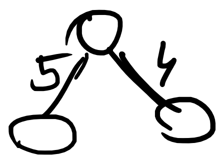
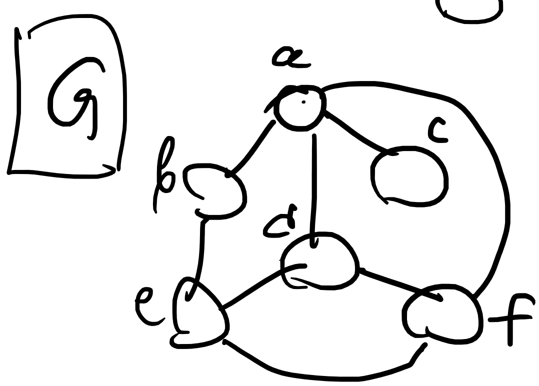
- 1. BFS - покриващо дърво
- 2. DFS - бързо намира цикли

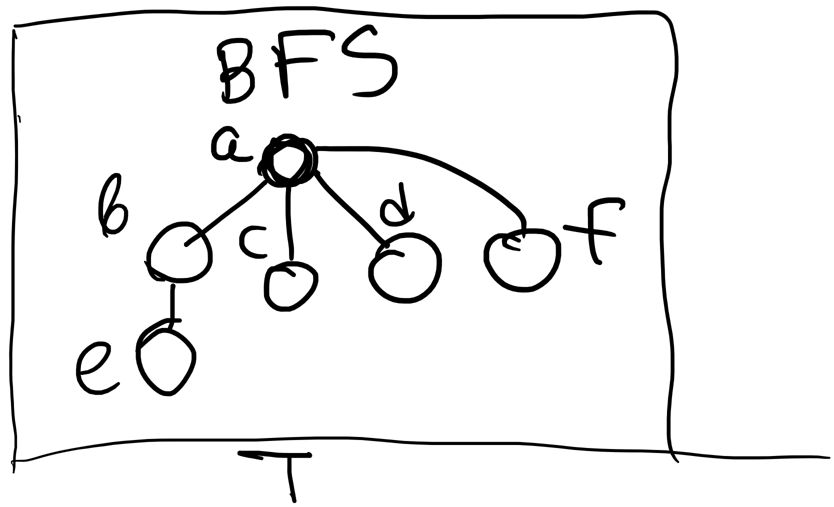
3. Прим
4. Крускал

минимално покр. д-во

5. Дийкстра - най-къси пътища от връх $u \in V$

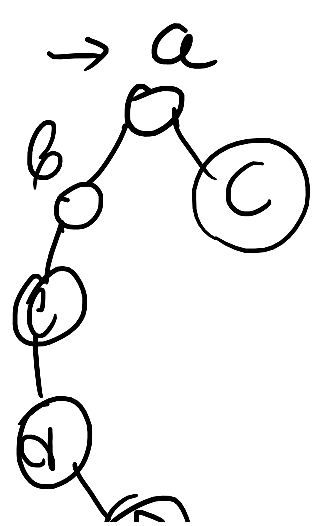






$T \subseteq G$

DFS



f

1. Диагонален метод на Кантор

2. Petersen

3. Булеви ф-ции

4. Хиперкуб

5. T на Бул

T1 Няма биекция $f: \mathbb{N} \to 2^{\mathbb{N}}$

инекция сюрекция

Ако, че имаме биекция $\mathbb{N} \to 2^{\mathbb{N}}$ ($2^{\mathbb{N}}$ е изброимо)

Деф Нека $A \subseteq \mathbb{N}$. За A дефинираме редицата:

$$\alpha_A = \alpha_0 \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_n \ldots ,$$

$$\alpha_i = \begin{cases} 0, \text{ ако } i \notin A \\ 1, \text{ ако } i \in A \end{cases}$$

$$\underbrace{\{0, 5, 7\}} \Rightarrow \begin{array}{cccccccccccc} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & \ldots & 0 & \ldots \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & & \uparrow & \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & & i & \end{array}$$

$$\begin{array}{rl} \rightarrow \alpha^0 = & \alpha_0^0 \ \alpha_1^0 \ \alpha_2^0 \ \ldots \ \alpha_n^0 \ \ldots \\ \rightarrow \alpha^1 = & \alpha_0^1 \ \alpha_1^1 \ \alpha_2^1 \ \ldots \ \alpha_n^1 \ \ldots \\ \alpha^2 = & \alpha_0^2 \ \alpha_1^2 \ \alpha_2^2 \ \ldots \ \alpha_n^2 \ \ldots \\ \vdots & \quad \vdots \qquad\qquad\qquad \vdots \\ \alpha^n = & \alpha_0^n \ \alpha_1^n \ \alpha_2^n \ \ldots \ \alpha_n^n \ \ldots \\ \beta = \alpha^i = & \alpha_0^i \ \alpha_1^i \ \alpha_2^i \ \ldots \ \alpha_i^i \ \ldots \\ \vdots & \end{array}$$

$$\beta = \overline{\alpha_0^0}\ \overline{\alpha_1^1}\ \overline{\alpha_2^2} \ldots \overline{\alpha_n^n} \ldots \overline{\alpha_i^i} \ldots$$

$\Rightarrow \exists \ \underline{M}$, чиято хар. редица е β

$\Rightarrow M \in 2^{\mathbb{N}}$

На коя позиция е M в таблицата???

Нека M е на позиция i в таблицата

но β се различава на i-та позиция от $\alpha_i^i \Rightarrow$ M да е представено на i-та позиция в таблицата. $(\forall i \in \mathbb{N})$ ↯

$\mathbb{N} \to \underbrace{\mathbb{N} \times \mathbb{N}}_{(x,y)}$ $\quad x, y \in \mathbb{N}$

0 0 1 0 0 1 . . . 0 0

$f: \mathbb{N} \to \mathbb{N} \times \mathbb{N}$

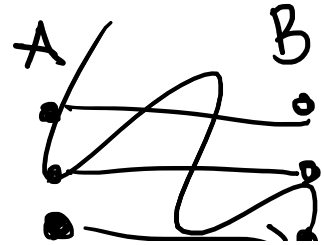
f е биекция

$\Rightarrow \mathbb{N} \times \mathbb{N}$ е изброимо

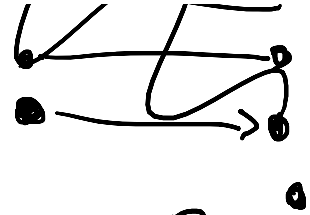
$f: A \to B$

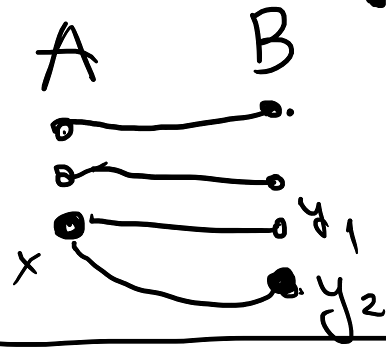
- f е инекция $\Leftrightarrow |A| \leq |B|$
- f е сюрекция $\Leftrightarrow |A| \geq |B|$
- f е биекция $\Leftrightarrow |A| = |B|$

f



$|A| > |B|$





$f(x) = y$

$x \in A \quad y \in B$

T2 Няма биекция $f: A \to 2^A$

$$\boxed{\begin{array}{l} M = \{X \mid X \notin X\} \\ M \in M\ ? \quad \lightning \quad \begin{array}{l} M \in M \Rightarrow M \notin M \\ M \notin M \Rightarrow M \in M \end{array} \end{array}}$$

$\boxed{2^{\mathbb{N}}}$

Нека, че $f: A \to 2^A$ е биекция.

Дефинираме и:

$$F = \{x \mid x \in A \land x \notin f(x)\}$$

$f(x) \in 2^A$

$\boxed{F \in 2^A}$

f е биекция $\Rightarrow$ f е сюрекция

$\Rightarrow F$ трябва да има първообраз

$\Rightarrow \exists\, x_0 \in A \ : \ f(x_0) = F$

$x_0 \in F$???

I сл. $x_0 \in F \Rightarrow \begin{array}{l} x_0 \in A \\ x_0 \notin f(x_0) \end{array} \Rightarrow x_0 \notin F$ ↯

II сл. $x_0 \notin F \Rightarrow x_0 \notin f(x_0) \Rightarrow x_0 \in F$ ↯

$\Rightarrow$ f не е сюрекция $\Rightarrow$
f не е биекция,

$M \to M \times M$, меизброимо

$N \to M$

$\boxed{N \to N \times N}$

$18 \to 2 \; 3 \; 3 \to$

$18 \to (1,2)$

$2^1 \cdot 3^2$

$30 \to \underline{2 \cdot 3 \cdot 5}$

$\to (x,y) \to 2^x 3^y \to$

$|N \times N| \leq |N|$ f.

$|\mathbb{N}|$

$f: \mathbb{N}\times\mathbb{N} \to \mathbb{N}$

$f = 2^x 3^y$

(1) f е инекция $\Rightarrow |\mathbb{N}\times\mathbb{N}| \le |\mathbb{N}|$

$g: \mathbb{N} \to \mathbb{N}\times\mathbb{N}$

$g(x) = (0, x)$

(2) g е инекция $\Rightarrow |\mathbb{N}| \le |\mathbb{N}\times\mathbb{N}|$

От (1) и (2) $\Rightarrow |\mathbb{N}| = |\mathbb{N}\times\mathbb{N}| \Rightarrow$

$\exists h: \mathbb{N} \to \mathbb{N}\times\mathbb{N}$

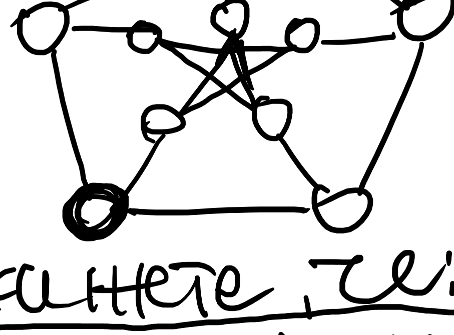
h е биекция

$f(x,y) = 2^x(2y+1) - 1$ е биекция

$f: \mathbb{N}\times\mathbb{N} \to \mathbb{N}$

Граф на Петерсен

3-регулярен

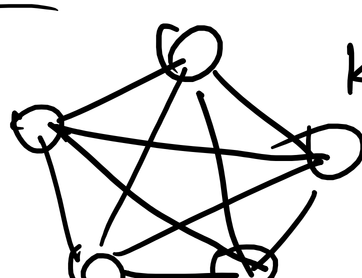


Докажете, че:

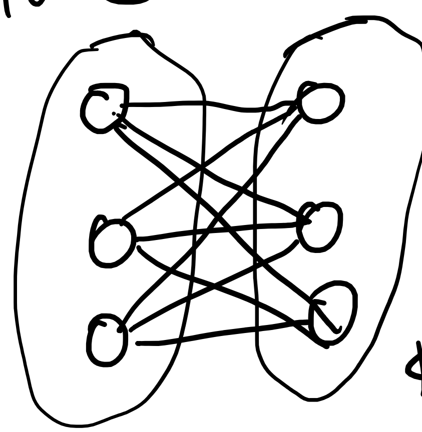
- а) не съдържа Хам. цикъл
- → б) съдържа Хам. път
- → в) не е планарен
- г) всичките му върхове са симетрични

(Т) на Куратовски

Краен граф е планарен $\Longleftrightarrow$ не съдържа подграф, който е ~~хо~~хомеоморфен (може да се свие) на $K_{3,3}$ или K_5.

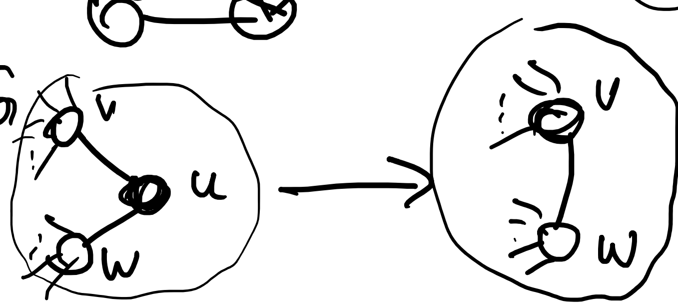


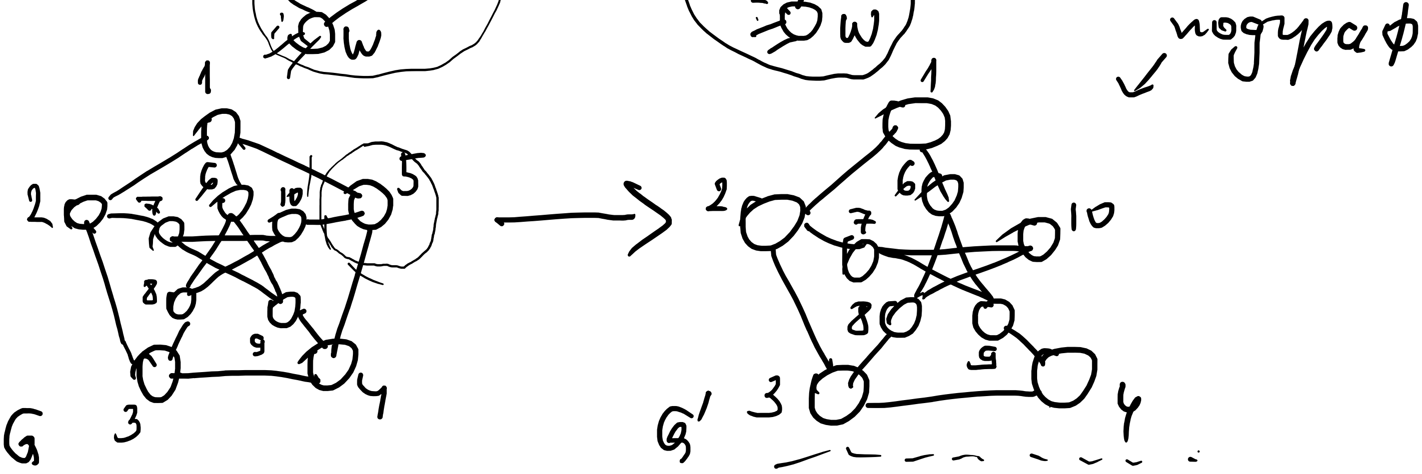
K_5

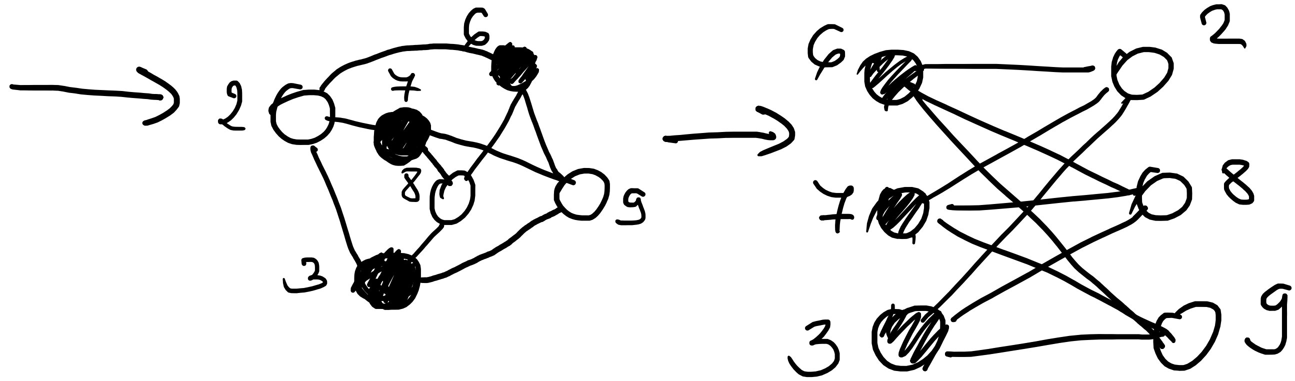


$K_{3,3}$

в G

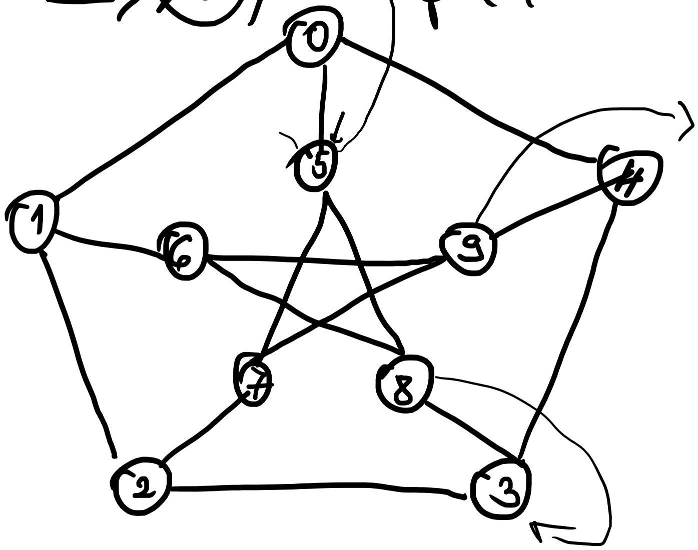


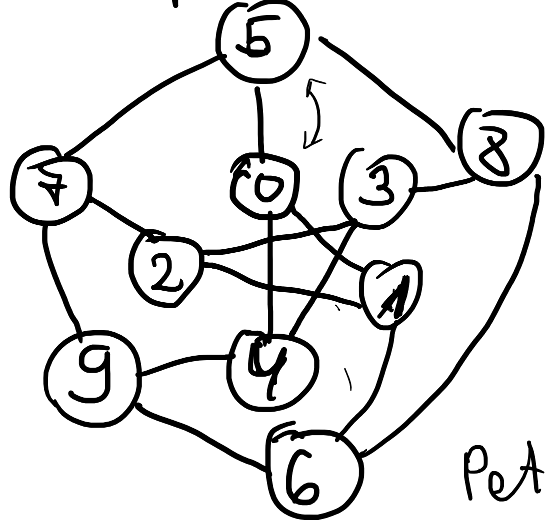




$K_{3,3}$

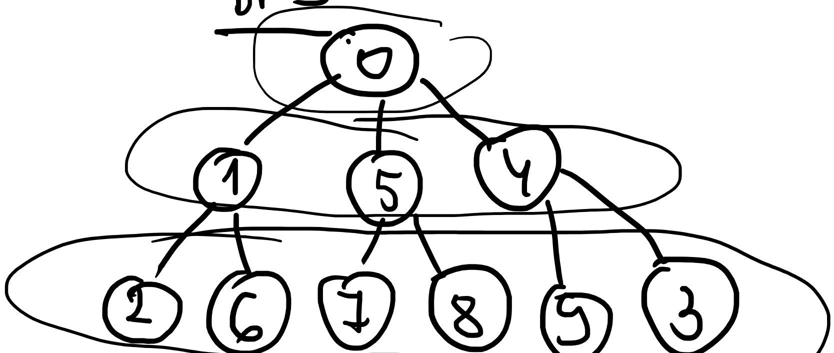
$\Rightarrow$ (T) – Графът на Петерсен не е планарен.





Petersen

BFS



$f(\tilde{x}^3) = (0\,1\,1\,1\,1\,1\,1\,0)$

СъвДНФ $\longrightarrow$ МДНФ

$\rightarrow x\tilde{\alpha}_i^n \vee \bar{x}\tilde{\alpha}^n = \tilde{\alpha}^n$

$$\overline{xyz}^{\,n+1} \vee \overline{x\overline{y}z}^{\,n+1} = xz$$

x_1	x_2	x_3	f	
0	0	0	0	
0	0	1	1	$\rightsquigarrow \overline{x}\,\overline{y}\,z \to f$
0	1	0	1	$\rightsquigarrow \overline{x}\,y\,\overline{z}$ $\vee$
0	1	1	1	$\rightsquigarrow \overline{x}\,y\,z$ $\vee$
1	0	0	1	$\rightsquigarrow x\,\overline{y}\,\overline{z}$ $\vee$
1	0	1	1	$\rightsquigarrow x\,\overline{y}\,z$ $\vee$
1	1	0	1	$\rightsquigarrow x\,y\,\overline{z}$
1	1	1	0	

$\boxed{\wedge \vee \neg}$

$f \longrightarrow$ СъвДНФ $\to$ композиция от м-вото на Бул

Нека $f \in \mathcal{F}_2^n \Rightarrow 2^n$ ст-сти на домейн

f (1) $\quad \boxed{f = \varnothing}$

$f(= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad$ 1 1 1 1 1 1

$\boxed{\text{СъвДНФ} = \ (x\overline{x})}$

$f(x) \to$ дали x е четно

$f == x == 0 \vee x == 2 \vee x == 4 \vee$
$x == 6 \vee \ldots$

$\cancel{\overline{x}}\,\overline{y}\,\overline{z}$	СъвДНФ $= 0$
	СъвДНФ $= \overline{x}\,\overline{y}\,z \vee x\,\overline{y}\,z$

$\bar{x}\bar{y}z$ ✓	$\bar{x}z$
$\bar{x}y\bar{z}$ ✓	$\bar{y}z$
$\bar{x}yz$ ✓	$y\bar{z}$
$x\bar{y}\bar{z}$ ✓	$\bar{x}y$
$x\bar{y}z$ ✓	$x\bar{y}$
$xy\bar{z}$ ✓	$x\bar{z}$

прости импликанти

	001	010	011	100	101	110
$t_1 = \bar{x}z$	V		V			
$t_2 = \bar{y}z$	V				V	
$t_3 = y\bar{z}$		V				V
$t_4 = \bar{x}y$		V	V			
$t_5 = x\bar{y}$				V	V	
$t_6 = x\bar{z}$				V		V

$$(t_1 \vee t_2)(t_3 \vee t_4)(t_1 \vee t_4)(t_5 \vee t_6)(t_2 \vee t_5)(t_3 \vee t_6) =$$

$$= (t_1 \vee t_2)(t_2 \vee t_5)(t_3 \vee t_4)(t_1 \vee t_4)$$

$$\ldots = $$

$$t_1t_2 \vee t_2 = t_2 \qquad (t_5 \vee t_6)(t_3 \vee t_6) =$$

$$= (t_1t_2 \vee \underbrace{t_2t_2}_{t_2} \vee t_1t_5 \vee t_2t_5)$$

$$(t_3t_1 \vee t_3t_4 \vee t_4t_1 \vee t_4t_4)$$

$$(t_5t_3 \vee t_5t_6 \vee t_6t_3 \vee t_6t_6) =$$

$$= (t_2 \vee t_1t_5)(t_4 \vee t_3t_1)(t_6 \vee t_5t_3) =$$

$$= (t_2t_4 \vee t_1t_2t_3 \vee t_1t_4t_5 \vee t_1t_3t_5)(t_6 \vee t_5t_3)$$

$$= t_2t_4t_6 \vee t_2t_3t_4t_5 \vee t_1t_2t_3t_6 \vee t_1t_2t_3t_5 \vee t_1t_4t_5t_6 \vee t_1t_3t_4t_5 \vee t_1t_3t_5t_6 \vee t_1t_3t_5$$

$\underline{t_2t_4t_6}$

$\hookrightarrow \bar{y}z \vee \bar{x}y \vee x\bar{z} = ΜΔΗΦ1$

$\underline{t_1t_3t_5}$

$\hookrightarrow \cancel{y\bar{z} \vee x\bar{y} \vee \bar{x}z = ΜΔΗΦ2}$

$\hookrightarrow$ ~~$y\bar{z} \vee x\bar{y} \vee \bar{x}z$ = МДНФ2~~

$x\bar{z} \vee y\bar{z} \vee x\bar{y}$ = МДНФ2

Зад

x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$\rightsquigarrow \bar{x}\bar{y}\bar{z}$

$\rightsquigarrow \bar{x}\bar{y}z$

$\alpha \preceq \beta$

T_0, T_1, S, L, M

$\bar{x}\bar{y}\bar{z} =$

$= (1\oplus x)(1\oplus y)(1\oplus z)$

а) СъвДНФ $= \bar{x}\bar{y}\bar{z} \vee \bar{x}\bar{y}z \vee \bar{x}yz \vee x\bar{y}z \vee x y\bar{z}$

$\oplus$

б) $\bar{x}\bar{y}\bar{z} \vee$ | $\bar{x}\bar{y}$

$\bar{x}\bar{y}z \vee$ | $\bar{x}z$

$\bar{x}yz \vee$ | $\bar{y}z$

$x\bar{y}z \vee$

$xy\bar{z}$

прости импликанти

импликанты

	000	001	011	101	110
$xy\bar z$					V
$\bar x\bar y$	V	V			
$\bar x z$		V	V		
$\bar y z$		V		V	

k n k-n

$$\text{МДНФ} = xy\bar z \vee \bar x\bar y \vee \bar x z \vee \bar y z$$

$1\oplus x = \bar x$ $\qquad$ $x\oplus x = 0$

в) ПН

$$\text{ПН} = (1\oplus x)(1\oplus y)(1\oplus z)\oplus(1\oplus x)(1\oplus y)z \oplus (1\oplus x)yz\oplus x(1\oplus y)z\oplus xy(1\oplus z) =$$

$$=(1\oplus x\oplus y\oplus xy)(1\oplus z)\oplus z(1\oplus x\oplus xy\oplus y \oplus yz\oplus \cancel{xyz}\oplus xz\oplus\cancel{xyz}\oplus xy\oplus xyz=$$

$$=1\oplus\cancel{z}\oplus x\oplus\cancel{xz}\oplus y\oplus yz\oplus\cancel{xy}\oplus\cancel{xyz}\oplus\cancel{z}\oplus\cancel{xz}\oplus\cancel{xyz}\oplus\cancel{yz}\oplus\cancel{yz}\oplus xz\oplus\cancel{xy}\oplus xyz = 1\oplus x\oplus y\oplus yz\oplus xz\oplus xyz$$

$$f = a_0 \oplus a_1 x \oplus a_2 y \oplus a_3 z \oplus a_4 xy \oplus a_5 xz \oplus a_6 yz \oplus a_7 xyz$$

$a_0 - a_7$

$\underline{a_0 = 1}$ $\quad a_0 \oplus a_3 = 1$

$\Rightarrow a_3 = 0$

$\vdots$

г)

F е шеферова $\overset{def}{\iff}$ {f} е пълно м-во

$\neg \ \wedge \ \vee$

$F = \{f_1, f_2 \ldots f_k\}, f_i \in \mathcal{F}_2$

Казваме, че F е пълно $\overset{def}{\iff}$ $\forall f \in \mathcal{F}_2$ може да се представи като композиция на елементите на F (на $f_1 \ldots f_k$)

Критерий на Пост-Яблонски

F е пълно $\overset{def}{\iff}$ F не е подм-во на

$\rightarrow T_0, T_1, S, L, M.$ $\qquad \alpha \preceq \beta \rightarrow$

$\to \underbrace{T_0}, T_1, S, L, M.$

$\alpha \preceq \beta \to f(\alpha) \leq f(\beta)$

$$f(\alpha^n) = \overline{f}(\overline{\alpha^n})$$

$00\ldots0$
$00\ldots1$
$00\ldots10$
$01\ldots1$
$10\ldots0$
$10\ldots1$
$\ldots$
$11\ldots0$
$11\ldots1$

$$a_0 \oplus a_1x_1 \oplus a_2x_2 \oplus \ldots a_nx_n$$

$$\underbrace{\cancel{a_i x_i x_j} \ldots}$$

x	y	$\wedge$
0	0	0
0	1	0
1	0	0
1	1	1

Когато $|F| = 1$!

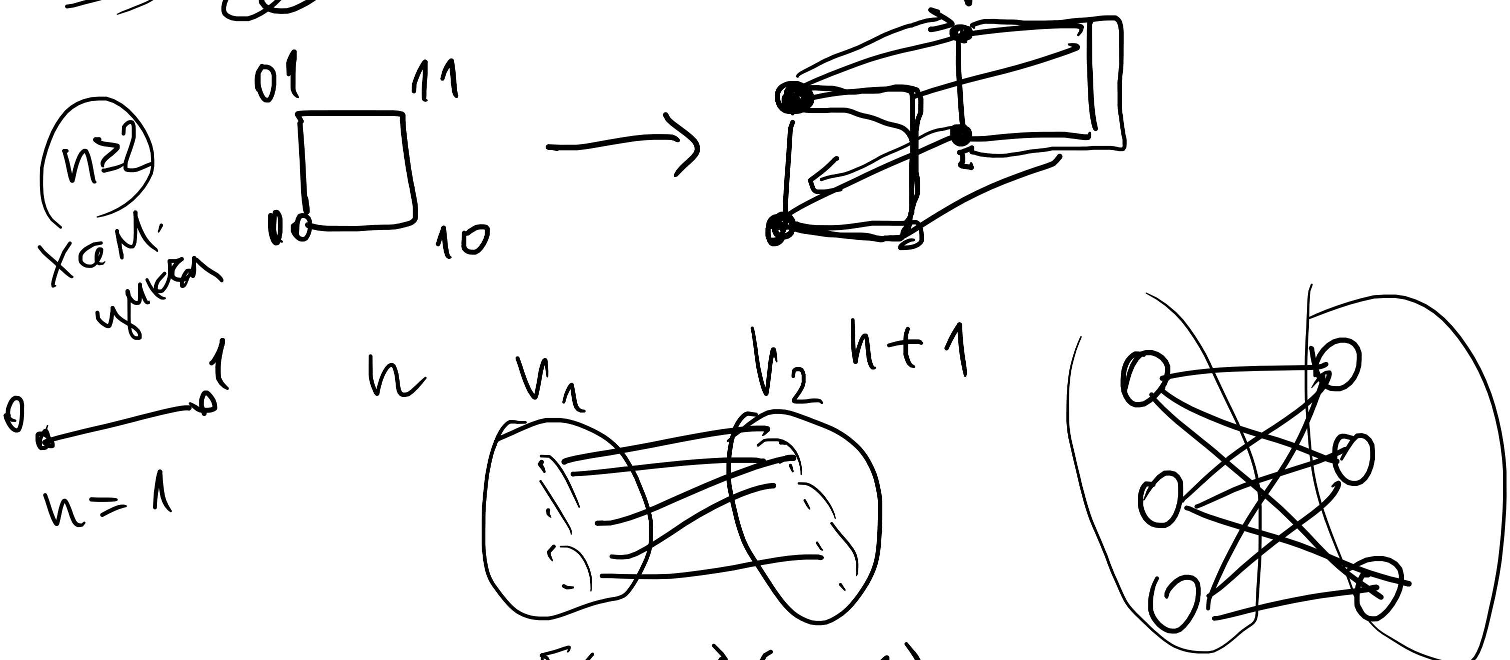
Достатъчно е да докажем, че

$f \notin T_0$, $f \notin T_1$, $f \notin S$, $f \in F$

за да твърдим, че F е пълно,

$\Rightarrow$ че f е шеферова.

⇒ ce t'e мефероба.



$$\frac{(n+1)(n+1)}{2}$$

level 1 $\quad x_1 + x_2 + x_3 + \ldots + x_n = k$

$$\rightarrow \binom{n+k-1}{n-1} = \binom{n+k-1}{k}$$

$$x_1 + x_2 + x_3 = 15 \qquad x_i \geq 0$$

$x > 0$

| 0 0 0 || 0 0 0 0 0 0 0 0 0 0 0 |

3 − 1 = 2

|| 0 0 0 $\quad \binom{15}{2} \quad$ 16.17 − (17)

$$11000 \quad (15 \quad 2) \quad \frac{16 \cdot 17}{2} = \binom{17}{2}$$

$$17 \begin{cases} 2 \text{ разд} \\ \underline{15 \text{ нули}} \end{cases} \quad \binom{17}{2}$$

level 2 $\quad x_1 + x_2 + \ldots + x_n \leq k$

$$x_1 + x_2 + \ldots + x_n + x_{n+1} = k$$

<u>level 3</u> $\quad 0 \leq k_1 \leq k_2 \leq n$

$$k_2 = k_1 + \alpha_1 \quad , \alpha_1 \geq 0$$

$$n = k_2 + \alpha_2 \quad , \alpha_2 \geq 0$$

$$\underline{k_1 + \alpha_1 + \alpha_2 = n} \quad , \left.\begin{matrix} k_1 \\ \alpha_1 \\ \alpha_2 \end{matrix}\right\} \geq 0$$