Modeling and Approximation of STOL Aircraft Longitudinal Aerodynamic Characteristics

S. N. Deepa[1] and G. Sudha[2]

Abstract: Flight dynamics deals principally with the natural response of the aircraft to longitudinal perturbations that typically consist of two underdamped oscillatory modes having rather different time scales. One of the modes has a relatively short period and is usually quite heavily damped. This is called the short period mode. The other mode has a much longer period and is rather lightly damped. This is called the phugoid mode. The required degree of dynamic stability is specified by the time it takes the motion to damp to half of its initial amplitude. The dynamic characteristics of an airplane is important in assessing its handling or flying qualities and for designing autopilots. This paper investigates the dynamic characteristics of short take off and landing (STOL) airplanes employing root locus technique and state space approach. The nonlinear differential equation of motion of an airplane developed from Newton's second law of motion was linearized using the small disturbance theory. Different phugoid approximations for longitudinal modes were developed and compared with the exact solution formulated using statistical software. Further research work about active control technology to improve aerodynamic efficiency which results in potential fuel savings should be carried out. **DOI: 10.1061/(ASCE)AS.1943-5525.0000403.**

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Introduction

The theoretical basis for the analysis of flight vehicle motion was developed almost concurrently with the successful demonstration of a powered flight of a human carrying airplane. As early as 1897, Frederick Lanchester was studying the motion of gliders, and he discovered that all flight vehicles possess certain natural frequencies or motions when disturbed from their equilibrium flight (Lanchester 1908). Lanchester called the oscillatory motion the phugoid motion. The mathematical modeling of flight vehicle was developed by G. H. Bryan (Bryan and Williams 1904). Bryan laid the foundation for airplane dynamic stability analysis and developed the concept of the aerodynamic stability derivatives (Bryan 1911). A more general formulation of linear unsteady aerodynamics in the aircraft longitudinal equations in terms of indicial functions was introduced by Tobak (Tobak and Schiff 1976). Later (Goman et al. 1990), Goman expressed the mathematical description of aircraft longitudinal aerodynamic characteristics. The analysis of flight motions is simplified, at least for small perturbations from certain equilibrium states (Shevell 1989), by the bilateral symmetry of most flight vehicles (Etkin and Reid 1998). This symmetry allows us to decompose motions into those involving longitudinal perturbations and those involving lateral/directional perturbations (Anderson 2000).

The ability of an aircraft to clear a 15 m (50 ft) obstacle within 450 m (1,500 ft) of commencing takeoff or in landing within 450 m (1,500 ft) after passing over a 15 m (50 ft) obstacle is called short take off and landing (STOL) aircraft. The STOL aircraft feature allows arrangements for use on runways with harsh conditions (such as high altitude or ice). The dynamic stability of STOL aircraft refers to how the aircraft behaves after it has been disturbed from the steady state flight conditions (DOD 2009). This paper focuses in determining the dynamic specifications such as oscillating motions described by two parameters, the period of time required for one complete oscillation and the time required to damp to half-amplitude. To achieve optimum performance of flying qualities damping and frequency of both short period and long period motions were determined in terms of aerodynamic stability derivatives. This influences the pilot's opinion of how easy or difficult the airplane is to fly. The main objective of the paper is to determine the typical motion of dynamic system by locating the longitudinal poles of the aircraft and to determine the time response of the STOL aircraft. The approach of this paper uses classical and modern control techniques in determining the longitudinal characteristics of STOL aircraft. The damping and frequency of various phugoid approximations, such as long period approximation, three-degree-of-freedom, Bairstow's approximation, Russell's approximation, and Phugoid candidate approximation are evaluated and compared.

Equations of Motion

The standard notation for describing the motion of, and the aerodynamic forces and moments acting on, a STOL flight vehicle is indicated in Fig. 1. The variables x, y, and z represent coordinates, with origin at the center of mass of the vehicle. The x-axis lies in the symmetry plane of the STOL vehicle (Nelson 1998) and points toward the nose of the vehicle. The z-axis also is assumed to lie in the plane of symmetry, perpendicular to the x-axis, and pointing approximately down. The y-axis completes a right-handed orthogonal system, pointing approximately out the right wing. The variables u, v, and w represent the instantaneous components of linear velocity in the directions of the x-axis, y-axis, and z-axis, respectively. The variables X, Y, and Z represent the components of aerodynamic

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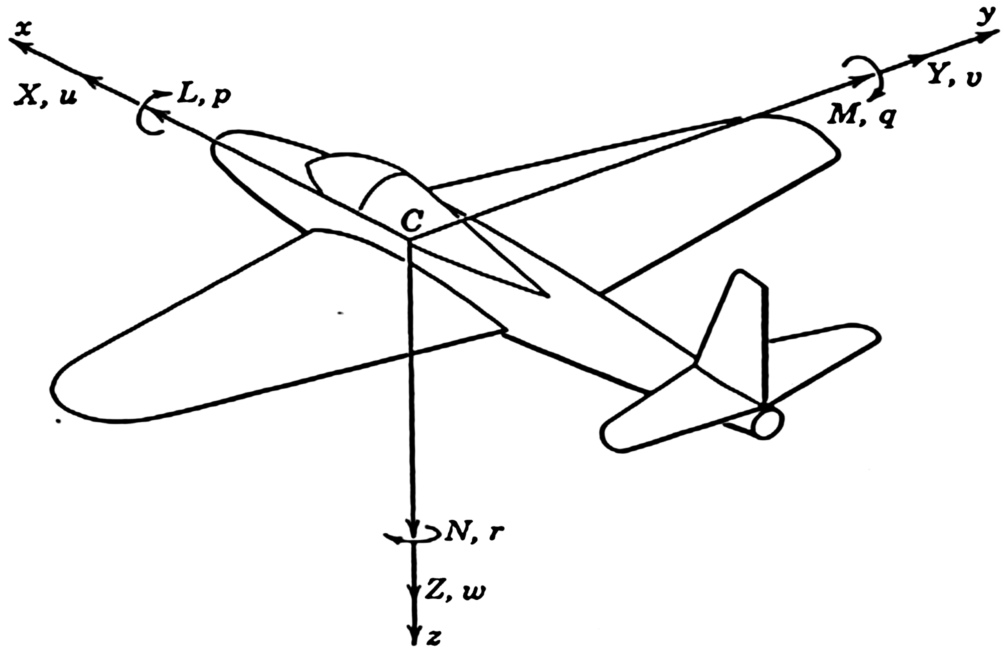

Fig. 1. Force, moments, and velocity components in a body fixed coordinate

force in the directions of the x-axis, y-axis, and z-axis, respectively. The variables p, q, and r represent the instantaneous components of rotational velocity about the x-axis, y-axis, and z-axis, respectively.

The aerodynamic forces and moments can be expressed as a function of all the motion variables. The complete set of equation of motion is given in Eqs. (1)–(3)

$$\left(\frac{d}{dt} - X_u\right)u + g_0 \cos\theta_0 - X_w w = X_{\delta e}\delta_e + X_{eT}\delta_T \tag{1}$$

$$-Z_u u + \left[(1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right]w - (u_0 + Z_q)q + g_0 \sin\theta_0 = Z_{\delta e}\delta_e + Z_{eT}\delta_T \tag{2}$$

$$-M_u u + \left[(M_{\dot{w}})\frac{d}{dt} - M_w\right]w - \left(\frac{d}{dt} + M_q\right)q = M_{\delta e}\delta_e + M_{eT}\delta_T \tag{3}$$

The variables L, M, and N represent the components of aerodynamic moments about the x-axis, y-axis, and z-axis, respectively. The variables φ, θ, and ψ, represent the angular rotations, relative to the equilibrium state, about the x-axis, y-axis, and z-axis, respectively (MacDonald et al. 1971). Thus, $p = \dot{\varphi}$, $q = \dot{\theta}$, and $r = \dot{\psi}$ where the dots represent time derivatives. The velocity components of the vehicle often are represented as angles, as indicated in Fig. 2. The velocity component w can be interpreted as the angle of attack

$$\alpha = \tan^{-1}\frac{w}{u} \tag{4}$$

and the velocity component v can be interpreted as the sideslip angle

$$\beta = \sin^{-1}\frac{v}{V} \tag{5}$$

Modern computer-based flight dynamics simulation is usually done in dimensional form, but the basic aerodynamic inputs are

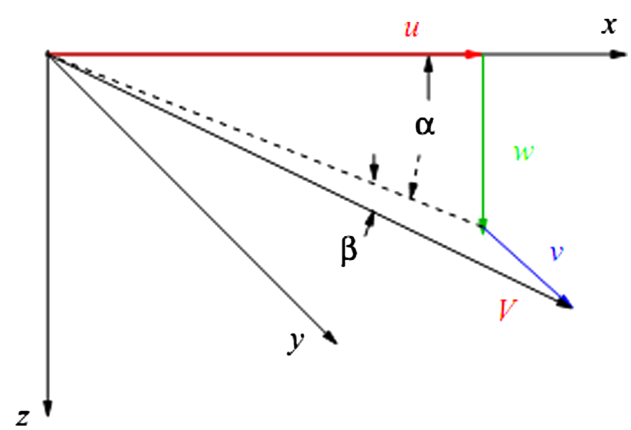

Fig. 2. Standard notation for aerodynamic forces and moment

best defined in terms of the classical nondimensional aerodynamic forms. These are defined using the dynamic pressure as follows:

$$Q = \frac{1}{2}\rho V^2 = \frac{1}{2}\rho SL V_{eq}^2 \tag{6}$$

where ρ = ambient density at the flight altitude and V_{eq} = equivalent airspeed, which is defined by the preceding equation in which ρSL = standard sea level value of the density. In addition, the vehicle reference area S, usually the wing platform area, wing mean aerodynamic chord $\overline{c}$, and wing span b are used to nondimensionalize forces and moments.

Longitudinal Equations of Motion (Stick Fixed)

The generalized longitudinal equation is nonlinear, and it can be linearized using small disturbance theory (Schmidt 1998). In applying small disturbance theory, it is assumed that the motion of an airplane consists of small deviations about a steady flight condition

(Barros and de Oliveira 2011). This theory is difficult to apply to problems in which large amplitude motions are to be expected. The large amplitude deviation is attributable to spinning or stalled flight. Yaw is causing the accidents as a stall and spin. The improper use of the rudder is the main cause of yaw. This can be avoided by proper yaw control mechanisms; however, in many cases, the small disturbance theory yields sufficient accuracy for practical engineering problems. Hence, the small disturbance theory is good in all flight conditions provided with suitable yaw control mechanisms. All the variables in the equation of motion are replaced by a reference value plus a perturbation or disturbance. The perturbations in aerodynamic forces and moments are functions of both, the perturbations in state variables and control inputs. The dependencies in Eqs. (7)–(9) describe the linearized small-disturbance longitudinal rigid body equation of motion

$$\Delta X = \frac{\partial X}{\partial u}u + \frac{\partial X}{\partial w}w + \frac{\partial X}{\partial \delta_e}\delta_e + \frac{\partial X}{\partial \delta_T}\delta_T \tag{7}$$

$$\Delta Z = \frac{\partial Z}{\partial u}u + \frac{\partial Z}{\partial w}w + \frac{\partial Z}{\partial \dot{w}}\dot{w} + \frac{\partial Z}{\partial q}q + \frac{\partial Z}{\partial \delta_e}\delta_e + \frac{\partial Z}{\partial \delta_T}\delta_T \tag{8}$$

$$\Delta \mathrm{M} = \frac{\partial Z\mathrm{M}}{\partial u}u + \frac{\partial \mathrm{M}}{\partial w}w + \frac{\partial \mathrm{M}}{\partial \dot{w}}\dot{w} + \frac{\partial \mathrm{M}}{\partial q}q + \frac{\partial \mathrm{M}}{\partial \delta_e}\delta_e + \frac{\partial \mathrm{M}}{\partial \delta_T}\delta_T \beta = \sin^{-1}\frac{v}{V} \tag{9}$$

In these equations, the control variables δ_e and δ_T correspond to perturbations from trim in the elevator and thrust (throttle) settings. The Z force and pitching moment M are assumed to depend on both the rate of change of angle of attack w and the pitch rate q, but the dependence of the X force on these variables is neglected.

Expressions for all the dimensional stability in terms of the dimensionless aerodynamic coefficient derivatives are summarized in the Appendix. These equations are nonlinear and coupled and generally can be solved only numerically, yielding relatively little insight into the dependence of the stability and controllability of the vehicle on basic aerodynamic parameters of the vehicle.

Longitudinal Approximations

Case 1: Exact Method

The longitudinal motion of an airplane (fixed control) disturbed from its equilibrium flight condition is characterized by two oscillatory modes of motion. One of the modes is lightly damped and has a long period. This motion is called long period or phugoid mode. The second basic motion is heavily damped and has a very short period called short period mode. The linearized longitudinal equation developed in Eqs. (7)–(9) is simple and are ordinary differential equations with constant coefficients (Wael and Quan 2011).

The coefficients in the differential equations are made up of the aerodynamic stability derivatives, mass, and inertia characteristics of the airplane. These equations can be written as a first order differential equation, called the state space equations, and are represented in Eq. (10)

$$\dot{X} = Ax + B\eta \tag{10}$$

where x = sate vector; η = control vector; and the matrices **A** and **B** contain aircrafts dimensional stability derivatives. A summary of the mass, geometric, and aerodynamic characteristics of the airplane were obtained from MacDonald et al. (1971) and Schmidt (1998). The stability coefficients are calculated using the values obtained from the Appendix. The values are $X_w = 0.0875$, $Z_w = -1.044$, $M_w = -1.965$, and $M_{\dot{w}} = -0.6913$, where X_w, Z_w, M_w and $M_{\dot{w}}$ are called stability derivatives, which are evaluated at the reference flight condition. The longitudinal state space matrix (Patel et al. 1977) for STOL transport for stick fixed is given in Eq. (11)

$$\mathbf{A} = \begin{bmatrix} -0.034 & 0.0875 & 0.0000 & -32.0 \\ -0.308 & -1.044 & 215.00 & 0.0000 \\ 0.2120 & -1.240 & -1.308 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{bmatrix} \tag{11}$$

The eigen values (Wilson et al. 1992; Faleiro and Lambregts 1999) of the longitudinal STOL transport are calculated (Sivanandam and Deepa 2007), and the values are in given in Eqs. (12) and (13)

$$\lambda_{1,2} = -0.0170 \pm i0.150 \text{ (phugoid)} \tag{12}$$

$$\lambda_{3,4} = -1.8865 \pm i0.813 \text{ (short period)} \tag{13}$$

The numerical parameters of primary interest are period, time, and number of cycles of half amplitude is calculated as given in Table 1, based on the computed eigen values. When the roots are real, there is of course no period, and the only parameter is the time to double or half. These are the times that must elapse during which any disturbance quantity will double or halve itself, respectively. When the modes are oscillatory, it is the envelope ordinate that doubles or halves. Because the envelope may be regarded as an amplitude modulation, it is thought to be the doubling or halving as applied to the variable amplitude. The stability of the airplane is governed by the real parts of the eigen values, the roots of the characteristics Eqs. (18) and (19).

Case II: Long Period and Short Period Approximation

The longitudinal motion of an airplane (fixed control) disturbed from its equilibrium flight condition is characterized by two oscillatory modes of motion. One of the modes is lightly damped and has a long period. This motion is called long period or phugoid mode. The second basic motion is heavily damped and has a very short period called short period mode. The long period or phugoid mode is a gradual interchange of potential and kinetic energy about the equilibrium altitude and airspeed. An approximation (Nelson 1998) to the long period mode, as in Eq. (14), can be obtained by neglecting the pitching moment equation and assuming that the change in angle of attack is zero

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \\ \frac{-Z_u}{u_0} & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta\theta \end{bmatrix} \tag{14}$$

where u = velocity component about X-axis; and g = gravitational acceleration.

Table 1. Summary of Period, Time, and Number of Half Cycles to Half Amplitude for Phugoid and Short Period Mode

Parameters	Phugoid (long period)	Short period
Period (P)	41.75 s	7.72 s
Time ($t\frac{1}{2}$)	40.58 s	0.3657 s
Number of cycles to half amplitude	0.967 cycles	0.047 cycles

In general, the short period approximation was found to be closer agreement with the exact solution than the phugoid approximation. An approximation to the short period mode of motion can be obtained by assuming $\Delta u = 0$ and dropping the X-force equation

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} \frac{Z_\alpha}{u_0} & 1 \\ M_\alpha + M_{\dot{\alpha}}\frac{Z_\alpha}{u_0} & M_q + M_{\dot{\alpha}} \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} \tag{15}$$

where α = angle of attack and q = angular rates of pitch axis.

The first derivatives of these variables are considered as state variables. Also it can denoted that $M_\alpha = u_0 Z_w$ and $M_{\dot{\alpha}} = u_0 M_{\dot{w}}$. The frequency and damping ratio are calculated and the summary is given in Table 2. This clearly shows short period mode is heavily damped and phugoid mode is lightly damped.

Case III: Evaluation of Different Phugoid Approximation

Three-Degree-of-Freedom Phugoid Approximation

Two degrees of approximation are modified by retaining static pitching moment terms $M_u u$ and $M_\alpha \alpha$. This leads to the additional degree of freedom α over u and θ (McRuer et al. 1973). The equation for phugoid frequency and damping is given by Eqs. (16) and (17)

$$\omega_p = \sqrt{\frac{g(M_u Z_\alpha - Z_u M_\alpha)}{u_0 M_\alpha}} \tag{16}$$

$$2\zeta_p \omega_p = -X_u + \frac{M_u(X_\alpha - g)}{M_\alpha} \tag{17}$$

The values of damping and frequency of three-degree-of-freedom phugoid approximation are computed as 0.079 and 0.214 rad/s.

Bairstow's Phugoid Approximation

Bairstow's method is based on an order of magnitude study of the terms in the fourth-order characteristic equation to arrive at an approximate quadratic equation (Northrop Corporation 1952). The equation for phugoid frequency and damping in terms of aerodynamic derivatives is given by Eqs. (18) and (19)

$$\omega_p = \sqrt{\frac{g(Z_u M_\alpha - M_u Z_\alpha)}{M_q Z_\alpha - u_0 M_\alpha}} \tag{18}$$

$$\begin{aligned} 2\zeta_p \omega_p = {} & \frac{1}{M_\alpha u_0 - M_q Z_\alpha} \times \left\{ X_u(-M_\alpha u_0 + M_q Z_\alpha) \right. \\ & \left. + Z_u \left[-X_\alpha M_q + \frac{g M_\alpha (u_0 M_{\dot{\alpha}} + M_q) + Z_\alpha}{M_\alpha u_0 - M_q Z_\alpha} \right] \right\} \\ & + \left\{ M_u \left[X_\alpha u_0 - \frac{g Z_\alpha (u_0 Z_{\dot{\alpha}} + Z_q) + M_\alpha {u_0}^2}{M_\alpha u_0 - M_q Z_\alpha} \right] \right\} \end{aligned} \tag{19}$$

The values of damping and frequency of Bairstow's phugoid approximation are computed as 0.223 and 0.149 rad/s.

Table 2. Summary of Longitudinal Approximations

Parameters	Long period (phugoid)	Short period
Frequency	$\omega_{np} = 0.214$ rad/s	$\omega_{nsp} = 2.07$ rad/s
Damping ratio	$\zeta_p = 0.0169$	$\zeta_{sp} = 0.913$

Russell's Approximation

In this approach (Russell 1996), the vertical and pitching accelerations, $\dot{w}$ and $\dot{q}$ are neglected on the basis that they are small in the phugoid mode. The equation for phugoid frequency and damping is given by Eqs. (20) and (21)

$$\omega_p = \sqrt{\frac{g(Z_u M_\alpha)}{M_q Z_\alpha - u_0 M_\alpha}} \tag{20}$$

$$2\zeta_p \omega_p = -X_u + \frac{Z_u(X_\alpha M_q)}{-M_q Z_\alpha + u_0 M_\alpha} \tag{21}$$

The values of damping and frequency of Russell's phugoid approximation are obtained as 0.214 and 0.149 rad/s.

Phugoid Candidate Approximation

This phugoid approximation is obtained by dividing the exact fourth order literal characteristic equation by the approximate literal short period equation (Kamesh and Pradeep 1998). The equation of phugiod damping and frequency (Pradeep 1998) is given by Eqs. (22) and (23)

$$\omega_p = \sqrt{\frac{g(-Z_u M_\alpha + M_u Z_\alpha)}{-M_q Z_\alpha + u_0 M_\alpha}} \tag{22}$$

$$\begin{aligned} 2\zeta_p \omega_p = {} & \frac{1}{M_\alpha u_0 - M_q Z_\alpha} [-g \sin\theta M_\alpha + X_u(g \sin\theta M_{\dot{\alpha}} \\ & - M_\alpha u_0 + M_q Z_\alpha)] \\ & + Z_u \left[-g M_{\dot{\alpha}} - X_\alpha M_q + \frac{g M_\alpha (u_0 M_{\dot{\alpha}} + M_q) + Z_\alpha}{M_\alpha u_0 - M_q Z_\alpha} \right] \\ & + M_u \left[u_0(X_\alpha - g) - \frac{g Z_\alpha (u_0 M_{\dot{\alpha}} + M_q) + Z_\alpha}{M_\alpha u_0 - M_q Z_\alpha} \right] \end{aligned} \tag{23}$$

The values of damping and frequency of phugoid candidate approximation are calculated as 0.017 and 0.15 rad/s.

Results and Discussions

The root locus was plotted as shown in Fig. 3 for the phugoid and short period eigen values. The damping ratio lies in between zero and one. The response of this will be underdamped exponentially decaying sinusoidal motion as shown in Fig. 4. From the response, it is clear that the maximum overshoot for a unit step input in the time domain is a function of only damping ratio ζ, and the resonance peak of the response is a function of damping ratio, ζ. Also the bandwidth is directly proportional to the natural damping frequency, ω_n. The higher the bandwidth is, the larger the resonant peak. The maximum peak is 17.4, settling time is 215 s and the final steady state value is 10.3. The overshoot and damping for short period mode is 0.074% and 0.917 (heavily damped). For phugoid mode overshoot is 70% and damping is 0.113 (lightly damped).

A summary of the results from the exact and approximate analysis for STOL aircraft is included in Table 3. In this analysis, short period approximation was found to be in closer agreement with the exact solution. Handling or flying qualities (Heffley and Jewell 1972), of an airplane are related to the dynamic and control characteristics of an airplane. According to the Cooper and Harper scale (Cooper and Harper 1969), the damping ratio of the short period mode is 0.917, and it is termed as good flying qualities of an STOL

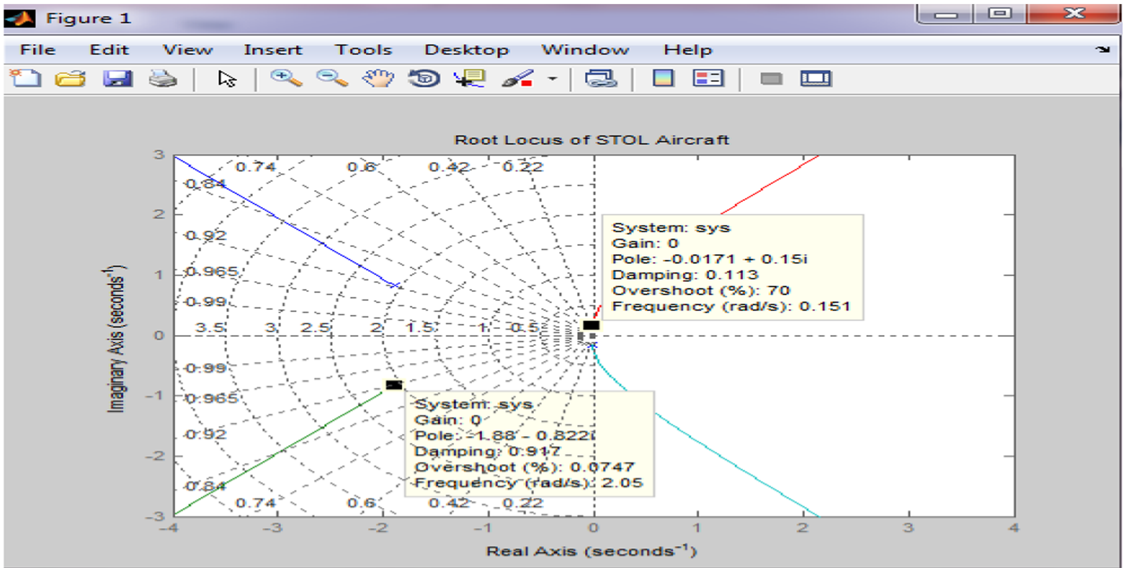

Fig. 3. Root locus plot for STOL transport

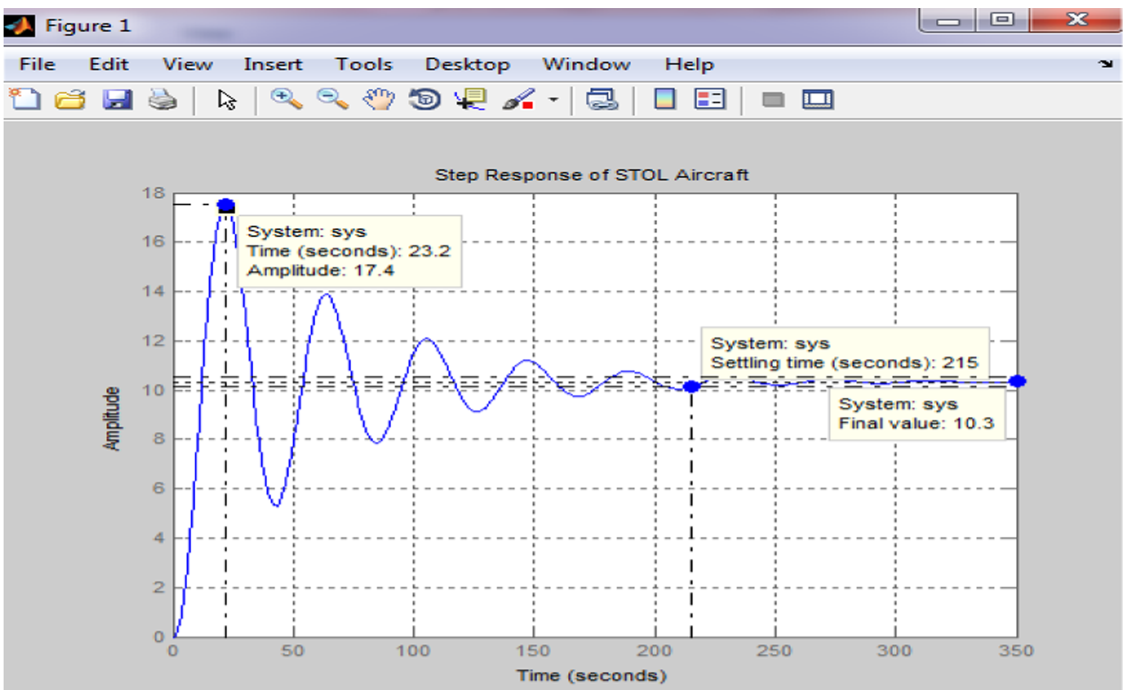

Fig. 4. Longitudinal step response of STOL transport

Table 3. Comparison of Exact and Short Period Approximation Method

Approximation method	Parameters	Exact method	Short period approximation method	Difference
Short period	$t_{\frac{1}{2}}$	0.367 s	0.365 s	0.002
	P	7.72 s	7.45 s	0.27

airplane. The results show that the short period approximation is more accurate than the one in phugoid mode.

The comparative quantitative analysis for STOL aircraft was formulated on Table 4. The approximation of three degree of freedom is poor. Baristow's and Russell's approximation shows good in damping and poor in frequency values. Long period approximation is better compared with the discussed methods, but this too deviates from the exact method. The phugoid candidate approximation gives more accurate solution compared with all the methods. This is true for STOL aircraft also which has very low error value, that is, an error value <0.1. The phugoid candidate approximation method is very close to the exact value.

A measure of the rate of growth or decay of the oscillation can be obtained from the time for halving or doubling the initial amplitude of the disturbances. When pilots are flying an STOL airplane under visual flight rules the phugoid damping can vary

Table 4. Comparison of Exact and Phugoid Approximation Methods

Approximation methods	Parameters	Exact method (s)	Three degree of freedom		Bairstow's approximation		Russell's approximation		Long period approximation		Phugoid candidate approximation	
			Values (s)	Error in %	Values (s)	Error in %	Values (s)	Error in %	Values (s)	Error in %	Values (s)	Error in %
Phugoid	$t_{\frac{1}{2}}$	40.58	8.73	78.4	3.09	92.3	3.22	92	40.82	0.58	40.5	0.19
	P	41.75	29.3	29.7	44.85	7.4	44.85	7.4	29.34	29.7	41.8	0.011

over a wide range $0 < \zeta < 1$, and they will still find the airplane acceptable to fly. The dynamics of aircraft will be oscillatory if the damping were too low, such as $\zeta < -1$. The performance of airplane can be improved by providing damping characteristics such as an automatic stabilization system.

Conclusions

This paper examines the damping ratio and frequency of long period approximations and short period approximations. Of the two characteristics modes, it is observed that the short period is fairly a true representation. The short period approximation is actually very good for a wide range of characteristics and flight conditions. The entire model was simulated in an Intel core processor i5-3210M, 2.5GHz speed, 4GB RAM using *MATLAB*. The time taken for carrying out this is 45 s. The various phugoid mode approximations were evaluated and the solutions of time period and time for halving the initial amplitude of the disturbances were compared. Long period approximation holds good in damping compared with three-degree- of-freedom, Baristow's approximation, and Russell's approximation. The oscillating parameters, damping, and frequency holds good in phugoid candidate approximation compared with the other methods. Although the pilot can correct STOL aircraft easily for the phugoid mode, it would become extremely fatiguing if the damping were too low. The phugoid damping is not at all well predicted by the other approximate solution. The long period approximation and new phugoid candidate approximation is very close to the exact value compared with other approximations and hence it controls flight dynamics. Future work could be focused on the active control technology such as automatic stabilization system to improve aerodynamic efficiency which results in potential fuel savings.

Appendix. Stability and Control Derivatives

The relation of dimensional stability derivatives for longitudinal motions to dimensionless derivatives of aerodynamic coefficients is given by the following equations:

$$X_u = \frac{QS}{mu_0}(2C_{X0} + C_{Xu}) \qquad Z_u = \frac{QS}{mu_0}(2C_{z0} + C_{zu})$$

$$M_u = \frac{QS\overline{c}}{I_y u_0} C_{mu} \qquad X_w = \frac{QS}{mu_0}(C_{x\alpha}) \qquad Z_w = \frac{QS}{mu_0}(C_{z\alpha})$$

$$M_w = \frac{QS\overline{c}}{I_y u_0} C_{mu} \qquad X_{\dot{w}} = 0 \qquad Z_{\dot{w}} = \frac{QS\overline{c^2}}{2\,mu_0^2} C_{z\dot{\alpha}}$$

$$M_{\dot{w}} = \frac{QS\overline{c^2}}{2I_y u_0} C_{m\dot{\alpha}} \qquad X_q = 0 \qquad Z_w = \frac{QS\overline{c}}{2\,mu_0} C_{zq} \qquad M_q = \frac{QS\overline{c^2}}{2I_y u_0} C_{mq}$$

Notation

The following symbols are used in this paper:

g = acceleration attributable to gravity;
M_q = dimensional variation of pitching moment with pitch rate;
M_u = dimensional variation of pitching moment with speed;
M_α = dimensional variation of pitching moment with angle of attack;
$M_{\dot{\alpha}}$ = dimensional variation of pitching moment with rate of change angle of attack;
q = perturbed pitch rate;
S = reference wing area;
T = thrust;
u = perturbed velocity along X;
u_0 = component of steady state velocity along X;
X_q = dimensional variation of X force with pitch rate;
X_{tu} = dimensional variation of X force due to thrust with speed;
X_u = dimensional variation of X force with speed;
X_α = dimensional variation of X force with angle of attack;
w = perturbed velocity along Z;
Z_q = dimensional variation of Z force with pitch rate;
Z_u = dimensional variation of Z force with speed;
Z_α = dimensional variation of Z force with angle of attack;
$Z_{\dot{\alpha}}$ = dimensional variation of Z force with rate of change angle of attack;
α = perturbed angle of attack;
ζ_p = damping ratio the phugoid;
ζ_{sp} = damping ratio the short period;
θ = disturbed pitch attitude angle;
θ_1 = steady state pitch attitude angle;
ρ = air density;
ω_{np} = frequency of phugoid; and
ω_{nsp} = frequency of short period.

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