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Requirements for Geometrically Optimized
Aircraft**

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**AIAA 16TH AEROSPACE
SCIENCES MEETING**

Huntsville, Alabama/January 16-18, 1978

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RELATING TAKE OFF GROSS WEIGHT AND MISSION
REQUIREMENTS FOR GEOMETRICALLY OPTIMIZED AIRCRAFT

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Abstract

The method of latin squares was applied to a three-dimensional mission space for a typical ground attack mission profile and independently to a three-dimensional design space to select input conditions for an aircraft sizing program. For each mission, the output take-off-gross-weights (TOGW) and corresponding design points determined a quadratic polynomial for TOGW in the three design variables by least squares regression analysis. The minimum TOGWs corresponding to each of the thirteen selected mission points were then determined and these were fit by a quadratic polynomial in the three mission variables. The result is a simple algebraic relationship between mission requirements and TOGW for geometrically optimized fighter aircraft. In a second application of these methods, two engine variables were included in the design space.

Nomenclature

AR	Aspect Ratio, independent design variable
BPR	Engine By-pass Ratio, independent design variable
DLN	Landing distance, dependent variable - feet
DTO	Take off distance, dependent variable - feet
GSS	Sustained normal load factor - "g's"
MACH	Dash Mach number
OPR	Operating engine pressure ratio, independent design variable
RNG	Dash range - nautical miles
STR	Internal expendable payload, independent mission variable - pounds
TAC	Acceleration time - minutes
TOGW	Take off gross weight, dependent mission variable - pounds
TW	Aircraft thrust-to-weight ratio, independent design variable
WOS	Aircraft wing loading, independent design variable - pounds per square foot

I. Introduction

This study demonstrates the applicability of statistical and optimization techniques to the task of relating minimum take-off gross weight to mission variables for fighter aircraft. This process is seen as an essential first step in the identification of the most cost-effective configuration for performing a particular task in a given scenario. It is expected that the techniques applied here to aircraft sizing will be equally applicable to aircraft costing and effectiveness studies if adequate cost and effectiveness models can be developed. The possibility then exists for analytically investigating the relationship between cost-effectiveness and mission variables.

The purpose of the effort was to demonstrate the application of latin squares, regression analysis, and optimization methods to determine the relationship between the take off gross weight and the mission requirements for an aircraft optimized in terms of its design geometry to yield the minimum gross weight required to perform the mission.

The following nine step approach was used in this study:

1. A mission profile was selected for simulation.
2. Three independent mission variables were selected and the range of their values defined. This three-dimensional space was called the "Mission Space."
3. Three independent design variables were selected and their range of values defined. This three-dimensional space was called the "Design Space."
4. The simple latin squares method was used to select several particular mission space points at which minimum weight designs were to be determined for points within the design space.
5. The simple latin squares method was used to select several particular design space points at which aircraft sizing was performed.
6. An aircraft sizing program was used to determine the take-off-gross-weight (TOGW), take-off-distance (DTO), and landing-distance (DLN) for each selected design point at each selected mission point.
7. For each mission point, TOGW was related to the design parameters by fitting a quadratic expression in the three design parameters to the sizing data using a regression analysis program, SURFIT.
8. Using these expressions, the minimum TOGW for each mission was determined with an optimization program, OAPEN.
9. Finally, SURFIT was used to obtain the desired relationship between these minimum TOGW's and their associated mission parameters.

In a second application of these methods, two engine parameters, OPR (operating pressure ratio) and BPR (by-pass ratio) were added to the design space and minimum TOGWs were determined for a single mission point subject to a succession of increasingly severe constraints on DTO, DLN, GSS (sustained normal load factor - "g's"), and TAC (acceleration time). Additional details regarding this study can be found in Reference 1.

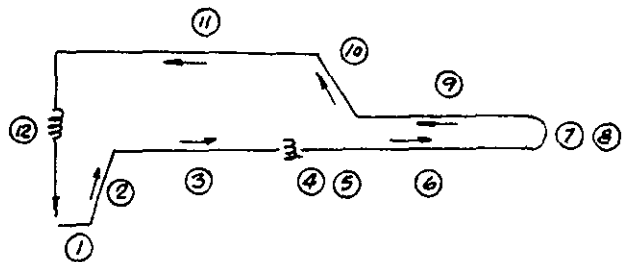
II. Mission Point and Design Point Selection

For the complete mission profile described in Fig. 1, the three parameters MACH (dash Mach number), RNG (dash range), and STR (internal expendable payload weight) were permitted to vary over the range specified in Table 1. Within this "mission space", twenty-seven (3^3) mission points were identified by the maximum, mean, and minimum values of each

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parameter. The simple latin squares method was used to select several points for further study from these candidate points.



1. Start engines, taxi, takeoff
2. Climb at .85M to 35000 feet
3. Outbound cruise - 275 NM, .85M, 35000 feet
4. Loiter - 30 minutes, .85M, 35000 feet
5. Accelerate to dash Mach number
6. Outbound dash - dash Mach number
7. Combat - 1 maximum "G" turn at dash Mach number
8. Expend internal stores
9. Inbound dash - dash Mach number
10. Climb to 45000 feet using afterburner
11. Inbound cruise - 325 NM, .85M
12. Descend to sea level, loiter for 20 minutes at .4M, land

Fig. 1 Mission profile

Table 1 Mission space

Variable	Range of Values
MACH	1.2 - 1.6
RNG	200 - 400 NM
STR	5000-10000 LBS

The aircraft and engine design parameters selected to be varied were WOS (wing loading), AR (aspect ratio), TW (aircraft thrust to weight ratio), OPR and BPR. The five-dimensional "design space" in which their variation was constrained is described in Table 2. The design variables were fixed at five equally spaced values for a total of 5^5 (3125) candidate design points for each of the 3^3 (27) mission points. The task of determining aircraft TOGWs at all possible combinations ($3^3 \times 5^5 = 84,375$) would have been formidable. The simple latin squares selection method was used to reduce this number to a manageable 675 (15 mission points \times 45 design points).

Table 2 Design space

Variable	Range of Values
OPR	10-30
BPR	0.2-2.2
WOS	80-160 LBS/FT ²
TW	0.6-1.0
AR	1.5-3.5

III. Simple Latin Squares Method

A statistical selection technique known as the simple latin squares method was used to logically identify representative subsets of the complete mission space and the complete design space. The method is based on random numbers, field algebra, and the algebra of integers, and yields a sequence of values for each variable which is formed by joining together permutations of the values of that variable. Hence, each variable is stepped through all of its values every k data points, where k is the number of values each variable is allowed to assume. The values of the variables are normalized on the interval $(-1, 1)$. For " n " independent variables, there are " n " matrices established to generate the design points or mission points.² The method identifies $2p^2-p$ points from the available candidates, where p is the smallest prime number in the set: $p \geq n$.

The 15 selected mission points are depicted in Fig. 2. It should be noted that Missions 8 and 9 are duplicates, as are Missions 12 and 15. Reference 3 describes an orthogonal latin squares method for which the selected points are always unique. Application of the simple latin squares method to the five dimensional design space produced the 45 unique points presented in Table 3.

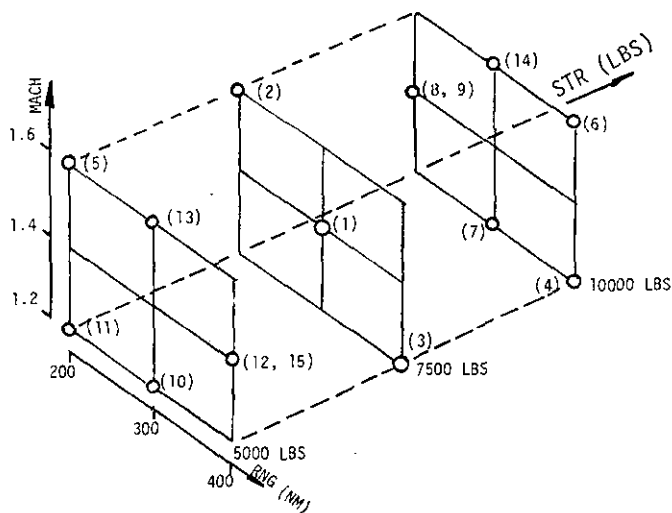


Fig. 2 Latin squares selected mission points

The latin squares method, simple or orthogonal, is probably not the best method for point selection. What is needed is a technique for choosing the best few points to fairly (with equal weighting) represent each subdomain of the entire space. Other "point scattering" techniques have been tried and found to produce satisfactory results.⁴ Of these the D-Optimal method shows considerable promise.⁵

IV. Aircraft Sizing

The study was completed in two parts using two different aircraft sizing programs furnished by the Air Force Flight Dynamics Laboratory Design Branch (AFFDL/FXB) to simulate the mission of Fig. 1. In Part A, a scalable, fixed cycle engine was used to reduce the design space to three dimensions (WOS, AR, TW). Consequently, the optimum TOGW relations were independent of OPR and BPR. In Part B a single mission point was selected for analysis of the full

Table 3 Latin squares selected design points

Case No.	WOS (LBS/FT ²)	AR	TW	OPR	BPR
1	120	2.5	.8	20	1.2
2	140	3.5	.6	15	1.2
3	160	2.0	.9	10	1.2
4	80	3.0	.7	30	1.2
5	100	1.5	1.0	25	1.2
6	160	3.0	.7	10	1.7
7	100	3.5	.6	25	2.2
8	140	1.5	1.0	15	.2
9	80	2.0	.9	30	.7
10	140	3.0	.9	25	1.7
11	160	1.5	.7	20	1.7
12	80	2.5	1.0	15	1.7
13	100	3.5	.8	10	1.7
14	120	2.0	.6	30	1.7
15	80	3.5	.8	15	1.7
16	120	1.5	.7	30	2.2
17	160	2.0	.6	20	.2
18	100	2.5	1.0	10	.7
19	160	3.5	1.0	30	2.2
20	80	2.0	.8	25	2.2
21	100	3.0	.6	20	2.2
22	120	1.5	.9	15	2.2
23	140	2.5	.7	10	2.2
24	100	1.5	.9	20	1.7
25	140	2.0	.8	10	2.2
26	80	2.5	.7	25	.2
27	120	3.0	.6	15	.7
28	80	1.5	.6	10	.2
29	100	2.5	.9	30	.2
30	120	3.5	.7	25	.2
31	140	2.0	1.0	20	.2
32	160	3.0	.8	15	.2
33	120	2.0	1.0	25	1.7
34	160	2.5	.9	15	2.2
35	100	3.0	.8	30	.2
36	140	3.5	.7	20	.7
37	100	2.0	.7	15	.7
38	120	3.0	1.0	10	.7
39	140	1.5	.8	30	.7
40	160	2.5	.6	25	.7
41	80	3.5	.9	20	.7
42	140	2.5	.6	30	1.7
43	80	3.0	1.0	20	2.2
44	120	3.5	.9	10	.2
45	160	1.5	.8	25	.7

five-dimensional design space. For the inclusion of OPR and BPR in the sizing program it was necessary for AFFDL/FXB personnel to develop twenty-five engine decks.

The twelve segment mission shown in Fig. 1 is potentially representative of future fighter aircraft designs. The specified mission was to be performed with a one-man crew, with all expendable stores carried internally. The aircraft was to make one 360 degree turn at the dash Mach number and altitude before expending the internal stores. Distance credit was given for all mission segments except subsonic loiter, combat, store expenditure, and pre-landing loiter. In Part A, the dash range (RNG) was the total distance covered in the Outbound and Inbound dash segments. In Part B, the RNG was fixed at 250 NM.

V. Surface Fit Approximation

In order to apply mathematical optimization methods to the aircraft designs generated by the sizing program, the dependent variables TOGW, DTO, and DLN were represented as analytic functions of the independent design variables WOS, AR, and TW. These "surface fit approximations" for the dependent variables were obtained through regression analysis using the computer program SURFIT (SURface FIT) supplied by the Air Force Aero Propulsion Laboratory (AFAPL). SURFIT was developed by McDonnell-Douglas Corporation, McDonnell Aircraft Company, St. Louis, Missouri.⁶

Although other options were available in SURFIT, it was decided that the dependent variables TOGW, DTO, and DLN would be represented by second order polynomials of the form:

$$\begin{aligned} \text{TOGW} = & A_0 + A_1(\text{WOS}) + A_2(\text{AR}) + A_3(\text{TW}) + A_{11}(\text{WOS})^2 \\ & + A_{12}(\text{WOS})(\text{AR}) + A_{13}(\text{WOS})(\text{TW}) + A_{22}(\text{AR})^2 \\ & + A_{23}(\text{AR})(\text{TW}) + A_{33}(\text{TW})^2 \end{aligned} \quad (1)$$

In general summation notation for "n" independent variables, Equation (1) can be written:

$$\text{TOGW} = A_0 + \sum_{i=1}^n A_i X_i + \sum_{i=1}^n \sum_{j=1}^n A_{ij} X_i X_j \quad (2)$$

where the A's are the coefficients and the X's are the independent variables. There were 10 coefficients for Part A and 21 coefficients for Part B.

Typical of the surface results obtained for all missions in Part A are those shown in Eqns (3), (4), and (5) for Mission 13. As is characteristic of surface fits developed by regression analysis, some of the ten terms were insignificant and do not appear.

$$\begin{aligned} \text{TOGW} = & 38850.416 - 28156.2033(\text{TW}) + .1780(\text{WOS})^2 \\ & - 16.0080(\text{WOS})(\text{AR}) + 450.7710(\text{AR}) \\ & - 1591.2582(\text{AR})(\text{TW}) \\ & + 26021.8945(\text{TW})^2 \end{aligned} \quad (3)$$

$$\begin{aligned} \text{DTO} = & 3239.8422 + 48.9005(\text{WOS}) - 7296.6377(\text{TW}) \\ & - .4827(\text{WOS})(\text{AR}) - 30.1409(\text{WOS})(\text{TW}) \\ & + 4549.4329(\text{TW})^2 \end{aligned} \quad (4)$$

$$\begin{aligned} \text{DLN} = & -954.8010 + 36.4208(\text{WOS}) + 2556.2243(\text{TW}) \\ & - .0238(\text{WOS})^2 + 1.1496(\text{WOS})(\text{AR}) - 39.7089(\text{AR})^2 \\ & + 118.2842(\text{AR})(\text{TW}) - 1649.1797(\text{TW})^2 \end{aligned} \quad (5)$$

Three dimensional plots of TOGW versus TW and AR were generated from the surface fit approximations in order to provide a check on the predicted minimum TOGW output from the optimizer. The plot presented in Fig. 3 is for Mission 13 at TW = .6 and was generated from Eqn. (3). While not suitable for determining numerical values of TOGW, these plots were useful for visualizing qualitative functional dependence. The variety of points from which a three dimensional surface can be viewed makes it imperative that the reader note the axis orientation for each plot. The three dimensional plots were generated on the CALCOMP Plotter using DISSPLA.⁷

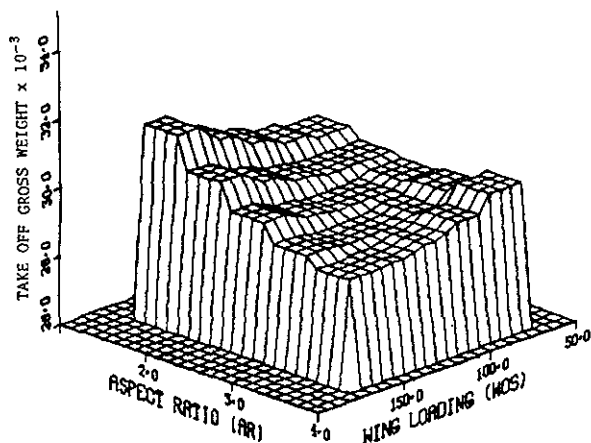


Fig. 3 3-D Plot of TOGW vs WOS and AR for Mission 13 (TW = .6)

VI. Optimization

The optimization problems considered in this study required the minimization of a performance function such as TOGW subject to the inequality constraint that another performance function such as DTO be equal to or less than some specified value. An additional "box constraint" was imposed such that the independent variables remained inside the design space. An object deck of the computer program OAPEN (Optimization Analysis by PENalty function) was supplied by the Air Force Aero Propulsion Laboratory (AFAPL) and was used to accomplish the various optimizations in this study. OAPEN was developed by The Boeing Aerospace Company, Seattle, Washington under contract F33615-73-2084 to AFAPL.⁸

The two optimizations performed for each mission were to: (1) minimize TOGW with no constraints on DTO and DLN other than they must be positive, and (2) minimize TOGW subject to the constraints that DTO must be equal to or less than 3500 feet and DLN must be equal to or less than 4500 feet. The independent variables WOS, AR, and TW were "box constrained" to not be outside the range of values noted in Table 2 for the design space.

OAPEN is designed to find an optimum design parameter vector using surface fit functions to approximate the true functions of the design parameters in an optimal design problem. In this study, the general optimization problem to be solved can be written as

$$\text{Minimize } f_1(\underline{X})$$

$$\text{Subject to } f_j(\underline{X}) \leq c_j, j = 2, m \quad (6)$$

where \underline{X} is the vector of independent variables, the f 's are the performance functions approximated by surface fits, and the c 's are the values of the upper limits for the constraint functions. OAPEN solves this problem by the penalty function method in which the inequalities of Equation (6) are used to establish a penalized cost function of the form

$$F(\underline{X}) = f_1(\underline{X}) + P_K \sum_{j=2}^m (CV_j)^2 \quad (7)$$

where $f_1(\underline{X})$ is as defined for Equation (6), P_K is a weight factor which modulates the severity of violating the constraints, and CV represents the violation of the constraint's inequalities. P_K corresponds

to the allowable tolerance on violation of the constraints. A default value of $P_K = 50$ exists in the program and corresponds to a two percent tolerance. A value of $P_K = 100$ (one percent tolerance) was used in this study. CV in Equation (7) has the form $(\text{DTO} - 3500)$ when DTO is constrained to be less than or equal to 3500 feet. CV is equal to zero if the constraint is satisfied and takes on the value $(\text{DTO} - 3500)$ when the constraint is violated (exceeded).

For the constrained minimization performed in this study, Equation (7) can be written

$$F(\underline{X}) = \text{TOGW} + 100(\text{DTO} - 3500)^2 + 100(\text{DLN} - 4500)^2 \quad (8)$$

OAPEN uses the Fletcher-Reeves conjugate gradient search method to find the minimum $F(\underline{X})$ given in Equation (8). The gradient of $F(\underline{X})$ is computed from the TOGW, DTO, and DLN surface fit approximations which are input to the program. The algorithm requires an initial value from which to begin the gradient search. The optimizations performed were based on surface fit approximations for TOGW, DTO, and DLN as quadratic functions of WOS, AR, and TW such as those given in Equations (3) through (5). The mean values of WOS, AR, and TW (120, 2.5, .8) were input as the starting point for the gradient search algorithm.

The results of constrained and unconstrained optimizations are presented in Table 4. In general, these particular constraints were satisfied at a cost of 300 - 500 pounds additional TOGW. Wing loadings (WOS) and aspect ratios (AR) were reduced while thrust-to-weight ratios (TW) remained the same with the exceptions of Mission 6 and Mission 13 where TW increased. Fig. 3 indicates the existence of an apparent minimum TOGW for Mission 13 in the general region predicted by OAPEN. The minimum was clearly confirmed by 2-D plots of TOGW vs. the design variables.

VII. Investigation Results

Part A

The optimum aircraft design results are presented in Table 4 for the thirteen unique missions of Part A. Surface fit approximations were developed to relate TOGW, DTO, and DLN for these optimum aircraft to the three independent mission variables MACH, RNG, and STR. The quadratic polynomial approximations technique discussed earlier and the program SURFIT were used for the regression analysis with the following results for the unconstrained case.

$$\begin{aligned} \text{TOGW} = & 97109.316 - 118597.9044(\text{MACH}) - 144.1725(\text{RNG}) \\ & + 2.6280(\text{STR}) + 42465.2574(\text{MACH})^2 + 86.9971(\text{MACH})(\text{RNG}) \\ & - .6542(\text{MACH})(\text{STR}) + .0735(\text{RNG})^2 \end{aligned} \quad (9)$$

$$\begin{aligned} \text{DTO} = & 1159.6500 + 10.6167(\text{RNG}) + .0770(\text{STR}) \\ & + 4.3653(\text{MACH})(\text{RNG}) - .0266(\text{RNG})^2 \end{aligned} \quad (10)$$

$$\begin{aligned} \text{DLN} = & 3756.6445 + .1771(\text{STR}) - 1246.7666(\text{MACH})^2 \\ & + 13.2589(\text{MACH})(\text{RNG}) - .0311(\text{RNG})^2 \end{aligned} \quad (11)$$

Some statistical analysis of each equation is presented in Table 5.

To help visualize the relationships given by Equations (9) through (11), three dimensional plots

Table 4 Summary of minimum TOGW results for unconstrained and constrained () optimization* for Part A

Mission				Optimum design points											
No.	Mach	RNG	STR	TOGW	(TOGW)	WOS	(WOS)	AR	(AR)	TW	(TW)	DTO	(DTO)	DLN	(DLN)
1	1.4	300	7500	27320	27554	141.79	111.55	3.5	2.98	.6	.6	4293	3506	5318	4237
2	1.6	200	7500	29871	29991	120.27	102.27	3.15	2.94	.6	.6	4036	3506	4787	4109
3	1.2	400	7500	25669	25758	137.21	120.58	3.35	3.06	.6	.6	3904	3503	4917	4353
4	1.2	400	10000	30074	30238	144.77	120.55	3.5	3.01	.6	.6	4086	3504	5260	4422
5	1.6	200	5000	26127	26127	111.58	103.01	3.5	3.5	.6	.6	3750	3500	4179	3866
6	1.6	400	10000	42182	43335	150.00	122.41	3.5	3.5	.69	.711	4595	3506	5816	4503
7	1.2	300	10000	28999	29154	144.28	120.22	3.5	3.07	.6	.6	4073	3493	5352	4503
8	1.4	200	10000	29982	30298	145.10	111.52	3.5	2.95	.6	.6	4382	3506	5728	4459
10	1.2	300	5000	20371	20395	134.66	120.90	3.49	3.27	.6	.6	3834	3502	4836	4364
11	1.2	200	5000	19736	19749	130.43	121.06	3.5	3.37	.6	.6	3729	3501	4816	4490
12	1.4	400	5000	25437	25523	129.73	111.53	3.24	3.00	.6	.6	3983	3505	4493	3892
13	1.6	300	5000	29035	29335	157.42	116.36	3.5	3.29	.648	.679	4779	3507	5567	4210
14	1.6	300	10000	36289	37360	149.38	102.15	3.5	2.74	.6	.6	4851	3512	5813	4052

*Constraints: DTO \leq 3500 feet and DLN \leq 4500 feet

Table 5 Multiple correlation and maximum error for Part A

	Multiple Correlation	Maximum % Error
TOGW	.99825	-2.63
DTO	.89380	-7.19
DLN	.90604	-9.82

of TOGW, DTO, and DLN for optimized aircraft with 5000 pounds of stores were generated using these equations for various values of MACH and RNG. Presented in Fig. 4 through Fig. 6, these plots dramatically depict trends over the allowable variation of MACH and RNG. The TOGW plot (Fig. 4) is particularly effective as a means of locating an apparent minimum or regions deserving more detailed analysis. For example, it can be seen from Fig. 4 that an apparent minimum exists for MACH = 1.2 in the vicinity of RNG = 275NM and not at RNG = 200NM, the minimum range in the mission space. This result contradicts the input data from Table 4 where the minimum TOGW did occur for MACH = 1.2 at RNG = 200NM (Mission 11). If Equation (9) was taken as exact, then it can be shown that the minimum TOGW of 19,775 pounds occurs for RNG = 270.58NM. For MACH = 1.2 and RNG = 200NM, Equation (9) predicts TOGW = 20,140 pounds. The difference of 365 pounds is, however, attributed to a buildup of error in the surface fits leading to Equation (9). The surface slope discontinuities of Fig. 4 through Fig. 6 and the viewpoint make it impossible to determine specific values of TOGW, DTO, or DLN from these plots.

Part B

In Part B of the study the same methods discussed above were applied to the case of a fixed mission of the same basic profile shown in Fig. 1. The mission parameters were fixed as follows: MACH = 1.95, RNG = 250NM, STR = 5500 LBS. The variation of the five independent design variables is given in Table 2 with the exception that the maximum WOS was reduced to 120 pounds/ft². The simple latin squares method selected 45 design points from the 5⁵ candidate points determined by 5 equally spaced values for each of the 5 variables.

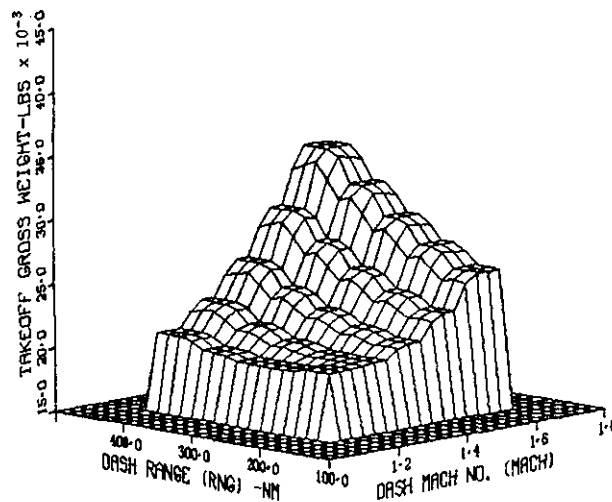


Fig. 4 3-D plot of Minimum TOGW vs MACH and RNG (STR = 5000 LBS)

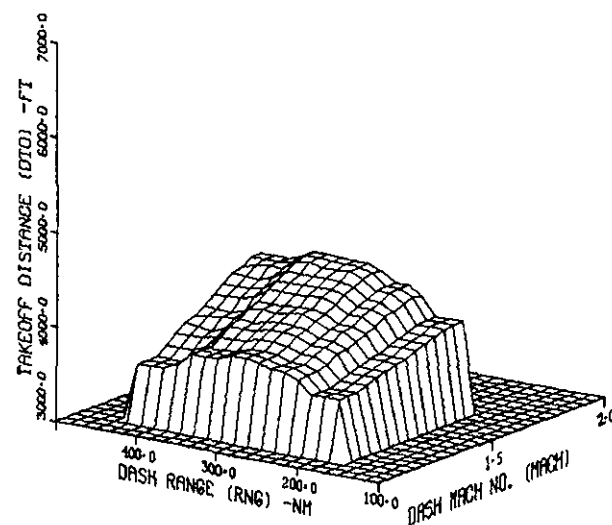


Fig. 5 3-D Plot of DTO vs MACH and RNG (STR = 5000 LBS)

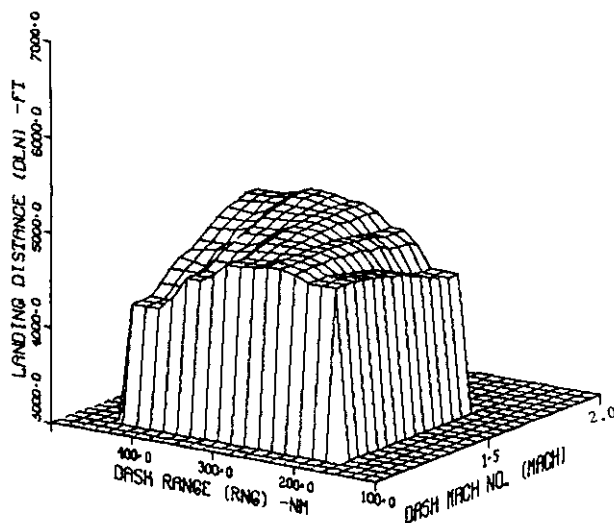


Fig. 6 3-D Plot of DLN vs MACH and RNG
(STR = 5000 LBS)

For each of these 45 design points and the mission profile, TOGW, DTO, DLN, TAC and GSS were computed by an AFFDL/FXB aircraft sizing program. The program required new engine tables (thrust and fuel flow vs Mach number and altitude) for each of the 25 combinations of OPR and BPR.

SURFIT was applied to sizing results to determine analytic expressions relating TOGW, DTO, DLN, GSS and TAC to the five independent design variables.

$$\begin{aligned} \text{TOGW} = & 45849.08 + 53.31(\text{OPR})^2 - 3988.15(\text{OPR})(\text{TW}) \\ & + 86838.3926(\text{TW})^2 - 900.9884(\text{AR}) \end{aligned} \quad (12)$$

$$\begin{aligned} \text{DTO} = & 9781.7277 + 77.6748(\text{WOS}) - 14341.0841(\text{TW}) \\ & - 3086.5461(\text{AR}) - 41.3483(\text{WOS}) \\ & - 7.4961(\text{WOS})(\text{AR}) + 6227.6061(\text{TW})^2 \\ & + 1605.3901(\text{TW})(\text{AR}) + 333.0961(\text{AR})^2 \end{aligned} \quad (13)$$

$$\begin{aligned} \text{DLN} = & 2848.4882 + 71.6588(\text{WOS}) - 1881.9125(\text{AR}) \\ & - 36.3198(\text{WOS})(\text{TW}) - 5.4750(\text{WOS})(\text{AR}) \\ & + 570.6408(\text{TW})(\text{AR}) + 245.5962(\text{AR})^2 \end{aligned} \quad (14)$$

$$\begin{aligned} \text{TAC} = & 4.9253 - .1225(\text{BPR}) - .008399(\text{WOS}) \\ & - 6.9671(\text{TW}) + .000013336(\text{OPR})^2 + .002236(\text{BPR})^2 \\ & + .08645(\text{BPR})(\text{TW}) + .00003341(\text{WOS})^2 \\ & + .003342(\text{WOS})(\text{TW}) - .0008496(\text{WOS})(\text{AR}) \\ & + 3.0129(\text{TW})^2 + .0159(\text{AR})^2 \end{aligned} \quad (15)$$

$$\begin{aligned} \text{GSS} = & .6559232 + .8677(\text{AR}) - .00588(\text{WOS})(\text{AR}) \\ & + .3618(\text{TW})(\text{AR}) \end{aligned} \quad (16)$$

As a result of the regression analysis, WOS and BPR terms do not appear in Equation (12) for TOGW, nor do OPR and BPR terms appear in Equations (13), (14), or (16) for DTO, DLN, or GSS.

The multiple correlation and maximum error for Equations (12) through (16) are presented in Table 6. Although the maximum error in TOGW was 20.26

percent, there were only seven of 45 data points at which the error was greater than ten percent, with most errors less than seven percent. For DTO, the maximum error was 15.08 percent, although there were only four points with errors greater than four percent. DLN had a maximum error of 13.08 percent, but there were only four cases where the error exceeded five percent. GSS had a maximum error of 12.09 percent and there were only two other cases where the error exceeded four percent. These errors may be attributed to the fact that TOGW varied between 40,591 and 99,643 pounds for the 45 design points. The simple polynomial surface fit approximation cannot adequately represent this large range of data. At this point the conscientious designer should redefine the design space and repeat the analysis to reduce errors.

Table 6 Multiple correlation and maximum error for Part B

Variable	Multiple Correlation	Maximum % Error
TOGW	.94306	20.26
DTO	.99626	-15.08
DLN	.98791	-13.08
TAC	.99888	-3.00
GSS	.99508	12.09

Based on Equations (12) through (16), several minimizations with a variety of constraints were performed using OAPEN. The results are presented in Table 7.

Table 7 Minimization results

Variable	Minimization Case				
	1	2	3	4	5
TOGW (LBS)	39224	39678	39678	39918	41514
DTO (FT)	2729	3002	3002	2442	1682
DLN (FT)	3537	3990	3990	3399	2797
TAC (MIN)	1.35	1.17	1.17	1.17	1.02
GSS (G's)	2.40	2.08	2.08	2.50	2.99
OPR	22.41	23.55	23.55	24.41	27.66
BPR	1.2	2.2	2.2	2.2	2.2
WOS (LBS/FT ²)	100	116.94	116.94	97.77	80
TW	.60	.63	.63	.65	.74
AR	3.5	3.5	3.5	3.5	3.5
Constraints					
DTO	None	<3000	<3000	<3000	<3000
DLN	None	None	<4000	<3500	<3000
TAC	None	<1.17	<1.17	<1.17	<1.17
GSS	None	None	>2.0	>2.5	>3.0

These results indicate that there is only 2290 pounds (5.8%) difference in TOGW between Case 1 (39224 lbs) where there were no constraints, and Case 5 (41514 lbs), where the most severe constraints were applied. The optimum design configurations were similar for all constrained cases in that there was very little change in OPR, BPR, TW, and AR. Note

that BPR, TW, and AR were either on or very near the boundary of the design space. Wing loading (WOS) was very sensitive to the constraints on landing distance (DLN) and acceleration time (TAC).

The variations of TOGW with OPR, AR, and TW for the unconstrained minimum case are shown on the three dimensional plots presented in Fig. 7 through 9. These plots also indicate that the minimum TOGW probably lies outside the design space and that higher aspect ratios (AR) and lower thrust-to-weight ratios (TW) should be considered for the unconstrained case. Also, higher by-pass ratios (BPR) should be considered, particularly if the constraints in Cases 2 through 5 become overriding requirements. Note that as TW requirements increase, as in Case 5, the optimum OPR also increases and may eventually exceed the design space limit of 30.

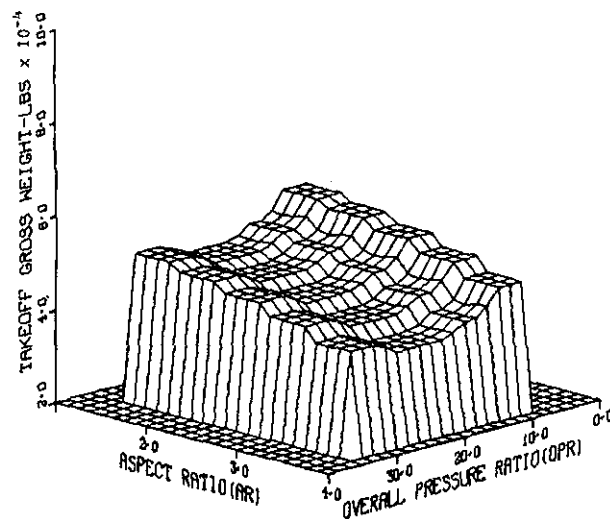


Fig. 7 3-D plot of minimum TOGW vs OPR and AR from optimized aircraft (TW = .6)

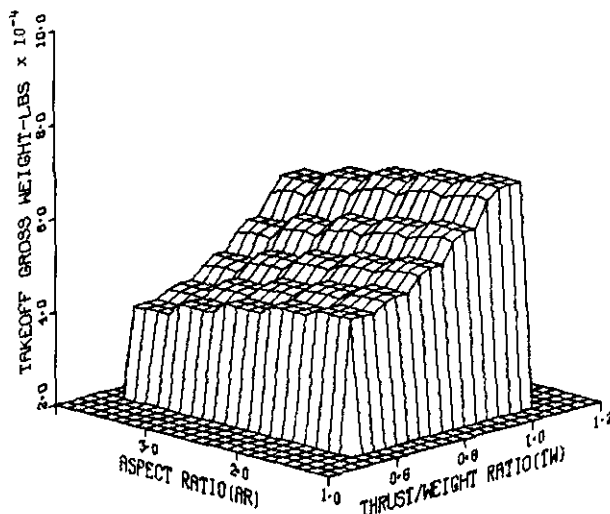


Fig. 8 3-D plot of TOGW vs TW and AR for optimized aircraft (OPR = 22.41)

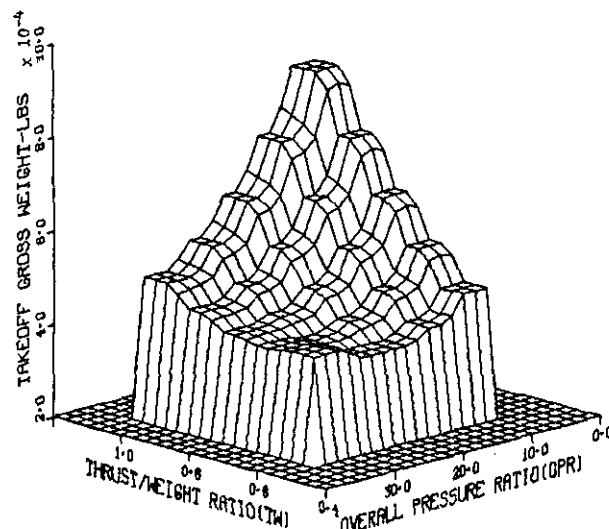


Fig. 9 3-D plot of TOGW vs TW and OPR for optimized aircraft (AR = 3.5)

VIII. Conclusions

The latin squares, surface fit, and optimization methodologies are very useful, flexible, and time saving tools for design and mission analysis. As more and more variables are included in such analyses, the need for these methods increases dramatically. Results from a single application of these methods may indicate the need for a second iteration with new limits (extended, limited, or shifted) on the design/mission variables. It is believed that the separate treatment of design and mission spaces used here yields more accurate results than if the mission and design variables had been all included together in a single "design and mission space". However, the current approach did require a three-fold increase in sizing points over the other approach.

IX. Acknowledgements

The support and assistance of AFAPL and AFFDL personnel are gratefully acknowledged.

X. References

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