HomeWork4

• Q1：线性支持向量机还可以写成如下形式，试求其对偶形式：

$$\min_{w,b,\xi} \quad \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{N}\xi_i^2$$

$$\text{s.t.} \quad y_i(w\cdot x_i + b) \geqslant 1-\xi_i, \quad i=1,2,\cdots,N \longrightarrow \alpha_i[y_i(w\cdot x_i+b)-1+\xi_i]$$

$$= \alpha_i[y_i(w\cdot x_i+b)] - \alpha_i + \alpha_i\xi_i$$

$$\xi_i \geqslant 0, \quad i=1,2,\cdots,N$$

Lagrange 函数

$$L = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{N}\xi_i^2 + \sum_{i=1}^{N}\alpha_i - \sum_{i=1}^{N}\alpha_i\xi_i - \sum_{i=1}^{N}\alpha_i y_i(w\cdot x_i+b) - \sum_{i=1}^{N}\beta_i\xi_i$$

分别求 w, b, ξ 的偏导数 $\quad \sum_{i=1}^{N}(\alpha_i+\beta_i)\xi_i$，化简时代入 (3)

(1) $\dfrac{\partial L}{\partial w} = w - \sum_{i=1}^{N}\alpha_i y_i x_i = 0 \iff w = \sum_{i=1}^{N}\alpha_i y_i x_i$

(2) $\dfrac{\partial L}{\partial b} = -\sum_{i=1}^{N}\alpha_i y_i = 0 \iff \sum_{i=1}^{N}\alpha_i y_i = 0$

(3) $\dfrac{\partial L}{\partial \xi_i} = 2C\xi_i - \alpha_i - \beta_i = 0 \iff \xi_i = \dfrac{\alpha_i+\beta_i}{2C}$

原问题转化为对 $\min\limits_{w,b,\xi} L$ 求 α 的极大 $\max\limits_{\alpha} L$

$$\max_{\alpha} L = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j y_i y_j x_i\cdot x_j + C\sum_{i=1}^{N}\left(\frac{\alpha_i+\beta_i}{2C}\right)^2 + \sum_{i=1}^{N}\alpha_i - \sum_{i=1}^{N}(\alpha_i+\beta_i)\beta_i$$

$$- \sum_{i=1}^{N}\alpha_i y_i(w\cdot x_i+b)$$

$$\sum_{i=1}^{N}\alpha_i y_i \cdot w x_i + 0$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j y_i y_j x_i\cdot x_j$$

$$= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j y_i y_j x_i\cdot x_j + \frac{1}{4C}\sum_{i=1}^{N}(\alpha_i+\beta_i)^2 - \sum_{i=1}^{N}\frac{(\alpha_i+\beta_i)^2}{2C} + \sum_{i=1}^{N}\alpha_i$$

$$= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j y_i y_j x_i \cdot x_j + \sum_{i=1}^{N}\alpha_i - \frac{1}{4C}\sum_{i=1}^{N}(\alpha_i+\beta_i)^2$$

$$\text{s.t.} \quad \sum_{i=1}^{N}\alpha_i y_i = 0$$

$$\alpha_i, \beta_i \geq 0 \qquad i = 1, 2, 3 \ldots N$$