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A JAVA SOFTWARE FOR AIRCRAFT FLIGHT DYNAMICS CALCULATION

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INDEX

SUMMARY

This Java program is able to determine the characteristics of dynamic stability for small perturbations in longitudinal and lateral-directional motion of an aircraft, from a set of input data supplied by the manufacturer or previously estimated.

Small perturbations allow to linearize the equations of motion system, generating a system of linear equations. This linear system can be decomposed into two subsystems: longitudinal dynamic equations and lateral-directional dynamic equations.

The program consists of three main *classes*: one for the stability and control derivatives calculation and the related matrices generation, another one is for eigenvalues and eigenvectors management, as well as the free response to small perturbations characteristics, and the last one is a *calculator* class, executed in the *main*. After reading from file, the calculator class estimates derivatives and free response characteristics, saves all results in *member variables* and prints them on the screen simultaneously.

This program was built in a “bottom-*up*" perspective. Thanks to its setting from *Object-Oriented Programming*, it can be easily implemented within a more complex code. Its *methods* and its *attributes* can be accessed at any time and can interact with other classes mostly oriented to the complete aircraft study.

1 INTRODUCTION

An aircraft is a deformable solid indeed and is not allowed to ignore the effects of its articulated subsystems, but is to be considered acceptable the hypothesis of rigid body for the study of flight dynamics.

In addition, the external forces and moments acting on the vehicle are not simple configuration and motion functions, especially aerodynamic actions.

At any given moment, they can be calculated if you know the current values of speed of the relative wind and angles of trim, but only with a certain approximation.

The level of detail necessary to determine the entity is the dominant theme of any formulation of the equations of motion.

The complete motion equations of a rigid aircraft in the atmosphere, subject to the aerodynamic actions, propulsion and weight force, are nonlinear and coupled. They can only be solved numerically and do not conceptually lend themselves to illustrate the dependence of stability and controllability characteristics of the aircraft from its geometric, inertial and aerodynamic properties.

1.1 LINEARIZED EQUATIONS SYSTEM

Mostly we can learn about aircraft behavior by analyzing the linear approximations of the complete motion equations. The solutions of the linearized equations are valid for small perturbations around to a given flight condition of reference, and they can be analyzed with tools well known by mathematical theory of dynamical systems.

The particular formulation of linear problem will depend on the reference condition choice. The nominal condition adopted lends itself particularly to the linearization procedure. It is a longitudinally symmetrical not accelerated movement along a straight line, at constant speed and rate of steady climb (in particular, zero), assuming the Earth as flat, namely a balanced flight condition, that perfectly meets the complete equations of motion.

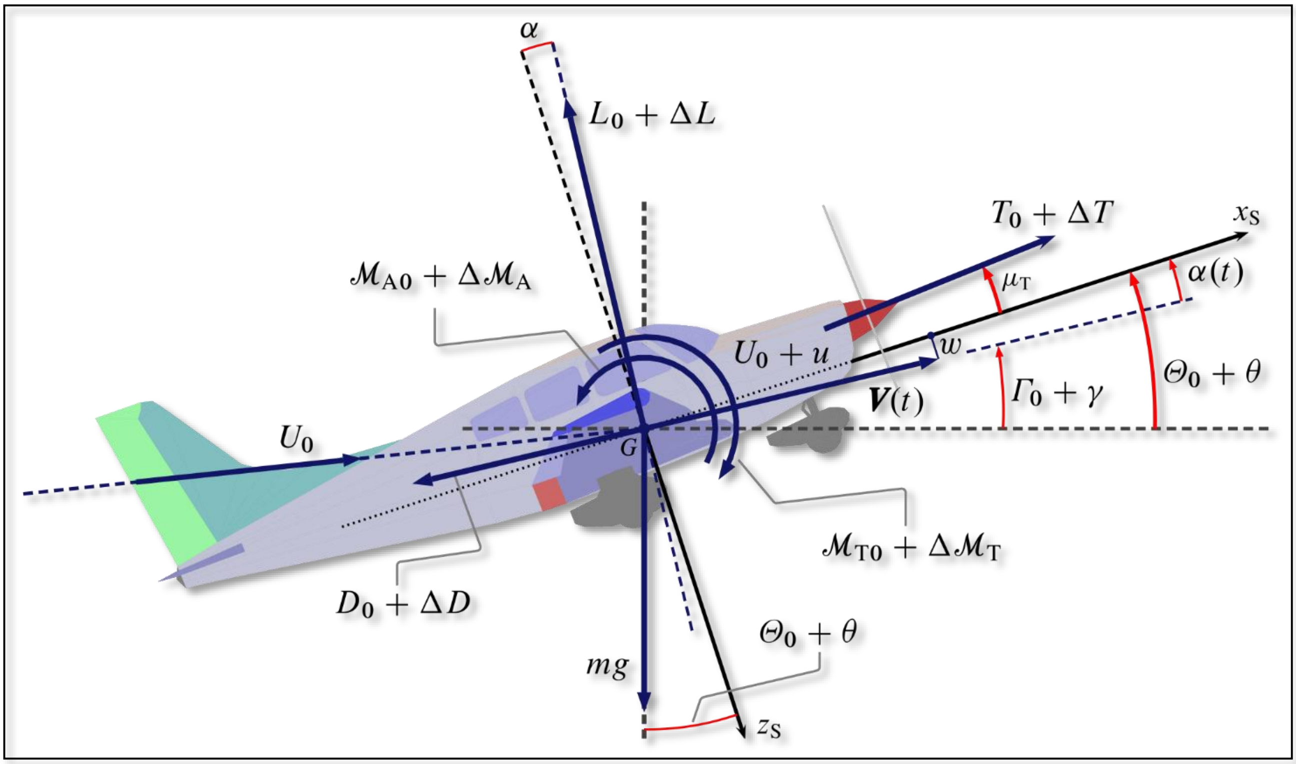


Figure 1.1- perturbed longitudinal motion (De Marco & Coiro, 2015)

Obviously, there are situations in which this approach has limitations, which occurs for all those motions characterized by large changes of the state variables or from not small assets and nonlinear aerodynamic effects.

Despite everything, the accuracy with which we can apply the small perturbations theory proves itself nevertheless acceptable in a wide spectrum of possible situations. This is essentially due to two circumstances:

- for a wide range of flight conditions of practical importance, aerodynamic forces and moments retain effectively a linear dependence with status and control variables;
- normal flight situations are actually perturbed motions of a certain amplitude, not infinitesimal, which correspond, indeed, to the combined effect of small linear and angular speed perturbations;

In fact, relatively small perturbation of the state variables can lead to flight conditions particularly 'violent' and this should normally be avoided.

1.1.1 LONGITUDINAL AND LATER-DIRECTIONAL EQUATIONS SYSTEMS

The longitudinal and lateral-directional linearized equations systems can be put in the following simplified form:

$$\boldsymbol{x'}_{LON} = \mathbf{A}_{LON}\, \boldsymbol{x}_{LON} + \mathbf{B}_{LON}\, \boldsymbol{u}_{LON}$$

$$\boldsymbol{x'}_{LD} = \mathbf{A}_{LD}\, \boldsymbol{x}_{LD} + \mathbf{B}_{LD}\, \boldsymbol{u}_{LD}$$

where $\mathbf{x}_{\mathrm{LON}}$ and $\mathbf{x}_{\mathrm{LD}}$ are perturbations of the state variables vectors, while $\mathbf{u}_{\mathrm{LON}}$ and $\mathbf{u}_{\mathrm{LD}}$ are perturbations of the control parameters vectors.

To better highlight the decoupling of longitudinal and lateral-directional motions, we rewrite the equations provided in matrix form:

$$\begin{Bmatrix} \boldsymbol{x}'_{\mathrm{LON}} \\ -- \\ \boldsymbol{x}'_{\mathrm{LD}} \end{Bmatrix} \approx \begin{bmatrix} [\mathbf{A}_{\mathrm{LON}}] & 0 \\ 0 & [\mathbf{A}_{\mathrm{LD}}] \end{bmatrix} \cdot \begin{Bmatrix} \boldsymbol{x}_{\mathrm{LON}} \\ -- \\ \boldsymbol{x}_{\mathrm{LD}} \end{Bmatrix} + \begin{bmatrix} [\mathbf{B}_{\mathrm{LON}}] & 0 \\ 0 & [\mathbf{B}_{\mathrm{LD}}] \end{bmatrix} \cdot \begin{Bmatrix} \boldsymbol{u}_{\mathrm{LON}} \\ -- \\ \boldsymbol{u}_{\mathrm{LD}} \end{Bmatrix}$$

Usually **x** vectors are composed by dynamic and cinematic state variables such as: u, v, w, q, p, r, $X_{E,G}$, $Y_{E,G}$, $Z_{E,G}$, θ, ϕ, ψ:

$$\boldsymbol{x} = \begin{Bmatrix} \boldsymbol{x}_{\mathrm{LON}} \\ -- \\ \boldsymbol{x}_{\mathrm{LD}} \end{Bmatrix} = \begin{Bmatrix} u \\ w \\ q \\ X_{e,g} \\ Z_{e,g} \\ \theta \\ -- \\ v \\ p \\ r \\ Y_{e,g} \\ \phi \\ \psi \end{Bmatrix} \quad ; \quad \boldsymbol{u} = \begin{Bmatrix} \boldsymbol{u}_{\mathrm{LON}} \\ -- \\ \boldsymbol{u}_{\mathrm{LD}} \end{Bmatrix} = \begin{Bmatrix} \delta_T \\ \delta_e \\ -- \\ \delta_a \\ \delta_r \end{Bmatrix}$$

As anticipated, we can work in simplified hypothesis, keeping a general validity for many disparate cases. In particular we assume our nominal condition as a longitudinally symmetrical (v_0=0) not accelerated movement along a straight line ($p_0 = q_0 = r_0 = 0$), at constant speed and rate of steady climb (in particular, null), null angular velocity ($\phi_0 = 0$), null initial prow angle ($\psi_0 = 0$) and assuming the Earth as flat.

Working under these hypotheses, our state vectors and coefficient matrices seem very simplified:

$$\boldsymbol{x}_{LON} = \begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} \quad ; \quad \boldsymbol{u}_{LON} = \begin{Bmatrix} \delta_T \\ \delta_e \end{Bmatrix}$$

$$\boldsymbol{x}_{LD} = \begin{Bmatrix} r \\ \beta \\ p \\ \phi \end{Bmatrix} \quad ; \quad \boldsymbol{u}_{LD} = \begin{Bmatrix} \delta_a \\ \delta_r \end{Bmatrix}$$

Matrices **A** and **B** are usually an articulated stability and control derivatives assembling but, adopting our simplified nominal condition, many coefficients get lightened. In particular, matrices **A** and **B** get reduced to (4×4) and (4×2) order matrices, assuming their well-known structure:

$$\mathbf{A}_{\mathrm{LON}} = \begin{bmatrix} \hat{X}_u & \hat{X}_w & 0 & -g \\ \frac{\hat{Z}_u}{1-\hat{Z}_{\dot{w}}} & \frac{\hat{Z}_w}{1-\hat{Z}_{\dot{w}}} & \frac{\hat{Z}_q+U_0}{1-\hat{Z}_{\dot{w}}} & \frac{-gS_\theta}{1-\hat{Z}_{\dot{w}}} \\ \hat{M}_u + \hat{k}\hat{Z}_u & \hat{M}_w + \hat{k}\hat{Z}_w & \hat{M}_q + \hat{k}\widehat{(Z_u + U_0)} & -\hat{k}gS_\theta \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{A}_{\mathrm{LD}} = \begin{bmatrix} N'_r & N'_\beta & N'_p & 0 \\ \frac{\hat{Y}_r}{U_0} - 1 & \frac{\hat{Y}_\beta}{U_0} & \frac{\hat{Y}_p}{U_0} & \frac{g}{U_0} \\ L'_r & L'_\beta & L'_p & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{B}_{\mathrm{LON}} = \begin{bmatrix} \hat{X}_{\delta_T} & \hat{X}_{\delta_e} \\ \frac{\hat{Z}_{\delta_T}}{1-\hat{Z}_{\dot{w}}} & \frac{\hat{Z}_{\delta_e}}{1-\hat{Z}_{\dot{w}}} \\ \hat{M}_{\delta_T} + \hat{k}\hat{Z}_{\delta_T} & \hat{M}_{\delta_e} + \hat{k}\hat{Z}_{\delta_e} \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_{\mathrm{LD}} = \begin{bmatrix} N'_{\delta_a} & N'_{\delta_r} \\ \frac{\hat{Y}_{\delta_a}}{U_0} & \frac{\hat{Y}_{\delta_r}}{U_0} \\ L'_{\delta_a} & L'_{\delta_r} \\ 0 & 0 \end{bmatrix}$$

1.1.2 STABILITY AND CONTROL DERIVATIVES

Derivative	Unit
$\hat{Y}_{\beta} = \dfrac{\bar{q}_0 S}{m} C_{Y_{\beta}}$	$\mathrm{m\,s^{-2}}$
$\hat{Y}_{p} = \dfrac{\bar{q}_0 S}{m} \left(\dfrac{b}{2U_0}\right) C_{Y_{p}}$	$\mathrm{m\,s^{-1}}$
$\hat{Y}_{r} = \dfrac{\bar{q}_0 S}{m} \left(\dfrac{b}{2U_0}\right) C_{Y_{r}}$	$\mathrm{m\,s^{-1}}$
$\hat{Y}_{\delta_a} = \dfrac{\bar{q}_0 S}{m} C_{Y_{\delta_a}}$	$\mathrm{m\,s^{-2}}$
$\hat{Y}_{\delta_r} = \dfrac{\bar{q}_0 S}{m} C_{Y_{\delta_r}}$	$\mathrm{m\,s^{-2}}$
$\hat{\mathcal{L}}_{\beta} = \dfrac{\bar{q}_0 S b}{I_{xx}} C_{\mathcal{L}_{\beta}}$	$\mathrm{s^{-2}}$
$\hat{\mathcal{L}}_{p} = \dfrac{\bar{q}_0 S b}{I_{xx}} \left(\dfrac{b}{2U_0}\right) C_{\mathcal{L}_{p}}$	$\mathrm{s^{-1}}$
$\hat{\mathcal{L}}_{r} = \dfrac{\bar{q}_0 S b}{I_{xx}} \left(\dfrac{b}{2U_0}\right) C_{\mathcal{L}_{r}}$	$\mathrm{s^{-1}}$
$\hat{\mathcal{L}}_{\delta_a} = \dfrac{\bar{q}_0 S b}{I_{xx}} C_{\mathcal{L}_{\delta_a}}$	$\mathrm{s^{-2}}$
$\hat{\mathcal{L}}_{\delta_r} = \dfrac{\bar{q}_0 S b}{I_{xx}} C_{\mathcal{L}_{\delta_r}}$	$\mathrm{s^{-2}}$
$\hat{\mathcal{N}}_{\beta} = \dfrac{\bar{q}_0 S b}{I_{zz}} C_{\mathcal{N}_{\beta}}$	$\mathrm{s^{-2}}$
$\hat{\mathcal{N}}_{p} = \dfrac{\bar{q}_0 S b}{I_{zz}} \left(\dfrac{b}{2U_0}\right) C_{\mathcal{N}_{p}}$	$\mathrm{s^{-1}}$
$\hat{\mathcal{N}}_{r} = \dfrac{\bar{q}_0 S b}{I_{zz}} \left(\dfrac{b}{2U_0}\right) C_{\mathcal{N}_{r}}$	$\mathrm{s^{-1}}$
$\hat{\mathcal{N}}_{\delta_a} = \dfrac{\bar{q}_0 S b}{I_{zz}} C_{\mathcal{N}_{\delta_a}}$	$\mathrm{s^{-2}}$
$\hat{\mathcal{N}}_{\delta_r} = \dfrac{\bar{q}_0 S b}{I_{zz}} C_{\mathcal{N}_{\delta_r}}$	$\mathrm{s^{-2}}$

Figure 1.2 lateral-directional stability and control derivatives (De Marco & Coiro, 2015)

The stability and control derivatives are the matrices **A** and **B** constituents. They derive directly from theories of aerodynamics and flight mechanics, and we generally will estimate them respectively to aircraft mass (in case of forces) or moments of inertia (in case of moments of forces). They are all reported in the following lists.

N.B.

The $\mathbf{A}_{LD}$ and $\mathbf{B}_{LD}$ coefficients are also known as "*primed derivatives*", and they result quite different from dimensional derivatives reported in the list on the left. *Primed derivatives* are obtained dividing dimensional derivatives for a linear combination of their moments and product of inertia:

$$I'_{xx} = \frac{I_{xx} I_{zz} - I_{xz}^2}{I_{zz}} \ , \quad I'_{zz} = \frac{I_{xx} I_{zz} - I_{xz}^2}{I_{xx}} \ , \quad I'_{xz} = \frac{I_{xx} I_{zz} - I_{xz}^2}{I_{xz}}$$

i.e.

$$\mathcal{L}'_p = \frac{\mathcal{L}_p}{I'_{xx}} + \frac{\mathcal{N}_p}{I'_{xz}}$$

Primed derivatives consider combined effect between Roll and Yaw.

Derivative	Unit
$\hat{X}_u = \begin{cases} -\dfrac{\bar{q}_0 S}{m U_0}\left(2\, C_D\vert_0 + M_0 \dfrac{\partial C_D}{\partial M}\Big\vert_0\right) & \text{constant thrust} \\ -\dfrac{\bar{q}_0 S}{m U_0}\left(3\, C_D\vert_0 + C_L\vert_0 \tan \Gamma_0 + M_0 \dfrac{\partial C_D}{\partial M}\Big\vert_0\right) & \text{constant power} \end{cases}$	s^{-1}
$\hat{X}_w = \dfrac{\bar{q}_0 S}{m U_0}\left(C_L\vert_0 - C_{D\alpha}\vert_0\right)$	s^{-1}
$\hat{X}_{\dot{w}} \approx 0$	–
$\hat{X}_q \approx 0$	ms^{-1}
$\hat{X}_{\delta_e} \approx 0$	ms^{-2}
$\hat{X}_{\delta_T} = \begin{cases} \dfrac{\bar{q}_0 S}{m}\left(C_{T_{Fix}} + k_V / U_0^2\right) & \text{constant thrust} \\ \dfrac{\bar{q}_0 S}{m}\left(C_{T_{Fix}} + k_V / U_0^3\right) & \text{constant power} \end{cases}$	ms^{-2}
$\hat{Z}_u = -\dfrac{\bar{q}_0 S}{m U_0}\left[2\, C_L\vert_0 + \dfrac{M_0^2}{1 - M_0^2}\, C_{L_M}\vert_0\right)$	s^{-1}
$\hat{Z}_w = -\dfrac{\bar{q}_0 S}{m U_0}\left(C_D\vert_0 + C_{L\alpha}\vert_0\right)$	s^{-1}
$\hat{Z}_{\dot{w}} = -\dfrac{1}{2}\dfrac{\rho_0 S \bar{c}}{2m}\, C_{L_{\dot{\alpha}}}\vert_0 = -\dfrac{1}{2\mu_0}\, C_{L_{\dot{\alpha}}}\vert_0 \approx 0$	–
$\hat{Z}_q = -\dfrac{U_0}{2\mu_0}\, C_{L_q}\vert_0 \approx 0$	ms^{-1}
$\hat{Z}_{\delta_e} = -\dfrac{\bar{q}_0 S}{m}\, C_{L_{\delta_e}}\vert_0 = -\dfrac{\bar{q}_0 S}{m}\, \eta_H \dfrac{S_H}{S}\, C_{L\alpha,H}\, \tau_e$	ms^{-2}
$\hat{\mathcal{M}}_u = \dfrac{\bar{q}_0 S \bar{c}}{I_{yy} U_0}\, C_{\mathcal{M}u}\vert_0 = \dfrac{\bar{q}_0 S \bar{c}}{I_{yy} U_0}\, M_0\, C_{\mathcal{M}_M}\vert_0$	$m^{-1}s^{-1}$
$\hat{\mathcal{M}}_w = \dfrac{\bar{q}_0 S \bar{c}}{I_{yy} U_0}\, C_{\mathcal{M}\alpha}\vert_0$	$m^{-1}s^{-1}$
$\hat{\mathcal{M}}_{\dot{w}} = \dfrac{\rho_0 S \bar{c}^2}{4 I_{yy}}\, C_{\mathcal{M}_{\dot{\alpha}}}\vert_0 = -2\, \dfrac{\rho_0 S \bar{c}^2}{4 I_{yy}}\, K\, \eta_H\, \bar{\mathcal{V}}_H\, \dfrac{l_H}{\bar{c}}\, \dfrac{d\epsilon}{d\alpha}\, C_{L\alpha,H}$	m^{-1}
$\hat{\mathcal{M}}_q = \dfrac{\rho_0 U_0 S \bar{c}^2}{4 I_{yy}}\, C_{\mathcal{M}_q} = -2\, \dfrac{\rho_0 U_0 S \bar{c}^2}{4 I_{yy}}\, K\, \eta_H\, \bar{\mathcal{V}}_H\, \dfrac{l_H}{\bar{c}}\, C_{L\alpha,H}$	s^{-1}
$\hat{\mathcal{M}}_{\delta_e} = \dfrac{\bar{q}_0 S \bar{c}}{I_{yy}}\, C_{\mathcal{M}_{\delta_e}}\vert_0 = -\dfrac{\bar{q}_0 S \bar{c}}{I_{yy}}\, \eta_H\, \bar{\mathcal{V}}_H\, C_{L\alpha,H}\, \tau_e$	s^{-2}

Figure 1.3 longitudinal stability and control derivatives (De Marco & Coiro, 2015)

1.2 AIRCRAFT DYNAMICS

The linear equations systems can be studied with classical methods of the *linear time-invariant* (LTI) theory. Study the system response will not be in our interest, rather we will just evaluate the dynamic *open-loop* characteristics, whose validity is extended to all specific cases in the presence of external forcing.

1.2.1 LONGITUDINAL DYNAMICS

It's our interest to analyze longitudinal *open-loop response* to estimate basic characteristics such as *damping coefficient* and *natural frequency*. To do so, we have to determine the characteristic polynomial of longitudinally symmetrical motion, which is the characteristic polynomial of dynamic matrix $\mathbf{A}_{LON}$ indeed, and, in terms of the eigenvalues of the matrix, can be expressed in the form:

$$\Delta_{LON}(s) = (s - \lambda_{SP})(s - \lambda^*_{SP})(s - \lambda_{PH})(s - \lambda^*_{PH})$$

$$\{\lambda_{SP}, \lambda^*_{SP}\} = \sigma_{SP} \pm i\,\omega_{SP} \qquad ; \qquad \{\lambda_{PH}, \lambda^*_{PH}\} = \sigma_{PH} \pm i\,\omega_{PH}$$

its roots are two pairs of complex and conjugated eigenvalues ($\lambda_{PH}, \lambda^*_{PH}$) and ($\lambda_{SP}, \lambda^*_{SP}$), which constitute two modal components of the dynamic, respectively known as ***phugoid*** (or *long-term*) and ***short-period*** modes. [1]

Introducing *natural frequency* and *damping coefficient*:

$$\zeta_\xi = \sqrt{\frac{1}{1+\left(\frac{\omega_\xi}{\sigma_\xi}\right)^2}} \qquad ; \qquad \omega_{n\xi} = -\,{}^{\sigma_\xi}\!/_{\zeta_\xi}$$

$$T_\xi = \frac{2\pi}{\omega_{n\xi}\sqrt{1-\zeta_\xi{}^2}} \qquad ; \qquad t_{\frac{1}{2}\xi} = \frac{\ln 2}{\omega_{n\xi}\zeta_\xi}$$

where ξ = SP, PH.

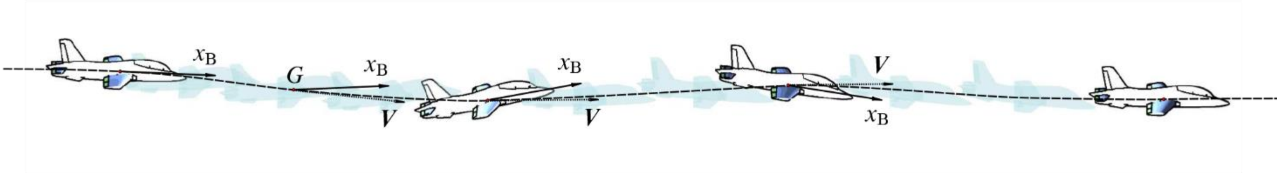


Figure 1.4 typical ***short-period*** *oscillation, with changes on both* $\boldsymbol{\alpha}$ *and* $\boldsymbol{\vartheta}$ *(De Marco & Coiro, 2015)*

1.2.2 LATERAL-DIRECTIONAL DYNAMICS

Similarly, we will analyze lateral-directional *open-loop response*. The matrix $\mathbf{A}_{LD}$ characteristic equation has the following roots:

$$\lambda_{ROLL} = \sigma_{ROLL} \qquad ; \qquad \lambda_{SPIRAL} = \sigma_{SPIRAL}$$

$$\{\lambda_{DR}, \lambda^*_{DR}\} = \sigma_{DR} \pm i\, \omega_{DR}$$

They are two real roots and a complex and conjugated one ($\lambda_{DR}, \lambda^*_{DR}$), which constitute three modal components of the dynamic, respectively known as ***roll***, ***spiral*** and ***dutch-roll*** modes.

Introducing *natural frequency* and *damping coefficient* for dutch-roll mode:

$$\zeta_{DR} = \sqrt{\frac{1}{1+\left(\frac{\omega_{DR}}{\sigma_{DR}}\right)^2}} \qquad ; \qquad \omega_{n_{DR}} = -\,{}^{\sigma_\xi}\!/_{\zeta_\xi}$$

$$T_{DR} = \frac{2\pi}{\omega_{n_{DR}}\sqrt{1-\zeta_{DR}{}^2}} \qquad ; \qquad t_{\frac{1}{2}_{DR}} = \frac{\ln 2}{\omega_{n_{DR}}\zeta_{DR}}$$

2 SOFTWARE

As a software for *Flight Dynamics Calculation*, our program consists of a series of routines able to handle sufficiently all the information related to the aircraft dynamics, which includes the stability derivatives calculation and the system response characteristics.

The program has been written in Java programming language, that is *concurrent*, *class-based* but, mostly, *object-oriented*: "*In the object-oriented (OO) paradigm, a program consists of interacting objects. An object encapsulates data and algorithms. Data defines the state of an object. Algorithms define the behavior of an object. An object communicates with other objects by sending messages to them*." (Sharan, 2014).

This is exactly the philosophy that our program embraces. In fact, it is organized in three main *classes*, each of which treats a particular aspect of the global analysis:

- *StabilityDerivativesCalc.java*: calculator of stability derivatives and matrices $\mathbf{A}_{LON}$, $\mathbf{B}_{LON}$, $\mathbf{A}_{LD}$, $\mathbf{B}_{LD}$;
- *DynamicStabilityCalculator.java*: calculator of eigenvalues, eigenvectors and response characteristics;
- *FlightDynamicsManager.java*: *Java Main Method* (*JVM*), *I/O* operator and *CalculateAll* method.

Our approach is to describe briefly the function of each class and their structure, showing some excerpts of code to better contextualize.

2.1 STABILITY DERIVATIVES CALCULATOR CLASS

The class *StabilityDerivativesCalc.java* employs more than forty different routines to estimate stability (and control) derivatives and to build longitudinal and lateral-directional matrices.

2.1.1 STABILITY AND CONTROL DERIVATIVES

The following type of *procedures* is based on aerodynamics theorems and considerations on flight mechanics. We will now see an example.

EXAMPLE 2.1

Xªu_CT:

$$\hat{X}_{u_{CP}} = -\frac{\bar{q}_0 S}{mU_0}\left(2C_D|_0 + M_0 \left.\frac{\partial C_D}{\partial M}\right|_0\right)$$

```
public static double calcXªu_CT (double rho0, double surf, double mass, double u0,
        double q0, double cd0, double m0, double cdM0) {

            return -q0*surf*(2*cd0 + m0*cdM0)/(mass*u0);

    }
```

Figure 2.1 - "Xªu_CT" calculation procedure

This type of method estimates all our stability and control derivatives.

2.1.2 LONGITUDINAL AND LATERAL-DIRECTIONAL MATRICES

This type of *procedures* is designed for longitudinal and lateral-directional matrices assembling.

EXAMPLE 2.2

Building A_{LON} matrix:

$$\mathbf{A}_{LON} = \begin{bmatrix} \hat{X}_u & \hat{X}_w & 0 & -g \\ \frac{\hat{Z}_u}{1-\hat{Z}_{\dot{w}}} & \frac{\hat{Z}_w}{1-\hat{Z}_{\dot{w}}} & \frac{\hat{Z}_q+U_0}{1-\hat{Z}_{\dot{w}}} & \frac{-gS_\theta}{1-\hat{Z}_{\dot{w}}} \\ \hat{M}_u + \hat{k}\hat{Z}_u & \hat{M}_w + \hat{k}\hat{Z}_w & \hat{M}_q + \hat{k}(\hat{Z}_u + U_0) & -\hat{k}gS_\theta \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

```
public static double[][] build_A_Lon_matrix (Propulsion propulsion_system,
   double rho0, double surf, double mass, double cbar, double u0, double q0,
   double cd0, double m0, double cdM0, double cl0, double cdAlpha0, double gamma0,
   double theta0_rad, double clAlpha0, double clAlpha_dot0, double cMAlpha0,
   double cMAlpha0_dot, double clQ0, double iYY, double cM_m0, double cMq) {

         double [][] aLon = new double [4][4];

         // Propulsion type in the Xªu calculation
         switch (propulsion_system)
                {
                case CONSTANT_TRUST:
                       aLon [0][0] = calcX_u_CT (rho0, surf, mass, u0, q0, cd0, m0,
                              cdM0);
                       break;
                case CONSTANT_POWER:
                       aLon [0][0] = calcX_u_CP (rho0, surf, mass, u0, q0, cd0, m0,
                              cdM0, cl0, gamma0);
                       break;
                default:
                       aLon [0][0] = calcX_u_CT (rho0, surf, mass, u0, q0, cd0, m0,
                              cdM0);
                       break;
                }

         double k = calcM_w_dot (rho0, mass, surf, cbar, iYY, cMAlpha0_dot)/(1 –
                calcZ_w_dot (rho0, surf, mass, cbar, clAlpha_dot0));

         // Construction of the Matrix [A Lon]
         aLon [0][1] = calcX_w(rho0, surf, mass, u0, q0, cl0, cdAlpha0);

         aLon [0][2] = 0;

         aLon [0][3] = -(9.8100)*Math.cos(theta0_rad);

         aLon [1][0] = calcZ_u (rho0, surf, mass, u0, q0, m0, cl0)/(1 –
                 calcZ_w_dot (rho0, surf, mass, cbar, clAlpha_dot0));

         aLon [1][1] = calcZ_w (rho0, surf, mass, u0, q0, m0, cd0, clAlpha0)/(1 –
                 calcZ_w_dot (rho0, surf, mass, cbar, clAlpha_dot0));

                                [...]

         aLon [2][3] = -k*(9.8100)*Math.sin(theta0_rad);

         aLon [3][0] = 0;

         aLon [3][1] = 0;

         aLon [3][2] = 1;

         aLon [3][3] = 0;

         return aLon;
   }
```

Figure 2.2 - "[A_Lon]" building procedure

EXAMPLE 2.3

Primed derivatives Calculation:

```
// Inertia coefficient calculation
        double i1 = iXZ/iXX;
        double i2 = iXZ/iZZ;

// Primed Derivatives calculation
        double Y_beta_1 = calcY_beta (rho0, surf, mass, u0, q0, cyBeta);
        double Y_p_1 = calcY_p (rho0, surf, mass, u0, q0, bbar, cyP);
        double Y_r_1 = calcY_r (rho0, surf, mass, u0, q0, bbar, cyR);
        double L_beta_1 = (calcL_beta (rho0, surf, bbar, iXX, u0, q0, cLBeta) +
                   i1*calcN_beta (rho0, surf, bbar, iZZ, u0, q0, cNBeta))/(1-
i1*i2);
                                    [...]
```

Figure 2.3 - "Primed Derivative" calculation for [A_Ld] and [B_Ld] assembling

2.1.3 FUNCTIONS AND SUBROUTINES

In this category, we have many *subroutines* designed to estimate some flight characteristics, such as the *dynamic pressure*, recalled inside our main routines.

EXAMPLE 2.4

Dynamic Pressure:

```
public static double calcDynamicPressure(double rho0, double u0) {

        return 0.5*rho0*Math.pow(u0,2);

  }
```

Figure 2.4 - "Dynamic Pressure" calculation

EXAMPLE 2.5

Propulsive Regime:

```
public enum Propulsion { CONSTANT_TRUST, CONSTANT_POWER, CONSTANT_MASS_FLOW,
 RAMJET }
```

Figure 2.5 - "Propulsive Regime" Enum type declaration

We have declared a new enumeration type called *Propulsion*, which describes our propulsive regime. This choose influences $\hat{X}_{u_{CP}}$ and $\hat{X}_{\delta_T}$ calculation by a *switch case* during $\mathbf{A}_{\mathrm{LON}}$ and $\mathbf{B}_{\mathrm{LON}}$ calculation (*see pag.14*).

2.2 DYNAMIC STABILITY CALCULATOR CLASS

The class *DynamicStabilityCalculator.java* employs two routines to build the eigenvectors and an eigenvalues matrix and other five different routines to estimate dynamic characteristics, such as *damping coefficient* or *natural frequency*.

2.2.1 EIGENVALUES AND EIGENVECTORS

The eigenvalues calculation is fundamental to get the dynamic characteristics. We obtain them by using a routine named *buildEigenValuesMatrix*, that includes many other *routines* imported from a package, *org.apache.commons.math3* (Apache Commons, s.d.), which allows you to generate two vectors, from matrices as $\mathbf{A}_{\mathrm{LON}}$ and $\mathbf{B}_{\mathrm{LON}}$, each one containing respectively the real and imaginary roots parts from the characteristic polynomial. Finally, these are reported in a matrix (in our case 4x2) which contains on each line the real and imaginary part of each eigenvector.

EXAMPLE 2.6

buildEigenValuesMatrix:

```java
public static double[][] buildEigenValuesMatrix (double aMatrix[][]) {

		RealMatrix aLonRM = MatrixUtils.createRealMatrix(aMatrix);
		EigenDecomposition aLonDecomposition = new EigenDecomposition(aLonRM);
		double[] reEigen = aLonDecomposition.getRealEigenvalues();
		double[] imgEigen = aLonDecomposition.getImagEigenvalues();

		double [][] lambda_Matrix = new double [4][2];

		for (int i=0 ; i < 4 ; i++) {
			lambda_Matrix[i][0] = reEigen [i];
			lambda_Matrix[i][1] = imgEigen [i];
		}

		return lambda_Matrix;
	}
```

Figure 2.6 - Eigenvalues Matrix composition procedure

On each row of the matrix we have *real part* and *imaginary part* (separated).

Equally important it is to get the eigenvectors obtained through a procedure called *buildEigenVector*, which is recalled as many times as the number of the eigenvectors to calculate, specifying each time the index for the i^{th} eigenvector.

EXAMPLE 2.7

buildEigenVector:

```
public static RealVector buildEigenVector (double aMatrix[][], int index) {
        RealMatrix aRM = new Array2DRowRealMatrix(aMatrix);
        EigenDecomposition eigDec = new EigenDecomposition(aRM);

        RealVector eigVec = eigDec.getEigenvector(index);

        return eigVec;
  }
```

Figure 2.7 – An Eigenvector composition procedure

2.2.2 DYNAMIC CHARACTERISTICS

These *procedures* are based on many classical methods from the *linear time-invariant* (LTI) theory. They are designed to calculate all the dynamic characteristics concerning an *open-loop* response and equally valid in all the other specific cases. They use as inputs all the elements from the i^{th} line of the eigenvalues matrix previously calculated.

EXAMPLE 2.8

Damping coefficient:

$$\zeta_{DR} = \sqrt{\frac{1}{1+(\frac{\omega_{DR}}{\sigma_{DR}})^2}}$$

```
public static double calcZeta (double sigma, double omega) {

        return Math.sqrt( 1 / ( 1 + Math.pow( omega/sigma , 2 )));

}
```

Figure 2.8 - "Damping Coefficient" calculation procedure

This is an *exact* value of mode characteristics (not *approximate*).

2.3 FLIGHT DYNAMICS MANAGER CLASS

The class *FlightDynamicsManager.java* contains all the necessary *procedures* to perform the dynamic stability calculation of the aircraft, reading from file and storing the read data as *global variables*, performing the procedures imported from the classes previously treated and printing their results.

2.3.1 GLOBAL ENVIRONMENT

A *global variable* is visible (hence accessible) throughout the program, unless shadowed. The set of all global variables is known as the *global environment* or *global state.* In compiled languages, global variables are generally static variables, whose *extent* (lifetime) is the entire runtime of the program. They are generally dynamically allocated when declared, since they are not known ahead of time but, once we will read our file, they will be reallocated with a specific value. Here is shown our *global variables list*.

EXAMPLE 2.9

Global Variables List:

```
Propulsion propulsion_system =
Propulsion.CONSTANT_TRUST;  // propulsion
regime type
    double rho0;             // air density
    double surf;             // wing area
    double mass;             // total mass
    double cbar;             // mean
aerodynamic chord
    double bbar;             // wingspan
    double u0;               // speed of the
aircraft
    double q0;               // dynamic
pressure
    double m0;               // Mach number
    double gamma0;           // ramp angle
    double theta0_rad =
Math.toRadians(gamma0);      // Euler angle
[rad] (assuming gamma0 = theta0)
    double iXX;              // lateral-
directional moment of inertia (IXX)
    double iYY;              // longitudinal
moment of inertia  (IYY)
    double iZZ;              // lateral-
directional moment of inertia (IZZ)
    double iXZ;              // lateral-
directional product of inertia (IXZ)
double cd0;                  // drag
coefficient at null incidence (Cdº) of the
aircraft
    double cdAlpha0;         // linear drag
gradient (CdAlphaº) of the aircraft

    double cdM0;             // drag
coefficient with respect to Mach (ClMº) of the
aircraft
    double cl0;              // lift
coefficient at null incidence (Clº) of the
aircraft
    double clAlpha0;         // linear lift
gradient (ClAlphaº) of the aircraft
    double clAlpha_dot0;     // linear lift
gradient time derivative (ClAlpha_dotº) of the
aircraft
    double clQ0;             // lift
coefficient with respect to q (ClQº) of the
aircraft
    double clM0;             // lift
coefficient with respect to Mach (ClMº) of the
aircraft
    double clDelta_T;        // lift
coefficient with respect to delta_T
(ClDelta_Tº) of the aircraft
    double clDelta_E;        // lift
coefficient with respect to delta_E
(ClDelta_Eº) of the aircraft
    double cMAlpha0;         // pitching moment
coefficient with respect to Alpha (CmAlphaº)
of the aircraft
```

Figure 2.9 – "Global Variables" list (part 1 of 3)

```
double cMAlpha_dot0;        // pitching moment
coefficient time derivative (CmAlpha_dotº) of
the aircraft
    double cM_m0;           // pitching moment
coefficient with respect to Mach number
    double cMq;             // pitching moment
coefficient with respect to q
    double cMDelta_T;       // pitching moment
coefficient with respect to delta_T
(CMDelta_Tº) of the aircraft
    double cMDelta_E;       // pitching moment
coefficient with respect to delta_E
(CMDelta_Eº) of the aircraft
    double cTfix;           // thrust
coefficient at a fixed point ( U0 = u ,
delta_T = 1 )
    double kv;              // scale factor of
the effect on the propulsion due to the speed
    double cyBeta;          // lateral force
coefficient with respect to beta (CyBeta) of
the aircraft
    double cyP;             // lateral force
coefficient with respect to p (CyP) of the
aircraft
    double cyR;             // lateral force
coefficient with respect to r (CyR) of the
aircraft
    double cyDelta_A;       // lateral force
coefficient with respect to delta_A
(CyDelta_A) of the aircraft
    double cyDelta_R;       // lateral force
coefficient with respect to delta_R
(CyDelta_R) of the aircraft
    double cLBeta;          // rolling moment
coefficient with respect to beta (CLBeta) of
the aircraft
    double cLP;             // rolling moment
coefficient with respect to a p (CLP) of the
aircraft
    double cLR;             // rolling moment
coefficient with respect to a r (CLR) of the
aircraft
    double cLDelta_A;       // rolling moment
coefficient with respect to a delta_A
(CLDelta_A) of the aircraft
    double cLDelta_R;       // rolling moment
coefficient with respect to a delta_R
(CLDelta_R) of the aircraft
    double cNBeta;          // yawing moment
coefficient with respect to a beta (CNBeta) of
the aircraft
    double cNP;             // yawing moment
coefficient with respect to p (CNP) of the
aircraft
    double cNR;             // yawing moment
coefficient with respect to r (CNR) of the
aircraft
    double cNDelta_A;       // yawing moment
coefficient with respect to delta_A
(CNDelta_A) of the aircraft
    double cNDelta_R;       // yawing moment
coefficient with respect to delta_R
(CNDelta_R) of the aircraft

    double x_u_CT;          // dimensional
derivative of force component X with respect
to "u" for Constant Thrust
    double x_u_CP;          // dimensional
derivative of force component X with respect
to "u" for Constant Power
    double x_w;             // dimensional
derivative of force component X with respect
to "w"
    double x_w_dot;         // dimensional
derivative of force component X with respect
to "w_dot"
    double x_q;             // dimensional
derivative of force component X with respect
to "q"
    double z_u;             // dimensional
derivative of force component Z with respect
to "u"
    double z_w;             // dimensional
derivative of force component Z with respect
to "w"
    double z_w_dot;         // dimensional
derivative of force component Z with respect
to "w_dot"
    double z_q;             // dimensional
derivative of force component Z with respect
to "q"
    double m_u;             // dimensional
derivative of pitching moment M with respect
to "u"
    double m_w;             // dimensional
derivative of pitching moment M with respect
to "w"
    double m_w_dot;         // dimensional
derivative of pitching moment M with respect
to "w_dot"
    double m_q;             // dimensional
derivative of pitching moment M with respect
to "q"
    double x_delta_T_CT;    // dimensional
control derivative of force component X with
respect to "delta_T" for Constant Thrust
    double x_delta_T_CP;    // dimensional
control derivative of force component X with
respect to "delta_T" for Constant Power
    double x_delta_T_CMF;   // dimensional
control derivative of force component X with
respect to "delta_T" for Constant Mass Flow
    double x_delta_T_RJ;    // dimensional
control derivative of force component X with
respect to "delta_T" for RamJet
    double x_delta_E;       // dimensional
control derivative of force component X with
respect to "delta_E"
    double z_delta_T;       // dimensional
control derivative of force component Z with
respect to "delta_T"
    double z_delta_E;       // dimensional
control derivative of force component Z with
respect to "delta_E"
    double m_delta_T;       // dimensional
control derivative of pitching moment M with
respect to "delta_T"
    double m_delta_E;       // dimensional
control derivative of pitching moment M with
respect to "delta_E"
    double y_beta;          // dimensional
derivative of force component Y with respect
to "beta"
    double y_p   ;          // dimensional
derivative of force component Y with respect
to "p"
    double y_r   ;          // dimensional
derivative of force component Y with respect
to "r"
    double l_beta;          // dimensional
derivative of rolling moment L with respect to
"beta"
```

Figure 2.10 - "global variables" list (part 2 of 3)

```
    double l_p   ;          // dimensional
derivative of rolling moment L with respect to
"p"
    double l_r   ;          // dimensional
derivative of rolling moment L with respect to
"r"
    double n_beta;          // dimensional
derivative of yawing moment N with respect to
"beta"
    double n_p   ;          // dimensional
derivative of yawing moment N with respect to
"p"
    double n_r   ;          // dimensional
derivative of yawing moment N with respect to
"r"
    double y_delta_A;       // dimensional
control derivative of force component Y with
respect to "delta_A"
    double y_delta_R;       // dimensional
control derivative of force component Y with
respect to "delta_R"
    double l_delta_A;       // dimensional
control derivative of rolling moment L with
respect to "delta_A"
    double l_delta_R;       // dimensional
control derivative of rolling moment L with
respect to "delta_R"
    double n_delta_A;       // dimensional
control derivative of yawing moment N with
respect to "delta_A"
    double n_delta_R;       // dimensional
control derivative of yawing moment N with
respect to "delta_R"
    double [][] aLon = new double [4][4];
                            // longitudinal
coefficients [A_Lon] matrix
    double [][] bLon = new double [4][2];
                            // longitudinal
control coefficients [B_Lon] matrix
    double [][] aLD = new double [4][4];
                            // lateral-
directional coefficients [A_LD] matrix
    double [][] bLD = new double [4][2];
                            // lateral-
directional control coefficients [B_LD] matrix
    double[][] lonEigenvaluesMatrix = new
double [4][2];              // longitudinal
eigenvalues matrix
    double[][] ldEigenvaluesMatrix = new
double [4][2];              // lateral-
directional eigenvalues matrix
    RealVector eigLonVec1;  // longitudinal
1st eigenvector
    RealVector eigLonVec2;  // longitudinal
2nd eigenvector
    RealVector eigLonVec3;  // longitudinal
3rd eigenvector
    RealVector eigLonVec4;  // longitudinal
4th eigenvector
    RealVector eigLDVec1;   // lateral-
directional 1st eigenvector
    RealVector eigLDVec2;   // lateral-
directional 2nd eigenvector
    RealVector eigLDVec3;   // lateral-
directional 3rd eigenvector
    RealVector eigLDVec4;   // lateral-
directional 4th eigenvector
    double zeta_SP;         // Short Period
mode damping coefficient
    double zeta_PH;         // Phugoid mode
damping coefficient
    double omega_n_SP;      // Short Period
mode natural frequency
    double omega_n_PH;      // Phugoid mode
natural frequency
    double period_SP;       // Short Period
mode period
    double period_PH;       // Phugoid mode
period
    double t_half_SP;       // Short Period
mode halving time
    double t_half_PH;       // Phugoid mode
halving time
    double N_half_SP;       // Short Period
mode number of cycles to halving time
    double N_half_PH;       // Phugoid mode
number of cycles to halving time
    double zeta_DR;         // Dutch-Roll mode
damping coefficient
    double omega_n_DR;      // Dutch-Roll mode
natural frequency
    double period_DR;       // Dutch-Roll mode
period
    double t_half_DR;       // Dutch-Roll mode
halving time
    double N_half_DR;       // Dutch-Roll mode
number of cycles to halving time
```

Figure 2.11 - "Global Variables" list (part 3 of 3)

2.3.2 INPUT MANAGEMENT CLASSES

The two *procedures* named *readDataFromExcelFile* and *cellToString* are designed for extracting and reinterpreting any information contained in specific cells of an *excel* file as a *string* value. This file has to follow a particular model or reading procedure will not succeeds. In fact, our reading *procedure* is designed to read only the 2nd column of the current sheet (sheet number can be selected within the *main method*) and to stop once reached the 47th line (as much lines as the data are).

EXAMPLE 2.10

A Correct Excel file output:

1	Variable Name	Variable Value	Unit Label	Description
2	propulsion_system	CONSTANT_TRUST	NONE	propulsion regime type
3	rho0	1,225	kg*m3	air density
4	surf	510,97	m^2	wing area
5	mass	255753	kg	total mass
6	cbar	8,32	m	mean aerodynamic chord
7	bbar	59,64	m	wingspan
8	u0	85,075	m*s	speed of the aircraft
9	m0	0,25	NONE	Mach number
10	gamma0	0	deg	ramp angle
11	theta0_rad	0	rad	Euler angle [rad] (assuming gamma0 = theta0)
[…]	*[…]*	*[…]*	*[…]*	*[…]*
46	cNDelta_A	0,0064	rad^(-1)	yawing moment coefficient with respect to delta_A (CNDelta_A) of the aircraft
47	cNDelta_R	-0,109	rad^(-1)	yawing moment coefficient with respect to delta_R (CNDelta_R) of the aircraft
48	*(empty)*	*(empty)*	*(empty)*	*(empty)*
49	*(empty)*	*(empty)*	*(empty)*	*(empty)*

Figure 2.12 – A correct "excel file" output

The last two rows are intentionally left *empty*.

EXAMPLE 2.11

readDataFromExcelFile:

```
public void readDataFromExcelFile(File excelFile, int sheetNum) {
        // Formats numbers up to 4 decimal places
        DecimalFormat df = new DecimalFormat("#,###,##0.0000");

        try {
              System.out.println("Input file: " + excelFile.getAbsolutePath());
              FileInputStream fis = new FileInputStream(excelFile);
              Workbook wb = WorkbookFactory.create(fis);
              Sheet ws = wb.getSheetAt(sheetNum);
              int rowNum = ws.getLastRowNum() + 1;
              System.out.println("Numero di righe: " + rowNum);
```

Figure 2.13 - Reading from excel file (part 1 of 3)

```
if(sheetNum == 0){
        System.out.println("-----------------------------------\n");
        System.out.println("\n BOEING 747 /// Flight Condition (2)”);
        System.out.println("___________________________________\n");
        System.out.println("DATA LIST: \n");
}
else if (sheetNum == 1){
        System.out.println("-----------------------------------\n");
        System.out.println("\n BOEING 747 /// Flight Condition (5)”);
        System.out.println("___________________________________\n");
        System.out.println("DATA LIST: \n");
}

for (int i = 0 ; i < rowNum ; i++) {
        Row row = ws.getRow(i);
        int colNum = ws.getRow(0).getLastCellNum();
        for (int j = 0 ; j < colNum-2 ; j++) {

                Cell cell = row.getCell(j);
                String value = cellToString(cell);
                switch (sheetNum){
                ///////////// 1st sheet ////////////
                case 0:
                        if ((i == 1) && (j == 1)) {
                        propulsion_system = Propulsion.valueOf(value);
                        switch (propulsion_system){
                           case CONSTANT_TRUST:
                               System.out.println(" PROPULSION SYSTEM:
                               CONSTANT TRUST \n");
                               break;
                                        [...]
                           default:
                           System.out.println(" PROPULSION SYSTEM:
                           CONSTANT TRUST \n");
                           break;
                           }
                        }

                        if ((i == 2) && (j == 1)) {
                        rho0 = Double.parseDouble(value);
                        System.out.println(" rho0          = " + rho0);
                        }

                        if ((i == 5) && (j == 1)) {
                        cbar = Double.parseDouble(value);
                        System.out.println(" cbar          = " + cbar);
                        }
                                        [...]

                        if ((i == 46) && (j == 1)) {
                        cNDelta_R = Double.parseDouble(value);
                        System.out.println(" cNDelta_R     = " +
                        cNDelta_R);
                        }

                        break;
```

This *method* extracts the rows number, using it as final value in the *for* cicle. During this cicle, for each row, the program reads the ith row and applies a new *for* cicle to read the jth column value. Then happens a *switch-case* cicle relative to sheet number and, right after, another *switch-case* cicle relative to the cell index. At this point a *sub-routine* saves the values in a *global variable* and prints it. Then the process is repeated for the 2nd sheet. If there's any need to read a file with more than two sheets, the code can be easily modified. Moreover, code could be even more simplified by using the same routine for 1st and 2nd sheet, but we preferred to keep them separated, in case that the second sheet present a different number or order of elements.

Figure 2.14 - Reading from excel file (part 2 of 3)

```
                                        ///////////// 2nd sheet /////////////
                                        case 0:
                                        if ((i == 1) && (j == 1)) {

                                                     [...]

                                        break;
                                        }
        }       }       }

        catch(Exception ioe) {
                ioe.printStackTrace();
        }       }
```

Figure 2.15 - Reading from excel file (part 3 of 3)

The second procedure, *cellToString*, will not be explained in details to be lighter on the discussion but it is important to know that it provides a method for converting almost any kind of variables types into a *String* type.

2.3.3 CONSTRUCTOR METHOD

Constructor in *Java* is a special type of *method* that is used to initialize the *object*.

```
public FlightDynamicsManager() {


   }
```

Figure 2.16 -"Constructor" method

Java constructor is invoked at the time of *object* creation. It constructs the values i.e. provides data for the object that is why it is known as *constructor*.

2.3.4 CALCULATOR METHOD

The *CalculateAll* method is definitely the biggest one. It has the task to recall all the other methods contained in the program and to save their results in the *global variables*, then it has to print them on screen to visualize the efficiency of the execution. Show in detail the entire *CalculateAll* structure would be unproductive, so is preferable to explain its single steps and extract some examples from each one.

"CalculateAll" **main structure** (for each *Longitudinal* and *Lateral-Directional dynamics*):

- Stability and Control Derivatives Calculation;
- Prints out the Stability and Control Derivatives List;
- Generates and Prints out the **A** and **B** matrices;
- Generates and Prints out the Eigenvalues matrix of **A** matrix;
- Generates and Prints out the Eigenvectors of **A** matrix;
- Calculates and Prints out the characteristics for *open-loop* modes;

We will now extract some examples for each category of tasks.

EXAMPLE 2.12

Stability and Control Derivatives Calculation (X_u_CT)

```
x_u_CT  = StabilityDerivativesCalc.calcX_u_CT(rho0, surf, mass, u0, q0, cd0, m0,
cdM0);
```

Figure 2.17 -"X_u_CT" derivative calculation

EXAMPLE 2.13

Stability and Control Derivatives List (X_u_CT)

```
// Formats numbers up to 4 decimal places
   DecimalFormat df = new DecimalFormat("#,###,##0.0000");

System.out.println("LONGITUDINAL STABILITY DERIVATIVES: \n");
   System.out.println(" Xªu_CT  = " + df.format(x_u_CT));
```

Figure 2.18 - "X_u_CT" derivative printing out

EXAMPLE 2.14

***A** and **B** matrices (**A**LON matrix)*

```
aLon = StabilityDerivativesCalc.build_A_Lon_matrix (propulsion_system, rho0, surf,
       mass, cbar, u0, q0, cd0, m0, cdM0, cl0, cdAlpha0, gamma0, theta0_rad,
       clAlpha0, clAlpha_dot0, cMAlpha0, cMAlpha_dot0, clQ0, iYY, cM_m0, cMq);

System.out.println(df.format(aLon[0][0])+"\t\t"+df.format(aLon[0][1])+"\t\t"+
df.format(aLon[0][2])+"\t\t"+df.format(aLon[0][3])+"\n");

                                    […]

System.out.println(aLon[3][0]+"\t\t"+aLon[3][1]+"\t\t"+aLon[3][2]+"\t\t"+aLon
[3][3]+"\n");
```

Figure 2.19 - "[A_Lon]" matrix calculation and printing out

EXAMPLE 2.15

Eigenvalues matrix of **A** *matrix (*$\mathbf{A}_{\mathrm{LON}}$*)*

```
lonEigenvaluesMatrix = DynamicStabilityCalculator.buildEigenValuesMatrix(aLon);

System.out.println("  SHORT PERIOD: "+df.format(lonEigenvaluesMatrix[0][0])+" ± j"+df.format(lonEigenvaluesMatrix[0][1])+"\n");

System.out.println("  PHUGOID:      "+df.format(lonEigenvaluesMatrix[2][0])+" ± j"+df.format(lonEigenvaluesMatrix[2][1])+"\n");
```

Figure 2.20 - Longitudinal eigenvalues generation and printing out

EXAMPLE 2.15

Eigenvectors of **A** *matrix (1st Eigenvector)*

```
System.out.println("________________________________________________________\n");
System.out.println("LONGITUDINAL EIGENVECTORS:\n");

eigLonVec1 = DynamicStabilityCalculator.buildEigenVector(aLon, 0);

System.out.println("EigenVector 1 = " + eigLonVec1);
```

Figure 2.21 - 1st longitudinal eigenvector generation and printing out

EXAMPLE 2.15

Open-Loop characteristics (Short Period)

```
zeta_SP = DynamicStabilityCalculator.calcZeta(lonEigenvaluesMatrix[0][0], lonEigenvaluesMatrix[0][1]);
omega_n_SP = DynamicStabilityCalculator.calcOmega_n(lonEigenvaluesMatrix[0][0], lonEigenvaluesMatrix[0][1]);
period_SP = DynamicStabilityCalculator.calcT(lonEigenvaluesMatrix[0][0], lonEigenvaluesMatrix[0][1]);
t_half_SP = DynamicStabilityCalculator.calct_half(lonEigenvaluesMatrix[0][0], lonEigenvaluesMatrix[0][1]);
N_half_SP = DynamicStabilityCalculator.calcN_half(lonEigenvaluesMatrix[0][0], lonEigenvaluesMatrix[0][1]);

System.out.println("SHORT PERIOD MODE CHARACTERISTICS\n");
System.out.println("Zeta_SP                       = "+df.format(zeta_SP)+"\n");
System.out.println("Omega_n_SP                    = "+df.format(omega_n_SP)+"\n");
System.out.println("Period                        = "+df.format(period_SP)+"\n");
System.out.println("Halving Time                  = "+df.format(t_half_SP)+"\n");
System.out.println("Number of cycles to Halving Time = "+df.format(N_half_SP)+"\n");
```

Figure 2.22 – "Short Period" open-loop characteristics

All customer examples cited well sums up all the statements that occur within the *method*. To better visualize how exactly does the *Calculator* works, we invite you to visualize the relative *APPENDIX*.

2.3.5 MAIN METHOD

In the Java language, when you execute a class with the Java *interpreter*, the runtime system starts by calling the class's *main()* method. The *Java Main Method* then calls all the other methods required to run your application.

EXAMPLE 2.15

Java Main Method

```
public static void main(String[] args) {

        FlightDynamicsManager theObj = new FlightDynamicsManager();

        System.out.println("---------------------------------------------------");
        System.out.println("Reading input data file (excel format)");
        String inputFileName = "AIRCRAFT_DATA.xlsx";
        File excelFile = new File (inputFileName) ;

        ////// select the excel sheet you want to read \\\\\\
                            int sheetNumber = 0;

        if (excelFile.exists()){

               System.out.println("File " + inputFileName + " found.");
               System.out.println("\n %%% start reading from file %%% ");

               // Read all data from file
               theObj.readDataFromExcelFile(excelFile, sheetNumber);
               System.out.println("\n %%% end of reading from file %%%");

               theObj.calculateAll();

        }
        else {
               System.out.println("File " + inputFileName + " not found.");

        }
   }
```

Figure 2.23 - Java Main Method

First step is to initialize the *Constructor*, then start reading from file.

Inside of it, we can select the *sheet number* (condition of flight). After that, *Calculator* starts. The *Main method* has been built as shortest as possible.

3 BOEING 747 – TEST

We are now ready to test the program on a complete aircraft model (*Boeing-747*) in two specific conditions of flight, making a comparison with actual results.

3.1 CONDITIONS OF FLIGHT

In the next figure, we will report the characteristics of the *Boeing-747*, a large *airliner* with turbofan engines, in two different flight conditions. The data were taken from the ***NASA report*** (Heffley & Jewell, December 1972). Numbering of the conditions considered ('2' and '5') corresponds to the same used in the original report. All the configurations appear in a clean configuration (flap not deflected and motors in function) except for condition '2', which presents the aircraft in *powered approach* configuration with a 20 deg flaps deflection.

Variable	Unit	Condition 2	Condition 5
h_0	m	0	6096
ρ_0	kg/m³	1,225	0,653
a_0	m/s	340,29	158,02
S	m²	510,97	510,97
m	kg	255753	288676
$\bar{c}$	m	8,32	8,32
$\bar{b}$	m	59,64	59,64
U_0	m/s	85,07	158,02
q_0	kg/m·s²	1098,42	8148,65
M_0	[adim.]	0,25	0,50
Γ_0	deg	0	0
I_{xx}	kg·m²	$1{,}94\cdot10^7$	$2{,}49\cdot10^7$
I_{yy}	kg·m²	$4{,}38\cdot10^7$	$4{,}49\cdot10^7$
I_{zz}	kg·m²	$6{,}14\cdot10^7$	$6{,}71\cdot10^7$
I_{xz}	kg·m²	$-3{,}02\cdot10^6$	$-3{,}74\cdot10^6$
$SM = (X_n - X_{cg})/\bar{c}$	[adim.]	0,22	0,22
α_B	deg	5,70	6,80
C_D	[adim.]	0,102	0,04

Figure 3.1 – Flight Conditions (part 1 of 2) (Heffley & Jewell, December 1972)

Variable	Unit	Condition 2	Condition 5
C_{D_α}	rad^{-1}	0,66	0,37
C_{D_M}	[adim.]	0	0
C_L	[adim.]	1,108	0,68
C_{L_α}	rad^{-1}	5,70	4,67
$C_{L_{\dot\alpha}}$	rad^{-1}	6,70	6,53
C_{L_M}		0	-0,09
C_{L_q}	rad^{-1}	5,40	5,13
$C_{L_{\delta_T}}$	[adim.]	0	0
$C_{L_{\delta_E}}$	rad^{-1}	0,338	0,356
C_{m_α}	rad^{-1}	-1,26	-1,15
$C_{m_{\dot\alpha}}$	rad^{-1}	-3,20	-3,35
C_{m_M}	[adim.]	0	0,12
C_{m_q}	rad^{-1}	-20,80	-20,7
$C_{m_{\delta_T}}$	rad^{-1}	0	0
$C_{m_{\delta_E}}$	rad^{-1}	-1,34	-1,43
$C_{T_{fix}}$	[adim.]	0	0
k_v	[adim.]	0	0
C_{Y_β}	[adim.]	-0,96	-0,9
C_{Y_p}	[adim.]	0	0
C_{Y_r}	[adim.]	0	0
$C_{Y_{\delta_A}}$	[adim.]	0	0
$C_{Y_{\delta_R}}$	[adim.]	0,175	0,1448
C_{l_β}	[adim.]	-0,22	-0,193
C_{l_p}	[adim.]	-0,45	-0,323
C_{l_r}	[adim.]	0,10	0,212
$C_{l_{\delta_A}}$	[adim.]	0,046	0,0129
$C_{l_{\delta_R}}$	[adim.]	0,007	0,0039
C_{n_β}	[adim.]	0,15	0,147
C_{n_p}	[adim.]	-0,12	-0,0687
C_{n_r}	[adim.]	-0,30	-0,278
$C_{n_{\delta_A}}$	[adim.]	0,0064	0,0015
$C_{n_{\delta_R}}$	[adim.]	-0,109	-0,1081

Figure 3.2 – Flight Conditions (part 2 of 2) (Heffley & Jewell, December 1972)

3.2 OUTPUT

We will now see a typical output for flight conditions '2' and '5'.

Condition 2

```
-----------------------------------------
Reading input data file (excel format)
File AIRCRAFT_DATA.xlsx found.

 %%% start reading from file %%%
Input file:
C:\workspace\newproj\AIRCRAFT_DATA.xlsx
rows number: 50
-----------------------------------------

 BOEING 747 /// Flight Condition (2)

DATA LIST:

 PROPULSION SYSTEM: CONSTANT TRUST

 rho0          = 1.225
 surf          = 510.9667
 mass          = 255753
                   […]
 cNR           = -0.3
 cNDelta_A     = 0.0064
 cNDelta_R     = -0.109

 %%% end of reading from file %%%

LONGITUDINAL STABILITY DERIVATIVES:

 Xªu_CT   = -0,0212
 Xªu_CP   = -0,0319
 Xªw      = 0,0466
 Xªw_dot  = 0,0000
 Xªq      = 0,0000
 Zªu      = -0,2307
 Zªw      = -0,6040
 Zªw_dot  = -0,0341
 Zªq      = -2,3389
 Mªu      = 0,0000
 Mªw      = -0,0064
 Mªw_dot  = -0,0008
 Mªq      = -0,4378


LONGITUDINAL CONTROL DERIVATIVES:

 Xªdelta_T_CT  = -0,0000
 Xªdelta_T_CP  = -0,0000
                   […]
 Zªdelta_E     = -2,9935
 Mªdelta_T     = 0,0000
 Mªdelta_E     = -0,5767
```

Condition 5

```
-----------------------------------------
Reading input data file (excel format)
File AIRCRAFT_DATA.xlsx found.

 %%% start reading from file %%%
Input file:
C:\workspace\newproj\AIRCRAFT_DATA.xlsx
rows number: 50
-----------------------------------------

 BOEING 747 /// Flight Condition (5)

DATA LIST:

 PROPULSION SYSTEM: CONSTANT TRUST

 rho0          = 0.6527
 surf          = 510.9667
 mass          = 288676
                   […]
 cNR           = -0.278
 cNDelta_A     = 0.0015
 cNDelta_R     = -0.1081

 %%% end of reading from file %%%

LONGITUDINAL STABILITY DERIVATIVES:

 Xªu_CT   = -0,0073
 Xªu_CP   = -0,0110
 Xªw      = 0,0283
 Xªw_dot  = 0,0000
 Xªq      = 0,0000
 Zªu      = -0,1240
 Zªw      = -0,4299
 Zªw_dot  = -0,0157
 Zªq      = -1,9482
 Mªu      = 0,0000
 Mªw      = -0,0056
 Mªw_dot  = -0,0004
 Mªq      = -0,4208


LONGITUDINAL CONTROL DERIVATIVES:

 Xªdelta_T_CT  = -0,0000
 Xªdelta_T_CP  = -0,0000
                   […]
 Zªdelta_E     = -5,1347
 Mªdelta_T     = 0,0000
 Mªdelta_E     = -1,1040
```

Figure 3.3 - Conditions '2' and '5' Output (part 1 of 5)

```
MATRIX [A_LON]:

-0,0212   0,0466   0,0000    -9,8100

-0,2231  -0,5841   80,0055   -0,0000

 0,0002  -0,0059  -0,5011     0,0000

  0.0      0.0      1.0         0.0

MATRIX [B_LON]:

-0,0000      0,0000

 0,0000     -2,8948

 0,000      -0,5744

  0.0         0.0

LONGITUDINAL EIGENVALUES

  SHORT PERIOD: -0,5515 ± j0,6879

  PHUGOID:      -0,0018 ± j0,1340

LONGITUDINAL EIGENVECTORS:

EigenVector 1 = {0,0062; -3,4144; -
0,0940; 0,0468}
EigenVector 2 = {0,8950; 10,7820; -
0,0224; 0,0991}
EigenVector 3 = {-41,9631; 6,24644; -
0,0622; 0,7458}
EigenVector 4 = {46,3490; -5,5770;
0,0991; 0,4545}

SHORT PERIOD MODE CHARACTERISTICS

Zeta_SP                          = 0,6255

Omega_n_SP                       = 0,8816

Period                           = 9,1341

Halving Time                     = 1,2569

Number of cycles to Halving Time = 0,1376
```

```
MATRIX [A_LON]:

-0,0073   0,0283   0,0000   -9,8100

-0,1195  -0,4233   153,65   -0,0000

 0,0003  -0,0054  -0,4870    0,0000

  0.0       0.0      1.0       0.0

MATRIX [B_LON]:

-0,0000      0,0000

 0,0000     -5,0554

 0,0000     -1,1018

  0.0         0.0

LONGITUDINAL EIGENVALUES

  SHORT PERIOD: -0,4567 ± j0,9119

  PHUGOID:      -0,0021 ± j0,0866

LONGITUDINAL EIGENVECTORS:

EigenVector 1 = {0,0137; -0,0158; 0,0080;
-0,0034}
EigenVector 2 = {-0,0429; -1,3620;
0,0001; -0,0071}
EigenVector 3 = {-89,2840; 1,0188; -
0,0669; 0,5293}
EigenVector 4 = {54,2225; 0,5551; 0,0442;
0,7607}

SHORT PERIOD MODE CHARACTERISTICS

Zeta_SP                          = 0,4478

Omega_n_SP                       = 1,0199

Period                           = 6,8901

Halving Time                     = 1,5177

Number of cycles to Halving Time = 0,2203
```

Figure 3.4 - Conditions '2' and '5' Output (part 2 of 5)

```
PHUGOID MODE CHARACTERISTICS                    PHUGOID MODE CHARACTERISTICS

Zeta_PH                          = 0,0132       Zeta_PH                          = 0,0238

Omega_n_PH                       = 0,1340       Omega_n_PH                       = 0,0866

Period                           = 46,905       Period                           = 72,555

Halving Time                     = 391,14       Halving Time                     = 336,90

Number of cycles to Halving Time = 8,3390       Number of cycles to Halving Time = 4,6435
```

Figure 3.5 - Conditions '2' and '5' Output (part 3 of 5)

As we can see from the *free response* characteristics and from the image below, it is easy to visualize the main differences between longitudinal response modes.

In particular, *short period mode* is characterized by a stronger damping coefficient and a higher natural frequency, which lead to a shorter period than *phugoid mode*.

It is also interesting to compare their values from condition '2' to '5', in dependence from altitude and Mach value.

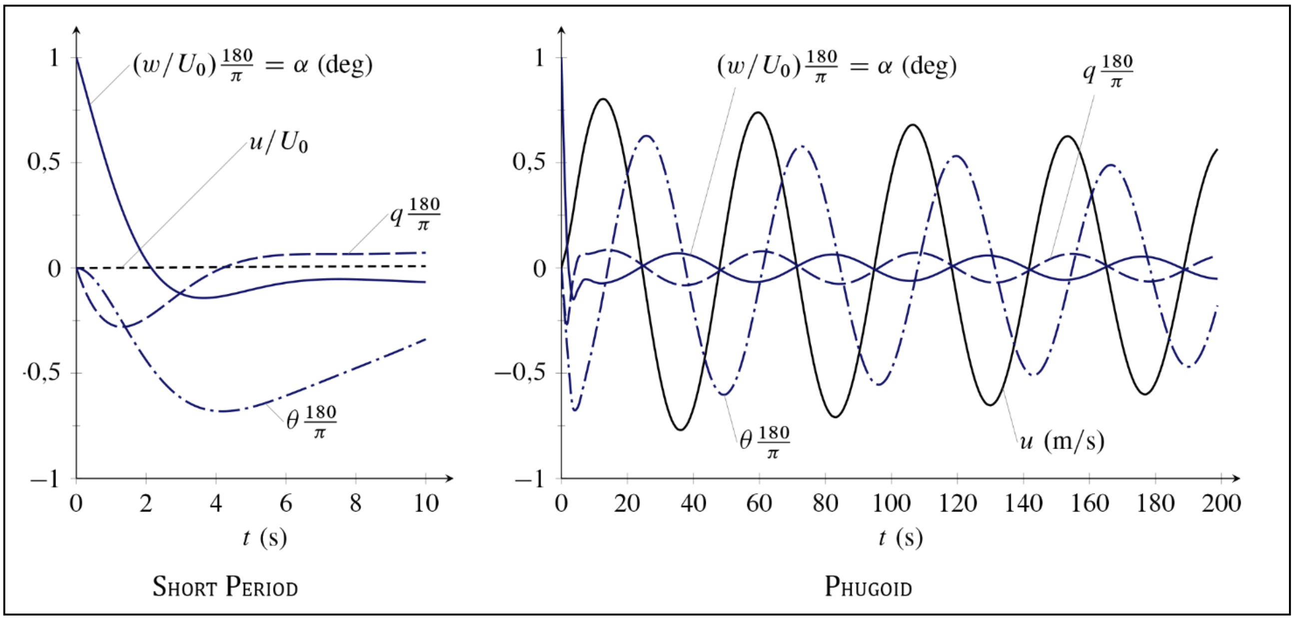


Figure 3.6 - Free responses of a Boeing-747 in the longitudinal disturbances of initial condition. The two responses were obtained exciting, respectively, only the Short Period mode (left) and the Phugoid mode (right) (De Marco & Coiro, 2015)

Condition 2

```
LATERAL-DIRECTIONAL STABILITY
DERIVATIVES:

 Yªbeta = -8,5023
 Yªp    = 0,0000
 Yªr    = 0,0000
 Lªbeta = -1,5399
 Lªp    = -1,0992
 Lªr    = 0,2467
 Nªbeta = 0,3299
 Nªp    = -0,0933
 Nªr    = -0,2313

LATERAL-DIRECTIONAL CONTROL DERIVATIVES:

 Yªdelta_A = 0,0000
 Yªdelta_R = 1,5499
 Lªdelta_A = 0,3212
 Lªdelta_R = 0,0488
 Nªdelta_A = 0,0141
 Nªdelta_R = -0,2398

MATRIX [A_LD]:

-0,2453   0,4089  -0,0395   0,0000

-1,0000  -0,0999   0,0000   0,1153

 0,2850  -1,6037  -1,0930   0,0000

  0.0      0.0      1.0      0.0

MATRIX [B_LD]:

-0,0017     -0,2440

 0,0000      0,0182

 0,3215      0,0868

  0.0         0.0

LATERAL-DIRECTIONAL EIGENVALUES

  ROLL:       -1,2306

  DUTCH-ROLL: -0,0806 ± j0,7433

  SPIRAL:     -0,0464
```

Condition 5

```
LATERAL-DIRECTIONAL STABILITY
DERIVATIVES:

 Yªbeta = -12,9810
 Yªp    = 0,0000
 Yªr    = 0,0000
 Lªbeta = -1,9212
 Lªp    = -0,6068
 Lªr    = 0,3983
 Nªbeta = 0,5439
 Nªp    = -0,0480
 Nªr    = -0,1941

LATERAL-DIRECTIONAL CONTROL DERIVATIVES:

 Yªdelta_A = 0,0000
 Yªdelta_R = 2,0885
 Lªdelta_A = 0,1284
 Lªdelta_R = 0,0388
 Nªdelta_A = 0,0056
 Nªdelta_R = -0,4000

MATRIX [A_LD]:

-0,2182   0,6566  -0,0143   0,0000

-1,0000  -0,0822   0,0000   0,0621

 0,4310  -2,0197  -0,6047   0,0000

  0.0      0.0      1.0      0.0

MATRIX [B_LD]:

-0,0016     -0,4056

 0,0000      0,0132

 0,1287      0,0997

  0.0         0.0

LATERAL-DIRECTIONAL EIGENVALUES

  ROLL:       -0,7414

  DUTCH-ROLL: -0,0729 ± j0,8562

  SPIRAL:     -0,0179
```

Figure 3.7 - Conditions '2' and '5' Output (part 4 of 5)

```
LATERAL-DIRECTIONAL EIGENVECTORS:

EigenVector 1 = {-0,1100; -0,6107;
0,7215; -0,4934}
EigenVector 2 = {0,3499; -0,0873; -
0,2928; -0,9171}
EigenVector 3 = {0,0052; 0,1090; 1,2600;
-1,0238}
EigenVector 4 = {-0,3019; -0,1348;
0,1244; -2,6813}

DUTCH-ROLL MODE CHARACTERISTICS

Zeta_DR                          = 0,1078

Omega_n_DR                       = 0,7477

Period                           = 8,4529

Halving Time                     = 8,5975

Number of cycles to Halving Time = 1,0171
```

```
LATERAL-DIRECTIONAL EIGENVECTORS:

EigenVector 1 = {0,2651; 0,2718; -0,8138;
0,0830}
EigenVector 2 = {-0,1770; 0,3066; 0,0023;
0,9435}
EigenVector 3 = {0,0623; -0,0794; -
1,3701; 1,8480}
EigenVector 4 = {-0,2514; -0,0751;
0,0738; -4,1280}

DUTCH-ROLL MODE CHARACTERISTICS

Zeta_DR                          = 0,0848

Omega_n_DR                       = 0,8593

Period                           = 7,3387

Halving Time                     = 9,5143

Number of cycles to Halving Time = 1,2964
```

Figure 3.8 - Conditions '2' and '5' Output (part 5 of 5)

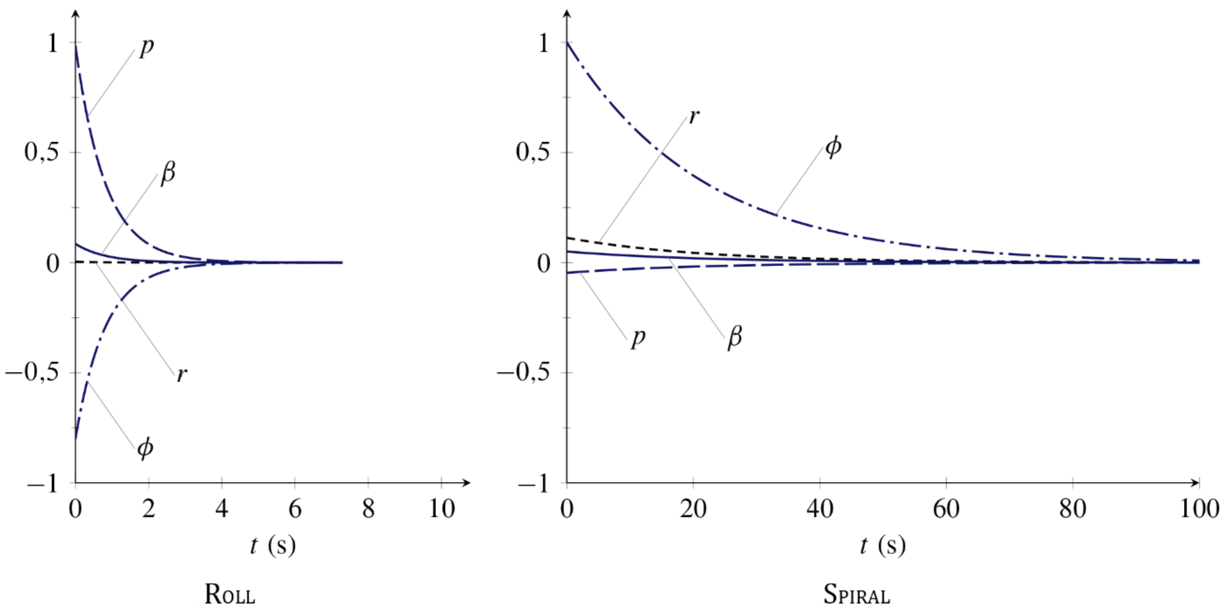
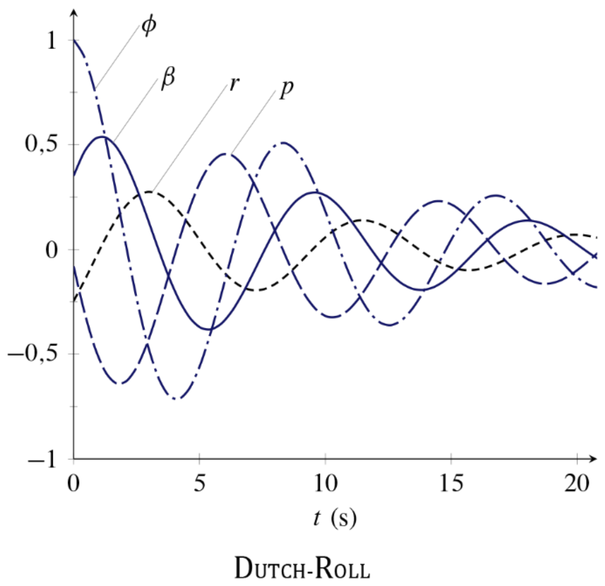


Figure 3.9 - Free responses of a Boeing-747 in the lateral-directional disturbances of initial condition. The two responses were obtained exciting, respectively, only the Roll mode (left) and the Spiral mode (right) (De Marco & Coiro, 2015)



The free responses shown in *Figures 3.6, 3.9* and *3.10* are obtained resolving the problem for an initial condition coincident, respectively, with the (left) eigenvector of free response modes.

Figure 3.10 - Free response of a Boeing 747 in lateral-directional perturbation, obtained exciting just the Dutch-Roll mode (De Marco & Coiro, 2015)FINAL REMARKS

3.3 FINAL REMARKS

Stability analysis transcends our discussion, in fact our program simply extrapolates dynamics key values without analyzing them in a system response optics. Nonetheless, it is immediate to note how all the eigenvectors are characterized by a negative real part, which supports the stability condition for small perturbations hypothesis. It is also clear how the values between condition '2' and '5' are comparable in order of magnitude. In addition, the differences between damping coefficient and natural frequency in longitudinal response modes explains why we easily distinguish the *short period* and *phugoid* modes according to their dynamic characteristics. Moreover, plotting damping coefficient in terms of natural frequency is of theoretical interest to determine a zone of response optimization, according to the *Thumb Print criterion (De Marco & Coiro, 2015)*.

Finally, we recall that all the results achieved are in exact form, obtained directly from the eigenvalues. There are also approximation formulas obtained instead manipulating the stability derivatives, which are more or less valid depending on the conditions of motion. Some cases, in particular the *dutch-roll mode*, are difficult to approximate, since they induce variations on all four state variables. That is why, for the purpose of a more appropriate and applicable analysis, we do not use these formulas to derive the characteristics of response modes but we prefer, instead, to obtain them from a more complex and accurate study on eigenvalues.

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NOTES

[1] Real roots λ_{HEIGHT} and λ_{RANGE} do not appear in our characteristic polynomial because matrix $\mathbf{A}_{\text{LON}}$ order has been reduced due to our simplified nominal condition. Their weight would have been negligible, anyway.