marker. Appropriate probe position resulted in the core region becoming clearly visible while the outer regions of the vortex remained unmarked. A laser sheet aligned normal to the wing surface highlighted the smoke-filled core. Kerosene smoke and $TiCl_4$ were used simultaneously in the photographs in Figs. 3b and 3c, with the laser sheet aligned parallel and normal, respectively, to the vortex core. These photographs again illustrate the effectiveness of an appropriately positioned $TiCl_4$ injection probe in placing tracer particles into the core region.

These preliminary observations indicate several interesting flow features. The core appears to maintain a more or less constant diameter upstream of breakdown. Similar results can be seen in the water-tunnel studies by Erickson[3] and Werle.[6] Because the probe sweeps over the wing showed no sign of particle entrainment of the outer vortex by the core region and the core fluid itself shows no apparent sign of exiting from that region, this would seem to indicate a more or less steady core flow and no mixing with the surrounding vortical flow.

Furthermore, previous arguments indicating the particle-scarce core region to be a result of the high circumferential velocity, or the migration of particles to a pressure matching region between the inner and outer flow regimes, seem questionable, because no particle, once in the core, was observed to exit from that region. It is more likely that the core fluid originates from the streamlines that pass through a small region near the apex. Unless the particle concentration in the core fluid is high at the beginning, acceleration and expansion of the core fluid may dilute the particle concentration, making the core fluid less effective in scattering light and resulting in a comparatively dark region.

## Concluding Remarks

The possibilities with the particle injection scheme should not be overlooked, especially with respect to LDV, which requires tracer smoke particles for its operation. With the proper injection techniques, the problem that plagues most LDV measurements on delta wings, namely, the lack of particles in the vortex core, can be eliminated. This method can help other localized flowfield studies, especially those void of particles by standard flow visualization methods. The vortex core region is only one application that can be investigated. Vortex trajectory mapping can also be accomplished by this method and, in particular, multiple vortex interaction.

These initial wind-tunnel investigations have indicated that the vortex core over a flat delta wing appears to maintain a more or less constant diameter until breakdown. The core may also be maintaining a constant velocity up to the breakdown point, as suggested by the very little mixing seen to occur between the core and the outer flow.

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# Drag Reduction Factor Due to Ground Effect

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## Introduction

NUMEROUS investigators[1–7] have studied and experimented with ground interference on the aerodynamic characteristics of wings. The general trend of this effect is an increasing lift-curve slope and a decreasing induced drag as the distance between the wing and ground is reduced. In this Note, we derive a theoretical working formula of this effect and compare it with existing formulas and experimental data.

The drag equation of an aircraft can be written as $C_D = C_{D0} + KC_L^2$, where $K = (\pi \mathcal{R} Ae)^{-1}$, $\mathcal{R}$ (aspect ratio) $= (2s)^2/A_s$, $A_s$ = span area. Here, $s$ and $e$ are the half-span and Oswald's wing efficiency, respectively. Usually, there are two ways to incorporate the ground effect in this drag equation, i.e.,

$$C_D = C_{D0} + \Phi K C_L^2, \quad 0 < \phi < 1 \tag{1a}$$

$$C_D = C_{D0} + C_L^2/(\pi e A_{\text{eff}}) \tag{1b}$$

where $A_{\text{eff}}$ is the effective aspect ratio resulting from the inclusion of the ground effect. These two are equivalent through the relation $A_{\text{eff}} = \mathcal{R}/\Phi$.

There exist several theoretical expressions of the drag reduction factor $\Phi$, namely Prandtl's[8]

$$\Phi(\text{Prandtl}) = \frac{8.72 + 0.1s/h}{7.4 + 2.1s/h}, \quad 2 \le s/h \le 15$$

and McCormick's[9]

$$\Phi(\text{McCormick}) = \frac{(8h/s)^2}{1 + (8h/s)^2}$$

However, in the present discussion, this factor has been derived rather differently, as shown below.

Because of the presence of the solid wall, the simplified horseshoe vortex system of an aircraft will accompany its image vortex system reflected in the plane boundary (runway).

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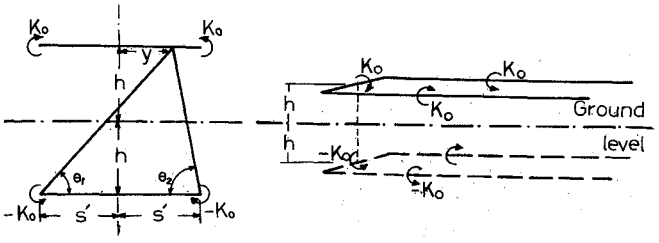


**Fig. 1 Simplified horseshoe vortex and its image vortex system reflected in the plane wall (runway).**

The situation is depicted in Fig. 1. The original elliptic loading is replaced by an equivalent rectangular loading with the same height $K_0$, but reduced span $2s' = \pi s/2$, i.e., the length of the equivalent bound vortex. The induced upwash due to the image trailing vortex system at the span location will be[10]

$$\Delta w_\uparrow = \frac{K_0}{4\pi}\left(\frac{s'+y}{r_1^2} + \frac{s'-y}{r_2^2}\right)$$

The lift gain of the span length $\delta y$ is $\delta\ell = \rho U K_0 \delta y$, which yields the induced drag reduction, $\delta d_v = \delta\ell\, \Delta W_\uparrow / U = \rho K_0 \delta y\, \Delta W_\uparrow$. The total drag reduction is

$$\Delta D_v = \int_{-s'}^{s'} \delta d_v = \frac{\rho K_0^2}{4\pi}\, \ell n \left[1 + \left(\frac{\pi s}{4h}\right)^2\right]$$

where the relations $r_1^2 = 4h^2 + (s'+y)^2$ and $r_2^2 = 4h^2 + (s'-y)^2$ have been used. Making use of the Kutta-Joukowsky theorem, $L = \rho U K_0 2s' = \rho U K_0 \pi s/2$, we finally obtain

$$\Delta D_v = \frac{\rho C_L^2 U^2 A_s^2}{4\pi^3 s^2}\, \ell n \left[1 + \left(\frac{\pi s}{4h}\right)^2\right] \tag{2}$$

Therefore, we write the net aerodynamic drag as $D_A = \frac{1}{2}\rho U^2 A_s C_D - \Delta D_v$. When this equation is put into the form of Eq. (1a), i.e.,

$$D_A = \tfrac{1}{2}\rho U^2 A_s (C_{D0} + \Phi K C_L^2)$$

we finally arrive at the following drag reduciton factor;

$$\Phi = 1 - \frac{2e}{\pi^2}\, \ell n \left[1 + \left(\frac{\pi s}{4h}\right)^2\right] \tag{3}$$

It is easily seen that $\lim_{h\to\infty}\Phi = 1$ (out-of-ground effect) as should be the case. Here, we notice that our expression [Eq. (3)] differs from those preceding in that Eq. (3) contains the explicit dependence on $e$, Oswald's wing efficiency as well as $s/h$. This dependence is experimenally supported by the earlier wind-tunnel test by Furlong and Boilech,[7] in which these investigators found that effects of sweepback and taper ratio show different characteristics from unswept and rectangular wings. Naturally, these wing parameters could lump into some form of the wing efficiency,[11] for instance, Weissinger planform efficiency factor $e'$.[12] It should be understood that the Oswald wing efficiency $e$ is the measure of departure from the ideal elliptical loading in the portion of the induced drag.

## Results and Discussion

Figure 2 compares the foregoing three expressions of $\Phi$ along with experimental data obtained by Fink and Lastinger,[13] in which the notion of $A_{\text{eff}}$ was employed. As are shown in Fig. 2, the McCormick factor underestimates the ground effect, whereas Prandtl factor somewhat overestimates the effect in the same $s/h$ range.

The present reduction factor [Suh-Ostowari, Eq. (3)] generally agrees with the wind-tunnel test when $e \sim 0.85$ and 0.9 are

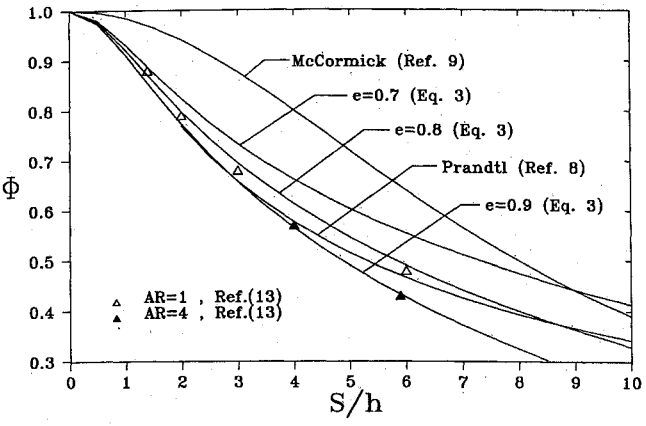


**Fig. 2 Effect of ground on the drag reduction factor Φ.**

assumed for $AR = 1$ and 4, respectively. These values of $e$ are quite feasible in the absence of wing/fuselage interference and for the unswept, untapered wings used in the experiment.[13] As shown in Fig. 2, the ground effect is more pronounced for wings of higher $e$ (Oswald's wing efficiency), i.e., smaller $\Phi$. Equation (3) generally agrees well with the results of wind-tunnel test for all practical $s/h$ ranges ($s/h < 7.5$). It can be noticed that at zero angle of attack, the lift coefficient $C_L$ nearly remains unchanged as $s/h$ varies.[13] Consequently, $C_L$ appearing in Eqs. (1a) and (1b) will be the same as that without ground effect if angle of attack is close to zero.

We conclude our discussion by noticing that Eq. (3) should provide a useful guide in the analysis of takeoff and/or landing performance.

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