EA614 - Análise de Sinais

EFC3 - Série de Fourier

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1 Parte Computacional

a)
No caso de $x(t)$ temos:

$$a_k = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} \frac{2}{T}te^{-jk\omega_0 t}dt \tag{1}$$

$T = 4s$ então podemos escrever como:

$$a_k = \frac{2}{T^2}\int_{-2}^{2} te^{-jk\omega_0 t}dt \tag{2}$$

Para $k = 0$:

$$a_0 = \frac{2}{T^2}\int_{-2}^{2} te^{0}dt = \frac{2}{T^2}\int_{-2}^{2} tdt \tag{3}$$

$$a_0 = 0 \tag{4}$$

Para $k \neq 0$ podemos resolver por partes:

$$\int udv = uv - \int vdu$$

$$u = t; \quad du = dt$$

$$v = \frac{-e^{jk\omega_0 t}}{jk\omega_0}; \quad dv = e^{jk\omega_0 t}dt$$

Então:

$$\int te^{-jk\omega_0 t}dt = -t\frac{e^{jk\omega_0 t}}{jk\omega_0} + \int \frac{e^{jk\omega_0 t}}{jk\omega_0}dt \tag{5}$$

$$a_k = \frac{2}{T^2}\left(\left[-t\frac{e^{jk\omega_0 t}}{jk\omega_0}\right]_{-2}^{2} + \int_{-2}^{2} \frac{e^{jk\omega_0 t}}{jk\omega_0}dt\right) = \frac{2}{T^2}\left[\frac{e^{jk\omega_0 t}}{jk\omega_0}\left(t + \frac{1}{jk\omega_0}\right)\right]_{-2}^{2} \tag{6}$$

$$a_k = \frac{2}{T^2}\left[\frac{-2}{jk\omega_0}\left(e^{jk\omega_0 2} + e^{-jk\omega_0 2}\right) + \frac{-1}{k^2\omega_0^2}\left(e^{jk\omega_0 2} - e^{-jk\omega_0 2}\right)\right] \tag{7}$$

Usando a fórmula de Euler temos:

$$\omega_0 = \tfrac{2\pi}{T} \text{ e } T = 4s$$

$$a_k = \frac{2}{4^2}\left[\frac{-8}{jk\pi}\cos(\pi k) - \frac{4j}{k^2\pi^2}\sin(\pi k)\right] \tag{8}$$

$$ak = \frac{2j}{k\pi}\cos(k\pi) \tag{9}$$

b)