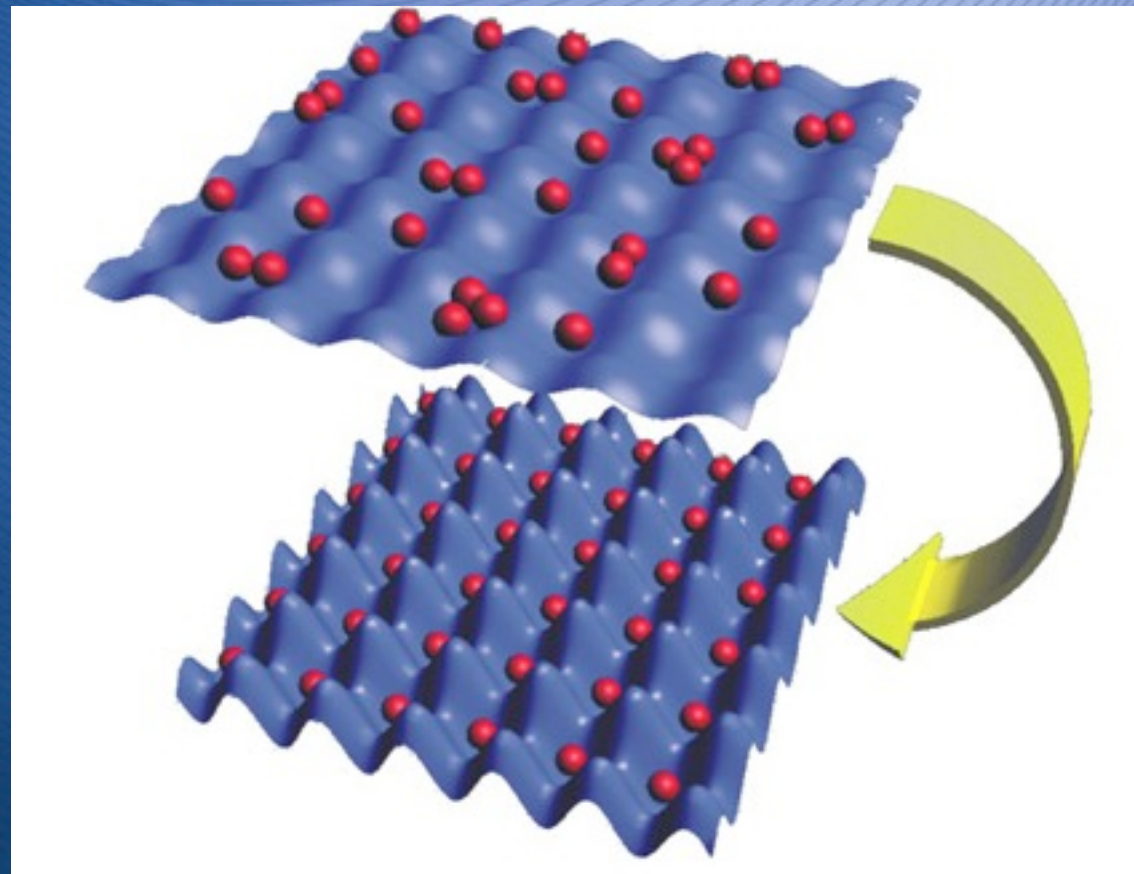


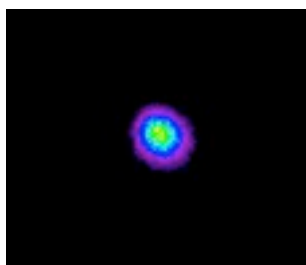
# Ultracold atomic gases as quantum simulators





# Feynman's quantum simulator

- We are able to control single quantum systems



Single Atoms and  
Ions

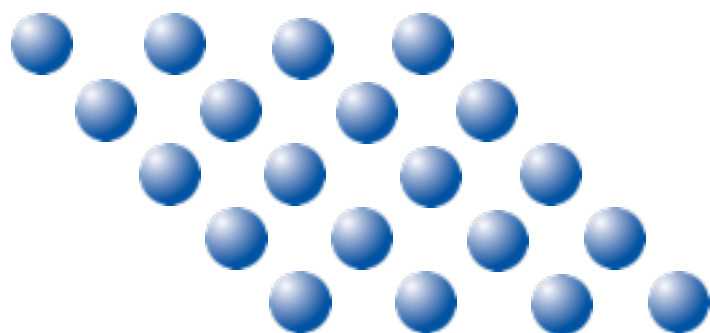


Photons



Quantum Dots

- New challenges:  
control, engineer and understand complex quantum system



## R. P. Feynman's Vision

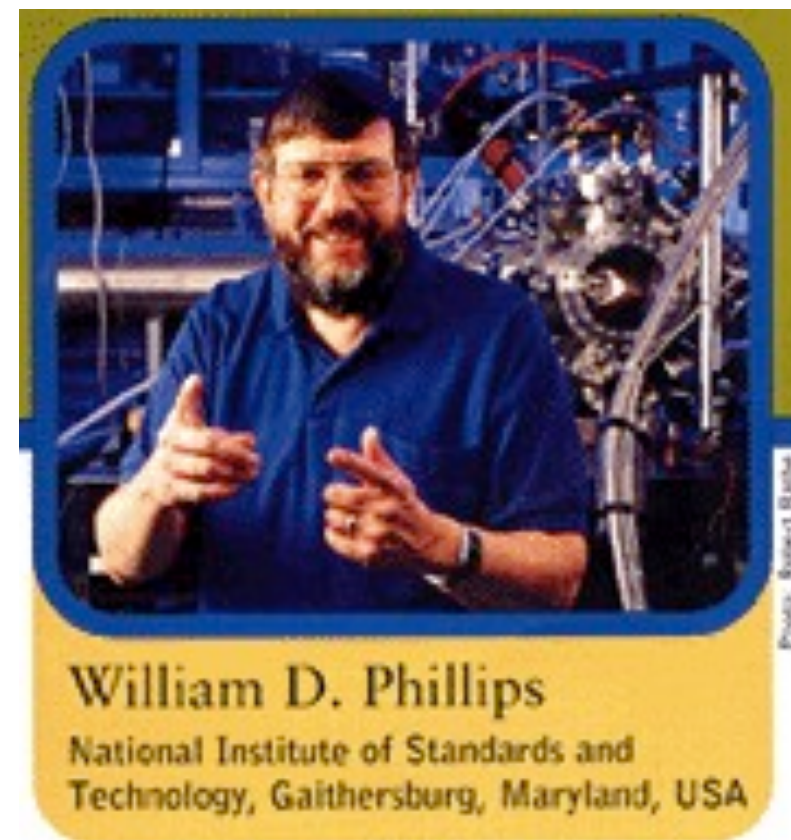
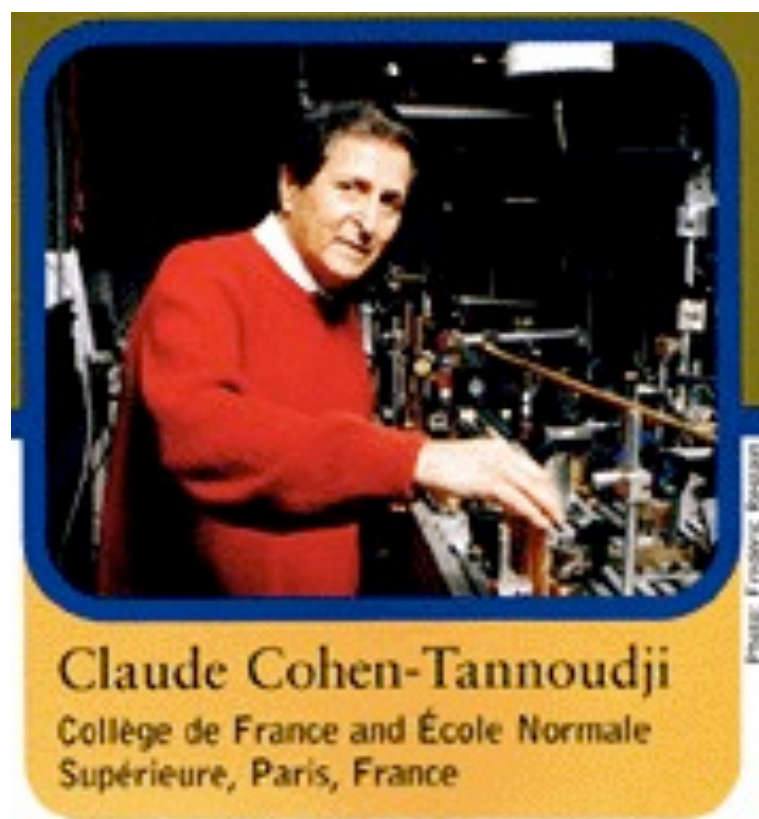
A Quantum Simulator to study  
the quantum dynamics  
of another system.

R.P. Feynman, Int. J. Theo. Phys. (1982)

R.P. Feynman, Found. Phys (1986)

# 1997 Nobel Prize in Physics

**Steven Chu, Claude Cohen-Tannoudji and William D. Phillips**  
share 1997 Nobel Prize for the development of methods to cool and trap atoms with laser light.

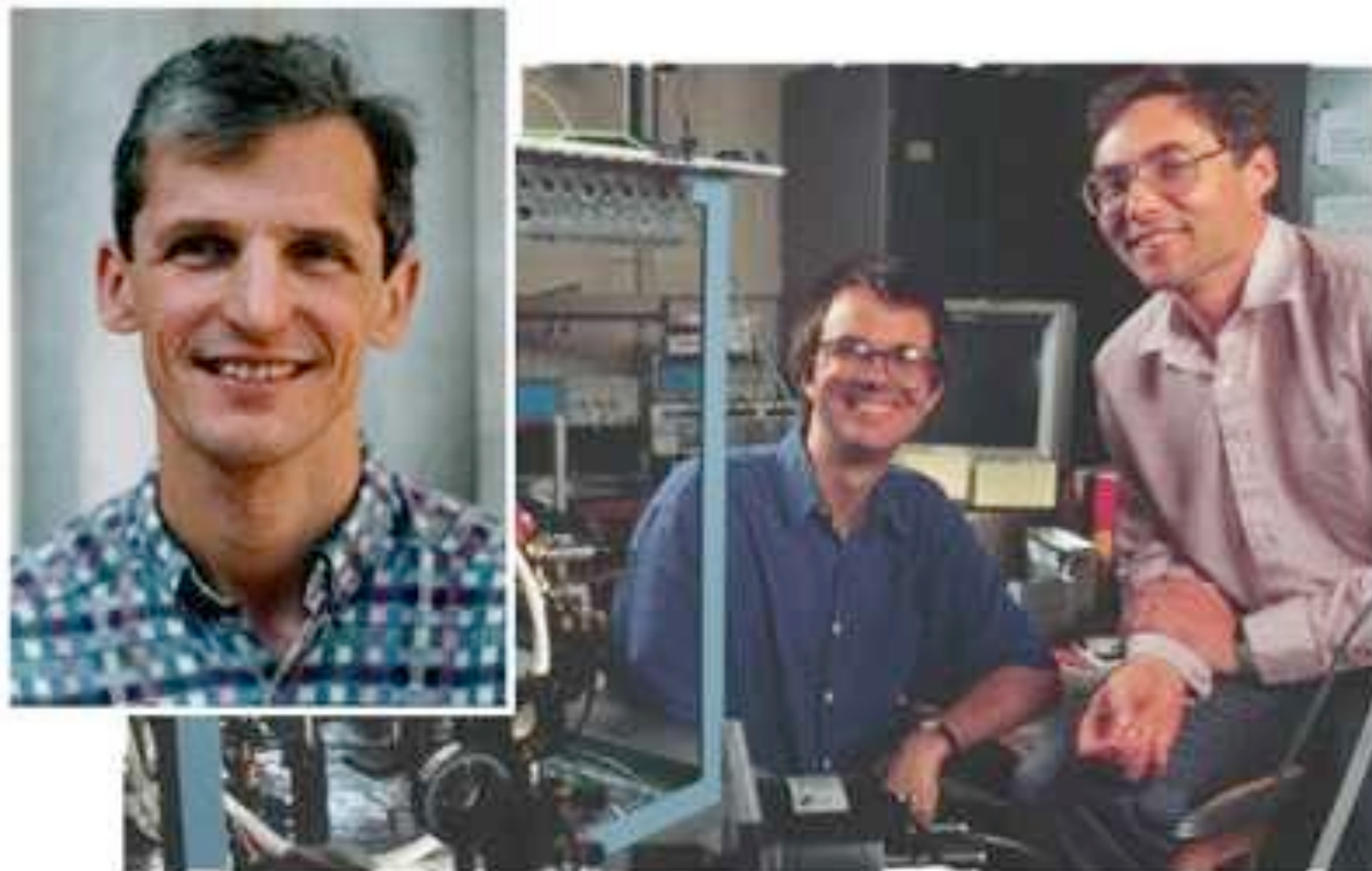




# 2001 Nobel Prize in Physics

**Carl Wieman , Eric Cornell and Wolfgang Ketterle**

share 2001 Nobel Prize for the achievement of BEC in dilute gases of alkali atoms and for the early fundamental studies of the properties of the condensates .



# Our Starting Point – Ultracold Quantum Gases

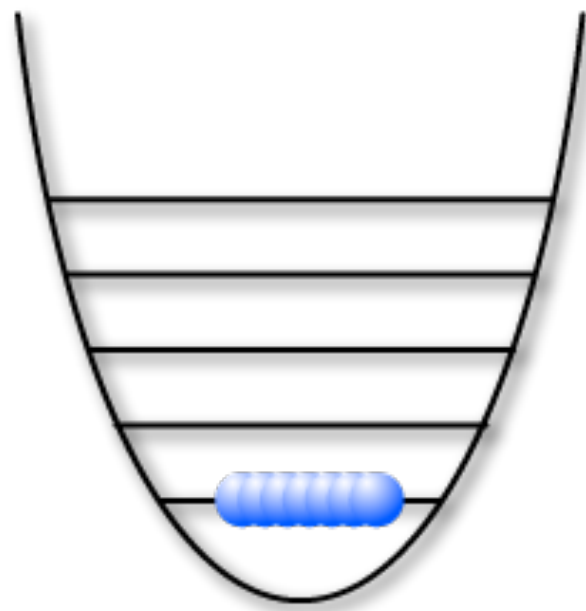
## Parameters:

Densities:  $10^{15} \text{ cm}^{-3}$

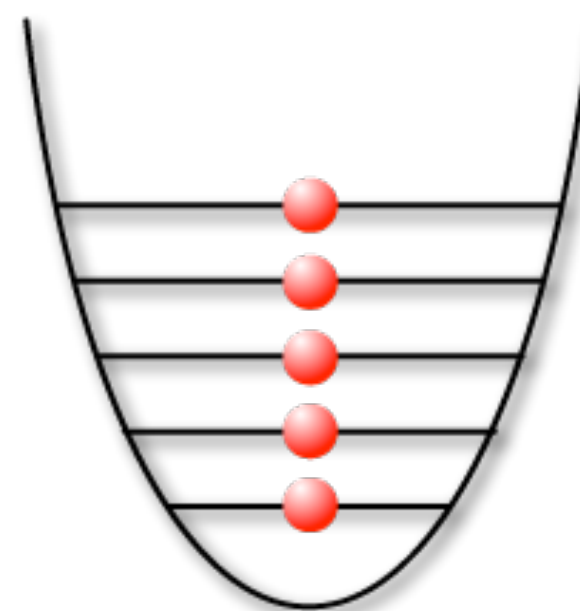
Temperatures: Nano Kelvin

Atom Numbers  $10^6$

Ground States at  $T=0$



Bose-Einstein  
Condensates e.g.  $^{87}\text{Rb}$



Degenerate Fermi Gases  
e.g.  $^{40}\text{K}$



A detailed 3D CAD model of an experimental setup for trapping and manipulating atoms. The setup is mounted on a grey perforated metal plate. It features a central horizontal silver tube. To the left, a blue rectangular block is labeled 'atom source'. Above the central tube, a large silver cylindrical component is labeled 'magneto-optical trap (cooling)'. To the right, a complex assembly of orange and silver parts is labeled 'optical trap and lattice'. The entire assembly is supported by various brackets, bolts, and a large silver flange on the right side.

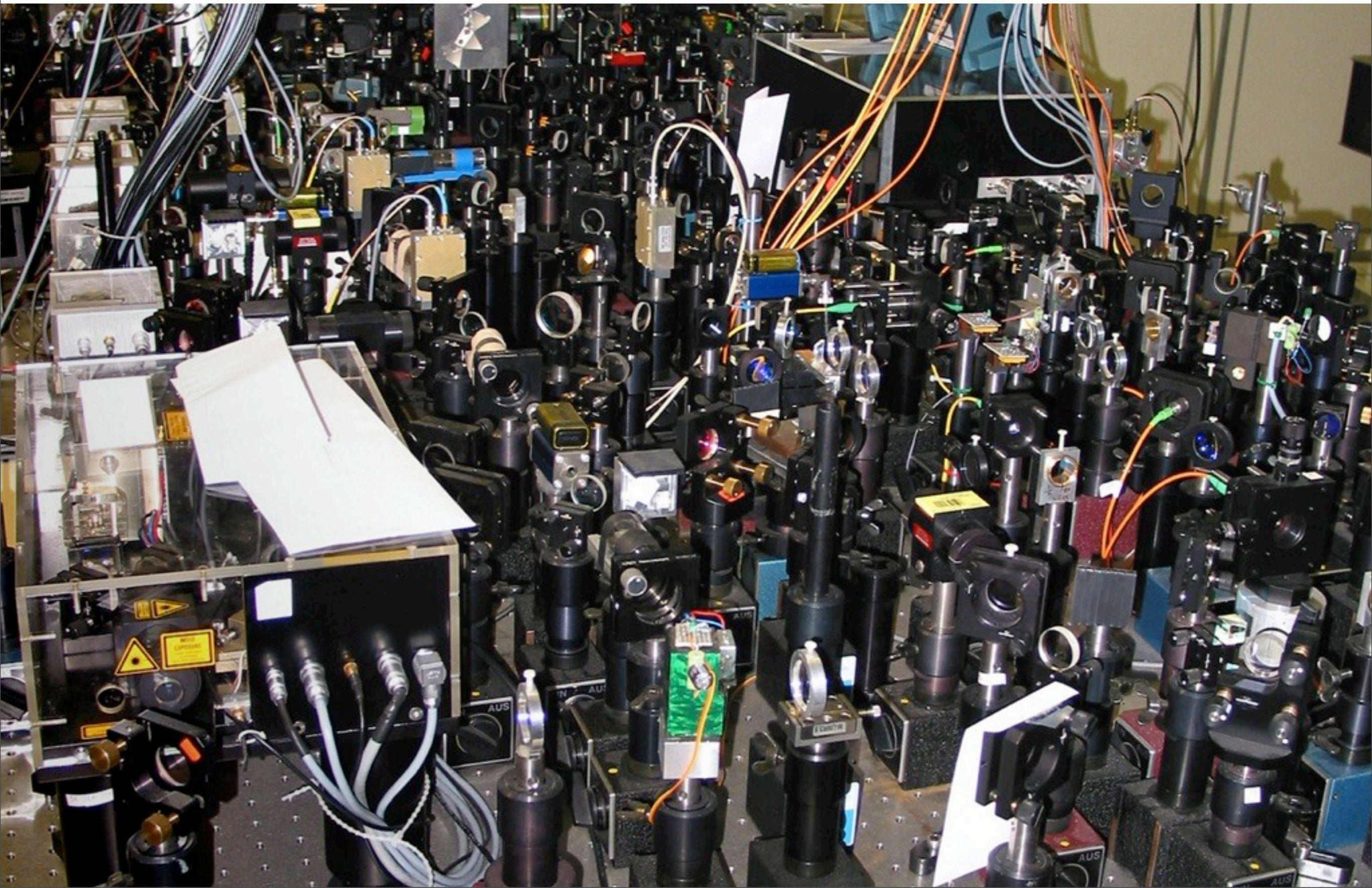
**magneto-optical trap  
(cooling)**

**atom  
source**

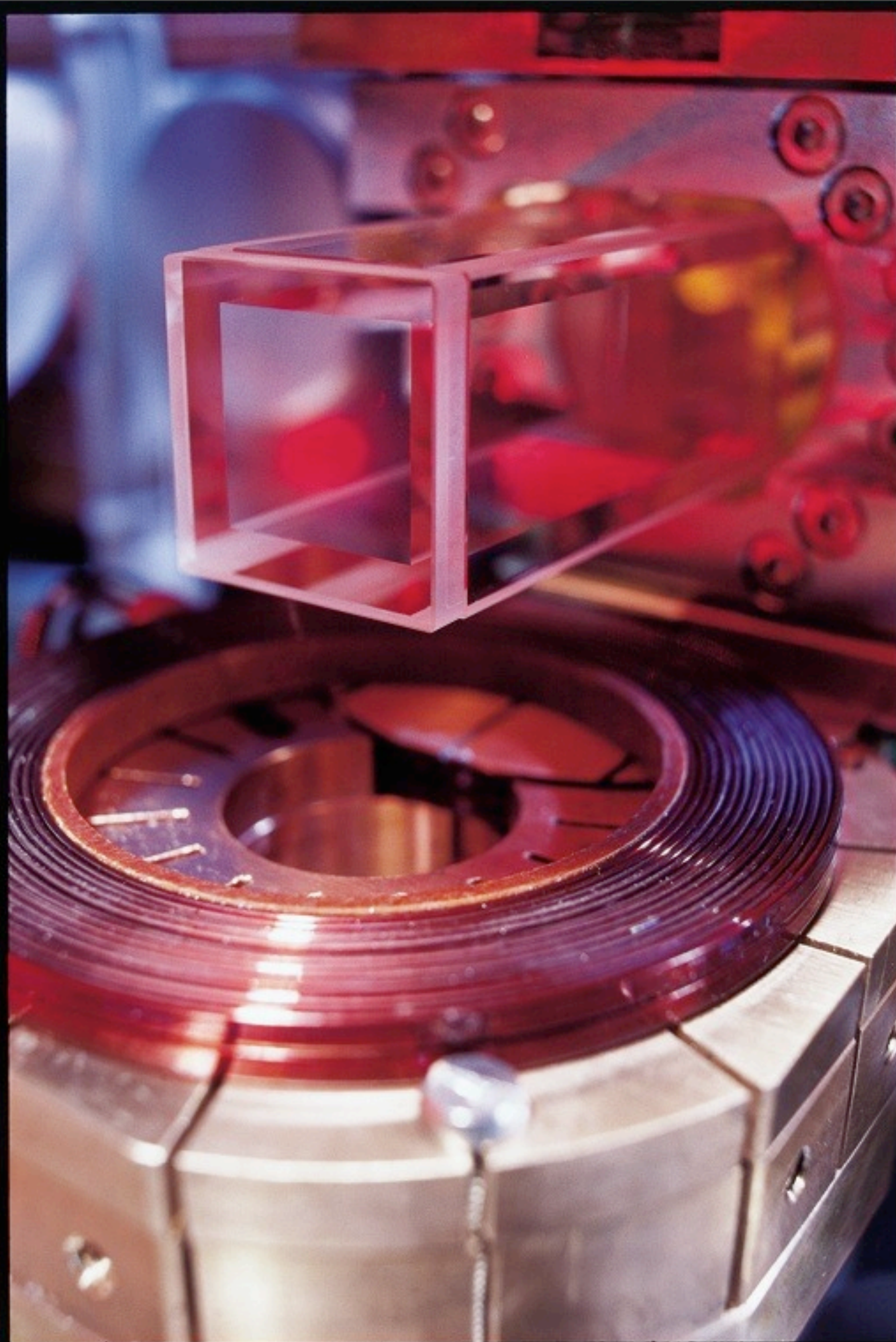
**optical trap  
and lattice**



**And a lot of optics and electronics !**







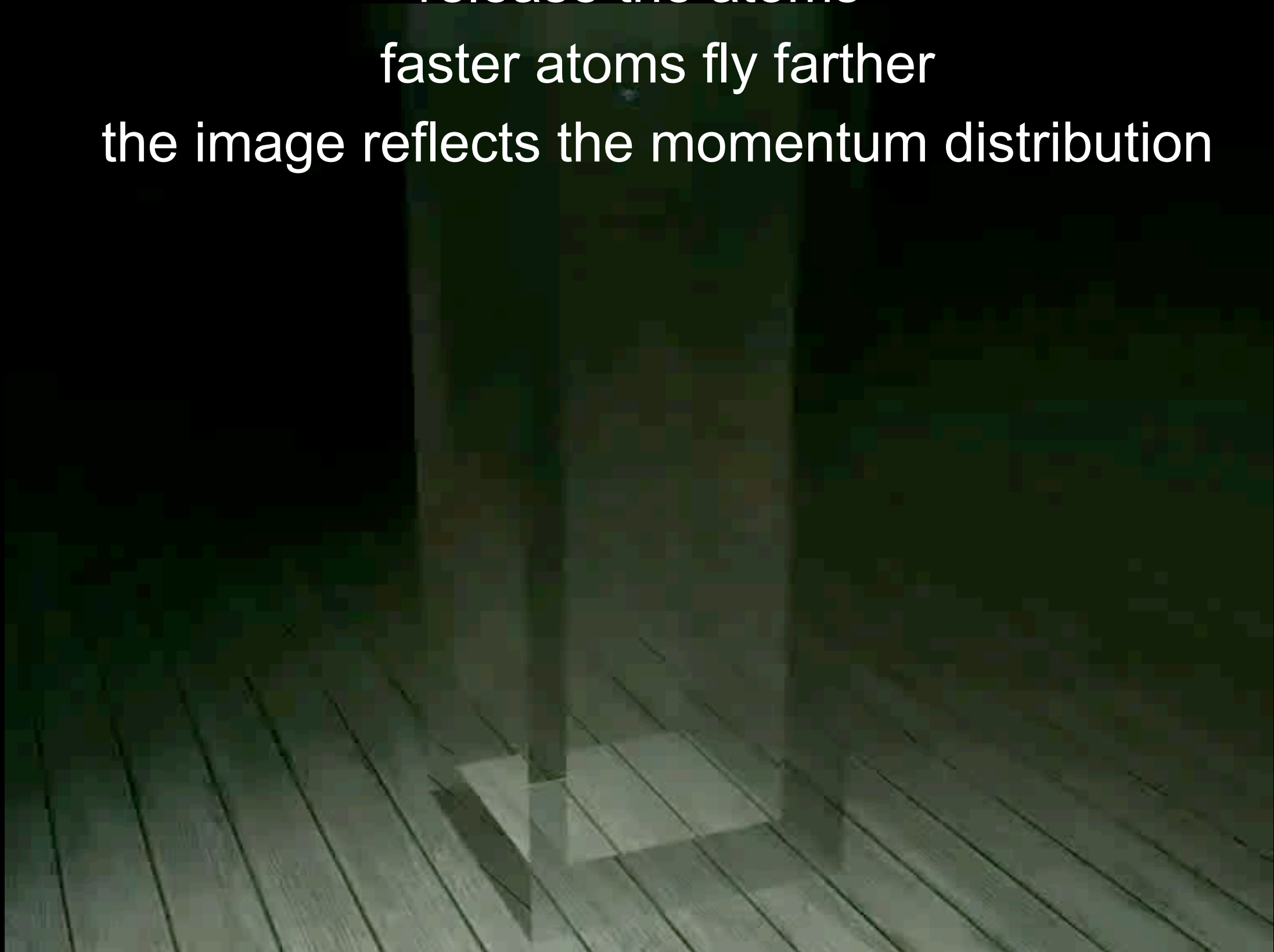


# How do we detect these quantum gases ?

release the atoms

faster atoms fly farther

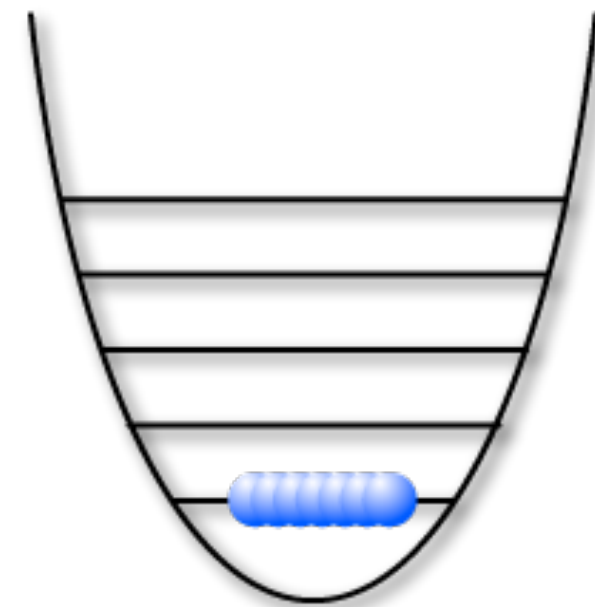
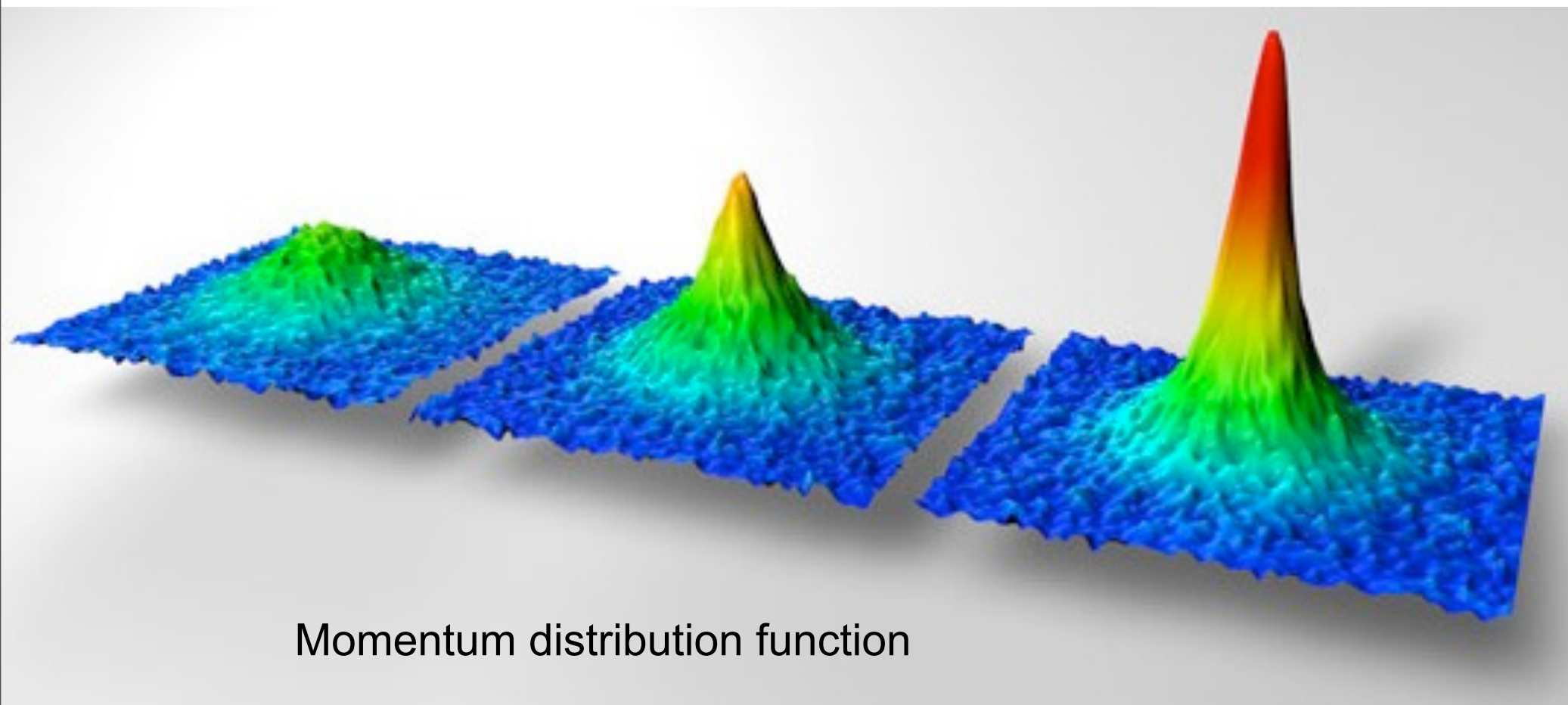
the image reflects the momentum distribution





# Bose-Einstein condensation in cold atomic gases

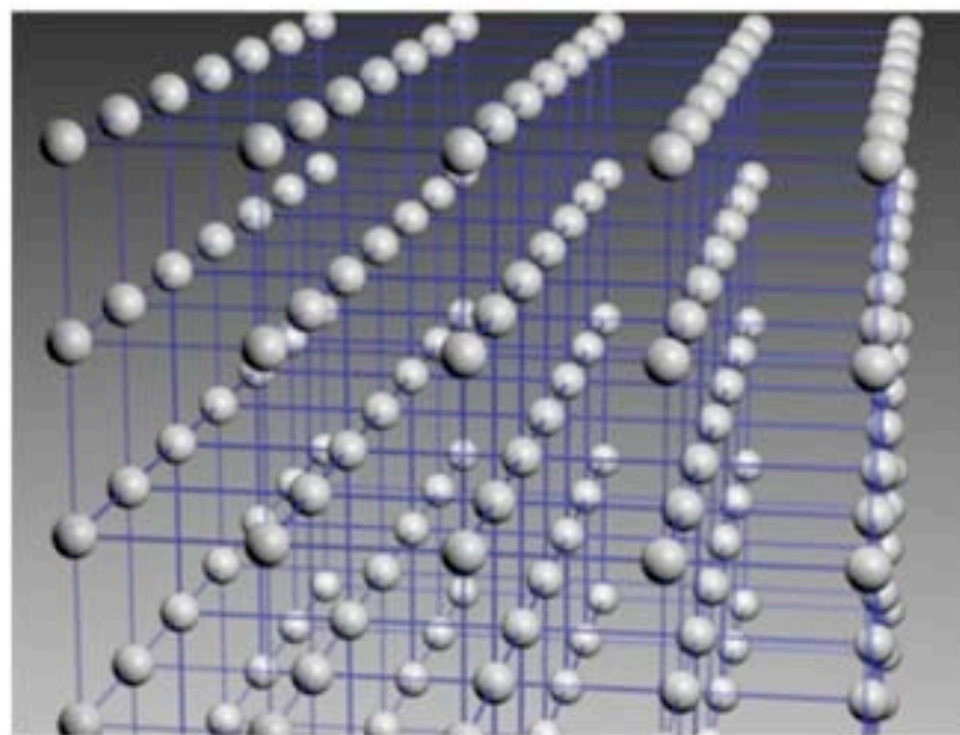
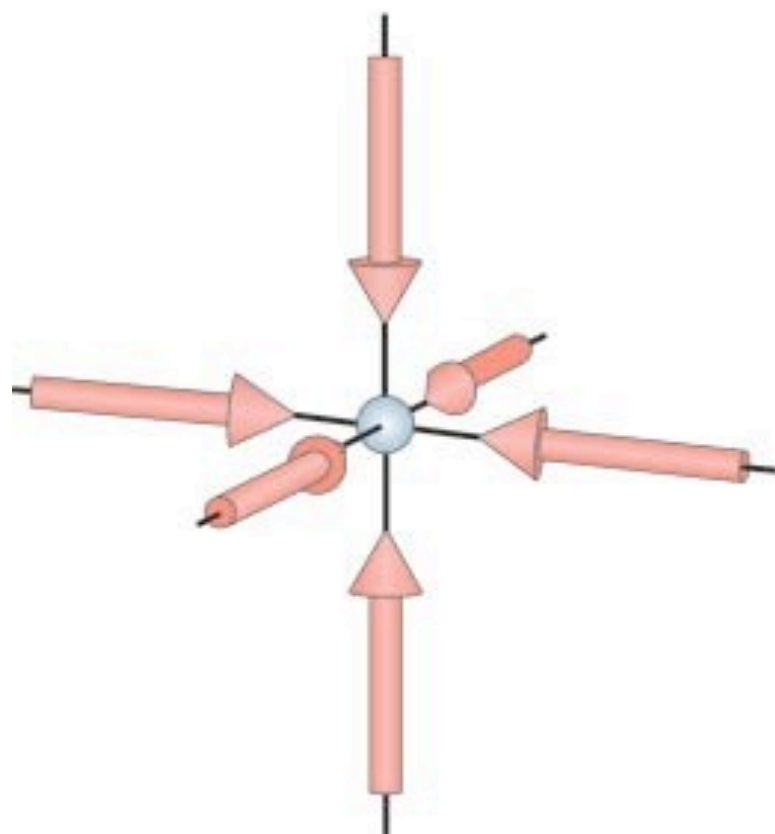
- At close to zero temperatures, a macroscopic fraction of all atoms in a Bose gas occupy the same quantum state
- A diverging occupation of the zero momentum state





# Optical lattices

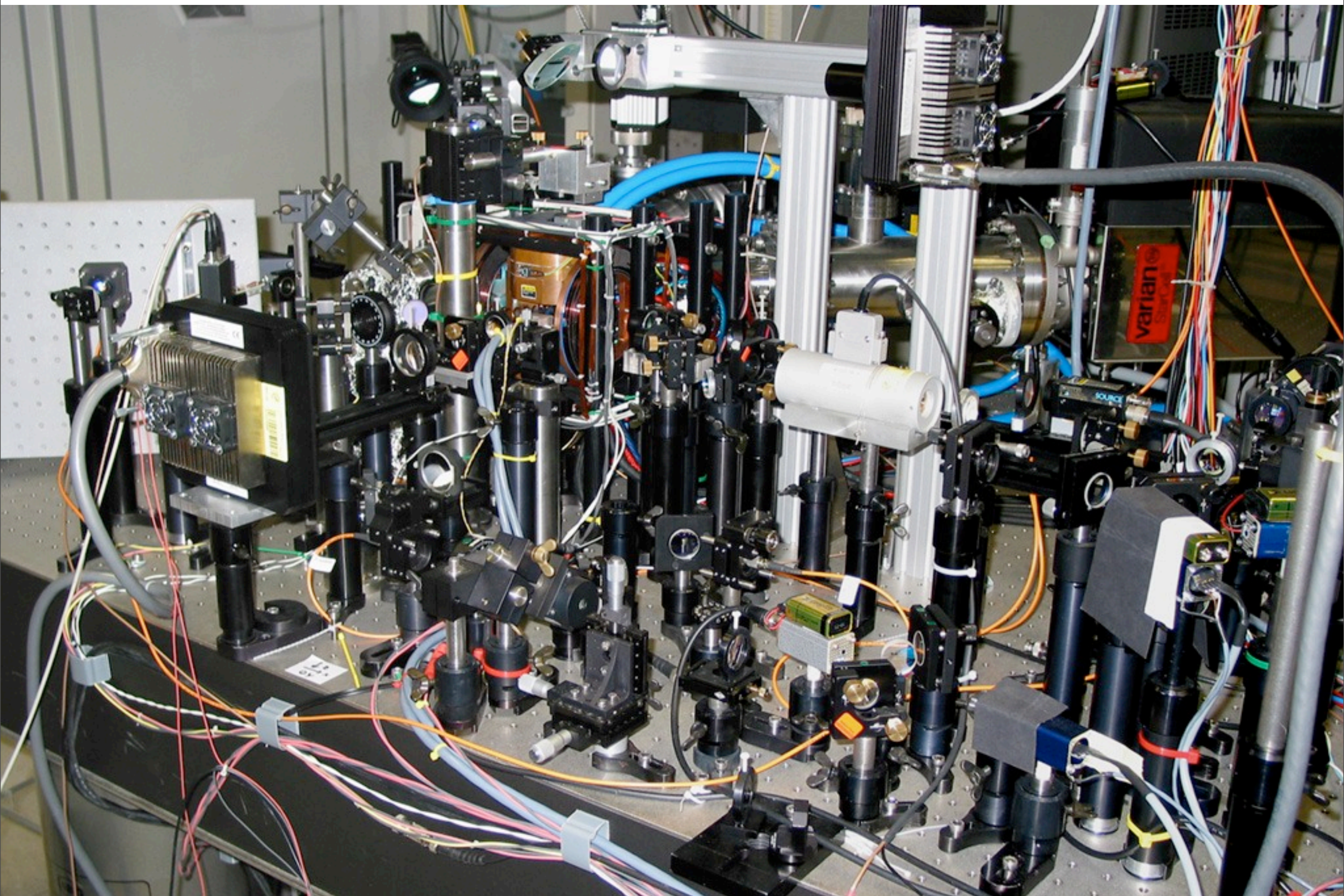
- formed by standing waves from three pairs of laser beams



- realize quantum **lattice** models of fermions or bosons



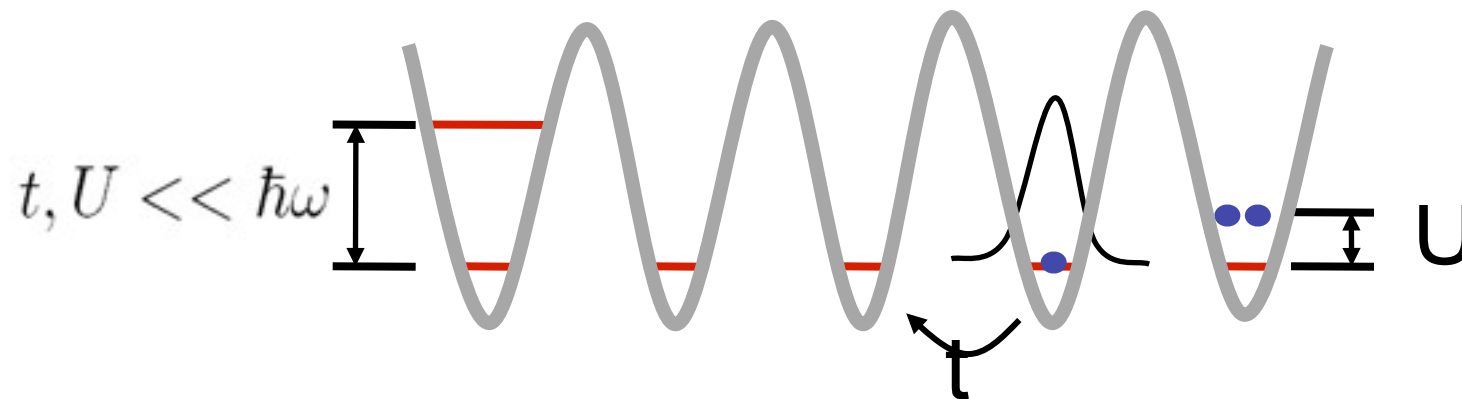
# Table 2





# Optical lattices and the Hubbard model

- Lasers couple to the dipole moment of the atoms
  - atoms prefer to sit at the amplitude maxima (AC Stark effect)
  - a periodic potential with periodicity half of the wave length
  - obtain a Hubbard model for the lowest band



- **Tunable and controlled**
  - Laser amplitude determines  $U$  and  $t$
  - Spatially varying couplings using optical superlattices
- **Flexible**
  - fermionic or bosonic atoms or mixtures are possible

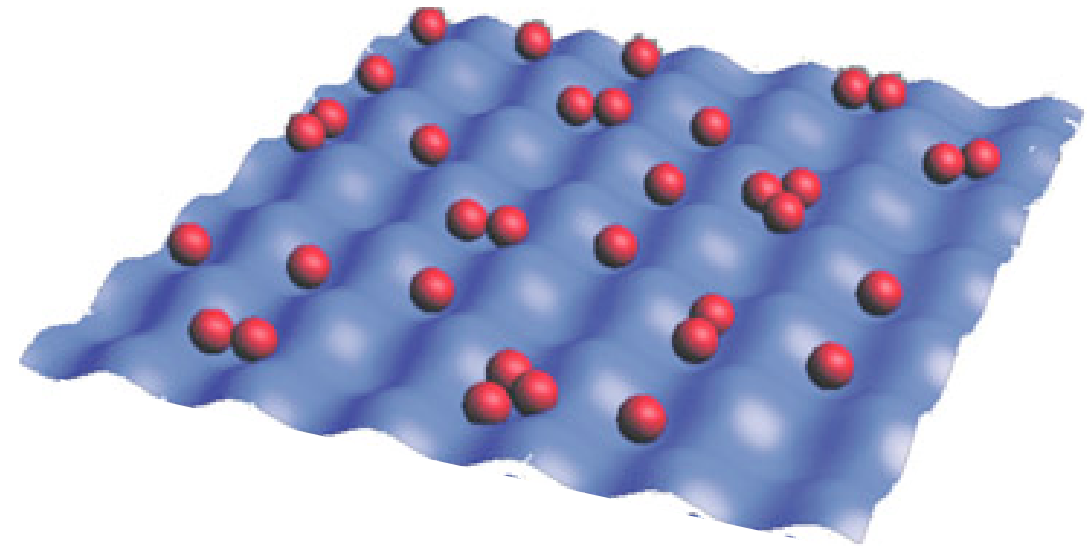


# Condensed matter phenomena in cold atoms



## Condensed Matter

- **Disordered**
- **Unknown interactions**
- **Little control**

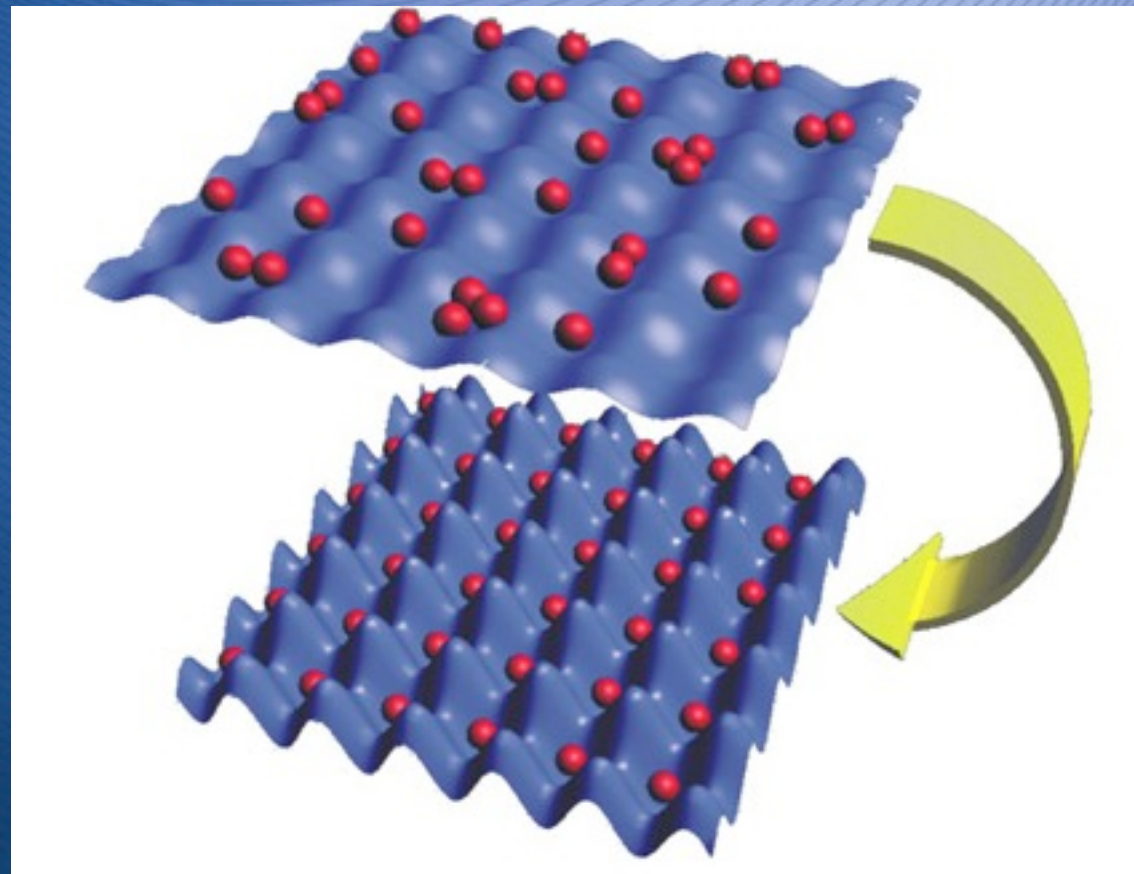


## Cold Atoms

- **Tunable dispersion**
- **Tunable interactions**
- **Perfect control**
- **Clean or controlled disorder**
- **Engineered Hamiltonians**



# Validating a quantum simulator: does it really work?





# DARPA Optical Lattice Emulator program

- Goal : build an optical lattice emulator to solve the prototypical models of condensed matter physics

implement  
**validate**

1. Validate / calibrate against known models (bosons)

design

2. Design a model with unknown physics (fermions)

new physics

3. Learn new physics assuming all parameters of the device are still under control

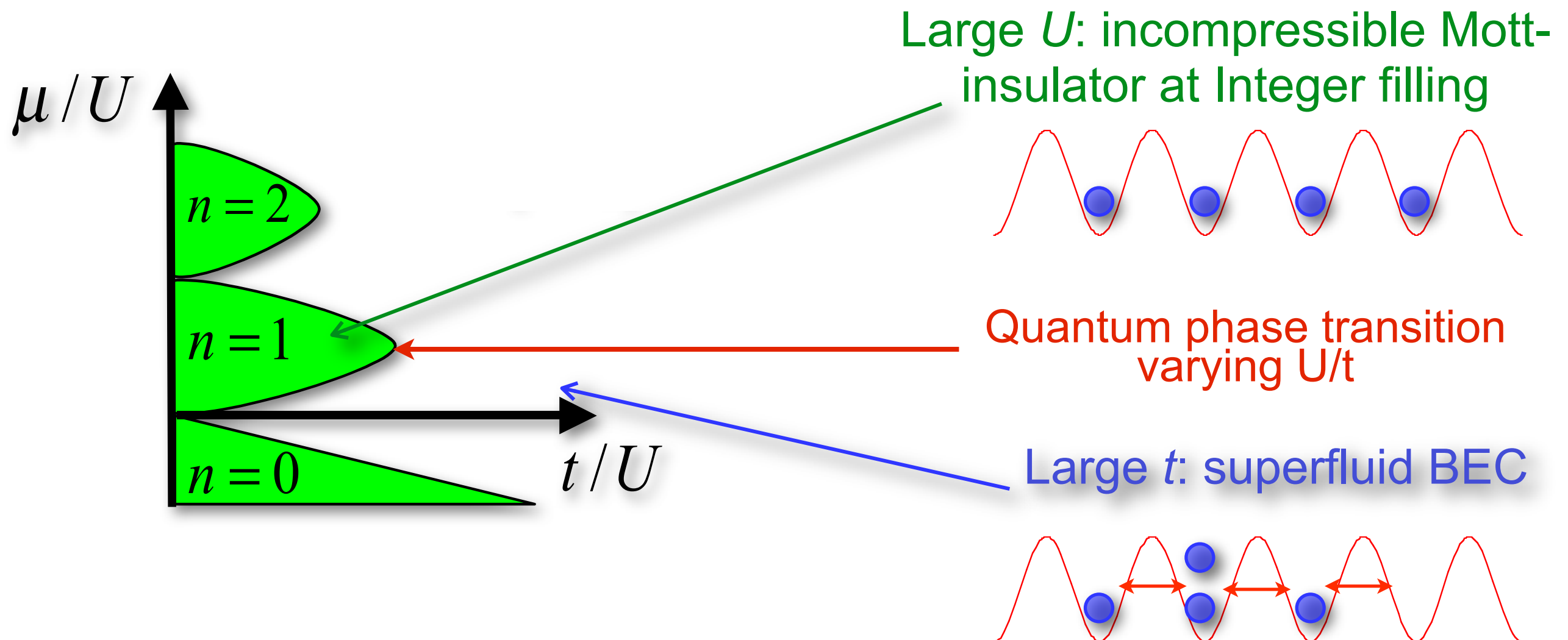


# Bose-Hubbard model

Fisher *et al*, PRB 1989

- Use bosonic atoms for validation

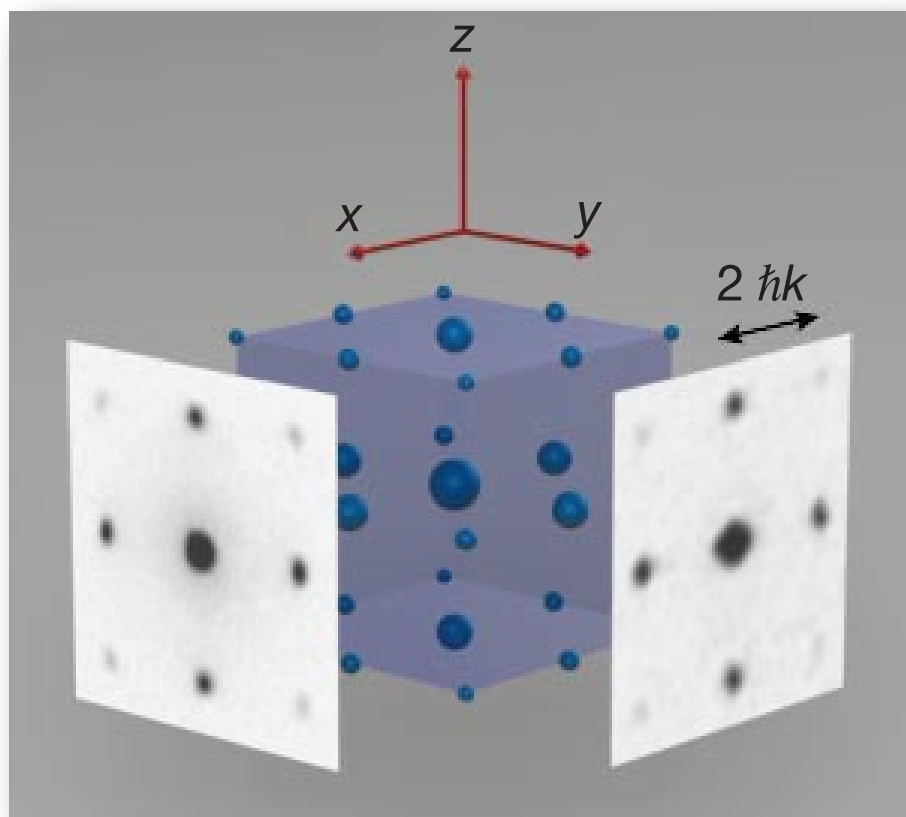
$$H = -t \sum_{\langle i,j \rangle} (b_i^\dagger b_j + b_j^\dagger b_i) + U \sum_i n_i(n_i - 1)/2 - \mu \sum_i n_i$$



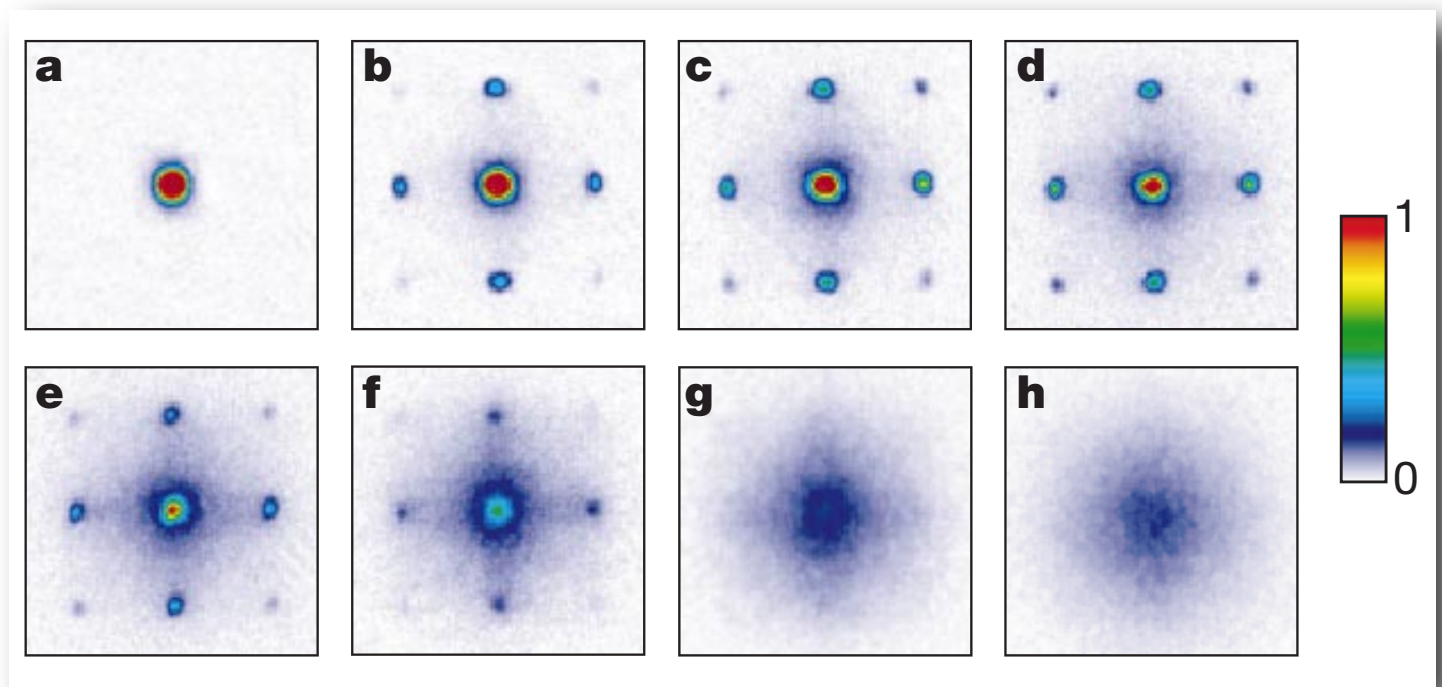


# The first optical lattice experiments

- Quantum phase transition as lattice depth is increased
  - Greiner et al, Nature (2002)
  - measuring the momentum distribution function in time-of-flight images



small  $U/t$ : condensate



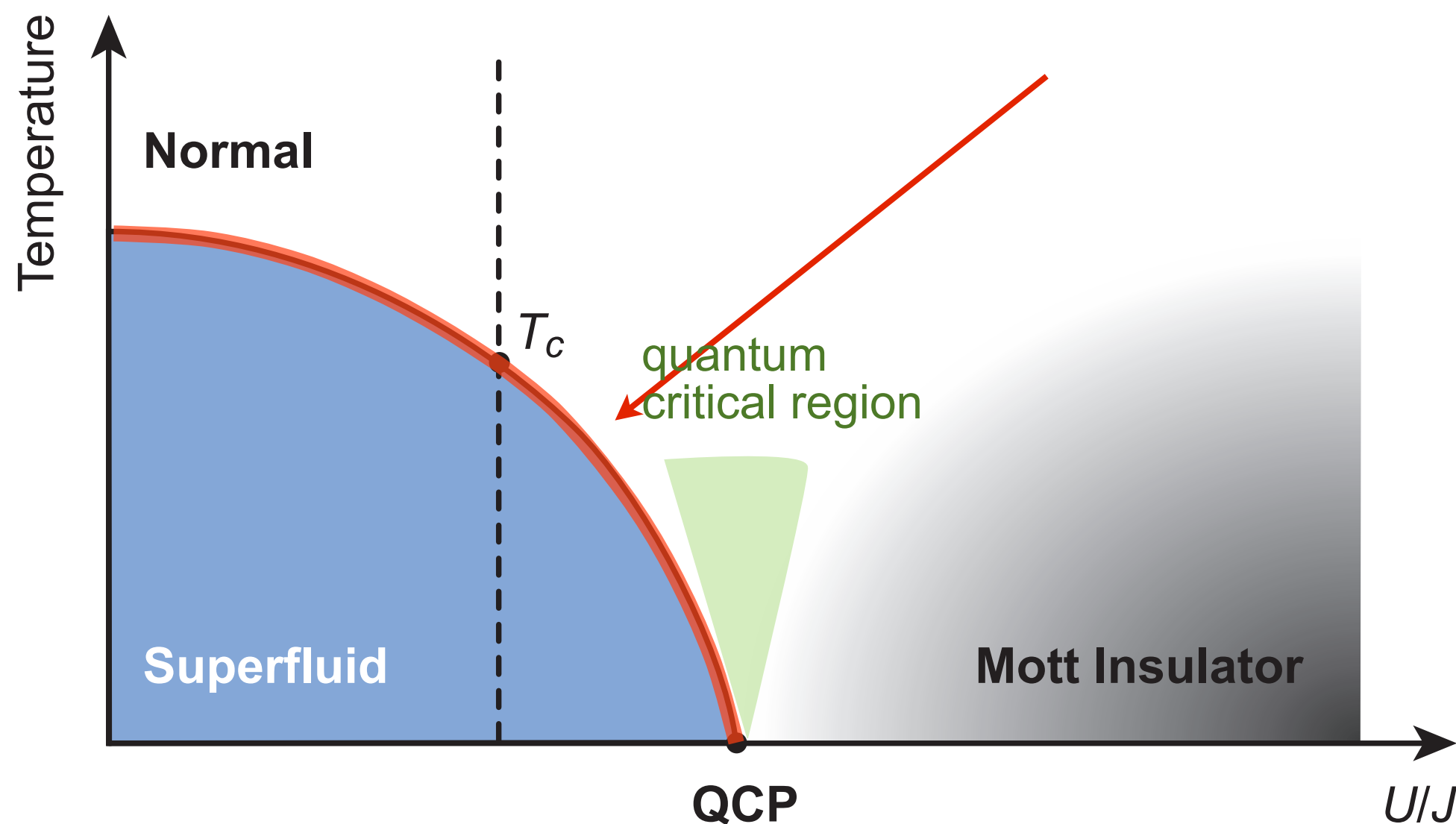
large  $U/t$ : Mott insulator

Can this be made more quantitative?



# Quantitative validation: the phase diagram

- Bosons in a 3D optical lattice at filling  $n = 1$
- Measure suppression of  $T_c$  close to the Mott insulator





# Validation by Quantum Monte Carlo simulations

- Approximation-free QMC simulations
  - worm algorithm
  - Prokof'ev, Svistunov and Tupitsyn, Sov. Phys. - JETP (1998)
  - up to 500,000 atoms
  - $220 \times 220 \times 200 \approx 10$  million sites
- We can model all details of the experiment
  - accurate microscopic model
  - same system size, particle numbers
  - temperature and entropy matched to experiment
  - measure quantities as observed in experiment
- Does the experiment agree with the QMC simulations?

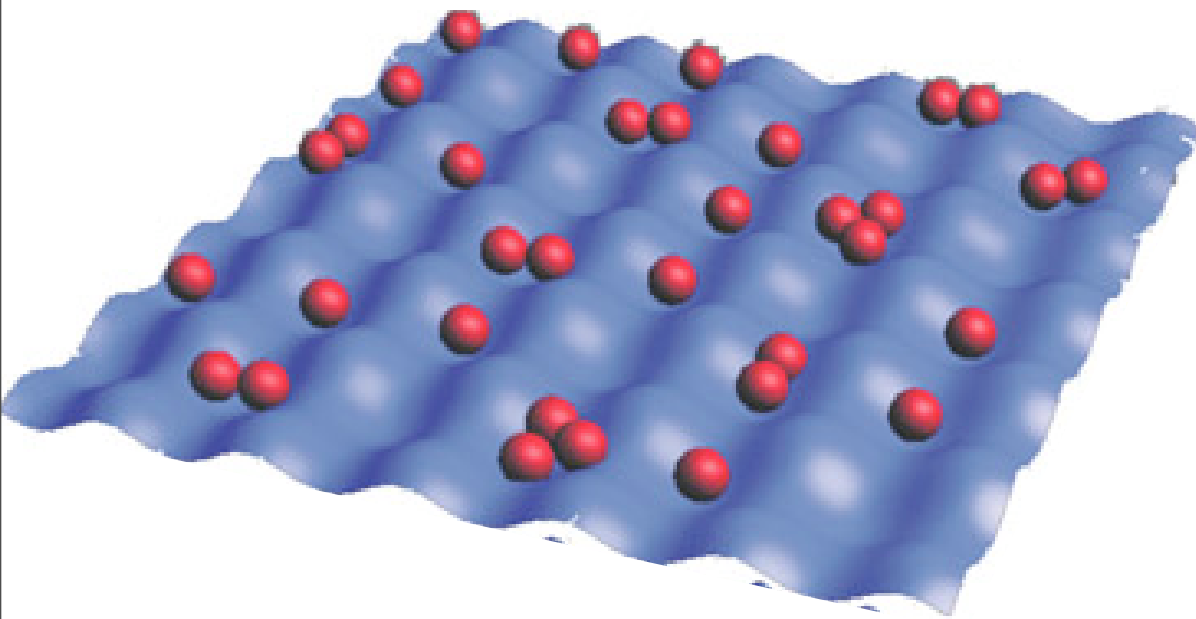


# Ab-initio mapping to the Hubbard model

$$H = \int d^3r \psi^\dagger(\vec{r}) \left( -\frac{\hbar^2}{2m} \Delta + V_{\text{opt}}(\vec{r}) \right) \psi(\vec{r}) + \frac{g}{2} \int d^3r \psi^\dagger(\vec{r}) \psi^\dagger(\vec{r}) \psi(\vec{r}) \psi(\vec{r})$$

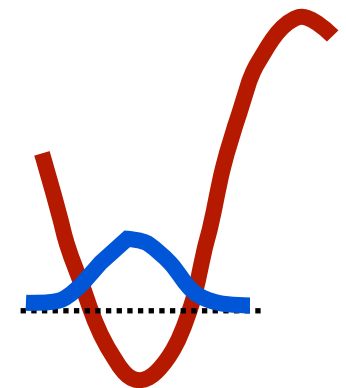
$$V_{\text{opt}}(r, z) = -V_0 e^{-2r^2/w^2(z)} \sin^2(kz)$$

$$g = \frac{4\pi\hbar^2 a_s}{m}$$



$$\psi(\vec{r}) = \sum_i w(\vec{r} - \vec{r}_i) b_i$$

express the bosonic field operator in terms of Wannier functions



$$H = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + \text{h.c.}) + U \sum_i n_i (n_i - 1)/2 - \mu \sum_i n_i + V \sum_i r_i^2 n_i$$

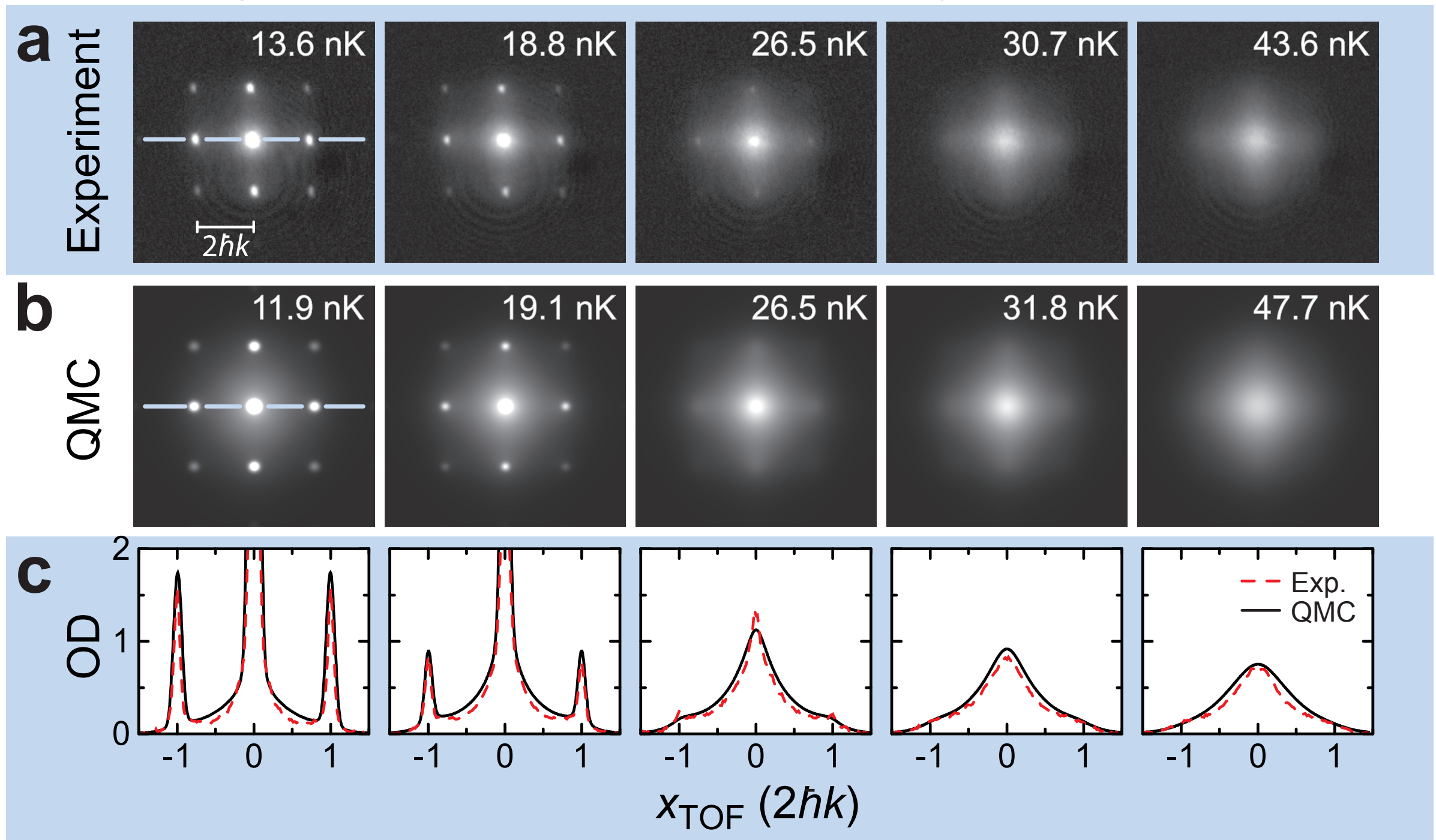
# Validation by Quantum Monte Carlo simulations

- Model all details of the experiment
  - accurate microscopic model
  - keeping the density at  $n = 1$
  - determining the temperature
  - calculate what the experiment should see
- Does the experiment agree with the QMC simulations?



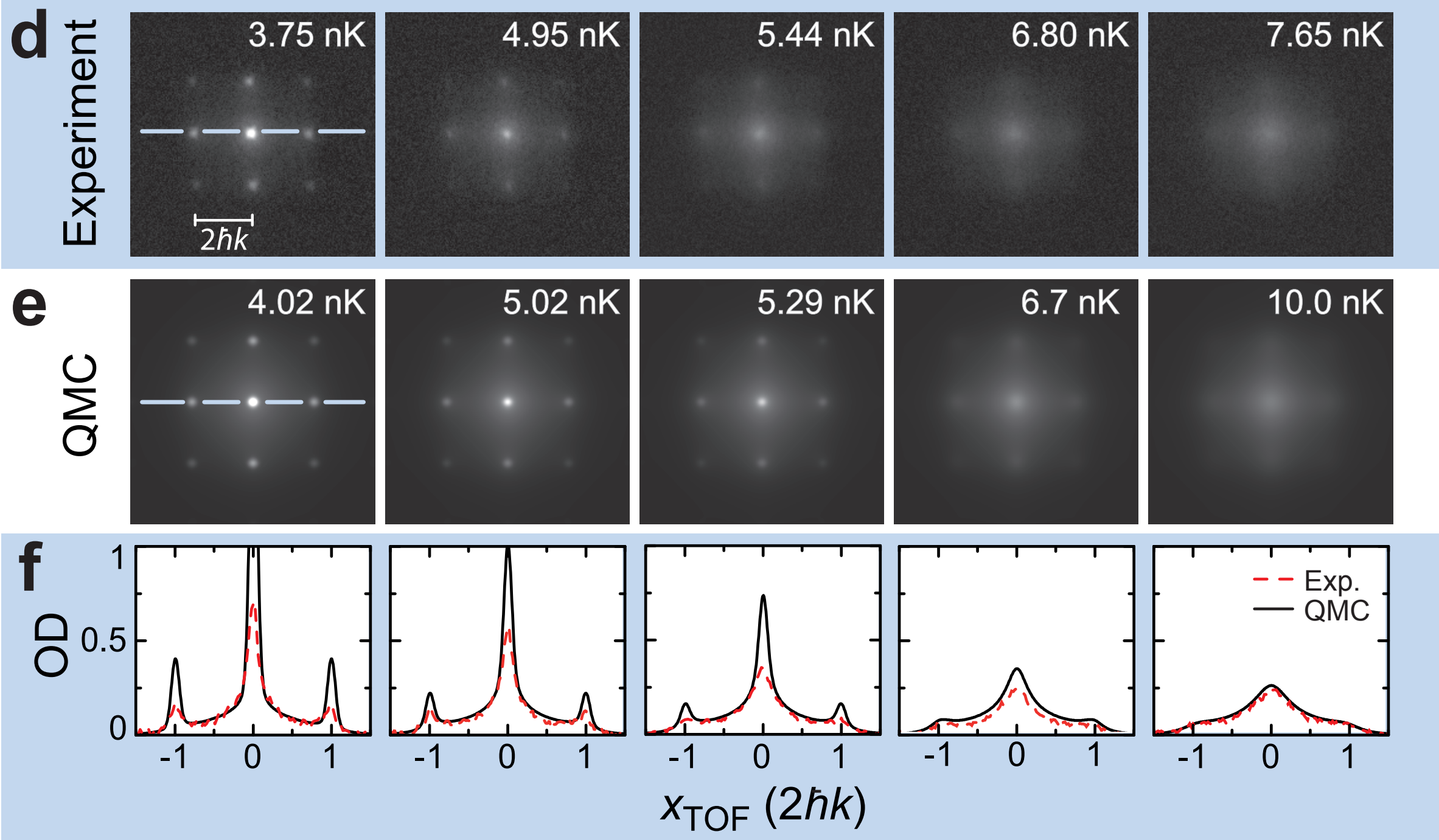
# Comparison QMC-experiment: small $U/t$

$$V_0 = 8E_r, \quad U/J = 8.11, \quad T_c = 26.5 \text{ nK}$$



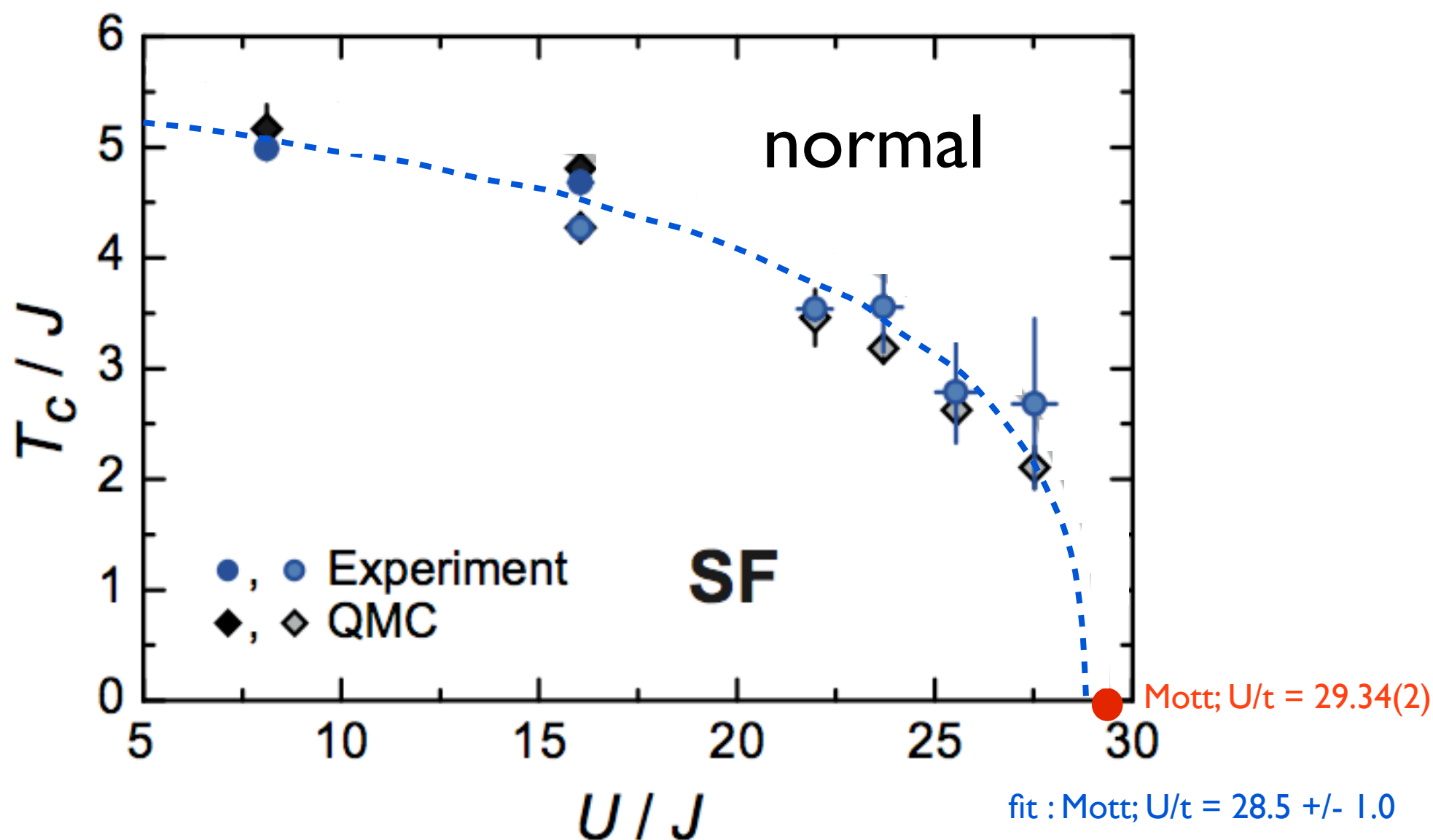
# Comparison QMC-experiment: large $U/t$

$$V_0 = 11.75E_r, \quad U/J = 27.5, \quad T_c = 5.31\text{nK}$$





# Phase diagram obtained by the quantum simulation



# Collaborators

## ■ Simulations

- Lode Pollet (ETH → UMass → Harvard)
- Nikolay Prokof'ev (UMass)
- Vito Scarola (ETH & UC Berkeley → VATech)
- Boris Svistunov (UMass)
- Ping Nang Ma (Hong Kong → ETH)



## ■ Experiments

- Stefan Trotzky (Mainz → Munich)
- Immanuel Bloch (Mainz → Munich)
- Ute Schnorrberger (Mainz → Munich)
- Fabrice Gerbier (Paris)

