Carnot Number

Input/Output Temperature Difference in Hydroelectric Power Stations

Seasonal Temperature Difference in Ocean/River Systems

Human Relevance

Grand Coulee Dam

- Hydro electric stations are the oldest form of grid scale power plant
- Largest power plant in the US ~ 7GW
- ~2× larger than the second largest power station (Palo Verde - Nuclear)
- Roosevelt Reservoir heats up the rivers waters
  - Much larger impact than factories or traditional power plants which use rivers/streams for cooling
- Salmon are particularly sensitive to the temperature of the river
  - >15°C strains the fish, >20°C can be fatal to the fish
- In 2019 ~ $\frac{1}{2}$ of all 500,00 sockeye salmon returning to the Columbia River basin died because of abnormally high temperatures
    - https://www.oregonlive.com/environment/2015/07/hot_water_killing_half_of_colu.html

Approximating The “Carnot Number”

Heat Drop Across Hydro-plant

Definitions

$$\eta \equiv \text{Turbine efficiency of power plant} \Rightarrow (1-\eta) \approx f \cdot 10^{-1}$$
$$g \equiv \text{Gravitational aceleration} \approx 10\ \frac{m}{s^2}$$
$$h \equiv \text{Height Difference between upper and lower reservoirs } \approx 10^2\ m$$
$$c_p \equiv \text{Specific heat of water } \approx f \cdot 10^3\ \frac{J}{\text{kg}\cdot{}^\circ\text{C}}$$
$$\dot{m} \equiv \text{Flow rate through plant}$$
$$t \equiv \text{time}$$
$$T \equiv \text{Temperature}$$

Governing Eqns.

$$\text{Power into Heat:} \quad P_H = (1-\eta)\,\dot{m}\,g\,h$$
$$\text{Work into heat:} \quad W_H = \int_0^{\Delta t} P_H\, d\tau = (1-\eta)\,\dot{m}\,g\,h\,\Delta t$$
$$\text{Heat to raise the temp. of water:} \quad \Delta Q = c_p(\dot{m}\,\Delta t)\,\Delta T$$
$$(1-\eta)\,\dot{m}\,g\,h\,\Delta t = c_p(\dot{m}\,\Delta t)\,\Delta T$$
$$\Rightarrow \quad \Delta T = \frac{(1-\eta)\,g\,h}{c_p} = \frac{f\cdot 10^{-1}\cdot 10\ \frac{m}{s^2}\cdot 10^2\ m}{f\cdot 10^3\ \frac{J}{\text{kg}\cdot{}^\circ\text{C}}} = 10^{-1}\ {}^\circ\text{C}$$

Checking the units:

Heat Change Over The Year

Intuition

We know that river’s in the north build up ice in the winter ($\approx 10^0$ °C) to being comfortable enough to swim in during the summer ($\approx 10^1$ °C – $f \cdot 10^1$ °C).
Thus the total temperature change is either 10^1 °C – $f \cdot 10^1$ °C.

To make a more mathematical guess, consider the temperature variation over the seasons to be a square wave rather than a sinusoid (Its either summer, or winter). Assume that the water in a dam’s reservoir originated entirely from underground aquifers, which were held at a constant temperature.

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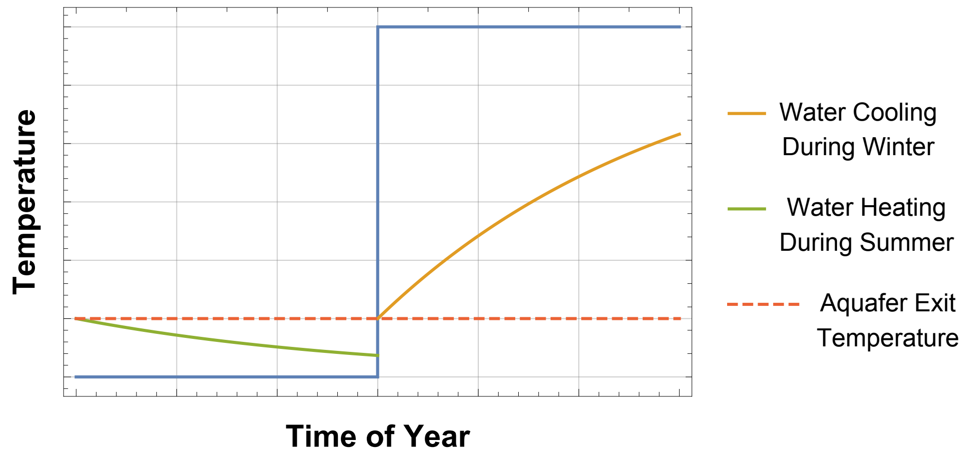

Saving Graphic

Definitions

$$h \equiv \text{Heat transfer coefficent of Air} \approx 10\ \frac{W}{m^2\,K}$$
$$c_p \equiv \text{Specific heat of water } \approx f \cdot 10^3\ \frac{J}{\text{kg}\cdot{}^\circ\text{C}}$$
$$m \equiv \text{Mass of body being cooled} \approx \text{Lake Roosevelt } 10^{13}\ \text{kg}$$
$$A \equiv \text{Exposed surface area of body being cooled} \approx \text{Lake Roosevelt } f \cdot 10^8\ m^2$$
$$T_a \equiv \text{Temperature of air } \approx f \cdot 10\ {}^\circ\text{C during the summer, } 0\ {}^\circ\text{C during the winter}$$
$$T_{\text{aq}} \equiv \text{Temperature of water in aquafer} \approx f\ {}^\circ\text{C year round}$$
$$T_w \equiv \text{Temperature of water in reservoir}$$
$$Q \equiv \text{Heat}$$

Governing Eqns.

$$\text{Newtonian cooling:} \qquad \frac{\partial Q}{\partial t} = h\,A\,(T_a - T_w(t))$$
$$\text{Heat to change the temp. of water:} \qquad \partial Q = c_p\, m\, \partial T_w$$
$$\frac{\partial T}{\partial t} = \frac{h\,A}{c_p\,m}\,(T_a - T_w(t))$$
$$T_w(t) = T_a - e^{-\frac{A\,h\,t}{c\,m}}\,(T_{\text{aq}} - T_a)$$

Summer

$$T_w(t) = f \cdot 10\ {}^\circ\text{C} - \exp\left(-\frac{f \cdot 10^8\ m^2 * 10\ \frac{W}{m^2\,K} * t}{f \cdot 10^3\ \frac{J}{\text{kg}\cdot{}^\circ\text{C}} * 10^{13}\ \text{kg}}\right)(f\ {}^\circ\text{C} - f \cdot 10\ {}^\circ\text{C})$$
$$T_w(t) = f \cdot 10\ {}^\circ\text{C} - \exp\left(-t * 10^{-7}\right)(f \cdot 10\ {}^\circ\text{C})$$
$$\text{After 6 months } t \approx f \cdot 10^7 \Rightarrow$$
$$T_w(t) = f \cdot 10\ {}^\circ\text{C} - \exp(-f)\,(f \cdot 10\ {}^\circ\text{C}) = f \cdot 10\ {}^\circ\text{C} - f \cdot 10^{-2} * (f \cdot 10\ {}^\circ\text{C}) = f \cdot 10\ {}^\circ\text{C}$$

Winter

$$T_w(t) = 0\,°\text{C} - \exp\left(-\frac{f\cdot 10^8\, m^2 * 10\, \frac{W}{m^2\, K} * t}{f\cdot 10^3\, \frac{J}{\text{kg}\cdot °\text{C}} * 10^{13}\, \text{kg}}\right)(f\,°\text{C} - 0\,°\text{C})$$
$$T_w(t) = -\exp\left(-t * 10^{-7}\right)(f\,°\text{C})$$
$$\text{After 6 months } t \approx f \cdot 10^7 \Rightarrow$$
$$T_w(t) = f \cdot 10^{-2} * (f\,°\text{C}) = 10^{-1}\,°\text{C}$$

Checking the units:

Approximate Carnot Number

$$\frac{10^{-1}\,°\text{C}}{f\cdot 10\,°\text{C}} = f \cdot 10^{-2}$$

What The Data Says

Columbia River - Grand Coulee Dam

Importing Data

Below Coulee

Above Coulee

Difference Across

Saving Data

Plotting Data

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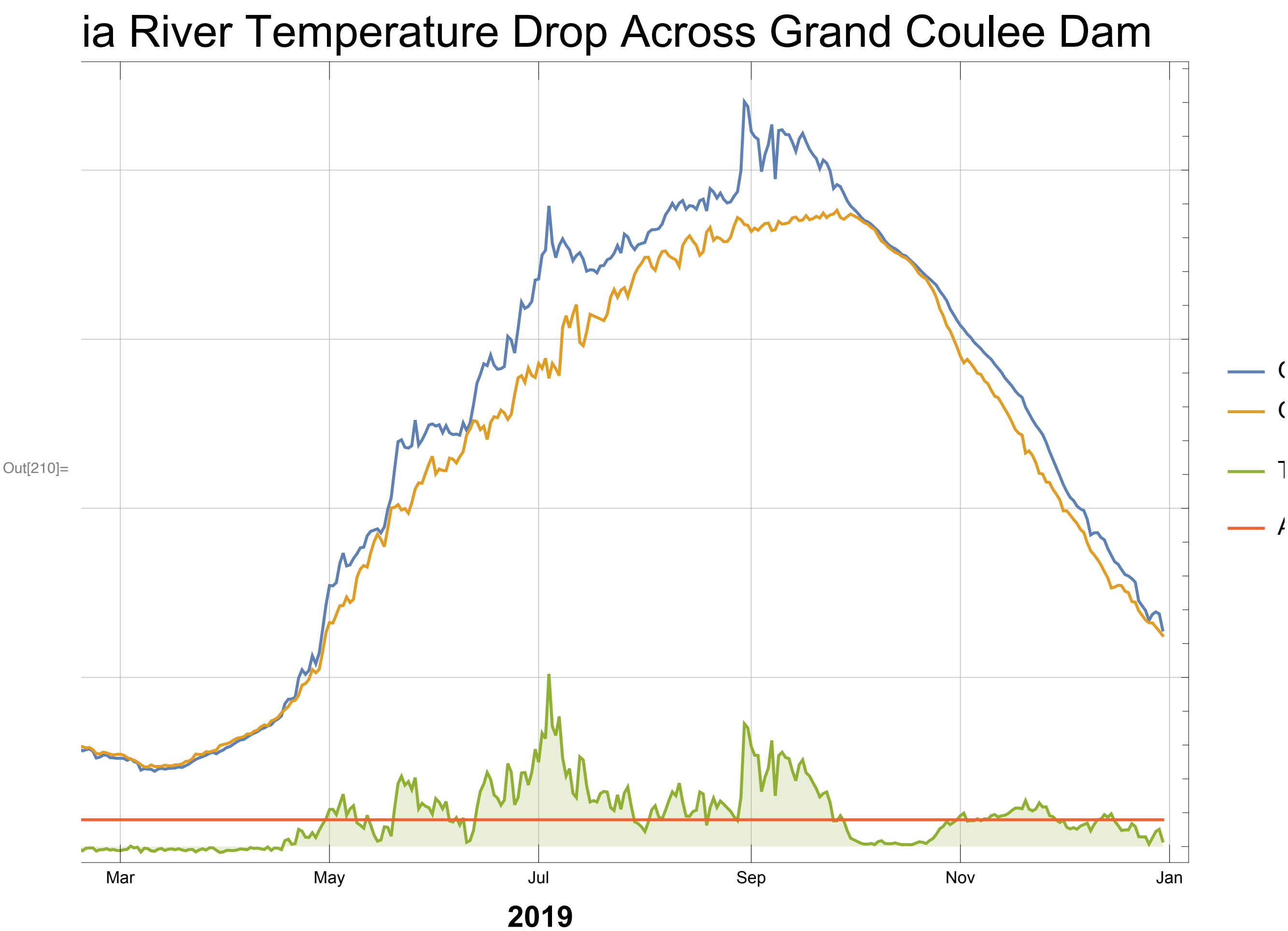

Saving Graphic

Data Source

"Columbia Basin Conditions for Temperature, Dissolved Gas Percent and Spill Percent"

⇒ http://www.cbr.washington.edu/dart/query/basin_conditions

Columbia Basin Research

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