}^x \int_{-\infty}^{\infty} f_{x,y}(x,y) dy dx$$

$$f_x(x) = P(A_1) f_{x|A_1}(x) + P(A_2) f_{x|A_2}(x) + \dots + P(A_m) f_{x|A_m}(x)$$

$$\int_{-\infty}^x f_x(t) dt = \sum_{i=1}^n P(A_i) \int_{-\infty}^x f_{x|A_i}(t) dt$$

$$F_x(x) = \sum_{i=1}^n P(A_i) F_{x|A_i}(x)$$

(x,y) \rightarrow st

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$$E[g(x_1, x_2, \dots, x_n)] = \int_{\mathbb{R}^n} g(x_1, x_2, \dots, x_n) f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) dx$$

Joint \rightarrow CDF

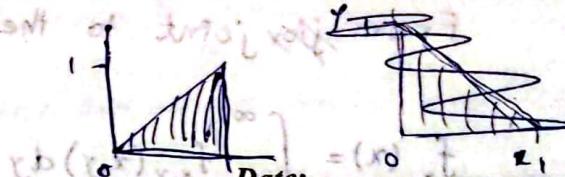
$$F_{x,y}(x,y) = P(X \leq x, Y \leq y) = \int_{-\infty}^x \int_{-\infty}^y f_{x,y}(x,y) dx_1 dy_2$$

$$\frac{\partial F_{x,y}(x,y)}{\partial y} = \int_{-\infty}^x f_{x,y}(x,y) dx_1, \quad \frac{\partial F_{x,y}(x,y)}{\partial x} = \int_{-\infty}^y f_{x,y}(x,y) dy_2$$

$$\frac{\partial^2 F_{x,y}(x,y)}{\partial x \partial y} = \frac{\partial^2 F_{x,y}(x,y)}{\partial y \partial x} = f_{x,y}(x,y)$$

conditional PDF.

$$f_{x|y}(x|y) = \frac{f_{x,y}(x,y)}{f_y(y)}$$



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$$f_y(y) f_{x|y}(x|y) = f_{x,y}(x,y) = (x \geq y) 9 = (x)_{x,y} 7$$

$$= \frac{f_{x,y}(x,y)}{f_x(x)}$$

$$f_y(y) f_{x|y}(x|y) = f_x(x) f_{y|x}(y|x)$$

$$P(x \in A | y = y) = \int_A f_{x|y}(x|y) dx$$

$$(x)_{x,y} 7 (A) 9 \Rightarrow (x)_{x,y} 7$$

$$f_{x,y}(x,y) = \begin{cases} cxy & \text{if } 0 \leq x \leq y \leq 1 \\ 0 & \text{else.} \end{cases}$$

$$\int_0^1 \int_0^x cxy dy dx = 1 \quad f_{x|y}(2|0.5) = 0$$

$$f_{x|y}(0.5|2) = 0$$

$$c \int_0^1 \int_0^x xy^2 dx dy = 1 \quad \int_0^1 \int_0^x cxy dy dx = 1 \Rightarrow c = \frac{1}{8}$$

$$c \int_0^1 \int_0^x x^3 dx dy = 1 \quad (x)_{x,y} 76$$

$$\frac{c}{2} \int_0^1 x^4 dx = 1 \quad (x)_{x,y} 76 = \frac{(x)_{x,y} 76}{x^6}$$

$$\frac{c}{2} \frac{1}{4} = 1 \quad x^6$$

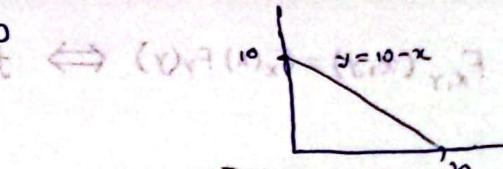
$$c = 8$$

$$f_{x,y}(x,y) = \begin{cases} 1/50 & \text{if } x,y \geq 0, x+y \geq 10 \\ 0 & \text{else.} \end{cases}$$

eccos/50/FC

10-y

$$f_y(y) = \int_0^{10-y} \frac{1}{50} dx = \frac{10-y}{50}$$



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$$f_{x|y}(x|y) = \frac{f_{x,y}(x,y)}{f_y(y)} \quad \text{if } x,y \geq 0, x+y \geq 10 \quad \Rightarrow \quad (x|x)_y = (x|x)_y$$

$$= \frac{1/50}{\frac{10-y}{50}}$$

$$f_{x|y}(x|2) = \frac{1}{8}$$

$$f_{x|y}(x|y) = \frac{1}{10-y}$$

$$(x|x)_y = (x|x)_y$$

General Random Variable.

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(x|x)

(x|x)

Independence.

$$f_{x,y}(x,y) = f_x(x) f_y(y) = (x|x) \cdot (y|y)$$

$$E[X|Y] = E[X] E[Y]$$

$$\text{var}(X+Y) = \text{var}(X) + \text{var}(Y)$$

$$f_{x,y}(x,y) = \begin{cases} \frac{1}{8}(1-x^2)(3-y) & \text{if } -1 \leq x \leq 1, 0 \leq y \leq 3 \\ 0 & \text{else.} \end{cases}$$

$$(x|x)_y = (3-x \geq y \geq 0) = (3-x \geq y \geq 0, x=y)$$

$$(3-x \geq y \geq 0, x=y) = (3-x \geq y \geq 0)$$

$$(x|x)_y = (x|x)_y$$

$$(x|x)_y = (x|x)_y$$

$$F_{x,y}(x,y) = F_x(x) F_y(y) \Leftrightarrow f_{x,y}(x,y) = f_x(x) f_y(y)$$

Bayes' Rule.

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$$f_{x|y}(x|y) = \frac{f_x(x) f_{y|x}(y|x)}{f_y(y)}$$

$$f_y(y) = \int f_x(x') f(y|x') dx' \quad 0 \leq x, x' \quad (x, x')_{\text{ext}} = (x, x)_{\text{int}}$$

If K is discrete and X is continuous random variable.

$$f_x(x) p_{K|x}(k|x) = p_k(k) f_{x|k}(x|k)$$

$$p_{K|x}(k|x) = \frac{f_x(x) p_k(k) f_{x|k}(x|k)}{f_x(x)} \quad \Rightarrow f_{x|k}(x|k) = \frac{f_x(x) p_{K|x}(k|x)}{p_k(k)}$$

$$f_x(x) = \sum_{k'} p_k(k') f_{x|k}(x|k')$$

$$p_k(k) = \int f_x(x') p_{K|x}(k|x') dx'$$

X
↑
discrete

Y
↑
continuous

$$P(X=x, y \leq Y \leq y+\delta) = P(X=x) P(y \leq Y \leq y+\delta | X=x)$$

$$\approx p_x(x) f_{y|x}(y|x) \delta$$

$$= P(y \leq Y \leq y+\delta) P(X=x | y \leq Y \leq y+\delta)$$

$$\approx f_y(y) \delta p_{x|y}(x|y)$$

$$p_x(x) f_{y|x}(y|x) = f_y(y) p_{x|y}(x|y)$$