From: **Aircraft Performance Analysis,**VDM Verlag Dr. Müller, 2009
M. Sadraey

9.6. V– n Diagram

Flight regime of any aircraft includes all permissible combinations of speeds, altitudes, weights, centers of gravity, and configurations. This regime is shaped by aerodynamics, propulsion, structure, and dynamics of aircraft. The borders of this flight regime are called flight envelope or maneuvering envelope. The safety of human onboard is guaranteed by aircraft designer and manufacturer. Pilots are always trained and warned through flight instruction manual not to fly out of flight envelope, since the aircraft is not stable, or not controllable or not structurally strong enough outside the boundaries of flight envelope. A mishap or crash is expected, if an aircraft is flown outside flight envelope.

The flight envelope has various types; each of which is usually the allowable variations of one flight parameter versus another parameter. These envelopes are calculated and plotted by flight mechanics engineers and employed by pilots and flight crews. For instance, the load masters of a cargo aircraft must pay extra caution to the center of gravity location whenever they distribute various loads on the aircraft. There are several crashes and mishaps that safety board's report indicated that load master are responsible, since they deployed more loads than allowed, or misplaced the load before take-off. Nose heavy and tail heavy are two flight concepts that pilots are familiar and experienced with, and are trained to deal with them safely.

Pilots are using several graphs and charts in their flight operations. Four important envelopes are as follows:

1. Diagram of variations of aircraft lift coefficient versus Mach number (C_L – M)
2. Diagram of variations of airspeed versus altitude (V – h)
3. Diagram of variations of center of gravity versus aircraft weight (X_{cg} – W)
4. Diagram of variations of airspeed versus load factor (V – n)

One of the most important diagrams is referred to as *flight envelope*. This envelope demonstrates the variations of airspeed versus load factor (V – n). In another word, it depicts the aircraft limit load factor as a function of airspeed. One of the primary reasons that this diagram is highly important is that, the maximum load factor; that is extracted from this graph; is a reference number in aircraft structural design. If the maximum load factor is under-calculated, the aircraft cannot withstand flight load safely. For this reason, it is recommended to structural engineers to recalculate the V-n diagram on their own as a safety factor.

In this section, details of the technique to plot the V- n diagram in introduced. Figure 9.10 shows a typical V-n diagram for a GA aircraft. This diagram is, in fact, a combination of two diagrams: 1. The V-n diagram without consideration of gust, 2. The V-n diagram on the effect of gust. In this section, we first have another look on the load factor and then present new concepts on load factor. Then the phenomena of gust and gust load are described. At the end of this section, the technique to plot V – n diagram is completely described. This description is supported by a solved example.

9.6.1. Load Factor

The load to the aircraft on the ground is naturally produced by the gravity (i.e. 1 times g). But, there are other sources of load to the aircraft during flight; one of which is the acceleration load. This load is usually normalized through load factor (i.e. "*n*" times g). In another word, aircraft load is expressed as a multiple of the standard acceleration due to gravity (g = 9.81 m/sec^2 = 32.17 ft/sec^2). Recall that we defined the load factor as the ratio between lift and weight.

$$n = \frac{L}{W} \tag{9.7}$$

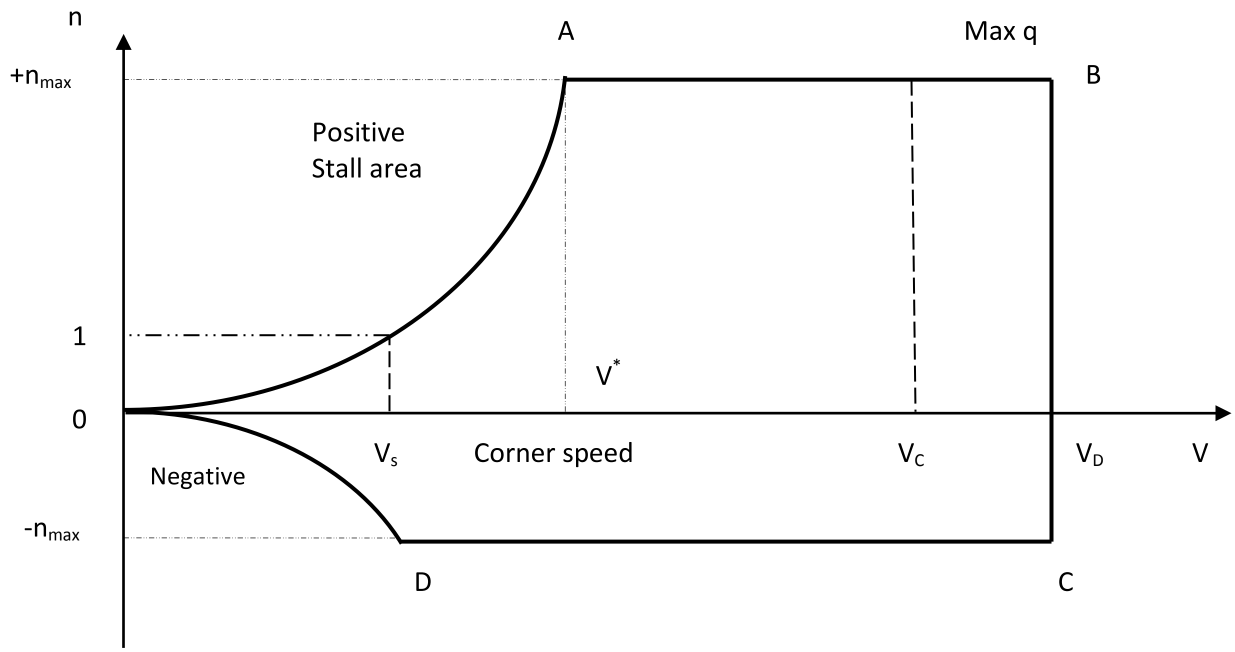

Figure 9.1. A typical V – n diagram for a GA aircraft

In some instances of flight such as turn and pull-up, the aircraft must generate a lift force such that it is more than weight. For instance, load factor in a pull-up from equation 9.86 can be re-written as:

$$n = \frac{a}{g} + 1 \tag{9.94}$$

where "*a*" is the centrifugal acceleration (V^2/R). As this acceleration increases; i.e. airspeed increases or radius of turn decreases; the load factor will increase too. For other flight operations, similar expressions can be drawn. In some instances; especially for missiles; this load factor may get as high as 30. Hence, the structure must carry this huge load safely. The aircraft structure must be strong enough to carry other loads including acceleration load such that aircraft is able to perform its mission safely. As the figure 9.11 illustrates, a low load factor fighter may end up getting targeted by a high load factor missile.

On the other hand, if the load is more than allowable design value, the structure will lose its integrity and may disintegrate during flight. Load factor is usually positive, but in some instances; including pull-down, or when encountering a gust; it may become negative. In general, the absolute value of maximum negative load factor must not exceed 0.4 times maximum positive load factor. Past experiences forced Federal Aviation Administration to regulate load factor on aircraft. Table 9.4 shows load factor for various types of aircraft. Table 9.5 demonstrates real values of load factor for several aircraft.

No	**Aircraft type**	**Maximum positive load factor**	**Maximum negative load factor**
1	Normal (non-acrobatic)	2.5 – 3.8	-1 to -1.5
2	Utility (semi-acrobatic	4.4	-1.8
3	Acrobatic	6	-3
4	Homebuilt	5	-2
5	Transport	3 – 4	-1 to -2
6	Highly maneuverable	6.5 – 12	-3 to -6
7	Bomber	2 – 4	-1 to -2

Table 9.4. Load factor for various types of aircraft

9.6.2. V – n Diagram without Gust Effect

As figure 9.10 shows, V-n diagram is an envelope that indicates the limits of load factor and speed for a safe flight. It is usually composed of two curves plus few lines. The two curves on the left hand side represent the aerodynamic limit on load factor imposed by stall (C_{Lmax}). The expression for the top curve is extracted from stall equation in turn (i.e. equation 9.10) as follows:

$$V_{s_t} = \sqrt{\frac{2nmg}{\rho S C_{L_{\max}}}} \qquad (9.10)$$

Hence

$$n_{\max} = \frac{V^2 \rho S C_{L_{\max}}}{2mg} \quad (9.95)$$

The top curve is literally a plot of equation 9.95. The region above this curve in the V-n diagram is the stall area. Since, no aircraft can fly continuously at a flight condition above this curve, so this is one of the limits on the aircraft maneuverability. Because the aircraft angle of attack will be above stall angle. Based on the equation 9.95, as the airspeed increases, the maximum load factor will increase proportionally to V^2. However, n_{max} cannot be allowed to increase indefinitely. It is constrained by the structural strength (structural limit load factor). The top horizontal line denotes the positive limit load factor in the V-n diagram.

No	**Aircraft**	**Engine**	**m_{TO} (kg)**	**P or T**	**+n**	**-n**
1	Eurofighter[1]	Turbofan	17000	2x90 kN	9	-3
2	Jaguar	Turbofan	15700	2x36 kN	8.6	-
3	Mirage 2000	Turbofan	10960	64.3 kN	13.5	-
4	SU-26M	Piston	800	360 hp	11	-9
5	BAe Hawk 60	Turbofan	8570	23.8 kN	8	-4
6	Boeing Skyfox	Turbofan	7365	2x16.5 kN	7.3	-3.5
7	Cessna 208	Turboprop	3311	600 hp	3.8	-1.52
8	Cessna 650	Turbofan	9979	2x16.2 kN	6.7	-1
9	Canadair CL-215	Turboprop	17100	2x2100 hp	3.25	-1
10	PITTS S-2A	Piston	680	200 hp	9	-4.5

Table 9.5. Statistical values of load factor for several aircraft

The flight velocity corresponding to the intersection between the left curve and top horizontal line (Point A) is referred to as *corner velocity*, and designated as V^* (V star). The corner velocity can be obtained by solving equation 9.95 for velocity, yielding:

$$V^* = \sqrt{\frac{2n_{\max} mg}{\rho S C_{L_{\max}}}} \quad (9.96)$$

[1] The aircraft is depicted in figure 8.3.

where the value of n_{max} corresponds to that at point A in figure 9.10. This speed sometimes is referred to as *maneuvering speed* (V_A), and is summarized as:

$$V_A = \sqrt{n_{max}} V_s \tag{9.97}$$

The point A is then called the *maneuver point*. At this point, both lift coefficient and load factor are simultaneously at their highest possible values. The corner velocity is an interesting velocity for fighter pilots. At speeds less than V^*, it is not possible to structurally damage the aircraft due to generation of load factor less than n_{max}. However, the bank angle is not high enough for a tight turn. In contrast, at speeds greater than V*, maneuverability decreases, since the speed is too high. Thus fighter pilots are recommended to select this speed for much of their maneuvering missions. For majority of the cases; and according to the discussions presented in sections 9.3 and 9.4; this point simultaneously corresponds to the tightest turn and fastest turn of an aircraft. Typical corner velocities of current advanced fighters are around 300 to 350 KEAS.

The right hand side of the V – n diagram, vertical line BC, is a high speed limit. This speed is usually selected to be the dive speed. At flight speeds higher than this limit, the dynamic pressure (q) is higher than the design value for the aircraft. At the speed above dive speed, destructive phenomena such as flutter, aileron reversal, and wing divergence, may happen that leads structural damage, or failure, or disintegration. This speed limit (dive speed) is a red-line speed for the aircraft; it should never be exceeded. The dive speed (V_D) is usually higher than aircraft maximum speed (V_{max}), and the aircraft maximum speed (V_{max}) is often higher than aircraft cruising speed (V_C). From FAR Part 23, the following regulations have been directly copied:

$$V_D \geq 1.4 V_C \quad \text{(Normal aircraft)} \tag{9.98a}$$

$$V_D \geq 1.5 V_C \quad \text{(Utility aircraft)} \tag{9.98b}$$

$$V_D \geq 1.55 V_C \quad \text{(Acrobatic aircraft)} \tag{9.98c}$$

The techniques to calculate maximum and cruising speeds have been presented in chapters 5 and 6. The bottom line of the V – n diagram, given by horizontal line CD corresponds with maximum negative limit load factor that is a structural limit when the aircraft is in a situation such as inverted flight. The bottom left curve corresponds to negative stall angle of attack. Since most wing airfoils have positive camber, their positive stall angles are often much higher than the absolute values of their negative stall angles. This curve defines the negative stall area. The Example 9.10 will show the details of plotting a V – n diagram for an aircraft.

9.6.3. Gust V – n Diagram

The atmosphere is a dynamic system that encompasses variety of phenomena. Some of these phenomena include turbulence, gust, wind shear, jet stream, mountain wave and thermal flow. In this section, we concentrate on only gust, since it is not predictable, but is happening during most high altitude flights. When an aircraft experiences a gust, the immediate effect is an increase or decrease in the angle of attack. Figure 9.11 shows the geometry of an upward gust. When an upward gust with a velocity of V_g, hits under the nose of an aircraft with the velocity of V, the instantaneous change (increase) in the angle of attack ($\Delta\alpha$), is determined through:

$$\Delta\alpha = \tan^{-1}\left(\frac{V_g}{V}\right) \approx \left(\frac{V_g}{V}\right) \quad (9.99)$$

Any sudden change (increase) in the angle of attack will produce a sudden change (increase) in the aircraft lift coefficient (ΔC_L):

$$\Delta C_L = C_{L_\alpha}\Delta\alpha \quad (9.100)$$

This in turn will generate a sudden change (increase) in lift (ΔL) as:

$$\Delta L = qS\Delta C_L \quad (9.101)$$

Recall the definition of load factor. This change in lift will create a change in load factor:

$$\Delta n = \frac{\Delta L}{W} \quad (9.102)$$

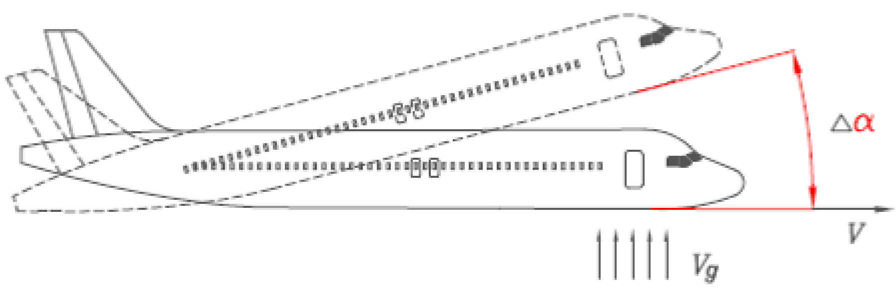

Figure 9.2. The geometry of an upward gust

This indicates that gust will change load factor and will generate a load called gust load. The loads experienced when an aircraft encounters a strong gust may sometimes exceed the maneuver load. Thus we must pay attention to gust load when plotting V-n diagram. As soon as we know the gust velocity, we are able to determine gust load. It is very hard to measure gust velocity, since it happens suddenly. The design requirements for gust velocities are extracted from flight test data.

There are various models for gust prepared by various researchers. Here, we refer to FAR for the gust modeling. According to FAR 23[2], a GA aircraft must be able to withstand gust with a velocity of 50 ft/sec from sea level up to 20,000 ft. From 20,000 ft to 50,000 ft the gust velocity decreases linearly to 25 ft/sec. an aircraft must safely fly at maneuver speed when encounters a gust with the velocity of 66 ft/sec. The aircraft must carry gust load during dive speed, if the gust speed is 25 ft/sec. These data are employed to plot the gust V-n diagram. FAR recommends using the following equation for modeling the "gust induced load factor" as a function of gust speed:

[2] For the current and updated standards and regulations, please refer to www.far.gov

$$n = 1 + \frac{k_g V_{gE} V_E a \rho S}{2W} \tag{9.103}$$

where k_g is a coefficient that is determined by the following expression:

$$k_g = \frac{0.88\mu_g}{5.3 + \mu_g} \tag{9.104}$$

and μ_g is called the aircraft mass ratio and is calculated through:

$$\mu_g = \frac{2m}{\rho \bar{C} a S} \tag{9.105}$$

In the above equations, m is aircraft mass, r is air density, C is wing mean aerodynamic chord, S is wing area, V_E is aircraft equivalent speed, V_{gE} is gust equivalent speed, and a is wing lift curve slope during gust encounter. Please note that the unit system in these equations is metric (i.e. SI system). The gust V-n diagram is plotted using lines based on the equation 9.103 for various speeds (i.e. 25, 50, and 66 ft/sec). Then the intersections between these three lines respectively with maneuver speed (V_A), cruising speed (V_C), and dive speed (V_D) must be marked. The gust V-n diagram is plotted for several altitudes to determine the highest load factor. Figure 9.12 shows a typical gust V-n diagram. This diagram is finally combined; in a special technique; with the basic V-n diagram, to obtain the final applicable V-n diagram.

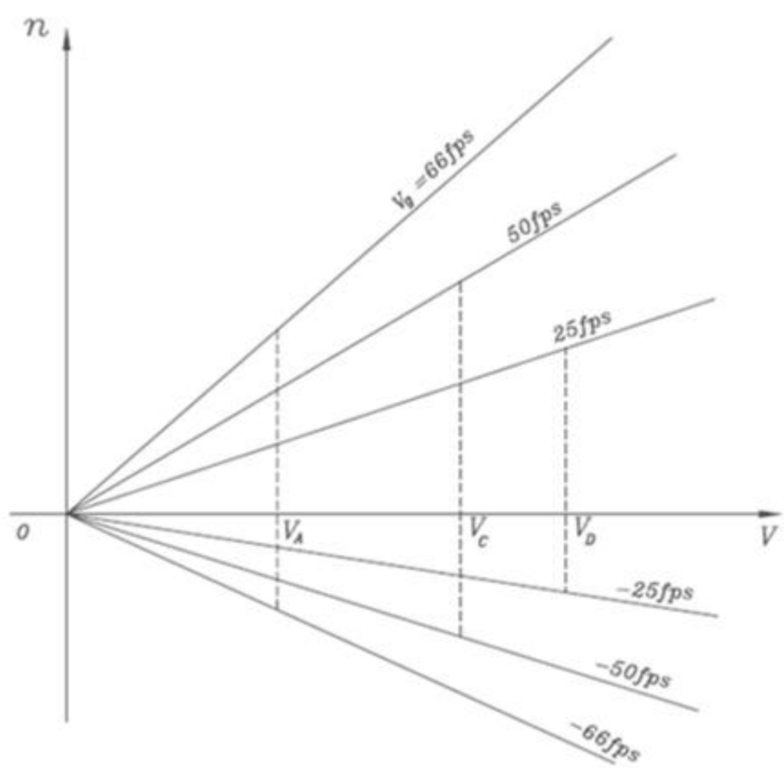

Figure 9.3. A typical gust V-n diagram

9.6.4. Combined V – n Diagram

Section 9.6.2 technique to plot the basic V-n diagram is introduced. In Section 9.6.3, the technique to plot the gust V-n diagram is presented. This section is about combination technique of basic V-n diagram with gust V-n diagram. Since the gust in the atmosphere is a true story, aircraft designers must predict the gust load and add them to the aircraft regular load (maneuver load), to have a safe and strong structure in flight operations. The maximum combined load factor is usually higher than separate load factor in each diagram. A typical combined V-n diagram for an aircraft is illustrated in figure 9.13.

The V-n diagram is unique for each aircraft, and pilots and flight crew are required to fly and operate inside this flight envelope. The following example demonstrates details of the technique to plot the combined V-n diagram for an acrobatic aircraft.

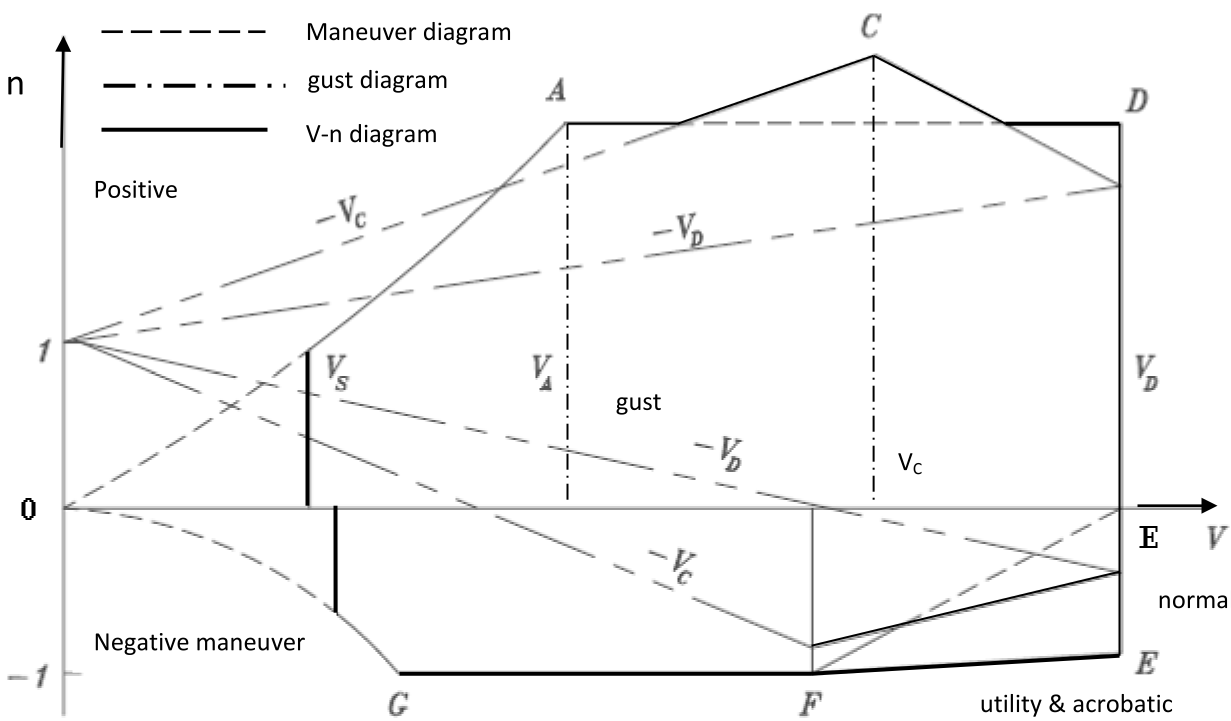

Figure 9.4. A typical combined V-n diagram for an aircraft

Example 9.10
Plot the combined V-n diagram for the following acrobatic aircraft. Then, determine the maximum load factor.

m = 2,300 kg, S = 19.33 m^2, C_{Lmax} = 2, - C_{Lmax} = -1.2, AR = 7, $C_{L\alpha}$ = a = 6.3 1/rad,
V_c = 310 KEAS (at 10,000 ft)

Solution:

The combined V-n diagram is plotted in three steps: 1. Basic V-n diagram, 2. Gust V-n diagram, 3. Combined V-n diagram.

1. **Basic V-n diagram**

The general shape of the combined V-n diagram resembles figure 9.10. We need to determine coordinates of points K, J, G, F, B, and A. since the aircraft type is acrobatic, maximum limit load factor, based on FAR 23 are as follows:

(Positive) n_{max} = +6
(Negative) n_{max} = - (0.0.5 x 6) = -3

The dive speed of this aircraft from equation 9.98c is:

V_D = 1.55 V_C = 1.55 x 310 = 480.5 knot

Hence, the coordinates of points F and G are (6, 480.5), and (-3, 480.5). To determine coordinates of points A, B, J, K, we need to derive two equation regarding C_{Lmax}.

$$V_s = \sqrt{\frac{2mg}{\rho S C_{L_{\max}}}} = \sqrt{\frac{2\times 2300\times 9.81}{1.225\times 19.33\times 2}} = 30.87 \quad \frac{m}{\sec} = 60 \quad KEAS \qquad (2.24)$$

The top curve or the load factor as a function of airspeed (in m/sec) is:

$$n = \frac{L}{W} = \frac{0.5\rho V^2 S C_{L_{\max}}}{W} = \frac{0.5\times 1.225\times V^2 \times 19.33\times 2}{2300\times 9.81} = 0.100105 V^2 \qquad (9.7)$$

For point B that the load factor is 6, the speed is as follows:

6 = 0.100105 V^2 => V = 75.6 m/sec = 147 KEAS

Thus, the coordinates of point B is (6, 147). With the same technique, we can derive the equation for the lower curve:

$$V_{s_i} = \sqrt{\frac{-2mg}{\rho S(-C_{L_{\max}})}} = \sqrt{\frac{-2\times 2300\times 9.81}{1.225\times 19.33\times (-1.2)}} = 39.85 \quad \frac{m}{\sec} = 77.5 \quad KEAS \qquad (2.24)$$

So, the coordinates of point K is (-1, 77.5). The lower curve or the load factor as a function of airspeed (in m/sec) is:

$$-n = \frac{-L}{W} = \frac{0.5\rho V^2 S\left(-C_{L_{\max}}\right)}{W} = \frac{0.5\times 1.225\times V^2\times 19.33\times(-1.2)}{2300\times 9.81} = -0.00063V^2 \qquad (9.7)$$

For point J that the load factor is -3, the speed is as follows:

$-3 = -0.00063\ V^2 \Rightarrow V = 69$ m/sec = 134.2 KEAS

Thus far, we have collected the following coordinates of figure 9.13:

O → (0, 0)
A → (1, 60)
B → (6, 147)
F → (6, 480.5)
G → (-3, 480.5)
J → (-3, 134.2)
K → (-1, 77.5)

By using these data, we can plot the basic V – n diagram as shown in figure 9.14.

1. Gust V-n diagram

Equation 9.103 shows the variations of load factor as a function of airspeed.

$$n = 1 + \frac{k_g V_{gE} V_E a\rho S}{2W} \qquad (9.103)$$

Since the cruising speed is for 10,000 ft, two flight conditions are considered for maximum load factor. Then we calculate n for both V_C and V_D.

a. Aircraft maximum weight at sea level

$$AR = \frac{b^2}{S} \Rightarrow b = \sqrt{AR\cdot S} = \sqrt{7\times 19.33} \Rightarrow b = 11.63 \quad m$$

$$\overline{C} = \frac{S}{b} = \frac{19.33}{11.63} \Rightarrow \overline{C} = 1.66 \quad m$$

$$\mu_g = \frac{2m}{\rho\overline{C}aS} = \frac{2\times 2300}{1.225\times 1.66\times 6.3\times 19.33} = 18.75 \qquad (9.105)$$

$$k_g = \frac{0.88\mu_g}{5.3+\mu_g} = \frac{0.88\times 18.75}{5.3+18.75} = 0.684 \qquad (9.104)$$

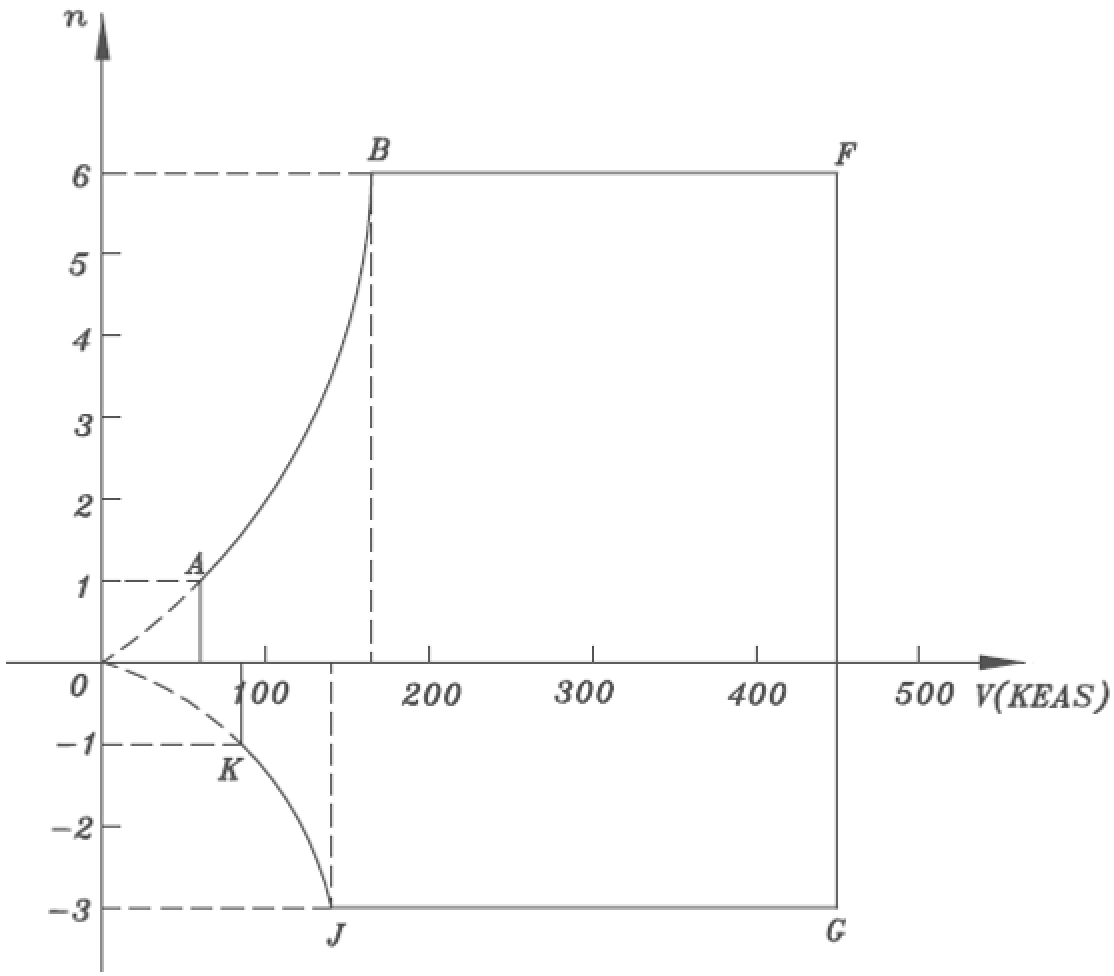

Figure 9.5. The basic V – n diagram for the acrobatic aircraft in Example 9.10

When gust velocity is ± 50 ft/sec (i.e. ± 15.25 m/sec), the load factor will be:

$$n = 1 + \frac{k_g V_{gE} V_E a \rho S}{2W} \tag{9.103}$$

$$n = 1 + \frac{0.684 \times (\pm 15.25) \times V \times 6.3 \times 1.225 \times 19.33}{2 \times 2300 \times 9.81} \Rightarrow n = 1 \pm 0.03436V \tag{9.103}$$

Since the cruising speed (V_C) is 310 KEAS, therefore:

$$n = 1 + 0.03436V = 1 + 0.03436 \times 310 \times 0.5144 = 6.48 \qquad \text{(positive value)}$$

$$n = 1 - 0.03436V = 1 - 0.03436 \times 310 \times 0.5144 = -4.48 \qquad \text{(negative value)}$$

We do the same for dive speed. When aircraft is flying with dive speed (V_D), the gust speed could be ± 25 ft/se (i.e. ± 7.5 m/sec). Hence, the load factor is:

$$n = 1 + \frac{k_g V_{gE} V_E a \rho S}{2W} \qquad (9.103)$$

$$n = 1 + \frac{0.684 \times (\pm 7.5) \times V \times 6.3 \times 1.225 \times 19.33}{2 \times 2300 \times 9.81} \Rightarrow n = 1 \pm 0.01688V \qquad (9.103)$$

Since the dive speed (V_D) is 480.5 KEAS, therefore:

$$n = 1 + 0.01688V = 1 + 0.01688 \times 480.5 \times 0.5144 = 1 + 4.173 = 5.173 \qquad \text{(positive value)}$$

$$n = 1 - 0.01688V = 1 - 0.01688 \times 480.5 \times 0.5144 = 1 - 4.173 = -3.173 \qquad \text{(negative value)}$$

b. Aircraft maximum weight at 10,000 ft

At 10,000 ft altitude, the air density is 0.9 kg/m^3. Parameters μ_g and k_g are.

$$\mu_g = \frac{2m}{\rho \bar{C} a S} = \frac{2 \times 2300}{0.9 \times 1.66 \times 6.3 \times 19.33} = 26.54 \qquad (9.105)$$

$$k_g = \frac{0.88 \mu_g}{5.3 + \mu_g} = \frac{0.88 \times 26.54}{5.3 + 26.54} = 0.733 \qquad (9.104)$$

When gust velocity is ± 50 ft/sec (i.e. ± 15.25 m/sec), the load factor will be:

$$n = 1 + \frac{0.733 \times (\pm 15.25) \times V \times 6.3 \times 0.9 \times 19.33}{2 \times 2300 \times 9.81} \Rightarrow n = 1 \pm 0.02715V \qquad (9.103)$$

Since the cruising speed (V_C) is 310 KEAS, therefore:

$$n = 1 + 0.02715V = 1 + 0.02715 \times 310 \times 0.5144 = 5.26 \qquad \text{(positive value)}$$

$$n = 1 - 0.02715V = 1 - 0.02715 \times 310 \times 0.5144 = -3.26 \qquad \text{(negative value)}$$

We do the same for dive speed. When aircraft is flying with dive speed (V_D), the gust speed could be ± 25 ft/se (i.e. ± 7.5 m/sec). Hence, the load factor is:

$$n = 1 + \frac{0.733 \times (\pm 7.5) \times V \times 6.3 \times 0.9 \times 19.33}{2 \times 2300 \times 9.81} \Rightarrow n = 1 \pm 0.01315V \qquad (9.103)$$

Since the dive speed (V_D) is 480.5 KEAS, therefore:

$$n = 1 + 0.01315V = 1 + 0.01315 \times 480.5 \times 0.5144 = 1 + 3.25 = 4.25 \qquad \text{(positive value)}$$

$n = 1 - 0.01315V = 1 - 0.01315 \times 480.5 \times 0.5144 = 1 - 3.25 = -2.25$ (negative value)

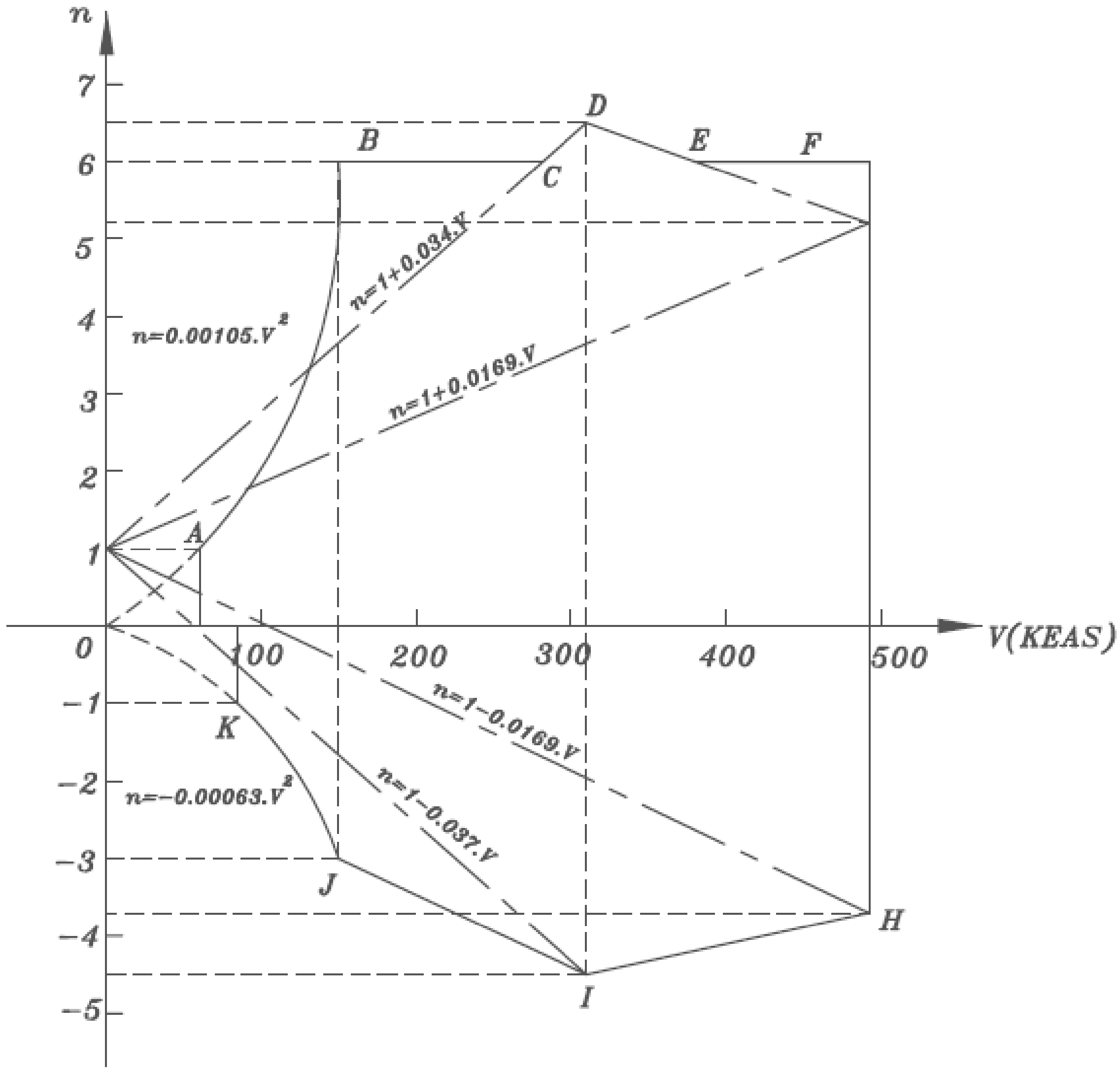

Figure 9.6. The combined (final) V-n diagram for the aircraft in Example 9.10

By comparison between the results of sections a and b, we see that, the load factor at sea level is higher than the load factor at 10,000 ft. Therefore, we can write:

$n_{max} = 6.48$

$-n_{max} = -4.48$

Thus, the coordinates of points D and I are respectively (6.48, 310), (-4.48, 310).

3. **Combined V-n diagram**

Now, we have sufficient data to plot the combined V-n diagram. Figure 9.15 demonstrate the final V-n diagram that includes the gust effect.