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Aircraft Stability and Control Calculations with a Mathcad-Excel Software Framework

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Alla mia famiglia,
a coloro che ci sono
e a chi non è più qui

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PREFACE

The goal of this thesis project has been improving, partially developing and completely restructuring an integrated software tool for preliminary analysis of subsonic aircraft stability and control parameters.
A large part of this tool has been developed in collaboration with my friend and colleague Carmine Varriale, exceptional person, to whom I extend my acknowledgements.
Improvements from the previous version were essential in order to:

- solve stability problems;
- correct a few inconsistencies in calculations;
- enhance the tool capabilities by adding several new features, especially concerning aircraft geometrical configurations;
- provide the tool with a fairly general analysis of lateral-directional parameters and a complete package of equations for taking into account engines effect on stability and control.

Engineering relations and formulas are mostly based on the book by Napolitano [3] and USAF DATCOM [1]. Software used mainly consists of Mathcad Prime 3.0, Microsoft Office Excel 2010 and a Hierarchical Data Format (HDF) database. A more in-depth description of these particular applications and of software tool structure and algorithms as a whole is provided in the following chapters.

Chapter 1

COMPUTER BASED AIRCRAFT PRELIMINARY DESIGN

1.1 Basic steps in aircraft design

For every new to-be-designed aircraft it is definitely imperative to establish, first of all, its performance requirements: these are assumed as *design conditions*. It is then fundamental to understand how deviations from design conditions affect each flight parameter, or, at least, the most critical of them; moreover, aircraft handling issues related to stability and control aspects must be accurately anticipated and their severity resolved: this means to estimate *performance sensitivity*.
Generally, the typical aircraft design process can be divided into five different phases:

(i) requirements establishing;

(ii) conceptual design;

(iii) preliminary design;

(iv) detailed design;

(v) proof-of-concept aircraft construction and testing.

This tool is intended to come in handy as far as the engineer is bound to the first three phases of the design process. In particular, the preliminary design phase is of special importance, as it ultimately checks whether the design conditions established in the first phase are fitted by the configuration chosen in the second one. It is consequently possible, in the third phase, to remark potential problems, try different solutions out and acquire more detailed information about parts geometry, dimensions and weight, and aircraft performance, stability and control.

It must be said that, for a more detailed and precise estimation of aerodynamic coefficients, wind tunnel tests on a prototype or CFD-based studies are strongly encouraged.

1.2 Complete aircraft aerodynamic model and semi-empirical formulas

The theoretical model adopted in the developing of this software is based on a set of formulas expressing the aerodynamic coefficients C_D, C_L, $C_{\mathcal{M}}$, C_Y, $C_{\mathcal{L}}$ and $C_{\mathcal{N}}$ at given flight conditions and for a specified plane geometry. Each one of these parameters is first calculated by taking into account the contributions of wing, fuselage, tail planes and engines separately; then, when the whole aircraft is virtually built-up, they are corrected by one or more appropriate factors (of semi-empirical nature) that have the purpose of taking into account aerodynamic disturbance phenomenons. The *semi-empirical* formulas trace back to the 1940s and 1950s, when NACA researchers led a massive wind tunnel experimentation program and managed, through a series of correlation studies (often supported by complex theories), to attain a number of closed-form relations for evaluating all the aerodynamic coefficients needed. In early times, the evaluation process was extremely awkward and time-consuming, for it, in some measure, consisted of checking data tables and graphs representing functions of one, two or more variables. Nowadays all of these tools have been transferred to commercial software applications, making it possible for engineers to run many different preliminary design tests and easily determine how aerodynamic parameters are affected by slight configuration alterations.

As a practical example, we can consider a complete aircraft in straight and level flight: if the engines contribution to lift is negligible, it is possible to express the lift coefficient C_L as

$$C_L = C_{L_\alpha}\alpha + C_{L_{\delta_\mathrm{e}}}\delta_\mathrm{e} + C_{L_{i_\mathrm{H}}} i_\mathrm{H}$$

The lift curve slope C_{L_α} can then be written down as

$$C_{L_\alpha} = C_{L_{\alpha,\mathrm{WB}}} + \eta_\mathrm{H}\frac{S_\mathrm{H}}{S_\mathrm{W}}C_{L_{\alpha,\mathrm{H}}}$$

where contributions from horizontal tail and wing-body set (also known as ‘partial plane’) are clearly separated. At the same time, however, effects due to the fact that we are dealing with a complete aircraft are underlined by the presence of previously mentioned semi-empirical factors: one of them (η_H) appears explicitly in the formula. This coefficient, e.g., encloses the interference between wing and horizontal tail.

In a similar way, it is possible to proceed to the modelling of every single other aerodynamic coefficient.

1.3 A brief description of software used

1.3.1 PTC Mathcad Prime 3.0

Mathcad (`http://www.ptc.com/product/mathcad/`) is a Windows-only software application, originally conceived for development, verification and presentation of technical data sheets. Due to its extremely flexible and user-friendly graphical interface, it can be handled both by entry-level and power users. Embracing the ‘What You See Is What You Get’ (WYSIWYG) philosophy, Mathcad is oriented around a digital worksheet where equations and expressions are created and manipulated in the same graphical format in which they are presented. This worksheet allows users to easily combine many elements of different types, such as plain text, images, plots and mathematical formulas. In addition to all of the purely mathematical features, Mathcad comes with a high-level programming language which allows users to deal with, for example, loops (for, while structures) and choices (if, else structures). It also comes with a package of hundreds of pre-defined functions, covering, among the others, curve fitting, experiments planning, statics and probability, signals elaboration, algebraic and differential equations. Moreover, it allows users to define their own custom functions, enhancing, as a matter of fact, its computational possibilities. One simple screenshot is shown in figure 1.1 on the facing page.

Mathcad reads data top to bottom and left to right and this allows for simple manipulation of input variables, assumptions, and expressions. It is generally accepted as the first computer application to automatically compute and check consistency of engineering units throughout the entire set of calculations. It works extremely fine, then, as a unit converter and it also manages different global default unit systems, such as the International System of Units (SI) or the US Customary System of Units (USCS). Mathcad is also able to handle a nested document structure, obtained by including different worksheets into a main one; furthermore, it imports partial or complete datasets from other file types, e.g. Comma Separated Values (CSV, extension `.csv`) or Microsoft Excel Data Sheets (extension `.xslx`) and, in a similar fashion, exports them, too.

Although a free version, named Mathcad Prime 3.0 Express, is also available, this project work has been carried out using the full version, namely PTC Mathcad Prime 3.0, for several advanced features were needed. Mathcad worksheets represent the calculation core of this software tool, and the ultimate environment where aerodynamic coefficients actual values are derived and presented to users.

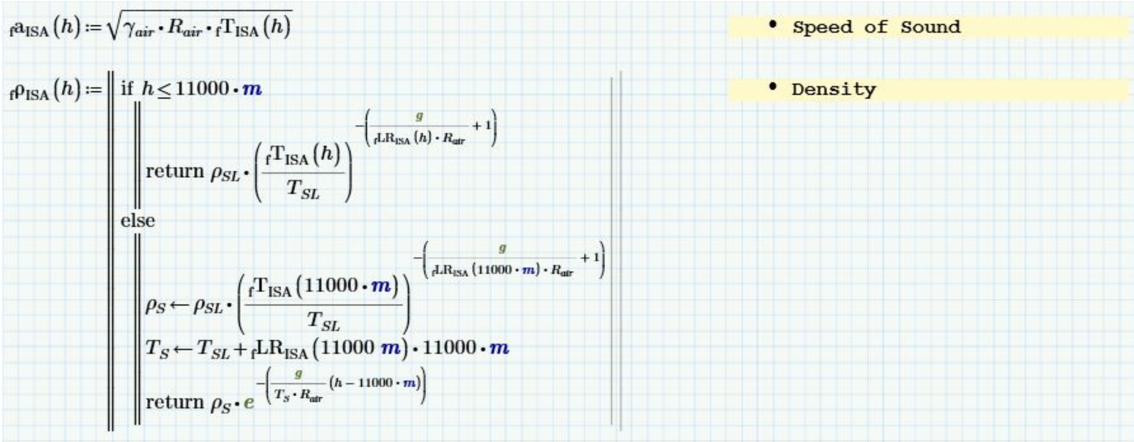

Figure 1.1: A screenshot from Mathcad Prime 3.0

	A	B	C	
1	Variable Name	Variable Value	Unit Label	Description
2	d_{B_W}	10,7	FT	Fuselage diameter at wing leading edge
3	d_B	9,14	FT	Mean fuselage diameter
4	l_B	92,4	FT	Fuselage length
5	S_{B_side}	930,85	FT^2	Fuselage side area
6	Z_1	12,1	FT	Fuselage vertical width at 0.25 fuselage length
7	Z_2	11,1	FT	Fuselage vertical width at 0.75 fuselage length
8	Z_{MAX}	10,7	FT	Max fuselage vertical width
9	ω_{MAX}	10,7	FT	Max fuselage vertical width from top view

Figure 1.2: A screenshot from Microsoft Excel 2010

1.3.2 Microsoft Office Excel 2010

Developed by Microsoft and available for both Windows and Mac operating systems, Excel is with no doubt the industry standard spreadsheet software application. Part of the Microsoft Office Suite, it is one of the fundamental instruments every scientist or engineer must know how to deal with. As any other spreadsheet software, it is structured as an (almost) infinite cell grid, used to organize and manipulate data: this is made possible by basic arithmetic operations and a battery of supplied functions covering statistical, financial, mathematical problems. In addition, Excel is able to display different kinds of data plots, from histograms and pie charts to some simple kind of 3D graphical display. Actually, its complexity makes it almost impossible to provide a full and satisfying brief presentation but, however, its popularity and wideness of use surely make it needless.

In this particular project work, Excel has been employed as a simple data sheet, with no use of any of its own pre-built specific functions (see figure 1.2). It has the role of acquiring input data directly from the user, of coordinating different Mathcad worksheets in a way that will be later explained, and of providing output data in a synthetic and more essential visual style than that provided by Mathcad.

1.3.3 Hierarchical Data Format

HDF, HDF4 and HDF5 standards

HDF, acronym for Hierarchical Data Format (`http://www.hdfgroup.org/`), is a solid freeware and completely portable standard for storing and managing data, libraries and files of various genres. The baseline distribution comes with the library, command-line utilities, tools for testing and a Java-based user interface named HDFView. Other tools exist, for manipulating and analysing data: these make HDF usefully extensible. The open-source format permits users or organizations both to easily share data across a wide variety of computational platforms and to generate structural interactions with applications written in different programming languages. It hence represents a sophisticated and robust data management tool to use for no charge, essential for project works lacking adequate technology resources.

HDF consists of two formats, the older being HDF4, the newer HDF5: both of them were originally designed as a generic technical format, which could be used in science unrelated areas, too. Although an official conversion tool is also available, they are both still widely used in industry, academia and government: there are more than 200 distinct relevant applications of the formats, and an estimated 1.6 million users of just NASA data. HDF is also the base format for a number of community standards, such as HDF-EOS[*] (NASA's Earth Observing System), and NeXus[†] (Neutron, Xray and Muon Science).

HDF5 is particularly good at dealing with data where complexity and scalability are important, and it does so by providing only two major types of objects in the file structure: datasets and groups. The former are multidimensional arrays containing homogeneous data: HDF supports n-dimensional arrays, and each element of the array may be a complex object itself, of virtually any type or size. The latter are container structures which can hold datasets and other groups, thus making HDF a truly hierarchical and file-system-like data format. For certain kinds of parallel I/O paradigms are supported, and thanks to its flexible structure, HDF5 files make it possible for applications to provide fast access to very large datasets, either reading or writing data. Eventually, since HDF5 places no special meaning to data stored in datasets, it is recommended, but not necessary, to employ additional built-in features to provide data with metadata, indexes, descriptions and other information, in order to give it full meaning.

HDFView

HDFView is a visual Java-based tool allowing users to browse through any HDF4 and HDF5 file, starting with a tree view of all top-level objects (mainly groups) in an HDF file's hierarchy (figure 1.3). Users can then scan and examine all data objects in the file, in a similar way as they would with their computer file explorer. Since HDFView was implemented by using the Java 2 platform, its graphical user interface (GUI) is machine-independent and has the same 'look and feel' wherever it is opened.
HDFView shows data content either as plain text, tables or images. Anyway, contents are loaded only in the moment of their selection, thus enhancing speed and efficiency (especially in the case of enormous datasets). Last, HDFView comes with some editing features, too: users are allowed to create, delete, and modify HDF object values and attributes in a more user-friendly way, thanks to the support of the practical and well organized user interface.

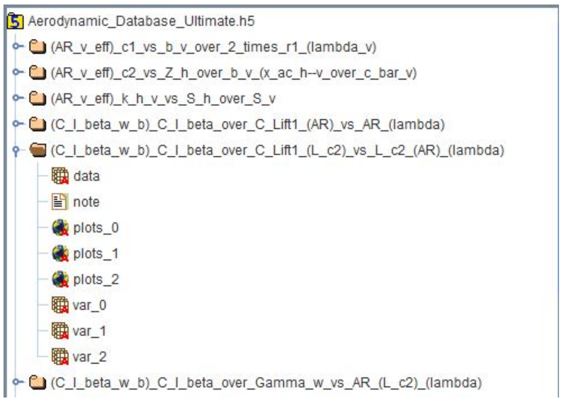

Figure 1.3: A screenshot from HDFView folder tree

1.3.4 LaTeX

LaTeX [5] is a mark-up language for typesetting professional quality documents, based on the TeX open-source asynchronous typesetting system. It is reckoned to be the language which best suits scientific writing standards. As it is, in fact, a proper programming language, first approaches to LaTeX might be a little harsh, though getting familiar with its rules turns out to give paper works an enormous potential, even at a beginner level. The following brief list of selected brilliant features makes it clear how using LaTeX is definitely worth the first struggles:

- it is possible to type complex mathematical formulas with little efforts using standard procedures;
- powerful algorithms automatically arrange content according to the most up to date typographic standards. This is made possible by LaTeX users themselves, who continuously update packages needed to customize one's work[‡];
- sections, tables and images numeration is automatically resolved and updated every time code is modified in the structure; indexes of contents, figures and tables are also generated with absolutely no effort;
- cross references of any different kind are easily manageable.

[*] HDF-EOS homepage: `http://eospso.gsfc.nasa.gov/`

[†] NeXus homepage: `http://www.nexusformat.org/`

[‡] See, for example, the Comprehensive TeX Archive Network (`http://www.ctan.org/`), the international TeX Users Group (`http://tug.org/`) or the Italian Gruppo degli Utilizzatori Italiani di TeX (`http://www.guitex.org/home/`).

```
%flight
\hASL        & \SI{\myAltitudeMT}{m}               & \SI{\myAltitudeFT}{ft}               &
\Mach        & $\myMachNumber$                     & {---}                                &
\qinf        & \SI{\myDynamicPressurePA}{Pa}       & \SI{\myDynamicPressureBAR}{bar}      &

%wing
\CbarW       & \SI{\myMACWingMT}{m}                & \SI{\myMACWingFT}{ft}                &
\SW          & \SI{\myAreaWingMTsquared}{m^2}      & \SI{\myAreaWingFTsquared}{ft^2}      &
\lamW        & $\myTaperRatioWing$                 & {---}                                &
\ARW         & $\myAspectRatioWing$                & {---}                                &
\eW          & \myInducedDragFactorWing            & {---}                                &
\alphazlW    & \SI{\myAlphaZeroLiftWingRAD}{rad}   & \SI{\myAlphaZeroLiftWingDEG}{deg}    &
\CLalphaW    & \SI{\myCLAlphaWingRAD}{rad^{-1}}    & \SI{\myCLAlphaWingDEG}{deg^{-1}}     &
\CMacW       & $\myCMacWing$                       & {---}                                &
```

Figure 1.4: Part of LaTeX code used at the end of this project work

Figure 1.4 shows part of the LaTeX code developed to generate the table on page 38. Numerical values associated to symbols are automatically updated once Mathcad files are changed, thus making it possible to create technical report fast and with ease.

1.4 A summary of the Software framework architecture

The whole software tool practically consists of a series of files for data input, calculation or output, and a simple structure of folders meant to organize them in the most logical way. As far as it concerns the purpose of this paper, the root folder, that stores everything needed for the tool to work correctly, will be referred to as the *main* folder. It contains four types of objects, with profoundly different roles in the program execution:

(i) `Aerodynamic_Database_Ultimate.h5`: an HDF5 file database storing all data needed to calculate complex aerodynamic coefficients;

(ii) a series of seven Mathcad `.mcdx` files, named `Default_Map_` *x*`.mcdx` (where *x* is a string such as *Wing* or *Lateral-Directional*), whose purpose is to collect data at the end of each calculation process and prepare it for output;

(iii) aircraft folders, where actual calculation happens;

(iv) `Default_Functions.mcdx`: a Mathcad sheet embracing only operative function definitions. This file will be included in each calculation sheet, as it will be later explained.

The (i) is a virtual version of those old-fashioned manuals filled with semi-empirical data plots and tables. Every coefficient is now computable by simply specifying its input parameters into a function form (for example, in a Mathcad sheet), saving, as a result, a lot of time and effort.
These files (ii) make absolutely no calculation, but only contain very long four-column matrices whose point is to assemble calculated data and prepare it for output and LaTeX processing. In order to do so, each one of these *maps* must be included at the very end of the corresponding Mathcad calculation sheet.
In (iii) there are two subfolders, two files for the input data and seven Mathcad files in which calculations happen. The first step is to insert the requested input data in the Excel workbook files. Mathcad files are numbered to allow the user to follow a logical order in calculations. One subfolder contains the output data in the Excel file. Output data is written here after every Mathcad sheet is fully run and the corresponding *map* is included. Other output files include:

- seven `.tex` documents, one for each Mathcad calculation sheet, only containing a bare list of TeX definitions. This files can be referred to from a TeX project folder, as a means to quickly build a whole library of new commands that gets updated every time the aircraft project is modified.

- a big series of miscellaneous `.csv` files, dealing with different parts of the plane, and embracing a wide range of purposes: from planform point coordinates, to variable linear laws values, to Shrenk's loading plots data.

For more informations and a very detailed description of how this software works and the every single steps that the user have to follow, see the bachelor thesis of Carmine Varriale [7].

Chapter 2

CALCULATION FORMULAS

To define completely a flight condition for an aircraft there are a large number of calculations that we have to do. The first step is to establish a flight condition in terms of altitude and Mach number. This two informations are the basis from that our analysis starts. The purpose of this software is to determine the deflection angles of the mobile surfaces, e.g. ailerons, rudder, elevator. These deflection angles are calculated imposing the longitudinal and lateral-directional equilibrium.
These calculations are divided in seven Mathcad worksheets:

- `1_Default_Flight_Calculations.mcdx`;
- `2_Default_Wing_Calculations.mcdx`;
- `3_Default_Fuselage_Calculations.mcdx`;
- `4_Default_HTail_Calculations.mcdx`;
- `5_Default_VTail_Calculations.mcdx`;
- `6_Default_Longitudinal_Calculations.mcdx`;
- `7_Default_Lateral_Directional_Calculations.mcdx`.

2.1 Flight calculations

The first step needed for study the problems related with the aircraft flight parameters definition is to establish the flight condition. These conditions are expressed by the atmospheric parameters, computed using the standard atmosphere model, well known as *ISA*, *International Standard Atmosphere*. In this model we considered some constant values at Sea Level altitude (indicated by the subscript *SL*). For example, we have that the temperature, the density and the dynamic viscosity are equal, respectively, to:

$$
\begin{aligned}
T_{SL} &= 288.16\,\mathrm{K}\\
\rho_{SL} &= 1.225\,\mathrm{kg\,m^{-3}}\\
\mu_{SL} &= 1.7894\times10^{-5}\,\mathrm{kg\,m^{-1}\,s^{-1}}
\end{aligned}
$$

The newly introduced variables are a function of altitude. Regarding to the temperature, we introduce the *Lapse Rate*:

$$
LR_{ISA}(h) = \begin{cases} -0.0065\,\mathrm{K\,m^{-1}}, & \text{if } h \le 11\,000\,\mathrm{m},\\ 0\,\mathrm{K\,m^{-1}}, & \text{otherwise.} \end{cases}
$$

Therefore, we obtain:

$$T_{ISA}(h) = \begin{cases} T_{SL} + LR_{ISA}h, & \text{if } h \leq 11\,000\,\text{m}, \\ T_{SL} + LR_{ISA}(11\,000\,\text{m}) \cdot 11\,000\,\text{m}, & \text{otherwise.} \end{cases}$$

The speed of sound is given by the following relationship:

$$a(h) = \sqrt{\gamma_{air} R_{air} T_{ISA}(h)}$$

where γ_{air} is the specific heat coefficient ratio (air adiabatic index) and R_{air} is the perfect gas constant. The density calculation is given by:

$$\rho_{ISA}(h) = \begin{cases} \rho_{SL}\left(\frac{T_{ISA}(h)}{T_{SL}}\right)^{-\left(\frac{g}{LR_{ISA}(h)R_{air}}+1\right)}, & \text{if } h \leq 11\,000\,\text{m}, \\ \rho_S e^{\frac{g}{T_S R_{air}}(h-11\,000\,\text{m})}, & \text{otherwise.} \end{cases}$$

where

$$\rho_S = \rho_{SL}\left(\frac{T_{ISA}(11\,000\,\text{m})}{T_{SL}}\right)^{-\left(\frac{g}{LR_{ISA}(11\,000\,\text{m})R_{air}}+1\right)}$$
$$T_S = T_{SL} + LR_{ISA}(11\,000\,\text{m}) \cdot 11\,000\,\text{m}$$

The viscosity is evaluated by:

$$\mu_{ISA}(h) = \left(1.458 \times 10^{-6}\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-1}\,\text{K}^{-0.5}\right)\frac{\sqrt{(T_{ISA}(h))^3}}{T_{ISA}(h) + 110.4\,\text{K}}$$

2.2 Wing calculations

The wing is the most important aerodynamic surface for an aircraft. Starting from some basic geometrical parameters and from the airfoils aerodynamic coefficients, we can define the parameters of the tridimensional wing.

2.2.1 Basic geometric and aerodynamic parameters

The basic parameters are the wing surface, the taper ratio, the aspect ratio and the mean aerodynamic chord, which are evaluated by the following relationship, respectively:

$$\lambda_{\text{W}} = \frac{c_{\text{W},tip}}{c_{\text{W},root}} \tag{2.1}$$

$$S_{\text{W}} = \frac{b_{\text{W}}}{2} c_{\text{W},root}(1 + \lambda_{\text{W}}) \tag{2.2}$$

$$Æ_{\text{W}} = \frac{{b_{\text{W}}}^2}{S_{\text{W}}} \tag{2.3}$$

$$\bar{c}_{\text{W}} = \frac{2}{3} c_{\text{W},root} \frac{1 + {\lambda_{\text{W}}}^2 + \lambda_{\text{W}}}{1 + \lambda_{\text{W}}} \tag{2.4}$$

The *mean aerodynamic chord* position along the three axes is given by:

$$X_{\bar{c}_{\text{W,LE}}} = \frac{b_{\text{W}}}{6} \frac{1 + 2\lambda_{\text{W}}}{1 + \lambda_{\text{W}}} \tan \Lambda_{LE}$$
$$Y_{\bar{c}_{\text{W}}} = \frac{b_{\text{W}}}{6} \frac{1 + 2\lambda_{\text{W}}}{1 + \lambda_{\text{W}}}$$
$$Z_{\bar{c}_{\text{W}}} = Y_{\bar{c}_{\text{W}}} \tan \Gamma_{\text{W}}$$

The subscript *LE* means that the $\bar{c}_W$ position along the X_S axis is evaluated from the wing apex. The global two-dimensional mean quantities of the wing are given by the following relationships:

$$(t/c_W)_{mean} = \frac{2}{S_W}\int_0^{b_W/2} c_W(y)\frac{t}{c_W}(y)\,\mathrm{d}y \tag{2.5}$$

$$C_{\ell\alpha_{W,mean}} = \frac{2}{S_W}\int_0^{b_W/2} c_W(y)C_{\ell\alpha_W}(y)\,\mathrm{d}y \tag{2.6}$$

$$\alpha_{0\ell_{W,mean}} = \frac{2}{S_W}\int_0^{b_W/2} c_W(y)\alpha_{0\ell_W}(y)\,\mathrm{d}y \tag{2.7}$$

$$C_{m,ac_{W,mean}} = \frac{2}{S_W\bar{c}_W}\int_0^{b_W/2} c_W^2(y)C_{m,ac_W}(y)\,\mathrm{d}y \tag{2.8}$$

The quantity 2.5 is the mean relative thickness respect with the wing chord; the quantity 2.6 is the mean two-dimensional lift curve slope; the quantity 2.7 is the mean alpha zero lift angle; the quantity 2.8 is the mean two-dimensional pitch moment coefficient respect with the aerodynamic center. All these quantities usually vary along the span. The three-dimensional wing alpha zero lift is evaluated by the following relationship:

$$\alpha_{0L,W} = \frac{2}{S_W}\int_0^{b_W/2} c_W(y)\left(\alpha_{0\ell_W}(y) - \varepsilon_{g_W}(y)\right)\mathrm{d}y \tag{2.9}$$

The wing sweep angle can be estimated at different point, which are expressed in percentage of the chord by the term x, as shown in the following relationship:

$$\tan\Lambda_x = \tan\Lambda_{LE} - \frac{4x(1-\lambda_W)}{AR_W(1+\lambda_W)} \tag{2.10}$$

The critical Mach number for the tridimensional wing is evaluated at the mean aerodynamic chord; in particular, we have:

$$M_{cr,3D_W} = \frac{M_{cr,2D_W}(Y_{\bar{c}_W})}{\cos\Lambda_{LE}}$$

2.2.2 Lift curve slope

The wing lift curve slope, $C_{L_{\alpha,W}}$, can be evaluated mainly in two different methods:

- classical method;
- *Polhamus* formula.

The classic formula is given by the following relationship:

$$\left(C_{L_{\alpha,W}}\right)_{classic} = \frac{C_{\ell\alpha_{W,mean}}}{\sqrt{1-M^2} + \frac{C_{\ell\alpha_{W,mean}}}{\pi AR_W e_W}} \tag{2.11}$$

Applying the Polhamus formula we have two different expressions depending on both flight condition and aircraft geometry. These conditions are:

(i) $M_\infty < M_{cr,3D_W}$

(ii) $\Lambda_{LE} < 32$ deg

(iii) $0.4 < \lambda_W < 1$

(iv) $3 < AR_W < 8$

If all these conditions are fulfilled, the Polhamus formula gives this expression to evaluate $C_{L_{\alpha,W}}$:

$$C_{L_{\alpha,W}} = \frac{2\pi AR_W}{2 + \sqrt{\frac{AR_W^2(1-M^2)}{k_p^2}\left(1 + \frac{\tan(\Lambda_{c/2})^2}{1-M^2}\right) + 4}} \tag{2.12}$$

where k_p is a function of AR_{W} and is computed as shown:

$$k_p = \begin{cases} 1 + \frac{\mathit{AR}_{\mathrm{W}}\left(1.87-0.000233\Lambda_{LE}\right)}{100}, & \text{if } \mathit{AR}_{\mathrm{W}} < 4, \\ 1 + \frac{\left(8.2-2.3\Lambda_{LE}\right)-\mathit{AR}_{\mathrm{W}}\left(0.22-0.153\Lambda_{LE}\right)}{100}, & \text{if } \mathit{AR}_{\mathrm{W}} \geq 4. \end{cases}$$

Whereas, if the above conditions (i) and (iv) are not fulfilled, we have

$$C_{L\alpha,\mathrm{W}} = \frac{a_0 \cos \Lambda_{LE}}{\sqrt{1-(M\cos\Lambda_{LE})^2 + \left(\frac{a_0\cos\Lambda_{LE}}{\pi \mathit{AR}_{\mathrm{W}}}\right)^2} + \frac{a_0\cos\Lambda_{LE}}{\pi \mathit{AR}_{\mathrm{W}}}} \tag{2.13}$$

where a_0 is equal to:

$$a_0 = \frac{C_{\ell\alpha_{\mathrm{W}}}(Y_{\bar{c}_{\mathrm{W}}})}{\sqrt{1-M^2\cos(\Lambda_{LE})^2}}$$

The wing lift coefficient at the initial condition, when $\alpha_{\mathrm{W}} = 0°$, is given by the following relationship:

$$C_{L0,\mathrm{W}} = C_{L\alpha,\mathrm{W}}(i_{\mathrm{W}} - \alpha_{0L,\mathrm{W}}) \tag{2.14}$$

2.2.3 Shrenk's method for basic and additional loading

To apply the Shrenk's method for basic and additional wing loading, we have to define the elliptical wing. The elliptical chord distribution is given by the relationship:

$$c_{\mathrm{ell}}(y) = c_{\mathrm{ell}_0}\sqrt{1-\left(\frac{y}{b_{\mathrm{W}}/2}\right)^2}$$

where the term c_{ell_0} is equal to:

$$c_{\mathrm{ell}_0} = \frac{4S_{\mathrm{W}}}{\pi b_{\mathrm{W}}}$$

The effective chord distribution is estimated as follow:

$$c_{\mathrm{eff}}(y) = \frac{c_{\mathrm{W}}(y)C_{\ell\alpha_{\mathrm{W}}}(y)}{C_{\ell\alpha_{\mathrm{W},mean}}}$$

The *basic* angle of attack is equal to:

$$\alpha_b(y) = \alpha_{0L,\mathrm{W}} - (\alpha_{0\ell_{\mathrm{W}}}(y) - \varepsilon_{g_{\mathrm{W}}}(y))$$

The basic wing loading function is given by the relationship:

$$cC_{\ell_b}(y) = \frac{1}{2}c_{\mathrm{W}}(y)C_{\ell\alpha_{\mathrm{W}}}(y)\alpha_b(y) \tag{2.15}$$

The additional wing loading function is given by:

$$cC_{\ell_a}(y) = \frac{1}{2}(c_{\mathrm{eff}}(y) + c_{\mathrm{ell}}(y)) \tag{2.16}$$

The total Shrenk's wing loading function is given by adding 2.15 and 2.16 functions. Integrating over the wing span we have:

$$C_{\mathrm{L}_b} = \frac{2}{S_{\mathrm{W}}}\int_0^{b_{\mathrm{W}}/2} cC_{\ell_b}(y)\,\mathrm{d}y \tag{2.17}$$

$$C_{\mathrm{L}_a} = \frac{2}{S_{\mathrm{W}}}\int_0^{b_{\mathrm{W}}/2} cC_{\ell_a}(y)\,\mathrm{d}y \tag{2.18}$$

2.2.4 Pitching moment coefficient

The pitching moment is an aerodynamic moment acting around the Y_S stability axis. There are two formulation for the tridimensional pitching moment coefficient, an exact formulation and an approximated one. The exact formulation is given by the following relationship:

$$C_{\mathcal{M}_{\mathrm{ac,W}}} = \frac{2}{S_{\mathrm{W}}\bar{c}_{\mathrm{W}}} \int_0^{b_{\mathrm{W}}/2} c C_{\ell_b}(y) x_{\mathrm{bW}}(y)\,\mathrm{d}y + \frac{2}{S_{\mathrm{W}}\bar{c}_{\mathrm{W}}} \int_0^{b_{\mathrm{W}}/2} C_{m,ac_{\mathrm{W}}}(y) {c_{\mathrm{W}}}^2(y)\,\mathrm{d}y \tag{2.19}$$

The approximate formula is provided by Roskam and it is given by the following relationship:

$$C_{\mathcal{M}_{\mathrm{ac,W}_{Roskam}}} = \frac{2\pi}{S_{\mathrm{W}}\bar{c}_{\mathrm{W}}} \int_0^{b_{\mathrm{W}}/2} \alpha_b(y) c_{\mathrm{W}}(y) x_{\mathrm{bW}}(y)\,\mathrm{d}y \tag{2.20}$$

In the two previous relationships the term x_{bW} is the moment arm, in particular, it is the distance between the section's aerodynamic center and the wing *3D* aerodynamic center. A mathematical expression for this distance is given by:

$$x_{\mathrm{bW}}(y) = X_{ac,\mathrm{W}} - \Big(y \tan\Lambda_{LE} + c_{\mathrm{W}}(y)\xi_{ac_{\mathrm{W,2D}}}(y)\Big)$$

where $X_{ac,\mathrm{W}}$ is the distance from the wing aerodynamic center and the wing apex.

2.2.5 Oswald factor

The calculation of the induced drag factor only due to the geometric effects is evaluated by the relationship:

$$e_{\mathrm{W}} = \frac{2}{2 - Æ_{\mathrm{W}} + \sqrt{4 + Æ_{\mathrm{W}}^2\Big(1 + \tan(\Lambda_{t_{max}})^2\Big)}} \tag{2.21}$$

The induced drag factor due to both geometric and aerodynamic effect is estimated using the relationship:

$$e_{\mathrm{W}} = \frac{1.1 C_{L_{\alpha},\mathrm{W}}}{R C_{L_{\alpha},\mathrm{W}} + (1-R)\pi Æ_{\mathrm{W}}} \tag{2.22}$$

where the term R is equal to:

$$R = 0.0004\lambda_e^3 - 0.008\lambda_e^2 + 0.0501\lambda_e + 0.8642$$

and the term λ_e is equal to:

$$\lambda_e = \frac{Æ_{\mathrm{W}}\lambda_{\mathrm{W}}}{\cos\Lambda_{LE}}$$

2.2.6 Downwash

The downwash is the change in direction of air deflected by the aerodynamic action of a part in motion (i.e. the wing, the airfoil, the blade rotor). Suppose we can approximate the downwash function using Taylor's series ended at first grade accuracy. Hence:

$$\varepsilon(M) \approx \varepsilon_0 + \frac{d\varepsilon}{d\alpha}(M)$$

where $\frac{d\varepsilon}{d\alpha}(M)$ is a given function of the Mach number:

$$\frac{d\varepsilon}{d\alpha}(M) = \frac{d\varepsilon}{d\alpha}(M=0)\sqrt{1-M^2}$$

In the Prandtl *Lifting Line Theory (LLT)* the downwash is estimated as follow:

$$\left(\frac{d\varepsilon}{d\alpha}\right)_{LLT}(M=0) = \frac{2C_{L_{\alpha},\mathrm{W}}}{\pi Æ_{\mathrm{W}} e_{\mathrm{W}}} \tag{2.23}$$

The downwash gradient is given by:

$$\left(\frac{d\varepsilon}{d\alpha}\right)_{LLT}(M) = \left(\frac{d\varepsilon}{d\alpha}\right)_{LLT}(M=0)\frac{1}{\sqrt{1-M^2}} \tag{2.24}$$

The term $(\varepsilon_0)_{LLT}$ is evaluated by:

$$(\varepsilon_0)_{LLT} = \frac{d\varepsilon}{d\alpha}(M)(i_{\text{W}} - \alpha_{0L,\text{W}})$$

In the DATCOM method the downwash is evaluated using some semi-empirical relationship. The downwash gradient at the condition $M = 0$ is given by the following semi-empirical formula:

$$\frac{d\varepsilon}{d\alpha}(M=0) = 4.44\left(K_{Æ_{\text{W}}} K_{\lambda_{\text{W}}} K_{MAC/4}\sqrt{\cos\Lambda_{c/4}}\right)^{1.19} \tag{2.25}$$

The coefficients in the above equation can be computed as follow:

$$K_{Æ_{\text{W}}} = \frac{1}{Æ_{\text{W}}} - \frac{1}{1 + Æ_{\text{W}}^{1.7}}$$

$$K_{\lambda_{\text{W}}} = \frac{10 - 3\lambda_{\text{W}}}{7}$$

$$K_{MAC/4} = \frac{1 - \frac{\Delta Z'}{b_{\text{W}}}}{\sqrt[3]{2\frac{\Delta X'}{b_{\text{W}}}}}$$

In the last coefficient, $K_{MAC/4}$, the terms $\Delta Z'$ and $\Delta X'$ are the vertical distance and the horizontal distance, respectively, between the horizontal tail quarter point of the mean aerodynamic chord and the wing quarter point of the mean aerodynamic chord, measured normal to root chord and parallel to root chord, respectively.

The term ε_0 is evaluated by:

$$\varepsilon_0 = \frac{d\varepsilon}{d\alpha}(M)(i_{\text{W}} - \alpha_{0L,\text{W}})$$

2.2.7 Control surfaces

The ailerons and the flaps are the main important control surfaces; they are part of the wing. For symmetrical configuration, the ailerons inner and outer stations are given by the following relationships, respectively:

$$y_{\text{a},in} = \eta_{\text{a},in}\frac{b_{\text{W}}}{2};\; y_{\text{a},out} = \eta_{\text{a},out}\frac{b_{\text{W}}}{2}$$

where the terms $\eta_{\text{a},in}$ and $\eta_{\text{a},out}$ are the dimensionless positions of the inner and outer stations along the semi-span, respectively. The ailerons surface is given by the formula:

$$S_{\text{a}} = 2c_{\text{a}}(y_{\text{a},out} - y_{\text{a},in})$$

In the same way, fot the flaps we have these relationships:

$$y_{\text{flap},in} = \eta_{\text{flap},in}\frac{b_{\text{W}}}{2}$$

$$y_{\text{flap},out} = \eta_{\text{flap},out}\frac{b_{\text{W}}}{2}$$

$$S_{\text{flap}} = 2c_{\text{flap}}(y_{\text{flap},out} - y_{\text{flap},in})$$

If the flaps are open, there is a variation in term of the zero lift angle; in particular we have:

$$\alpha_{0L_{\text{W},flap}} = \alpha_{0L,\text{W}} + \frac{S_{\text{flap}}}{S_{\text{W}}}\Delta\alpha_{0\ell_{\text{W},flap}}$$

2.3 Fuselage calculations

For the calculation of some semi-empirical coefficients we have to introduce the *Fuselage Fineness Ratio*, *FRR*:

$$FRR = \frac{l_B}{d_B}$$

To compute the aerodynamic coefficients of the fuselage we use the *Multhopp* method. This method divides the fuselage in several sections and the total contribution is given by the sum of the different terms (see figure 2.1 in the next page). The presence of the fuselage produces a change of downwash, in particular, we have by the DATCOM formulation:

$$\left(\frac{d\varepsilon}{d\alpha}\right)_{Multhopp}(M=0) = 4.44\left(K'_{AR_W} K'_{\lambda_W} K'_{MAC/4}\sqrt{\cos\Lambda_{c/4}}\right)^{1.19} \tag{2.26}$$

in which $K'_{AR_W} = K_{AR_W}$ and $K'_{\lambda_W} = K_{\lambda_W}$ are evaluated in the section 2.2.6 on page 19. The term $K'_{MAC/4}$ is given by the following relationship:

$$K'_{MAC/4} = \frac{1}{\sqrt[3]{2\frac{\Delta X'}{b_W}}}$$

where the term $\Delta X'$ has the meaning explained in the section 2.2.6 on page 19. The term $\Delta Z'$ in this case is equal to zero. The downwash gradient is given by:

$$\left(\frac{d\varepsilon}{d\alpha}\right)_B = \left(\frac{d\varepsilon}{d\alpha}\right)_{Multhopp}(M=0)\sqrt{1-M^2} \tag{2.27}$$

The fuselage contribution to the pitching moment coefficient is evaluated by the following relationship:

$$C_{\mathcal{M}0,B} = \frac{\pi(K_2-K_1)}{2S_W\bar{c}_W}\sum_{k=0}^{N_{B_0}-2} W_{B_k}^2\left(-i_W + \alpha_{0L,W} + i_{cl_k}\right)\Delta x \tag{2.28}$$

where the term $(K_2 - K_1)$ is the fuselage apparent mass coefficient and it is evaluated graphically (see [4], figure 3.6); the term N_{B_0} is the fuselage number of division (+1); the term W_{B_k} is the local fuselage width/diameter; the term i_{cl_k} is the incidence angle of the fuselage camberline relative to the fuselage reference line; the term Δx indicates the fuselage section length and it is defined as $\Delta x = l_B/(N_{B_0} - 1)$. The gradient of the pitching moment coefficient is given by the following relationship:

$$C_{\mathcal{M}\alpha,B} = \frac{\pi}{2S_W\bar{c}_W}\sum_{k=0}^{N_{B_1}-2} W_{B_1,k}^2\left(\varepsilon u_{\alpha 1_k}+1\right)\Delta x_1 + \frac{\pi}{2S_W\bar{c}_W}\sum_{k=0}^{N_{B_2}-2} W_{B_2,k}^2\frac{X_1}{\Delta L_{H_{ac},W_{LE}}}\left(1-\left(\frac{d\varepsilon}{d\alpha}\right)_{Multhopp}\right)\Delta x_2 \tag{2.29}$$

In this equation the terms $(N_{B_1} - 2)$ and $(N_{B_2} - 2)$ are the fuselage number of division (+1) from the fuselage nose to wing leading edge and from the wing leading edge to fuselage tail, respectively. The terms $W_{B_1,k}$ and $W_{B_2,k}$ have the same meaning expressed in the equation 2.28. The term $\varepsilon u_{\alpha 1_k}$ represents the upwash for front fuselage sections and it is evaluated graphically (see [4], figures 3.9a and 3.9b). The terms Δx_1 and Δx_2 are a section length of the fuselage from the fuselage nose to wing leading edge and from the wing leading edge to fuselage tail, respectively. The term X_1 is the distance between the fuselage section centroid and the wing trailing edge; the term $\Delta L_{H_{ac},W_{LE}}$ is the distance (measured parallel to the wing root chord) between the wing leading edge and the horizontal tail aerodynamic center. Considering the contribution from the fuselage, the aerodynamic center position is changed. In particular, in the wing-body (WB) configuration, the aerodynamic center is equal to:

$$\xi_{AC,WB} = \xi_{AC,W} + \Delta\xi_{AC,WB} \tag{2.30}$$

where the term $\Delta\xi_{AC,WB}$ is given by the following relationship:

$$\Delta\xi_{AC,WB} = -\frac{C_{\mathcal{M}\alpha,B}}{C_{L_{\alpha,W}}}$$

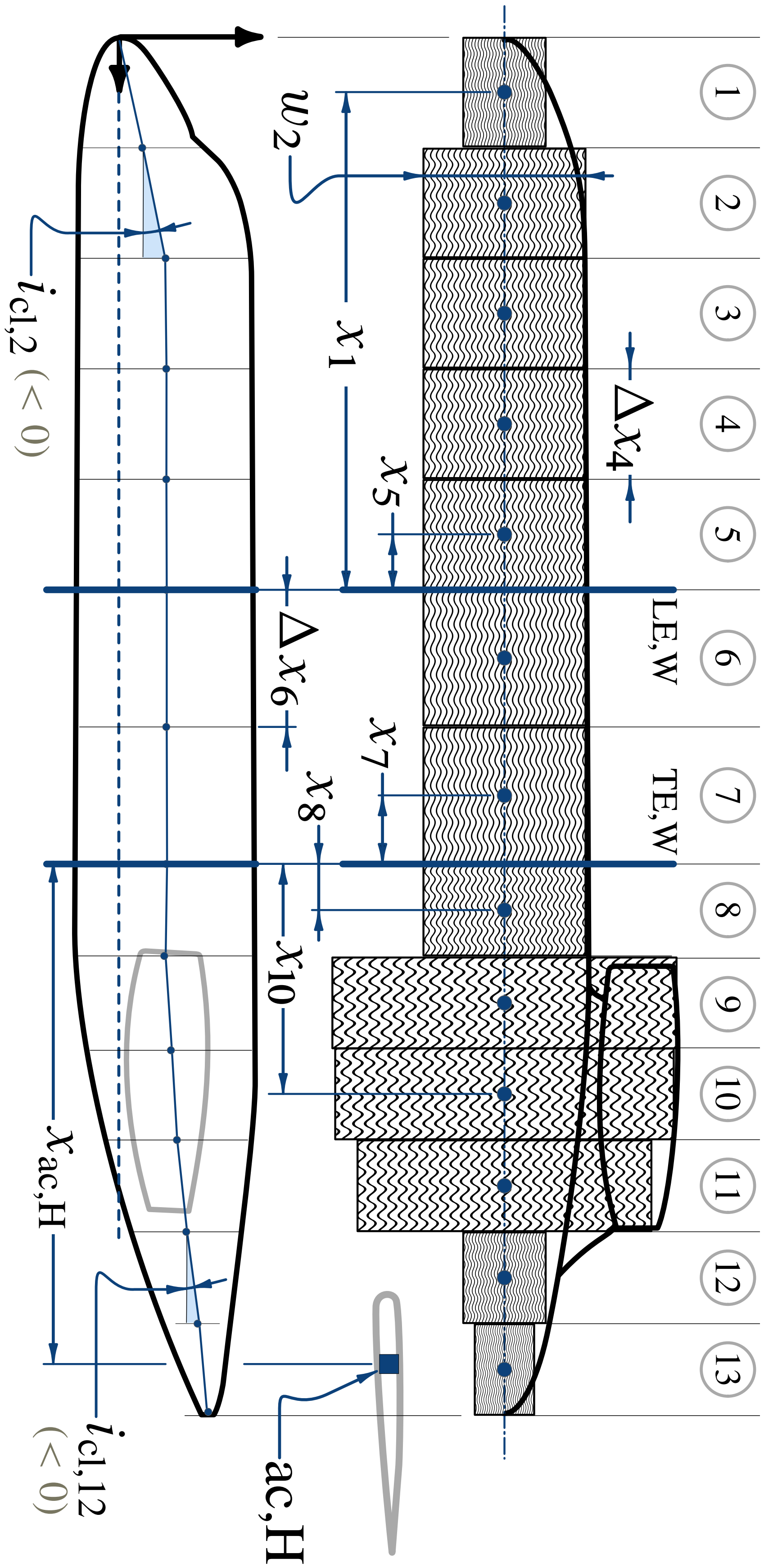

Figure 2.1: Multhopp strip method of Douglas DC-9-10

2.4 Horizontal Tail calculations

To compute all parameters needed with regard to the Horizontal Tail, see Section 2.2 on page 16. Please notice, that in this case the root profile related parameters are always referred to the fuselage centre line.

2.5 Vertical Tail calculations

To compute all parameters needed with regard to the Vertical Tail, see Section 2.2 on page 16. Please notice, that in this case the vertical tale is considered as a mirrored wing (hence b_{V} is doubled).

2.6 Longitudinal equilibrium calculations

To completely define the longitudinal equilibrium we have to consider both the steady-state both the unsteady-state flight condition. In addition, we can consider two different configurations for the aircraft: stick fixed and stick free. To consider the stick free condition we have to introduce the free elevator factor that is evaluated by the following relationship:

$$F = 1 - \tau_{\mathrm{e}} \frac{C_{\mathcal{H}_{\alpha_{\mathrm{e}}}}}{C_{\mathcal{H}_{\delta_{\mathrm{e}}}}} \tag{2.31}$$

The unsteady-state is caused by the perturbation terms in all the components of the linear and angular velocities.

2.6.1 Stick fixed Drag coefficient

The steady-state aerodynamic force acting along the X_S axis is the drag. An expression for the drag is given by:

$$D = C_D \bar{q} S_{\mathrm{W}} \tag{2.32}$$

where:

- C_D is the aircraft dimensionless drag coefficient at steady-state condition;
- $\bar{q}$ is the dynamic pressure acting on the aircraft;
- S_{W} is the reference wing surface.

For a given Mach number and for the Reynolds number associated with the aerodynamic design, the drag coefficient can be expressed as function of the following variables:

$$C_D = f(\alpha, \delta_e, i_{\mathrm{H}})$$

The introduction of a Taylor expansion with a first-order approximation around the condition $[\alpha_0 = \delta_{e_0} = i_{H_0} = 0°]$ leads to:

$$C_D = C_{D_0} + C_{D_\alpha}\alpha + C_{D_{\delta_{\mathrm{e}}}}\delta_e + C_{D_{i_{\mathrm{H}}}} i_{\mathrm{H}}$$

The drag stability derivatives $C_{D_{\delta_{\mathrm{e}}}}$ and $C_{D_{i_{\mathrm{H}}}}$, with respect to the elevator deflection and stabilator deflection respectively, can be approximated to be negligible. In this way we have $C_D \approx C_{D_0} + C_{D_\alpha}\alpha$. The stability derivative with respect to the angle of attack is given by the following relationship:

$$C_{D_\alpha} = \frac{2 C_L C_{L_\alpha}}{\pi Æ_{\mathrm{W}} \varepsilon}$$

An alternative formula for the modeling of the overall C_D coefficient is the well-known Prandtl relationship:

$$C_D = C_{D0} + \frac{C_L^2}{\pi Æ_{\mathrm{W}} \varepsilon} \tag{2.33}$$

2.6.2 Stick fixed Lift coefficient

The steady-state aerodynamic force acting along the Z_S axis is the lift. An expression for the lift is given by:

$$L = C_L \bar{q} S_{\mathrm{W}} \tag{2.34}$$

For a given Mach number and a given Reynolds number, the lift coefficient can be expressed as function of the followmg vanables:

$$C_L = (\alpha, \delta_e, i_\mathrm{H})$$

The introduction of a Taylor expansion with a first-order approximation around the condition $[\alpha_0 = \delta_{e_0} = i_{H_0} = 0°]$ leads to:

$$C_L = C_{L0} + C_{L\alpha}\alpha + C_{L_{\delta_\mathrm{e}}}\delta_e + C_{L_{i_\mathrm{H}}} i_\mathrm{H}$$

For a conventional subsonic aircraft, the lift of the entire aircraft at steady-state condition is given by:

$$L = L_{WB} + L_H$$

In term of coefficients we have:

$$C_L = C_{L_\mathrm{WB}} + C_{L_\mathrm{H}}\eta_\mathrm{H}\frac{S_\mathrm{H}}{S_\mathrm{W}} \tag{2.35}$$

The coefficient C_{L0} is evaluated at the initial condition and is given by:

$$C_{L0} = C_{L0_\mathrm{WB}} - \eta_\mathrm{H}\frac{S_\mathrm{H}}{S_\mathrm{W}} C_{L_{\alpha,\mathrm{H}}}\varepsilon_0$$

The stability derivative with respect to the angle of attack is evaluated by the following relationship:

$$C_{L_\alpha} = C_{L_{\alpha,\mathrm{WB}}} + C_{L_{\alpha,\mathrm{H}}}\eta_\mathrm{H}\frac{S_\mathrm{H}}{S_\mathrm{W}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

Wing-body contribution

A Taylor expansion of C_{L_WB} with a first order approximation around the condition $[\alpha_0 = 0°]$ is given by:

$$C_{L_\mathrm{WB}} = C_{L0_\mathrm{WB}} + C_{L_{\alpha,\mathrm{WB}}}\alpha_\mathrm{WB} \tag{2.36}$$

where C_{L0_WB} is the lift coefficient evaluated at the initial condition $[\alpha_0 = 0°]$ and is given by the following relationship:

$$C_{L0_\mathrm{WB}} = C_{L_{\alpha,\mathrm{WB}}} i_\mathrm{W}$$

The second coefficient in the equation 2.36 is the lift stability derivative of the WB configuration with respect to the angle of attack. This coefficient is given by:

$$C_{L_{\alpha,\mathrm{WB}}} = K_\mathrm{WB} C_{L_{\alpha,\mathrm{W}}} \tag{2.37}$$

where the term K_{WB} is equal to:

$$K_\mathrm{WB} = 1 + 0.025\frac{d_\mathrm{B}}{b_\mathrm{W}} - 0.25\left(\frac{d_\mathrm{B}}{b_\mathrm{W}}\right)^2$$

Horizontal tail contribution

A Taylor expansion of C_{L_H} with a first order approximation around the condition $[\alpha_0 = \delta_{e_0} = i_{H_0} = 0°]$ is given by:

$$C_{L_\mathrm{H}} = C_{L0_H} + C_{L_{\alpha,\mathrm{H}}}\alpha_\mathrm{H} + C_{L_{\alpha,\mathrm{H}}}\tau_\mathrm{e}\delta_\mathrm{e} + C_{L_{\alpha,\mathrm{H}}} i_\mathrm{H} \tag{2.38}$$

Usually the coefficient C_{L0_H} is equal to zero because of symmetrical profiles are used for the design of the horizontal tail. The angles δ_e and i_H are the elevator and the stabilator deflections, respectively.
In general the stabilator and the elevator contributions are given by the following relationships, respectively:

$$C_{L_{i_\mathrm{H}}} = \eta_\mathrm{H}\frac{S_\mathrm{H}}{S_\mathrm{W}} C_{L_{\alpha,\mathrm{H}}}$$

$$C_{L_{\delta_\mathrm{e}}} = C_{L_{i_\mathrm{H}}}\tau_\mathrm{e}$$

2.6.3 Stick fixed Moment coefficient

The steady-state aerodynamic moment acting around the Y_S axis is the pitching moment. An expression for the pitching moment is given by:

$$M = C_{\mathcal{M}}\bar{q}S_{\mathrm{W}} \tag{2.39}$$

For a given Mach number and a given Reynolds number, the pitching moment coefficient can be expressed as function of the followmg vanables:

$$C_{\mathcal{M}} = (\alpha, \delta_e, i_{\mathrm{H}})$$

The introduction of a Taylor expansion with a first-order approximation around the condition $[\alpha_0 = \delta_{e_0} = i_{H_0} = 0°]$ leads to:

$$C_{\mathcal{M}} = C_{\mathcal{M}0} + C_{\mathcal{M}_\alpha}\alpha + C_{\mathcal{M}_{\delta_\mathrm{e}}}\delta_e + C_{\mathcal{M}_{i_\mathrm{H}}}i_{\mathrm{H}}$$

where:

- $C_{\mathcal{M}0}$ is the pitching moment coefficient evaluated at the condition $[\alpha_0 = \delta_{e_0} = i_{H_0} = 0°]$;
- $C_{\mathcal{M}_\alpha}$ is the pitching moment stability derivative with respect to the angle of attack;
- $C_{\mathcal{M}_{\delta_\mathrm{e}}}$ is the pitching moment stability derivative with respect to the elevator deflection;
- $C_{\mathcal{M}_{i_\mathrm{H}}}$ is the pitching moment stability derivative with respect to the stabilator deflection.

For a conventional subsonic aircraft the pitching moment coefficient for the entire aircraft is given by:

$$\begin{aligned} M = M_{AC,\mathrm{WB}} + L_{\mathrm{WB}}(x_{\mathrm{CG}} - x_{\mathrm{AC}})\cos\alpha_{\mathrm{W}} + D_{\mathrm{WB}}(x_{\mathrm{CG}} - x_{\mathrm{AC}})\sin\alpha_{\mathrm{W}} + \\ - L_H(x_{\mathrm{AC,H}} - x_{\mathrm{CG}})\cos\alpha_{\mathrm{H}} + D_H(x_{\mathrm{AC,H}} - x_{\mathrm{CG}})\sin\alpha_{\mathrm{H}} \end{aligned} \tag{2.40}$$

in which $\alpha_{\mathrm{H}} = \alpha_{\mathrm{W}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$, is the horizontal tail angle of attack. At small values for the angles of attack we can approximate $\cos\alpha_{\mathrm{W}} \approx 1$, $\cos\alpha_{\mathrm{H}} \approx 1$, $\sin\alpha_{\mathrm{W}} \approx \alpha_{\mathrm{W}}$ and $\sin\alpha_{\mathrm{H}} \approx \alpha_{\mathrm{H}}$. Therefore, in terms of coefficient and introducing the dimensionless longitudinal distances we have:

$$\begin{aligned} C_{\mathcal{M}} = C_{\mathcal{M}_{\mathrm{ac,WB}}} + (C_{L0_{\mathrm{WB}}} + C_{L\alpha,\mathrm{WB}}\alpha_{\mathrm{WB}})\left(\xi_{\mathrm{CG}} - \xi_{\mathrm{AC,WB}}\right) + \\ - C_{L\alpha,\mathrm{H}}\left[\left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha_{\mathrm{W}} + \tau_{\mathrm{e}}\delta_{\mathrm{e}} + i_{\mathrm{H}}\right]\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right) \end{aligned} \tag{2.41}$$

Isolating the terms which are functions of $(\alpha, \delta_{\mathrm{e}}, i_{\mathrm{H}})$ respectively leads to:

$$\begin{aligned} C_{\mathcal{M}0_{\mathrm{WB}}} &= C_{\mathcal{M}_{\mathrm{ac,WB}}} + C_{\mathcal{M}0,\mathrm{B}} + C_{L0_{\mathrm{WB}}}\left(\xi_{\mathrm{CG}} - \xi_{\mathrm{AC,WB}}\right) \\ C_{\mathcal{M}0} &= C_{\mathcal{M}0_{\mathrm{WB}}} + \eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\frac{\bar{c}_{\mathrm{H}}}{\bar{c}_{\mathrm{W}}}C_{\mathcal{M}_{\mathrm{ac,H}}} + \eta_{\mathrm{H}}C_{L\alpha,\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right)\varepsilon_0 \\ C_{\mathcal{M}_\alpha} &= C_{L\alpha,\mathrm{WB}}\left(\xi_{\mathrm{CG}} - \xi_{\mathrm{AC,WB}}\right) - C_{L\alpha,\mathrm{H}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right) \\ C_{\mathcal{M}_{i_\mathrm{H}}} &= -C_{L\alpha,\mathrm{H}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right) \\ C_{\mathcal{M}_{\delta_\mathrm{e}}} &= -C_{L\alpha,\mathrm{H}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right)\tau_{\mathrm{e}} \end{aligned}$$

2.6.4 Engines contribution

The presence of the engines provides contributions to both the translation both the rotation with respect to the stability axes. In the steady-state flight condition the engines trust is equal to the total drag: $T = D$. To obtain the trust coefficient we use the following relationship:

$$T_{\mathrm{C}} = \frac{T}{2\bar{q}{d_{eng}}^2}$$

The engine surface is evaluated by the relationship: $S_{eng} = {d_{eng}}^2\pi/4$. The stability derivative with respect to the angle of attack is evaluated by the following relationship:

$$C_{N_{\alpha,eng}} = N_{eng} C_{N_{\alpha',eng}} \left(1 + \varepsilon_{\alpha,eng}\right) \tag{2.42}$$

The pitching moment coefficient evaluated at the condition $[\alpha_0 = \delta_{e_0} = i_{H_0} = 0°]$ is given by the relationship:

$$C_{\mathcal{M}0_{eng}} = 2T_{\mathrm{C}} \frac{{d_{eng}}^2}{S_{\mathrm{W}}} \frac{Z_{eng}}{\bar{c}_{\mathrm{W}}} \tag{2.43}$$

where the term Z_{eng} is the vertical distance between the aircraft center of gravity and the engine axis. The stability derivative with respect to the angle of attack is given by:

$$C_{\mathcal{M}_{\alpha,eng}} = C_{N_{\alpha,eng}} \frac{S_{eng}}{S_{\mathrm{W}}} \frac{X_{eng}}{\bar{c}_{\mathrm{W}}} \tag{2.44}$$

where the term X_{eng} is the horizontal distance between the aircraft center of gravity and the engines.

2.6.5 Stick free Lift coefficient

This coefficient is evaluated doing the same considerations made in the stick fixed case. The difference consists in a coefficient, the free elevator factor, F, given by the equation 2.31.
The aircraft contribution in the condition $[\alpha_0 = \delta_{e_0} = i_{H_0} = 0°]$ is given by the following relationship:

$$C_{L0_{free}} = C_{L0_{\mathrm{WB}}} - \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} C_{L_{\alpha,\mathrm{H}}} \varepsilon_0 F \tag{2.45}$$

The stability derivative with respect to the angle of attack is evaluated by the following relationship:

$$C_{L_{\alpha,free}} = C_{L_{\alpha,\mathrm{WB}}} + C_{L_{\alpha,\mathrm{H}}} \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} \left(1 - \frac{d\varepsilon}{d\alpha}\right) F \tag{2.46}$$

The contribution of the stabilator is given by the following relationship:

$$C_{L_{i_{\mathrm{H},free}}} = \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} C_{L_{\alpha,\mathrm{H}}} F \tag{2.47}$$

The final expression of the coefficient $C_{L_{free}}$ is given by:

$$C_{L_{free}} = C_{L0_{free}} + C_{L_{\alpha,free}} \alpha_{free} + C_{L_{i_{\mathrm{H},free}}} i_{\mathrm{H}}$$

2.6.6 Stick free Drag coefficient

This coefficient is evaluated doing the same considerations made in the stick fixed case. The difference consists in a coefficient, the free elevator factor, F, given by the equation 2.31.
The stability derivative with respect to the angle of attack is evaluated by the following relationship:

$$C_{D_{\alpha,free}} = \frac{2 C_{L_{free}} C_{L_{\alpha,free}}}{\pi Æ_{\mathrm{W}} \varepsilon} \tag{2.48}$$

An alternative formula for the modeling of the overall $C_{D_{free}}$ coefficient is the well-known Prandtl relationship:

$$C_{D_{free}} = C_{D0} + \frac{C_{L_{free}}^2}{\pi Æ_{\mathrm{W}} \varepsilon} \tag{2.49}$$

2.6.7 Stick free Moment coefficient

This coefficient is evaluated doing the same considerations made in the stick fixed case. The difference consists in a coefficient, the free elevator factor, F, given by the equation 2.31.

The aircraft contribution in the condition $[\alpha_0 = \delta_{e_0} = i_{H_0} = 0°]$ is given by the following relationship:

$$C_{\mathcal{M}0_{free}} = C_{\mathcal{M}0_{\mathrm{WB}}} + \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} \frac{\bar{c}_{\mathrm{H}}}{\bar{c}_{\mathrm{W}}} C_{\mathcal{M}_{\mathrm{ac,H}}} + \eta_{\mathrm{H}} C_{L_{\alpha,\mathrm{H}}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} \left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right) \varepsilon_0 F \tag{2.50}$$

The pitching moment stability derivative with respect to the angle of attack is evaluated by:

$$C_{\mathcal{M}_{\alpha,free}} = C_{L_{\alpha,\mathrm{WB}}} \left(\xi_{\mathrm{CG}} - \xi_{\mathrm{AC,WB}}\right) - C_{L_{\alpha,\mathrm{H}}} \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} \left(1 - \frac{d\varepsilon}{d\alpha}\right) \left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right) F \tag{2.51}$$

The pitching moment stability derivative with respect to the stabilator deflection is given by:

$$C_{\mathcal{M}_{i_{\mathrm{H},free}}} = -C_{L_{\alpha,\mathrm{H}}} \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} \left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right) F \tag{2.52}$$

2.6.8 Unsteady-state coefficients

The next objective is the modeling of the small perturbation components of the lift and drag forces and the pitching moment. The origin of these components is associated with the perturbation terms in all the components of the linear and angular velocities. The perturbation in terms of the vertical linear velocity component can also be interpreted as a perturbation in the angle of attack.

Angle of attack rate effect

The contributions from the fuselage and the wing can be neglected, therefore for $C_{L_{\dot{\alpha}}}$ we have the approximation $C_{L_{\dot{\alpha}}} \approx C_{L_{\dot{\alpha},\mathrm{H}}}$. A closed-form expression for $C_{L_{\dot{\alpha},\mathrm{H}}}$ is given by:

$$C_{L_{\dot{\alpha},\mathrm{H}}} = 2 C_{L_{\alpha,\mathrm{H}}} \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} \left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right) \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha} \tag{2.53}$$

Also for the coefficient $C_{\mathcal{M}_{\dot{\alpha}}}$ the contributions from the fuselage and the wing can be neglected, so we have $C_{\mathcal{M}_{\dot{\alpha}}} \approx C_{\mathcal{M}_{\dot{\alpha},\mathrm{H}}}$. A closed-form relationship is given by:

$$C_{\mathcal{M}_{\dot{\alpha},\mathrm{H}}} = -C_{L_{\dot{\alpha},\mathrm{H}}} \left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right) = -2 C_{L_{\alpha,\mathrm{H}}} \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} \left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right)^2 \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha} \tag{2.54}$$

The drag coefficient $C_{D_{\dot{\alpha}}}$ is always negligible.

Pitch rate effect

For C_{L_q} we have that only the contribution from the fuselage is negligible. Therefore, we can consider $C_{L_q} \approx C_{L_{q,\mathrm{W}}} + C_{L_{q,\mathrm{H}}}$. A closed-form relationship for the evaluation of $C_{L_{q,\mathrm{W}}}$ is given by:

$$C_{L_{q,\mathrm{W}}} = \frac{Æ_{\mathrm{W}} + 2\cos\Lambda_{c/4}}{Æ_{\mathrm{W}} B + 2\cos\Lambda_{c/4}} C_{L_{q,\mathrm{W}}}\Big|_{M=0} \tag{2.55}$$

where

$$B = \sqrt{1 - M^2 \left(\cos\Lambda_{c/4}\right)^2}$$

$$C_{L_{q,\mathrm{W}}}\Big|_{M=0} = \left(\frac{1}{2} + 2\left|\xi_{\mathrm{AC,W}} - \xi_{\mathrm{CG}}\right|\right) C_{L_{\alpha,\mathrm{W}}}\Big|_{M=0}$$

The contribution from the horizontal tail is given by the following relationship:

$$C_{L_{q,\mathrm{H}}} = 2 C_{L_{\alpha,\mathrm{H}}} \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} \left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right) \tag{2.56}$$

Considering both the equations 2.55 and 2.56 we have for C_{L_q} the following expression:

$$C_{L_q} = \frac{\mathit{AR}_{\mathrm{W}} + 2\cos\Lambda_{c/4}}{\mathit{AR}_{\mathrm{W}}\sqrt{1 - M^2\left(\cos\Lambda_{c/4}\right)^2} + 2\cos\Lambda_{c/4}}\left(\frac{1}{2} + 2\left|\xi_{\mathrm{AC,W}} - \xi_{\mathrm{CG}}\right|\right) C_{L_{\alpha,\mathrm{W}}}\Big|_{M=0} + \\ + 2C_{L_{\alpha,\mathrm{H}}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right) \quad (2.57)$$

For the coefficient $C_{\mathcal{M}_q}$ the contribution from the fuselage can be typically negligible. Therefore, we have $C_{\mathcal{M}_q} \approx C_{\mathcal{M}_{q,\mathrm{W}}} + C_{\mathcal{M}_{q,\mathrm{H}}}$. A closed-form relationship for the wing contribution is given by:

$$C_{\mathcal{M}_{q,\mathrm{W}}} = \frac{\frac{\mathit{AR}_{\mathrm{W}}^3\tan^2\Lambda_{c/4}}{\mathit{AR}_{\mathrm{W}}+6\cos\Lambda_{c/4}} + \frac{3}{B}}{\frac{\mathit{AR}_{\mathrm{W}}^3\tan^2\Lambda_{c/4}}{\mathit{AR}_{\mathrm{W}}+6\cos\Lambda_{c/4}} + 3} C_{\mathcal{M}_{q,\mathrm{W}}}\Big|_{M=0} \quad (2.58)$$

where $B = \sqrt{1 - M^2\left(\cos\Lambda_{c/4}\right)^2}$.

The term $C_{\mathcal{M}_{q,\mathrm{W}}}\Big|_{M=0}$ is found using the following relationship:

$$C_{\mathcal{M}_{q,\mathrm{W}}}\Big|_{M=0} = -K_q C_{L_{\alpha,\mathrm{W}}}\Big|_{M=0}\cos\Lambda_{c/4}C$$

where the coefficient C is given by:

$$C = \frac{\mathit{AR}_{\mathrm{W}}\left(0.5\left|\xi_{\mathrm{AC,W}} - \xi_{\mathrm{CG}}\right| + 2\left|\xi_{\mathrm{AC,W}} - \xi_{\mathrm{CG}}\right|^2\right)}{\mathit{AR}_{\mathrm{W}} + 2\cos\Lambda_{c/4}} + \frac{1}{24}\frac{\mathit{AR}_{\mathrm{W}}^3\tan^2(\Lambda_{c/4})}{\mathit{AR}_{\mathrm{W}} + 6\cos\Lambda_{c/4}} + \frac{1}{8}$$

while the coefficient K_q is evaluated graphically (see [3], figure 3.12).
The contribution from the horizontal tail is given by the following closed-form relationship:

$$C_{\mathcal{M}_{q,\mathrm{H}}} = -2C_{L_{\alpha,\mathrm{H}}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right)^2 \quad (2.59)$$

Considering the equations 2.58 and 2.59, we have for $C_{\mathcal{M}_q}$ this final expression:

$$C_{\mathcal{M}_q} = \frac{\frac{\mathit{AR}_{\mathrm{W}}^3\tan^2\Lambda_{c/4}}{\mathit{AR}_{\mathrm{W}}+6\cos\Lambda_{c/4}} + \frac{3}{\sqrt{1-M^2\left(\cos\Lambda_{c/4}\right)^2}}}{\frac{\mathit{AR}_{\mathrm{W}}^3\tan^2\Lambda_{c/4}}{\mathit{AR}_{\mathrm{W}}+6\cos\Lambda_{c/4}} + 3}\left[-K_q C_{L_{\alpha,\mathrm{W}}}\Big|_{M=0}\cos\Lambda_{c/4}\right]\cdot \\ \cdot\frac{\mathit{AR}_{\mathrm{W}}\left(0.5\left|\xi_{\mathrm{AC,W}} - \xi_{\mathrm{CG}}\right| + 2\left|\xi_{\mathrm{AC,W}} - \xi_{\mathrm{CG}}\right|^2\right)}{\mathit{AR}_{\mathrm{W}} + 2\cos\Lambda_{c/4}} + \frac{1}{24}\frac{\mathit{AR}_{\mathrm{W}}^3\tan^2\Lambda_{c/4}}{\mathit{AR}_{\mathrm{W}} + 6\cos\Lambda_{c/4}} + \frac{1}{8} + \\ - 2C_{L_{\alpha,\mathrm{H}}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right)^2 \quad (2.60)$$

Also in this case the drag coefficient C_{D_q} is always negligible.

2.6.9 Neutral point and Static Stability Margin

The aircraft aerodynamic center is defined as the point with respect to which the pitching moment for the entire aircraft does not change with variations in the longitudinal angle of attack. Mathematically, this implies:

$$C_{\mathcal{M}_\alpha} = 0$$

Stick fixed

Therefore, setting to zero the previously introduced $C_{\mathcal{M}_\alpha}$ relationship:

$$C_{\mathcal{M}_\alpha} = 0 = C_{L_{\alpha,\mathrm{WB}}}\left(\xi_{\mathrm{CG}} - \xi_{\mathrm{AC,WB}}\right) - C_{L_{\alpha,\mathrm{H}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right)$$

leads to the following expression:

$$\xi_{\mathrm{N}} = \frac{\xi_{\mathrm{AC,WB}} + \frac{C_{L_{\alpha,\mathrm{H}}}}{C_{L_{\alpha,\mathrm{WB}}}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\xi_{\mathrm{AC,H}}}{1 + \frac{C_{L_{\alpha,\mathrm{H}}}}{C_{L_{\alpha,\mathrm{WB}}}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)} \tag{2.61}$$

The Static Stability Margin can be evaluated through two separate reports that converge to the same result:

$$SSM = \xi_{\mathrm{CG}} - \xi_{\mathrm{N}} \tag{2.62}$$

$$SSM = \frac{C_{\mathcal{M}_\alpha}}{C_{L_\alpha}} \tag{2.63}$$

Stick free

Setting to zero the previously introduced $C_{\mathcal{M}_\alpha}$ relationship:

$$C_{\mathcal{M}_{\alpha,free}} = 0 = C_{L_{\alpha,\mathrm{WB}}}\left(\xi_{\mathrm{CG}} - \xi_{\mathrm{AC,WB}}\right) - C_{L_{\alpha,\mathrm{H}}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(\xi_{\mathrm{AC,H}} - \xi_{\mathrm{CG}}\right)F$$

leads to the following expression:

$$\xi_{\mathrm{N}_{free}} = \frac{\xi_{\mathrm{AC,WB}} + \frac{C_{L_{\alpha,\mathrm{H}}}}{C_{L_{\alpha,\mathrm{WB}}}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\xi_{\mathrm{AC,H}}F}{1 + \frac{C_{L_{\alpha,\mathrm{H}}}}{C_{L_{\alpha,\mathrm{WB}}}}\eta_{\mathrm{H}}\frac{S_{\mathrm{H}}}{S_{\mathrm{W}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)F} \tag{2.64}$$

The Static Stability Margin can be evaluated by the following relationship:

$$SSM_{free} = \xi_{\mathrm{CG}} - \xi_{\mathrm{N}_{free}} \tag{2.65}$$

Engines contribution

Whereas also the engines contribution, the position of the neutral point has changed compared to the previous case. In particular:

$$\Delta\xi_{\mathrm{N}_{eng}} = \frac{C_{\mathcal{M}_{\alpha,eng}}}{C_{L_\alpha}}$$

Therefore, we have:

$$\xi_{\mathrm{N}_{eng}} = \xi_{\mathrm{N}} + \Delta\xi_{\mathrm{N}_{eng}} \tag{2.66}$$

The Static Stability Margin in this case is equal to:

$$SSM_{eng} = \xi_{\mathrm{CG}} - \xi_{\mathrm{N}_{eng}} \tag{2.67}$$

2.6.10 Equilibrium system

To determine the equilibrium angles α_{WB} and δ_{e} we impose the equilibrium conditions to the vertical translation and the equilibrium to the rotation around the Y_S axis. These two equations can be write in terms of matrix and vectors. In particular, we have:

$$\begin{bmatrix} C_{L_\alpha} + C_{N_{\alpha,eng}} & C_{L_{\delta_\mathrm{e}}} \\ C_{\mathcal{M}_\alpha} + C_{\mathcal{M}_{\alpha,eng}} & C_{\mathcal{M}_{\delta_\mathrm{e}}} \end{bmatrix}\begin{bmatrix} \alpha_{\mathrm{WB}} \\ \delta_{\mathrm{e}} \end{bmatrix} = \begin{bmatrix} C_L - (C_{L0} + C_{L_{i_\mathrm{H}}} i_{\mathrm{H}}) \\ C_{\mathcal{M}} - (C_{\mathcal{M}0} + C_{\mathcal{M}0_{eng}} + C_{\mathcal{M}_{i_\mathrm{H}}} i_{\mathrm{H}}) \end{bmatrix}$$

2.7 Lateral-Directional equilibrium calculations

To completely define the lateral-directional equilibrium we have to consider both the steady-stady both the unsteady-state. The unsteady-state is caused by the perturbation terms in all the components of the linear and angular velocities.

2.7.1 Rolling moment coefficient

The steady-state rolling moment can be evaluated by the relationship:

$$\mathcal{L} = C_{\mathcal{L}} \bar{q} S_{\mathrm{W}} b_{\mathrm{W}} \tag{2.68}$$

where the rolling moment coefficient is expressed by

$$C_{\mathcal{L}} = f(\beta, \delta_A, \delta_R)$$

The first order approximation for the Taylor expansion gives the following expression for $C_{\mathcal{L}}$:

$$C_{\mathcal{L}} = C_{\mathcal{L}_0} + C_{\mathcal{L}_\beta}\beta + C_{\mathcal{L}_{\delta_a}}\delta_a + C_{\mathcal{L}_{\delta_r}}\delta_r$$

in which the term $C_{\mathcal{L}_0}$ is zero if the aircraft is symmetric with respect to the *XZ* plane.

Dihedral effect

The dihedral effect can be quantified by the coefficient $C_{\mathcal{L}_\beta}$. The starting relationship for $C_{\mathcal{L}_\beta}$ is:

$$C_{\mathcal{L}_\beta} = C_{\mathcal{L}_{\beta,\mathrm{WB}}} + C_{\mathcal{L}_{\beta,\mathrm{H}}} + C_{\mathcal{L}_{\beta,\mathrm{V}}} \tag{2.69}$$

In the equation 2.69 there are the contributions of the wing-body configuration, of the horizontal tail and of the vertical tail. In turn, the contributions of the wing-body configuration and the horizontal tail consist of three terms: the first one is due to the wing geometric dihedral angle; the second one is due to an aerodynamic phenomenon asscociated with high-wing aircraft, in particular it is associated with the sideslip angle; the third one is due to the geometric wing sweep angle.
A closed-form expression for the modeling $C_{\mathcal{L}_{\beta,\mathrm{WB}}}$ is given by:

$$\begin{aligned} C_{\mathcal{L}_{\beta,\mathrm{WB}}} = 57.3 C_L &\left[\left(\frac{C_{\mathcal{L}_\beta}}{C_L}\right)_{\Lambda_{c/2}} K_{M_\Lambda} K_f + \left(\frac{C_{\mathcal{L}_\beta}}{C_L}\right)_{AR}\right] + \\ &+ 57.3\left\{\Gamma_W\left[\frac{C_{\mathcal{L}_\beta}}{\Gamma_W} K_{M_\Gamma} + \frac{\Delta C_{\mathcal{L}_\beta}}{\Gamma_W}\right] + \left(\Delta C_{\mathcal{L}_\beta}\right)_{Z_W} + \varepsilon_W \tan\Lambda_{c/4}\left(\frac{\Delta C_{\mathcal{L}_\beta}}{\varepsilon_W \tan\Lambda_{c/4}}\right)\right\} \end{aligned} \tag{2.70}$$

In the formula 2.70 there are some semi-empirical coefficients:

(i) $(C_{\mathcal{L}_\beta}/C_L)_{\Lambda_{c/2}}$ is the contribution associated with the wing sweep angle;

(ii) K_{M_Λ} is a correction factor associated with the Mach number and the wing sweep angle;

(iii) K_f is a correction factor associated with the length of the forward portion of the fuselage;

(iv) $(C_{\mathcal{L}_\beta}/C_L)_{AR}$ is the contribution associated with the wing aspect ratio;

(v) $C_{\mathcal{L}_\beta}/\Gamma_W$ is the contribution associated with the wing dihedral angle;

(vi) K_{M_Γ} is a correction factor associated with the Mach number and the wing dihedral angle;

(vii) $\Delta C_{\mathcal{L}_\beta}/\Gamma_W$ is a correction factor associated with the size of the fuselage;

(viii) $(\Delta C_{\mathcal{L}_\beta})_{Z_W}$ is a correction factor associated with the location of the fuselage with respect to the wing;

(ix) $\Delta C_{\mathcal{L}_\beta}/(\varepsilon_W \tan\Lambda_{c/4})$ is a correction factor associated with the twist angle ε between the zero-lift lines of the wing sections at the tip and at the root stations.

The terms (i), (ii), (iii), (iv), (v), (vi) and (ix) are evaluated graphically (see [3], figures 4.39, 4.40, 4.41, 4.42, 4.43, 4.44, 4.46, respectively).
The term (vii) is modeled using the relationship:

$$\frac{\Delta C_{\mathcal{L}_\beta}}{\Gamma_W} = -0.0005 \text{Æ}_{\mathrm{W}} \left(\frac{d_{\mathrm{B}}}{b_{\mathrm{W}}}\right)^2$$

The factor (viii) is modeled using the relationship:

$$(\Delta C_{\mathcal{L}_\beta})_{Z_{\mathrm{W}}} = \frac{1.2\sqrt{\text{Æ}_{\mathrm{W}}}}{57.3} \frac{Z_W}{b_{\mathrm{W}}} \left(\frac{2d_{\mathrm{B}}}{b_{\mathrm{W}}}\right)$$

The horizontal tail can be considered as a wing operating at a lower dynamic pressure, with a smaller surface and a smaller wing span. Therefore, a relationship for $C_{\mathcal{L}_{\beta,\mathrm{H}}}$ is given by:

$$C_{\mathcal{L}_{\beta,\mathrm{H}}} = C_{\mathcal{L}_{\beta,\mathrm{WB}}}\Big|_H \eta_{\mathrm{H}} \frac{S_{\mathrm{H}}}{S_{\mathrm{W}}} \frac{b_{\mathrm{H}}}{b_{\mathrm{W}}} \tag{2.71}$$

where the $C_{\mathcal{L}_{\beta,\mathrm{WB}}}|_H$ is the previously introduced $C_{\mathcal{L}_{\beta,\mathrm{WB}}}$ evaluated with the geometric parameters of the horizontal tail.

The vertical tail contribution to the dihedral effect is expressed in the following formula:

$$C_{\mathcal{L}_{\beta,\mathrm{V}}} = -k_{Y_V} |C_{L_{\alpha,\mathrm{V}}}| \eta_{\mathrm{V}} \left(1 + \frac{d\sigma}{d\beta}\right) \frac{S_{\mathrm{V}}}{S_{\mathrm{W}}} \frac{Z_V \cos\alpha_{\mathrm{W}} - X_V \sin\alpha_{\mathrm{W}}}{b_{\mathrm{W}}} \tag{2.72}$$

where k_{Y_V} is an empirical factor evaluated graphically (see [3], figure 4.13).

Ailerons deflection effect

The ailerons are an asymmetric control surface. A positive deflection of the ailerons implies a trailing edge down deflection of the left aileron and a trailing edge up deflection of the right aileron. The combined result of these deflections is a positive rolling moment. The mathematical expression of $C_{\mathcal{L}_{\delta_a}}$ is:

$$C_{\mathcal{L}_{\delta_a}} = C'_{\mathcal{L}_{\delta_a}} \tau_{\mathrm{a}} \tag{2.73}$$

where $C'_{\mathcal{L}_{\delta_a}}$ is given by the following relationship:

$$C'_{\mathcal{L}_{\delta_a}} = -\frac{\Delta RME\, k_M}{\sqrt{1-M^2}}$$

in which ΔRME is a factor evaluated graphically (see [3], figures from 4.51 to 4.54) and in this parameter it is made the assumption that the entire wing section between the aileron inner and outer stations is deflected. The term τ_{a} is a control surface effectiveness factor, evaluated graphically (see [3], figure 4.55). The term k_M is equal to:

$$k_M = \frac{C_{L_{\alpha,\mathrm{W}}}\sqrt{1-M^2}}{2\pi}$$

Rudder deflection effect

The rudder is a control surface. Its contribution to the rolling moment originates from the lateral force associated with the deflection of the rudder through its moment arm with respect to the aircraft center of gravity. A mathematical relationship for this moment coefficient is:

$$C_{\mathcal{L}_{\delta_r}} = |C_{L_{\alpha,\mathrm{V}}}| \eta_{\mathrm{V}} \frac{S_{\mathrm{V}}}{S_{\mathrm{W}}} K_r \tau_{\mathrm{r}} \frac{Z_r \cos\alpha_{\mathrm{W}} - X_r \sin\alpha_{\mathrm{W}}}{b_{\mathrm{W}}} \tag{2.74}$$

In this relationship the term $|C_{L_{\alpha,\mathrm{V}}}|$ is the lift-curve slope for the vertical tail, τ_{r} is a control surface effectiveness factor (see [3], figure 4.26) and K_r is a correction factor associated with the span of the rudder within the span of the vertical tail, evaluated graphically (see [3], figure 4.27).

2.7.2 Yawing moment coefficient

The steady-state yawing moment can be evaluated by the relationship:

$$\mathcal{N} = C_{\mathcal{N}} \bar{q} S_{\mathrm{W}} b_{\mathrm{W}} \tag{2.75}$$

where the rolling moment coefficient is expressed by

$$C_{\mathcal{N}} = f(\beta, \delta_A, \delta_R)$$

The first order approximation for the Taylor expansion gives the following expression for $C_{\mathcal{N}}$:

$$C_{\mathcal{N}} = C_{\mathcal{N}_0} + C_{\mathcal{N}_\beta} \beta + C_{\mathcal{N}_{\delta_a}} \delta_a + C_{\mathcal{N}_{\delta_r}} \delta_r$$

in which the term $C_{\mathcal{N}_0}$ is zero if the aircraft is symmetric with respect to the XZ plane.

Weathercock effect

The aerodynamic coefficient $C_{\mathcal{N}_\beta}$ is known as weathercock effect. The starting relationship for its evaluation is:

$$C_{\mathcal{N}_\beta} = C_{\mathcal{N}_{\beta,\mathrm{W}}} + C_{\mathcal{N}_{\beta,\mathrm{B}}} + C_{\mathcal{N}_{\beta,\mathrm{H}}} + C_{\mathcal{N}_{\beta,\mathrm{V}}} + C_{\mathcal{N}_{\beta,eng}} \tag{2.76}$$

The contribution from the wing and the horizontal tail are negligible for all configurations. The fuselage contribution is evaluated using the relationship:

$$C_{\mathcal{N}_{\beta,\mathrm{B}}} = -57.3 K_N K_{Re_B} \frac{S_{\mathrm{B,side}}}{S_{\mathrm{W}}} \frac{l_{\mathrm{B}}}{b_{\mathrm{W}}} \tag{2.77}$$

where the coefficient K_N is an empirical factor, estimated graphically (see [3], figure 4.68), related to the geometric coefficients of the axial cross section of the fuselage and the coefficient K_{Re_B}, evaluated graphically (see [3], figure 4.69), is related to the Reynolds number.
The most significant contribution to $C_{\mathcal{N}_\beta}$ is provided by the vertical tail. This contribution is evaluated by:

$$C_{\mathcal{N}_{\beta,\mathrm{V}}} = k_{Y_V} |C_{L_{\alpha,\mathrm{V}}}| \eta_{\mathrm{V}} \left(1 + \frac{d\sigma}{d\beta}\right) \frac{S_{\mathrm{V}}}{S_{\mathrm{W}}} \frac{X_V \cos\alpha_{\mathrm{W}} + Z_V \sin\alpha_{\mathrm{W}}}{b_{\mathrm{W}}} \tag{2.78}$$

Ailerons deflection effect

The asymmetric deflection of the left and right ailerons also generate small but not negligible drag force leading to a small negative yawing moment. A relationship for modeling $C_{\mathcal{N}_{\delta_a}}$ is given by:

$$C_{\mathcal{N}_{\delta_a}} = \Delta(K_{n_a}) C_L C_{\mathcal{L}_{\delta_a}} \tag{2.79}$$

where C_L is the aircraft lift coefficient, the term $C_{\mathcal{L}_{\delta_a}}$ is the rolling moment coefficient due to ailerons deflection and K_{n_a} is an empirical factor, estimated graphically (see [3], figures 4.72 and 4.73).

Rudder deflection effect

This coefficient is related to the contribution given by the rudder deflection. The mathematical expression is:

$$C_{\mathcal{N}_{\delta_r}} = -|C_{L_{\alpha,\mathrm{V}}}| \eta_{\mathrm{V}} \frac{S_{\mathrm{V}}}{S_{\mathrm{W}}} K_r \tau_{\mathrm{r}} \frac{X_r \cos\alpha_{\mathrm{W}} + Z_r \sin\alpha_{\mathrm{W}}}{b_{\mathrm{W}}} \tag{2.80}$$

Engines contribution

The presence of the engines provides contribution to the translation around the Z_S axis. The engine surface is evaluated by the relationship: $S_{eng} = {d_{eng}}^2 \pi/4$. The yawing moment coefficient due to the sideslip angle is given by the following relationship:

$$C_{\mathcal{N}_{\beta,eng}} = -C_{\mathcal{N}_{\alpha,eng}} \frac{S_{eng}}{S_{\mathrm{W}}} \frac{X_{eng}}{\bar{c}_{\mathrm{W}}} \tag{2.81}$$

where the meaning of these terms is explained in the section 2.6.4 on page 25.

2.7.3 Unsteady-state rolling moment coefficient

Roll rate effect

The coefficient $C_{\mathcal{L}_p}$ models the contribution of the rolling moment coefficient due to roll rate. The mathematical relationship is given by:

$$C_{\mathcal{L}_p} = C_{\mathcal{L}_{p,\mathrm{WB}}} + C_{\mathcal{L}_{p,\mathrm{H}}} + C_{\mathcal{L}_{p,\mathrm{V}}} \tag{2.82}$$

The contribution from the fuselage is negligible, so $C_{\mathcal{L}_{p,\mathrm{WB}}} \approx C_{\mathcal{L}_{p,\mathrm{W}}}$, and the relationship for this coefficient is given by:

$$C_{\mathcal{L}_{p,\mathrm{W}}} = RDP\frac{k}{\beta} \tag{2.83}$$

where $\beta = \sqrt{1-M^2}$ and RDP is the rolling damping parameter, evaluated graphically (see [3], figures 4.80 and 4.81). The coefficient $C_{\mathcal{L}_{p,\mathrm{H}}}$ is evaluated using the relationship 2.83, in which we put the horizontal tail geometric parameters. This coefficient is often negligible.The contribution from the vertical tail is:

$$C_{\mathcal{L}_{p,\mathrm{V}}} = 2|C_{L\alpha,\mathrm{v}}|\eta_{\mathrm{V}}\left(1+\frac{d\sigma}{d\beta}\right)\frac{S_{\mathrm{V}}}{S_{\mathrm{W}}}\left(\frac{Z_V}{b_{\mathrm{W}}}\right)^2 \tag{2.84}$$

Yaw rate effect

The contribution to the rolling moment due to the yaw rate is contained in the coefficient $C_{\mathcal{L}_r}$. The modeling for this coefficient starts from the following relationship:

$$C_{\mathcal{L}_r} = C_{\mathcal{L}_{r,\mathrm{WB}}} + C_{\mathcal{L}_{r,\mathrm{H}}} + C_{\mathcal{L}_{r,\mathrm{V}}} \tag{2.85}$$

The fuselage and the horizontal tail do not significantly contribute to this coefficient, so we have $C_{\mathcal{L}_r} \approx C_{\mathcal{L}_{r,\mathrm{W}}} + C_{\mathcal{L}_{r,\mathrm{V}}}$. The wing contribution is given by:

$$C_{\mathcal{L}_{r,\mathrm{W}}} = \left(\frac{C_{\mathcal{L}_r}}{C_L}\right)\Bigg|_{M,C_L=0} C_L + \left(\frac{\Delta C_{\mathcal{L}_r}}{\Gamma_W}\right)\Gamma_W + \left(\frac{\Delta C_{\mathcal{L}_r}}{\varepsilon}\right)\varepsilon \tag{2.86}$$

The coefficient $\left(C_{\mathcal{L}_r}/C_L\right)\big|_{M,C_L=0}$ is given by:

$$\left(\frac{C_{\mathcal{L}_r}}{C_L}\right)\Bigg|_{M,C_L=0} = D\left(\frac{C_{\mathcal{L}_r}}{C_L}\right)\Bigg|_{M=0,C_L=0}$$

where

$$D = \frac{1+\frac{\mathit{Æ}_{\mathrm{W}}(1-B^2)}{2B[\mathit{Æ}_{\mathrm{W}}\cdot B+2\cos\Lambda_{c/4}]}+\frac{\mathit{Æ}_{\mathrm{W}}B+2\cos\Lambda_{c/4}}{\mathit{Æ}_{\mathrm{W}}B+4\cos\Lambda_{c/4}}\frac{\tan^2\Lambda_{c/4}}{8}}{1+\frac{\mathit{Æ}_{\mathrm{W}}+2\cos\Lambda_{c/4}}{\mathit{Æ}_{\mathrm{W}}+4\cos\Lambda_{c/4}}\frac{\tan^2\Lambda_{c/4}}{8}}$$

in which

$$B = \sqrt{1-M^2\cos^2\Lambda_{c/4}}$$

and $\left(C_{\mathcal{L}_r}/C_L\right)\big|_{M=0,C_L=0}$ is evaluated graphically (see [3], figure 4.85) using wing geometric parameters. In the equation 2.86 the term $\Delta C_{\mathcal{L}_r}/\Gamma_W$ is a factor due to the wing dihedral angle; the term $\Delta C_{\mathcal{L}_r}/\varepsilon$ is a factor due to the wing twist angle. The first factor is modeled using the relationship:

$$\frac{\Delta C_{\mathcal{L}_r}}{\Gamma_W} = \frac{1}{12}\frac{\pi\mathit{Æ}_{\mathrm{W}}\sin\Lambda_{c/4}}{\mathit{Æ}_{\mathrm{W}}+4\cos\Lambda_{c/4}}$$

the second one is evaluated graphically (see [3], figure 4.87). The vertical tail contribution into the equation 2.85 is given by:

$$C_{\mathcal{L}_{r,\mathrm{V}}} = -2|C_{L\alpha,\mathrm{v}}|\eta_{\mathrm{V}}\left(1+\frac{d\sigma}{d\beta}\right)\frac{S_{\mathrm{V}}}{S_{\mathrm{W}}}\frac{X_V\cos\alpha_{\mathrm{W}}+Z_V\sin\alpha_{\mathrm{W}}}{b_{\mathrm{W}}}\frac{Z_V\cos\alpha_{\mathrm{W}}-X_V\sin\alpha_{\mathrm{W}}}{b_{\mathrm{W}}} \tag{2.87}$$

2.7.4 Unsteady-state yawing moment coefficient

Roll rate effect

The coefficient $C_{\mathcal{N}_p}$ models the contribution to the yawing moment due to the roll rate. The relationship for this coefficient is:

$$C_{\mathcal{N}_p} = C_{\mathcal{N}_{p,\mathrm{WB}}} + C_{\mathcal{N}_{p,\mathrm{H}}} + C_{\mathcal{N}_{p,\mathrm{V}}} \tag{2.88}$$

Because the fuselage and the horizontal tail do not significantly contribute to this coefficient, we have that $C_{\mathcal{N}_p} \approx C_{\mathcal{N}_{p,\mathrm{W}}} + C_{\mathcal{N}_{p,\mathrm{V}}}$. A relationship for the wing contribution is given by the following relationship:

$$C_{\mathcal{N}_p} = -C_{\mathcal{L}_{p,\mathrm{W}}} \tan(\alpha_\mathrm{W}) + C_{\mathcal{L}_p} \tan(\alpha_\mathrm{W}) + \left(\frac{C_{\mathcal{N}_p}}{C_L}\right)\Bigg|_{M,C_L=0} C_L + \left(\frac{\Delta C_{\mathcal{N}_p}}{\varepsilon}\right)\varepsilon \tag{2.89}$$

where $C_{\mathcal{L}_{p,\mathrm{W}}}$ and $C_{\mathcal{L}_p}$ are described in 2.83 and 2.84, respectively. The coefficient $\left(C_{\mathcal{N}_p}/C_L\right)\big|_{M,C_L=0}$ is given by:

$$\left(\frac{C_{\mathcal{N}_p}}{C_L}\right)\Bigg|_{M,C_L=0} = C\left(\frac{C_{\mathcal{N}_p}}{C_L}\right)\Bigg|_{M=0,C_L=0}$$

where

$$C = \frac{AR_\mathrm{W} + 4\cos\Lambda_{c/4}}{AR_\mathrm{W} B + 4\cos\Lambda_{c/4}} \, \frac{AR_\mathrm{W} B + \frac{1}{2}[AR_\mathrm{W} B + 4\cos\Lambda_{c/4}]\tan^2\Lambda_{c/4}}{AR_\mathrm{W} + \frac{1}{2}[AR_\mathrm{W} + 4\cos\Lambda_{c/4}]\tan^2\Lambda_{c/4}}$$

in which

$$B = -\frac{1}{6}\,\frac{AR_\mathrm{W} + 6(AR_\mathrm{W} + \cos\Lambda_{c/4})\left[(\xi_\mathrm{CG} - \xi_\mathrm{AC})\frac{\tan\Lambda_{c/4}}{AR_\mathrm{W}} + \frac{\tan^2\Lambda_{c/4}}{12}\right]}{AR_\mathrm{W} + \cos\Lambda_{c/4}}$$

The term $\Delta C_{\mathcal{N}_p}/\varepsilon$ is associated with the wing twist angle and is evaluated graphically (see [3], figure 4.83). The vertical tail contribution into the equation 2.88 is given by:

$$C_{\mathcal{L}_{r,\mathrm{V}}} = -2|C_{L_{\alpha,\mathrm{V}}}|\eta_\mathrm{V}\left(1 + \frac{\mathrm{d}\sigma}{\mathrm{d}\beta}\right)\frac{S_\mathrm{V}}{S_\mathrm{W}}\,\frac{X_V\cos\alpha_\mathrm{W} + Z_V\sin\alpha_\mathrm{W}}{b_\mathrm{W}}\,\frac{Z_V\cos\alpha_\mathrm{W} - X_V\sin\alpha_\mathrm{W} - Z_V}{b_\mathrm{W}} \tag{2.90}$$

Yaw rate effect

The coefficient $C_{\mathcal{N}_r}$ models the contribution to the yawing moment due to the yaw rate. A relationship for this coefficient is:

$$C_{\mathcal{N}_r} = C_{\mathcal{N}_{r,\mathrm{WB}}} + C_{\mathcal{N}_{r,\mathrm{H}}} + C_{\mathcal{N}_{r,\mathrm{H}}} \tag{2.91}$$

The fuselage and the horizontal tail do not significantly contribute to this coefficient; therefore, we have that $C_{\mathcal{N}_r} \approx C_{\mathcal{N}_{r,\mathrm{W}}} + C_{\mathcal{N}_{r,\mathrm{V}}}$. The wing contribution is evaluated by the relationship:

$$C_{\mathcal{N}_{r,\mathrm{W}}} = \left(\frac{C_{\mathcal{N}_r}}{C_L^2}\right)C_L^2 + \left(\frac{C_{\mathcal{N}_r}}{C_{D_0}}\right)C_{D_0} \tag{2.92}$$

in which the terms $C_{\mathcal{N}_r}/C_L^2$ and $C_{\mathcal{N}_r}/C_{D_0}$ are evaluated graphically (see [3], figures 4.89 and 4.90, respectively). The vertical tail contribution is given by:

$$C_{\mathcal{L}_{r,\mathrm{V}}} = -2|C_{L_{\alpha,\mathrm{V}}}|\eta_\mathrm{V}\left(1 + \frac{\mathrm{d}\sigma}{\mathrm{d}\beta}\right)\frac{S_\mathrm{V}}{S_\mathrm{W}}\,\frac{(X_V\cos\alpha_\mathrm{W} + Z_V\sin\alpha_\mathrm{W})^2}{{b_\mathrm{W}}^2} \tag{2.93}$$

2.7.5 Equilibrium system

To determine the equilibrium angles δ_a and δ_r we impose the equilibrium conditions to the rotations around the X_S and Z_S axes. These two equations can be write in terms of matrix and vectors. In particular, we have:

$$\begin{bmatrix} C_{\mathcal{L}_{\delta_a}} & C_{\mathcal{L}_{\delta_r}} \\ C_{\mathcal{N}_{\delta_a}} & C_{\mathcal{N}_{\delta_r}} \end{bmatrix}\begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} C_\mathcal{L} - (C_{\mathcal{L}_0} + C_{\mathcal{L}_\beta}\beta + C_{\mathcal{L}_p}p + C_{\mathcal{L}_r}r) \\ C_\mathcal{N} - (C_{\mathcal{N}_0} + C_{\mathcal{N}_\beta}\beta + C_{\mathcal{N}_p}p + C_{\mathcal{N}_r}r) \end{bmatrix}$$

Chapter 3

EXAMPLE OF APPLICATION ON DOUGLAS DC-9-10

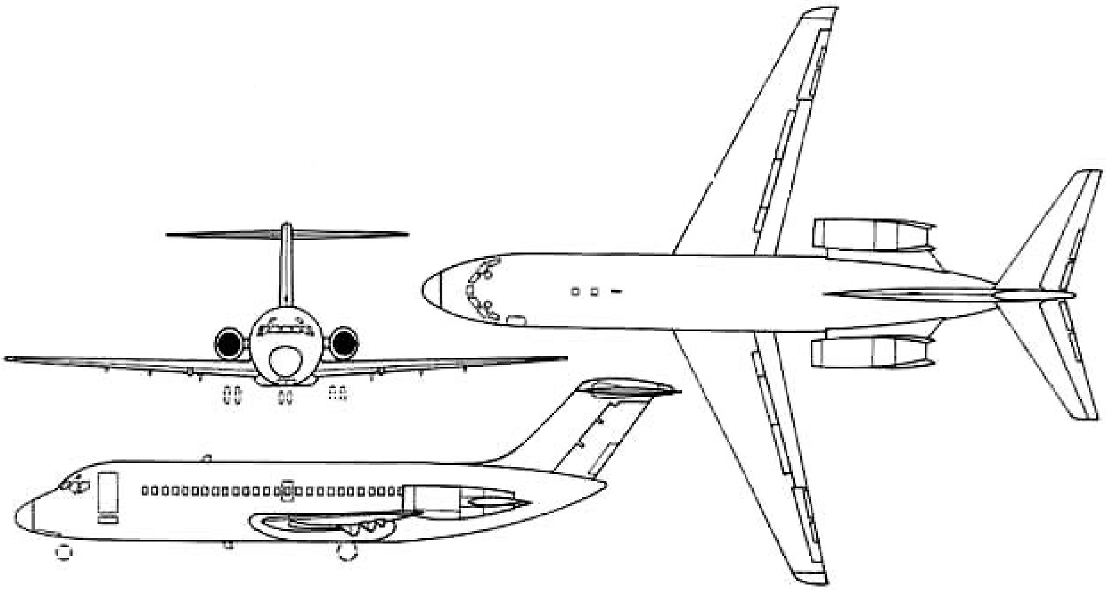

Figure 3.1: Three views of Douglas DC-9-10

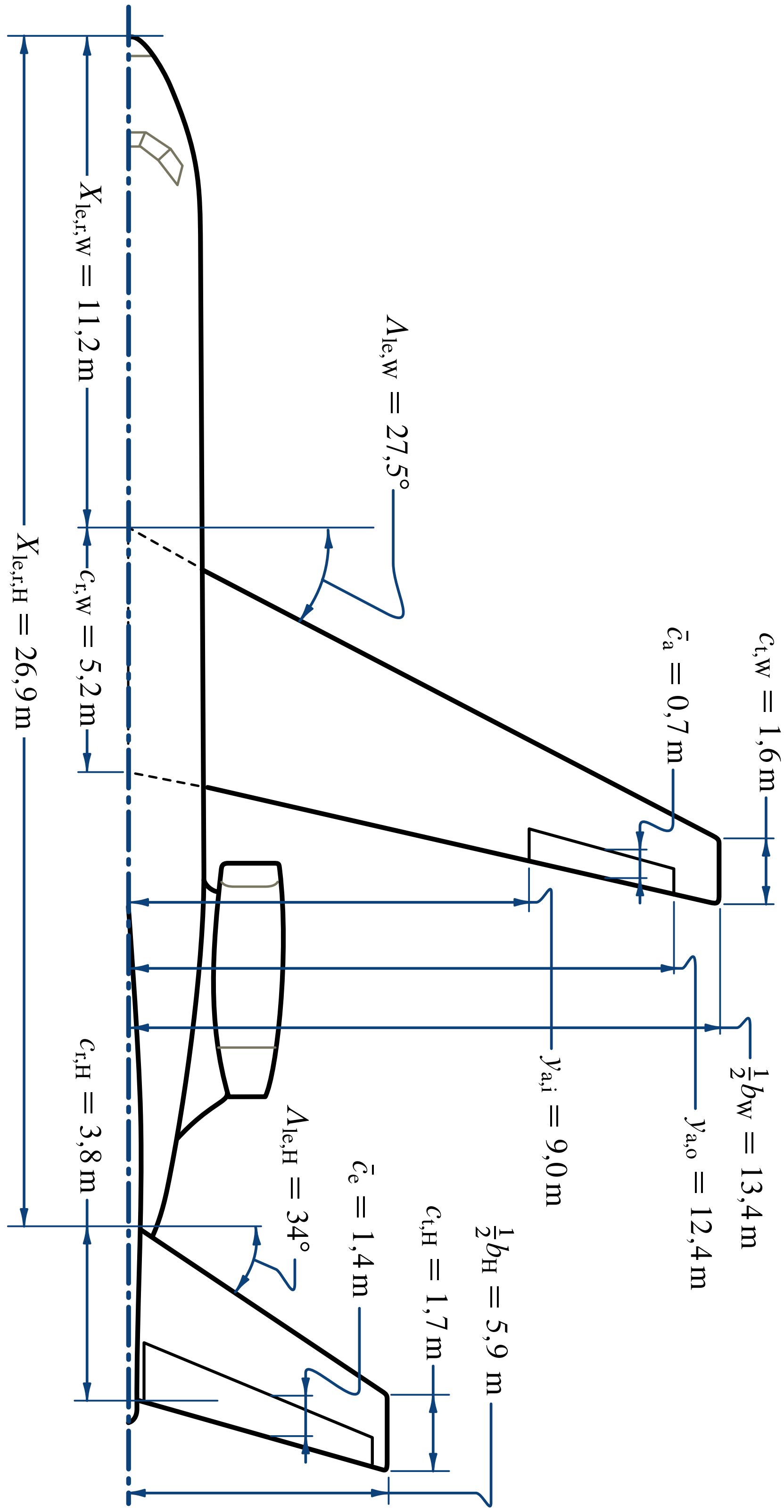

Figure 3.2: Top View of Douglas DC-9-10

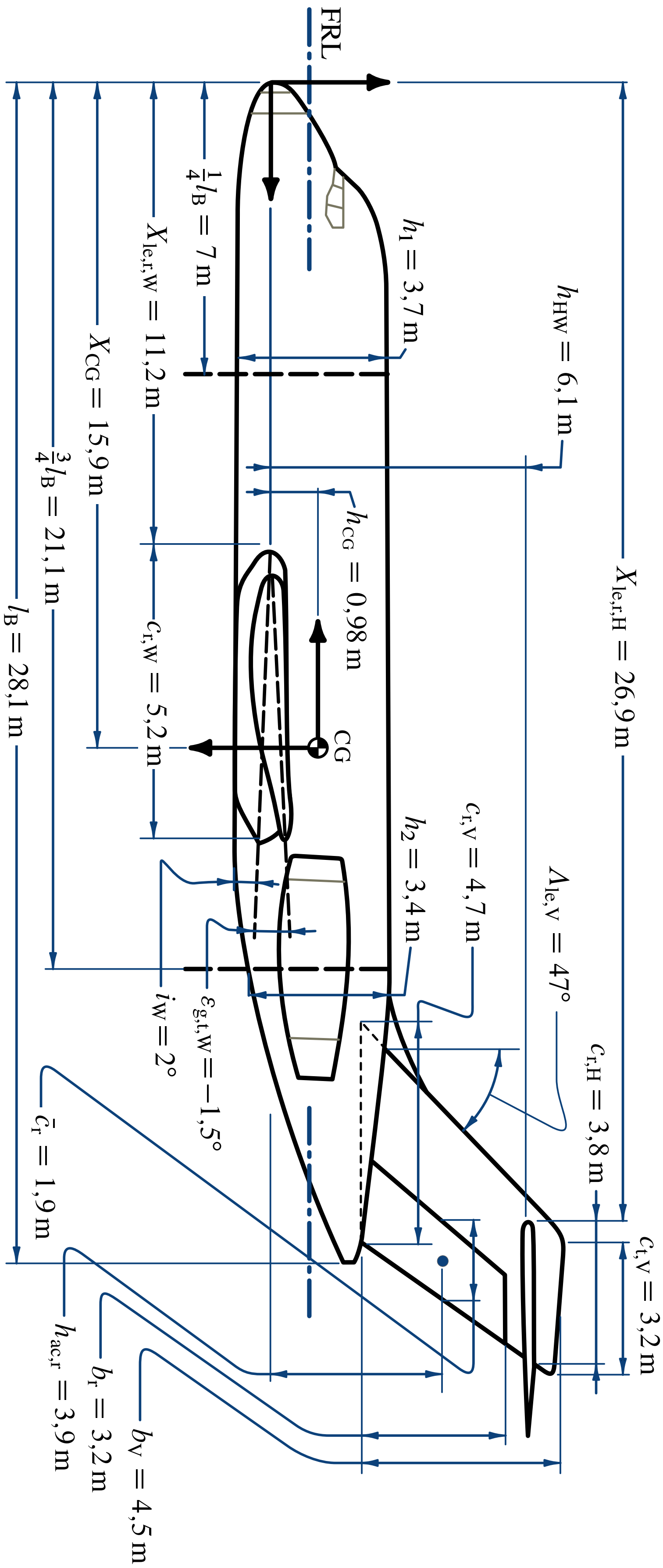

Figure 3.3: Side View of Douglas DC-9-10

Table 3.1: Notable calculated data for Douglas DC-9-10

VARIABLE	SI UNITS	USCS UNITS	DESCRIPTION
h_{ASL}	10 000 m	32 808.4 ft	Altitude
M_∞	0.696	—	Mach number
q_∞	8961.2 Pa	0.0896 bar	Dynamic pressure
$\bar{c}_{\mathrm{W}}$	3.642 m	11.947 ft	Wing m.a.c.
S_{W}	87.62 m^2	943.17 ft^2	Wing area
λ_{W}	0.227	—	Wing taper ratio
$Æ_{\mathrm{W}}$	8.474	—	Wing aspect ratio
e_{W}	0.918	—	Wing Oswald factor
$\alpha_{0\ell_{\mathrm{W}}}$	−0.0333 rad	−1.91 deg	Wing zero-lift angle of attack
$C_{L\alpha,\mathrm{W}}$	6.227 rad^{-1}	0.1087 deg^{-1}	Wing lift coefficient slope
$C_{\mathcal{M}_{\mathrm{ac,W}}}$	−0.057	—	Wing pitching moment coeff.
$S_{\mathrm{B,side}}$	86.48 m^2	930.86 ft^2	Fuselage lateral area
FFR	10.109	—	Fuselage fineness ratio
$C_{\mathcal{M}0,\mathrm{B}}$	−0.10091	—	Fus. zero-lift pitching moment coeff.
$C_{\mathcal{M}\alpha,\mathrm{B}}$	1.155 54 rad^{-1}	0.02 deg^{-1}	Fus. pitching moment coeff. slope
$\bar{c}_{\mathrm{H}}$	2.433 m	7.982 ft	Hor. tail m.a.c.
S_{H}	25.47 m^2	274.16 ft^2	Hor. tail area
λ_{H}	0.367	—	Hor. tail taper ratio
$Æ_{\mathrm{H}}$	4.94	—	Hor. tail aspect ratio
e_{H}	0.965	—	Hor. tail Oswald factor
$\alpha_{0L,\mathrm{H}}$	0 rad	0 deg	Hor. tail zero-lift angle of attack
$C_{L\alpha,\mathrm{H}}$	4.634 rad^{-1}	0.0809 deg^{-1}	Hor. tail lift coefficient slope
$C_{\mathcal{M}_{\mathrm{ac,H}}}$	−0.07	—	Hor. tail pitching moment coeff.
$\bar{c}_{\mathrm{V}}$	4.334 m	14.218 ft	Ver. tail m.a.c.
S_{V}	20.54 m^2	221.11 ft^2	Ver. tail area
λ_{V}	0.613	—	Ver. tail taper ratio
$Æ_{\mathrm{V}}$	2.272	—	Ver. tail aspect ratio
e_{V}	−0.005	—	Ver. tail Oswald factor
$C_{L\alpha,\mathrm{V}}$	5.618 rad^{-1}	0.098 deg^{-1}	Ver. tail lift coefficient slope
$C_{\mathcal{M}_{\mathrm{ac,V}}}$	−0.09	—	Ver. tail pitching moment coeff.
α_{WB}	0.0556 rad	3.186 deg	Wing-Body angle of attack
δ_{e}	−0.05 rad	−2.864 deg	Elevator deflection angle
α_{H}	−0.0246 rad	−1.41 deg	Hor. tail angle of attack
C_L	0.562	—	Lift coefficient
C_D	0.0399	—	Drag coefficient
x_{CG}	1.002 m	3.286 ft	CG distance from m.a.c. l.e.
x_{N}	2.704 m	8.872 ft	Neutral point distance from m.a.c. l.e.
SSM	−0.467	—	Static Stability Margin
T_c	1.201	—	Thrust Coefficient
$\bar{\mathcal{V}}_{\mathrm{H}}$	1.211	—	Hor. tail volume ratio
$C_{\mathcal{H}_{\alpha_{\mathrm{e}}}}$	−0.0067 rad^{-1}	−0.000 12 deg^{-1}	Elevator hinge coeff. α-slope
$C_{\mathcal{H}_{\delta_{\mathrm{e}}}}$	−0.0127 rad^{-1}	−0.000 22 deg^{-1}	Elevator hinge coeff. δ_{e}-slope
τ_{e}	0.5	—	Elevator effectiveness coeff.
C_{L0}	0.203	—	Lift coeff. for $\alpha_{\mathrm{WB}} = \delta_{\mathrm{e}} = 0$

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VARIABLE	SI UNITS	USCS UNITS	DESCRIPTION
$C_{L\alpha,\mathrm{WB}}$	$6.227\,\mathrm{rad}^{-1}$	$0.1087\,\mathrm{deg}^{-1}$	Wing-Body lift coefficient slope
$C_{L\alpha}$	$7.296\,\mathrm{rad}^{-1}$	$0.1273\,\mathrm{deg}^{-1}$	Lift coefficient slope
$C_{L_{i_\mathrm{H}}}$	$1.2797\,\mathrm{rad}^{-1}$	$0.022\,33\,\mathrm{deg}^{-1}$	Lift coefficient i_H-slope
$C_{L_{\delta_\mathrm{e}}}$	$0.6398\,\mathrm{rad}^{-1}$	$0.011\,17\,\mathrm{deg}^{-1}$	Lift coefficient δ_e-slope
$C_{\mathcal{M}\alpha}$	$-3.411\,\mathrm{rad}^{-1}$	$-0.059\,53\,\mathrm{deg}^{-1}$	Pitching moment coeff. slope
$C_{\mathcal{M}_{i_\mathrm{H}}}$	$-5.331\,\mathrm{rad}^{-1}$	$-0.0931\,\mathrm{deg}^{-1}$	Pitching moment coeff. i_H-slope
$C_{\mathcal{M}_{\delta_\mathrm{e}}}$	$-2.666\,\mathrm{rad}^{-1}$	$-0.0465\,\mathrm{deg}^{-1}$	Pitching moment coeff. δ_e-slope
F	0.736	—	Free elevator factor
δ_ef	0.02 rad	1.007 deg	Floating elevator deflection angle
SSM_free	−0.319	—	Free-stick Static Stability Margin
β	0.03 rad	2 deg	Angle of sideslip
τ_a	0.597	—	Ailerons effectiveness coeff.
τ_r	0.68	—	Rudder effectiveness coeff.
C_{Y_β}	$0.334\,\mathrm{rad}^{-1}$	$0.0058\,\mathrm{deg}^{-1}$	Lateral force coeff. slope
$C_{\mathcal{L}\beta}$	$-0.2222\,\mathrm{rad}^{-1}$	$-0.003\,878\,\mathrm{deg}^{-1}$	Dihedral effect
$C_{\mathcal{L}\delta_a}$	$-0.1186\,\mathrm{rad}^{-1}$	$-0.002\,07\,\mathrm{deg}^{-1}$	Rolling moment coeff. δ_a-slope
$C_{\mathcal{L}\delta_r}$	$0.0842\,\mathrm{rad}^{-1}$	$0.001\,469\,\mathrm{deg}^{-1}$	Rolling moment coeff. δ_r-slope
$C_{\mathcal{N}\beta}$	$0.2573\,\mathrm{rad}^{-1}$	$0.004\,491\,\mathrm{deg}^{-1}$	Yawing moment coeff. slope
$C_{\mathcal{N}\delta_a}$	$0.0016\,\mathrm{rad}^{-1}$	$2.81 \times 10^{-5}\,\mathrm{deg}^{-1}$	Yawing moment coeff. δ_a-slope
$C_{\mathcal{N}\delta_r}$	$-0.2887\,\mathrm{rad}^{-1}$	$-0.005\,038\,9\,\mathrm{deg}^{-1}$	Yawing moment coeff. δ_r-slope
$C_{\mathcal{L}p}$	−0.4556	—	Roll. mom. coeff. slope due to roll rate
$C_{\mathcal{N}p}$	0.1638	—	Yaw. mom. coeff. slope due to roll rate
$C_{\mathcal{L}r}$	0.2287	—	Roll. mom. coeff. slope due to yaw rate
$C_{\mathcal{N}r}$	−0.2673	—	Yaw. mom. coeff. slope due to yaw rate
δ_a	−0.042 rad	−2.4083 deg	Ailerons deflection angle
δ_r	0.0304 rad	1.7432 deg	Rudder deflection angle

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