5.5 DOWNWASH

All lifting surfaces affect the flow aft of those surfaces. In subsonic flow, a lifting surface also affects the flow forward of that surface. This effect is referred to as *downwash*. The downwash behind a wing in subsonic flow is a consequence of the wing's trailing-vortex system, shown in Figure 5.33. A *vortex sheet* is shed from a lifting wing, and the sides of the sheet roll up into *tip vortices*. For high-aspect-ratio wings the sheet is relatively flat, but for low-aspect-ratio and/or highly swept wings the sheet may be bowed up from wing root to tip.

The main effect of this trailing-vortex system is to deflect the airflow (behind the wing) downward relative to the free-stream flow $\mathbf{V}_\infty$. This flow deflection reduces the local angle of attack on any lifting surfaces located behind the wing, such as a horizontal stabilizing surface as shown in Figures 5.33. As shown in Figure 5.34, the local angle of attack on the aft lifting surface is reduced by the *downwash angle* ε, which in turn depends on the location of the aft surface

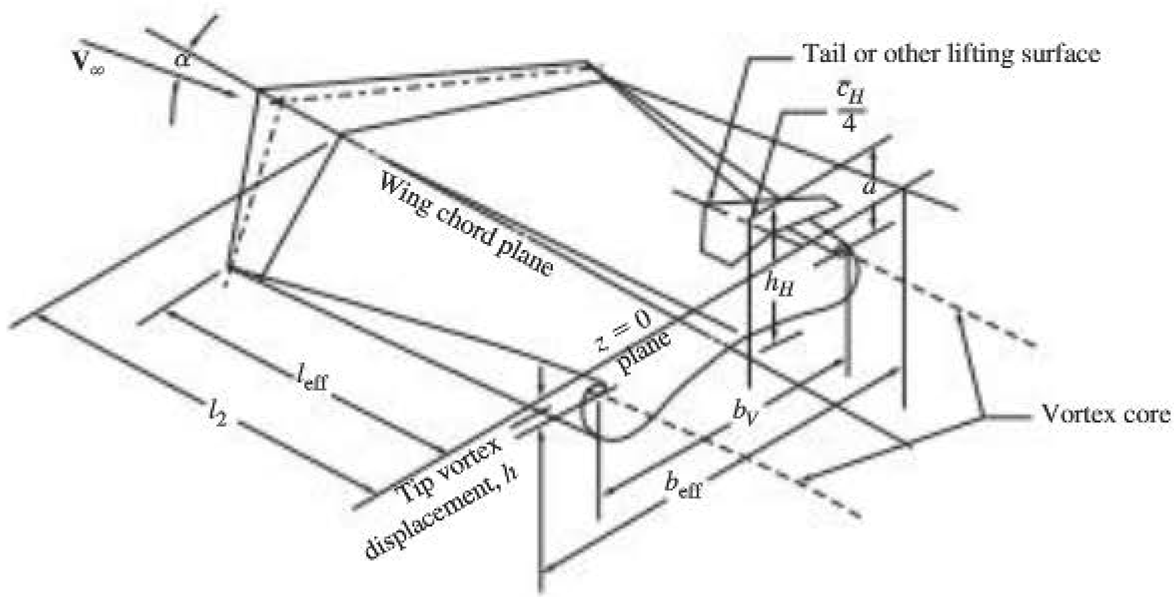

Figure 5.33 Wing trailing-vortex system and aft lifting surface.

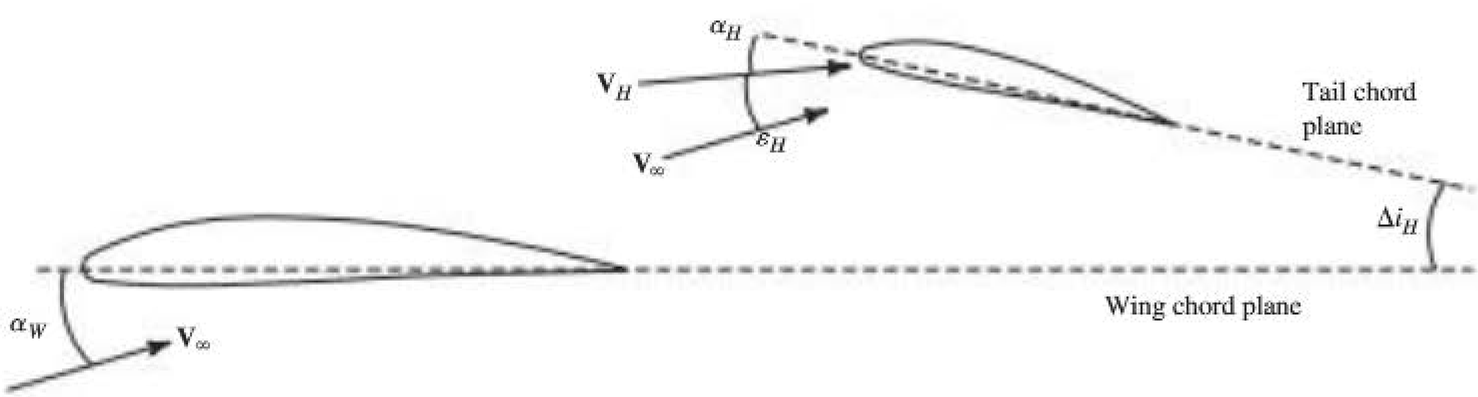

Figure 5.34 Influence of wing downwash on tail angle of attack.

relative to the wing, namely h_H and l_2 shown in Figure 5.33. In terms of the downwash angle, the angle of attack on the aft lifting surface α_H may be written as

$$\alpha_H = \alpha_W + \Delta i_H - \varepsilon_H \tag{5.90}$$

where

α_W is the wing angle of attack (= angle of attack of wing root chord)
Δi_H is the tail incidence angle (= angle between the wing and tail root chords)
ε_H is the downwash angle at the aft lifting surface

Equal in importance to the downwash angle is the change in downwash angle with wing angle of attack, or $d\varepsilon/d\alpha_W$. This parameter, known as the *downwash gradient,* is also a function of location aft of the wing. Theoretically, the downwash gradient is

$$\left.\frac{d\varepsilon}{d\alpha_W}\right|_{\text{theory}} = \begin{cases} 1 & \text{At the wing's trailing edge} \\ \dfrac{2C_{L_{\alpha_W}}}{\pi A} & \text{At infinity behind the wing} \end{cases} \tag{5.91}$$

Note that the angle of attack of the aft lifting surface may now be written in terms of the following linear expression

$$\alpha_H = \left(1 - \frac{d\varepsilon_H}{d\alpha_W}\right)\alpha_W + \Delta i_H \tag{5.92}$$

In addition, when the wing angle of attack is at α_0, the wing lift is zero by definition. Therefore, assuming the downwash angle varies linearly with wing angle of attack, the downwash angle aft of the wing may be written as

$$\boxed{\varepsilon = \frac{d\varepsilon}{d\alpha_W}\left(\alpha_W - \alpha_{0_W}\right)} \tag{5.93}$$

At intermediate distances behind the wing, $d\varepsilon/d\alpha_W$ may be estimated from Figure 5.35. This figure shows the downwash gradient $d\varepsilon/d\alpha_W$ in the wing's plane of symmetry and at the height of the vortex core. The gradient is given as a function of position of the aft lifting surface behind the wing in wing semispans $l_2/(b/2)$, effective wing aspect ratio A_{eff}, (which equals Ae), sweep of the wing's quarter chord line $\Lambda_{c/4}$, and the downwash gradient at infinity, found from Equation (5.91). The dashed lines show an example for $\Lambda_{c/4} = 45°$, $A_{\text{eff}} = 3$, $l_2/(b/2) = 0.8$, and a downwash gradient at infinity of 0.5. If the aft surface is located well above or below the height of the vortex core, as shown in Figure 5.33, the downwash gradient will be somewhat reduced.

As noted previously, ahead of a wing in subsonic flow, the flow is deflected upward relative to the free-stream flow, and this upward angle of deflection is referred to as the *upwash angle* ε_u. Figure 5.36 may be used to estimate the

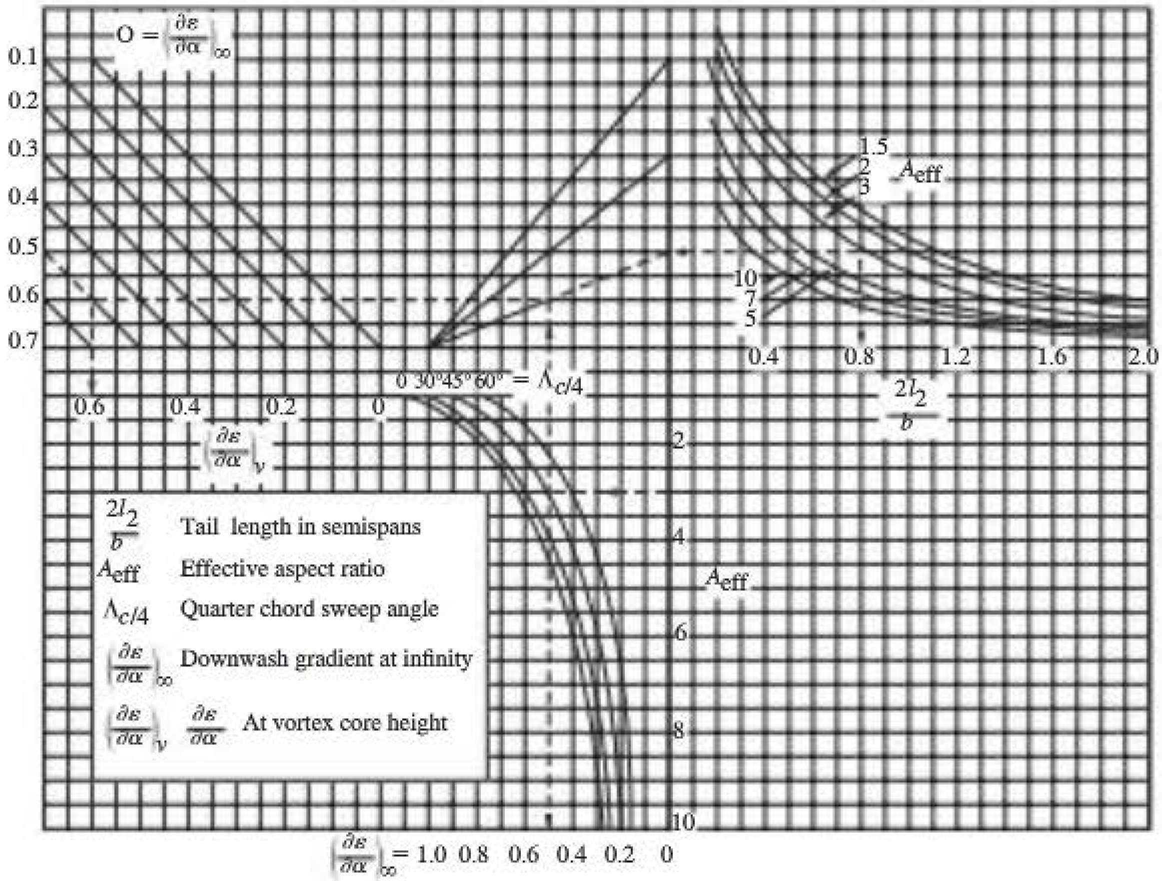

Figure 5.35 Downwash gradient aft of a wing.

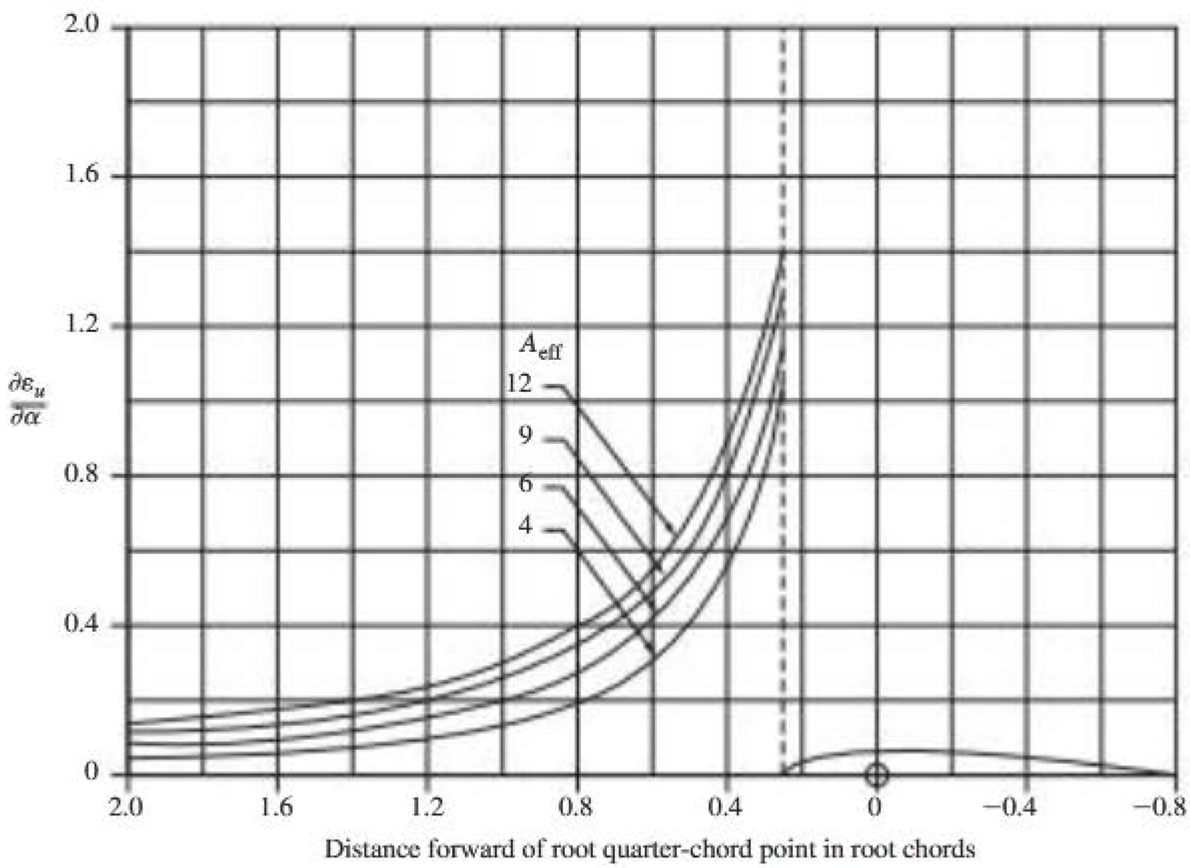

Figure 5.36 Upwash gradient forward of a wing.

upwash gradient $d\varepsilon_u/d\alpha_W$ in the wing's plane of symmetry, at various distances ahead of a wing with effective aspect ratio Ae. Analogous to the case with downwash, the local angle of attack forward of a wing may be expressed as

$$\alpha_{\text{forward}} = \left(1 + \frac{d\varepsilon_u}{d\alpha_W}\right)\alpha_W + \Delta i_{\text{forward}} \tag{5.94}$$

where $\Delta i_{\text{forward}}$ is the incidence angle between the root chord of the wing and the root chord of any lifting surface located forward of the wing (such as a canard).

EXAMPLE 5.7

Downwash Aft of Wing

Again consider the wing analyzed in Example 5.4. If the wing's angle of attack is 2 deg, determine the downwash angle at a point located at the height of the vortex core, and 2.5 root-chord lengths aft of the wing apex. Also, if the aerodynamic center of a horizontal tail is located at the above point, and the tail's geometric incidence angle $\Delta i_H = 2$ deg, find the local angle of attack of the horizontal tail (see Figure 5.34).

■ Solution

For the given wing, the aspect ratio is $A = 5.33$, the sweep of the quarter-chord line is $\Lambda_{c/4} \approx 25$ deg, and the lift effectiveness is $C_{L_\alpha} = 4.19$ /rad. Consequently, from Equation (5.91), the downwash gradient an infinite distance behind the wing is

$$\frac{d\varepsilon}{d\alpha_W}\bigg|_\infty = \frac{2C_{L_\alpha}}{\pi A} = \frac{2(4.19)}{\pi(5.33)} = 0.5$$

The point of interest is located 2.5 wing-root-chord lengths behind the wing apex. Hence the distance l_2 in Figure 5.33 is 1.5 root-chord lengths, and the nondimensionalized distance is

$$\frac{2l_2}{b} = \frac{(7.5 \times 1.5)}{(15)} = 0.75$$

From Figure 5.35 (starting from the bottom axes in the upper-right and lower-left quadrants), we find that the downwash gradient at the location of the tail is

$$\frac{d\varepsilon}{d\alpha_W} = 0.57$$

The downwash angle aft of the wing may be estimated from Equation (5.93), or

$$\varepsilon = \frac{d\varepsilon}{d\alpha_W}(\alpha_W - \alpha_{0_W})$$

So for the wing in question, the downwash angle at the tail when the wing angle of attack is 2 deg is

$$\varepsilon_H = 0.57(2 - 1.33) = 0.38 \text{ deg}$$

For the tail incidence angle given, using Equation (5.90) we find that the local angle of attack at the horizontal tail is then

$$\alpha_H = \alpha_W + \Delta i_H - \varepsilon_H = 2 + 2 - 0.38 = 3.62 \text{ deg}$$