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Trim Equations of Motion For Aircraft Design: Rolling Performance and Take-Off Rotation

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ABSTRACT

The development of performance-optimal stability and control design solutions for advanced conventional and unconventional aircraft configuration shapes pose a particular challenge during the conceptual design phase. This design challenge can be attributed to a lack of design methods available, chronic data shortage when dealing with novel configuration shapes, and permanent time pressure during the early conceptual design segment. The situation is particularly apt when evaluating symmetric or asymmetric aircraft shapes in asymmetric flight conditions.

When evaluating the stability and control characteristics of a flight vehicle, all design decision-making needs to be based on a trimmed flight vehicle, ideally in six degrees-of-freedom. The task of trimming the vehicle in symmetric and asymmetric flight conditions represents, however, a non-trivial task during the conceptual design phase. This paper presents the development and discusses the application of the trim equations of motion (steady state equations of motion) for rolling performance and take-off rotation applicable to conventional and unconventional aircraft shapes of symmetric and asymmetric layout. The fully developed six-degree-of-freedom formulation aims to trim those flight conditions key to sizing the vehicle's control effectors during the conceptual design stage. The asymmetric-oblique-wing aircraft has been selected as the development 'benchmark vehicle', because of its unequaled development potential, the inherent inclusion of the range of symmetric aircraft types, and the fact that the majority of critical flight conditions for the design of controls are asymmetric flight conditions. Overall, the primary research aim is the development of a generic analytical framework with the capability to trim the aircraft in six degrees-of-freedom for the assessment of control power required/available.

NOMENCLATURE

b	=	span
$\bar{c}$	=	mean aerodynamic chord (m.a.c.)
C_D	=	drag coefficient (aircraft)
C_{D_0}	=	drag coefficient (aircraft) for zero angle-of-attack
C_{D_p}	=	variation of aircraft drag coefficient with roll rate
C_{D_α}	=	variation of aircraft drag coefficient with angle of attack
C_l	=	rolling moment coefficient (aircraft)
C_{l_0}	=	rolling moment coefficient (aircraft) for zero angle-of-attack
C_{l_p}	=	variation of aircraft rolling moment coefficient with roll rate
C_{l_β}	=	variation of aircraft rolling moment coefficient with angle of sideslip
C_L	=	lift coefficient (aircraft)
C_{L_0}	=	lift coefficient (aircraft) for zero angle-of-attack
C_{L_p}	=	variation of aircraft lift coefficient with roll rate
C_{L_α}	=	variation of aircraft lift coefficient with angle of attack
C_m	=	pitching moment coefficient (aircraft)
C_{m_0}	=	pitching moment coefficient (aircraft) for zero angle-of-attack
C_{m_p}	=	variation of aircraft pitching moment coefficient with roll rate
C_{m_q}	=	variation of aircraft pitching moment coefficient with pitch rate
C_{m_α}	=	variation of aircraft pitching moment coefficient with angle of attack
C_n	=	yawing moment coefficient (aircraft)
C_{n_0}	=	yawing moment coefficient (aircraft) for zero angle-of-attack
C_{n_p}	=	variation of aircraft yawing moment coefficient with roll rate

C_{n_β}	=	variation of aircraft yawing moment coefficient with angle of sideslip
C_Y	=	side force coefficient (aircraft)
C_{Y_0}	=	side force coefficient (aircraft) for zero angle-of-attack
C_{Y_p}	=	variation of aircraft sideforce coefficient with roll rate
F_B	=	body axes: F_B (*c.g., x, y, z*)
F_E	=	frame of reference (inertial system) attached to the Earth: F_E (O_E, x_E, y_E, z_E)
$F_{T_x}, F_{T_y}, F_{T_z}$	=	scalar components of $\vec{T}$
g	=	acceleration due to gravity
$\vec{G}$	=	resultant external moment vector, about the mass center
$\vec{h}'$	=	angular momentum vector of spinning rotors with respect to rotor mass center
h'_x, h'_y, h'_z	=	scalar components of $\vec{h}'$ in F_B
i	=	aerodynamic control effector variable incidence stabilizer angle (trimmable CE)
I_B	=	inertia matrix
I_x, I_y, I_z	=	moments of inertia
I_{xy}, I_{yz}, I_{xz}	=	products of inertia
L, M, N	=	scalar components of $\vec{G}$ in F_B, thrust moments
m	=	mass
p, q, r	=	scalar components of $\vec{\omega}$ in F_B
$\dot{p}, \dot{q}, \dot{r}$	=	scalar components of $\dot{\vec{\omega}}$ in F_B, rate of change of aircraft angular velocity
$\overline{q}$	=	aircraft dynamic pressure
S	=	area; wing reference area
$\vec{T}$	=	thrust vector
T_x, T_y, T_z	=	scalar components of $\vec{T}$
u, v, w	=	scalar components of $\vec{V}$ in F_B, perturbed values of *U, V and W*
U, V, W	=	scalar velocity components of $\vec{V}$
W	=	weight (aircraft)
W_{rp}	=	load at the rotation point
$\vec{V}$	=	aircraft velocity vector
x_E, y_E, z_E	=	coordinates of aircraft mass center relative to fixed axes (inertial system F_E)
x_T, y_T, z_T	=	thrust line coordinates relative to the aircraft c.g.
$\hat{x}, \hat{z}$	=	coordinates of c.g. relative to rotation point (main gear axel)
X, Y, Z	=	components of resultant force
α , β	=	angle of attack, angle of sideslip
δ	=	control effector deflection angle
γ	=	flight path angle
Δ	=	increment (perturbation) of a parameter; non-zero reference value
μ_x, μ_y	=	tire-to-runway friction coefficient
τ_X, τ_M, τ_N	=	corrections for propulsive installation
ϕ, θ, ψ	=	Euler angles
$\vec{\omega}$	=	angular velocity vector

Subscripts

A	=	aerodynamic
B, b	=	body frame F_B
D	=	drag increment due to inoperative engine
DiCE	=	Directional CE
i	=	variation with CE incidence angle
i,j,k/l,m,n	=	variable indices
LaCE	=	Lateral CE
LoCE	=	Longitudinal CE
s	=	stability axes
T	=	thrust
δ	=	variation with CE deflection

Superscripts

E	=	inertial system

INTRODUCTION

CONTEXT. A study has been conducted over a four-year period to develop a generic stability and control (s&c) methodology *AeroMech* to consistently size the control effectors of fixed-wing aircraft of conventional but in particular unconventional configuration shapes at the aircraft conceptual design level, see [1] and [2]. Overall, the true complexity of the research undertaking has been hidden in the objective, to develop a generic (vehicle configuration independent) methodology concept and algorithm. The inclusion of asymmetric flight conditions and asymmetric aircraft configurations, in addition to the reasonably well-known symmetric aircraft types, contributed significantly to the overall technical complexity and level of abstraction. In addition, control effectors (CEs) are not designed for the cruise design point. Instead, they are sized in the 'grey-areas' of the flight envelope, where non-linear aerodynamics prevails, demanding specific information during the initial conceptual design phase. Overall, it has been a clear development target to develop a methodology concept for conceptual design application with the consequent intent *"... things should be as simple as possible, but no simpler ..."*.

DESIGN FOR STABILITY AND CONTROL. In *AeroMech*, all design decision-making is based on the evaluation of the trimmed aircraft in six degrees-of-freedom (6-DOF), an exception being the take-off rotation maneuver. The trim (steady state) EOM are solved to determine control power available/required for a range of pre-defined critical flight conditions (DCFC). The dynamic (small perturbation) EOM are solved to estimate the gain constants required to restore stiffness and damping for reduced stability to unstable airframes. The dynamic stability characteristics are analyzed using the results generated by the dynamic EOM. An off-line analysis sequence evaluates the dynamic stability characteristics of the vehicle with reduced order models. This trend-information enables the designer to gain physical insights into the mode drivers. The output file finally contains design-relevant information, which ensures the balance between control power and static-, maneuver-, and dynamic stability.

Control power available/required is defined with (a) the volume coefficient (geometry), (b) stability derivative coefficients (aerodynamics), and (c) the CE deflection angle required (operation). For more information, see [1].

BECHMARK VEHICLE. The term 'generic' implies, that the asymmetric aircraft type is considered to be the most general aircraft arrangement, whereby symmetric types represent rather 'simplified' or special cases where certain simplifying assumptions are valid. Clearly, functionality of the methodology concept for asymmetric aircraft ascertains functionality for the range of symmetric aircraft. An important by-product of this approach is the capability, to enable handling of asymmetric flight conditions of symmetric and asymmetric aircraft configurations, a non-typical ability for a conceptual design method.

Technically, asymmetric aircraft configurations resemble the most demanding aircraft type, which might, however, provide an unmatched performance potential. Asymmetric aircraft types, in particular the OFWC, are the single correct choice for minimum wave drag and minimum induced drag due to lift. In addition, the structural efficiency of the OFWC is superior due to its span-loader concept and volumetric efficiency. However, the real complications are their inherent stability and control characteristics. In contrast to symmetric aircraft types, asymmetric aircraft represent highly coupled systems due to inertia coupling and aerodynamic coupling effects. For introductory reading, Nelms [3] has produced an excellent summary of oblique-wing technology programmes. A more recent summary of oblique flying wing studies is presented by Li, Seebass, and Sobieczky [4].

MODELING APPROACH. There are two general flight conditions for which solutions of the equations of motion (EOM) are of primary interest. The steady state EOM form the basis for studying vehicle *controllability* problems (control power), whereby the perturbed state EOM form the basis for studying aircraft *dynamic stability and response* problems and *automatic flight control theory and application*. The present context is only concerned with studying controllability aspects.

There exist three principal approaches in analytically modelling the asymmetric aircraft type:

(a) Decoupling of the longitudinal and lateral-directional motions and neglecting the cross-coupling terms finally leads to the classical three degree-of-freedom (3-DOF) approach.
(b) Separation of the analysis into the longitudinal and the lateral-directional motions without decoupling (inclusion of cross-coupling terms), see Thelander [5] and Maine [6].
(c) Formulation of the fully coupled 6-DOF EOM including primary cross-coupling effects.

The implications of the above three schemes are briefly discussed. All three approaches do not demand any particular computing power. The primary issue of interest is simplicity. It is a constant quest in aircraft conceptual design that one strives for an analytical model complicated enough to adequately represent the system. Once a certain complexity level has been surpassed, extra complication in the model almost invariably degrades the result. The major complications one can foresee are twofold:

i. The aerodynamic estimation is not adequate for the complexity level selected.
ii. Excessive computation difficulties arise.

A combined longitudinal and lateral-directional model seems, at a first glance, far more complicated than two separate ones. However, it must be realized that both approaches might use the same aerodynamic data available, thus the only complication left is of computational character. If we assume that modern numerical methods are able to solve three equations simultaneously, then six equations do not pose a specific problem. Still, the 6-DOF approach provides more opportunity for things to go wrong. However, it should be recalled that modern CFD methods, finite element (FE) methods, or simulation software, are far more complex than the method developed here. The argumentation therefore has to concentrate on the issue, of how well the 3-DOF approach represents the physics of interest, or whether or not the 6-DOF approach provides more trustworthy information with more inherent potential in its approach for future applications (provision of a properly trimmed aircraft, consideration of all flight conditions of interest, consistent static and dynamic investigations, etc.).

It has been decided, to derive the underlying mathematical framework of the generic stability and control methodology *AeroMech* based on the 6-DOF static- and dynamic equations of motion (EOM). With this analytical framework in place it is possible, to evaluate the range of design-constraining flight conditions (DCFCs) for sizing CEs, see [1].

STEADY STATE EQUATIONS OF MOTION

GENERAL EULER EOM. Steady state flight is characterized by having zero rates of change of the linear and angular velocity components with time relative to the body-fixed axis system in an atmosphere of constant density.

$$\dot{\vec{V}} = 0 \tag{1}$$

$$\dot{\vec{\omega}} = 0 \tag{2}$$

The following flight cases have been modeled for the asymmetric aircraft type, see [1]:

1. *Steady State Straight Line Flight*;
2. *Steady State Turning Flight*;
3. *Steady State Pull-Up and Push-Over Flight*;
4. *Steady State Rolling Performance*;
5. *Quasi-Steady Take-Off Rotation Maneuver*.

The quasi-steady take-off rotation flight case cannot be considered a steady state flight case, since $\dot{q} \neq 0$. However, the instant at which the rotation is evaluated (*q=0*) permits the grouping of this flight case with the steady state flight cases. This paper discusses the *Steady State Rolling Performance* and *Quasi-Steady Take-Off Rotation Maneuver* cases. The underlying equations for modeling the above steady state flight conditions are the *General Euler Equations Of Motion With Spinning Rotors* (derivation see [1]).

$$X_A + X_T - mg\sin\theta = m\left(\dot{u}^E + qw^E - rv^E\right) \tag{3a}$$

$$Y_A + Y_T + mg\cos\theta\sin\phi = m\left(\dot{v}^E + ru^E - pw^E\right) \tag{3b}$$

$$Z_A + Z_T + mg\cos\theta\cos\phi = m\left(\dot{w}^E + pv^E - qu^E\right) \tag{3c}$$

$$L_A + L_T = I_x\dot{p} - I_{yz}\left(q^2 - r^2\right) - I_{zx}\left(\dot{r} + pq\right) - I_{xy}\left(\dot{q} - rp\right) - \left(I_y - I_z\right)qr + qh'_z - rh'_y \tag{4a}$$

$$M_A + M_T = I_y\dot{q} - I_{zx}\left(r^2 - p^2\right) - I_{xy}\left(\dot{p} + qr\right) - I_{yz}\left(\dot{r} - pq\right) - \left(I_z - I_x\right)rp + rh'_x - ph'_z \tag{4b}$$

$$N_A + N_T = I_z\dot{r} - I_{xy}\left(p^2 - q^2\right) - I_{yz}\left(\dot{q} + rp\right) - I_{zx}\left(\dot{p} - qr\right) - \left(I_x - I_y\right)pq + ph'_y - qh'_x \tag{4c}$$

$$p = \dot{\phi} - \dot{\psi}\sin\theta \tag{5a}$$

$$q = \dot{\theta}\cos\phi + \dot{\psi}\sin\phi\cos\theta \tag{5b}$$

$$r = -\dot{\theta}\sin\phi + \dot{\psi}\cos\phi\cos\theta \tag{5c}$$

$$\dot{\phi} = p + q\left(\sin\phi + r\cos\phi\right)\tan\theta \tag{6a}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{6b}$$

$$\dot{\psi} = \left(q\sin\phi + r\cos\phi\right)\sec\theta \tag{6c}$$

$$\dot{x}_E = u^E\cos\theta\cos\psi + v^E\left(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi\right) + w^E\left(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi\right) \tag{7a}$$

$$\dot{y}_E = u^E\cos\theta\sin\psi + v^E\left(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi\right) + w^E\left(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi\right) \tag{7b}$$

$$\dot{z}_E = -u^E\sin\theta + v^E\sin\phi\cos\theta + w^E\cos\phi\cos\theta \tag{7c}$$

The above equations contain the following assumptions:

1. The Earth is treated flat and stationary in inertial space, thus rotational velocity is neglected.
2. The equations are valid for any orthogonal axis system fixed at the c.g. of the aircraft (body axes).
3. The aircraft is a rigid body ($\dot{I}_B = 0$), having attached to it any number of rigid spinning rotors.
4. The spinning rotors have constant angular speed relative to the body axes ($\dot{h}'_B = 0$). The axis of any spinning rotor is fixed in direction relative to the body axes. This assumption is valid for thrust vectoring with a movable nozzle (usual), where the thrust vector alters direction but the axes of the spinning rotors stay constant. The assumption of spinning rotors with fixed axes requires to be reviewed, when applied to the OFWC with engines pivoted dependent on wing sweep adjustment during flight.
5. The wind velocity is zero, so that $\vec{V}^E = \vec{V}$.

The usual assumptions like, (i) the existence of a plane of symmetry (C_{xz}), (ii) neglection of aerodynamic cross-coupling, (iii) the absence of rotor gyroscopic effects, have not been accepted in the present context.

THRUST FORCES AND MOMENTS. At first it is required to model the thrust forces and moments acting on the airframe.

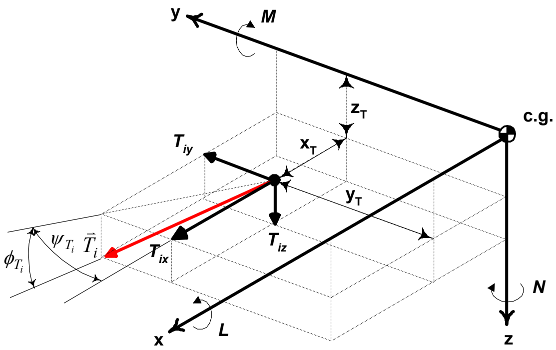

Fig. 1. Thrust force component break-down.

The thrust terms in stability axis in their final form (derivation see [1]):

$$F_{T_{x_s}} = \sum_{i=1}^{n} T_i \left(\cos\phi_{T_i} \cos\psi_{T_i} \cos\alpha + \sin\phi_{T_i} \sin\alpha \right) \tag{8a}$$

$$F_{T_{y_s}} = \sum_{i=1}^{n} T_i \left(\cos\phi_{T_i} \sin\psi_{T_i} \right) \tag{8b}$$

$$F_{T_{z_s}} = \sum_{i=1}^{n} T_i \left(-\cos\phi_{T_i} \cos\psi_{T_i} \sin\alpha + \sin\phi_{T_i} \cos\alpha \right) \tag{8c}$$

$$L_s = \sum_{i=1}^{n} \begin{bmatrix} T_i \cos\alpha \left(-\cos\phi_{T_i} \sin\psi_{T_i} z_T + \sin\phi_{T_i} y_T \right) \\ + T_i \sin\alpha \left(\cos\phi_{T_i} \sin\psi_{T_i} x_T - \cos\phi_{T_i} \cos\psi_{T_i} y_T \right) \end{bmatrix} \tag{9a}$$

$$M_s = \sum_{i=1}^{n} \left[T_i \left(\cos\phi_{T_i} \cos\psi_{T_i} z_T - \sin\phi_{T_i} x_T \right) \right] \tag{9b}$$

$$N_s = \sum_{i=1}^{n} \begin{bmatrix} T_i \sin\alpha \left(\cos\phi_{T_i} \sin\psi_{T_i} z_T - \sin\phi_{T_i} y_T \right) \\ + T_i \cos\alpha \left(\cos\phi_{T_i} \sin\psi_{T_i} x_T - \cos\phi_{T_i} \cos\psi_{T_i} y_T \right) \end{bmatrix} \tag{9c}$$

Note, the angle ϕ_{T_i} in Figure 1 is negative.

STEADY STATE ROLLING PERFORMANCE. Any discussion of this flight case requires consideration of the kinematics of the motion. Since any aircraft generally tends to follow the path of least resistance, there are two basic possibilities for roll:

(A) Roll about the wind axis (flight direction);
(B) Roll about the forward minimum inertia axis (principal axis).

Conventional aircraft configurations (e.g., the classical tail-aft configuration - TAC), having modest inertias and strong aerodynamic stiffness, tend to roll about the wind axis (flight path). Throughout this flight case the aircraft remains in trim, since the angle-of-attack remains at the trimmed value and the sideslip angle stays constant. In contrast, highly loaded slender aircraft with weaker stability characteristics tend to roll about the minimum inertia axis, resulting in an interchange of angle-of-attack and sideslip angles every 90° of roll. As a result, the motion is of oscillatory character. Clearly, the analysis of such motion has more physical significance, when the EOM are referred to principal axes. However, since the roll maneuver primarily initiates course changes, the roll performance should be determined in stability axes, which is consistent with the previous choice of axis system (see Reference [1]).

The ‘quasi-non-oscillatory’ condition at the instant $\phi = 0$ applies to both aircraft types (slender and non-slender). Clearly, the roll subsidence mode is not a substitute for a real time simulation, which has to take the different kinematics of slender and non-slender aircraft into account. The complexity of the roll case becomes obvious, when observing the various roll-loading conditions (roll kinematics) involved: (a) steady level flight; (b) roll initiation; (c) steady roll rate; (d) roll arresting; or (e) reverse roll. It has been decided that steady state roll is the convenient condition to be evaluated during conceptual design. The steady state roll is achieved when the roll damping moment generated by the airframe is equal to the applied increment in rolling moment.

The inherent complexity of the range of classical to novel flight vehicle configurations defines the analytical modeling framework, from the 1-DOF model sufficient for the conventional TAC to possibly 6-DOF for the asymmetric aircraft type, see Table 1.

TABLE 1. Configuration Complexity for Roll Analysis

	Non-Slender Symmetric Aircraft	*Slender Symmetric Aircraft*	*Non-Slender Asymmetric Aircraft*	*Slender Asymmetric Aircraft*
Roll about principal axis	no	yes	no	yes
Roll around stability axis	yes	no	yes	no
Oscillatory motion relative to wind axis	no	yes	no	yes
Aerodynamic CE coupling	no	no	yes	yes
Inertia coupling	no	no	yes	yes
Aircraft in trim during roll	yes	no	no	no

Table 1 indicates that the non-slender symmetric aircraft (e.g., A340) is the simplest aircraft configuration considered, whereby the slender asymmetric aircraft (e.g., AD-1) is the most complicated one. Simplified conceptual design analysis is usually formulated for the non-slender symmetric aircraft, modeling roll performance as a single-degree-of-freedom problem (the rolling convergence is a motion of almost a single degree-of-freedom rotation about the stability x-axis). An exception to this are aircraft with highly swept low aspect ratio wings, where the roll-yaw-pitch coupling requires a complete 6-DOF simulation. Clearly, the 6-DOF analyses becomes obligatory when discussing asymmetric aircraft types.

It is possible to specify roll performance via the following figures of merits: (a) roll helix angle, $pb/2V$ [rad]; (b) roll rate, p [deg/s]; (c) roll acceleration, $\dot{p}$ [deg/s^2]; (d) wing tip velocity, V_{tip} [m/s]; (e) ratio C_l / C_L [-]; or (f) roll mode time constant; time to bank, t_ϕ [deg/s]. The criteria relevant in the present context are the roll helix angle, roll rate, and time to bank, which provide sufficient information for sizing the LaCEs.

The roll performance case is a special design-constraining flight condition (DCFC), because it usually requires a full simulation by solving the 6-DOF dynamic EOM. In the present context, the 6-DOF trim EOM are solved for a prescribed roll helix angle or roll rate, and a 1-DOF model estimates the time to bank. Thus, to ensure consistency with the approach taken so far, the following process is suggested in context with *AeroMech*:

(a) Solve 6-DOF trim EOM for a prescribed roll helix angle $pb/2V$ (trim, physical visibility);
(b) Solve 1-DOF dynamic EOM for time to bank (step control input);
(c) Solve 6-DOF dynamic EOM (dynamic response analysis)

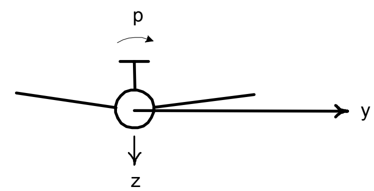

Fig. 2. *Roll performance at ϕ =0.*

Steps (a) and (b) are presented below. Step (c) is enabled with the 6-DOF small perturbation EOM described in Reference [1]. The following presents the 6-DOF trim EOM, utilised to estimate the CE deflection required for a prescribed roll helix angle $pb/2V$ with a value of, e.g., 0.07 (for a discussion of the Gilruth-Criterion, see Gilruth and Turner [7] and Abzug and Larrabee [8]. The EOM have to be solved for a trimmed condition, and in particular for the aileron deflection required enforcing the prescribed roll rate, p, see Figure 2.

The flight condition of interest is horizontal flight ($\theta = \gamma = 0$). The aircraft performs a steady roll maneuver and the situation of particular interest is, when the aircraft rolls through $\phi = 0$. However, $\phi \neq 0$ is permitted to trim the aircraft in y-direction (consider only small angles of ϕ). This steady state flight condition leads to the non-linear <u>*6-DOF Trim EOM for Steady State Rolling Flight*</u> written in stability axes (derivation see Reference 1).

<u>X-Force:</u>

$$0 = -\begin{bmatrix} C_{D_0} + C_{D_\alpha}\alpha + C_{D_p}\dfrac{p\bar{c}}{2V} \\ + C_{D_{i_{LoCE}}} i_{LoCE} + C_{D_{\delta_{LoCE}}} \delta_{LoCE} \\ + C_{D_{i_{DiCE}}} i_{DiCE} + C_{D_{\delta_{DiCE}}} \delta_{DiCE} \\ + C_{D_{i_{LaCE}}} i_{LaCE} + C_{D_{\delta_{LaCE}}} \delta_{LaCE} \end{bmatrix} \bar{q}S + \sum_{i=1}^{n} T_i \left(\cos\phi_{T_i} \cos\psi_{T_i} \cos\alpha + \sin\phi_{T_i} \sin\alpha \right) \tag{10a}$$

<u>Y-Force:</u>

$$mg\sin\phi = \begin{bmatrix} C_{Y_0} + C_{Y_p}\dfrac{pb}{2V} \\ + C_{Y_{i_{LoCE}}} i_{LoCE} + C_{Y_{\delta_{LoCE}}} \delta_{LoCE} \\ + C_{Y_{i_{DiCE}}} i_{DiCE} + C_{Y_{\delta_{DiCE}}} \delta_{DiCE} \\ + C_{Y_{i_{LaCE}}} i_{LaCE} + C_{Y_{\delta_{LaCE}}} \delta_{LaCE} \end{bmatrix} \bar{q}S \tag{10b}$$

<u>Z-Force:</u>

$$-mg\cos\phi + mpV\sin\beta = -\begin{bmatrix} C_{L_0} + C_{L_\alpha}\alpha + C_{L_p}\dfrac{p\bar{c}}{2V} \\ + C_{L_{i_{LoCE}}} i_{LoCE} + C_{L_{\delta_{LoCE}}} \delta_{LoCE} \\ + C_{L_{i_{DiCE}}} i_{DiCE} + C_{L_{\delta_{DiCE}}} \delta_{DiCE} \\ + C_{L_{i_{LaCE}}} i_{LaCE} + C_{L_{\delta_{LaCE}}} \delta_{LaCE} \end{bmatrix} \bar{q}S + \sum_{i=1}^{n} T_i \left(-\cos\phi_{T_i} \cos\psi_{T_i} \sin\alpha + \sin\phi_{T_i} \cos\alpha \right) \tag{10c}$$

L-Moment:

$$0=\begin{bmatrix} C_{l_0}+C_{l_p}\frac{pb}{2V} \\ +C_{l_{i_{LoCE}}}i_{LoCE}+C_{l_{\delta_{LoCE}}}\delta_{LoCE} \\ +C_{l_{i_{DiCE}}}i_{DiCE}+C_{l_{\delta_{DiCE}}}\delta_{DiCE} \\ +C_{l_{i_{LaCE}}}i_{LaCE}+C_{l_{\delta_{LaCE}}}\delta_{LaCE} \end{bmatrix}\bar{q}Sb \tag{11a}$$

M-Moment:

$$I_{zx}p^2-ph_z'=\begin{bmatrix} C_{m_0}+C_{m_\alpha}\alpha+C_{m_p}\frac{p\bar{c}}{2V} \\ +C_{m_{i_{LoCE}}}i_{LoCE}+C_{m_{\delta_{LoCE}}}\delta_{LoCE} \\ +C_{m_{i_{DiCE}}}i_{DiCE}+C_{m_{\delta_{DiCE}}}\delta_{DiCE} \\ +C_{m_{i_{LaCE}}}i_{LaCE}+C_{m_{\delta_{LaCE}}}\delta_{LaCE} \end{bmatrix}\bar{q}S\bar{c}+\sum_{i=1}^{n}T_i\left(\cos\phi_{T_i}\cos\psi_{T_i}z_T-\sin\phi_{T_i}x_T\right) \tag{11b}$$

N-Moment:

$$-I_{xy}p^2+ph_y'=\begin{bmatrix} C_{n_0}+C_{n_p}\frac{pb}{2V} \\ +C_{n_{i_{LoCE}}}i_{LoCE}+C_{n_{\delta_{LoCE}}}\delta_{LoCE} \\ +C_{n_{i_{DiCE}}}i_{DiCE}+C_{n_{\delta_{DiCE}}}\delta_{DiCE} \\ +C_{n_{i_{LaCE}}}i_{LaCE}+C_{n_{\delta_{LaCE}}}\delta_{LaCE} \end{bmatrix}\bar{q}Sb \tag{11c}$$

Equations (10) and (11) are non-linear equations with respect to the state variables α, β, and ϕ. The above two sets of steady state equations form the basis for studying vehicle controllability design-aspects during rolling flight. The following rolling flight conditions can be modelled for the full range of symmetric and asymmetric aircraft configurations and concepts:

(A) $\beta \geq\leq 0$, *Thrust Symmetry*

[symmetric and asymmetric rolling with all engines operative, certain systems failed]

(B) $\beta \geq\leq 0$, *Thrust Asymmetry*

[asymmetric rolling with engine failure(s), certain systems failed]

Equations (10) and (11) can be solved for any combination of the following design- and state-variables for steady-state rolling performance:

α, β, ϕ, V, $\bar{\rho}$, p, $\sum i_{LoCE,DiCE,LaCE}$, $\sum\delta_{LoCE,DiCE,LaCE}$, T_i, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Since there are six equations, several of the design- and state variables listed above have to be specified before the system can be solved using iterative matrix techniques. The following example flight cases indicate the potential of Equations (10) and (11).

CASE (A) ($\beta \geq\leq 0$, *Thrust Symmetry*)

(i) *Control Power Required to Attain Prescribed Roll Rate*

Pre-Selection: $\beta=0$, ϕ, V, $\bar{\rho}$, p, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-3}\delta_i$, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Numerical Solution: α_{trim}, $\delta_{LoCE_{i_{trim}}}$, $\delta_{DiCE_{i_{trim}}}$, $\delta_{LaCE_{i_{trim}}}$, T_i

Problem Description: Having specified a roll rate, p, or helix angle, $pb/2V$, for the symmetric aircraft, the 6-DOF CE deflection angles required are estimated.

(ii) *Achievable Roll Rate With Control Power Available*

Pre-Selection: $\beta\neq 0$, ϕ, V, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-2}\delta_i$, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Numerical Solution: α_{trim}, $\delta_{LoCE_{i_{trim}}}$, $\delta_{DiCE_{i_{trim}}}$, p, T_i

Problem Description: Having commanded maximum deflection of the LaCEs, the maximum attainable roll rate gets estimated. This case is valid for the symmetric aircraft with $\beta\neq 0$ and the asymmetric aircraft with symmetric thrust setting.

CASE (B) ($\beta \geq\leq 0$, *Thrust Asymmetry*)

(i) *Control Power Required to Attain a Prescribed Roll Rate*

Pre-Selection: $\beta=0$, ϕ, V, $\bar{\rho}$, p, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-3}\delta_i$, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Numerical Solution: α_{trim}, $\delta_{LoCE_{i_{trim}}}$, $\delta_{DiCE_{i_{trim}}}$, $\delta_{LaCE_{i_{trim}}}$, T_i

Problem Description: Having specified a roll rate, p, or helix angle, $pb/2V$, for the symmetric/asymmetric aircraft with thrust asymmetry, the 6-DOF CE deflection angles required are estimated.

***(ii)** Achievable Roll Rate With Control Power Available*

<u>Pre-Selection:</u> $\beta \neq 0$, ϕ, V, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-2} \delta_i$, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

<u>Numerical Solution:</u> α_{trim}, $\delta_{LoCE_{i_{trim}}}$, $\delta_{DiCE_{i_{trim}}}$, p, T_i

<u>Problem Description:</u> Having commanded maximum deflection of the LaCEs, the maximum attainable roll rate gets estimated. This case with thrust asymmetry is valid for the symmetric aircraft with $\beta \neq 0$ and the asymmetric aircraft.

The estimation of time to bank is a complicated dynamic problem. The result is affected by flight control system (FCS) dynamics, control rate limiting, aeroelasticity, coupling effects typical for asymmetric aircraft types, etc. For simplicity reasons it has been decided, to consider the single-degree-of-freedom roll response for a specified control input, although rather optimistic results have to be expected compared to results provided by complicated transfer functions. The derivation in Reference [1] yields the following well-known single-degree-of-freedom model. The maximum steady state roll rate for the particular magnitude of LaCE step input is given with

$$p = -\frac{2V}{b}\frac{C_{l_{\delta_{LaCE}}}\delta_{LaCE}}{C_{l_p}} \tag{13a}$$

and for the helix angle follows

$$\frac{pb}{2V} = -\frac{C_{l_{\delta_{LaCE}}}\delta_{LaCE}}{C_{l_p}} \tag{13b}$$

The bank angle is obtained by integrating the roll rate given with (13a):

$$\int p dt = \phi(t) = -\frac{2V}{b}\frac{C_{l_{\delta_{LaCE}}}\delta_{LaCE}}{C_{l_p}}\left[t + \frac{1}{L_p}\left(1 - e^{L_p t}\right)\right] \tag{14}$$

The bank angle response given with Equation (14) consists of a first term varying linearly with time, and a second term varying exponentially with time. The second term vanishes for infinite time, resulting in an overall linear bank angle response relationship (constant roll rate maneuver) with time. Note that the 1-DOF approximation alone does not consider a trimmed aircraft, since no rudder deflection is commanded to maintain a coordinated rolling motion, etc.

In summary, the 6-DOF analysis estimates for a pre-defined roll helix angle relevant roll performance parameters ($V, C_{l_p}, C_{l_{\delta_{LaCE}}}, \delta_{LaCE}$), which are not dependent on roll dynamics. By feeding this information into Equation (14), an estimate of the time-to-bank capability can be obtained. Clearly, the procedure couples the 6-DOF model with the 1-DOF analysis. The advantage is that the data provided to the 1-DOF analysis represents a trimmed aircraft, taking aerodynamic and inertia coupling effects into account. Thus, reasonably accurate results may be obtained without solving the dynamic EOM at this stage. As a result, physical visibility is maximum.

STRAIGHT TAKE-OFF ROTATION. The take-off rotation maneuver is not a steady state condition since $\dot{q} \neq 0$. The instant of interest during the maneuver is, when nose-wheel lift-off is commanded, see Figure 3.

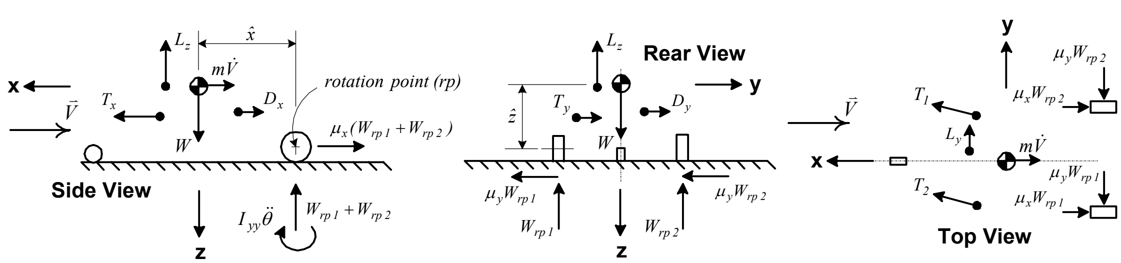

***Fig. 3.** Take-off rotation 'snap-shot'.*

This special DCFC models the instant, when the vehicle has reached lift-off speed, $V = V_{lift\text{-}off}$. It is then that the LoCE commands nose wheel lift-off. The pitching moment generated by the LoCE just balances the vehicle with the nose gear fully extended (no weight on the nose gear), thus no contact of the nose gear with the runway. The rotational speed is assumed to be still zero, $q = 0$, but the angular acceleration, $\ddot{\theta}$, is maximum. The aircraft is in a forward acceleration process, $\dot{V} \neq 0$, with the load factor still one, $n = 1$.

The primary interest lies in determining the control power required to generate a predefined angular acceleration $\ddot{\theta}$ about the main gear axel (no moments are transmitted through the axel) during the beginning of the take-off rotation maneuver where $\dot{\theta}=0$. Figure 3 illustrates the instant during the take-off rotation maneuver considered.

This special DCFC applies to aircraft with a tricycle landing gear and taildraggers can be considered. A take-off run along a horizontal runway is modeled, $\gamma=0$, since the runway slope has an insignificant effect on LoCE control power required. The c.g. is assumed to lie laterally between the main gear contact points; thus the aircraft weight is equally distributed. For the symmetric aircraft it is assumed that $\phi=0$ and $\beta=0$, whereby the asymmetric aircraft assumes $\phi=0$ and $\beta\neq 0$ (The asymmetric aircraft is modelled with a perfectly aligned landing gear relative to the runway, whereby the airframe asymmetry is aerodynamically characterised with *$\beta\neq 0$).*

It is of primary interest to estimate the control power required to generate a predefined angular acceleration, $\ddot{\theta}$, about the main gear axel during initiation of the take-off rotation maneuver with $\dot{\theta}=0$. The take-off rotation problem is modeled without sideward drift (the aircraft is arrested in y-direction), thereby reducing the 6-DOF problem to the non-linear <u>*5-DOF Trim EOM for the Quasi-Steady Take-Off Rotation Maneuver*</u> written in stability axis (derivation see Reference [1]):

<u>X-Force:</u>

$$m\dot{V}+\mu_x W_{rp}=-\begin{bmatrix} C_{D_0}+C_{D_\alpha}\alpha \\ +C_{D_{i_{LoCE}}}i_{LoCE}+C_{D_{\delta_{LoCE}}}\delta_{LoCE} \\ +C_{D_{i_{DiCE}}}i_{DiCE}+C_{D_{\delta_{DiCE}}}\delta_{DiCE} \\ +C_{D_{i_{LaCE}}}i_{LaCE}+C_{D_{\delta_{LaCE}}}\delta_{LaCE} \end{bmatrix}\bar{q}S+\sum_{i=1}^{n}T_i\left(\cos\phi_{T_i}\cos\psi_{T_i}\cos\alpha+\sin\phi_{T_i}\sin\alpha\right) \tag{15a}$$

<u>Z-Force:</u>

$$mg-W_{rp}=\begin{bmatrix} C_{L_0}+C_{L_\alpha}\alpha \\ +C_{L_{i_{LoCE}}}i_{LoCE}+C_{L_{\delta_{LoCE}}}\delta_{LoCE} \\ +C_{L_{i_{DiCE}}}i_{DiCE}+C_{L_{\delta_{DiCE}}}\delta_{DiCE} \\ +C_{L_{i_{LaCE}}}i_{LaCE}+C_{L_{\delta_{LaCE}}}\delta_{LaCE} \end{bmatrix}\bar{q}S-\sum_{i=1}^{n}T_i\left(-\cos\phi_{T_i}\cos\psi_{T_i}\sin\alpha+\sin\phi_{T_i}\cos\alpha\right) \tag{15b}$$

<u>L-Moment:</u>

$$I_{xy}\ddot{\theta}=-\begin{bmatrix} C_{l_0}+C_{l_\beta}\beta \\ +C_{l_{i_{LoCE}}}i_{LoCE}+C_{l_{\delta_{LoCE}}}\delta_{LoCE} \\ +C_{l_{i_{DiCE}}}i_{DiCE}+C_{l_{\delta_{DiCE}}}\delta_{DiCE} \\ +C_{l_{i_{LaCE}}}i_{LaCE}+C_{l_{\delta_{LaCE}}}\delta_{LaCE} \end{bmatrix}\bar{q}Sb \tag{16a}$$

<u>M-Moment:</u>

$$I_y\ddot{\theta}+W_{rp}\left(\mu_x\hat{z}+\hat{x}\right)=\begin{bmatrix} C_{m_0}+C_{m_\alpha}\alpha+C_{m_q}\dfrac{q\bar{c}}{2V} \\ +C_{m_{i_{LoCE}}}i_{LoCE}+C_{m_{\delta_{LoCE}}}\delta_{LoCE} \\ +C_{m_{i_{DiCE}}}i_{DiCE}+C_{m_{\delta_{DiCE}}}\delta_{DiCE} \\ +C_{m_{i_{LaCE}}}i_{LaCE}+C_{m_{\delta_{LaCE}}}\delta_{LaCE} \end{bmatrix}\bar{q}S\bar{c}+\sum_{i=1}^{n}T_i\left(\cos\phi_{T_i}\cos\psi_{T_i}z_T-\sin\phi_{T_i}x_T\right) \tag{16b}$$

<u>N-Moment:</u>

$$I_{xy}\dot{q}=-\begin{bmatrix} C_{n_0}+C_{n_\beta}\beta \\ +C_{n_{i_{LoCE}}}i_{LoCE}+C_{n_{\delta_{LoCE}}}\delta_{LoCE} \\ +C_{n_{i_{DiCE}}}i_{DiCE}+C_{n_{\delta_{DiCE}}}\delta_{DiCE} \\ +C_{n_{i_{LaCE}}}i_{LaCE}+C_{n_{\delta_{LaCE}}}\delta_{LaCE} \end{bmatrix}\bar{q}Sb \tag{16c}$$

Equations (15) and (16) are non-linear equations with respect to the state variable α. The above two sets of quasi-steady equations form the basis for studying vehicle controllability design-aspects during initiation of the take-off rotation maneuver. The following flight conditions can be modelled for the full range of symmetric and asymmetric aircraft configurations and concepts:

(A) $\beta=0\,/\,\beta\neq 0$, *Thrust Symmetry*

[take-off rotation with all engines operative; valid for the symmetric/asymmetric types of aircraft]

(B) $\beta = 0 / \beta \neq 0$, *Thrust Asymmetry*

[take-off rotation with engine(s) inoperative; valid for the symmetric/asymmetric types of aircraft]

Equations (15) and (16) can be solved for any combination of the following design- and state-variables for the quasi-steady, straight take-off rotation maneuver:

$$\alpha\,,\ \beta\,, V,\ \dot{V}\,,\ \overline{\rho}\,,\ \ddot{\theta}\,,\ \sum i_{LoCE,DiCE,LaCE}\,,$$
$$\sum \delta_{LoCE,DiCE,LaCE}\,,\ T_i\,,\ x_{T_i}\,,\ y_{T_i}\,,\ z_{T_i}\,,\ \phi_{T_i}\,,\ \psi_{T_i}$$

Since there are five equations, several of the design- and state variables listed above have to be specified before the system can be solved using iterative matrix techniques. The following example flight cases indicate the potential of Equations (15) and (16).

CASE (A) ($\beta = 0 / \beta \neq 0$, *Thrust Symmetry*)

(i) *Control Power Required to Attain Prescribed Pitch Acceleration (Symmetric Aircraft)*

Pre-Selection: $\alpha,\ \beta = 0\,, V,\ \dot{V}\,,\ \overline{\rho}\,,\ \ddot{\theta}\,,\ \sum_{j=1}^{n} i_j\,,\ \sum_{i=1}^{m-1} \delta_i\,,$

$$x_{T_i}\,,\ y_{T_i}\,,\ z_{T_i}\,,\ \phi_{T_i}\,,\ \psi_{T_i}$$

Numerical Solution: $\delta_{LoCE_{i_{trim}}}\,,\ T_i$

Problem Description: Having pre-selected an angular acceleration, $\ddot{\theta}$, for the symmetric aircraft with thrust symmetry, the LoCE deflection angles required are estimated.

(ii) *Control Power Required to Attain Prescribed Pitch Acceleration (Asymmetric Aircraft)*

Pre-Selection: $\alpha,\ \beta \neq 0\,, V,\ \dot{V}\,,\ \overline{\rho}\,,\ \ddot{\theta}\,,\ \sum_{j=1}^{n} i_j\,,\ \sum_{i=1}^{m-3} \delta_i\,,$

$$x_{T_i}\,,\ y_{T_i}\,,\ z_{T_i}\,,\ \phi_{T_i}\,,\ \psi_{T_i}$$

Numerical Solution: $\delta_{LoCE_{i_{trim}}}\,,\ \delta_{DiCE_{i_{trim}}}\,,\ \delta_{LaCE_{i_{trim}}}\,,\ T_i$

Problem Description: Having pre-selected an angular acceleration, $\ddot{\theta}$, for the asymmetric aircraft with thrust symmetry, the CE deflection angles required are estimated. Clearly, the landing gear is symmetrically aligned with the runway to avoid lateral drift, whereby the airframe is of asymmetric geometry layout.

CASE (B) ($\beta = 0 / \beta \neq 0$, *Thrust Asymmetry*)

(i) *Control Power Required to Attain Prescribed Pitch Acceleration (Symmetric Aircraft)*

Pre-Selection: $\alpha,\ \beta = 0\,, V,\ \dot{V}\,,\ \overline{\rho}\,,\ \ddot{\theta}\,,\ \sum_{j=1}^{n} i_j\,,\ \sum_{i=1}^{m-3} \delta_i\,,$

$$x_{T_i}\,,\ y_{T_i}\,,\ z_{T_i}\,,\ \phi_{T_i}\,,\ \psi_{T_i}$$

Numerical Solution: $\delta_{LoCE_{i_{trim}}}\,,\ \delta_{DiCE_{i_{trim}}}\,,\ \delta_{LaCE_{i_{trim}}}\,,\ T_i$

Problem Description: Having pre-selected an angular acceleration, $\ddot{\theta}$, for the symmetric aircraft, the CE deflection angles required are estimated. The thrust asymmetry condition needs to be trimmed to maintain $\beta = 0$.

(ii) *Control Power Required to Attain Prescribed Pitch Acceleration (Asymmetric Aircraft)*

Pre-Selection: $\alpha,\ \beta \neq 0\,, V,\ \dot{V}\,,\ \overline{\rho}\,,\ \ddot{\theta}\,,\ \sum_{j=1}^{n} i_j\,,\ \sum_{i=1}^{m-3} \delta_i\,,$

$$x_{T_i}\,,\ y_{T_i}\,,\ z_{T_i}\,,\ \phi_{T_i}\,,\ \psi_{T_i}$$

Numerical Solution: $\delta_{LoCE_{i_{trim}}}\,,\ \delta_{DiCE_{i_{trim}}}\,,\ \delta_{LaCE_{i_{trim}}}\,,\ T_i$

Problem Description: Having pre-selected an angular acceleration, $\ddot{\theta}$, for the asymmetric aircraft, the CE deflection angles required are estimated ($\beta \neq 0$ indicates the geometric asymmetry of the airframe under investigation; it is, however, not meant that the asymmetric aircraft skids with a sideslip angle of the landing gear relative to the runway, thereby violating the assumption of no lateral skid).

CONCLUSIONS

The presented formulation for steady state rolling performance and quasi-steady take-off rotation maneuver allows the aircraft designer to evaluate a range of flight and certification-critical flight conditions with a minimum set of input data at conceptual design level. This fully developed six- and five-degree-of-freedom formulation of the equations of motion enables the designer to consistently evaluate conventional and unconventional fixed-wing flight vehicles in symmetric and asymmetric flight conditions. The design aim, to arrive at a simple but generic formulation valid for a range of flight vehicles, has been achieved.

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