

VERTICAL TAIL PARAMETERS INITIALIZATION

Hidden Area --> Import of Excel INPUT V-Tail Data

Hidden Area --> Preliminary Mapping of imported Data

Hidden Area --> Import and preliminary mapping of OTHER Excel Data

INPUT V-TAIL PARAMETERS LIST

Input parameters

$b_V = 4.831 \text{ m}$	$\Lambda_{V_{LE}} = 47 \text{ deg}$	$\Gamma_V = 0 \text{ deg}$
$\xi_{tmax_V} = 0.4$	$\varepsilon_V = 0 \text{ deg}$	$\eta_V = 0.97$
$c_{V_r} = 5.273 \text{ m}$	$t_{over_c_{V_r}} = 0.11$	$C_{l\alpha_{V_r}} = 0.105 \frac{1}{\text{deg}}$
$\xi_{ac_{V_r}} = 0.25$	$C_{m_{ac_{V_r}}} = -0.07$	$M_{cr_{V_{2D_r}}} = 0.68$
$c_{V_t} = 3.231 \text{ m}$	$t_{over_c_{V_t}} = 0.11$	$C_{l\alpha_{V_t}} = 0.105 \frac{1}{\text{deg}}$
$\xi_{ac_{V_t}} = 0.25$	$C_{m_{ac_{V_t}}} = -0.07$	$M_{cr_{V_{2D_t}}} = 0.68$
$\eta_{rd_{in}} = 0$	$\eta_{rd_{out}} = 0.656$	$c_{rd} = 1.89 \text{ m}$

Imported parameters

$\Delta X_{W_{LE_Nose}} = 11.13 \text{ m}$	$\Delta X_{VT_{LE_Nose}} = 22.3 \text{ m}$	$\Delta Z_{W_{LE_Nose}} = -0.94 \text{ m}$
$\Delta Z_{HT_{ac_{W_{ac}}}} = 6.83 \text{ m}$		
$M_1 = 0.696$	$c_{W_r} = 5.243 \text{ m}$	$i_W = 2 \text{ deg}$
$S_W = 87.62 \text{ m}^2$	$MAC_W = 3.642 \text{ m}$	$\Lambda_{W_{c4_{eqv}}} = 2 \text{ deg}$
$AR_W = 8.474$	$r_1 = 1.52 \text{ m}$	$h_1 = 2.35 \text{ m}$
$b_H = 11.217 \text{ m}$	$S_H = 25.47 \text{ m}^2$	$MAC_H = 2.433 \text{ m}$
$X_{MAC_{LE_H}} = 1.66 \text{ m}$	$X_{ac_{WB}} = 3.265 \text{ m}$	

VTAIL PARAMETERS CALCULATIONS

V-Tail basic parameters

$$b_{2V} := 2 \cdot b_V = 9.662 \text{ m}$$

• Considering Vertical Tail like other wings

$$b_{2V} = 9.662 \text{ m}$$

$$\lambda_V := \frac{c_{V_t}}{c_{V_r}} = 0.613$$

$$\lambda_V = 0.613$$

$$S_V := \frac{b_V}{2} \cdot c_{V_r} \cdot (1 + \lambda_V) = 20.542 \text{ m}^2$$

$$S_V = 20.542 \text{ m}^2$$

$$S_{2V} := \frac{b_{2V}}{2} \cdot c_{V_r} \cdot (1 + \lambda_V) = 41.083 \text{ m}^2$$

$$S_{2V} = 41.083 \text{ m}^2$$

$$AR_{2V} := \frac{b_{2V}^2}{S_{2V}} = 2.272$$

$$AR_{2V} = 2.272$$

$$AR_V := AR_{2V}$$

$$AR_V = 2.272$$

$$MAC_V := \frac{2}{3} \cdot c_{V_r} \cdot \left(\frac{1 + \lambda_V^2 + \lambda_V}{1 + \lambda_V} \right) = 4.334 \text{ m}$$

$$MAC_V = 4.334 \text{ m}$$

$$X_{MAC_LE_V} := \frac{b_{2V}}{6} \cdot \frac{(1 + 2 \cdot \lambda_V)}{(1 + \lambda_V)} \cdot \tan(\Lambda_{V_LE}) = 2.383 \text{ m}$$

$$X_{MAC_LE_V} = 2.383 \text{ m}$$

$$Y_{MAC_V} := \frac{b_{2V}}{6} \cdot \frac{1 + 2 \cdot \lambda_V}{1 + \lambda_V} = 2.222 \text{ m}$$

$$Y_{MAC_V} = 2.222 \text{ m}$$

$$Z_{MAC_V} := Y_{MAC_V} \cdot \tan(\Gamma_V) = 0 \text{ m}$$

$$Z_{MAC_V} = 0 \text{ m}$$

V-Tail, sweep angles

$$f\Lambda(x, \Lambda_{le}, AR, \lambda) := \text{if} \left(AR = 0, \Lambda_{le}, \text{atan} \left(\tan(\Lambda_{le}) - \frac{4 \cdot x \cdot (1 - \lambda)}{AR \cdot (1 + \lambda)} \right) \right)$$

• Sweep angle function

$$\Lambda_{V_LE} := f\Lambda(0, \Lambda_{V_LE}, AR_V, \lambda_V) = 0.82$$

$$\Lambda_{V_LE} = 47 \text{ deg}$$

$$\Lambda_{V_TE} := f\Lambda(1, \Lambda_{V_LE}, AR_V, \lambda_V) = 0.576$$

$$\Lambda_{V_TE} = 33.01 \text{ deg}$$

$$\Lambda_{V_c4} := f\Lambda(0.25, \Lambda_{V_LE}, AR_V, \lambda_V) = 0.768$$

$$\Lambda_{V_c4} = 44.03 \text{ deg}$$

$$\Lambda_{V_c2} := f\Lambda(0.5, \Lambda_{V_LE}, AR_V, \lambda_V) = 0.711$$

$$\Lambda_{V_c2} = 40.729 \text{ deg}$$

$$\Lambda_{V_tmax} := f\Lambda(\xi_{tmax_V}, \Lambda_{V_LE}, AR_V, \lambda_V) = 0.735$$

$$\Lambda_{V_tmax} = 42.091 \text{ deg}$$

Hidden Area --> V-Tail, linear laws coefficients

V-Tail, linear laws defined over the whole semi-span

$$f_{c_V}(y) := A_{c_V} \cdot y + B_{c_V}$$

$$f_{t_over_c_V}(y) := A_{t_{c_V}} \cdot y + B_{t_{c_V}}$$

$$f_{C_{l_{\alpha_V}}}(y) := A_{C_{l_{\alpha_V}}} \cdot y + B_{C_{l_{\alpha_V}}}$$

$$f_{C_{m_ac_2D_V}}(y) := A_{C_{m0_V}} \cdot y + B_{C_{m0_V}}$$

$$f_{\epsilon_{g_V}}(y) := A_{\epsilon_V} \cdot y + B_{\epsilon_V}$$

$$f_{\xi_{ac_2D_V}}(y) := A_{\xi_{ac_V}} \cdot y + B_{\xi_{ac_V}}$$

$$f_{M_{cr_V_2D}}(y) := A_{M_{cr_V}} \cdot y + B_{M_{cr_V}}$$

V-Tail, 2D mean quantities

$$t_{over_c_V_mean} := \frac{1}{S_V} \cdot \int_0^{b_V} f_{c_V}(y) \cdot f_{t_over_c_V}(y) dy = 0.084$$

$$t_{over_c_V_mean} = 0.084$$

$$C_{m_ac_V_mean} := \frac{1}{S_V \cdot MAC_V} \cdot \int_0^{b_V} f_{c_V}(y)^2 \cdot f_{C_{m_ac_2D_V}}(y) dy = -0.045$$

$$C_{m_ac_V_mean} = -0.045$$

$$C_{l_{\alpha_V_mean}} := \frac{1}{S_V} \cdot \int_0^{b_V} f_{c_V}(y) \cdot f_{C_{l_{\alpha_V}}}(y) dy = 4.571$$

$$C_{l_{\alpha_V_mean}} = 0.08 \text{ deg}^{-1}$$

MISCELLANEOUS DATA CALCULATIONS

$$\Delta X_{VT_{LE-W_{LE}}} := \Delta X_{VT_{LE-Nose}} - \Delta X_{W_{LE-Nose}} = 11.17 \text{ m}$$

$$\Delta X_{VT_{cA_r-W_{cA_r}}} := \Delta X_{VT_{LE-W_{LE}}} + \frac{c_{V_r}}{4} - \frac{c_{W_r}}{4} = 11.178 \text{ m}$$

$$\Delta Z_{W_{cA_r-B_{CL}}} := \Delta Z_{W_{LE-Nose}} + \sin(i_W) \cdot \frac{c_{W_r}}{4} = -0.894 \text{ m}$$

$$\Delta Z_{HT_{cA_r-B_{CL}}} := \Delta Z_{W_{cA_r-B_{CL}}} - \Delta Z_{HT_{ac-W_{ac}}} = -7.724 \text{ m}$$

$$x_{AC_{HV}} := X_{MAC_{LE-H}} + 0.25 \cdot MAC_H = 2.268 \text{ m}$$

Rudder inner and outer stations and area

$$y_{rd_in} := \eta_{rd_in} \cdot b_V = 0 \text{ m}$$

$$y_{rd_in} = 0 \text{ m}$$

$$y_{rd_out} := \eta_{rd_out} \cdot b_V = 3.169 \text{ m}$$

$$y_{rd_out} = 3.169 \text{ m}$$

$$c_{V_mean_@rd} := f_{c_V} \left(\frac{y_{rd_in} + y_{rd_out}}{2} \right) = 3.933 \text{ m}$$

$$c_{V_mean_@rd} = 3.933 \text{ m}$$

$$S_{rd} := 2 \cdot c_{rd} \cdot (y_{rd_out} - y_{rd_in}) = 11.978 \text{ m}^2$$

$$S_{rd} = 11.978 \text{ m}^2$$

@Aerodynamic Database ---> (control_surface) tau_e vs c_control_surface_over_c_horizontal_tail

$$\tau_{rd} = 0.68$$

Induced drag factors, only due to geometric effects

$$e_{V_alt} := \frac{2}{2 - AR_V + \sqrt{4 + AR_V^2 (1 + \tan(A_{V_tmax})^2)}} = 0.591$$

$$e_{V_alt} = 0.591$$

$$e_{V_alt_A0} := 1.78 \cdot (1 - 0.045 \cdot AR_V^{0.68}) - 0.64 = 1$$

• Alternative
formula: valid
for unswept
wings

$$e_{V_alt_A} := 4.61 \cdot (1 - 0.045 \cdot AR_V^{0.68}) \cdot \cos(A_{V_LE})^{0.15} - 3.1 = 0.911$$

• Alternative
formula: valid
for swept wings

3d critical Mach number at mean aerodynamic chord

$$M_{cr_V_3D_@MAC_V} := \frac{f_{cr_V_2D}(Y_{MAC_V})}{\cos(A_{V_LE})} = 0.997$$

$$M_{cr_V_3D_@MAC_V} = 0.997$$

VERTICAL TAIL EFFECTIVE ASPETC RATIO

@Aerodynamic Database ---> (control_surface) tau_e vs c_control_surface_over_c_horizontal_tail

$$c_1 = 1.606$$

@Aerodynamic Database ---> (AR_v_eff)_c2_vs_Z_h_over_b_v(x_ac_h--v_over_c_bar_v)

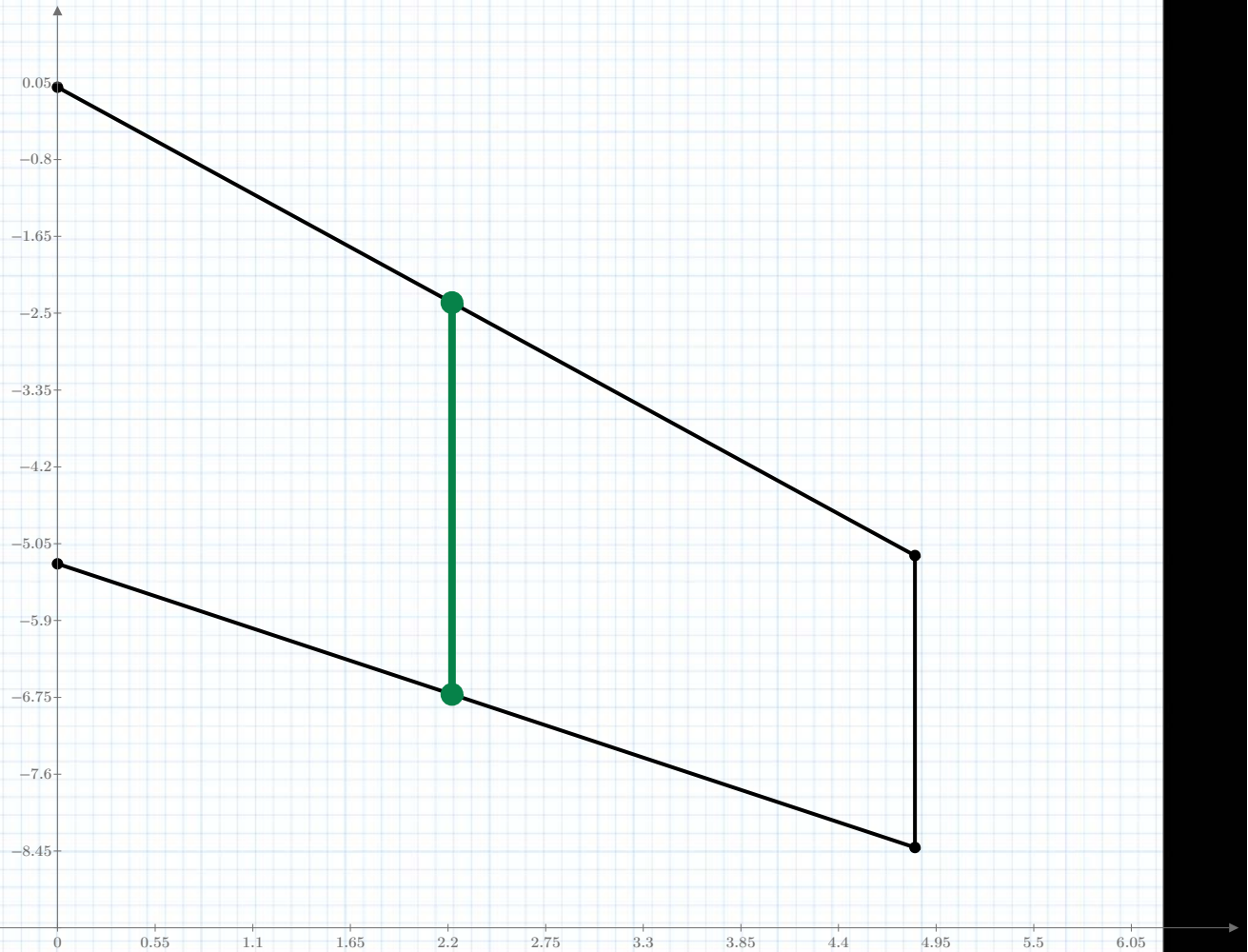
$$c_2 = -7.943$$

@Aerodynamic Database ---> (AR_v_eff)_k_h_v vs S_h_over_S_v

$$K_{HV} = 0.988$$

$$AR_{V_eff} := c_1 \cdot AR_V \cdot (1 + K_{HV} \cdot (c_2 - 1)) = -28.583$$

$$AR_{V_eff} = -28.583$$



VERTICAL TAIL LIFT-CURVE SLOPE

V-Tail Lift Curve Slope, function definitions

$f_{k_{Polhamus}}(M, M_{cr_{3D}}, A_{LE}, \lambda, AR) :=$

- if $(M < M_{cr_{3D}}) \wedge (A_{LE} < 32 \text{ deg}) \wedge (\lambda > 0.4) \wedge (\lambda < 1) \wedge (AR > 3) \wedge (AR < 8)$
 - if $AR < 4$
 - return $1 + \frac{AR \cdot (1.87 - 0.000233 \cdot A_{LE})}{100}$
 - else
 - return $1 + \frac{((8.2 - 2.3 \cdot A_{LE}) - AR \cdot (0.22 - 0.153 \cdot A_{LE}))}{100}$
- else (---> Polhamus Formula is not valid)
 - return 100

• Polhamus
Formula
Coefficient

$f_{C_{L\alpha_V}}(M, k_P, AR, \Lambda_{c2}, C_{l\alpha@MAC}, \Lambda_{LE}) :=$ if $k_P \neq 100$ (---> use Polhamus Formula)

• General Formula for Lift Curve Slope

$$\begin{aligned} & \text{return } \frac{2 \cdot \pi \cdot AR}{2 + \sqrt{\left(\left(\frac{AR^2 \cdot (1-M^2)}{k_P^2} \left(1 + \frac{\tan(\Lambda_{c2})}{(1-M^2)} \right) \right) + 4 \right)}} \\ & \text{else (---> use alternative formula)} \\ & a_0 \leftarrow \frac{C_{l\alpha@MAC}}{\sqrt{1-M^2 \cdot \cos(\Lambda_{LE})^2}} \\ & \text{return } \frac{a_0 \cdot \cos(\Lambda_{LE})}{\sqrt{1 - (M \cdot \cos(\Lambda_{LE}))^2 + \left(\frac{a_0 \cdot \cos(\Lambda_{LE})}{\pi \cdot AR} \right)^2} + \frac{a_0 \cdot \cos(\Lambda_{LE})}{\pi \cdot AR}} \end{aligned}$$

V-Tail Lift Curve Slope, general formula for inner/outer panel and whole wing

$$k_{Polhamus_V} := f_{k_{Polhamus}}(M_1, M_{cr_V_3D@MAC_V}, \Lambda_{V_LE}, \lambda_V, AR_V) = 100$$

$$k_{Polhamus_V} = 100$$

$$C_{l\alpha_V@MAC_V} := f_{C_{l\alpha_V}}(Y_{MAC_V}) = 6.016$$

$$C_{l\alpha_V@MAC_V} = 0.105 \text{ deg}^{-1}$$

$$C_{L\alpha_V@M0} := f_{C_{L\alpha_V}}(0, k_{Polhamus_V}, AR_V, \Lambda_{V_c2}, C_{l\alpha_V@MAC_V}, \Lambda_{V_LE})$$

$$C_{L\alpha_V@M0} = 2.374 \text{ rad}^{-1}$$

$$C_{L\alpha_V@M0} = 0.041 \text{ deg}^{-1}$$

$$C_{L\alpha_V} := f_{C_{L\alpha_V}}(M_1, k_{Polhamus_V}, AR_{V_eff}, \Lambda_{V_c2}, C_{l\alpha_V@MAC_V}, \Lambda_{V_LE})$$

$$C_{L\alpha_V} = 5.618 \text{ rad}^{-1}$$

$$C_{L\alpha_V} = 0.098 \text{ deg}^{-1}$$

Induced drag factor, due to both geometric and aerodynamic effects

$$\begin{aligned} f_e(C_{L\alpha}, AR, \lambda, \Lambda_{LE}) := & \left\| \lambda_e \leftarrow \frac{AR \cdot \lambda}{\cos(\Lambda_{LE})} \right. \\ & \left\| R \leftarrow 0.0004 \cdot \lambda_e^3 - 0.008 \cdot \lambda_e^2 + 0.0501 \cdot \lambda_e + 0.8642 \right. \\ & \left\| \text{return if} \left(AR = 0, 0, \frac{1.1 \cdot C_{L\alpha}}{R \cdot C_{L\alpha} + (1-R) \pi \cdot AR} \right) \right\| \end{aligned}$$

• Function for calculating wing induced drag factor, including aerodynamic and geometric effects

$$e_V := f_e(C_{L\alpha_V}, AR_{V_eff}, \lambda_V, \Lambda_{V_LE}) = -0.005$$

$$e_V = -0.005$$

V-Tail Lift Curve Slope, classic formula

$$C_{L\alpha_V_classic} := \frac{C_{l\alpha_V_mean}}{\sqrt{1-M_1^2} + \frac{C_{l\alpha_V_mean}}{\pi \cdot AR_V \cdot e_V}} = -0.035$$

$$C_{L\alpha_V_classic} = -0.035$$

V-TAIL AERODYNAMIC CENTER - GRAPHICAL METHOD (DATCOM/NAPOLITANO)

@Aerodynamic Database ---> (x_bar_ac_w)_k1_vs_lambda

$$K1_{ac_V_Datcom} = 1.213$$

@Aerodynamic Database ---> (x_bar_ac_w)_k2_vs_L_LE_(AR)_(lambda)

$$K2_{ac_V_Datcom} = 0.454$$

@Aerodynamic Database ---> (x_bar_ac_w)_x'_ac_over_root_chord_vs_tan_(L_LE)_over_beta_(AR_times_tan_(L_LE))_(lambda)

$$X_{ac_over_c_r_V_Datcom} = 0.664$$

Aerodynamic center positions

$$\xi_{ac_V} := K1_{ac_V_Datcom} \cdot (X_{ac_over_c_r_V_Datcom} - K2_{ac_V_Datcom}) = 0.255$$

$$X_{ac_V} := \xi_{ac_V} \cdot MAC_V + X_{MAC_LE_V} = 3.487 \text{ m}$$

$$x_{ac_V} := X_{ac_V} - X_{MAC_LE_V} = 1.104 \text{ m}$$

$$X_{ac_V} = 3.487 \text{ m}$$

$$x_{ac_V} = 1.104 \text{ m}$$

V-Tail Volume Ratio based on aerodynamic centers distance

$$\Delta X_{VT_{ac}W_{TE}} := \Delta X_{VT_{LE}Nose} - \Delta X_{W_{LE}Nose} - c_{W_r} + X_{ac_V} = 9.414 \text{ m}$$

$$\Delta X_{VT_{ac}W_{ac}} := \Delta X_{VT_{LE}Nose} - \Delta X_{W_{LE}Nose} - X_{ac_{WB}} + X_{ac_V} = 11.392 \text{ m}$$

$$VolumeRatio_{V_{ac}} := \frac{S_V}{S_W} \cdot \frac{\Delta X_{VT_{ac}W_{ac}}}{MAC_W} = 0.733$$

3D PITCHING MOMENT COEFFICIENT ABOUT V-TAIL AERODYNAMIC CENTER

Approximated formulation (Roskam)

$$C_{Mac_V} := \frac{2}{S_V \cdot MAC_V} \cdot \int_0^{b_V} C_{m_{ac_{2D_V}}}(y) \cdot c_V(y)^2 dy = -0.09$$

$$C_{Mac_V} = -0.09$$

SIDEWASH EFFECT

$$\eta_{V_times_1_plus_d\sigma_over_d\beta} := 0.724 + 3.06 \cdot \frac{\frac{S_V}{S_W}}{1 + \cos(\Lambda_{W_{c4_{eqv}}})} + 0.4 \cdot \frac{\Delta Z_{W_{c4_r}B_{CL}}}{h_1} + 0.009 \cdot AR_W = 1.007$$

MAPPING AND OUTPUT CREATION

Includi << ..\Default_Map_VTail.mcdx

Excel Writing

$$First_Row_{V_1} := 4$$

$$Block_{V_1} := \text{map_matrix_transform}(\text{mVTail_Data_Map}_{imported})$$

$$Excel_Output_{V_1} := \text{write_full_output}(sOutput_Excel_File, Block_{V_1}, n_{sheet}, First_Row_{V_1})$$

$$First_Row_{V_2} := First_Row_{V_1} + \text{rows}(Block_{V_1}) + 2 = 40$$

$$Block_{V_2} := \text{map_matrix_transform}(\text{mVTail_Data_Map}_{input})$$

$$Excel_Output_{V_2} := \text{write_full_output}(sOutput_Excel_File, Block_{V_2}, n_{sheet}, First_Row_{V_2})$$

$$First_Row_{V_3} := First_Row_{V_2} + \text{rows}(Block_{V_2}) + 2 = 72$$

$$Block_{V_3} := \text{map_matrix_transform}(\text{mVTail_Data_Map})$$

$$Excel_Output_{V_3} := \text{write_full_output}(sOutput_Excel_File, Block_{V_3}, n_{sheet}, First_Row_{V_3})$$

$First_Row_{V_4} := First_Row_{V_3} + \text{rows}(Block_{V_3}) + 2 = 142$

$Block_{V_4} := \text{fmap_matrix_transform}(\text{mVTail_Data_Map}_{LLCoeffs})$

$Excel_Output_{V_4} := \text{fwrite_full_output}(\text{sOutput_Excel_File}, Block_{V_4}, n_{sheet}, First_Row_{V_4})$

$First_Row_{V_5} := First_Row_{V_4} + \text{rows}(Block_{V_4}) + 2 = 171$

$Block_{V_5} := \text{fmap_matrix_transform}(\text{mVTail_Data_Map}_{Misc})$

$Excel_Output_{V_5} := \text{fwrite_full_output}(\text{sOutput_Excel_File}, Block_{V_5}, n_{sheet}, First_Row_{V_5})$

TeX Macro writing on .tex

${}_vcomplete_macros_V := \text{stack}(Block_{V_1}^{(2)}, Block_{V_2}^{(2)}, Block_{V_3}^{(2)}, Block_{V_4}^{(2)}, Block_{V_5}^{(2)})$

${}_vtex_V := \text{fwrite_matrix}(\text{".\Output\VTAIL_TeX_Macros.tex"}, {}_vcomplete_macros_V, \text{" "})$