

PtrHash: Minimal Perfect Hashing at RAM Throughput

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Abstract

Motivation. Given a set K of n keys, a minimal perfect hash function (MPHF) is a collision-free bijective map H_{mpfh} from K to $\{0, \dots, n-1\}$. These functions have uses in databases, search engines, and are used in bioinformatics indexing tools such as Pufferfish (using BBHash), and Piscem (PHash). PHash is also used in SHash, a data structure on k -mers that supports membership queries. PHash only takes around 5% of the total space of SHash, and thus, trading slightly more space for faster queries is beneficial. Thus, this work presents a (minimal) perfect hash function that first prioritizes query throughput, while also allowing efficient construction for 10^9 or more elements using 2.4 bits of memory per key.

Contributions. Both PHash and PHOBIC first map all n keys to $n/\lambda < n$ buckets. Then, each bucket stores a *pilot* that controls the final hash value of the keys mapping to it. PtrHash builds on this by using 1) fixed-width (uncompressed) 8-bit pilots, 2) a construction algorithm similar to Cuckoo hashing to find suitable pilot values. Further, it partitions the keys, so that keys in each part map to their own set of *slots*. PtrHash 3) uses the same number of buckets and slots for each part, with 4) a *single* remap table to map intermediate positions $\geq n$ to $< n$, 5) encoded using per-cacheline Elias-Fano coding. Lastly, 6) PtrHash supports *streaming* queries, where we use prefetching to answer a stream of multiple queries more efficiently than one-by-one processing.

Results. With default parameters, PtrHash takes 2.4 bits per key. On 300 million string keys, PtrHash is as fast or faster to build than other MPHFs at a similar size, and at least $2.1\times$ faster to query. When streaming multiple queries, this improves to $3.3\times$ speedup over the fastest alternative, while also being significantly faster to construct. When using 10^9 integer keys instead, query times are as low as 12 ns/key when iterating in a for loop, or even down to 8 ns/key when using the streaming approach, just short of the 7.4 ns inverse throughput of random memory accesses.

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1 Introduction

Given a set of n keys $\{k_0, \dots, k_{n-1}\}$, a *hash function* maps them to some co-domain $[m] := \{0, \dots, m-1\}$. When $m \geq n$ and the hash is injective (collision-free), it is also called *perfect*. When additionally $m = n$ and it is surjective onto $[n]$, it is *minimal*. Thus, a *minimal perfect hash function* (MPHF) bijectively maps a set of n keys onto $[n]$.

Metrics. Various aspects of MPHF data structures can be optimized. First, one could minimize its space usage and try to approach the $\log_2(e) = 1.4427$ bits/key lower bound [31]. Indeed, there are many recent works in this direction, such as Bipartite ShockHash-RS, which uses under 1.5 bits/key [25, 23, 21], and CONSENSUS-RecSplit [26], which goes as low as 1.444 bits/key.

In this paper, we focus primarily on optimizing for query throughput and secondarily on construction speed, while relaxing space usage up to 3 bits/key. This continues the line of work of FCH [16], PTHash [36, 37], and PHOBIC [19], that all provide relatively fast queries.

Problem statement. Construct a *minimal perfect hash function* data structure H_{mpfh} that is fast to query, ideally using one memory access per lookup, and fast to construct, while staying below 3 bits/key of space.

Motivation. Our main motivating application is to optimize the use of PTHash in SSSHash [35], a data structure to index a set of k -mers (sequences of k DNA bases). There, the MPHF only takes around 5% of the total space. Thus, a slightly increased space usage of the MPHF has little effect on the total space, while faster lookups could significantly improve the overall query speed. In this application, k -mers are typically encoded as 64-bit integers, and thus we will focus our attention on integer keys.

Further applications can be found in domains such as networking [29], databases [7], and full-text indexing [2], where one could imagine hashing IP addresses, URLs, or (compact) suffix-trie edge labels.

Contributions. We introduce PtrHash, a minimal perfect hash function that is primarily optimized for query throughput and construction speed, at the cost of slightly more memory usage. It builds on the same principles as PTHash(-HEM) and Phobic: first, keys are partitioned into *parts*. Then, the keys in each part are further split into *buckets*, and each bucket is assigned a *pilot* that controls the values (*slots*) that the keys in the bucket hash to.

Compared to PTHash and PHOBIC, the main novelties of PtrHash are:

1. a fixed number of buckets *and slots* per part, removing the need for a part-offset lookup;
2. the use of fixed-width 8-bit *pilots*, so that no compact encoding is needed;
3. a pilot search based on Cuckoo hashing;
4. remapping using a *single* remap table, again simplifying lookups;
5. a remap table based on a per-cacheline Elias-Fano encoding [10, 13], CacheLineEF;
6. the use of *prefetching* to *stream* multiple queries in parallel.

Results. When using 300 million string keys, PtrHash with default parameters takes 2.4 bits/key and is nearly as fast to construct as the fastest other methods, while being much faster to query. Compared to the next-fastest method to query, PtrHash provides $2.1\times$ faster queries when looping naively, or $3.3\times$ faster when streaming.

When using 10^9 integer keys instead, PtrHash can achieve an inverse throughput¹ as low as 12 ns/key when looping over queries, or even 8 ns/key when streaming.

The hardware used for benchmarking has a maximum single-threaded memory bandwidth of 7.4 ns per cache line. Thus, under the assumption² that almost every query requires reading at least one new cache line from main memory, our method is close to the maximum possible query throughput. Likewise, in a multi-threaded setting, PtrHash can fully saturate the DDR4 memory bandwidth while answering around 1 query per fetched cache line.

2 Related work

There is a vast amount of literature on (minimal) perfect hashing, going back to e.g. [30]. Here we only give a highlight of recent approaches. We refer the reader to Section 2 of [37] and Sections 4 and 8 of the thesis of Hans-Peter Lehmann [21], which contains a nice overview of different approaches taken by various tools.

Space lower bound. There is a lower bound of $n \log_2(e)$ bits to store a minimal perfect hash function on n random keys [31]. To get some feeling for this bound, consider any hash function. Intuitively the probability that this is an MPHF is $n!/n^n$. From this, it follows that at most, around $\log_2(n^n/n!) \approx n \log_2(e)$ bits of information are needed to “steer” the hash function in the right direction. Now, a naive approach is to use a seeded hash function, and try $O(e^n)$ seeds until a perfect hash function is found. However, that is not feasible in practice. The method that currently gets closest to the lower bound is CONSENSUS-RecSplit [26], which goes as low as 1.444 bits/key.

Bucket placement. PtrHash builds on methods that first group the keys into buckets of a few keys. Then, keys in the buckets are assigned their hash value one bucket at a time, such that newly assigned values do not collide with previously taken values. All methods iterate different possible key assignments for each bucket until a collision-free one is found, but differ in the way hash values are determined. To speed up this search, large buckets are assigned a hash before small buckets, since smaller buckets are easier to place when many slots are already taken.

FCH [16] uses a fixed number of bits to encode the seed for each bucket and uses a *skew* distribution of bucket sizes. The seed stored in each bucket determines how far the keys are *displaced* (rotated) to the right from their initially hashed positions. A fallback hash can be used if needed, and construction can fail if that also does not work. CHD [1] uses uniform bucket sizes, but uses a variable-width encoding for the seeds. PTHash [36] combines these two ideas and introduces a number of compression schemes for the seed values, that are called *pilots*. Instead of directly generating an MPHF, it first generates a PHF to $[n']$ for $n' = n/\alpha \approx n/0.99$, and values mapping to positions $\geq n$ are *remapped* to the skipped values in $[n]$. PTHash-HEM [37] first partitions the keys, and uses this to build multiple parts in parallel. This also enables external-memory construction. Lastly, PHOBIC [19] improves from the simple *skew* distribution of FCH to an *optimal bucket assignment function*, which speeds up construction and enables smaller space usage. Secondly, it partitions the input

¹ For interpretability and consistency with latency numbers, we report the inverse throughput in nano-seconds per key, rather than keys per second. We will still refer to this as *throughput*, rather than *inverse throughput*, following the [intel instruction manual](#).

² This is a strong assumption, and indeed, PtrHash with the cubic bucket assignment function already slightly breaks this assumed lower bound.

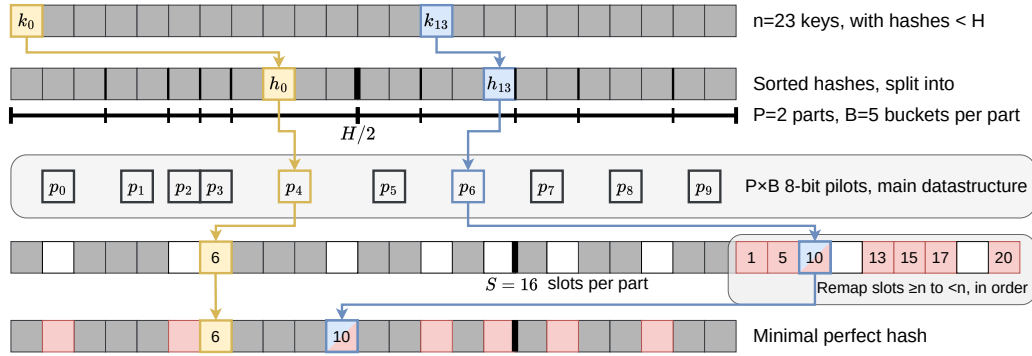


Figure 1 Overview of PtrHash on $n = 23$ keys. The keys are hashed into $[H] = [2^{64}]$ and this range is split into $P = 2$ parts and $B = 5$ buckets per part. The key highlighted in yellow has a the 9'th smallest hash, and ends up in *bucket 4* (starting at index 0). The corresponding *pilot* p_4 hashes the key to *slot 6*. The array of pilots (grey background) is the main component of the PtrHash data structure, and ensures that all keys hash to different slots. The blue key has a hash in the second part (upper half) of hashes, in bucket 6. It gets hashed to slot 25, which is larger than the number of keys $n = 23$. Thus, it is *remapped* (along with the other red cells) into an empty slot $< n$ via a (compressed) list of free slots, which is the second main component of the data structure.

into parts of expected size 2500 and uses the same number of buckets for each part. Then, it uses that the pilot values of the i 'th bucket of each part follow the same distribution, and encodes them together. Together, this saves 0.17 bits/key over PTHash. Lastly, some of the ideas in PtrHash (fixed 8-bit pilots and cuckoo hashing) have been independently proposed in [18].

3 PtrHash

The core design goal of PtrHash³ is to simplify PTHash to speed up both query speed and construction time, at the cost of possibly using slightly more memory.

3.1 Overview

Before going into details, we first briefly explain the fully constructed PtrHash data structure and how to query it, see Figure 1. We also highlight differences to PTHash [36] and PHOBIC [19].

Parts and buckets. The input is a set of n keys $\{k_0, \dots, k_{n-1}\}$ that we want to hash to n slots $[n] := \{0, \dots, n-1\}$. We first hash the keys using a 64-bit hash function h into $\{h(k_0), \dots, h(k_{n-1})\}$. The total space of hashes $[2^{64}]$ is equally partitioned into P parts, and the part of a key is easily found as $\lfloor P \cdot h(k_i) / 2^{64} \rfloor = \text{hi}(P \cdot h(k_i))$ [27], where $\text{hi}(a \cdot b)$ returns the high 64 bits of the product of two 64-bit integers, and likewise, $\text{lo}(a \cdot b)$ returns the low 64 bits. Then, the expected n/P keys in each part are further split into exactly B non-uniform

³ The PT in PTHash stand for *Pilot Table*. The author of the present paper mistakenly understood it to stand for Pibiri and Trani, the authors of the PTHash paper. Due to the current author's unconventional last name, and PTKG not sounding great, the first initial (R) was appended instead, doubling as a hint that PtrHash is written in Rust. As things go, nothing is as permanent as a temporary name. Furthermore, we follow the Google style guide and avoid a long run of uppercase letters, and write PtrHash instead of PTRHash.

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buckets: each key has a *relative position* x inside the part, and this is passed through a *bucket assignment function* $\gamma : [0, 1) \mapsto [0, 1)$ such as $\gamma(x) = x^2$ that controls the distribution of expected bucket sizes [19], as explained in detail in Section 3.3. The result is then scaled to a bucket index in $[B]$:

$$\begin{aligned} \text{part}(k_i) &:= \text{hi}(P \cdot h(k_i)), \\ x &:= \text{lo}(P \cdot h(k_i)) / 2^{64}, \\ \text{bucket}(k_i) &:= \text{hi}(B \cdot (2^{64} \cdot \gamma(x))). \end{aligned} \tag{1}$$

Slots and pilots. Now, the goal and core of the data structure is to map the n/P expected keys in each part to $S \approx (n/P)/\alpha$ slots, where $\alpha \approx 0.99$ gives us $\approx 1\%$ extra slots to play with. The pilot for each bucket controls to which slots its keys map. PtrHash uses fixed-width 8-bit *pilots* [18] $\{p_0, \dots, p_{P \cdot B - 1}\}$, one for each bucket. Specifically, key k_i in bucket $\text{bucket}(k_i)$ with pilot $p_{\text{bucket}(k_i)}$ maps to slot

$$\text{slot}(k_i) := \text{part}(k_i) \cdot S + \text{reduce}(h(k_i) \oplus h_p(p_{\text{bucket}(k_i)}), S), \tag{2}$$

where $\text{reduce}(\cdot, S)$ maps the random 64-bit integer into $[S]$ as explained below, and \oplus denotes xor.

Compared to PHOBIC and PTHash(-HEM) [37], there are two differences here. First, while we still split the input into parts, we assign each part not only the same number of buckets, but also the *same* number of slots, instead of scaling the number of slots with the *actual* size of each part. This removes the need store a prefix sum of part sizes, and avoids one memory access at query time to look up the offset of the key's part. This idea was recently independently introduced as ε -cost sharding [39]. Second, previous methods search for arbitrary large pilot values that require some form of compression to store efficiently. Our 8-bit pilots can simply be stored in an array so that lookups are simple.

We now go over some specific details.

Hash functions. The 8-bit pilots p_b are hashed into pseudo-random 64-bit integers by using FxHash [6] for h_p , which simply multiplies the pilot with a *mixing constant* C after xoring by a global seed:

$$h_p(p) := C \cdot (p \oplus \text{seed}). \tag{3}$$

When the keys are 64-bit integers, we use this same FxHash algorithm to hash them ($h(k) := C \cdot k$), since multiplication by an odd constant is invertible modulo 2^{64} (since $\text{gcd}(C, 2^{64}) = 1$) and hence collision-free. For other types of keys, the hash function depends on the number of elements. When the number of elements is not too far above 10^9 , the probability of hash collisions with a 64-bit hash function is sufficiently small, and, following PHast [4], we use the 64-bit variant of GxHash [17], a hash function based on AES hardware instructions. When the number of keys goes beyond $2^{32} \approx 4 \cdot 10^9$, the probability of 64-bit hash collisions increases. In this case, we use the 128-bit variant of GxHash. The high 64-bits determine the part and bucket in Equation (1), and the low 64-bits are used in Equation (2) to determine the slot.

The reduce function. To obtain the slot inside the current part, we must reduce the hash based on the key and its pilot to a number in $\{0, \dots, S - 1\}$. One way of doing this is to use “fast mod” [28], which uses two multiplications when the modulus (the number of slots per part S) is less than 2^{32} .

When S is a power of two, we can instead use $\text{reduce}(x, S) = \text{hi}(C \cdot x) \bmod S$, which only needs a single multiplication and a bitmask. The multiplication by the mixing constant C ensures that all bits of x are used. In practice, this is the method we use.

When the number of parts is small, a drawback of limiting S to powers of two is that this could cause up to 50% empty slots. In this case, fast mod can be used for reliability. Then, that S must *not* a power of two, so that $x \bmod S$ depends on all⁴ bits of x . Additionally, we can only use a single part, simplifying queries.

Remapping. Since each part has slightly ($\approx 1\%$) more slots than keys, some keys will map to an index $\geq n$, leading to a *non-minimal* perfect hash function. To fix this, those are *remapped* back into the “gaps” left behind in slots $< n$ using a (possibly compressed) lookup table. This is explained in detail in Section 3.4.

Whereas PTHash-HEM uses a separate remap *per part*, PtrHash only has a single “global” remap table.

Construction. The main difficulty of PtrHash is during construction (Section 3.2), where we must find values of the pilots p_j such that all keys indeed map to different slots. Like other methods, PtrHash processes multiple parts in parallel. Within each part, it sorts the buckets from large to small and “greedily” assigns them the smallest pilot value that maps the keys in the bucket to slots that are still free. Unlike other methods though, PtrHash only allows pilots up to 255. When no suitable pilot is found, we use a method similar to (blocked) Cuckoo hashing [33, 15]: a pilot with a minimal number of collisions is chosen, and the colliding buckets are “evicted” and will have to search for a new pilot.

Parameter values. In practice, we usually use $\alpha = 0.99$. Similar to PHOBIC, the number of buckets per part is set to $B = \lceil (\alpha \cdot S) / \lambda \rceil$, where λ is the expected size of each bucket and is around 3 to 4. The number of parts is $P = \lceil n / (\alpha S) \rceil$. Smaller parts fit better in cache and hence are faster to construct, while too small parts have too much variance in their size, causing some parts to possibly have more than S keys in them. Thus, we would like to choose S as the smallest size for which the probability that any part is over-subscribed is sufficiently small. Vigna [39, eq. 3] shows that in practice, the following formula works well:

$$P \approx n / (\alpha S) \leq \frac{n \varepsilon^2 / 2}{\ln(n \varepsilon^2 / 2)},$$

where we use $\varepsilon = (1 - \alpha) / 2$ to ensure that all parts have at last half of the average number of free slots. For $\alpha = 0.99$, this reduces to

$$\alpha S \geq 80\,000 \cdot \ln(n / 80\,000), \quad (4)$$

and so this is the number of key per part αS we choose, with a minimum of 80 000 for when $n \leq 80\,000$.

Streaming queries. PtrHash supports *streaming* queries, where multiple queries are processed in parallel. This allows prefetching pilots from memory, and thus increases throughput and better uses the available memory bandwidth. This is explained and evaluated in Appendix A.

⁴ Only depending on the $\lg_2 S$ low bits is not good enough, since the **part** and **bucket** functions only depend on the high $\lg_2(P \cdot B)$ bits, leaving some bits in the middle unused.

Sharding. When the number of keys is so large that their hashes do not fit into memory, one of three sharding strategies can be used: in-memory, on-disk, or hybrid. These are explained and evaluated in Appendix B.

3.2 Construction

Both PTHash-HEM and PHOBIC first partition the keys into parts, and then build an MPHF part-by-part, optionally in parallel on multiple threads. Within each part, the keys are randomly split into *buckets* of average size λ (Figure 1). Since $\lambda \leq 4$ in practice, the variance on bucket sizes is quite large. Thus, the buckets are sorted from large to small, and one-by-one *greedily* assigned a *pilot*, such that the keys in the bucket map to *slots* not yet covered by earlier buckets.

As more buckets are placed, there are fewer remaining empty slots, and searching for pilots becomes harder. Hence, PTHash uses $n/\alpha > n$ slots to ensure there sufficiently many empty slots for the last pilots. This speeds up the search and reduces the values of the pilots. PHOBIC, on the other hand, uses relatively small parts of expected size 2500, so that the search for the last empty slot usually should not take much more than 2500 attempts. Nevertheless, a drawback of the greedy approach is that pilots values have an uneven distribution, making it somewhat harder to compress them while still allowing fast access (e.g., requiring the interleaved coding of PHOBIC).

Hash-evict⁵. In PtrHash, we instead use *fixed width*, single byte pilots. To achieve this, we use a technique resembling Cuckoo hashing [33] that was also independently found in [18, Section 4.5]. As before, buckets are greedily *inserted* from large to small. For some buckets, there may be no pilot in $[2^8]$ such that all its keys map to empty slots. When this happens, a pilot is found with the lowest weighted number of *collisions*. The weight of a collision with an element of a bucket of size s is s^2 , to prevent *evicting⁶* large buckets, as those are harder to place. The colliding buckets are evicted by emptying the slots they map to and pushing them back onto the priority queue of remaining buckets. Then, the new bucket is inserted, and the next largest remaining or evicted bucket is processed.

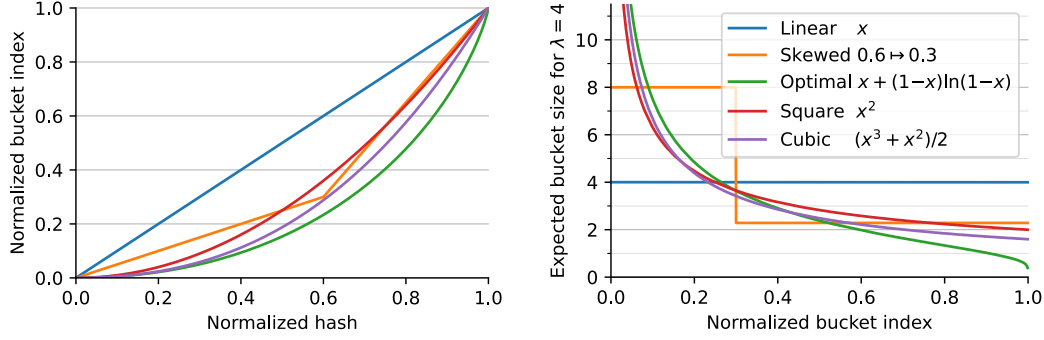
In order to efficiently search for pilot vectors, we use a bitvector of taken slots. Additionally, we avoid infinite loops of evicted buckets by storing the 16 most recently placed buckets, and never displacing those.

3.3 Bucket Assignment Functions

During construction, slots fill up as more buckets are placed. Because of this, the first buckets are much easier to place than the later ones, when only few empty slots are left. To compensate for this, we can introduce an uneven distribution of bucket sizes, so that the first buckets are much larger and the last buckets are smaller. FCH [16] accomplishes this by a *skew* mapping that assigns 60% of the elements to 30% of the buckets, so that those 30% are *large* buckets while the remaining 70% is *small* (Figure 2). This is also the scheme used by PTHash.

⁵ We would have preferred to call this method hash-displace, as *displace* is the term used instead of *evict* in e.g. the Cuckoo filter [12]. Unfortunately, *hash and displace* is already taken by compressed hash-and-displace [32, 1]

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■ **Figure 2** The left shows various bucket assignment functions γ , such as the piecewise linear function (skewed) used by FCH and PTHash, and the optimal function introduced by PHOBIC. Flatter slopes at $x = 0$ create larger buckets, while steeper slopes at $x = 1$ create more small buckets, as shown on the right, as the distribution of expected bucket sizes given by $(\gamma^{-1})'$ when the expected bucket size is $\lambda = 4$.

The optimal bucket function. PHOBIC [19] provides a more thorough analysis and uses the optimal function $\gamma_p(x) = x + (1 - x) \ln(1 - x)$ when the target load factor is $\alpha = 1$. A small modification is optimal for $\alpha < 1$ [20, Appendix B], but for simplicity we only consider the original γ_p . This function has derivative 0 at $x = 0$, so that many x values map close to 0. In practice, this causes the largest buckets to have size much larger than \sqrt{S} . Such buckets are hard to place, because by the birthday paradox they are likely to have multiple elements hashing to the same slot. To fix this, PHOBIC ensures the slope of γ is at least $\varepsilon = 1/(5\sqrt{S})$ by using $\gamma_{p,\varepsilon}(x) = x + (1 - \varepsilon)(1 - x) \ln(1 - x)$ instead. For simplicity in the implementation, we fix $\varepsilon = 1/2^8$, which works well in practice.

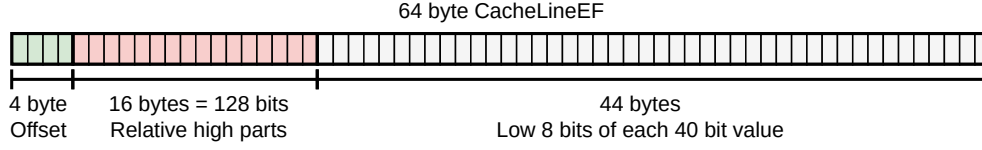
Approximations. For PtrHash, we aim for high query throughput, and thus we would like to only use simple computations and avoid additional lookups as much as possible. To this end, we replace the $\ln(1 - x)$ by its first order Taylor approximation at $x = 0$, $\ln(1 - x) \approx -x$, giving the quadratic $\gamma_2(x) := x^2$. Using the second order approximation $\ln(1 - x) \approx -x - x^2/2$ results in the cubic $\gamma(x) = (x^2 + x^3)/2$. This version again suffers from too large buckets, so in practice we use $\gamma_3(x) = \frac{2^8 - 1}{2^8} \cdot (x^2 + x^3)/2 + \frac{1}{2^8} \cdot x$. We also test the trivial $\gamma_1(x) := x$.

These values can all be computed efficiently by using that the input and output of γ are 64-bit unsigned integers representing a fraction of 2^{64} , so that e.g. x^2 can simply be computed as $\text{hi}(x \cdot x)$.

3.4 Remapping using CacheLineEF

Like PTHash, PtrHash uses a parameter $0 < \alpha \leq 1$ to use a total of $n' = n/\alpha$ slots, introducing $n' - n$ additional free slots. As a result of the additional slots, some, say R , of the keys will map to positions $n \leq q_0 < \dots < q_{R-1} < n'$, causing the perfect hash function to not be *minimal*.

Remapping. Since there are a total of n keys, this means there are exactly R empty slots (“gaps”) left behind in $[n]$, say at positions L_0 to L_{R-1} . We *remap* the keys that map to positions $\geq n$ to the empty slots at positions $< n$ to obtain a *minimal* perfect hash function.



■ **Figure 3** Overview of the CacheLineEF data structure.

A simple way to store the remap is as a plain array F , such that $F[q_i - n] = L_i$. PTHash encodes this array using Elias-Fano coding [10, 13], after setting undefined positions of F equal to their predecessor. The benefit of a plain F array is fast and cache-local lookups, whereas Elias-Fano coding provides a more compact encoding that typically requires multiple lookups to memory.

CacheLineEF. We would like to answer each query by reading only a single cache line from memory. To do this, we use a method based on *interleaving* data. First, the list of non-decreasing F positions is split into chunks of $C = 44$ values $\{v_0, \dots, v_{43}\}$, with the last chunk possibly containing fewer values. We assume that values are at most 40 bits, and that the average difference between adjacent values in each chunk is not more than 500. Then, each chunk is encoded into 64 bytes that can be stored as single cache line, as shown in Figure 3.

We first split all values into their 8 *low* bits ($v_i \bmod 2^8$) and 32 *high* bits ($\lfloor v_i/2^8 \rfloor$). Further, the high part is split into an *offset* (the high part of v_0) and the *relative* high part:

$$v_i = 2^8 \cdot \underbrace{\lfloor v_0/2^8 \rfloor}_{\text{Offset}} + 2^8 \cdot \underbrace{(\lfloor v_i/2^8 \rfloor - \lfloor v_0/2^8 \rfloor)}_{\text{Relative high part}} + \underbrace{(v_i \bmod 2^8)}_{\text{Low bits}}. \quad (5)$$

This is stored as follows.

- First, the 32 bit offset $\lfloor v_0/2^8 \rfloor$ is stored.
- Then, the relative high parts are encoded into 128 bits. For each $i \in [44]$, bit $i + \lfloor v_i/2^8 \rfloor - \lfloor v_0/2^8 \rfloor$ is set to 1. Since the v_i are increasing, each i sets a distinct bit, for a total of 44 set bits.
- Lastly, the low 8 bits of each v_i are directly written to the 44 trailing bytes.

Lookup. The value at position i is found by summing the terms of Equation (5). The offset and low bits can be read directly. This relative high part can be found as $2^8 \cdot (\text{select}(i) - i)$, where $\text{select}(i)$ gives the position of the i 'th 1 bit in the 128-bit-encode relative high parts. In practice, this can be implemented efficiently using a `popcount` to go into the high or low half, followed by the `PDEP` instruction⁷ provided by the BMI2 bit manipulation instruction set [34].

Limitations. CacheLineEF uses $64/44 \cdot 8 = 11.6$ bits per value, which is more than the usual Elias-Fano, which for example takes $8 + 2 = 10$ bits per value for data with an average *stride* (gap between consecutive integers) of 2^8 . Furthermore, values are limited to 40 bits, covering 10^{12} items. The range could be increased to 48 bit numbers by storing 5 bytes of the offset, but this has not been necessary so far. Lastly, each CacheLineEF can only span a

⁷ Unfortunately, while AMD Zen 2 does support this instruction, it is very slow in practice.

range of around $(128 - 44) \cdot 2^8 = 21\,504$, or an average stride of 500. This means that for PtrHash, we only use CacheLineEF when $\alpha \leq 0.99$, so that the average distance between empty slots is 100 and the average stride of 500 is not exceeded in practice. When $\alpha > 0.99$, a simple plain array can be used without much overhead.

4 Results

We now evaluate PtrHash construction and query throughput for different parameters, and compare PtrHash to other minimal perfect hash functions. All experiments are run on an Intel Core i7-10750H CPU with 6 cores and hyper-threading disabled. The frequency is pinned to 2.6 GHz. Cache sizes are 32 KiB L1 and 256 KiB L2 per core, and 12 MiB shared L3 cache. Main memory is 64 GiB DDR4 at 3200 MHz, split over two 32 GiB banks.

In Section 4.1, we compare the effect of various parameters and configurations on the size, construction speed, and query speed of PtrHash. In Section 4.2, we compare PtrHash to other methods.

Further, Appendix A.2 evaluates the effect of prefetching with batching and streaming queries. We select streaming with prefetching 32 iterations ahead as the default. We also show that in a multi-threaded setting, this can fully exhaust the available memory bandwidth.

Lastly, in Appendix B.1 we state the results of constructing PtrHash on 50 billion keys using various sharding strategies.

4.1 Construction

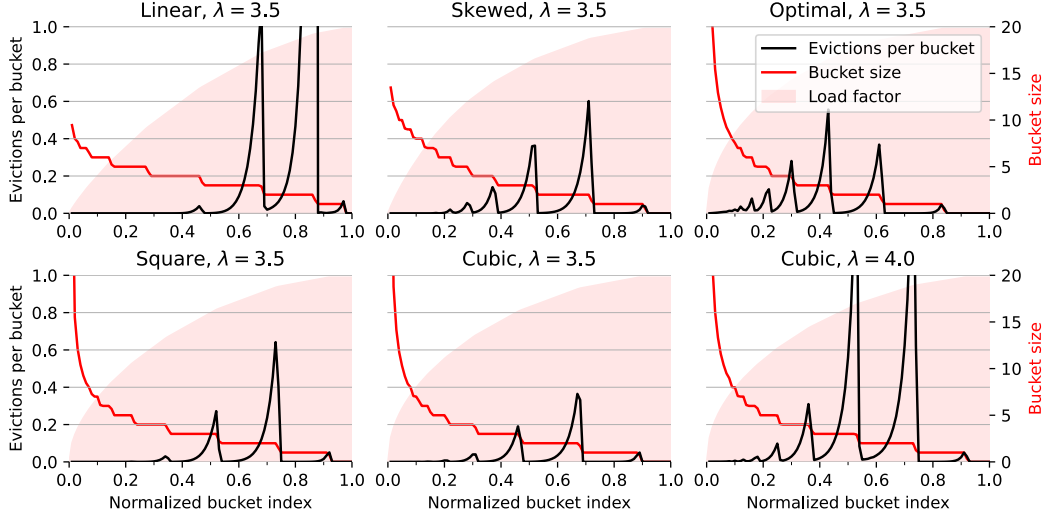
The construction experiments use 10^9 random 64-bit integer keys, for which the data structure takes around 300 MB and thus is much larger than L3 cache. Unless otherwise mentioned, construction is in parallel using 6 cores. For the query throughput experiments, we also test on 20 million keys, for which the data structure take around 6 MB and easily fit in L3 cache. To avoid the time needed for hashing keys, and since our motivating application is indexing k -mers that fit in 64 bits, we always use random 64-bit integer keys, and hash them using FxHash.

Without using the external memory construction, memory usage during construction is dominated by the size of the input keys and their hashes, which are typically much larger than the few bits per key needed for the construction itself.

4.1.1 Bucket Functions

In Figure 4, we compare the performance of different bucket assignment functions γ in terms of the bucket size distribution and the number of evictions for each additionally placed bucket. We see that the linear $\gamma_1(x) = x$ has a lot of evictions for the last buckets of size 3 and 2, but like all methods it is fast for the last buckets of size 1 due to the load factor $\alpha < 1$. The optimal distribution of PHOBIC performs only slightly better than the skewed one of FCH and PTHash, and can be seen to create more large buckets since the load factor increases fast for the first buckets. The cubic γ_3 is clearly much better than all other functions, and is also tested with larger buckets of average size $\lambda = 4$, where all other functions fail.

In the remainder, we will test the linear γ_1 for simplicity and lookup speed, and the cubic γ_3 for space efficiency.



■ **Figure 4** Bucket size distribution (red) and average number of evictions (black) per additionally placed bucket during construction of the pilot table, for different bucket assignment functions. Parameters are $n = 10^9$ keys, $S = 2^{18}$ slots per part, and $\alpha = 0.99$, and the red shaded load factor ranges from 0 to α . In the first five plots $\lambda = 3.5$ so that the pilots take 2.29 bits/key. For $\lambda = 4.0$ (bottom-right), the linear, skewed, and optimal bucket assignment functions cause endless evictions, and construction fails. The cubic function does work, resulting in 2.0 bits/key for the pilots.

4.1.2 Tuning Parameters for Construction

In Figure 5 we compare the multi-threaded construction time and space usage of PtrHash on $n = 10^9$ keys for various parameters $\gamma \in \{\gamma_1, \gamma_3\}$, $2.7 \leq \lambda \leq 4.2$, $\alpha \in \{0.98, 0.99, 0.995, 0.998\}$, and plain remapping or CacheLineEF. We see that for fixed γ and α , the construction time appears to increase exponentially as λ increases. At too large λ , some parts fail to build after a total of $10S$ evictions, which is a hard limit we impose to avoid running into eviction cycles. Load factors α closer to 1 (thinner lines) achieve smaller overall data structure size, but take longer to construct and time out at smaller λ . The cubic γ_3 is faster to construct than the identity γ_1 for small $\lambda \leq 3.5$. Unlike γ_1 , it also scales to much larger λ up to 4, and thereby achieves significantly smaller overall size.

We note that for small λ , construction time does converge to around 19 ns/key. A rough time breakdown is that for each key, 1 ns is spent on hashing, 5 ns on sorting all the keys, 12 ns to search for pilots, and lastly 1 ns on remapping to empty slots.

Recommended parameters. Based on these results, we choose three sets of parameters for further evaluation, as indicated with blue, black, and red dots in Figure 5:

- **Fast** (blue), aiming for query speed: using the linear γ_1 , $\lambda = 3.0$, $\alpha = 0.99$, and a plain vector for remapping. Construction takes only just over 20 ns/key, close to the apparent lower bound, and space usage is 3 bits/key. This can be used when n is small, or more generally when memory usage is not a bottleneck.
- **Default** (black), a trade-off between fast construction and small space: using cubic γ_3 , $\lambda = 3.5$, and $\alpha = 0.99$, with CacheLineEF remapping.
- **Compact** (red), aiming for small space: using the cubic γ_3 , $\lambda = 4.0$, $\alpha = 0.99$, and CacheLineEF remapping. Construction now takes around 50 ns/key, but the data structure only uses 2.12 bits/key. In practice, this configuration sometimes ends up in endless eviction cycles, and $\lambda = 3.9$ may be better.

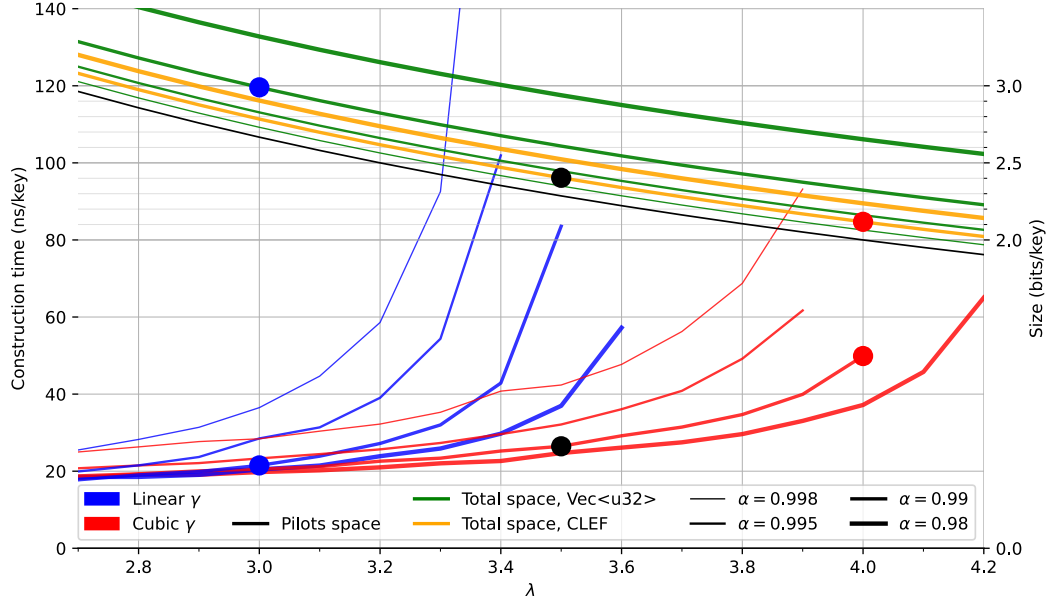


Figure 5 This plot shows the construction time (blue and red, left axis) and data structure size (black, green, and yellow, right axis) as a function of λ for $n = 10^9$ keys. Parallel construction time on 6 threads is shown for both the linear and cubic γ , and for various values of α (thickness). The curves stop because construction times out when λ is too large. For each λ , the black line shows the space taken by the array of pilots. For larger λ there are fewer buckets, and hence the pilots take less space. The total size including the remap table is shown in green (plain vector) and yellow (CacheLineEF) for various α . The blue (fast), black (default), and red (compact) dots highlight the chosen parameter configurations.

4.1.3 Remap

In Table 1, we compare the space usage and query throughput of the different remap data structures for both the fast and compact parameters, for $n = 10^9$ keys. We observe that the overhead of CacheLineEF is $2.75\times$ smaller than a plain vector, and only 40% larger than Elias-Fano encoding as implemented in the sux library [38].

The speed of non-minimal (PHF) queries that do not remap does not depend on the remap structure used.

For *minimal* (MPHF) queries with the for loop, with fast parameters, EF is significantly slower (14.2 ns) with the fast parameters than the plain vector (12.5 ns), while CacheLineEF (12.9 ns) is only slightly slower. The difference is much smaller with the compact parameters, because the additional computations for the cubic γ_3 reduce the number of iterations the processor can work ahead. When streaming queries, for both parameter choices CacheLineEF is less than 0.1 ns slower than the plain vector, while EF is 1 ns slower.

In the end, we choose CacheLineEF when using compact parameters, but prefer the simpler and slightly faster plain vector for fast parameters. Since $\alpha = 0.99$ is close to 1, the remap structure is not accessed much, and the performance improvement of CacheLineEF over plain EliasFano coding is not too large.

■ **Table 1** Comparison of space usage (bits/key) and query throughput (ns/query) of PtrHash when using the recommended parameters with different remap structures. Query throughput is shown both for perfect hashing (without remap), and for minimal perfect hashing (with remap). Additionally, query throughput is shown both for a for-loop and for streaming.

Configuration	Pilots	Query PHF		Remap		Query MPHF	
	Space	Loop	Stream	Type	Space	Loop	Stream
Fast $\alpha = 0.99, \lambda = 3.0$, linear γ_1	2.67	11.5	8.6	Vec<u32>	0.33	12.5	8.8
				CacheLineEF	0.12	12.9	8.8
				EF	0.09	14.2	9.7
Default $\alpha = 0.99, \lambda = 3.5$, cubic γ_3	2.29	17.6	7.9	Vec<u32>	0.33	20.0	8.6
				CacheLineEF	0.12	21.0	8.7
				EF	0.09	21.2	9.6
Compact $\alpha = 0.99, \lambda = 4.0$, cubic γ_3	2.00	17.7	8.0	Vec<u32>	0.33	20.3	8.6
				CacheLineEF	0.12	20.9	8.6
				EF	0.09	21.7	9.7

4.2 Comparison to Other Methods

In Table 2 we compare the performance of PtrHash against other methods on short, random strings. In particular, we compare against methods and configurations that are reasonably fast to construct: SIMDRecSplit [11, 5], Bipartite ShockHash-Flat [25, 23], Consensus-RecSplit [26], FMPH and FMPHGO [3], FiPS [21], SicHash [24], CHD [1], PTHash [36, 37], and PHOBIC [19]. The specific parameters are based on Table 1 of [19], Table 8.1 of [21], and Table 3 of [3]. These results were obtained using the excellent MPHF-Experiments library [22] by Hans-Peter Lehmann. Construction is done on 6 threads in parallel when supported. By default, the framework queries one key at a time. For PtrHash with streaming queries, we modified this to query all keys at once.

Input. The input is 300 million random strings of random length between 10 and 50 characters. This input size is such that the MPHF data structures take around 75 MB, which is much larger than the 12 MB L3 cache.

PtrHash. As expected, the space usage of PtrHash matches the numbers of Table 1. In general, PtrHash can be slightly larger due to rounding in the number of parts and slots per part, but for large inputs like here this effect is small. Construction times per key are slightly slower than as predicted by Figure 5, while we might expect slightly faster construction due to the lower number of keys. Likely, the slowdown is caused by hashing the input strings. The hashing of input strings has a much worse effect on query throughput. In Table 1, we obtained query throughput of 12 ns and 18 ns for the fast and compact configurations when looping over integer keys, and as low as 8 ns when streaming queries. With string inputs, these numbers increase to 33 ns resp. 35 ns when looping, and 16 ns (resp. 23 ns) when streaming. A similar effect can be seen when comparing Tables 3 and 4 of [3].

Speed. We observe that PtrHash with fast parameters is the fastest to construct alongside SIMDRecSplit (27 ns/key and 26 ns/key) and FiPS (16 ns/key, assuming optimal scaling to 6 threads), resulting in around 3 bits/key for all three methods. However, query throughput of PtrHash is $9\times$ (SIMDRecSplit) resp. $3.3\times$ (FiPS) faster, going up to $19\times$ resp. $6.8\times$ faster when streaming all queries at once. Compared to the next-fastest method to query,

■ **Table 2** Performance comparison of MPHf methods on 300 million random string keys of uniform length between 10 and 50. Construction time is shown for 6 threads. A * indicates single-threaded timings (optimistic 6-fold speedup in parentheses). Near-optimal values in each column are bolded.

	Approach	Configuration	Space bits/key	Construction 6t, ns/key	Query ns/query
Brute force	SIMDRecSplit	$n=5, b=5$	2.96	26	310
		$n=8, b=100$	1.81	66	258
	Bip. ShockHash-Flat	$n=64$	1.62	2140* (357)	201
	Consensus-RecSplit	$k=256, \varepsilon=0.10$	1.58	521* (87)	565
		$k=512, \varepsilon=0.03$	1.49	1199* (200)	528
Fingerprinting	FMPH	$\gamma=2.0$	3.40	44	168
		$\gamma=1.0$	2.80	69	236
	FMPHGO	$s=4, b=16, \gamma=2.0$	2.86	298	160
		$s=4, b=16, \gamma=1.0$	2.21	423	212
	FiPS	$\gamma=2.0$	3.52	93* (16)	109
		$\gamma=1.5$	3.12	109* (18)	124
Graph	SicHash	$p_1=0.21, p_2=0.78, \alpha=0.90$	2.41	48	149
		$p_1=0.45, p_2=0.31, \alpha=0.97$	2.08	63	141
Bucket placement	CHD	$\lambda=3.0$	2.27	1059* (177)	542
	PThash	$\lambda=4.0, \alpha=0.99, \text{C-C} + \text{HEM}$	3.19	403 173	77
		$\lambda=5.0, \alpha=0.99, \text{EF} + \text{HEM}$	2.17	765 323	156
	PHOBIC	$\lambda=3.9, \alpha=1.0, \text{IC-C}$	4.14	62	116
		$\lambda=4.5, \alpha=1.0, \text{IC-R}$	2.34	80	179
		$\lambda=6.5, \alpha=1.0, \text{IC-C}$	2.44	220	108
		$\lambda=7.0, \alpha=1.0, \text{IC-R}$	1.86	446	157
	PtrHash	Fast $\lambda=3.0, \alpha=0.99, \gamma_1, \text{Vec} + \text{streaming}$	2.99	27	33 16
		Default $\lambda=3.5, \alpha=0.99, \gamma_3, \text{CLEF} + \text{streaming}$	2.40	32	37 23
		Compact $\lambda=4.0, \alpha=0.99, \gamma_3, \text{CLEF} + \text{streaming}$	2.12	63	35 23

PThash-CC (HEM), PtrHash is twice faster to query (or nearly $5\times$ when streaming), is $6.5\times$ faster to build, and even slightly smaller.

With default parameters, PtrHash is $2.1\times$ faster to query than the fastest configuration of PThash, and $3.3\times$ faster when using streaming, while being over $5\times$ faster to construct. Indeed, the speedup in query speed is explained by the fact that only a single memory access is needed for most queries (compared to ≥ 2 for PtrHash-HEM and PHOBIC), and generally by the fact that the code for querying is short.

Space. PtrHash with the fast parameters is larger (2.99 bits/key) than some other methods, but compensates by being significantly faster to construct and/or query. When space is of importance, the compact version can be used (2.12 bits/key). This takes $2.4\times$ longer to build at 63 ns/key, and has only slightly slower queries. Compared to methods that are smaller, PtrHash is over $3\times$ faster to build than PHOBIC. Consensus, SIMDRecSplit, and SicHash achieve smaller space of 1.58, 1.81, and 2.08 bits/key in comparable time (63-87 ns/key), but again are at least $3\times$ slower to query, or over $6\times$ compared to streaming queries.

5 Conclusions and Future Work

We have introduced PtrHash, a minimal perfect hash function that builds on PTHash and PHOBIC. Its main novelty is the use of fixed-width 8-bit pilots that simplify queries. To make this possible, we use *hash-and-evict*, similar to Cuckoo hashing: when there is no pilot that leads to a collision-free placement of the corresponding keys, some other pilots are *evicted* and have to search for a new value.

The result is an MPHf with twice faster queries (37 ns/key) than any other method (at least 77 ns/key) for datasets larger than L3 cache. Further, due to its simplicity, queries can be processed in *streaming* fashion, giving another two times speedup (as low as 16 ns/key). At this point, the hashing of string inputs becomes a bottleneck. For integer keys, such as k -mers, much higher throughput of up to 8 ns/key can be obtained, close to the 7.4 ns per cache line bandwidth, or when using multiple cores even saturating the main memory (2.5 ns/key).

Future work. A theoretical analysis of our method is currently missing. While the hash-evict strategy works well in practice, we currently have no relation between the bucket size λ , load factor α , and the number of evicts arising during construction. Such an analysis could help to better understand the optimal bucket assignment function, like PHOBIC [19] did for the case without eviction. Possibly, the analysis of [18, Section 5] could be extended to fully cover our method.

Second, the size of pilots could possibly be improved by further parameter tuning. In particular we use 8-bit pilots, while slightly fewer or more bits may lead to smaller data structures. An experiment with 4-bit pilots was not promising, however.

To further improve the throughput, we suggest that more attention is given to the exact input format. As already seen, hashing all queries at once can provide significant performance gains via prefetching. For string input specifically, it is more efficient when the strings are consecutively packed in memory rather than separately allocated, and it might be more efficient to explicitly hash multiple strings in parallel. More generally, applications should investigate whether they can be rewritten to take advantage of streaming queries. Furthermore, current throughput is limited by the fact that nearly every query needs to fetch a new cache line. It would be interesting to design an MPHf that only requires, say, half a cache line per query, or to disprove the existence of such an MPHf.

Lastly, we refer the reader to PHast [4], an MPHf that introduces a number of interesting simplifications, leading to a datastructure that is both smaller and faster to query than PtrHash, although it is somewhat slower to construct. It remains an open problem whether it is possible to construct an MPHf with space within 0.1 bits/key from the lower bound that is as fast to query as PtrHash and PHast.

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A Query Throughput

A.1 Batching and Streaming

Throughput. In practice in bioinformatics applications such as SSHash, we expect many independent queries to the MPHf. This means that queries can be answered in parallel, instead of one by one. Thus, we should optimize for query *throughput* rather than individual query latency. We report throughput as *inverse throughput* in amortized nanoseconds per query, rather than the usual queries per second.

Out-of-order execution. An MPHf on 10^9 keys requires memory at least $1.5 \text{ bits/key} \times 10^9 \text{ keys} = 188 \text{ MB}$, which is much larger than the L3 cache of size around 16 MB. Thus, most queries require reading a pilot from main memory (RAM), which usually has a latency around 80 ns. Nevertheless, existing MPHfs such as FCH [16] achieve an inverse throughput as low as 35 ns/query on such a dataset [36]. This is achieved by *pipelining* and the *reorder buffer*. For example, Intel Skylake CPUs can execute over 200 instructions ahead while waiting for memory to become available [41, 8]. This allows the CPU to already start processing future queries and fetch the required cache lines from RAM while waiting for the current query. Thus, when each iteration requires less than 100 instructions and there are no branch-misses, this effectively makes up to two reads in parallel. A large part of speeding up queries is then to reduce the length of each iteration so that out-of-order execution can fetch memory more iterations ahead.

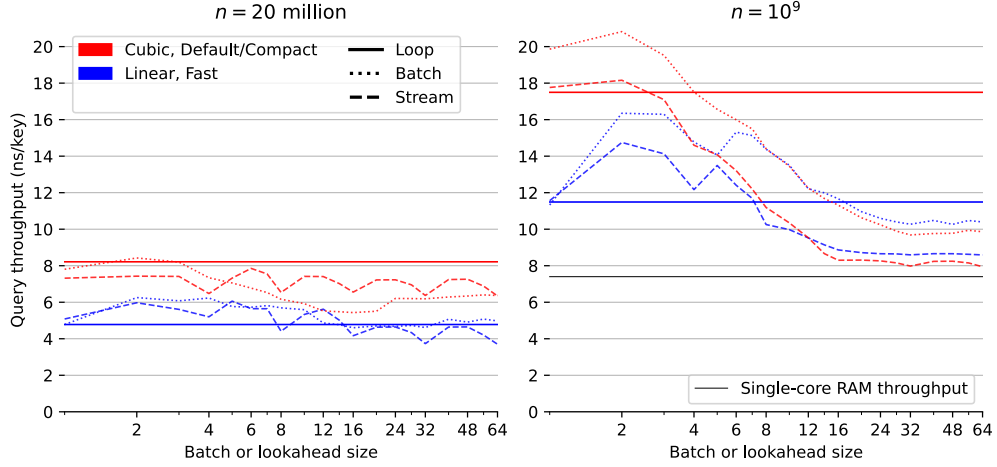
Prefetching. Instead of relying on the CPU hardware to parallelize requests to memory, we can also explicitly *prefetch*⁸ cache lines from our code. Each prefetch requires a *line fill buffer* to store the result before it is copied into the L1 cache. Skylake has 12 line fill buffers [9], and hence can support up to 12 parallel reads from memory. In theory, this gives a maximal random memory throughput around $80/12 = 6.67 \text{ ns per read from memory}$, but in practice experiments show that the limit is 7.4 ns per read. Thus, our goal is to achieve a query throughput of 7.4 ns.

We consider two models to implement prefetching: batching and streaming.

Batching. In this approach, the queries are split into batches (chunks) of size B , and are then processed one batch at a time. In each batch, two passes are made over all keys. In the first pass, each key is hashed, its bucket is determined, and the cache line containing the corresponding pilot is prefetched. In the second pass, the hashes are iterated again, and the corresponding slots are computed.

Streaming. A drawback of batching is that at the start and end of each batch, the memory bandwidth is not fully saturated. Streaming fixes this by prefetching the cache line for the pilot B iterations ahead of the current one, and is able to sustain the maximum possible number of parallel prefetches throughout, apart from at the very start and end.

⁸ There are multiple types of prefetching instructions that prefetch into a different level of the cache hierarchy. We prefetch into all levels of cache using `prefetcht0`. Other prefetch variants give similar results.



■ **Figure 6** Query throughput of prefetching via batching (dotted) and streaming (dashed) with various batch/lookahead sizes, compared to a plain for loop (solid), for $n = 20 \cdot 10^6$ (left) and $n = 10^9$ (right) keys. Blue shows the results for the fast parameters, and red for the compact parameters. Default parameters give performance nearly identical to the compact parameters, since the main differentiating factor is the use of γ_1 versus γ_3 . All times are measured over a total of 10^9 queries, and for (non-minimal) perfect hashing only, *without* remapping.

A.2 Evaluation

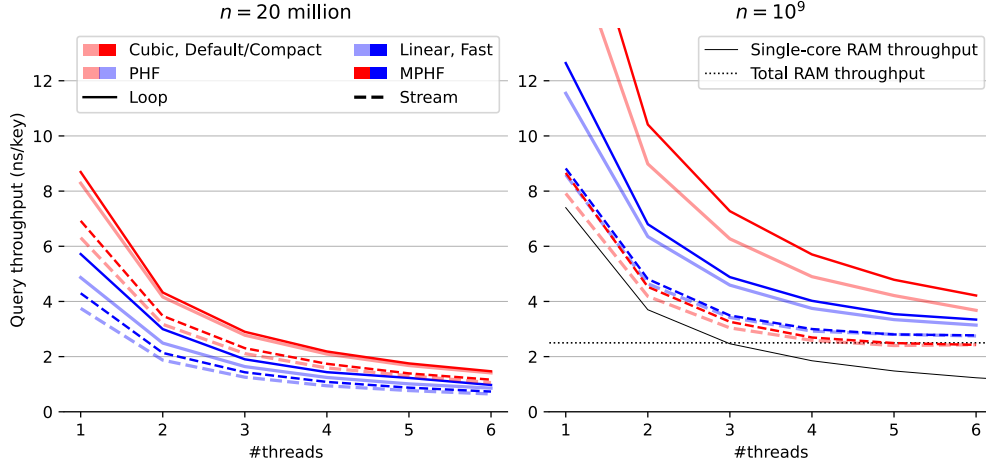
A note on benchmarking throughput. To our knowledge, all recent papers on (minimal) perfect hashing measure query speed by first creating a list of keys, and then querying all keys in the list, as in `for key in keys { ptr_hash.query(key); }`. One might think this measures the average latency of a query, but that is not the case, as the CPU will execute instructions from adjacent iterations at the same time. Indeed, as can be seen in Table 1, this loop can be as fast as 12 ns/key for $n = 10^9$, which is over 6 times faster than the RAM latency of around 80 ns (for an input of size 300 MB), and thus, at least 6 iterations are being processed in parallel.

Hence, we argue that existing benchmarks measure (and optimize for) throughput and that they assume that the list of keys to query is known in advance. We make this assumption explicit by changing the API to benchmark all queries at once, as in `ptr_hash.query_all(keys)`. This way, we can explicitly process multiple queries in parallel.

We also argue that properly optimizing for throughput is relevant for applications. SShash, for example, queries all minimizers of a DNA sequence, which can be done by first computing and storing those minimizers, followed by querying them all at once.

We now explore the effect of the batch size and number of parallel threads on query throughput.

Batching and Streaming. In Figure 6, we compare the query throughput of a simple for loop with the batching and streaming variants with various batch/lookahead sizes. We see that both for small $n = 20 \cdot 10^6$ and large $n = 10^9$, the fast parameters yield higher throughput than the compact parameters when using a for loop. This is because of the overhead of computing $\gamma_3(x)$. For small n , batching and streaming do not provide much benefit, indicating that memory latency is not a bottleneck. However, for large n , both batching and streaming improve over the plain for loop. As expected, streaming is faster than batching here. For streaming, throughput saturates when prefetching around 16 iterations



■ **Figure 7** In this plot we compare the throughput of a for loop (solid) versus streaming (dashed) for multiple threads, for both non-minimal (dimmed) and minimal (bright) perfect hashing. The left shows results for $n = 20 \cdot 10^6$, and the right shows results for $n = 10^9$. In blue are the results for the fast parameters with γ_1 , while results for the compact parameters with γ_3 are in red, which performs identical to the default parameters. On the right, the solid black line shows the maximum throughput based on 7.4 ns per random memory access per thread, and the solid black line shows the maximum throughput based on the total memory bandwidth of 25.6 GB/s.

ahead. At this point, memory throughput is the bottleneck, and the difference between the compact and fast parameters disappears. In fact, compact parameters with γ_3 are slightly *faster*. This is because γ_3 has a more skew distribution of bucket sizes with more large buckets. When the pilots for these large buckets are cached, they are more likely to be hit by subsequent queries, and hence avoid some accesses to main memory.

For further experiments we choose streaming over batching, and use a lookahead of 32 iterations. The final throughput of 8 ns per query is very close to the optimal throughput of 7.4 ns per random memory read.

A.3 Multi-threaded Throughput.

In Figure 7 we compare the throughput of the fast and compact parameters for multiple threads. When $n = 20 \cdot 10^6$ is small and the entire data structure fits in L3 cache, the scaling to multiple threads is nearly perfect. As expected, minimal perfect hashing (bright) tends to be slightly slower than perfect hashing (dimmed), but the difference is small. The fast γ_1 is faster than the compact γ_3 , and streaming provides only a small benefit over a for loop. For large $n = 10^9$, all methods converge towards the limit imposed by the full RAM throughput of 25.6 GB/s. Streaming variants hit this starting at around 4 threads, and remain faster than the for loop. As before, the compact version is slightly faster because of its more efficient use of the caches, and is even slightly better than the maximum throughput of random reads to RAM. Minimal perfect hashing is only slightly slower than perfect hashing.

B Sharding

When the number of keys is large, say over 10^{10} , their 64-bit (or 128-bit) hashes may not all fit in memory at the same time, even though the final PtrHash data structure (the list of pilots) would fit. Thus, we can not simply sort all hashes in memory to partition them. Instead, we split the set of all n hashes into, say $s = \lceil n/2^{32} \rceil$ shards of $\approx 2^{32}$ elements each, where the i 'th shard corresponds to hash values in $s_i := [2^{64} \cdot i/s, 2^{64} \cdot (i+1)/s)$. Then, shards are processed one at a time. The hashes in each shard are sorted and split into parts, after which the parts are constructed as usual. This way, the shards only play a role during construction, and the final constructed data structure is independent of which sharding strategy was used.

In-memory sharding. The first approach to sharding is to iterate over the set of keys s times. In the i 'th iteration, all keys are hashed, and only those hashes in the corresponding interval s_i are stored and processed. This way, no disk space is needed for construction.

On-disk sharding. A drawback of the first approach is that keys are potentially hashed many times. This can be avoided by writing hashes to disk. Specifically, we can create one file per shard and append hashes to their corresponding file. These files are then read and processed one by one.

Hybrid sharding. A hybrid of the two approaches above only requires disk space for $D < s$ shards. This iterates and hashes the keys $\lceil s/D \rceil$ times, and in each iteration writes hashes for D shards to disk. Those are then processed one by one as before.

On-disk PtrHash. When the number of keys is so large that even the pilots do not fit in memory, they can also be stored to disk and read on-demand while querying. This is supported using ε -serde [40, 14].

B.1 Evaluation

We tested the in-memory and hybrid sharding by constructing PtrHash with default parameters on $5 \cdot 10^{10}$ random integer keys on a laptop with only 64 GB of memory, using 6 cores in parallel. All 64-bit hashes would take 400 GB, so we use 24 shards of around 2^{31} keys, that each take 16 GB. The final data structure takes 2.40 bits/key, or 15 GB in total, and the peak memory usage is around 50 GB.

The in-memory strategy iterates through and hashes the integer keys 24 times, and takes 3098 seconds in total or 129 s per shard. Of this, 67 s (52%) is spent on hashing the keys, 14 s (11%) is spent sorting hashes into buckets, and 45 s (35%) is spent searching for pilots.

The hybrid strategy is allowed to use up to 128 GB of disk space, and thus writes hashes to disk in 3 batches of 8 shards at a time. This brings the total time down to 2494 s (19% faster), and uses 104 s per shard. Of this, an amortized 31 s (30%) per shard is spent writing hashes to disk, and 9 s (9%) is spent reading hashes from disk, which together is faster than the 67 s that was previously spent on hashing all keys.