FLOYAO.JL

JAN LUKAS BOSSE

TLDR

Slogan

Fermionic linear optics circuits are a class of quantum circuits that can be simulated in polynomial time and space.

FLOYao.jl brings this power to the Yao.jl eco system

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FERMIONIC LINEAR OPTICS CIRCUITS

THE MAJORANA OPERATORS

Define the <u>Majorana operators</u> as

$$\gamma_{2i-1} := \prod_{j=1}^{i-1}(-Z_j)X_i \qquad \text{and} \qquad \gamma_{2i} := -\prod_{j=1}^{i-1}(-Z_j)Y_i$$

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Products of two Majorana operators take the form

$$P_i \prod_{i<k<j} Z_k P_j \qquad \text{and} \qquad Z_i$$

with $P_i, P_j \in \{X, Y\}$

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RELATION TO FERMIONIC OPERATORS

Defining

$$c_i := \frac{1}{2}(\gamma_{2i-1} + i\gamma_{2i}) \qquad \text{and} \qquad c_i^\dagger = \frac{1}{2}(\gamma_{2i-1} - i\gamma_{2i})$$

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we get the normal Fermionic operators under the Jordan-Wigner transformation.
They satisfy the canonical anti-commutation relations

$$\{c_i, c_j\} = \{c_i^\dagger, c_j^\dagger\} = 0 \qquad \text{and} \qquad \{c_i, c_j^\dagger\} = \delta_{ij}.$$

and annihilate the vacuum

$$c_i \left|0 \cdots 0\right\rangle = c_i \left|\Omega\right\rangle = 0 \quad \forall i$$

FLO GATES

We call a unitary U a fermionic linear optics (FLO) gate / operator if

$$U\gamma_i U^\dagger = R_i^j \gamma_j$$

with a $2n \times 2n$-matrix R_i^j.

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Examples:

- X_j, Y_j and Z_j, since they all (anti-)commute with all γ_i
- $e^{-it(X_j X_{j+1} + Y_j Y_{j+1})} = e^{-it(c_j^\dagger c_{j+1} + c_j c_{j+1}^\dagger)}$ (fermions hopping on a lattice)
- $e^{-itX_j X_{j+1}}$ and e^{-itZ_j} (transverse field Ising model)
- e^{-itH} with $H = H^{ij} c_i^\dagger c_j + \Delta^{ij} c_i c_j + \overline{\Delta^{ij}} c_j^\dagger c_i^\dagger$ (all free fermion models)

FREE FERMIONIC EVOLUTION

Consider a hamiltonian quadratic in the fermionic operators

$$H = \frac{i}{4} A^{kl} \gamma_k \gamma_l$$

and the Heisenberg equation

$$\dot{\gamma}_i = [\gamma_i, H] = \cdots = -\frac{1}{2} A^{ki} \gamma_k$$

which has the solution

$$\gamma_i(t) = (e^{-\frac{t}{2}A})_i^j \gamma_j$$

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SIMULATION OF `APPLY!(REG, GATE)`

- Check if `gate` is a FLO gate
- Find matrix *R* corresponding to `gate`
- Update `reg.R` to `R * reg.R`
- Return the updated `reg`

SIMULATION OF `EXPECT(HAM, REG => CIRCUIT)`

Is simply done in the Heisenberg picture. Say

$$H = \gamma_{i_1} \cdots \gamma_{i_k} \qquad \text{and} \qquad U = U^{(1)} \ldots U^{(p)}$$

then

$$\begin{aligned} \langle\Omega| U^\dagger H U |\Omega\rangle &= \langle\Omega| U^\dagger \gamma_{i_1} U \cdots U^\dagger \gamma_{i_k} U |\Omega\rangle \\ &= R^{j_1}_{i_1} \cdots R^{j_k}_{i_k} \langle\Omega| \gamma_{i_1} \cdots \gamma_{i_k} |\Omega\rangle \end{aligned}$$

with

$$R^i_j = R^{(p)i}_{a_p} \cdots R^{(1)a_2}_j \qquad \text{satisfying} \qquad U^{(a)\dagger}\gamma_i U^{(a)} = R^{(a)j}_i \gamma_j$$

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SIMULATION OF `MEASURE(REG, QUBITS)`

...is also possible in poly-time, but does not really fit onto this slide. If you are interested nevertheless, have a look at Lemma 4 in `https://arxiv.org/abs/1108.3845`.

DEMO

Off to the Notebook we go...