A small-angle approximation that does provide an estimate for the y-coordinate of the aerodynamic center can be obtained from Eqs. (4.8.29) and (4.8.30). This is bases on approximating the average downwash angle for each lifting surface as being linearly proportional to the lift developed on the other surface,

$$\varepsilon_w \cong C_{\varepsilon w} C_{L_h}, \quad \varepsilon_h \cong C_{\varepsilon h} C_{L_w} \tag{4.8.37}$$

Following Example 4.7.3, using Eq. (4.8.37) in Eqs. (4.7.4)–(4.7.11), and applying the results together with the traditional approximations $C_D \approx 0$, $\cos(\alpha) \approx 1$, and $\sin(\alpha) \approx \alpha$ to Eqs. (4.8.29) and (4.8.30) yields

$$\bar{x}_{ac} = \frac{x_w C_{w,\alpha} + x_h C_{h,\alpha}}{C_{w,\alpha} + C_{h,\alpha}} + \frac{(y_h - y_w)[C_{w,\alpha} C_{h0} - C_{h,\alpha} C_{w0}]}{(C_{w,\alpha} + C_{h,\alpha})^2} \tag{4.8.38}$$

$$\bar{y}_{ac} = \frac{y_w C_{w,\alpha} + y_h C_{h,\alpha}}{C_{w,\alpha} + C_{h,\alpha}} \tag{4.8.39}$$

$$C_{w,\alpha} \equiv \frac{C_{L_w,\alpha}(1 - C_{\varepsilon w} C_{L_h,\alpha})}{1 - (C_{\varepsilon w} C_{L_w,\alpha})(C_{\varepsilon h} C_{L_h,\alpha})} \tag{4.8.40}$$

$$C_{h,\alpha} \equiv \frac{S_h}{S_w} \eta_h \frac{C_{L_h,\alpha}(1 - C_{\varepsilon h} C_{L_w,\alpha})}{1 - (C_{\varepsilon w} C_{L_w,\alpha})(C_{\varepsilon h} C_{L_h,\alpha})} \tag{4.8.41}$$

$$C_{w0} \equiv \frac{C_{L_w,\alpha}[(\alpha_{0w} - \alpha_{L0w}) - C_{\varepsilon w} C_{L_h,\alpha}(\alpha_{0h} - \alpha_{L0h} + \varepsilon_e \delta_e)]}{1 - (C_{\varepsilon w} C_{L_w,\alpha})(C_{\varepsilon h} C_{L_h,\alpha})} \tag{4.8.42}$$

$$C_{h0} \equiv \frac{S_h}{S_w} \eta_h \frac{C_{L_h,\alpha}[(\alpha_{0h} - \alpha_{L0h} + \varepsilon_e \delta_e) - C_{\varepsilon h} C_{L_w,\alpha}(\alpha_{0w} - \alpha_{L0w})]}{1 - (C_{\varepsilon w} C_{L_w,\alpha})(C_{\varepsilon h} C_{L_h,\alpha})} \tag{4.8.43}$$

where (x_w, y_w) and (x_h, y_h) are the coordinates of the aerodynamic center of the main wing and the horizontal control surface, respectively. For a detailed development of this result see Phillips, Alley, and Niewoehner (2008).

4.9. Effect of the Fuselage, Nacelles, and External Stores

The contribution of the fuselage, nacelles, and external stores to the static stability of an airplane is typically destabilizing. Furthermore, in some cases these effects can be quite significant. Thus, the fuselage, nacelles, and external stores should be included in the trim and stability analysis during all phases of the airplane design process.

The aerodynamic forces and moments generated on the fuselage, nacelles, and external stores are extremely complex, and are significantly affected by aerodynamic interactions with the wing and tail. To estimate these forces and moments accurately, we must make use of computer simulations and/or wind tunnel tests. Such methods should always be employed in the later phases of the airplane design process. However, for the purpose of preliminary design, it is useful to have some method that can be used to easily estimate these effects.

As a first approximation, the effect of the fuselage, nacelles, and external stores on lift is usually neglected. However, even for preliminary design, some estimation of the effect of the fuselage, nacelles, and external stores on airplane moments should be made. The method to be presented here is taken from Hoak (1960), and is presented in terms of the fuselage, but the same approach can be used for the nacelles and external stores.

Consider the fuselage geometry shown in Fig. 4.9.1. As has been our usual convention, l will indicate a distance measured aft of the CG and c will indicate the total length of an object in the chordwise direction. While the characteristic area for a lifting surface such as the wing or tail is always taken as the planform area, the characteristic area for a fuselage or nacelle is typically defined to be the projected frontal area. This is the maximum cross-sectional area of the body in a plane normal to the axial chord.

A rough estimate of the fuselage pitching moment coefficient about the airplane center of gravity can be made from the experimental correlation,

$$C_{m_f} \equiv \frac{m_{cg_f}}{\frac{1}{2}\rho V_\infty^2 S_f c_f} = -2\frac{l_f}{c_f}\left[1-1.76\left(\frac{d_f}{c_f}\right)^{3/2}\right]\alpha_f \tag{4.9.1}$$

where S_f is the maximum cross-sectional area of the fuselage, c_f is the chord length of the fuselage, l_f is the distance that the fuselage center of pressure is aft of the airplane center of gravity, d_f is the diameter of a circle having the same area as the maximum cross-sectional area of the fuselage,

$$d_f \equiv 2\sqrt{S_f/\pi} \tag{4.9.2}$$

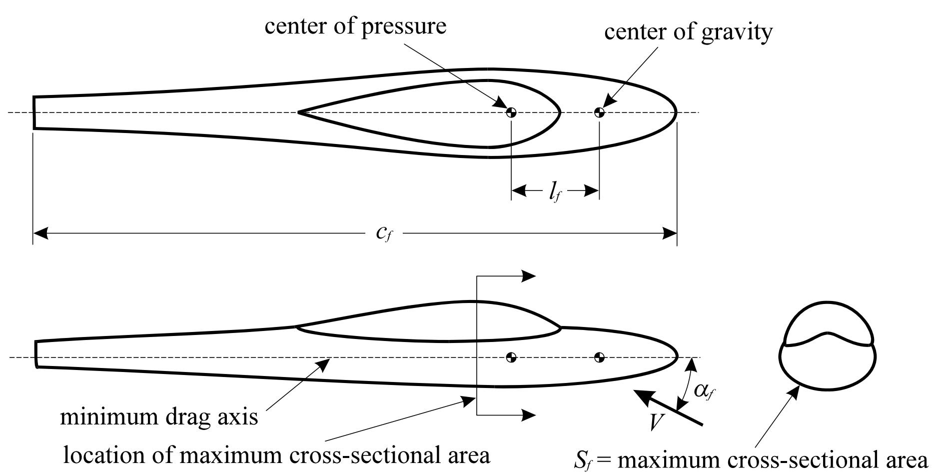

Figure 4.9.1. Fuselage geometry.

DRAFT

and α_f is the angle of attack for the minimum drag axis of the fuselage. As a first estimate, the fuselage center of pressure can be assumed to be halfway between the nose and the point of maximum cross-sectional area. Since this point is normally forward of the airplane's center of gravity, l_f is typically negative.

The pitching moment computed from Eq. (4.9.1) can be added to the contributions from the wing and the tail to obtain an estimate for the complete airframe. However, the characteristic area for the complete airplane was defined to be the area of the main wing, and the characteristic length was defined to be the mean chord length of the main wing. Thus, the contribution of the fuselage to the total airplane pitching moment coefficient can be approximated by multiplying the result from Eq. (4.9.1) by the product $S_f c_f$ and dividing by the product $S_w \bar{c}_w$:

$$(\Delta C_m)_f \equiv \frac{m_{cg_f}}{\frac{1}{2}\rho V_\infty^2 S_w \bar{c}_w} = \frac{S_f c_f}{S_w \bar{c}_w} C_{m_f} = -2\frac{S_f l_f}{S_w \bar{c}_w}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{3/2}\right]\alpha_f \quad (4.9.3)$$

This can be written as

$$(\Delta C_m)_f = (\Delta C_{m0})_f + (\Delta C_{m,\alpha})_f \alpha \quad (4.9.4)$$

where

$$(\Delta C_{m0})_f = -2\frac{S_f l_f}{S_w \bar{c}_w}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{3/2}\right]\alpha_{0f} \quad (4.9.5)$$

$$(\Delta C_{m,\alpha})_f = -2\frac{S_f l_f}{S_w \bar{c}_w}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{3/2}\right] \quad (4.9.6)$$

α is the airplane angle of attack relative to the fuselage reference line, and α_{0f} is the angle that the minimum drag axis of the fuselage makes with the fuselage reference line. Because the fuselage center of pressure is usually forward of the airplane center of gravity, making l_f negative, the fuselage contribution to the pitch stability derivative is typically destabilizing.

It is important to remember that this experimental correlation gives only a rough estimate of the aerodynamic pitching moment generated on the fuselage, nacelles, and external stores. It should only be used as a first approximation for preliminary design. Wind tunnel tests and/or computer simulations should always be used in the later phases of design. Because the aerodynamic moments generated on the fuselage, nacelles, and external stores are primarily the result of pressure forces, panel methods will give a reasonable estimate for these moments. However, for best results, viscous computational fluid dynamics (CFD) or wind tunnel tests should be used. Although nacelles and external stores are typically much smaller than the fuselage, their effect on stability can be very significant and should not be neglected.

EXAMPLE 4.9.1. The airplane described in Example 4.4.1 has a fuselage that is 23 ft long. The point of maximum cross-section is located 9 ft aft of the nose and the airplane center of gravity is 8 ft aft of the nose. The maximum cross-sectional area of the fuselage is 21 ft^2. Estimate the static margin in the linear lift range for the complete airframe, including the effect of the fuselage.

Solution. For this airplane we have

$$S_w = 180\ \text{ft}^2,\quad b_w = 33\ \text{ft},\quad C_{L_w,\alpha} = 4.44,\quad l_w = -0.71\ \text{ft},$$
$$S_t = 36\ \text{ft}^2,\quad b_t = 12\ \text{ft},\quad C_{L_t,\alpha} = 3.97,\quad l_t = 14.29\ \text{ft},$$
$$\eta_t = 1.0,\quad \varepsilon_{d,\alpha} = 0.44,\quad S_f = 21\ \text{ft}^2,\quad d_f = 2\sqrt{21\,\text{ft}^2/\pi} = 5.17\ \text{ft},$$
$$c_f = 23\ \text{ft},\quad l_f = (9\ \text{ft}/2) - 8\ \text{ft} = -3.5\ \text{ft}$$

By definition, the moment slope relative to the stick-fixed neutral point is zero. Thus, using Eq. (4.9.6) to estimate the fuselage contribution yields

$$\frac{\partial C_{m_{np}}}{\partial \alpha} = -\frac{l_w - l_{np}}{\bar{c}_w} C_{L_w,\alpha} - \frac{S_t(l_t - l_{np})}{S_w \bar{c}_w}\eta_t(1-\varepsilon_{d,\alpha})C_{L_t,\alpha} - 2\frac{S_f(l_f - l_{np})}{S_w \bar{c}_w}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{3/2}\right] = 0$$

Solving for the static margin gives

$$\frac{l_{np}}{\bar{c}_w} = \frac{l_w C_{L_w,\alpha} + \dfrac{S_t l_t}{S_w}\eta_t(1-\varepsilon_{d,\alpha})C_{L_t,\alpha} + 2\dfrac{S_f l_f}{S_w}\left[1 - 1.76\left(\dfrac{d_f}{c_f}\right)^{3/2}\right]}{\bar{c}_w\left\{C_{L_w,\alpha} + \dfrac{S_t}{S_w}\eta_t(1-\varepsilon_{d,\alpha})C_{L_t,\alpha} + 2\dfrac{S_f}{S_w}\left[1 - 1.76\left(\dfrac{d_f}{c_f}\right)^{3/2}\right]\right\}}$$

$$= \frac{(-0.71)4.44 + \dfrac{36(14.29)}{180}1.0(1-0.44)3.97 + 2\dfrac{21(-3.5)}{180}\left[1 - 1.76\left(\dfrac{5.17}{23}\right)^{3/2}\right]}{\dfrac{180}{33}\left\{4.44 + \dfrac{36}{180}1.0(1-0.44)3.97 + 2\dfrac{21}{180}\left[1 - 1.76\left(\dfrac{5.17}{23}\right)^{3/2}\right]\right\}}$$

$$= \underline{9\%}$$

Comparing this result with that from Example 4.4.1, we see that the destabilizing effect of the fuselage was to move the neutral point forward by about 3 percent of the wing chord. This is clearly significant in view of the fact that the design static margin is often on the order of 5 percent of the wing chord.

DRAFT