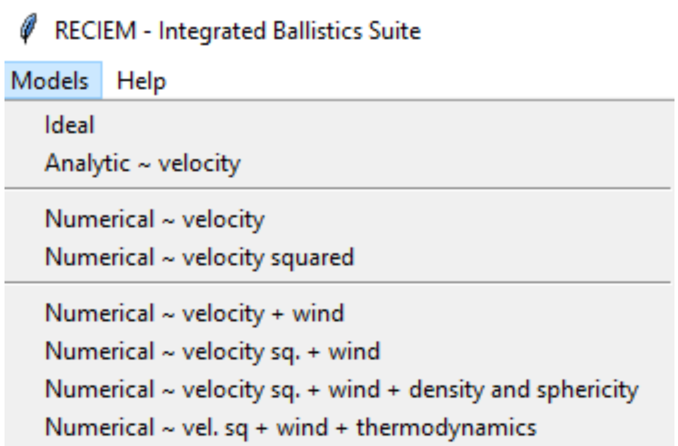
User Manual for BALISTICA soft,
authored by José Brenes-André, and coded by Santiago Nuñez

Overall view

This document pretends to be a basic users guide of a suite of 8 softs to be used in the analysis of ballistical movement of fragments, mainly caused by volcanic eruptions, although it can also be applied to sediment deposition. The suite seeks to appeal both to physicist and vulcanologists hoping that at some point they will find common ground and be more productive together.

On account of that, several precautions were taken to ease its use: screen set up, data input format, graphical data display, as well as numerical result of some characteriscal data have been kept as similar as possible.

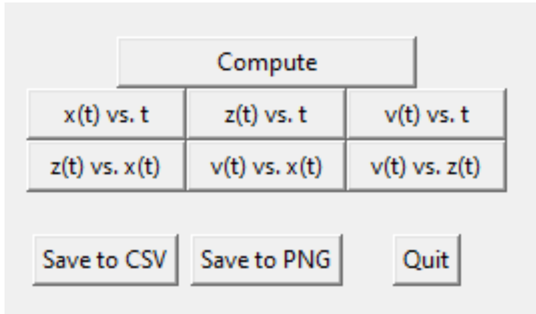
A look at the Models menu, reveals that conceptual complexity increases downwards, and that it has been divided into 3 groups



The ideal case has been set apart, and it is solved using the standard phormulae, found in a regular Physics textbook, for movement affected only by constant gravity, as a limiting case

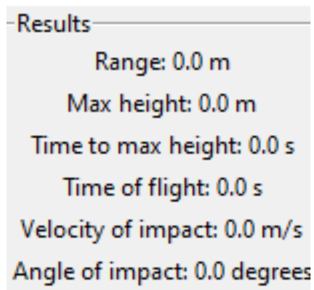
A second group is composed by the movement of a body affected by drag with constant C_d propotional to linear velocy, and it is solved both using mathematical equations resulting from solving the differenttial equation (denoted by Analytic ~ velocity), and by Euler method (denoted by Numerical ~ velocity), so as to allow the user compare the corresponding results, The group alos includes the movement of a body affected by drag with constant C_d but proportional to velocity squared. (denoted by Numerical ~ velocity squared), which is only solve by Euler method.

Once the input values have been input, the user is to click the *Compute* button to obtain the numerical results, which can be save in CSV format by clicking on the *Save to CSV* button. Several variables can be combined to graph by cliking on the corresponding button; *x(t) vs. T*, etc. Graph can be save as a PNG file.



User is the offer more options than the regular height vs range (z(t), x(t)) option.

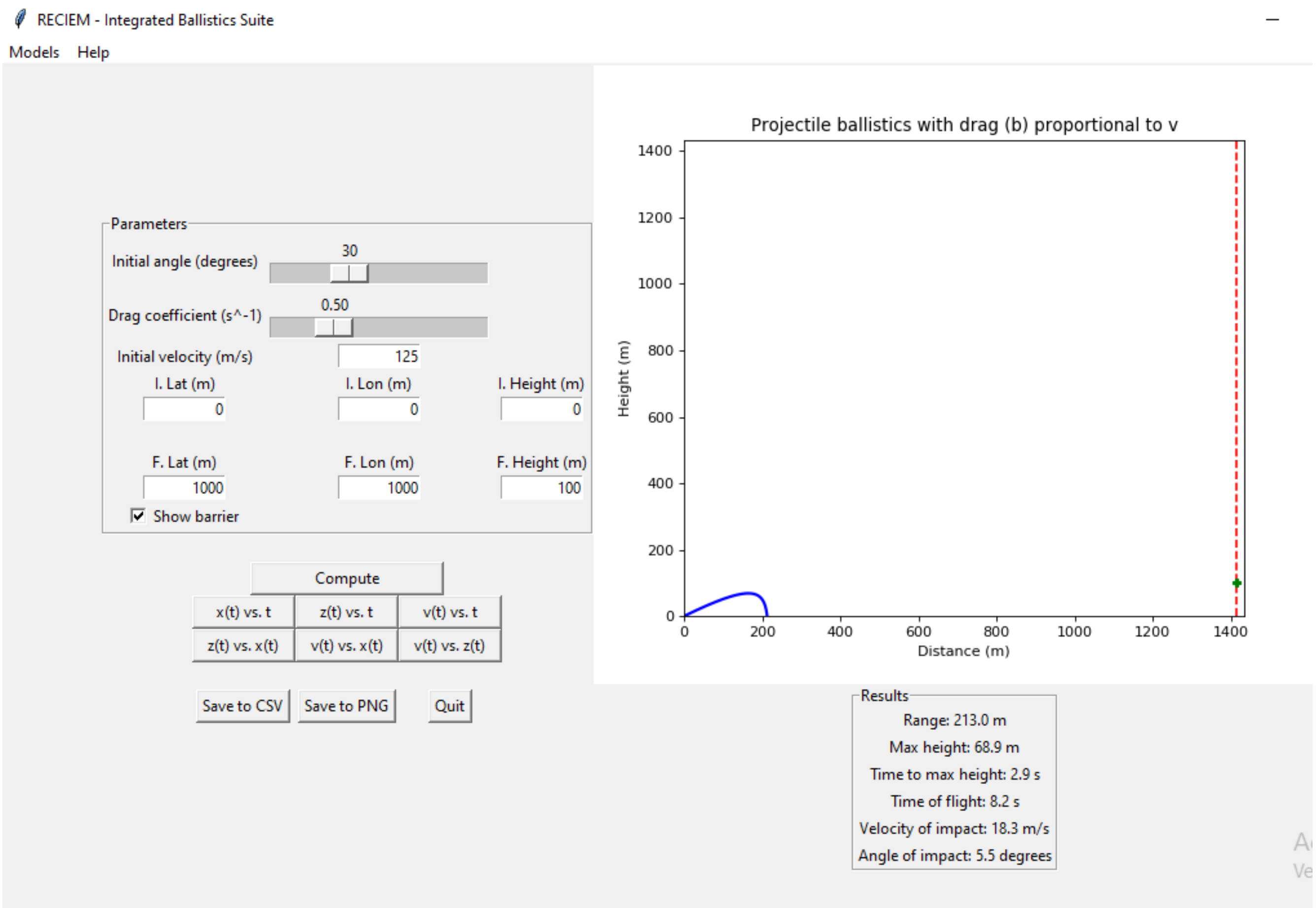
More precise values for chateristics points of the trajectory are included in table below the graph.



Positive values of impact angle are measure counterclockwise from a horizontal.

In order to provide the user with a more geological flavor, initial and final positions are input using any geographical coordinate system based on meters, such as Lambert.

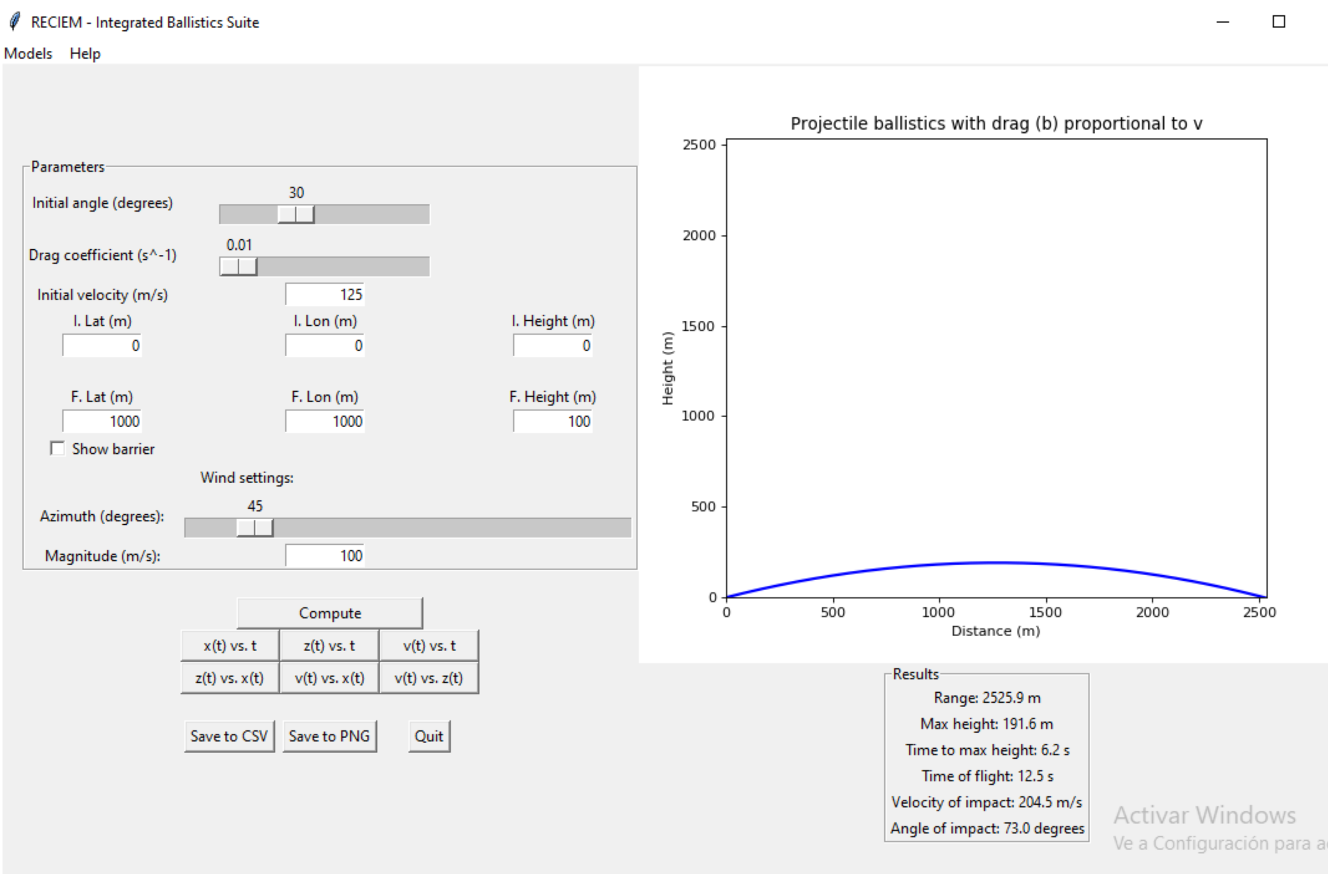
If needed the user can activate the "*show barrier*"option by means of which the target point si represented by a vertical line located at a rage calculated on a horizontal plane (x,y), over which a green asterisk is located at a z value equal to the diference in heigh of initial and final points. Negative height values correspond to final point lower than initial point.

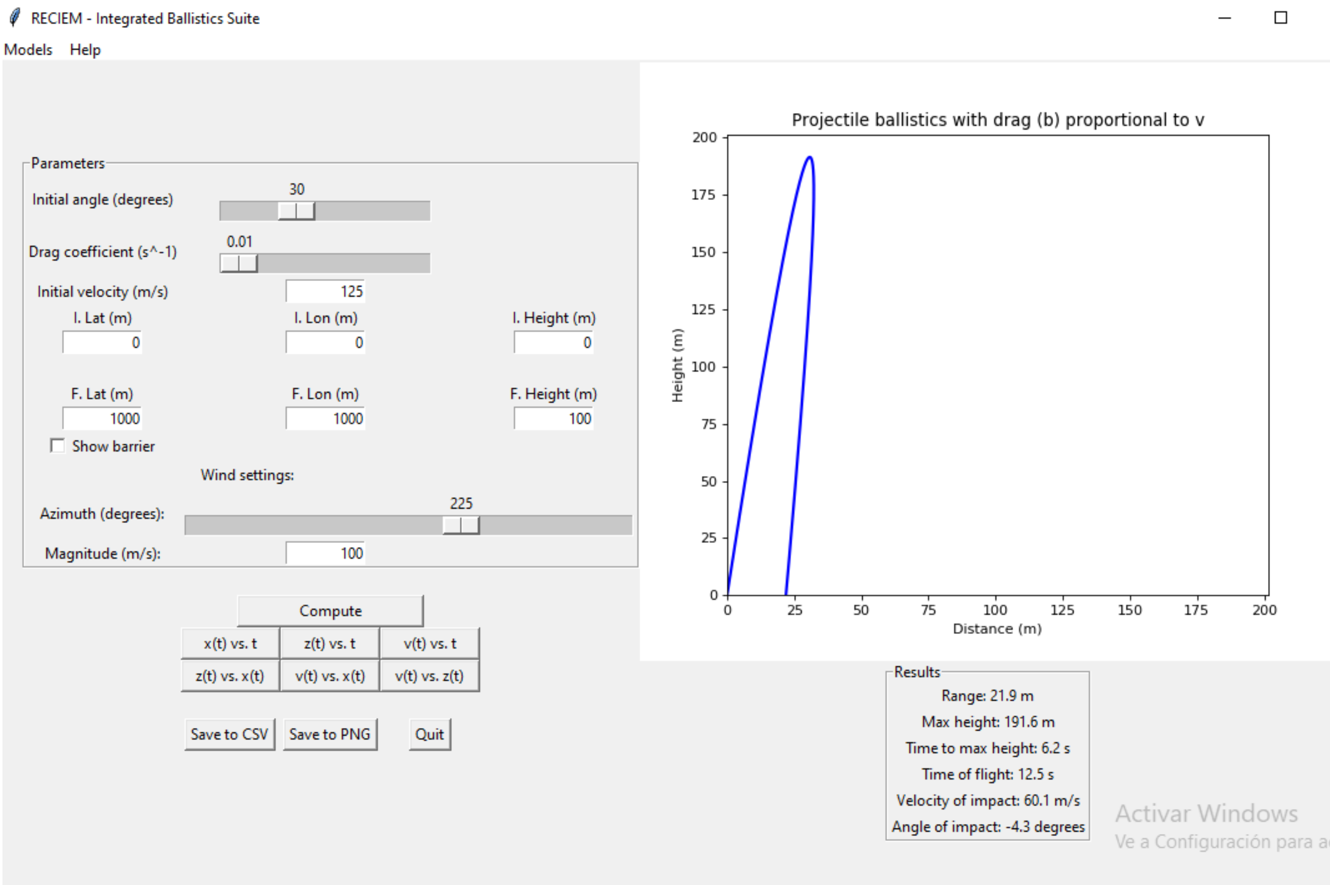


The graph above correspond to a fragment launch with a velocity of 125 m/s at an initial angle of 30° Fragment is subjected to a drag linearly dependent to velocity. Drag coefficient of 0,5 s^{-1} correspond to (b/m) where b is the regular drag coefficient, but now divided by mass.

Constant horizontal velocity wind effects can easily be analysed. Wind velocity direction is input as the weel known azimuth. Let us examine the case of a 100 m/s wind with 4 different velocities.

In our example, for a 45° azimuth fragment and wind will move in the same (x,z) plane. The corresponding differental equation is solved taking into account that, because of the curvature of the trajectory, the angle between the direction of the wind and the instant velocity of the fragment varies as the fragment moves. Thus , only a fraction of the wind velocity adds up to the fregament in any one moment. The overaal added effect is shown to result in a range of 2525,9 meters, compare to a mere 21,9 meters for an azimuth of 225°, which is also in the same (x,z) plane.

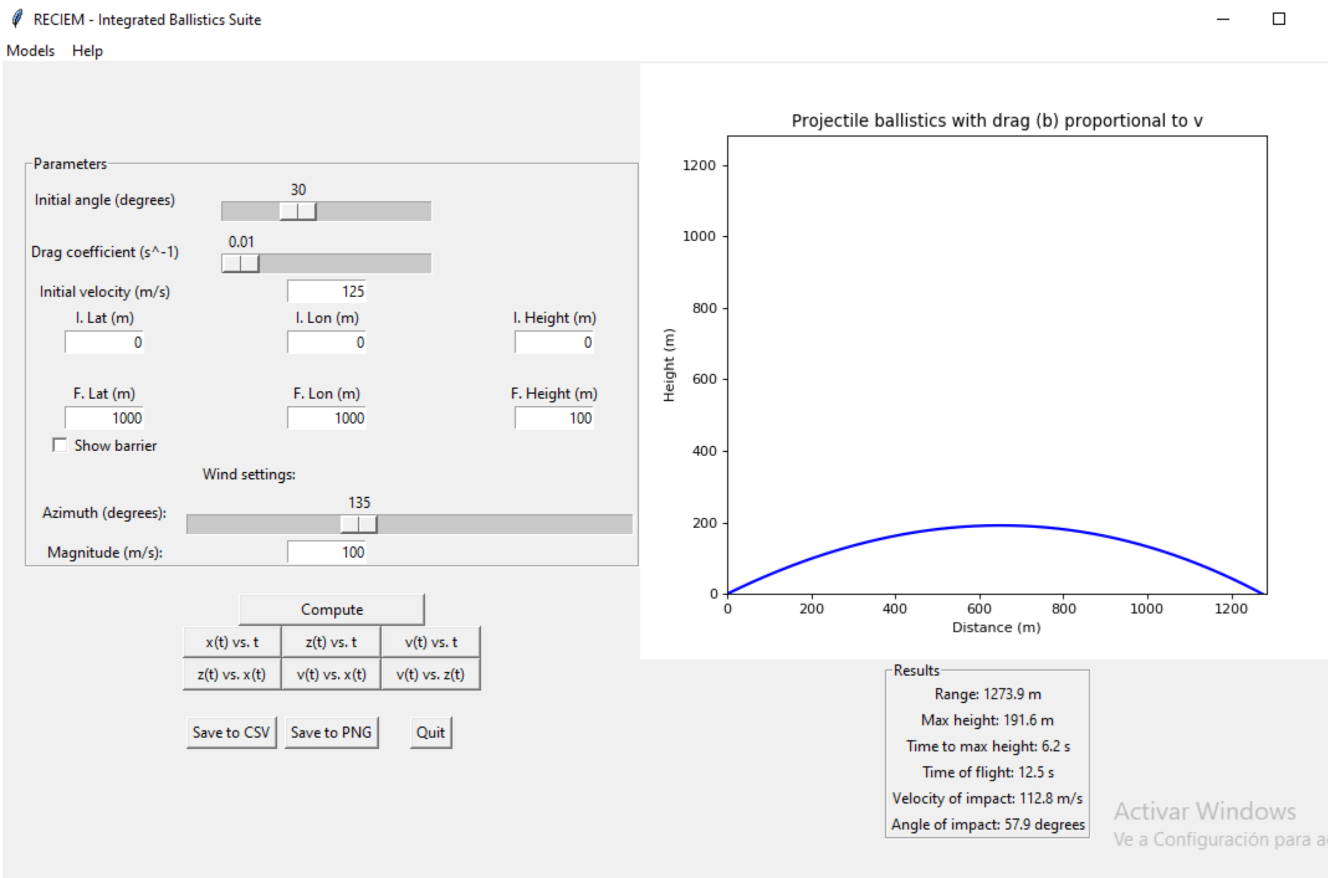




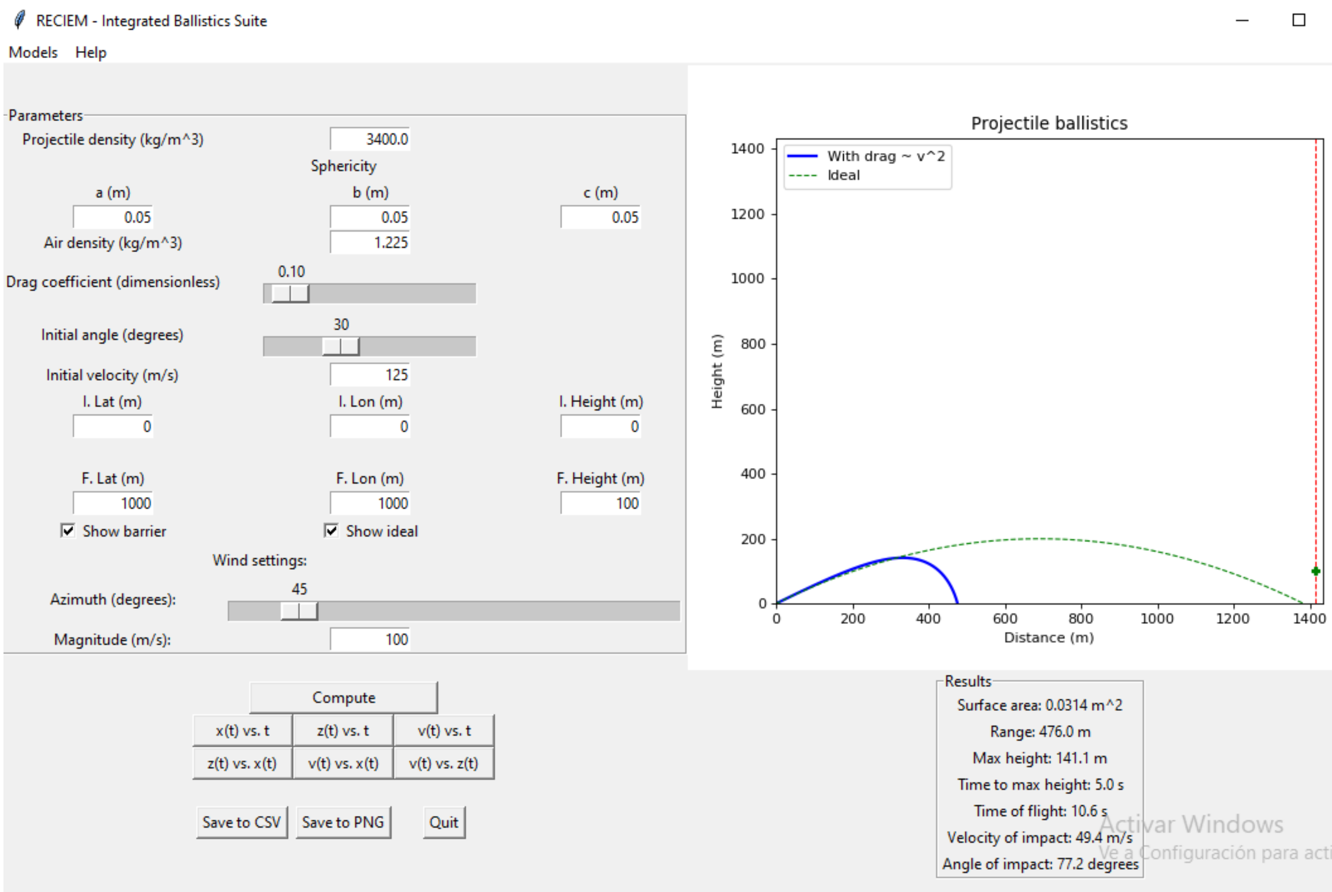
Please notice that the negative impact angle of – 4,3° is now measure from the vertical rather than the horizontal.

The user is waned that any other azimuth value will imply the existance of a wind component perpendicular to such (x,z) plane. In fact azimuths of 135° and 315° in our example will correspond to wind blowing totally perpendicular to such a plane. In reality this situation will deflect the trajectory in a manner similar to that of Coriolis effect, increasing or diminishing it, in a second order effect that we rather neglect, Because of this, none of our solutions include this second order effect.

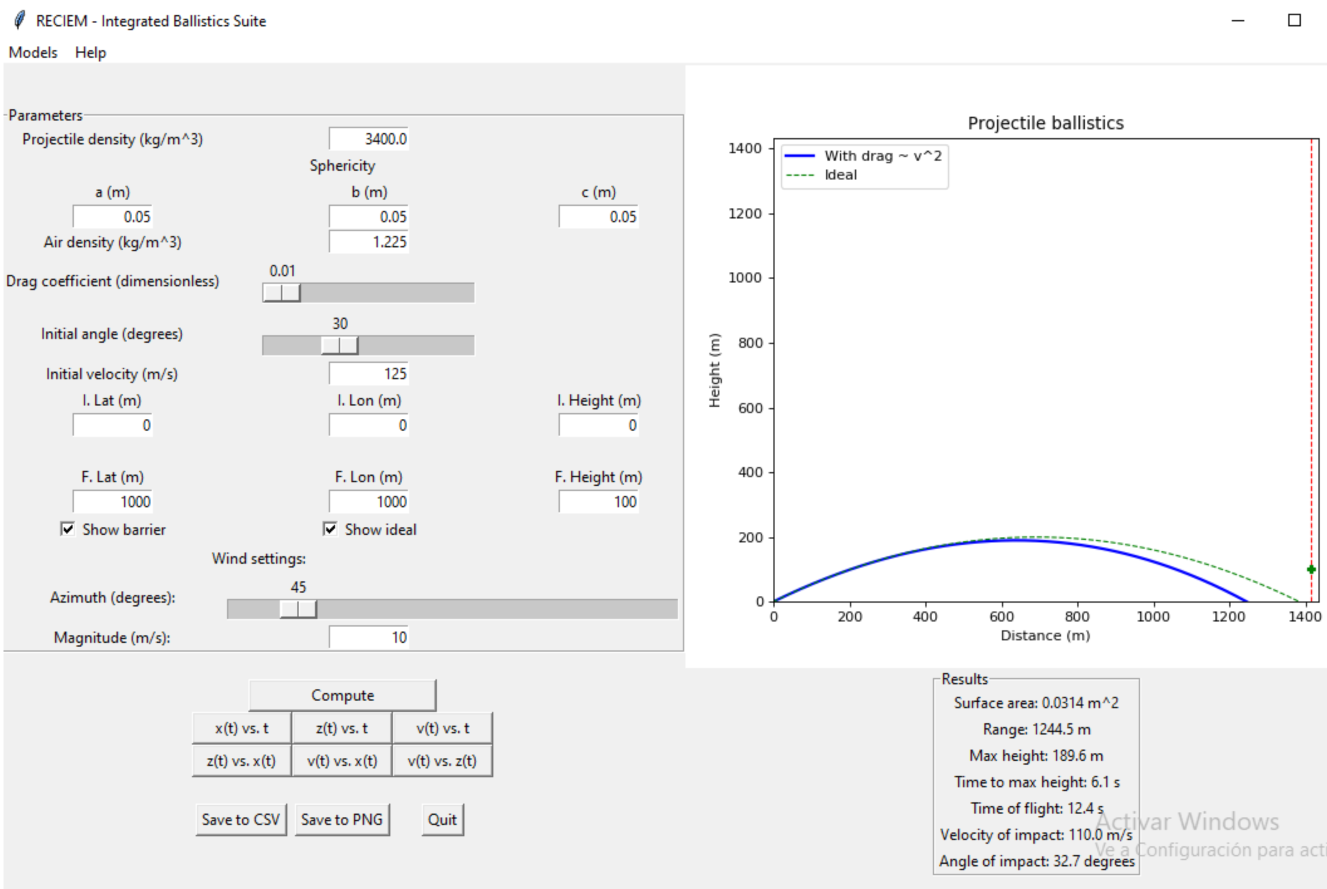
Thus the corresponding rage of 1273,9 meters would then correspond to the no-wind situation, shown in the following graph.

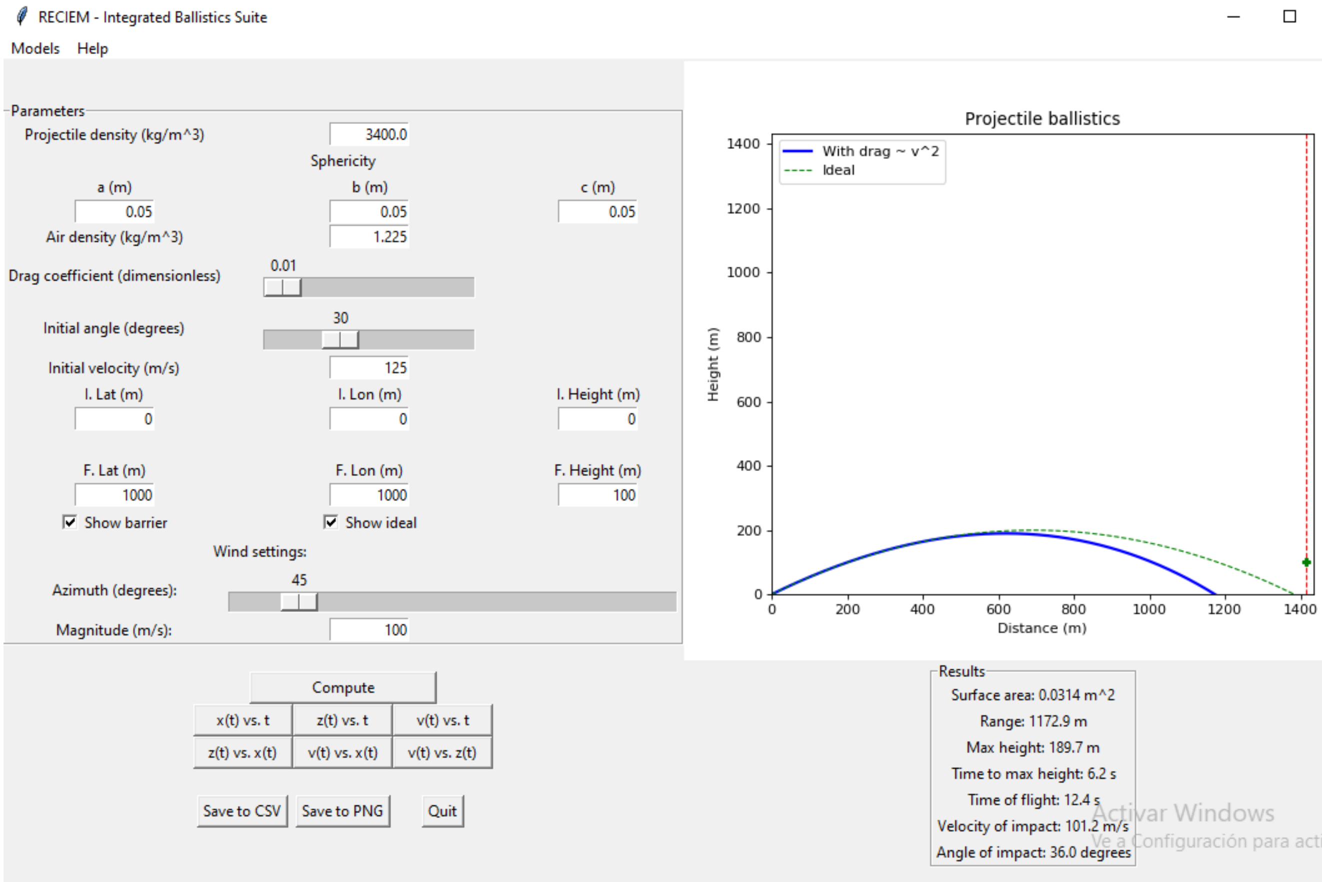


More complete analysis can be carried out if user chooses to include, in the same graph the "*show barrier*" and the "*ideal*" options, as shown in the following graph, as well as an approximate shape (fragments is considered as a ellipsoid) via the sphericity. Please notice that drag coefficient was increased tenfold from 0,01 to 0,1, so a dramatic change is effected.



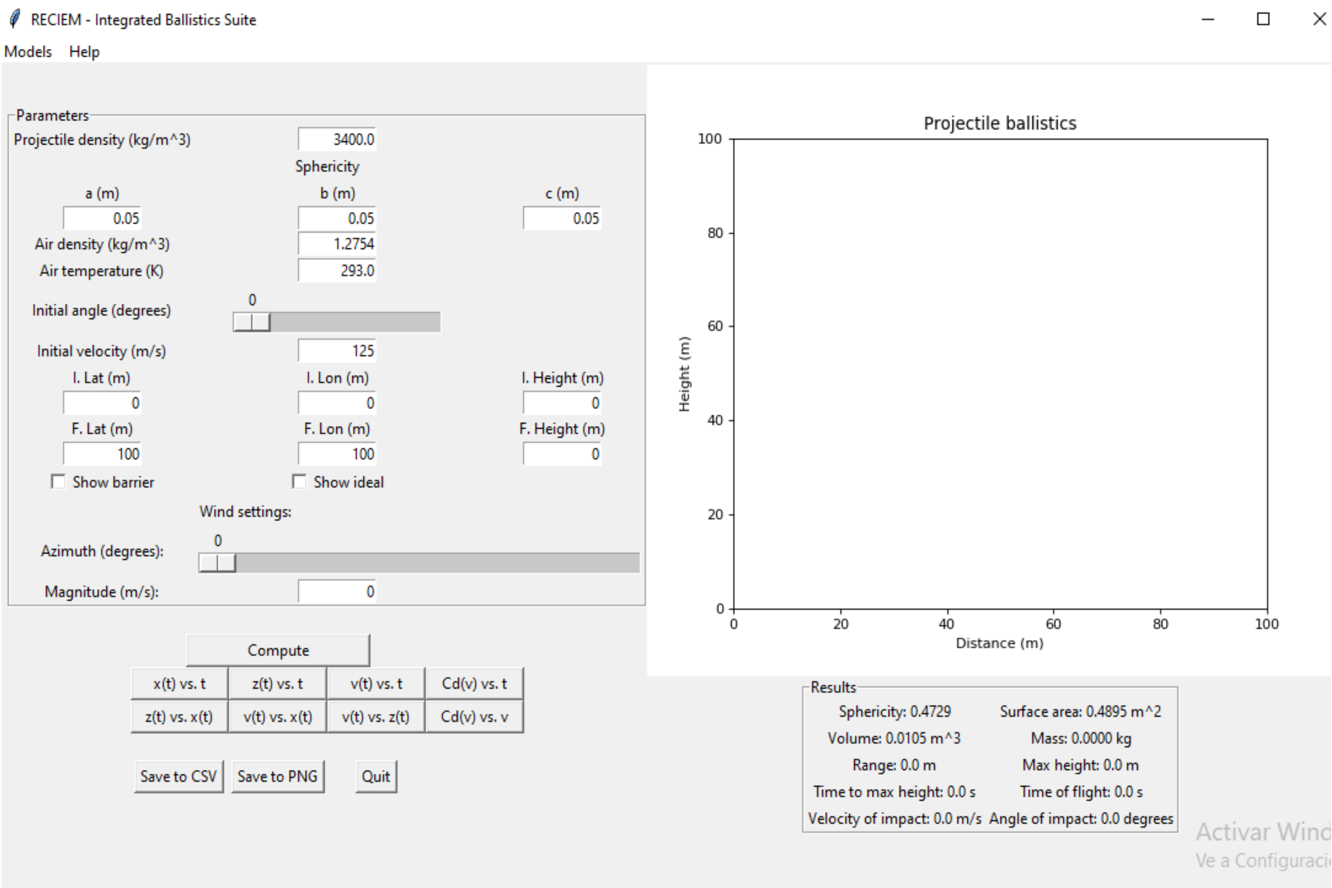
A small drag coefficient of only 0,01 would result in a nearly ideal case as seen in the following graph.


RECIEM - Integrated Ballistics Suite
Models Help
Parameters
Projectile density (kg/m^3) 3400.0
Sphericity
a (m) 0.05
b (m) 0.05
c (m) 0.05
Air density (kg/m^3) 1.225
Drag coefficient (dimensionless) 0.01
Initial angle (degrees) 30
Initial velocity (m/s) 125
I. Lat (m) 0
I. Lon (m) 0
I. Height (m) 0
F. Lat (m) 1000
F. Lon (m) 1000
F. Height (m) 100
Show barrier
Show ideal
Wind settings:
Azimuth (degrees): 45
Magnitude (m/s): 10
Projectile ballistics
With drag ~ v^2
Ideal
Height (m)
Distance (m)
Compute
x(t) vs. t
z(t) vs. t
v(t) vs. t
z(t) vs. x(t)
v(t) vs. x(t)
v(t) vs. z(t)
Save to CSV
Save to PNG
Quit
Results
Surface area: 0.0314 m^2
Range: 1244.5 m
Max height: 189.6 m
Time to max height: 6.1 s
Time of flight: 12.4 s
Velocity of impact: 110.0 m/s
Angle of impact: 32.7 degrees
Activar Windows


RECIEM - Integrated Ballistics Suite
Models
Help
Parameters
Projectile density (kg/m^3)
3400.0
Sphericity
a (m)
0.05
b (m)
0.05
c (m)
0.05
Air density (kg/m^3)
1.225
Drag coefficient (dimensionless)
0.01
Initial angle (degrees)
30
Initial velocity (m/s)
125
I. Lat (m)
0
I. Lon (m)
0
I. Height (m)
0
F. Lat (m)
1000
F. Lon (m)
1000
F. Height (m)
100
Show barrier
Show ideal
Wind settings:
Azimuth (degrees):
45
Magnitude (m/s):
100
Projectile ballistics
With drag ~ v^2
Ideal
Height (m)
Distance (m)
Compute
x(t) vs. t
z(t) vs. t
v(t) vs. t
z(t) vs. x(t)
v(t) vs. x(t)
v(t) vs. z(t)
Save to CSV
Save to PNG
Quit
Results
Surface area: 0.0314 m^2
Range: 1172.9 m
Max height: 189.7 m
Time to max height: 6.2 s
Time of flight: 12.4 s
Velocity of impact: 101.2 m/s
Angle of impact: 36.0 degrees
Activar Windows
Ve a Configuración para activ

In depth view

A more realistic analysis can be performed using the last sof the suite. A standard atmosphereand Sutherland relationship, with a reasonable value of air temperature at crater, are used to find air density and viscosity, allowing to calculate Reynolds number at any given point of the fragment trajectory.
In this way a variable C_d can be estimated at each point, for different values of sphericity, allowing to calculate fragment trajectory in a more realistic way.



Volcanic ballistic is an area of geology that, for multiple reasons, have received very little attention. Physical aspects such as the different type of forces involved, flight velocities, Stokes and Newton regimes existant during the fragment trayectory, drag coefficient variability united to the mathematical complexities to obtain a reasonably real movement, added to the few real data that can be gathered on the field definitively explains a good part of such a desinterest.

The following software package pretends to reverse this trend, by addressing several of these aspects in a concertated manner by disecting in several parts organized in an increasing level of difficulty. Although this partial independence allows any user to run the softwares in any order (s)he may find suitable, (s)he is encouraged to initially explore them in the presented in order to get an overview.

In all levels, the user is to input the initial data in the Parameters Box, either by sliding a pointer or else directly writting the numbers (a decimal point, rather than a comma, is expected). User can jump from one box to the next by using the TAB key. Click the COMPUTE button once your a finished.
The user can navigate among the 6 possible graphics by just clicking on the desired graphic label, if no initial data has been changed.

However, if one or more of the input data has been changed, the user is to click again on the COMPUTE button to incorporate the new data, before choosing any of the 6 possible graphs.

LEVEL 1

Initially, the simplest case of the parabolic motion without forces other than gravity, the way it is tackled in introductory courses of Phyics, is presented. Emphasis is placed in understanding the physics behind the terms: Range, Flight time, Maximum height, Initial and Final velocities, Initial and Final angle.

A detour: **DRAG**

Drag is a word that will be present from here on, so we better introduce it in a more formal manner. Let us start with a daily life example.

You can jump into a swimming poll in basically two radically different ways: in a belly flop, or else in an olympic style dive. In the former (not recommended!) a huge amount of water would have to be displaced in a short span of time, that is, would have to be accelerated to make room for you. The apparently high 'stiffness' of water that you feel reminds you that inertial effects are playing a very important role. Diving, on the contrary, means that small amounts of water are being displaced at a slow rate and viscous stress will dominate. Either way you experience what is technically known as *drag:* the resistance that a fluid (liquid or gas) exerts on a body opposing its movement. It is the result of two effects: the *skin drag,* consisting of the vicous friction between the body and the fluid that surrounds it; and the *form drag,* that arises due to fluid pressure variations across the body, and as expected it depends heavily on the shape of the object.

When diving, drag may be linearily proportional to your relative velocity with respect to water, in what is known as *creeping flow,* or *Stocke´s regime.* By choosing the right fluid viscosity and density, depending on its shape any body can be force to move under a such a regime. A bateria, for instance, would move under those conditions in a water drop. More precisely, creeping flow is expected when Re < 1, being Re the Reynolds number defined below.

At large Reynolds number, the body leaves behind a wake composed of highly disturbed turbulent fluid. To estimate the drag force we notice that the momentum density of the fluid would be of the order of ρv (ρ being the fluid density), and because body with cross-sectional area A will (per unit time) cover a volume Av, gives as a result a change of momentum with time (force according to Newton) of $A\rho v^2$. However, as the fluid in the wake is not completly at rest, the momentum change will be only a fraction of ρv. For the case of a sphere, for example, in the range of $10^3 < Re < 10^5$ only some 20% of the momentum is lost when the fluid impinges the sphere. This fraction varies for other Re. Hence, an empirically obtained dimensionless *drag coefficient* C_d has to be included when calculating the drag force.

In a more technical way, drag force is formulated as

$F_d = ½ \rho A v^2 C_d$, where

F_d = drag force

ρ = fluid mass density
A = cross-secctional area
v = velocity of the object relative to the fluid
C_d = drag coeficient

A quick search will show you that spheres are ubiquosly used for $A = \pi r^2$ comes very handy. However this approximation is out of question for volcanic debry, as calculation of A requires a previous thorough discussion of what is meant by the words *"shape"* and *"texture"* , and going to the lab to obtain the numerical values associated to them. In the next levels some practical ways to circunvent these problems will be presented.

In summary: The v depedence regime is known as Stocke´s regime or flow, the v^2 dependence is also called Newtonian´s flow or regime, and in between is the transition or Pradtl´s regime. An adimensional number (Reynold´s number) is used to tell them apart. To avoid scaring the user, we will wait to Level 6 to show how this is done in a practical way.

The set of equations associated to fluid dynamics, the so-called Navier-Stokes equations, are highly non-linear and can be solved under very restrictive conditions, defined by different adimensional numbers. Experimental research with "wind tunnels" was the only way out to get the qualitative aspects right (air planes were so designed), until recently when computational fluid dynamics is on the rising.

In this work, some of those solutions will be put to practical use, in such a way that transition from well known ideal case such as Level 1 to the comprehensive analysis of Level 6 will then be done in small steps to help grasp some of important (to our purpose!) points. For this matter, graphical presentation of results will be emphasized. Researcher willing to delve into details is invited to read the reference material.

It is expected that physics students will find this collage of softwares useful to get a better understanding of fluid dynamics, possibly finding volcanic applications interesting enough to give a helping hand. Vulcanologists, on the other hand, are invited to explore ballistic from a more quantitative point of view, and eventually meeting their physics counterpart mid way in the process.

LEVEL 2

An extra force is now added: drag force in its simplest form b**v** (drag proportional to velocity, a regime known as Stoke´s regime). At this point the user is to explore how the trajectory as well as the mathematical results vary as the value of the coefficient b is varied. At this level no hint is given as to how to calculate the b value for a particular case. Of course, nothing prevents the more advanced user to previously calculate a reasonable value of b for the problem at hand.

Just be known that the drag increases as the value of the coefficient b increases.

This is the general way this concept is dealt in the introductory Physics courses, with no particular problem in mind. The software here presented intends to motivate the exploration of different combination of the initial parameters and its consequences.

The mathematically inclined user may consult the mathematical appendix where the equations used in the soft as well as hints as to how find the velocity and position in terms of time.

LEVEL 3

A more complex case is now tackled: the Newtonian regime where drag is now proportional to velocity squared (bv^2). In the mathematical Appendix you can find how the mathematical equations relating position and velocity as functions of time are obtained. User is warned that a solid knowledge of Introductory Calculus is required. Again, nothing is said as to how to find a reasonable value of b for the problem at hand. The independence of each level allows the user to apply this software to problems other tha the volcanic ballistics.

The user is strongly invited to compare the results obtained in this two levels while keeping all other paremeters constant.

LEVEL 4

From a volcanical point of view, it has been that prevailing constant wind velocity does have an impact on the trayectory and velocity of the fragment while in flight. Following the trend, at this level wind velocity paralell to the ground is added to the equation of a fragment moving in a Stocke´s regime.

Mathematics gets now more involved, and a standard approximate technique known as Ruge Kutta method has to be used to trace the ballistic trajectory as well as obtain the final results. Hence, no close equations can be presented at this point. However, the user can still examine how the initial parabolic trajectory (only gravity acting), later modify by Stoke´s drag is further modified when wind is taken into account.

LEVEL 5

At this stage, the user can explore how wind affects the trajectory and instant velocity of a mass already moving in a Newtin´s drag regime. From the mathematical point of view, all it is required is to apply Runge Kutta to a new equation including a bv^2 term rather than the bv term of level 4. From the physics point of view, the effect is more pronounced as can be observed if the corresponding results (all other terms equal) for level 4 and level 5 are compared.

LEVEL 6

The time to submerge ourselves into vulcanology has now arrived!. For that we have to calculate a next to real coeficient b used to calculate the drag force, considered constant in the previous levels 3 nad 4. We will use the custommary definition $b = ½ C_d A \rho$ where C_d stands for the drag coefficient, A is the fragment object area on which the drag force is exerted, and ρ refers to the average density of the fluid (normally air) through which the fragment moves, and explore each of the parameters involved.

The irregular shape of volcanic fragments makes it practically impossible to find the exact value of A. To make things even worse, it is experimentally found that the drag coefficient C_d has a complex velocity dependence.

The author has deviced a self contained approach that hopefully would allow researchers both have a much better understanding of ballistic, and solve some of the practical problems present in ballistic vulcanology.

Reynold´s number.

Under certain conditions, the flow pattern in the vecinity of a body and the non-coefficients- are identical in air, water, or in other liquid or gaseous fluids. It is a standard practice to use scale model (that placed in the wind tunnel) and scale up the results to apply them to real object, of a much bigger size. That is why values for C_d found experimentally using wind tunnels are presented as graphics of C_d as functions of an adimensional number known as Reynold´s number. Due to the small size of the model required Reynold´s number can be obtained using fluids like air, water or even glicerin.

Reynold´s number, Re, is defined as

Reynolds number	Re	$\mathbf{Re} = \frac{UL\rho}{\mu} = \frac{UL}{\nu}$	fluid mechanics (ratio of fluid inertial and viscous forces)[4]

the ratio of the inertial force to the viscous force acting on the ballistic. U is the relative velocity of the model to the fluid; L is a characteristic length of the ballistic; ρ is the fluid density, and μ and υ the viscosity and dynamical viscosity, respectively, of the fluid used in the tests.

In addition, experimentally it is found that fluids with associated Re less than a critical value R_c flow in a laminar regime, whereas turbulence would set in for Re values greater than R_c, and drag effects are much different. If the fluid is the same in both cases, and the ballistic does not deform, the only free parameter is then the instantaneous velocity U, which in fact is one of the unknown the reasercher seeks to define!!

Fragment shape

In volcanic ballistic studies, due to its complexity and inherent dangers, very scant filed data can be obtained. Fragments can not be painted, for instance, to associate them to a given eruption; effect of an eruption on the already ejected fragments are unavoidable, as it is the possibility of eventual fragmentation upon collision which prevents the researcher to know the actual shape of the fragment from the moment it was ejected to just before landing. Hence the researcher can only hope that the samples collected did not deform in flight, nor on landing.

Wadell (1934) defined the sphericity, Ψ, of an object as the ratio of the nominal surface area (surface area of a sphere having the same volume as the object) to the actual surface area of the object. This ratio is known as true sphericity index. The sphericity index of a sphere is 1 and, by the isoperimetric inequality, any object which is not a sphere will have a sphericity value less than 1.

$\Psi = S_n / S$

S and S_n are the surface area and nominal surface area of the object respectively

A word of warning by Smart (2013)

"***the insistence by Wadell, 1932, p. 446, on "a clear mathematical distinction between shape and roundness" was both a great step forward and an inhibiting influence. It has since been suggested that 'shape' is not a unique concept (e.g. Blott and Pye, 2008); and 'shape' has been divided below into 'anisotropy' and 'regularity'. Straight edges and flat facets are treated separately. Then follow, 'roundness' and 'roughness'; after which under the heading 'Experimental', some remarks about concepts which are important but for which established methods are yet to emerge."***

In fact, in Blott and Pye (2008) it is observed:

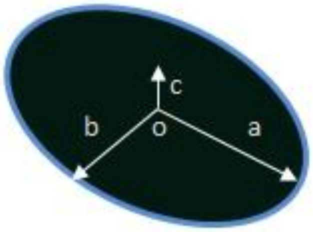
Shape is a fundamental property of all objects, including sedimentary particles, but it remains one of the most difficult to characterize and quantify for all but the simplest of shapes. Despite a large literature on the subject, there remains widespread confusion regarding the meaning and relative value of different measures of particle shape

Due to the relative easyness in the field as a first approximating every fragment would be considered an ellipsoid of mutually perpendicular major semi-axis **a**; intermediate semi-axis **b**, and minor semi-axis **c.**

The calculation of the exact surface area of an smooth ellipsoid, needed to calculate sphericity ψ is rather cumbersome In this work, we opted to use the

Knud Thomsen's Formula

According to wikipedia.org the surface area of a general ellipsoid cannot be expressed exactly by an elementary function. However an approximate formula can be used and is shown below:



a, b and c defines the vertical distances from the origin of the ellipsoid to its surface.

$$SA = 4 \cdot \pi \left(\frac{a^p b^p + a^p c^p + b^p c^p}{3} \right)^{1/p}$$

Where $p \approx 1.6075$ yields a relative error of at most 1.061% (Knud Thomsen's formula)

π defines the ratio of any circle's circumference to its diameter and is approximately equal to 3.141593, however the value 3.14 is often used.

Example: determine the surface area of a ellipsoid that has following properties:
a = 2 m
b = 3 m
c = 4 m

$SA = 4 \cdot \pi \cdot ((a^{1.6075}b^{1.6075} + a^{1.6075}c^{1.6075} + b^{1.6075}c^{1.6075})/3)^{1/1.6075} = 111.604\ m^2$

Online Surface Area Calculator, click on the link will open a new window.

Cross section area A.

The area A that apperas in the definition b = ½ C_d A ρ is the projected area perpendicular to the velocity at each instant. In the case of a sphere of radius r is inmediately found that $A = \pi r^2$. But, as soon as the fragment is approximated to an ellipsoid a lot of practical problems show up. Ballistic trajectories usually last less than 20 seconds, a short time to empirically determined if the fragment rotated in such a way as to move in the most aerodynamical manner, in the event that it does it!. Air turbulence near the volcanic plume may force the fragment to tumble. Fragment collisions among themselves may eject one of them rotating along one of its axes. Researchers can think of a couple of more problems.

According to the IUPAC definition, the equivalent diameter of a non-spherical particle is equal to a diameter of a spherical particle that exhibits identical properties (e.g., aerodynamic, hydrodynamic, optical, electrical) to that of the investigated non-spherical particle.

The aerodynamic equivalent diameter is that of a unit density sphere having the same settling velocity (due to gravity) as the particle of interest of whatever shape and density. For particles in non-turbulent motion, the equivalent diameter is identical to the diameter encountered in the Stokes law because $v_t = (g\rho/b)\ V$, being V the the particle volume. We here propose that volume $V = (4\pi/3)\ \boldsymbol{abc}$ implies an average cross sectional area $A = \pi\ (\boldsymbol{abc})^{2/3}$. This is another way of saying that we will use the geometric average of the 3 semi-axis as the equivalent radius, rather than choosing the arithmetic average.

Calculation of drag coeficient C_d

Yang (2003) pointed out that the drag coefficient would be a function of the same properties used to calculate the drag force (C_d would be a function of the particle´s Reynolds) only when fluid is Newtonian. However, surface roughness and small-scale vesicularity can also significantly decrease the drag coefficient.

In spite of these caveats, some correlation between C_d and particle shape is needed to advance in the analyzis of the volcanic ballistic processes. The way out proposed here is based on results that naturally takes into account the main problems in a reasonable way.

Laminar (also known as Stoke´s) regime ($Re < 1$) requires the use of a drag term linearly proportional to the velocity. For the turbulent regime ($Re > 10^3$) (usually denoted as Newtonian in the sedimentology area) a velocity square dependence for the drag is required. Both results should merge smoothly in the intermediate regime ($1 < Re < 10^3$). The Clift & Gauvin (1970) relationship between the drag coefficient C_d and the Reynold´s number Re is stated as

$$C_D = \frac{24}{Re}(1 + A Re^B) + \frac{C}{1 + \frac{D}{Re}}$$

automatically solves all these problems. By fitting (uncertainties less than 5%) the above relation to experimental values of 400 experimental data, Haider & Levenspiel (1989) showed that value of parameters A, B, C and D do not vary when fluid changes from laminar to turbulent regimes, which permits calculations in the very large range from $Re = 0$ till $Re < 2 \times 10^5$.

A theoretically derivation of C_d from the Navier-Stoke´s equations can be found in Concha (2009), applied to regular shaped solids. Although it includes sphericity, it is there observed that **Concha and Christiansen (1986) found it necessary to define a *hydrodynamic shape factor* to be used with the above equations, since the usual methods to measure sphericity did not give good results.** And a few lines below, it was explained that **"The hydrodynamic sphericity may be obtained by performing sedimentation, or fluidization experiments, calculating the drag coefficient for the particle using a volume equivalent diameter and obtaining the sphericity (defined for isometric particles) that fit the experimental value."**

In other words, C_d as a function of sphericity ψ was deduced, and a value of ψ has to be found to fit the experimental C_d . In addition, no proper correlation have been developed for the prediction of drag over arbitrarily shaped particles (Yang, 2003: p.17). Isaacs & Thodos (1967) , as well as Thompson & Clark (1991) proposed that sphericity is inadequate to describe drag flow over irregular particles. Our proposal may reduce these objections for a given value of ψ can now be used, and parameters A, B, C and D are now adjusted to fit the experimental C_d .

Hence the only uncertainty left is how good does sphericity ψ reflects the fragment movement. It is expected that the irregularity of volcanic fragments would cause a little or even none drag crisis since they have a consistent bluff-body separation point throughout a wide range of Reynold´s number Re, as mentioned in Loth (2008). The sudden decrease in the drag coefficient as the fragment speed increases is commonly known as the 'drag crisis'. Cylinders and spheres are considered bluff bodies because at large Reynolds numbers the drag is dominated by the pressure losses in the wake. (Cross, 2016). However, this same irregularity, plus expected surface roughness would definitevely cause that the real surface area be different from theoretically calculated, corresponding to a smooth surface. In addition, several different shapes may be asociated to a same sphericity.

Roughness is another source of concern. Golf balls, for example, are designed with dimples on its surface to augment the turbulence very close to the surface, to increase the pressure behind the ball by bringing the high speed air stream closer. Thus, for this same reason it is reasonable to expect that the excess of pressure in front of the ejected volcanic debris with respect to the back, may be reduced by the roughness and re-entrant parts, causing a drag reduction and even a deviation from a straight line course as a consecuence (Cross, 2016)

How all this was used in the soft.

Haider & Levenspiel (1989) included two very important points for our soft.

First, several tens of samples of a given sphericity ϕ (gathered from different papers) were used to obtain experimental values for C_d as Reynolds number Re were swept, and values of A, B, C and D were obtained by best-fitting.The resulting values are shown i the following table

TABLE 2
Best values of parameters to be used in eqn. (4) for predicting C_D for particles of various sphericities

ϕ	A	B	C	D
1.000	0.1806	0.6459	0.4251	6880.95
0.906	0.2155	0.6028	0.8203	1080.835
0.846	0.2559	0.5876	1.2191	1154.13
0.806	0.2734	0.5510	1.406	762.39
0.670	0.4531	0.4484	1.945	101.178
0.230	2.5	0.21	15	30
0.123	4.2	0.16	28	19
0.043	7	0.13	67	7
0.026	11	0.12	110	5

It was expected that a fitting done using 408 experimental data reported by different authors would result in near to real life averages.

Second, given mathematical equations relating parameters A, B, C and D to any given sphericity were derived by bestfitting values given in Table 2. The uncertainty inherent in guessing A, B, C and D from eye inspection is then reduced, allowing to include such mathematical relations in a software.

$$A = \exp(2.3288 - 6.4581\,\phi + 2.4486\,\phi^2) \quad (1$$

$$B = 0.0964 + 0.5565\,\phi \quad (1$$

$$C = \exp(4.905 - 13.8944\,\phi + 18.4222\,\phi^2 - 10.2599\,\phi^3) \quad (1$$

$$D = \exp(1.4681 + 12.2584\,\phi - 20.7322\,\phi^2 + 15.8855\,\phi^3) \quad (1$$

For coherence, the original equations will be kept, rather than trying to find our own.
No uncertainty for these parameters was not quoted in the article.

Fluid caracterization

The last element of b = ½ C_d A ρ to be examined is the density of the fluid ρ, and the viscosity μ and υ (included in Reynold´s number).

Viscosity will be calculated using Sutherland result

Sutherland's law

In 1893 William Sutherland, an Australian physicist, published a relationship between the dynamic viscosity, μ, and the absolute temperature, T, of an ideal gas. This formula, often called Sutherland's law, is based on kinetic theory of ideal gases and an idealized intermolecular-force potential. Sutherland's law is still commonly used and most often gives fairly accurate results with an error less than a few percent over a wide range of temperatures. Sutherland's law can be expressed as:

$$\mu = \mu_{ref}\left(\frac{T}{T_{ref}}\right)^{3/2}\frac{T_{ref}+S}{T+S}$$

T_{ref} is a reference temperature.
μ_{ref} is the viscosity at the T_{ref} reference temperature
S is the Sutherland temperature

Some authors instead express Sutherland's law in the following form:

$$\mu = \frac{C_1 T^{3/2}}{T+S}$$

Comparing the formulas above the C_1 constant can be written as:

$$C_1 = \frac{\mu_{ref}}{T_{ref}^{3/2}}(T_{ref}+S)$$

Sutherland's law coefficients:

Gas	$\mu_0[\frac{kg}{ms}]$	$T_0[K]$	$S[K]$	$C_1[\frac{kg}{ms\sqrt{K}}]$
Air	1.716×10^{-5}	273.15	110.4	1.458×10^{-6}

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- **Sutherland, W. (1893)**, "The viscosity of gases and molecular force", Philosophical Magazine, S. 5, 36, pp. 507-531 (1893).

Air density will be calculated using a lapse of 0,00065 K/m, according to the following equation

Summary

In principle, a laptop and the Runge Kutta method would be enough to calculate the ballistic instantaneous velocity at any time. In brief, the initial velocity value v_o provided by the reasercher is used as seed value to calculate the net force acting on the ballistic (gravity, wind and drag bv^2), at $t = 0$. Newton´s second law net force $= m\, dv/dt$ can be used to calculate the velocity change (dv) a short time dt afterwards. Sample semi-axis a,b and c. are used to obtain the corresponding sphericity, Clift & Gauvin (1970) model is next used to obtain C_d . In this way the indirect velocity dependence of the drag force ($\frac{1}{2} C_d A \rho v^2$), due to C_d dependence on the Reynold´s number Re can be resolved. Defining $v = v_o + dv$ as a new initial velocity this procedure can be iterated over and over to find the instantaneous velocity dependence with time for that sample.

Although thisis a round about method, it has five strong points: a) the shape of each given particle can be taken into account, rather than having to arbitrarily assimilate them to just 3 fixed cases (esphere, high cube, and low cube) as required in EJECT!; b) the parameter dependency on esphericity was obtained from a fitting to experimental data which includes other experimental conditions such as surface roughness, c) as it was said above, Reynold´s number is used to allow extend the dynamical findings (in our case, the possibility of using the same C_d experimentally found in wind tunnels can also be used for bigger fragments. This can be done if there exists a dimensional similarity which, by the definition of spherity coefficient, is automatically fulfill for the coefficient does not change if all 3 dimensions of the fragment are multiply by the same factor; d) the method here proposed is based on an approximation valid for both the Stoke´s and the Newtonian regimes, so no a priori distintiction between the two has to be performed, e) this method is more comprehensive than the one used, for instance, in software EJECT!, where, for simplicity, the C_d for shapes other than a sphere is fetched from experimental curve of C_d versus Mach number (ratio of velocity of ballistic to the velocity of sound). Velocity of sound is readily calculated using $\sqrt{\gamma}RT$.

LEVEL 7

Leaning even more to the geological side, the last program is included to ease the exploration of different scenarios that may minimized ballistic hazard. Although the required mathematical equations are simpler than those of level 5 and 6, a deeper geological insight is required. The adjoint paper shows both the mathematical and the geological aspects. User is warned that the program is designed to help explore different scenarios, but the final decisions are totally the user´s responsability.

EXAMPLES, OBSERVATIONS, TIPS

LEVEL 1

Choose an angle θ such that $0 < \theta < 45°$, with a given velocity
Observe in the Results box that initial angle is the same as the final angle, and that time required to reach maximum height is the same required to reach ground starting from the maximum height

This happens regardless of the angle, and is a consequence of the energy conservation.

By using the same initial velocity but a new initial angle $\varphi = 90° - \theta$ test that the range (distance comprised from launch to arrival point) is the same for both cases. Test also that time required from launch to reach maximum height equals that required to touch ground starting from maximun height. Notice that this does NOT imply that flight time for initial angle φ is the same as that for initial angle θ. Explain physically why one is bigger than the other.

LEVEL 2.

Before actually running the software, what qualitative changes you expect if, using a constant initial angle and velocity, you run the program using b values from 0,1 to 0,5 in steps of 0,1. Did you succeed in your forecast? If not, what do you think you did not consider?

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