HomeWork5

- Q1：P189 习题10.2

10.2 考虑盒子和球组成的隐马尔可夫模型 $\lambda=(A,B,\pi)$，其中，

$$A=\begin{bmatrix}0.5 & 0.1 & 0.4\\ 0.3 & 0.5 & 0.2\\ 0.2 & 0.2 & 0.6\end{bmatrix},\quad B=\begin{bmatrix}0.5 & 0.5\\ 0.4 & 0.6\\ 0.7 & 0.3\end{bmatrix},\quad \pi=(0.2,0.3,0.5)^{\mathrm{T}}$$

设 $T=8$，$O=$(红,白,红,红,白,红,白,白)，用前向后向概率计算 $P(i_4=q_3|O,\lambda)$.

给定模型 λ 和观测 O，在时刻 t 处状态 q_i 的概率：

$$P(i_t=q_i|O,\lambda)=\frac{\alpha_t(i)\beta_t(i)}{P(O|\lambda)}=\frac{\alpha_t(i)\beta_t(i)}{\sum_{j=1}^{N}\alpha_t(j)\beta_t(j)}$$

故由 $P(i_4=q_3|O,\lambda)=\dfrac{\alpha_4(3)\beta_4(3)}{\sum_{j=1}\alpha_4(j)\beta_4(j)}$

(1)

计算 α；(前向)

$\alpha_1(1)=\pi_1 b_1(O_1)=0.2\times0.5=0.1$

$\alpha_1(2)=\pi_2 b_2(O_1)=0.3\times0.4=0.12$

$\alpha_1(3)=\pi_3 b_3(O_1)=0.5\times0.7=0.35$

$\alpha_2(1)=\left[\sum_{i=1}^{3}\alpha_1(i)a_{i1}\right]b_1(O_2)=$

(后略去该步) $[\alpha_1(1)a_{11}+\alpha_1(2)a_{21}+\alpha_1(3)a_{31}]\cdot0.5=0.156\times0.5=0.078$

$\alpha_2(2)=\left[\sum_{i=1}^{3}\alpha_1(i)a_{i2}\right]b_2(O_2)=0.14\times0.6=0.084$

$\alpha_2(3)=\left[\sum_{i=1}^{3}\alpha_1(i)a_{i3}\right]b_3(O_2)=0.274\times0.3=0.0822$

$\alpha_3(1)=\left[\sum_{i=1}^{3}\alpha_2(i)a_{i1}\right]b_1(O_3)=0.08064\times0.5=0.04032$

$\alpha_3(2)=\left[\sum_{i=1}^{3}\alpha_2(i)a_{i2}\right]b_2(O_3)=0.06624\times0.4=0.026496$

$\alpha_3(3)=\left[\sum_{i=1}^{3}\alpha_2(i)a_{i3}\right]b_3(O_3)=0.09732\times0.7=0.068124$

$\alpha_4(1)=\left[\sum_{i=1}^{3}\alpha_3(i)a_{i1}\right]b_1(O_4)=0.0417336\times0.5=0.0208668$

$\alpha_4(2)=\left[\sum_{i=1}^{3}\alpha_3(i)a_{i1}\right]b_2(O_4)=0.0309048\times0.4=0.01236192$

$\alpha_4(3)=\left[\sum_{i=1}^{3}\alpha_3(i)a_{i1}\right]b_3(O_4)=0.0623016\times0.7=0.04361112$

(2)

计算 β (后向)

$$A=\begin{bmatrix}0.5 & 0.1 & 0.4\\ 0.3 & 0.5 & 0.2\\ 0.2 & 0.2 & 0.6\end{bmatrix},\quad B=\begin{bmatrix}0.5 & 0.5\\ 0.4 & 0.6\\ 0.7 & 0.3\end{bmatrix},\quad \pi=(0.2,0.3,0.5)^{\mathrm{T}}$$

$\beta_8(i)=1 \quad i=1,2,3$

$\beta_7(1)=\sum_{j=1}^{3} a_{1j} b_j(O_8)\beta_8(j)=0.5\times0.5\times1+0.1\times0.6\times1+0.4\times0.3\times1=0.43$

$\beta_7(2)=\sum_{j=1}^{3} a_{2j} b_j(O_8)\beta_8(j)=0.3\times0.5\times1+0.5\times0.6\times1+0.2\times0.3\times1=0.51$

$\beta_7(3)=\sum_{j=1}^{3} a_{3j} b_j(O_8)\beta_8(j)=0.2\times0.5\times1+0.2\times0.6\times1+0.6\times0.3\times1=0.4$

$\beta_6(1)=\sum_{j=1}^{3} a_{1j} b_j(O_7)\beta_7(j)=0.1861$ （计算过程同上）

$\beta_6(2)=\sum_{j=1}^{3} a_{2j} b_j(O_7)\beta_7(j)=0.2415$

$\beta_6(3)=\sum_{j=1}^{3} a_{3j} b_j(O_7)\beta_7(j)=0.1762$

$\beta_5(1)=\sum_{j=1}^{3} a_{1j} b_j(O_6)\beta_6(j)=0.105521$

$\beta_5(2)=\sum_{j=1}^{3} a_{2j} b_j(O_6)\beta_6(j)=0.100883$

$\beta_5(3)=\sum_{j=1}^{3} a_{3j} b_j(O_6)\beta_6(j)=0.111934$

$\beta_4(1)=\sum_{j=1}^{3} a_{1j} b_j(O_5)\beta_5(j)=0.04586631$

$\beta_4(2)=\sum_{j=1}^{3} a_{2j} b_j(O_5)\beta_5(j)=0.05280909$

$\beta_4(3)=\sum_{j=1}^{3} a_{3j} b_j(O_5)\beta_5(j)=0.04280618$

$\therefore P(i_4=q_3 \mid O,\lambda)=\dfrac{0.04361112\times0.04280618}{\sum_{j=1}\alpha_4(j)\beta_4(j)}$

$\sum_{j=1}\alpha_4(j)\beta_4(j)=\alpha_4(1)\beta_4(1)+\alpha_4(2)\beta_4(2)+\alpha_4(3)\beta_4(3)$

$\therefore P(i_4=q_3 \mid O,\lambda)=0.5369518$