Combining Behavioral Choice with a Branching Process Model of Disease

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Disease Spread

How Contagion Spreads

- (Newman 2002) describes a class of networks on which an SIR model can be solved exactly.
- Social network is an infinite random graph described by degree distribution $\{p_k\}$
- Contagion can spread along each edge with probability T

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$$T_c = \frac{\sum_k (p_k k)}{\sum_k (p_k k(k-1))}$$

- If $T < T_c$, epidemic occurs with zero probability.

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- An "epidemic" occurs if the contagion spreads to an infinite number of nodes. (A non-zero portion.)
- Given degree distribution, there is a critical transmissibility threshold T_c
- When $T > T_c$, the probability an epidemic occurs equals the expected portion of nodes which become infected. Denoted R_∞

$$R_\infty = 1 - \sum_k \left(p_k \cdot (1 - (1 - \upsilon) T)^k \right)$$

where $\upsilon \in (0, 1)$ is the solution to

$$\upsilon = \frac{\sum_k \left(p_k k \cdot (1 - (1 - \upsilon) T)^k \right)}{\sum_k (p_k k)}$$

Important Variables so Far

- $\{p_k\}$is the degree distribution of the network.
- T is transmissibility.
- T_c is the critical transmissibility threshold.
- R_∞ is the probability and size of epidemic when $T > T_c$
- υ can be thought of as the chance a random neighbor remains uninfected.
- Finally, define the risk of disease from a neighbor Ψ as

$$\psi \equiv \begin{cases} 0 & \text{if } T \leq T_c \\ (1-\upsilon)T & \text{if } T > T_c \end{cases}$$

Individual Choice and Equilibrium

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- Each person *makes this choice exactly once*, when news of a potential epidemic arrives.

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- The critical transmissibility threshold is given by:

$$T_c(\{N_i\}) = \frac{\sum_i \alpha_i N_i}{\sum_i \alpha_i N_i^2}$$

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- The probability and size of the epidemic when $T > T_c$ is given by

$$R_\infty = 1 - \sum_i \left[\alpha_i e^{-(1-\upsilon)TN_i} \right]$$

where υ is the solution to

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- And finally, let $\Psi^* (\{N_i\})$be the value of Ψ, taken as a function of the set of choices.
  - When $T \leq T_c (\{N_i\})$, $\Psi^* (\{N_i\}) = 0$
  - When $T > T_c (\{N_i\})$, $\Psi^* (\{N_i\})$ is the solution $\Psi \in (0, 1)$ to:

$$\Psi = T \frac{\sum_i A_i N_i (1 - e^{-\Psi N_i})}{\sum_i A_i N_i}$$

People like being friendly, but dislike disease risk.

- The payoff for a person of type i is

$$U_i(N_i; \Psi) = u_i(N_i) - \delta_i \cdot \left(1 - e^{-\Psi N_i}\right)$$

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- $1 - e^{-\Psi N_i}$ is the probability of getting sick during this outbreak.
- δ_i is the disutility from getting sick.

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  - The total payoff $U_i(N_i;\Psi)$ is continuous and concave down,
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- If $\delta_i = 1$ for all i, then the following function has these properties:

$$u_i(N) = \ln\left(\frac{N}{\theta_i}\right) - \frac{N}{\theta_i}$$

where θ_i is the person's optimal choice when $\Psi = 0$

Equilibrium

Given exogenous $T, \{\alpha_i\}$, an equilibrium in this model consists of Ψ, N_i such that

$$\Psi = \Psi^* \left(\{N_i\}\right)$$

$$N_i = N_i^*(\Psi) \equiv \arg\max U_i(N_i; \Psi)$$

Equilibrium Existence

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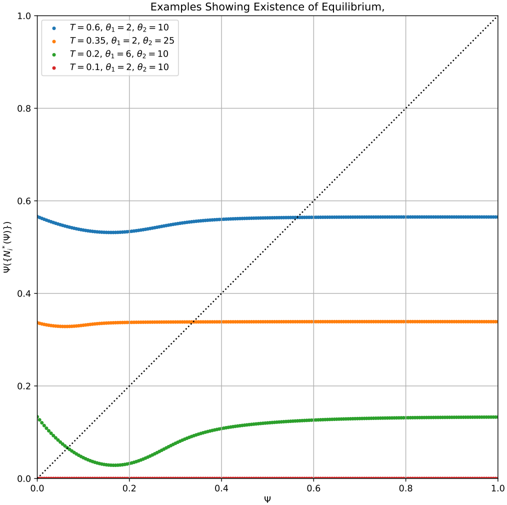

Equilibrium Existence

- **Proposition 1:** If for each i, the optimal policy function $N_i^*(\Psi)$is a continuous non-negative function on $\Psi \in [0,1]$,then an equilibrium exists.
- **Proposition 2:** Iff $T \leq T_c\left(\left\{N_i^*(0)\right\}\right)$, then there is an equilibrium without any risk of epidemic exists, where $\Psi = 0$ and $N_i = N_i^*(0)$ for all i.

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- When $\Psi > \frac{1}{N_i}$, an *increase* in disease risk may lead to individuals trying *less* hard to avoid getting sick.

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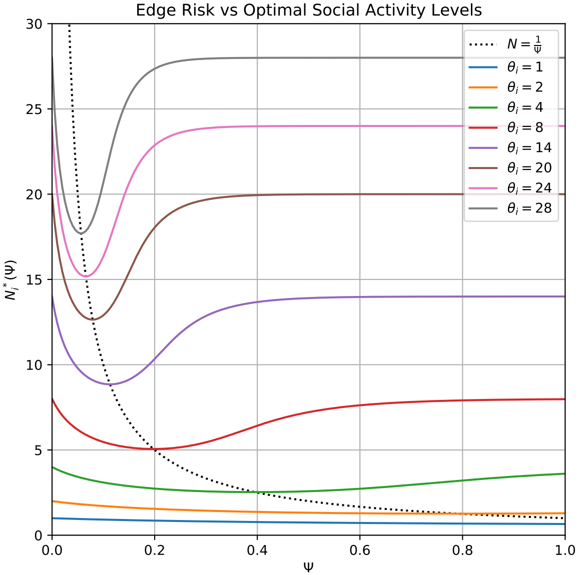

When can N_i have *positive* externalities?

- **Proposition 3:** Suppose $\{N_i\}$ is such that $T > T_c(\{N_i\})$. In this case,

$$\frac{\partial \Psi^*(\{N_i\})}{\partial N_j} < 0$$

$$\Updownarrow$$

$$\left(1 - e^{-\Psi^*(\{N_i\})N_j}\right) < \frac{\Psi(\{N_i\})}{T}\left(1 - TN_j e^{-\Psi(\{N_i\})N_j}\right)$$

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Related Literature

- (Newman 2002) Describes a model of disease spread based on branching processes and uses it to explicitly solve SIR models for a class of networks.
- (Meyers et al. 2005) shows that this works well to approximate the behavior of complex social networks.
- (Kremer 1996) Demonstrates similar results regarding fatalism and counter-intuitive externalities in a model of the steady-state of an endemic disease, rather than a disease outbreak.