

VERTICAL TAIL PARAMETERS INITIALIZATION

Hidden Area --> Import of Excel INPUT V-Tail Data

Hidden Area --> Preliminary Mapping of imported Data

Hidden Area --> Import and preliminary mapping of OTHER Excel Data

INPUT V-TAIL PARAMETERS LIST

Input parameters

$b_V = 10.68 \text{ m}$	$\Lambda_{V_{LE}} = 44.5 \text{ deg}$	$\Gamma_V = 0 \text{ deg}$
$\xi_{tmax_V} = 0.35$	$\varepsilon_V = 0 \text{ deg}$	$\eta_V = 1$
$c_{V_r} = 8.1 \text{ m}$	$t_{over_c_{V_r}} = 0.12$	$C_{l\alpha_{V_r}} = 0.12 \frac{1}{\text{deg}}$
$\xi_{ac_{V_r}} = 0.25$	$C_{m_{ac_{V_r}}} = 0$	$M_{cr_{V_{2D_r}}} = 0.7$
$c_{V_t} = 2.51 \text{ m}$	$t_{over_c_{V_t}} = 0.09$	$C_{l\alpha_{V_t}} = 0.12 \frac{1}{\text{deg}}$
$\xi_{ac_{V_t}} = 0.25$	$C_{m_{ac_{V_t}}} = 0$	$M_{cr_{V_{2D_t}}} = 0.7$
$\eta_{rd_in} = 0$	$\eta_{rd_out} = 0.95$	$c_{rd} = 1.83 \text{ m}$

Imported parameters

$\Delta X_{W_{LE_Nose}} = 25.05 \text{ m}$	$\Delta X_{VT_{LE_Nose}} = 61.96 \text{ m}$	$\Delta Z_{W_{LE_Nose}} = -0.75 \text{ m}$
$\Delta Z_{HT_{ac_{W_{ac}}}} = 1.65 \text{ m}$		
$M_1 = 0.65$	$c_{W_r} = 15.57 \text{ m}$	$i_W = 2 \text{ deg}$
$S_W = 468.83 \text{ m}^2$	$MAC_W = 9.505 \text{ m}$	$\Lambda_{W_{c4_{eqv}}} = 2 \text{ deg}$
$AR_W = 7.916$	$r_1 = 3.9 \text{ m}$	$h_1 = 6.33 \text{ m}$
$b_H = 21.96 \text{ m}$	$S_H = 105.41 \text{ m}^2$	$MAC_H = 5.2 \text{ m}$
$X_{MAC_{LE_H}} = 3.705 \text{ m}$	$X_{ac_{WB}} = 8.057 \text{ m}$	

VTAIL PARAMETERS CALCULATIONS

V-Tail basic parameters

$$b_{2V} := 2 \cdot b_V = 21.36 \text{ m}$$

• Considering Vertical Tail like other wings

$$b_{2V} = 21.36 \text{ m}$$

$$\lambda_V := \frac{c_{V_t}}{c_{V_r}} = 0.31$$

$$\lambda_V = 0.31$$

$$S_V := \frac{b_V}{2} \cdot c_{V_r} \cdot (1 + \lambda_V) = 56.657 \text{ m}^2$$

$$S_V = 56.657 \text{ m}^2$$

$$S_{2V} := \frac{b_{2V}}{2} \cdot c_{V_r} \cdot (1 + \lambda_V) = 113.315 \text{ m}^2$$

$$S_{2V} = 113.315 \text{ m}^2$$

$$AR_{2V} := \frac{b_{2V}^2}{S_{2V}} = 4.026$$

$$AR_{2V} = 4.026$$

$$AR_V := AR_{2V}$$

$$AR_V = 4.026$$

$$MAC_V := \frac{2}{3} \cdot c_{V_r} \cdot \left(\frac{1 + \lambda_V^2 + \lambda_V}{1 + \lambda_V} \right) = 5.796 \text{ m}$$

$$MAC_V = 5.796 \text{ m}$$

$$X_{MAC_LE_V} := \frac{b_{2V}}{6} \cdot \frac{(1 + 2 \cdot \lambda_V)}{(1 + \lambda_V)} \cdot \tan(\Lambda_{V_LE}) = 4.326 \text{ m}$$

$$X_{MAC_LE_V} = 4.326 \text{ m}$$

$$Y_{MAC_V} := \frac{b_{2V}}{6} \cdot \frac{1 + 2 \cdot \lambda_V}{1 + \lambda_V} = 4.402 \text{ m}$$

$$Y_{MAC_V} = 4.402 \text{ m}$$

$$Z_{MAC_V} := Y_{MAC_V} \cdot \tan(\Gamma_V) = 0 \text{ m}$$

$$Z_{MAC_V} = 0 \text{ m}$$

V-Tail, sweep angles

$${}_f\Lambda(x, \Lambda_{le}, AR, \lambda) := \text{if} \left(AR = 0, \Lambda_{le}, \text{atan} \left(\tan(\Lambda_{le}) - \frac{4 \cdot x \cdot (1 - \lambda)}{AR \cdot (1 + \lambda)} \right) \right)$$

• Sweep angle function

$$\Lambda_{V_LE} := {}_f\Lambda(0, \Lambda_{V_LE}, AR_V, \lambda_V) = 0.777$$

$$\Lambda_{V_LE} = 44.5 \text{ deg}$$

$$\Lambda_{V_TE} := {}_f\Lambda(1, \Lambda_{V_LE}, AR_V, \lambda_V) = 0.431$$

$$\Lambda_{V_TE} = 24.669 \text{ deg}$$

$$\Lambda_{V_c4} := {}_f\Lambda(0.25, \Lambda_{V_LE}, AR_V, \lambda_V) = 0.706$$

$$\Lambda_{V_c4} = 40.426 \text{ deg}$$

$$\Lambda_{V_c2} := {}_f\Lambda(0.5, \Lambda_{V_LE}, AR_V, \lambda_V) = 0.625$$

$$\Lambda_{V_c2} = 35.791 \text{ deg}$$

$$\Lambda_{V_tmax} := {}_f\Lambda(\xi_{tmax_V}, \Lambda_{V_LE}, AR_V, \lambda_V) = 0.674$$

$$\Lambda_{V_tmax} = 38.642 \text{ deg}$$

Hidden Area --> V-Tail, linear laws coefficients

V-Tail, linear laws defined over the whole semi-span

$$f_{c_V}(y) := A_{c_V} \cdot y + B_{c_V}$$

$$f_{t_over_c_V}(y) := A_{t_{c_V}} \cdot y + B_{t_{c_V}}$$

$$f_{C_{l_{\alpha_V}}}(y) := A_{C_{l_{\alpha_V}}} \cdot y + B_{C_{l_{\alpha_V}}}$$

$$f_{C_{m_ac_2D_V}}(y) := A_{C_{m0_V}} \cdot y + B_{C_{m0_V}}$$

$$f_{\epsilon_{g_V}}(y) := A_{\epsilon_V} \cdot y + B_{\epsilon_V}$$

$$f_{\xi_{ac_2D_V}}(y) := A_{\xi_{ac_V}} \cdot y + B_{\xi_{ac_V}}$$

$$f_{M_{cr_V_2D}}(y) := A_{M_{cr_V}} \cdot y + B_{M_{cr_V}}$$

V-Tail, 2D mean quantities

$$t_{over_c_V_mean} := \frac{1}{S_V} \cdot \int_0^{b_V} f_{c_V}(y) \cdot f_{t_over_c_V}(y) \, dy = 0.053$$

$$t_{over_c_V_mean} = 0.053$$

$$C_{m_ac_V_mean} := \frac{1}{S_V \cdot MAC_V} \cdot \int_0^{b_V} f_{c_V}(y)^2 \cdot f_{C_{m_ac_2D_V}}(y) \, dy = 0$$

$$C_{m_ac_V_mean} = 0$$

$$C_{l_{\alpha_V_mean}} := \frac{1}{S_V} \cdot \int_0^{b_V} f_{c_V}(y) \cdot f_{C_{l_{\alpha_V}}}(y) \, dy = 3.253$$

$$C_{l_{\alpha_V_mean}} = 0.057 \, \text{deg}^{-1}$$

MISCELLANEOUS DATA CALCULATIONS

$$\Delta X_{VT_{LE}W_{LE}} := \Delta X_{VT_{LE}Nose} - \Delta X_{W_{LE}Nose} = 36.91 \, m$$

$$\Delta X_{VT_{c4_r}W_{c4_r}} := \Delta X_{VT_{LE}W_{LE}} + \frac{c_{V_r}}{4} - \frac{c_{W_r}}{4} = 35.043 \, m$$

$$\Delta Z_{W_{c4_r}B_{CL}} := \Delta Z_{W_{LE}Nose} + \sin(i_W) \cdot \frac{c_{W_r}}{4} = -0.614 \, m$$

$$\Delta Z_{HT_{c4_r}B_{CL}} := \Delta Z_{W_{c4_r}B_{CL}} - \Delta Z_{HT_{ac}W_{ac}} = -2.264 \, m$$

$$x_{AC_{HV}} := X_{MAC_{LE}H} + 0.25 \cdot MAC_H = 5.005 \, m$$

Rudder inner and outer stations and area

$$y_{rd_in} := \eta_{rd_in} \cdot b_V = 0 \, m$$

$$y_{rd_in} = 0 \, m$$

$$y_{rd_out} := \eta_{rd_out} \cdot b_V = 10.146 \, m$$

$$y_{rd_out} = 10.146 \, m$$

$$c_{V_mean_@rd} := f_{c_V} \left(\frac{y_{rd_in} + y_{rd_out}}{2} \right) = 2.79 \, m$$

$$c_{V_mean_@rd} = 2.79 \, m$$

$$S_{rd} := 2 \cdot c_{rd} \cdot (y_{rd_out} - y_{rd_in}) = 37.134 \, m^2$$

$$S_{rd} = 37.134 \, m^2$$

$$\tau_{rd} = 0.793$$

Induced drag factors, only due to geometric effects

$$e_{V_alt} := \frac{2}{2 - AR_V + \sqrt{4 + AR_V^2 \left(1 + \tan^2(\Lambda_{V_tmax})\right)}} = 0.571$$

$$e_{V_alt} = 0.571$$

$$e_{V_alt_A0} := 1.78 \cdot \left(1 - 0.045 \cdot AR_V^{0.68}\right) - 0.64 = 0.933$$

• **Alternative formula: valid for unswept wings**

$$e_{V_alt_A} := 4.61 \cdot \left(1 - 0.045 \cdot AR_V^{0.68}\right) \cdot \cos(\Lambda_{V_LE})^{0.15} - 3.1 = 0.774$$

• **Alternative formula: valid for swept wings**

3d critical Mach number at mean aerodynamic chord

$$M_{cr_V_3D_@MAC_V} := \frac{f_{cr_V_2D}(Y_{MAC_V})}{\cos(\Lambda_{V_LE})} = 0.981$$

$$M_{cr_V_3D_@MAC_V} = 0.981$$

VERTICAL TAIL EFFECTIVE ASPETC RATIO

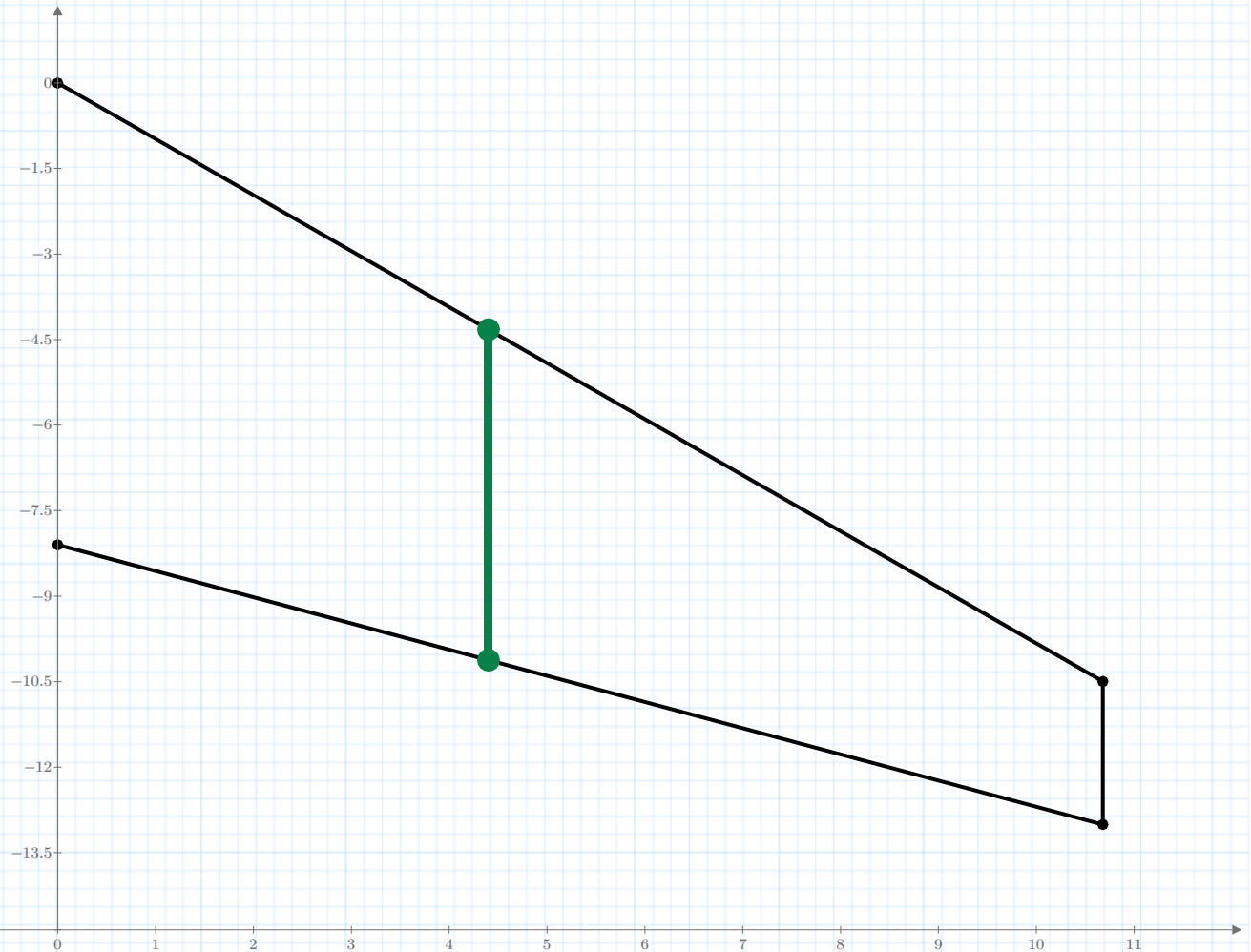
$$c_1 = 1.607$$

$$c_2 = 1.105$$

$$K_{HV} = 1.118$$

$$AR_{V_eff} := c_1 \cdot AR_V \cdot \left(1 + K_{HV} \cdot (c_2 - 1)\right) = 7.225$$

$$AR_{V_eff} = 7.225$$



VERTICAL TAIL LIFT-CURVE SLOPE

V-Tail Lift Curve Slope, function definitions

$f_{\text{Polhamus}}(M, M_{cr_3D}, A_{LE}, \lambda, AR) :=$

- Polhamus Formula Coefficient

```

if (M < Mcr_3D) ∧ (ALE < 32 deg) ∧ (λ > 0.4) ∧ (λ < 1) ∧ (AR > 3) ∧ (AR < 8)
  if AR < 4
    return 1 +  $\frac{AR \cdot (1.87 - 0.000233 \cdot A_{LE})}{100}$ 
  else
    return 1 +  $\frac{((8.2 - 2.3 \cdot A_{LE}) - AR \cdot (0.22 - 0.153 \cdot A_{LE}))}{100}$ 
  else
    (---> Polhamus Formula is not valid)
    return 100

```

$fC_{L\alpha_V}(M, k_P, AR, \Lambda_{c2}, C_{l\alpha@MAC}, \Lambda_{LE}) :=$ if $k_P \neq 100$ (---> use Polhamus Formula)

• General Formula for Lift Curve Slope

$$\begin{aligned} & \text{return } \frac{2 \cdot \pi \cdot AR}{2 + \sqrt{\left(\left(\frac{AR^2 \cdot (1 - M^2)}{k_P^2} \left(1 + \frac{\tan(\Lambda_{c2})^2}{(1 - M^2)} \right) \right) + 4 \right)}} \\ & \text{else (---> use alternative formula)} \\ & a_0 \leftarrow \frac{C_{l\alpha@MAC}}{\sqrt{1 - M^2 \cdot \cos(\Lambda_{LE})^2}} \\ & \text{return } \frac{a_0 \cdot \cos(\Lambda_{LE})}{\sqrt{1 - (M \cdot \cos(\Lambda_{LE}))^2 + \left(\frac{a_0 \cdot \cos(\Lambda_{LE})}{\pi \cdot AR} \right)^2} + \frac{a_0 \cdot \cos(\Lambda_{LE})}{\pi \cdot AR}} \end{aligned}$$

V-Tail Lift Curve Slope, general formula for inner/outer panel and whole wing

$$k_{Polhamus_V} := f k_{Polhamus}(M_1, M_{cr_V_3D@MAC_V}, \Lambda_{V_LE}, \lambda_V, AR_V) = 100$$

$$k_{Polhamus_V} = 100$$

$$C_{l\alpha_V@MAC_V} := f C_{l\alpha_V}(Y_{MAC_V}) = 6.875$$

$$C_{l\alpha_V@MAC_V} = 0.12 \text{ deg}^{-1}$$

$$C_{L\alpha_V@M0} := f C_{L\alpha_V}(0, k_{Polhamus_V}, AR_V, \Lambda_{V_c2}, C_{l\alpha_V@MAC_V}, \Lambda_{V_LE})$$

$$C_{L\alpha_V@M0} = 3.358 \text{ rad}^{-1}$$

$$C_{L\alpha_V@M0} = 0.059 \text{ deg}^{-1}$$

$$C_{L\alpha_V} := f C_{L\alpha_V}(M_1, k_{Polhamus_V}, AR_{V_eff}, \Lambda_{V_c2}, C_{l\alpha_V@MAC_V}, \Lambda_{V_LE})$$

$$C_{L\alpha_V} = 4.76 \text{ rad}^{-1}$$

$$C_{L\alpha_V} = 0.083 \text{ deg}^{-1}$$

Induced drag factor, due to both geometric and aerodynamic effects

$$f e(C_{L\alpha}, AR, \lambda, \Lambda_{LE}) := \begin{cases} \lambda_e \leftarrow \frac{AR \cdot \lambda}{\cos(\Lambda_{LE})} \\ R \leftarrow 0.0004 \cdot \lambda_e^3 - 0.008 \cdot \lambda_e^2 + 0.0501 \cdot \lambda_e + 0.8642 \\ \text{return if } \left(AR = 0, 0, \frac{1.1 \cdot C_{L\alpha}}{R \cdot C_{L\alpha} + (1 - R) \pi \cdot AR} \right) \end{cases}$$

• Function for calculating wing induced drag factor, including aerodynamic and geometric effects

$$e_V := f e(C_{L\alpha_V}, AR_{V_eff}, \lambda_V, \Lambda_{V_LE}) = 0.941$$

$$e_V = 0.941$$

V-Tail Lift Curve Slope, classic formula

$$C_{L\alpha_V_classic} := \frac{C_{l\alpha_V_mean}}{\sqrt{1 - M_1^2} + \frac{C_{l\alpha_V_mean}}{\pi \cdot AR_V \cdot e_V}} = 3.148$$

$$C_{L\alpha_V_classic} = 3.148$$

V-TAIL AERODYNAMIC CENTER - GRAPHICAL METHOD (DATCOM/NAPOLITANO)

@Aerodynamic Database ---> (x_bar_ac_w)_k1_vs_lambda

$$K1_{ac_V_Datcom} = 1.393$$

@Aerodynamic Database ---> (x_bar_ac_w)_k2_vs_L_LE_(AR)_(lambda)

$$K2_{ac_V_Datcom} = 0.537$$

@Aerodynamic Database ---> (x_bar_ac_w)_x'_ac_over_root_chord_vs_tan_(L_LE)_over_beta_(AR_times_tan_(L_LE))_(lambda)

$$X_{ac_over_c_r_V_Datcom} = 0.762$$

Aerodynamic center positions

$$\xi_{ac_V} := K1_{ac_V_Datcom} \cdot (X_{ac_over_c_r_V_Datcom} - K2_{ac_V_Datcom}) = 0.312$$

$$X_{ac_V} := \xi_{ac_V} \cdot MAC_V + X_{MAC_LE_V} = 6.137 \text{ m}$$

$$x_{ac_V} := X_{ac_V} - X_{MAC_LE_V} = 1.811 \text{ m}$$

$$X_{ac_V} = 6.137 \text{ m}$$

$$x_{ac_V} = 1.811 \text{ m}$$

V-Tail Volume Ratio based on aerodynamic centers distance

$$\Delta X_{VT_{ac}W_{TE}} := \Delta X_{VT_{LE}Nose} - \Delta X_{W_{LE}Nose} - c_{W_r} + X_{ac_V} = 27.477 \text{ m}$$

$$\Delta X_{VT_{ac}W_{ac}} := \Delta X_{VT_{LE}Nose} - \Delta X_{W_{LE}Nose} - X_{ac_{WB}} + X_{ac_V} = 34.99 \text{ m}$$

$$VolumeRatio_{V_{ac}} := \frac{S_V}{S_W} \cdot \frac{\Delta X_{VT_{ac}W_{ac}}}{MAC_W} = 0.445$$

3D PITCHING MOMENT COEFFICIENT ABOUT V-TAIL AERODYNAMIC CENTER

Approximated formulation (Roskam)

$$C_{Mac_V} := \frac{2}{S_V \cdot MAC_V} \cdot \int_0^{b_V} C_{m_{ac_{2D_V}}}(y) \cdot r_{c_V}(y)^2 dy = 0 \quad C_{Mac_V} = 0$$

SIDEWASH EFFECT

$$\eta_{V_times_1_plus_d\sigma_over_d\beta} := 0.724 + 3.06 \cdot \frac{\frac{S_V}{S_W}}{1 + \cos(\Lambda_{W_{c4_{eqv}}})} + 0.4 \cdot \frac{\Delta Z_{W_{c4_r}B_{CL}}}{h_1} + 0.009 \cdot AR_W = 0.941$$

MAPPING AND OUTPUT CREATION

Includi << ..\Default_Map_VTail.mcdx

Excel Writing

$$First_Row_{V_1} := 4$$

$$Block_{V_1} := \text{rmap_matrix_transform} \left({}_mVTail_Data_Map_{imported} \right)$$

$$Excel_Output_{V_1} := \text{rwrite_full_output} \left({}_sOutput_Excel_File, Block_{V_1}, n_{sheet}, First_Row_{V_1} \right)$$

$$First_Row_{V_2} := First_Row_{V_1} + \text{rows} \left(Block_{V_1} \right) + 2 = 40$$

$$Block_{V_2} := \text{rmap_matrix_transform} \left({}_mVTail_Data_Map_{input} \right)$$

$$Excel_Output_{V_2} := \text{rwrite_full_output} \left({}_sOutput_Excel_File, Block_{V_2}, n_{sheet}, First_Row_{V_2} \right)$$

$$First_Row_{V_3} := First_Row_{V_2} + \text{rows} \left(Block_{V_2} \right) + 2 = 72$$

$$Block_{V_3} := \text{rmap_matrix_transform} \left({}_mVTail_Data_Map \right)$$

$$Excel_Output_{V_3} := \text{rwrite_full_output} \left({}_sOutput_Excel_File, Block_{V_3}, n_{sheet}, First_Row_{V_3} \right)$$

$$First_Row_{V_4} := First_Row_{V_3} + rows \left(Block_{V_3} \right) + 2 = 142$$

$$Block_{V_4} := \text{fmap_matrix_transform} \left({}_m VTail_Data_Map_{LLCoeffs} \right)$$

$$Excel_Output_{V_4} := \text{fwrite_full_output} \left({}_s Output_Excel_File, Block_{V_4}, n_{sheet}, First_Row_{V_4} \right)$$

$$First_Row_{V_5} := First_Row_{V_4} + rows \left(Block_{V_4} \right) + 2 = 171$$

$$Block_{V_5} := \text{fmap_matrix_transform} \left({}_m VTail_Data_Map_{Misc} \right)$$

$$Excel_Output_{V_5} := \text{fwrite_full_output} \left({}_s Output_Excel_File, Block_{V_5}, n_{sheet}, First_Row_{V_5} \right)$$

TeX Macro writing on .tex

$${}_v complete_macros_V := \text{stack} \left(Block_{V_1}^{(2)}, Block_{V_2}^{(2)}, Block_{V_3}^{(2)}, Block_{V_4}^{(2)}, Block_{V_5}^{(2)} \right)$$

$${}_v tex_V := \text{fwrite_matrix} \left(".\backslash Output \backslash VTAIL_TeX_Macros.tex", {}_v complete_macros_V, " " \right)$$