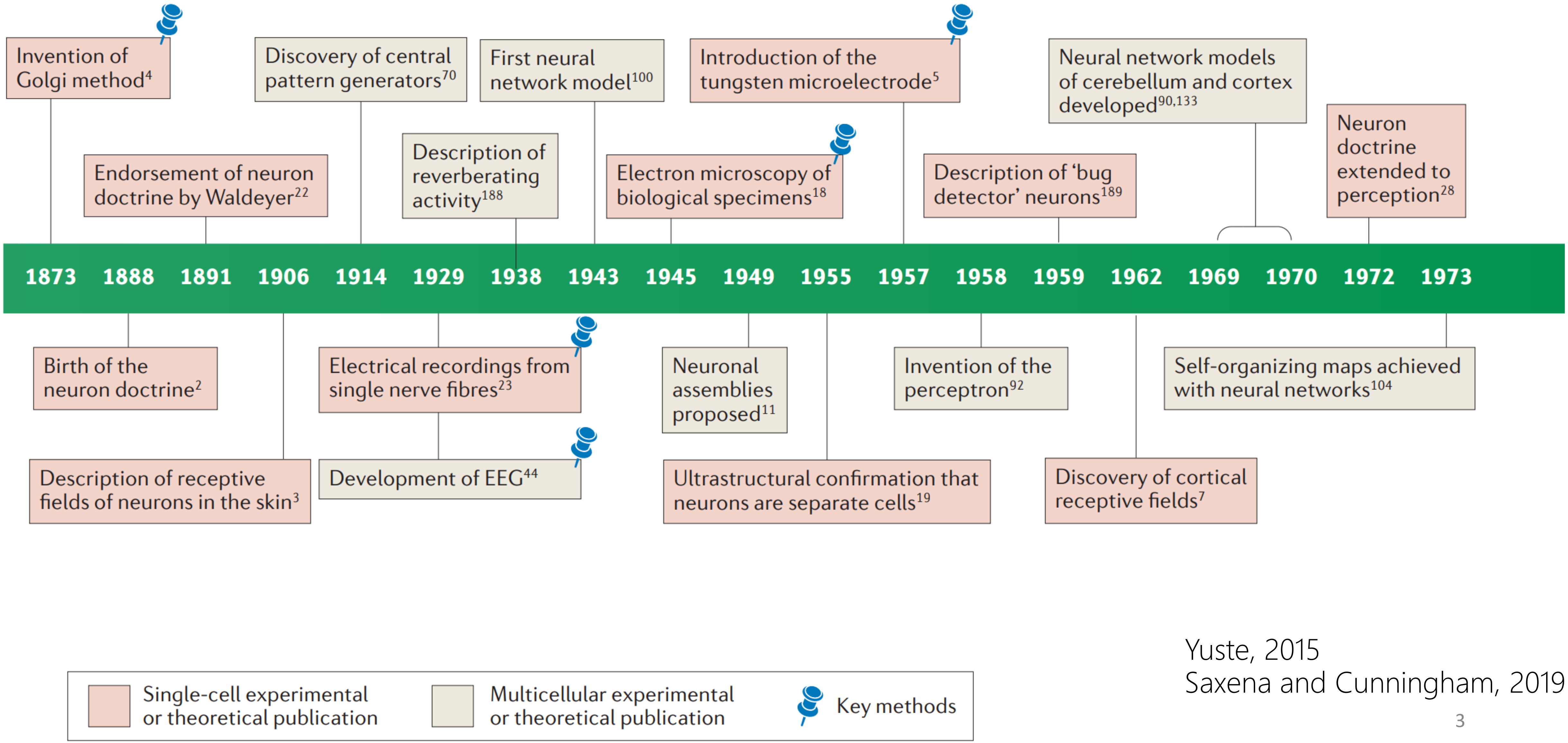
Geometry of abstraction

Valeria Fascianelli

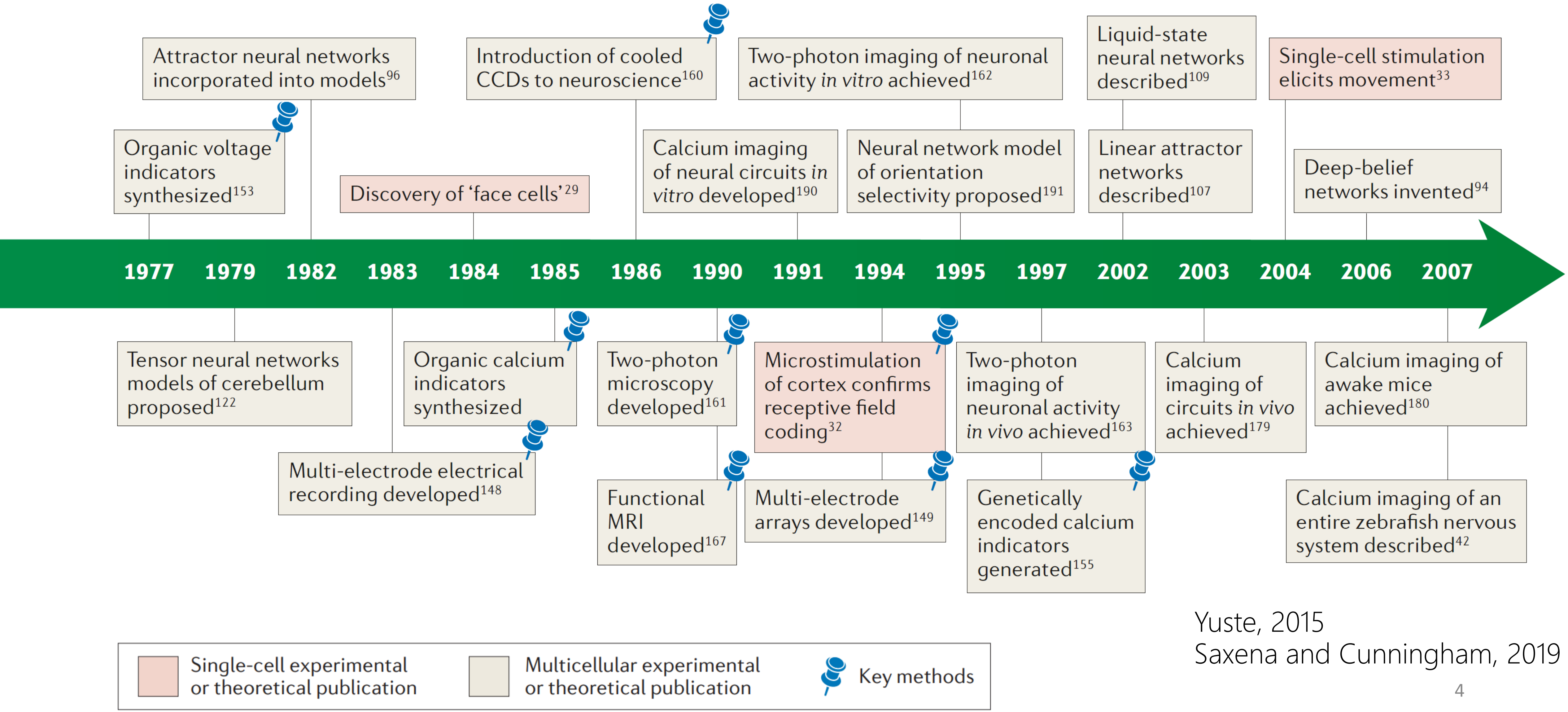
Advanced theory course, CTN
February 22nd, 2022

Outline

1. Pure selectivity neurons, mixed selectivity neurons, neural representation
2. Format of the neural representation: **geometry**, **generalization**, **abstraction**



Yuste, 2015
Saxena and Cunningham, 2019



Yuste, 2015
Saxena and Cunningham, 2019

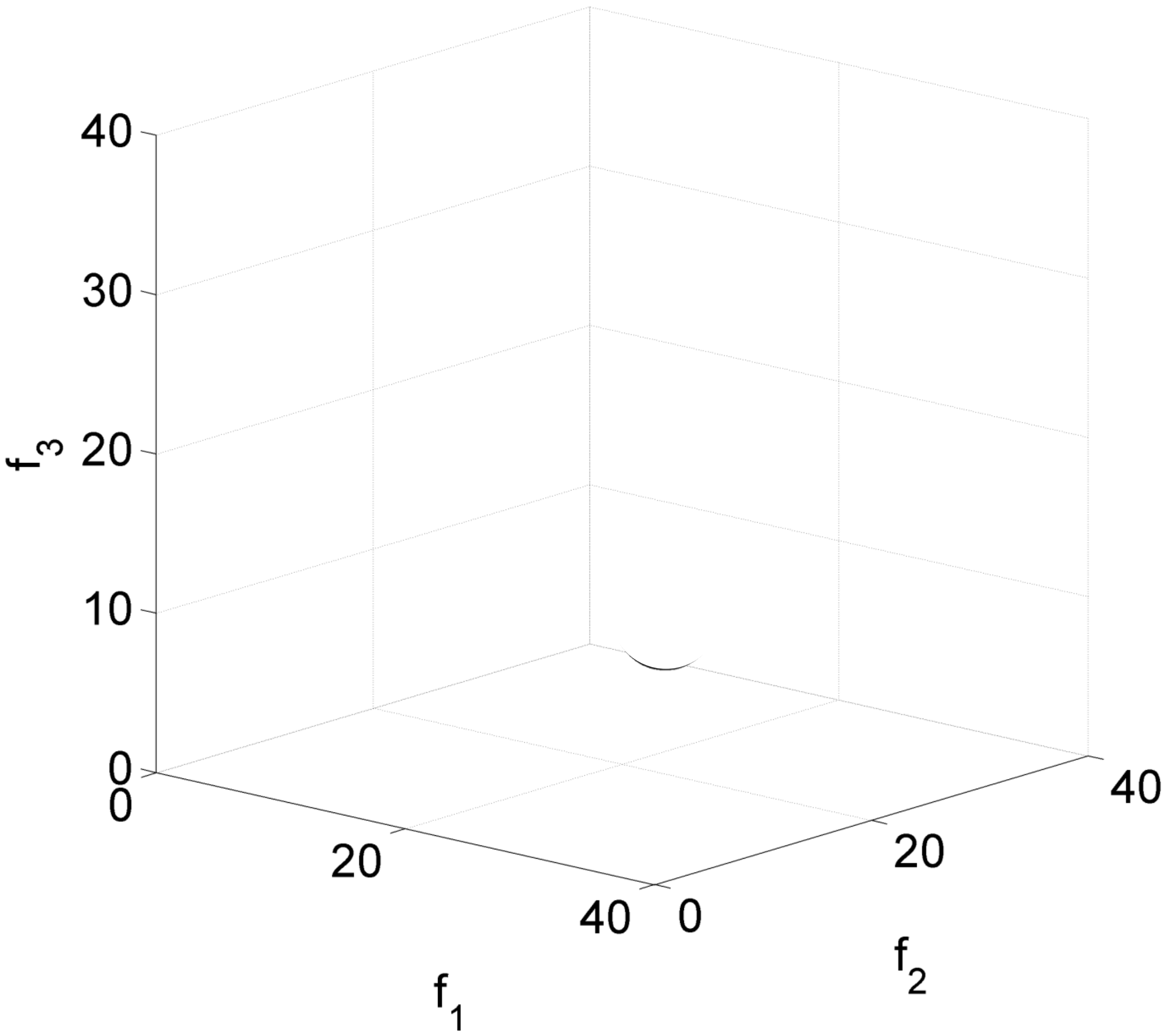
- Thanks to the advance in recording techniques, analyses tools moved from single-neuron to population frameworks

- Quantify and decode information represented across many neurons

- Ability to solve complex cognitive task could rely on neurons showing selectivity to multiple task variables (mixed selectivity, Rigotti et al., 2013)

Pure selectivity neurons

$f_1 = 60\,a, \quad f_2 = 60\,b, \quad f_3 = 60 - 50a$



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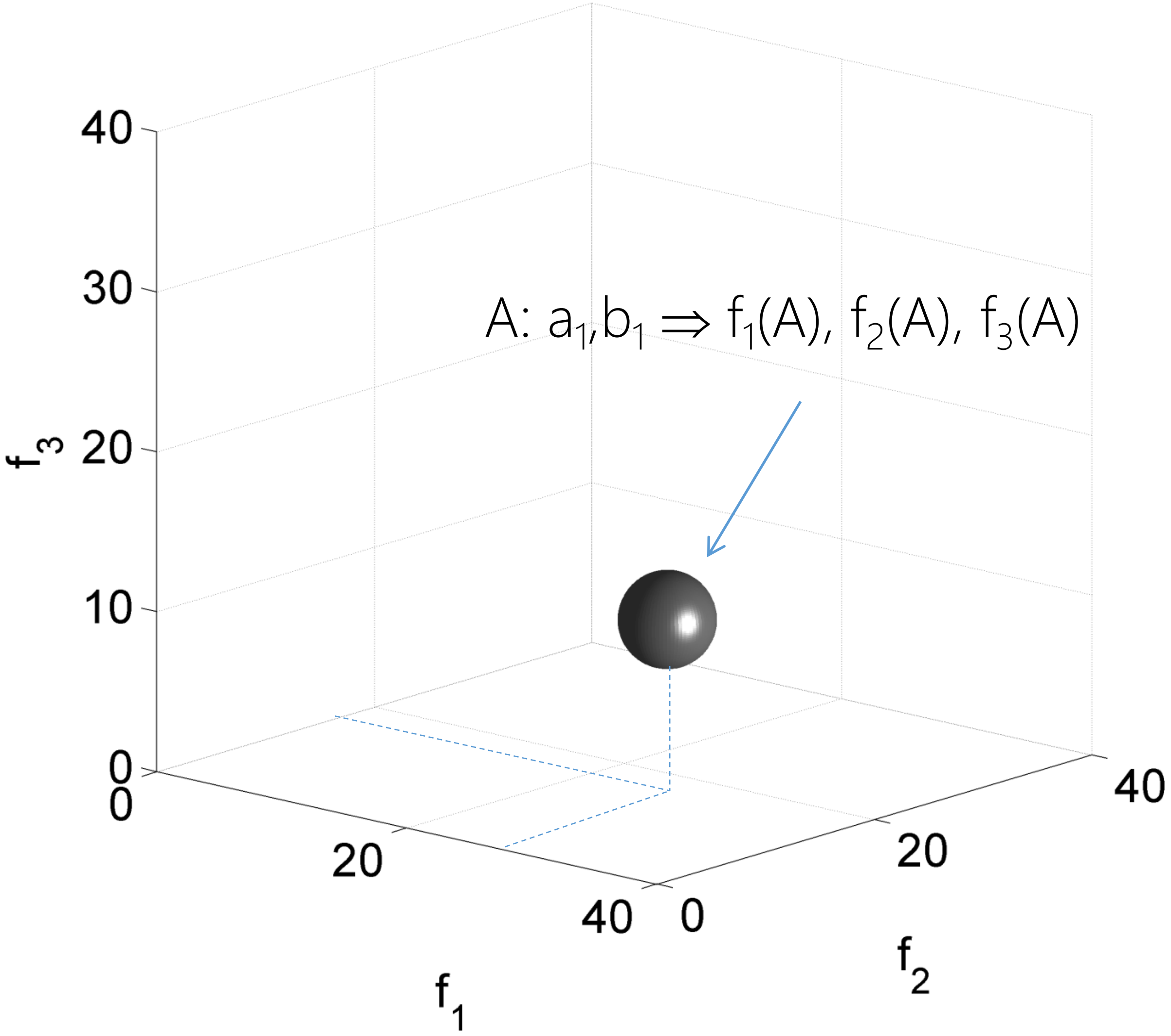
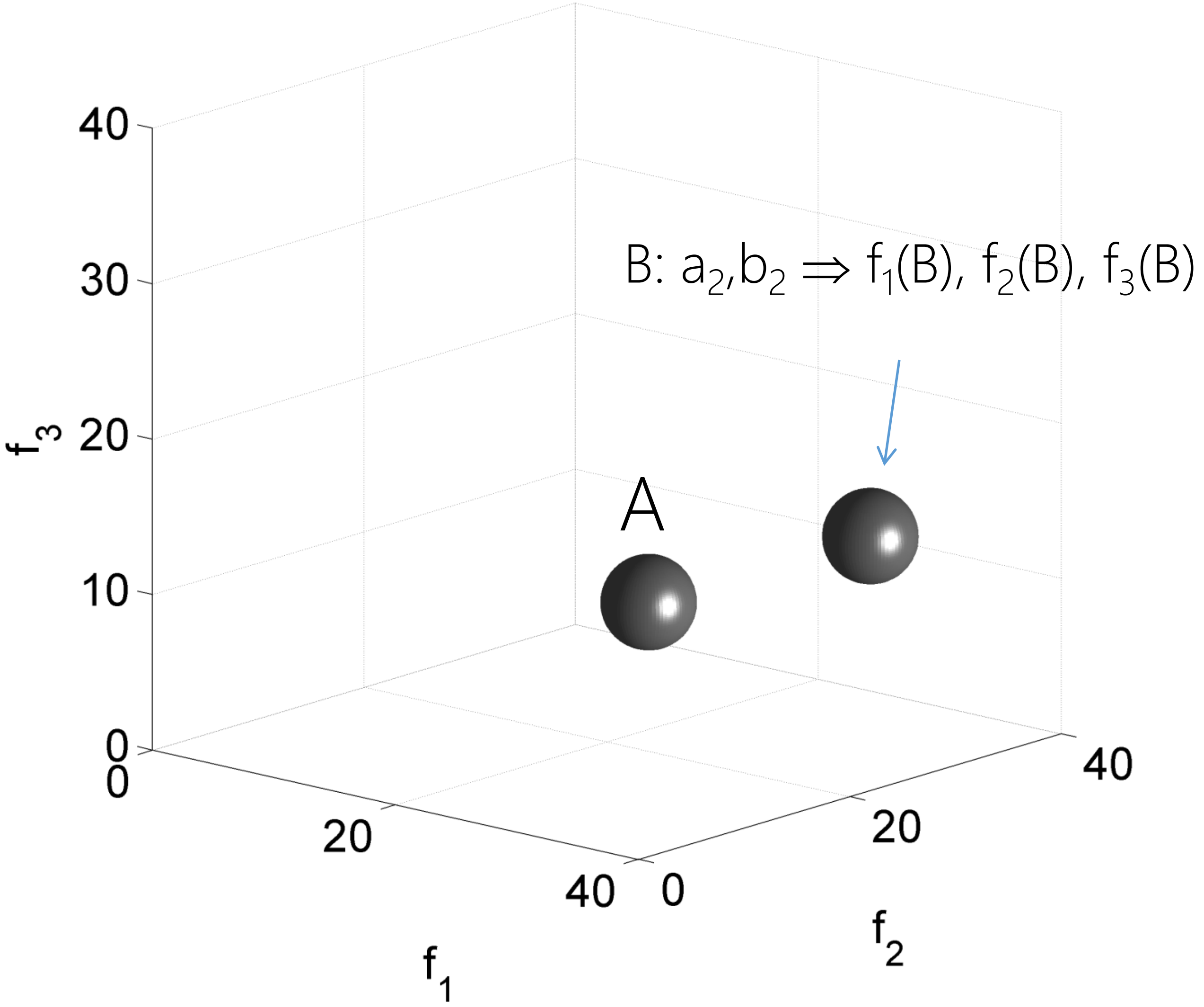
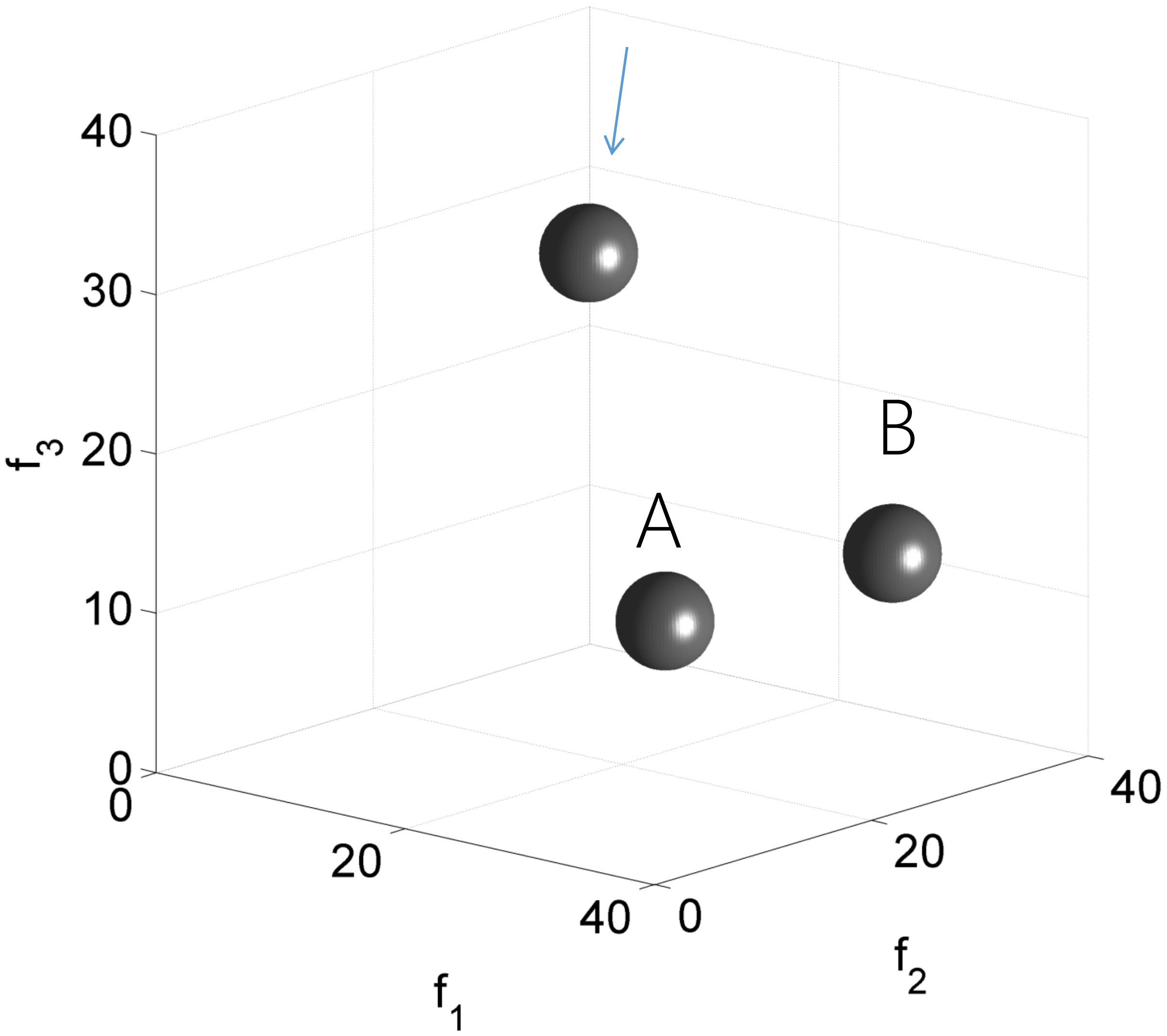
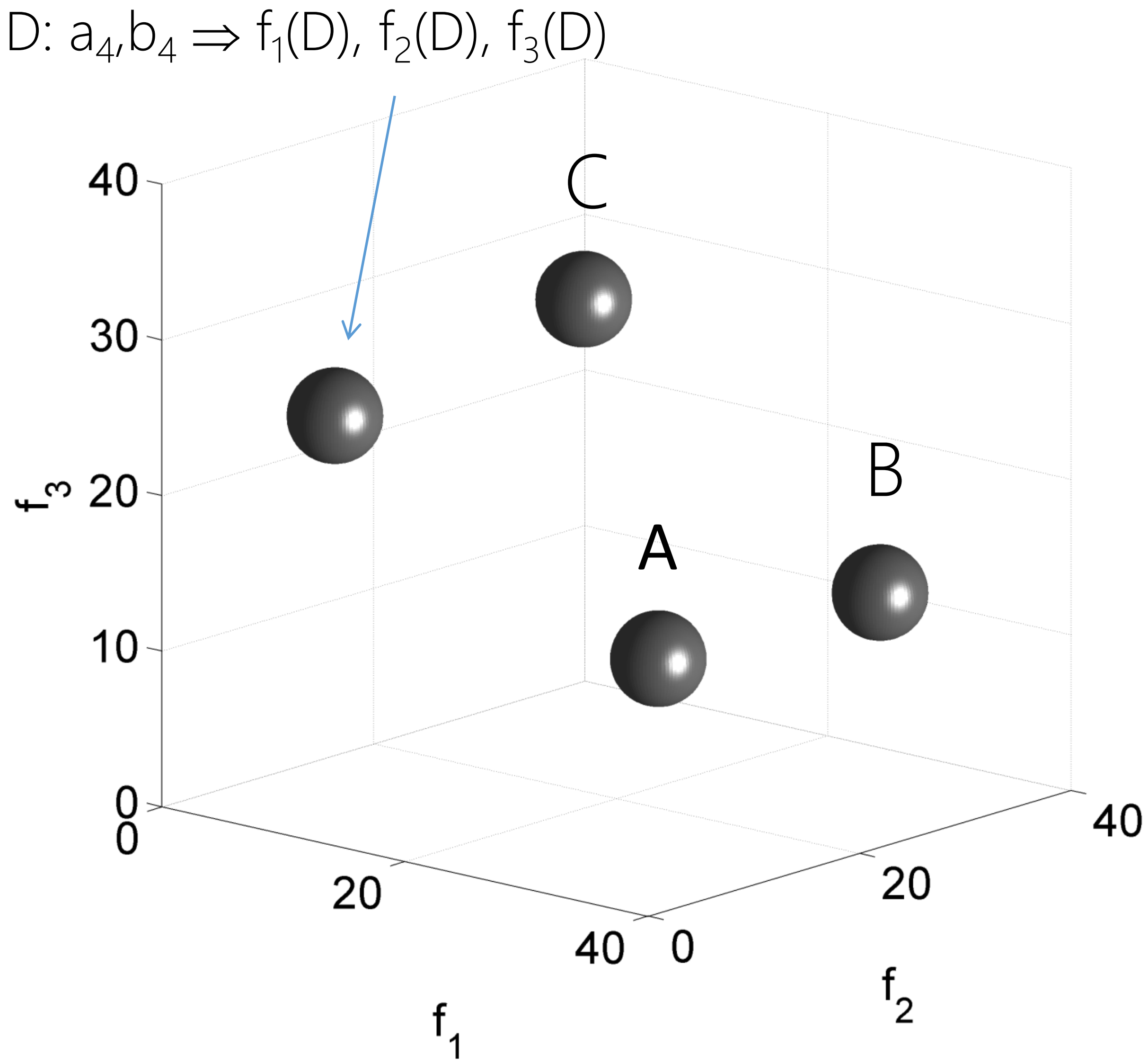




B: $a_2,b_2 \Rightarrow f_1(B), f_2(B), f_3(B)$
A
f_3
40
30
20
10
0
0
20
40
f_1
0
20
40
f_2

$C: a_3, b_3 \Rightarrow f_1(C), f_2(C), f_3(C)$





D: a_4,b_4 ⇒ $f_1(D)$, $f_2(D)$, $f_3(D)$
C
B
A
f_3
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0
20
40
f_1
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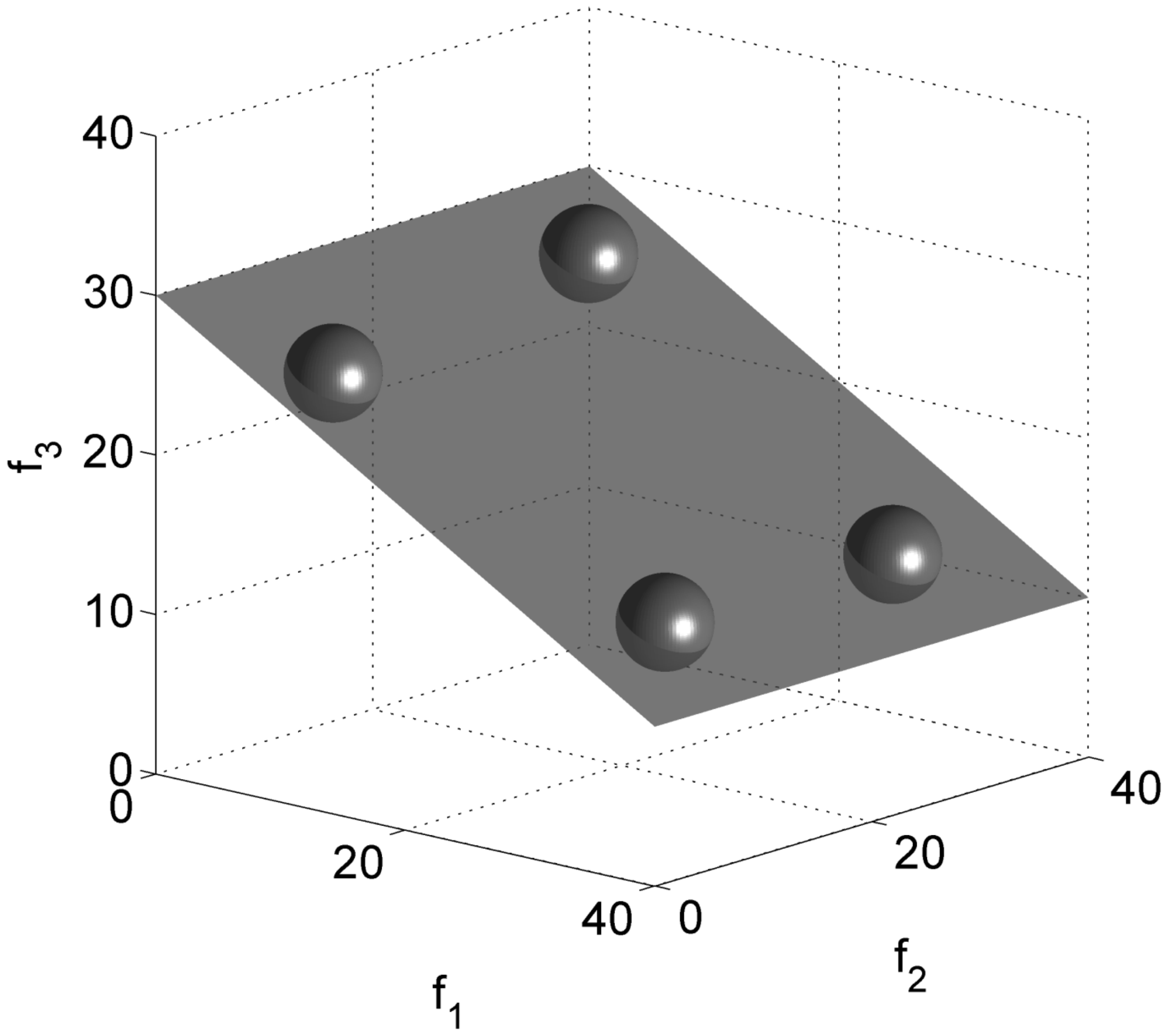
20
40
f_2



40
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f_3
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f_1
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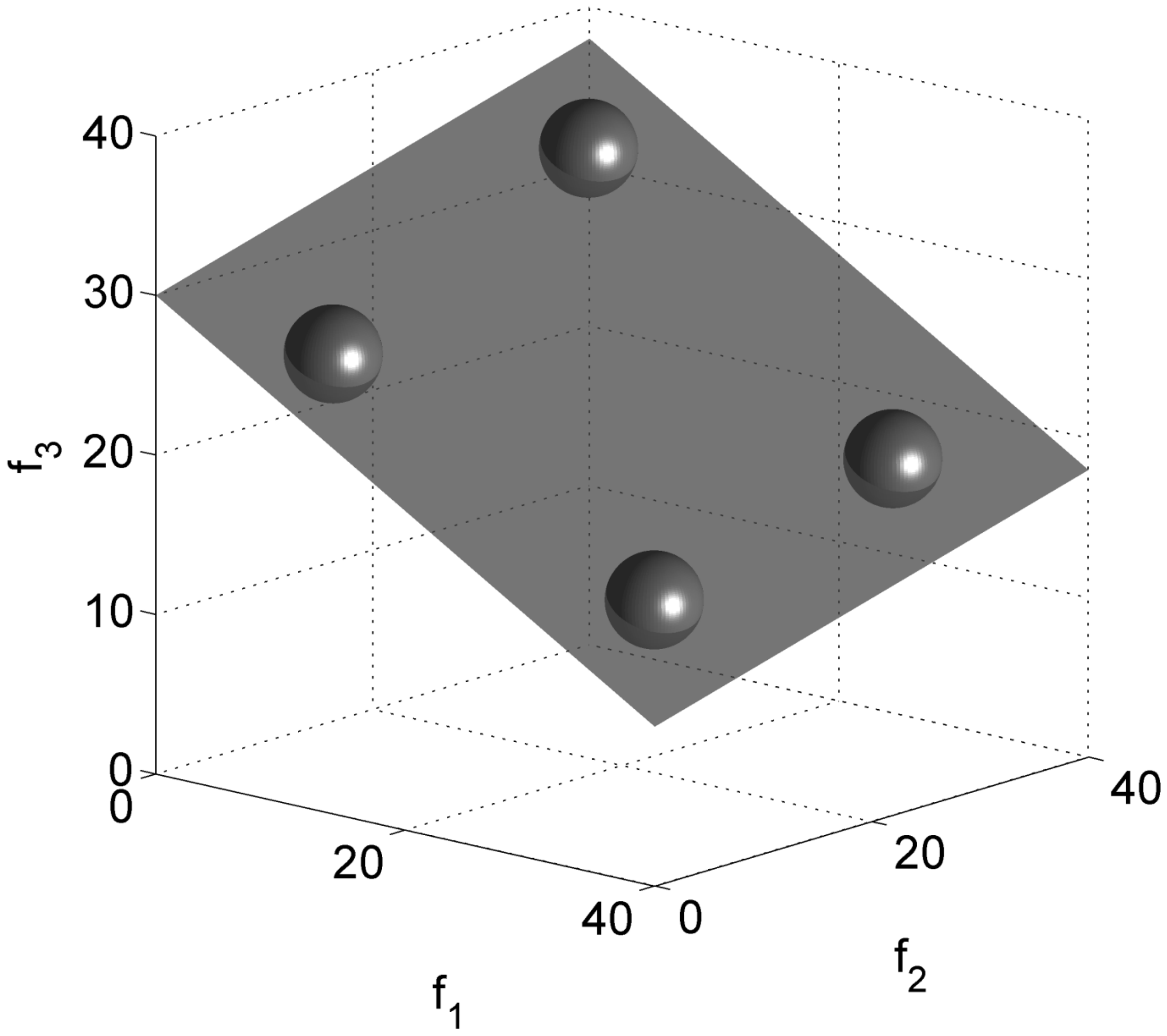
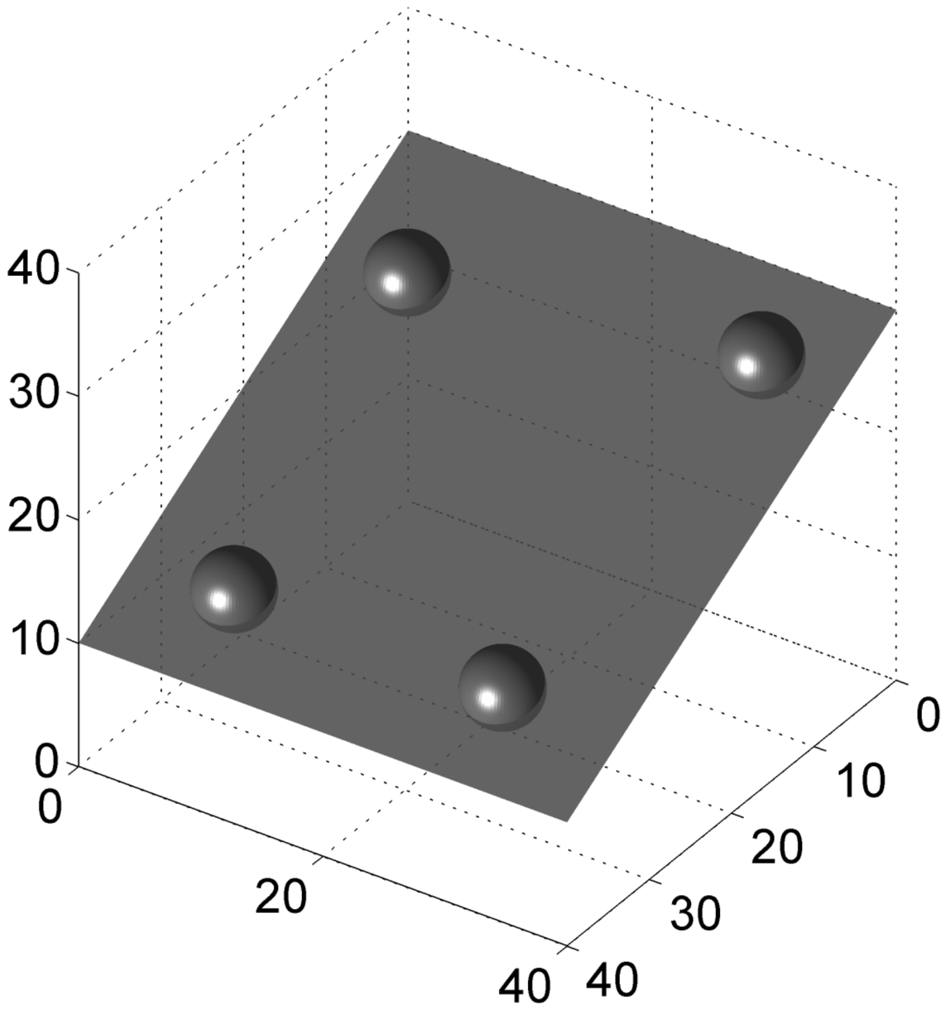
20
40
f_2

Linear mixed selectivity neuron: f_3=30-15a+6b



Quality of neural representation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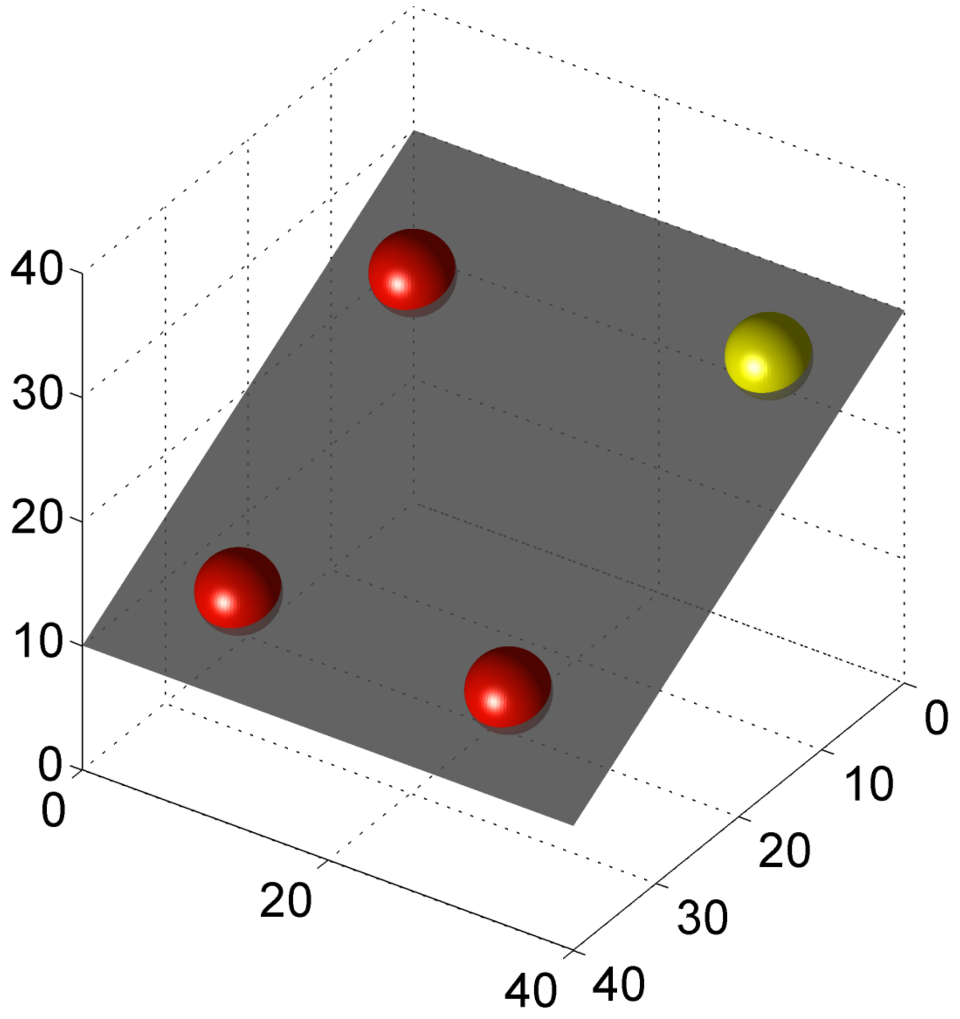

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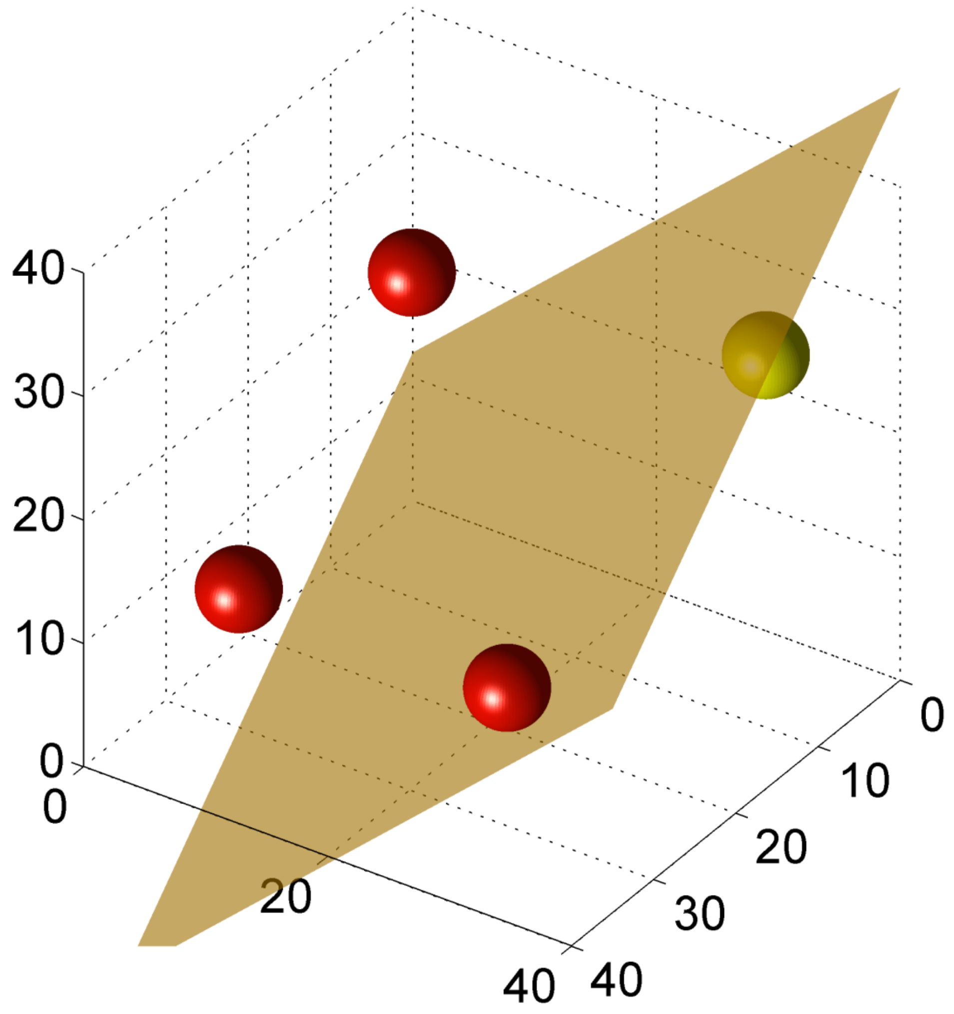
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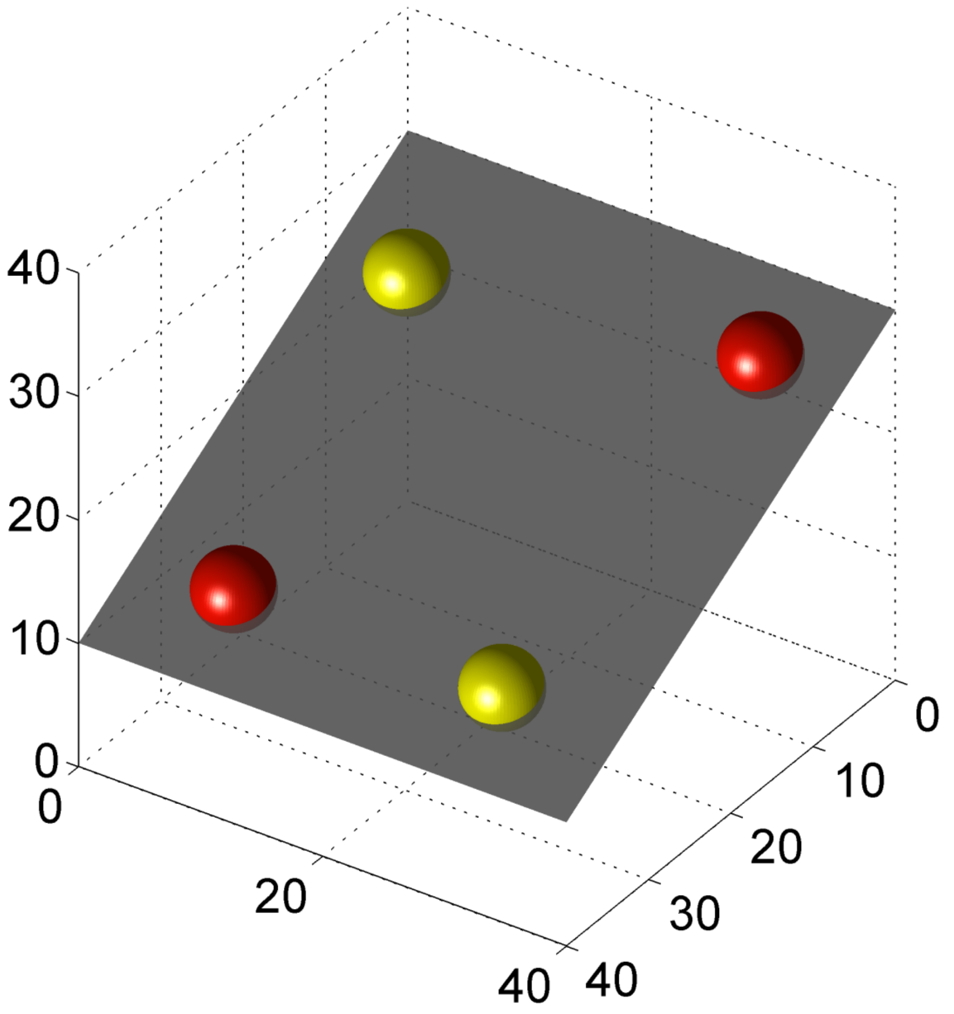
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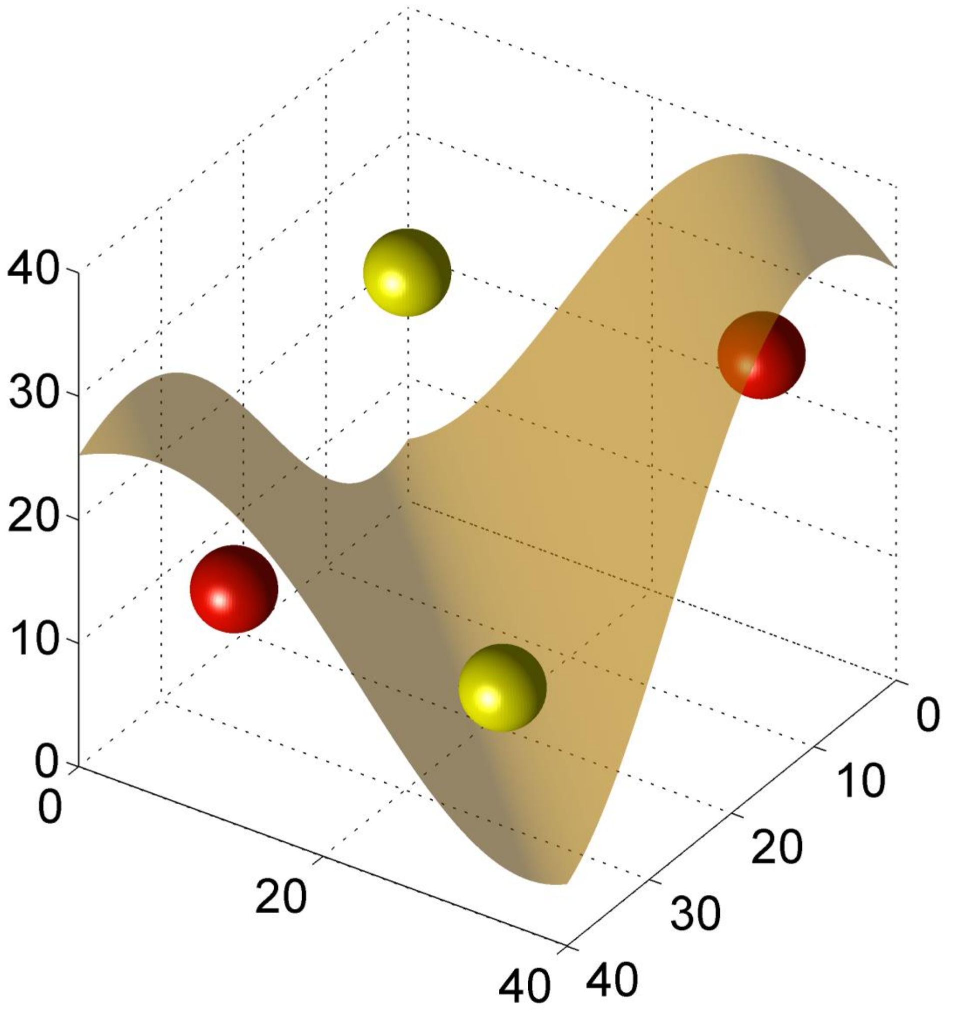
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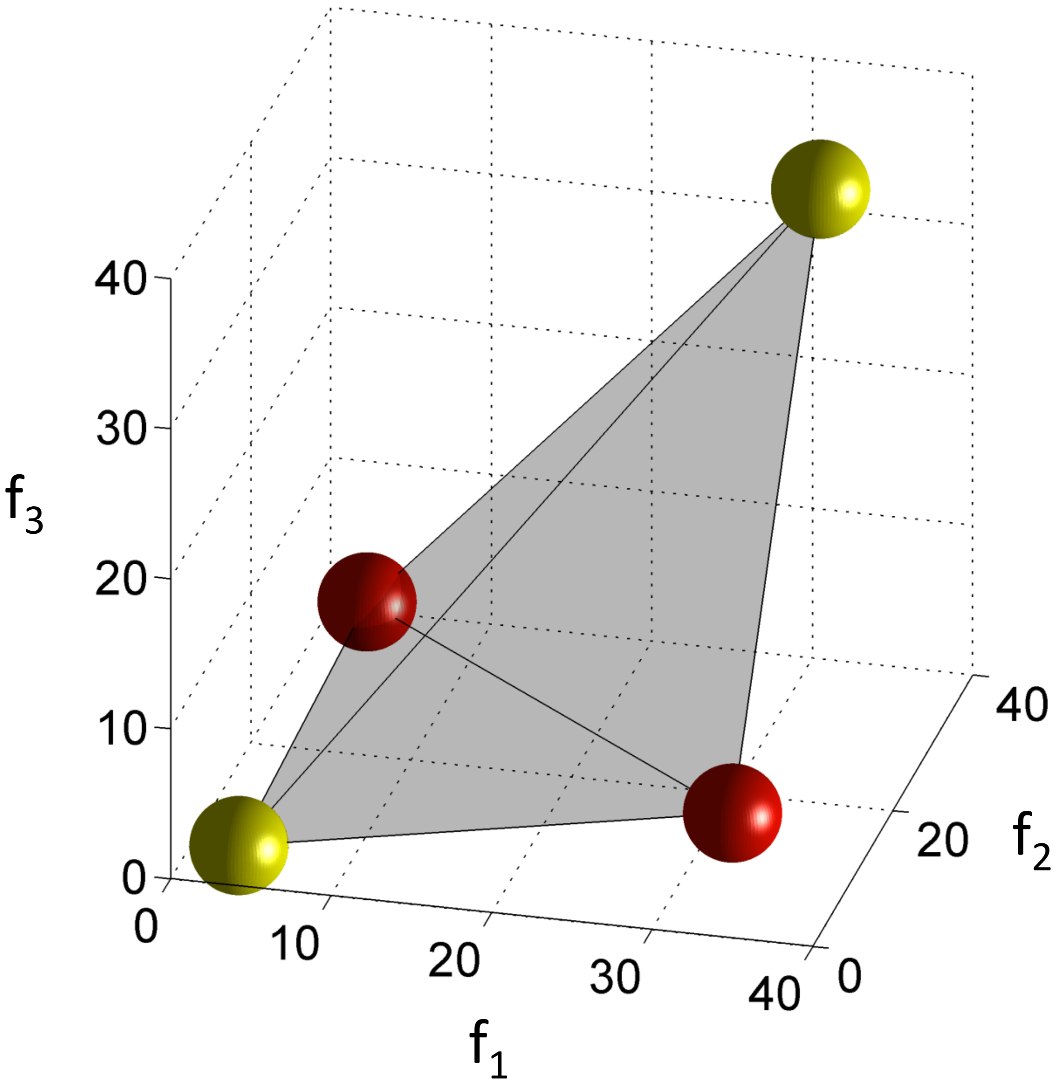
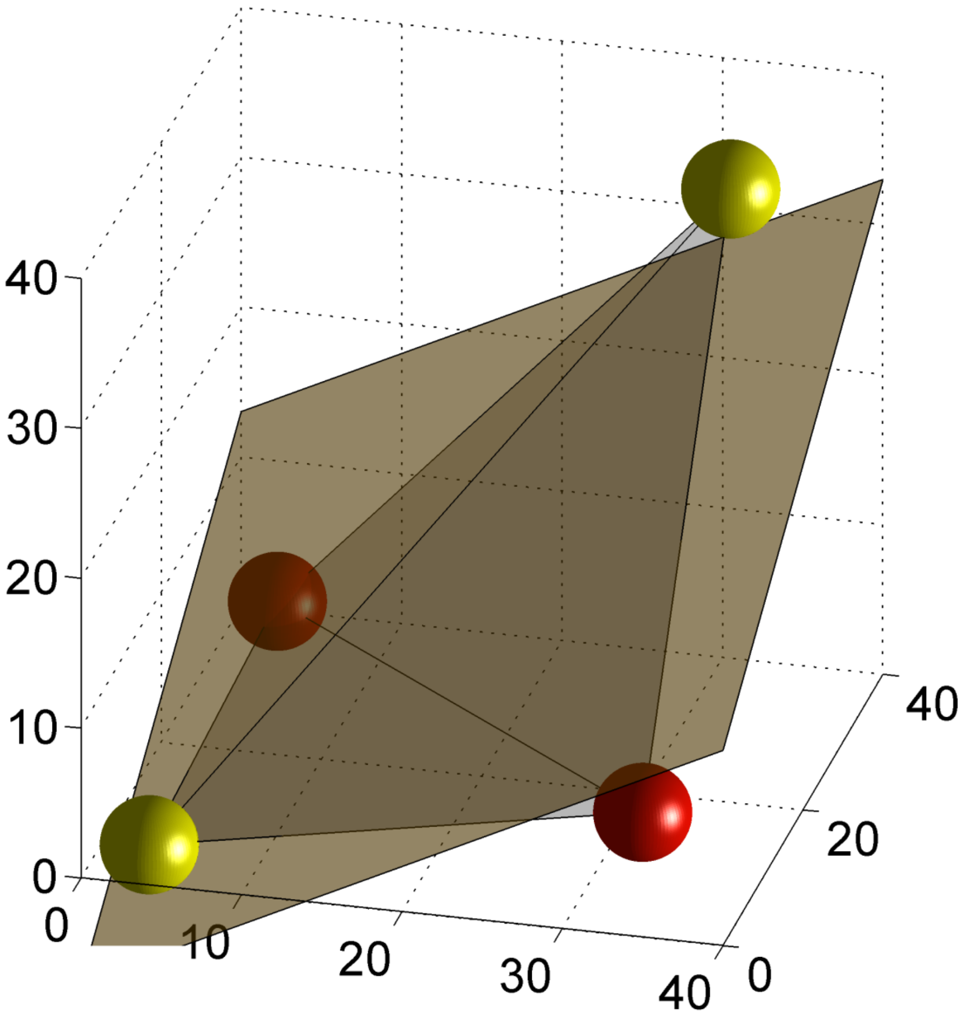
40

Non-linear mixed selectivity

$f_1 = 60\ a, \quad f_2 = 60\ b, \quad f_3 = \phi(a,b)$

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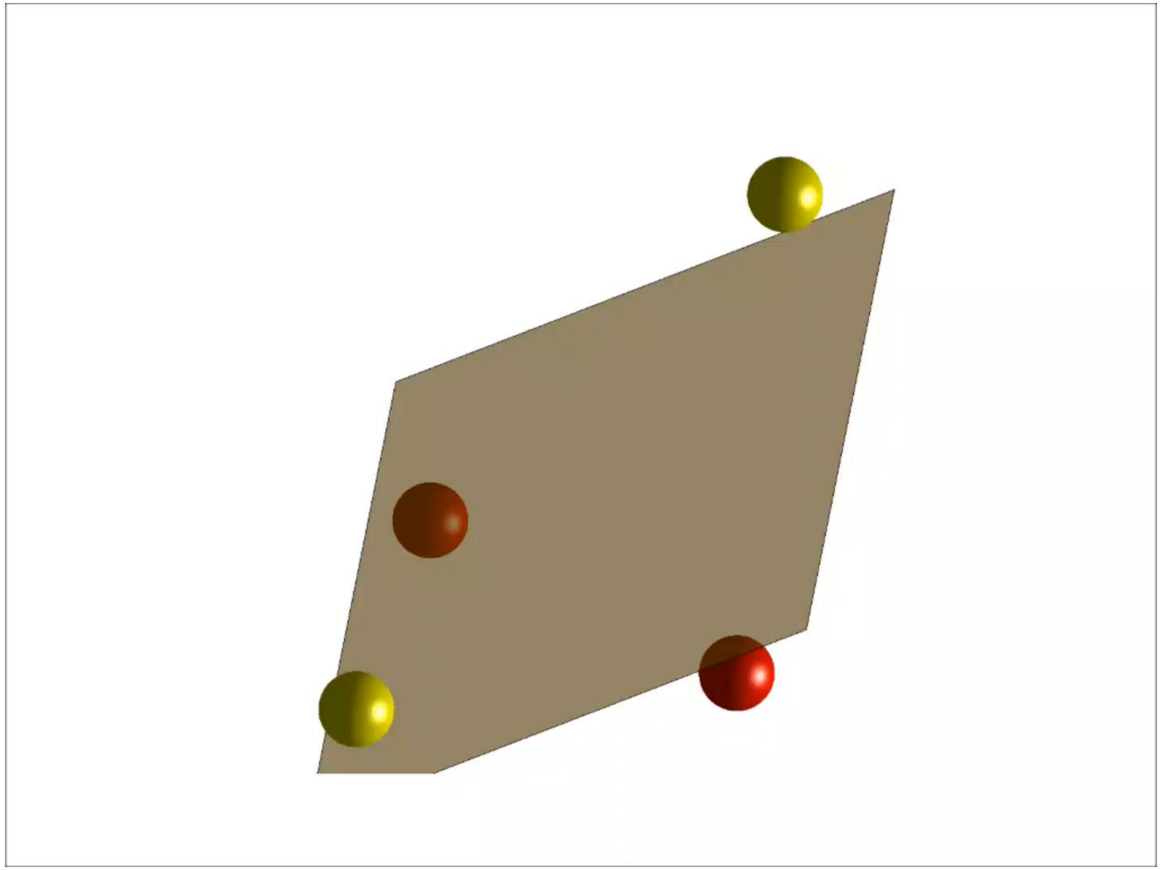
40



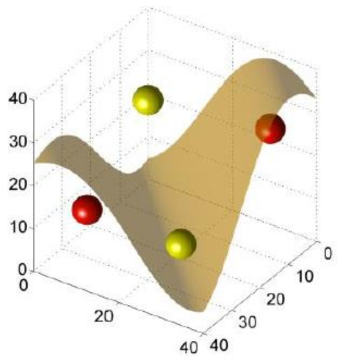
Summary

Neural response properties:

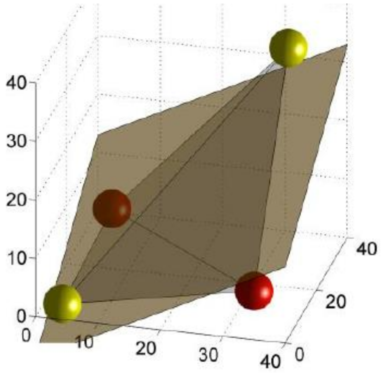
Pure and linear
mixed selectivity

Non-linear
mixed selectivity

Geometry of neural representation:



Low dimensionality



High dimensionality

Rigotti et al., Nature 2013

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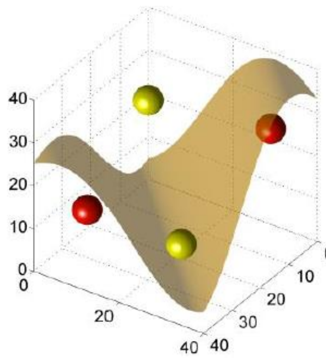
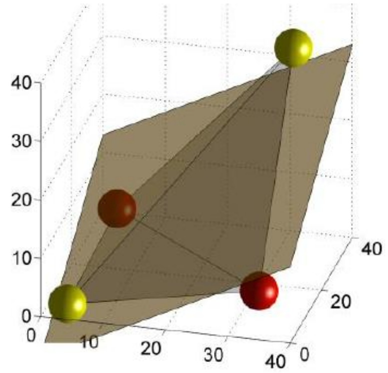




Low dimensionality

High dimensionality

Readout:

Complex

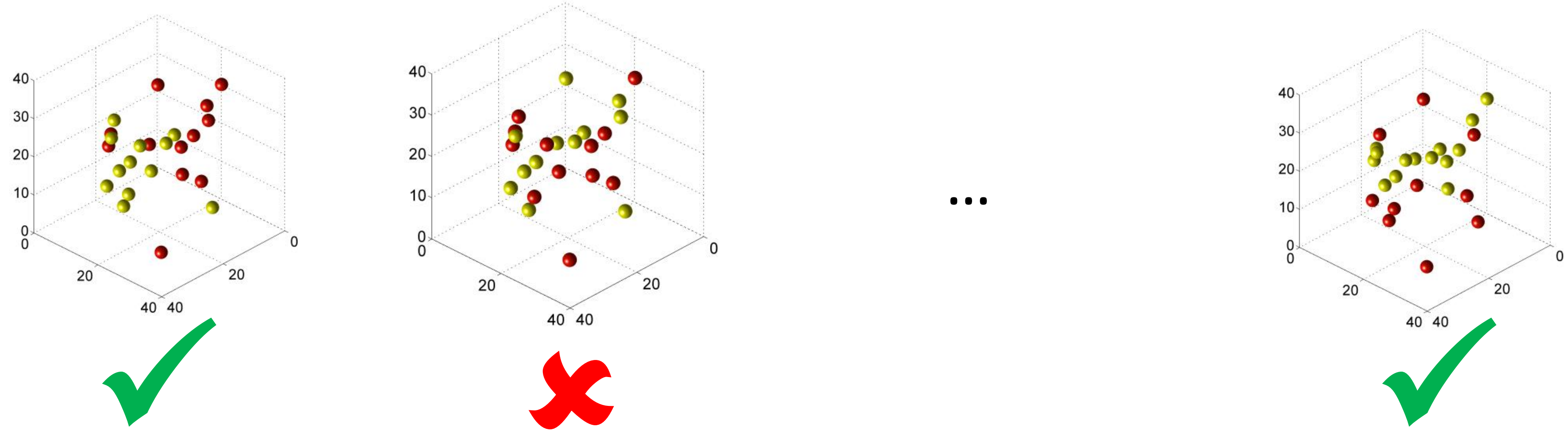
Simple (linear)

Quantitative measure of the quality of the representation

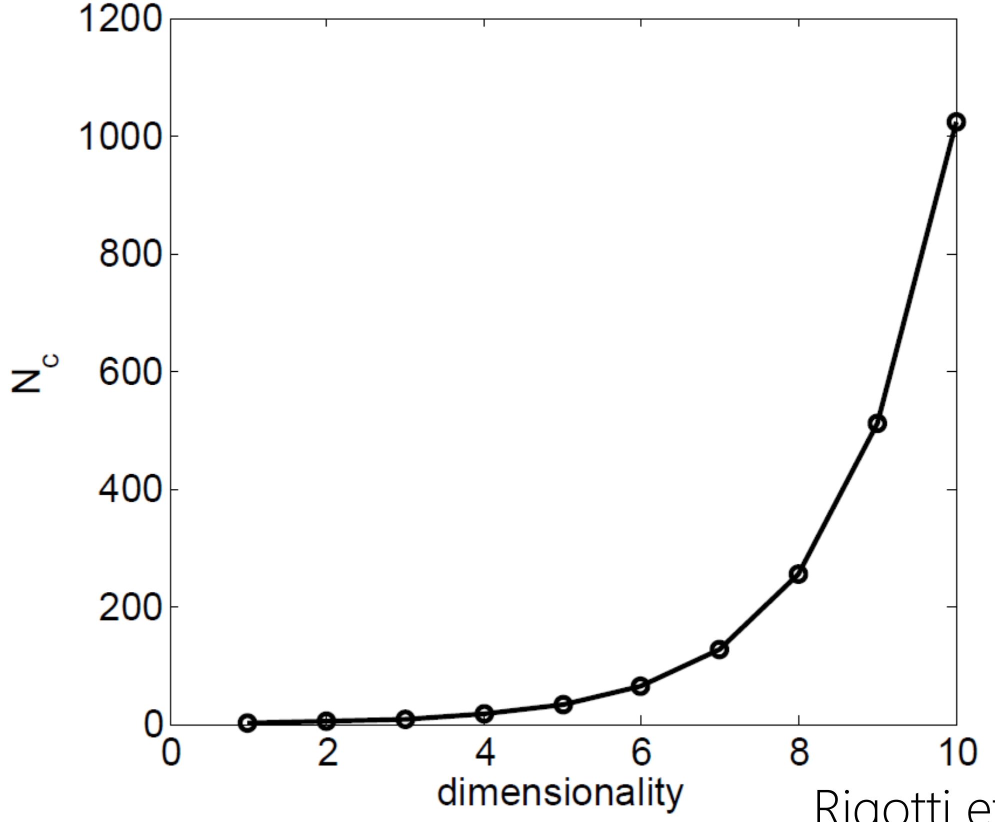
?

?

Rigotti et al., Nature 2013

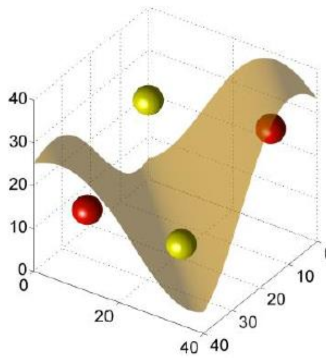
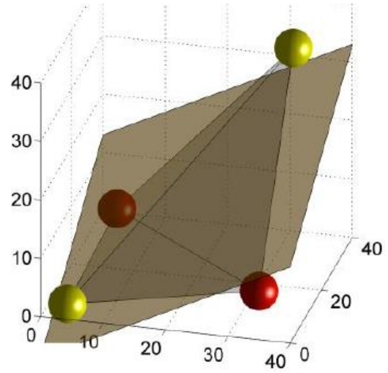


N_c = Number of binary classifications that are implementable by a simple linear readout



Rigotti et al., Nature 2013

Summary

	Pure and linear mixed selectivity	Non-linear mixed selectivity	
Neural response properties:			
Geometry of neural representation:			
	Low dimensionality	High dimensionality	d ~log(N)
Readout:	Complex	Simple (linear)	
Quality of neural representation:	Low	High	N ~exp(d)

Rigotti et al., Nature 2013

High dimensionality: 1. non linear mixed selectivity
2. diversity

Different mixed selectivity neurons should exhibit different response properties.
The neural activation should be heterogeneous enough to guarantee
a large coverage of the response space

High or low dimensionality?

Dimensionality is a global property of the **geometry.**

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Dimensionality reduction is important for example in object recognition, where it is desirable that different views of the same object (due to slight changes in pose, position, and so on) all result in the same classification response. The resulting representation clearly allows for a potentially large reduction in sample complexity, since the label provided by a training sample can be **generalized** over all the inputs corresponding to the same object (DiCarlo et al., 2007).

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Dimensionality and **behavior**: collapse of dimensionality in error trials in frontal areas (Rigotti et al., 2013, Fusi et al., 2016).

Geometry- definition

The shape defined by the relative positions of patterns in the high-dimensional space of neural activity

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Still an open question: **Is geometry all you need? The importance of representational geometry across brain areas and artificial neural networks** (Ramon Nogueira, Valeria Fascianelli, Lorenzo Posani, Mario Dipoppa, Worskhop at Cosyne 2022)

Abstraction

abstract

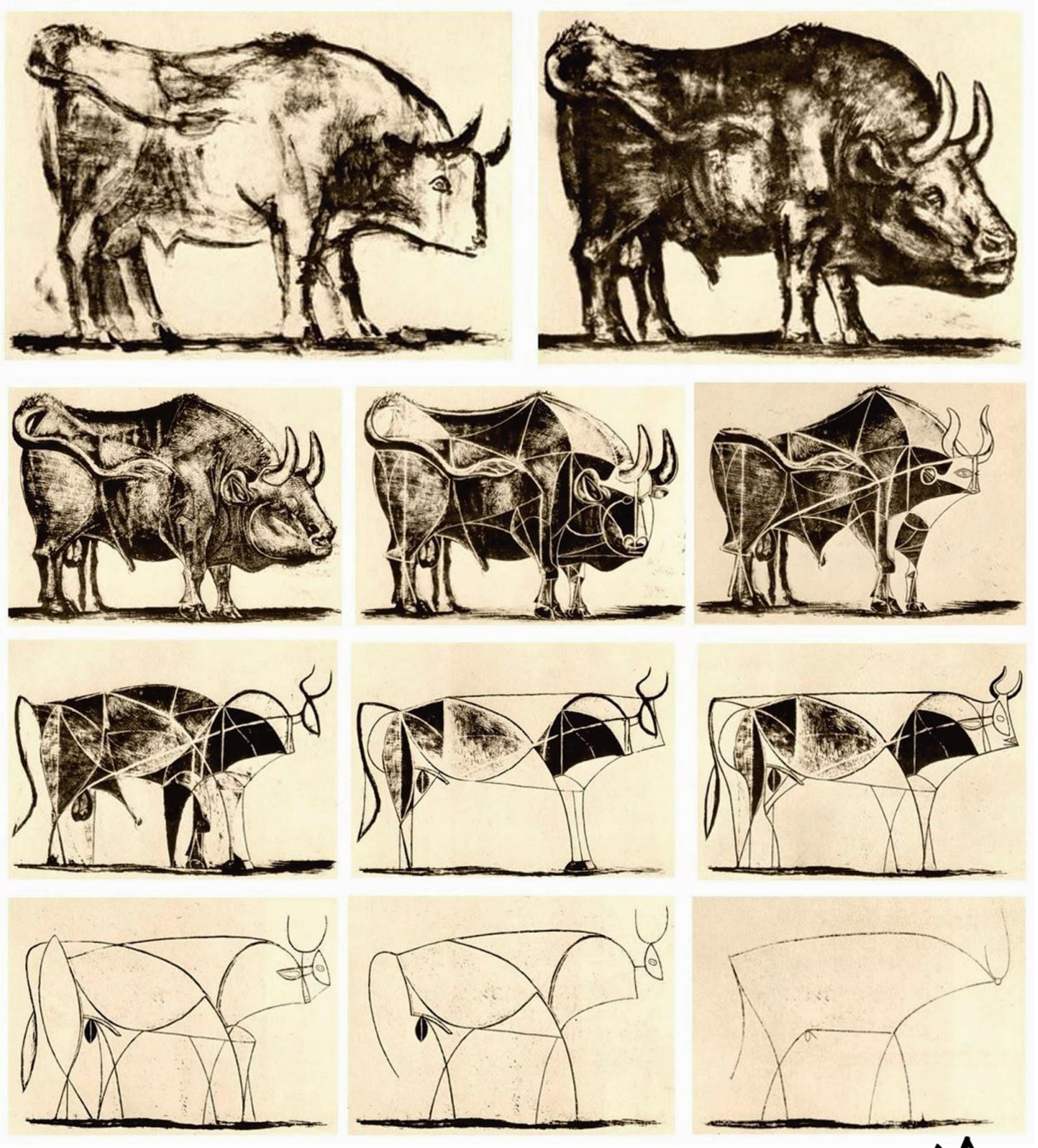
[adjective]

ab·stract | \ ab-ˈstrakt , ˈab-ˌ \

Definition of *abstract*

(Entry 1 of 3)

1a: disassociated from any specific instance



Picasso

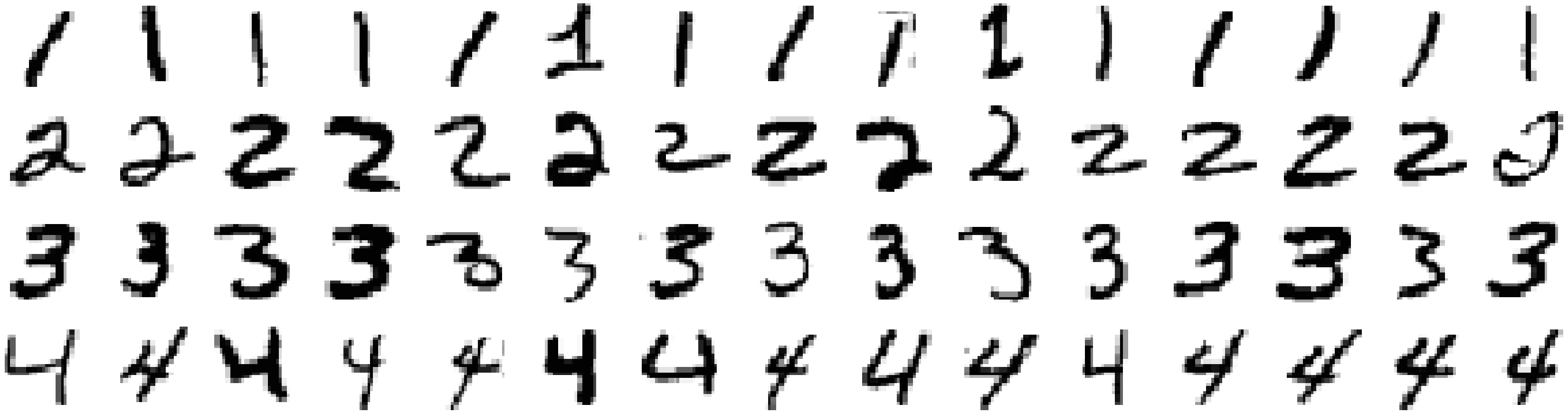
Cell

Article

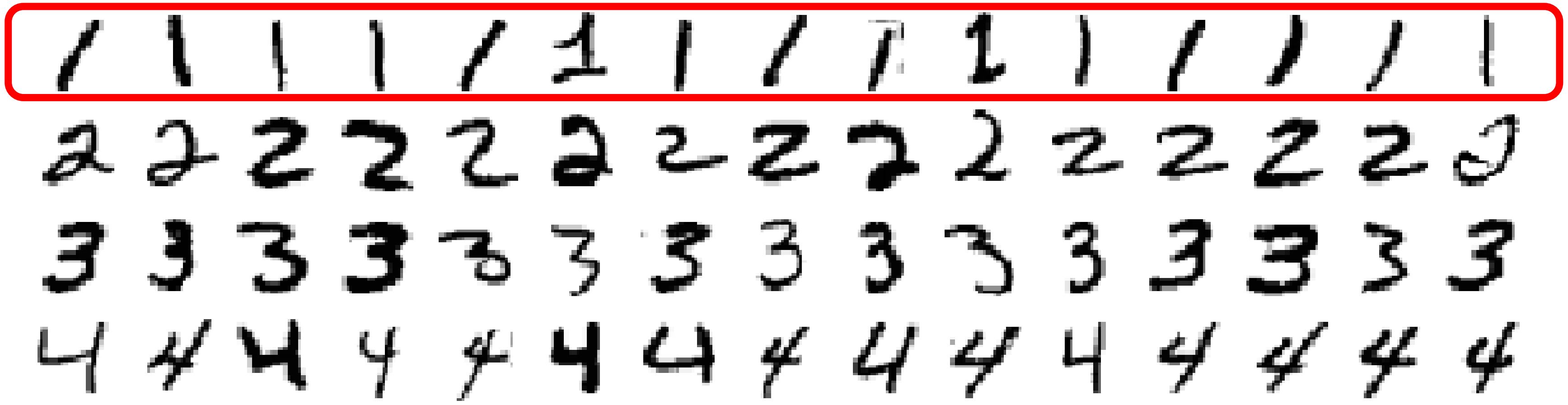
The Geometry of Abstraction in the Hippocampus and Prefrontal Cortex

Silvia Bernardi,[2,3,5,8,10] Marcus K. Benna,[1,4,5,9,10] Mattia Rigotti,[7,10] Jérôme Munuera,[1,10,12] Stefano Fusi,[1,4,5,6,11,*] and C. Daniel Salzman[1,2,5,6,8,11,13,*]

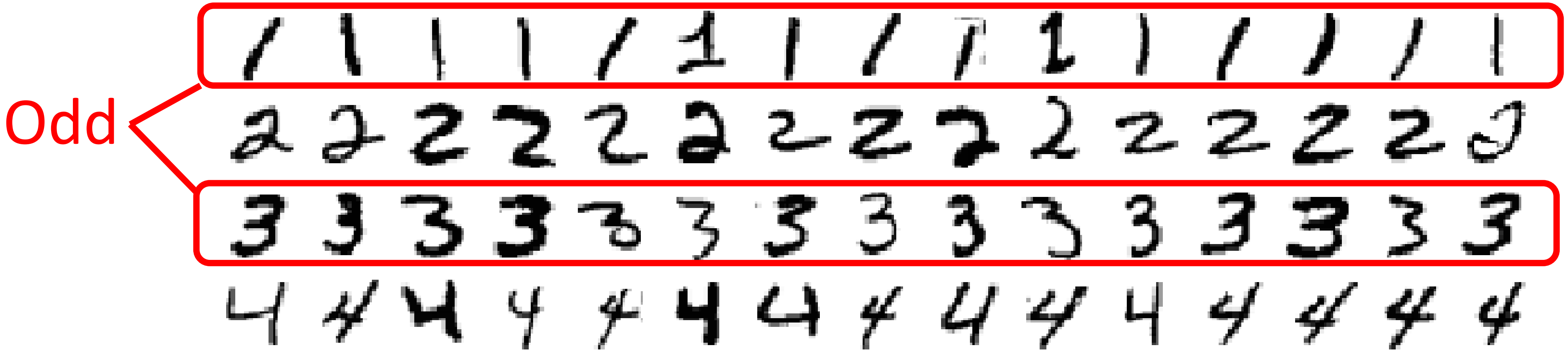
MNIST Input



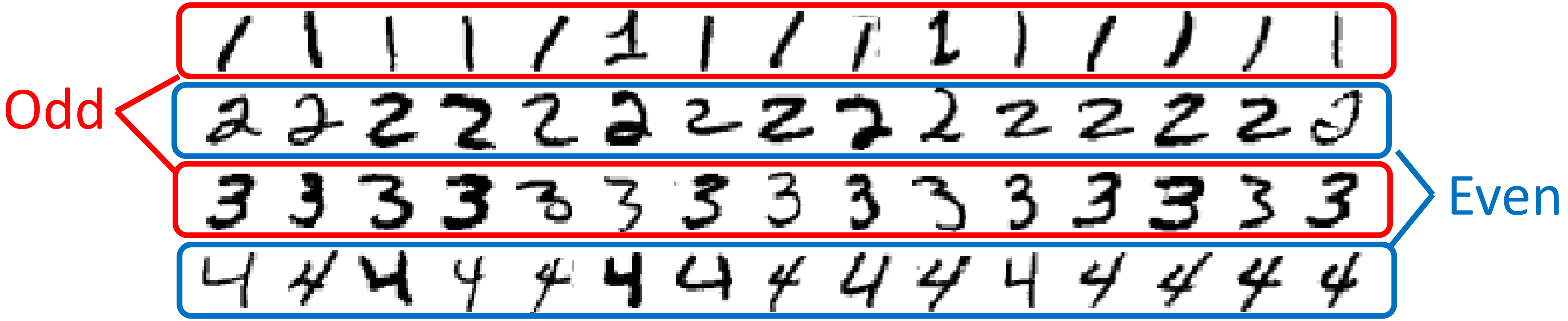
MNIST Input

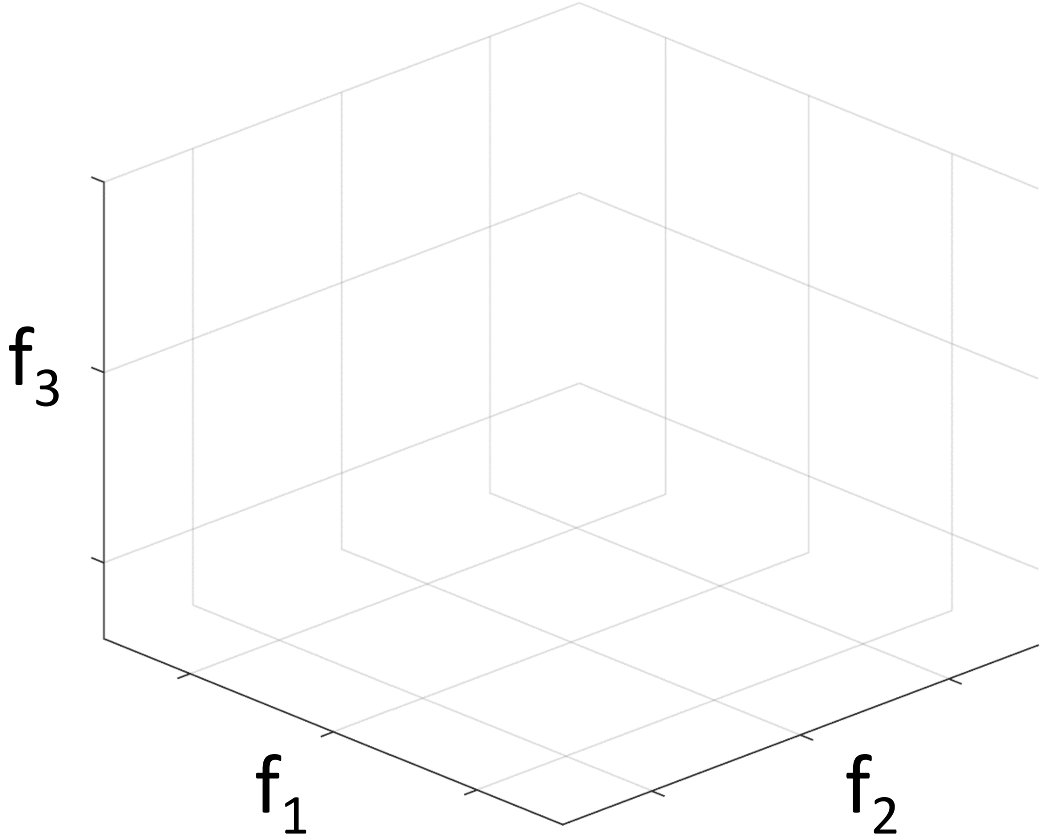


MNIST Input



MNIST Input




f_3
f_1
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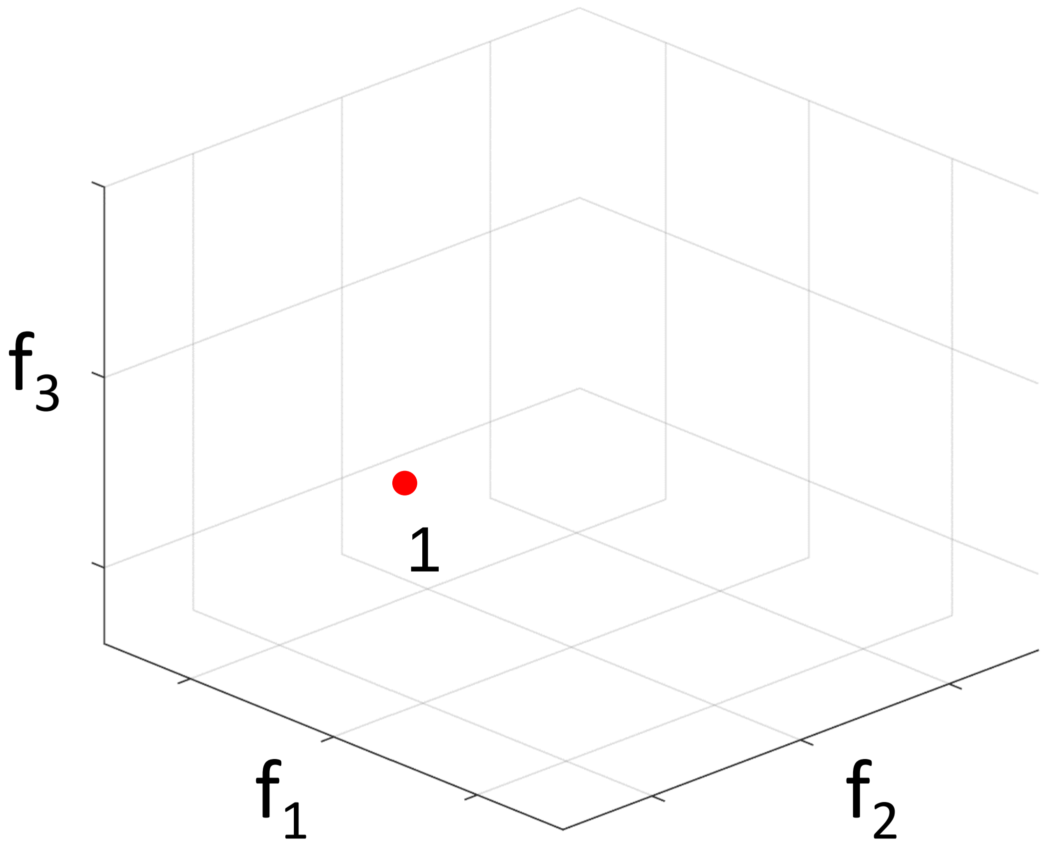

f_3
1
f_1
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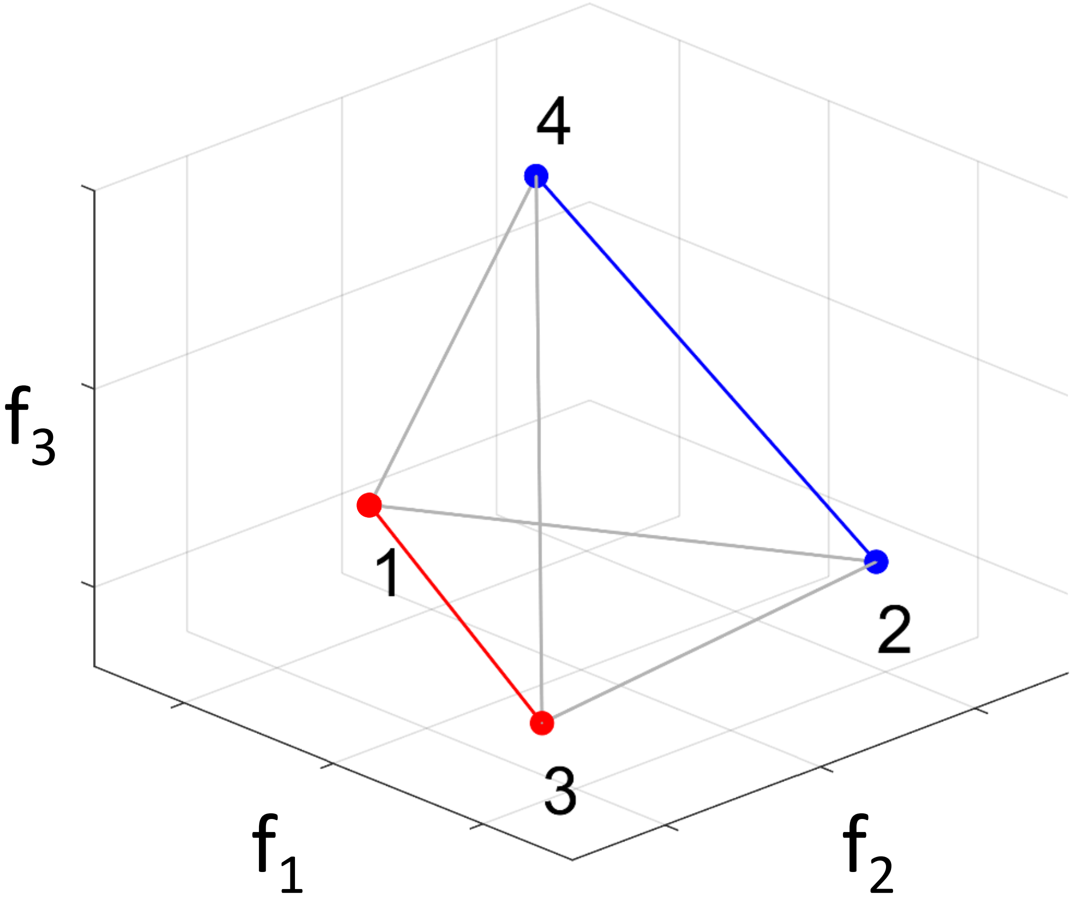
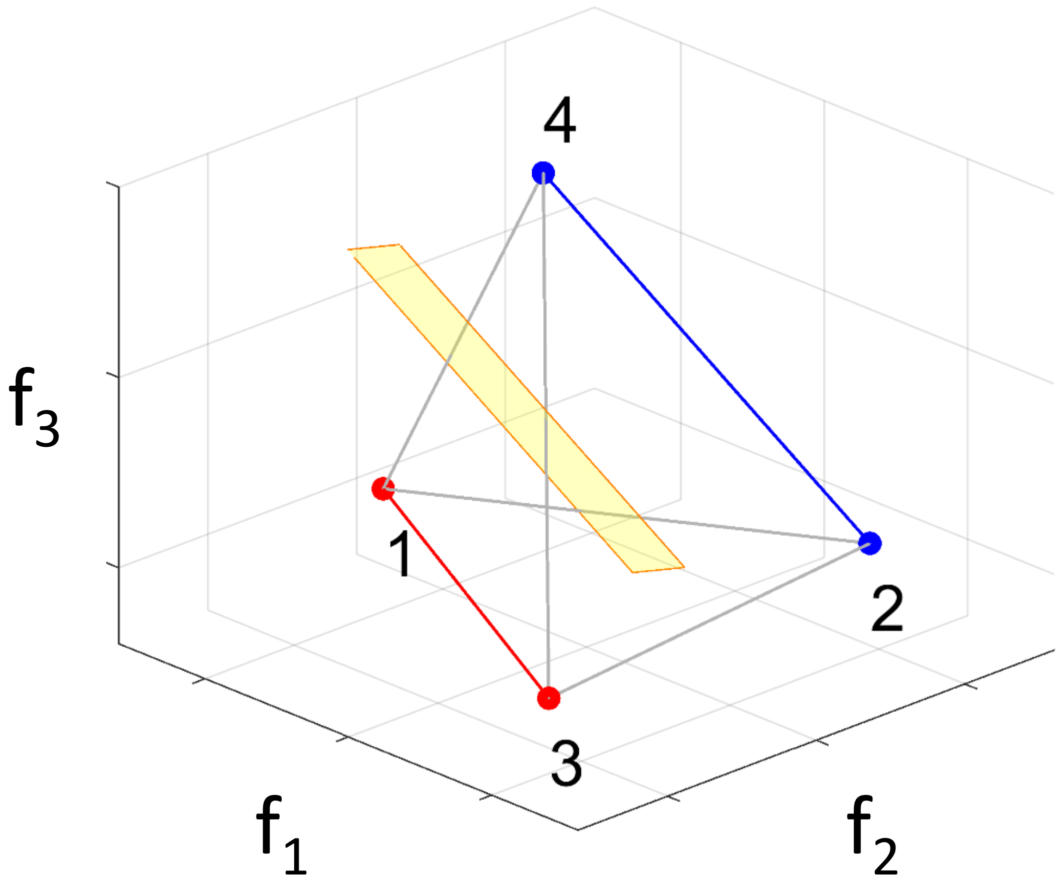


Parity

1, 3 instances of "ODD"

2, 4 instances of "EVEN"

Points in random positions in the firing rate space, non linear mixed selectivity



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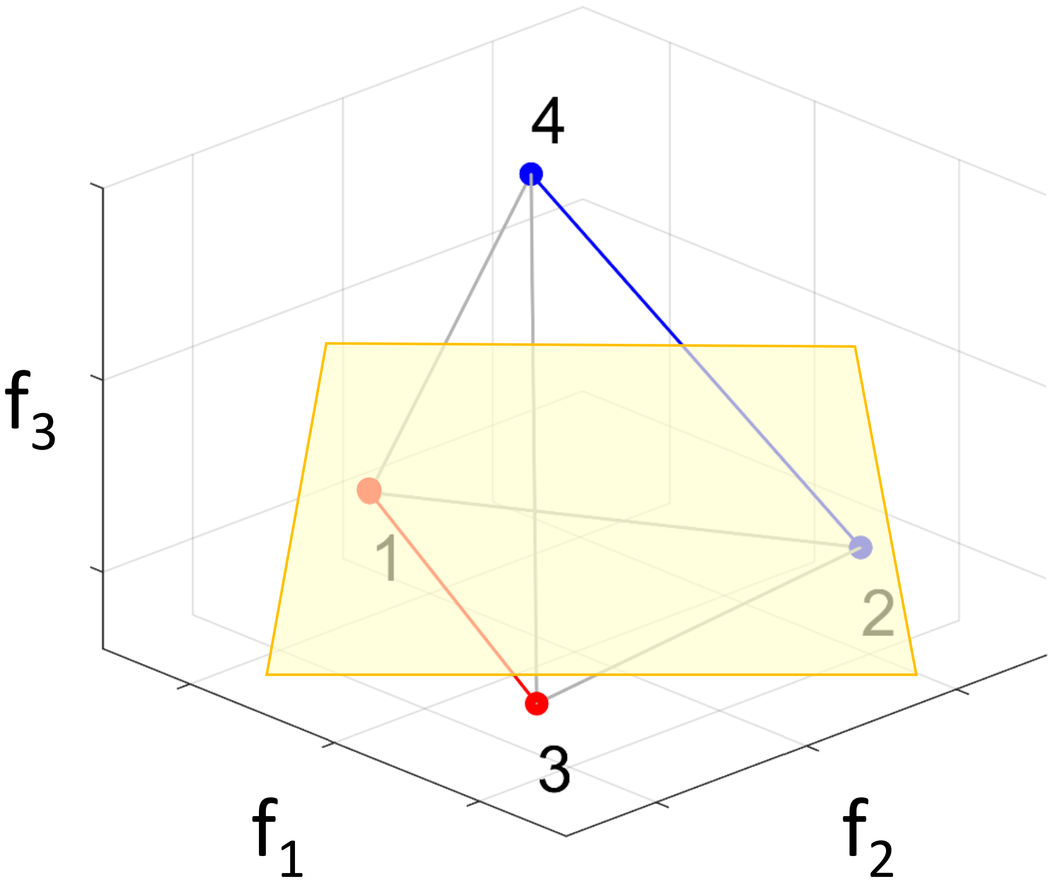
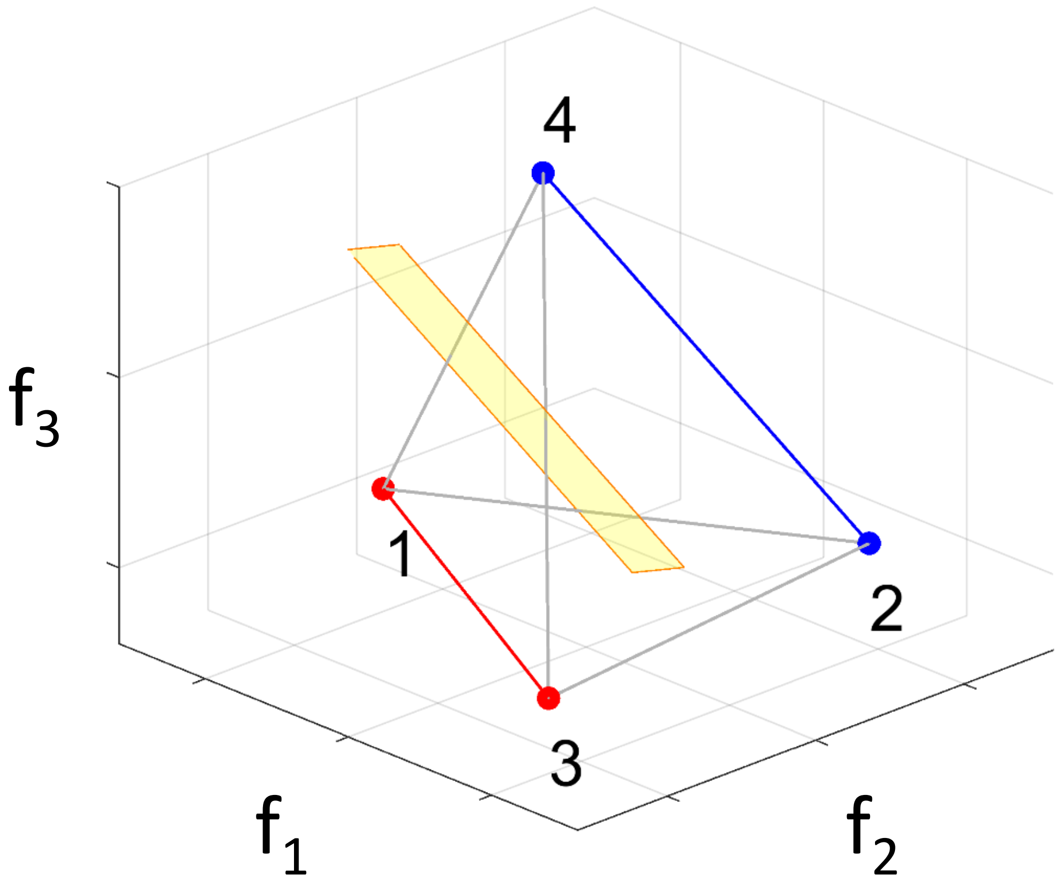


Magnitude

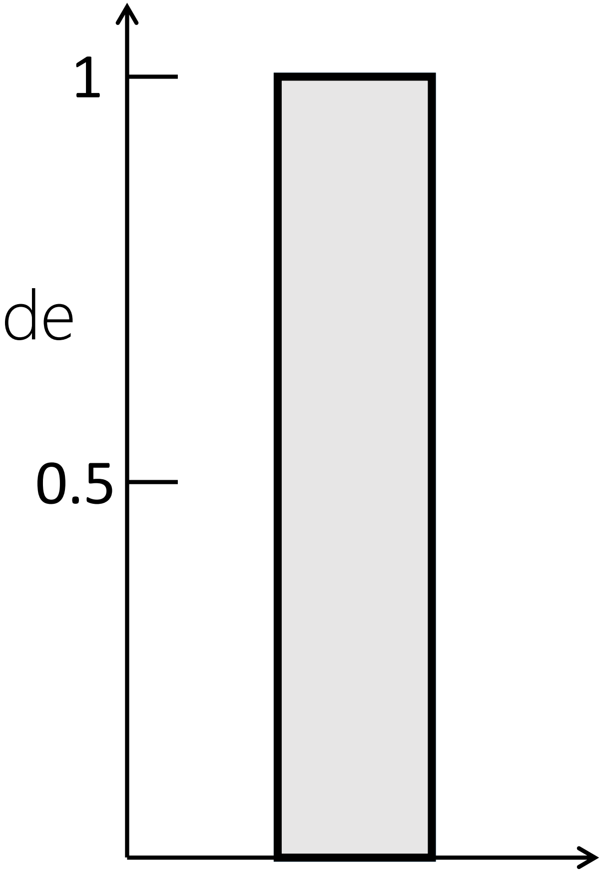
1, 2 instances of "SMALL"

3,4 instances of "LARGE"



(1,3) vs (2,4): Parity
(1,2) vs (3,4): Magnitude
(1,4) vs (2,3): ???
…

shattering dimensionality



Rigotti et al. *Nature*, 2013

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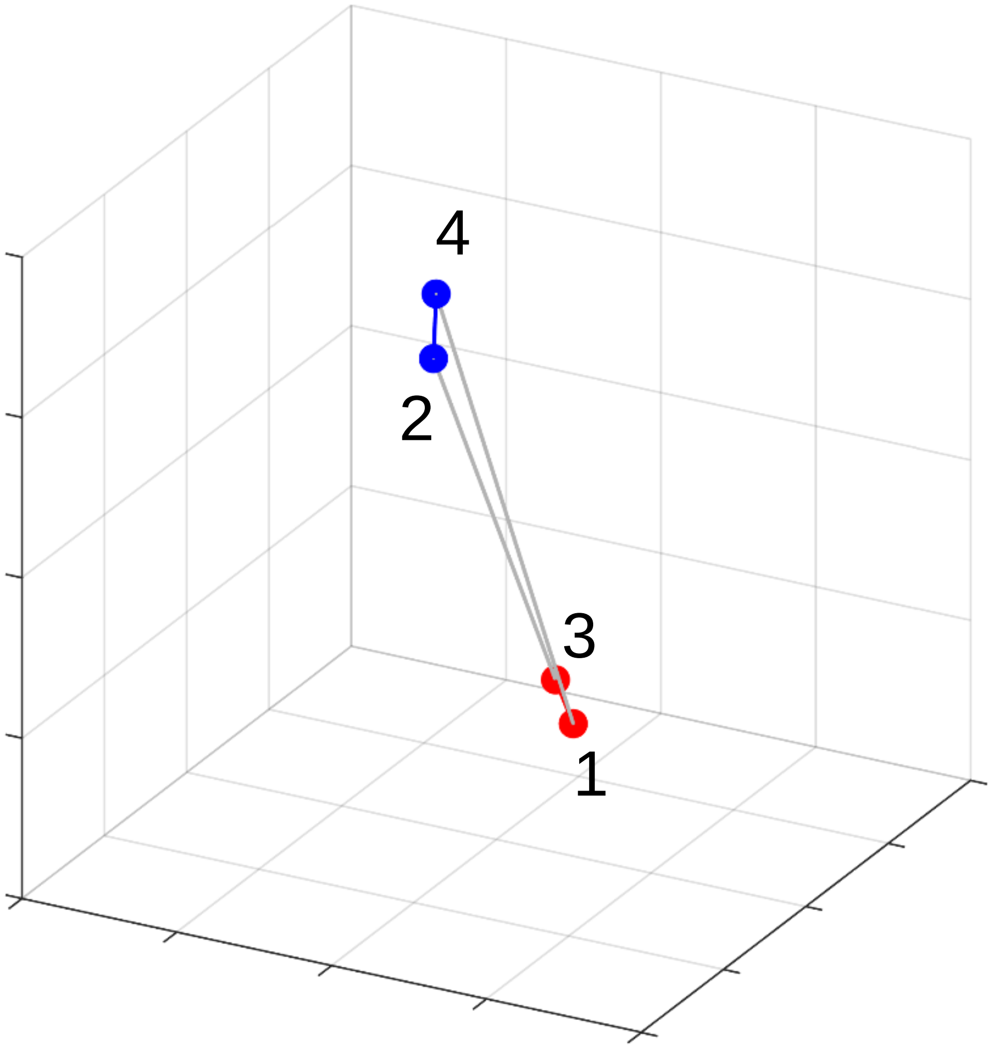
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2
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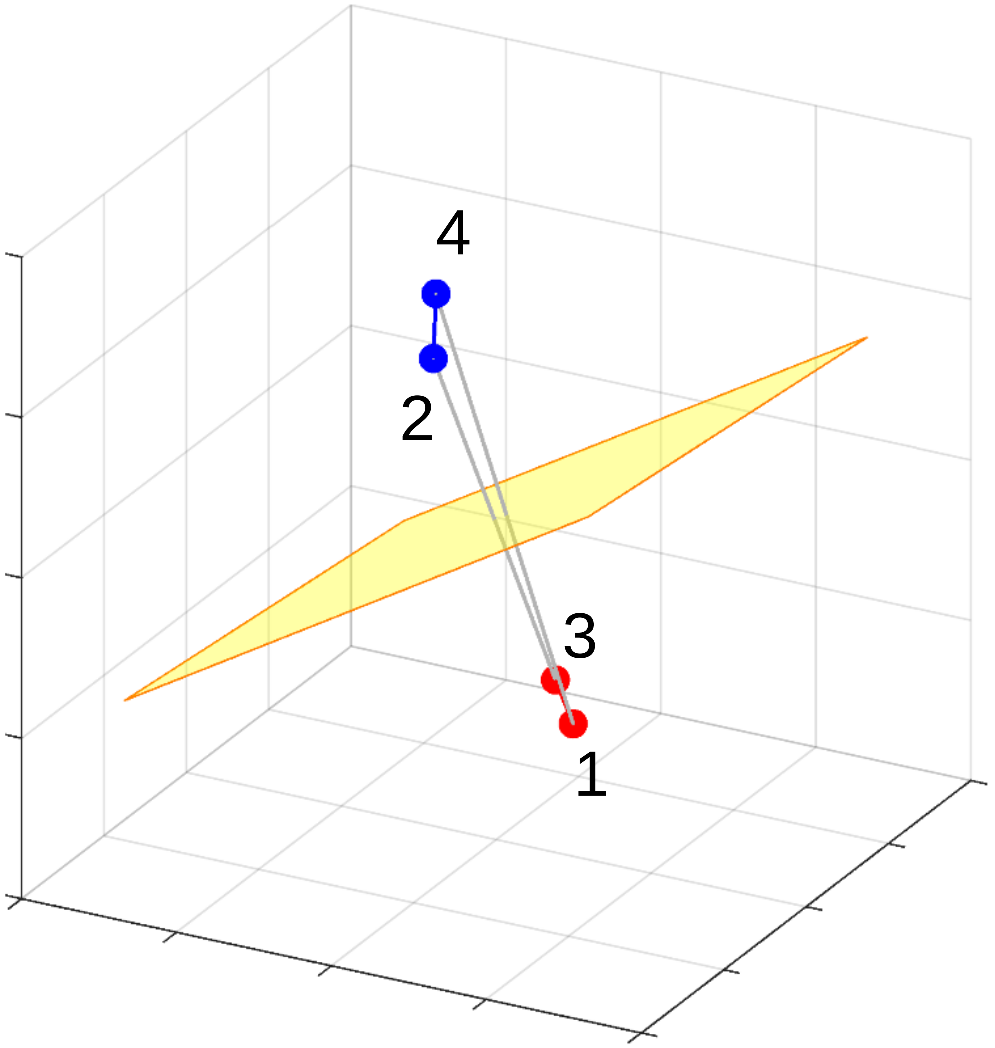
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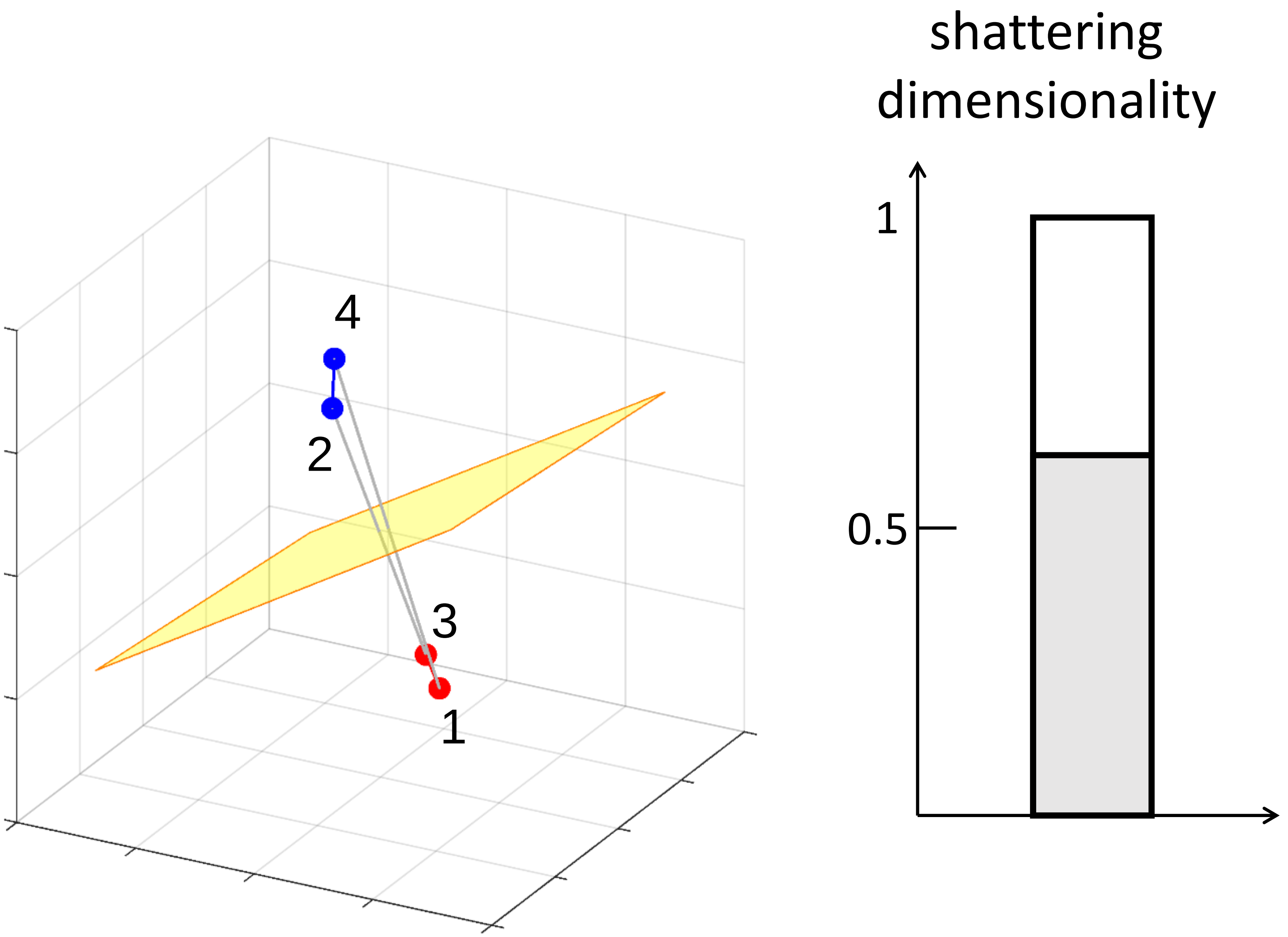
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shattering
dimensionality
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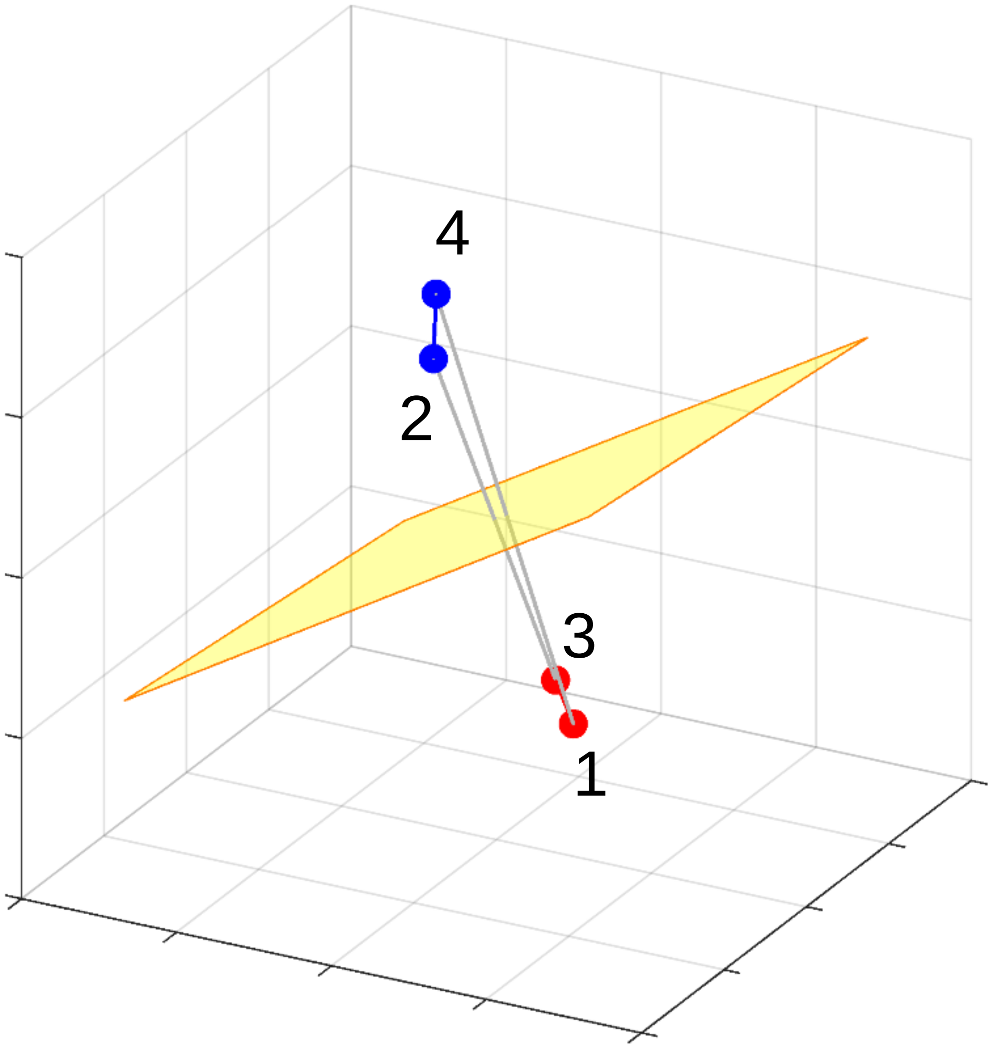
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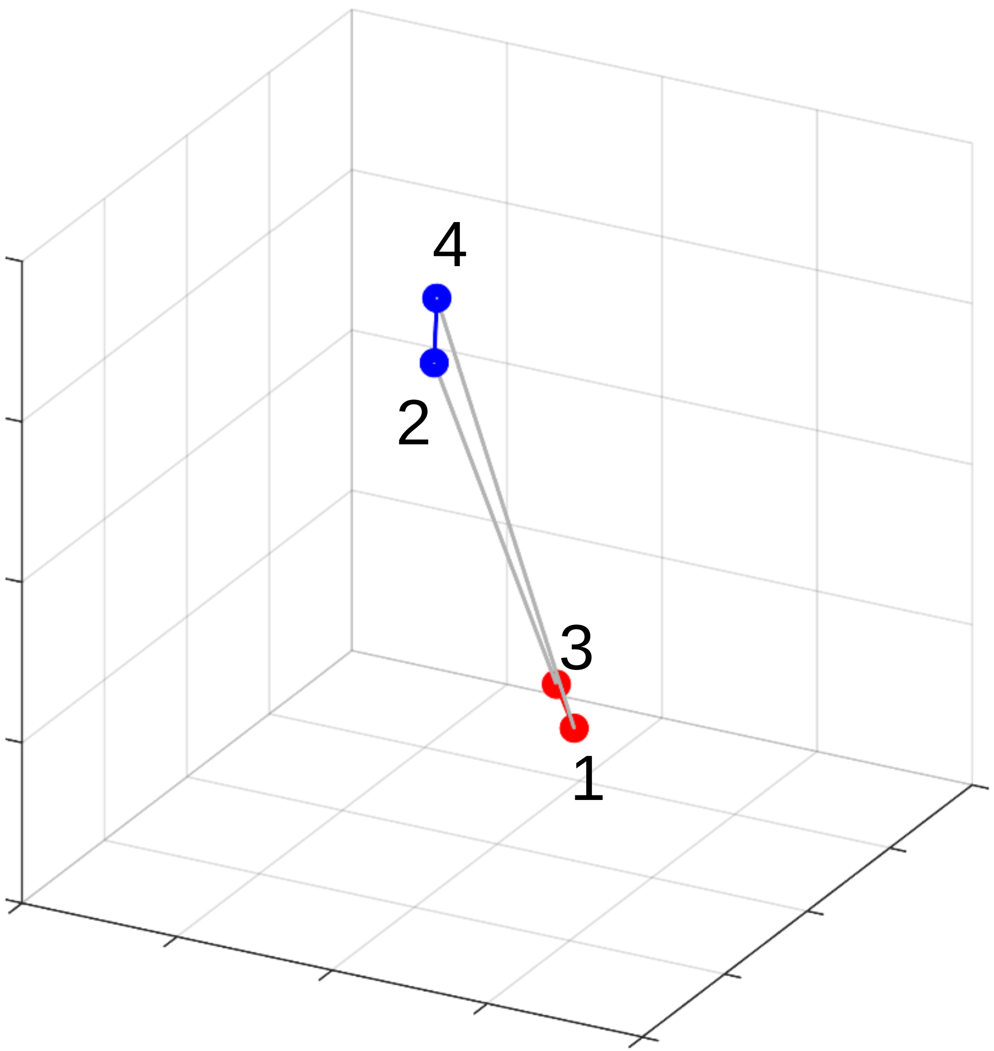
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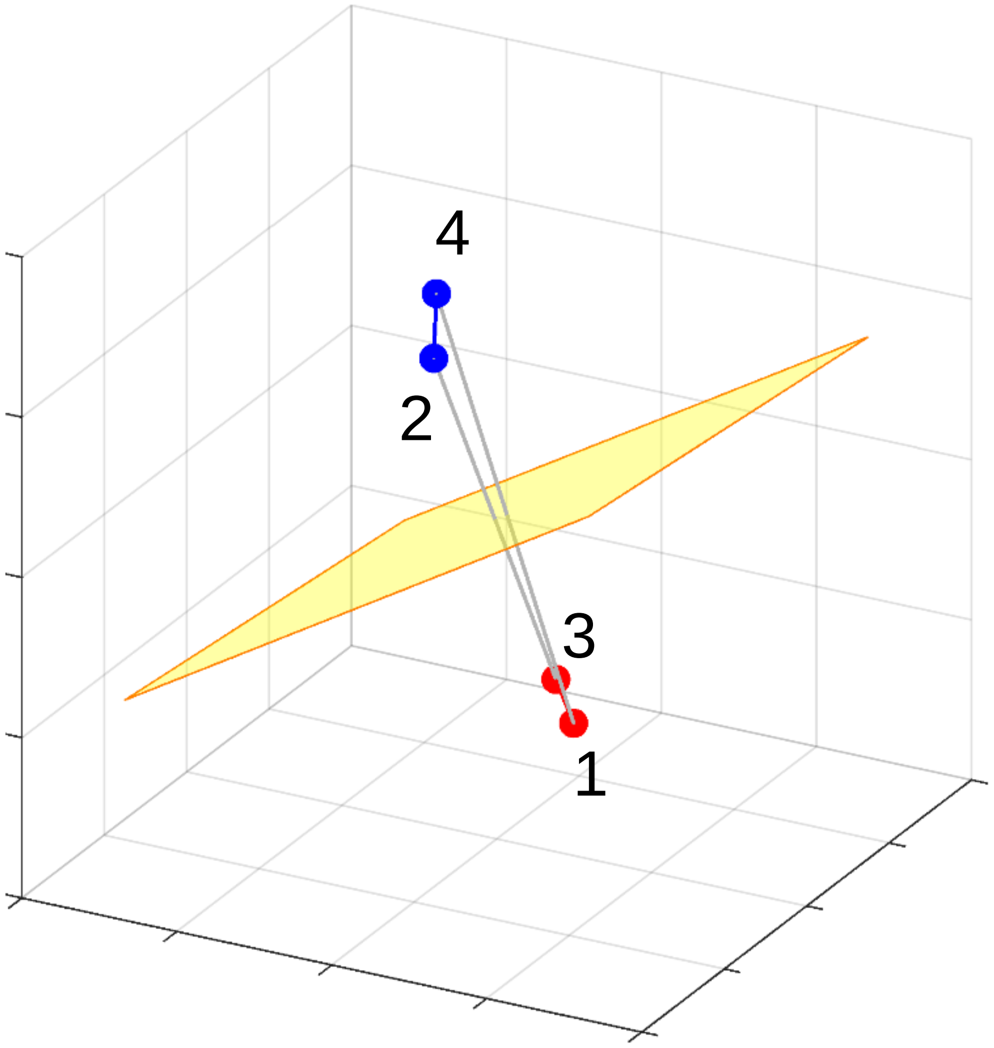
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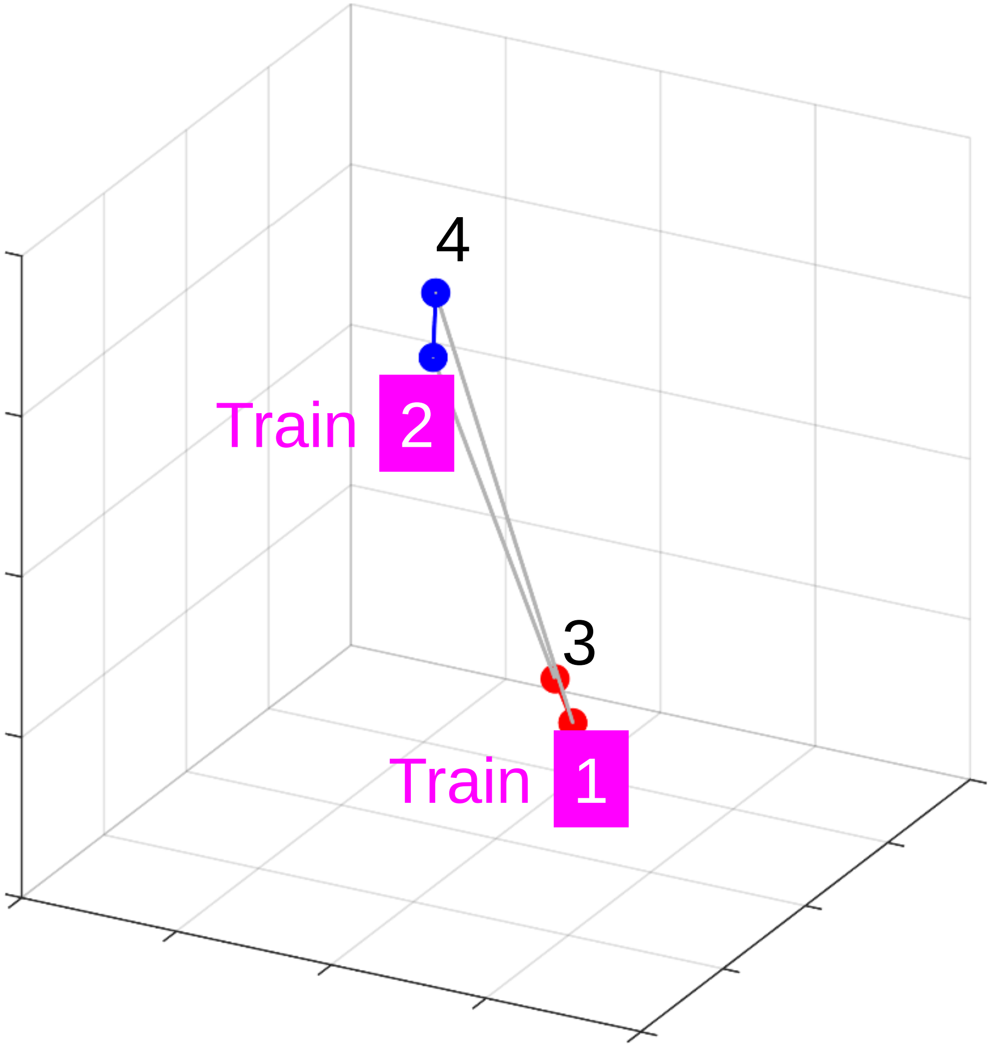
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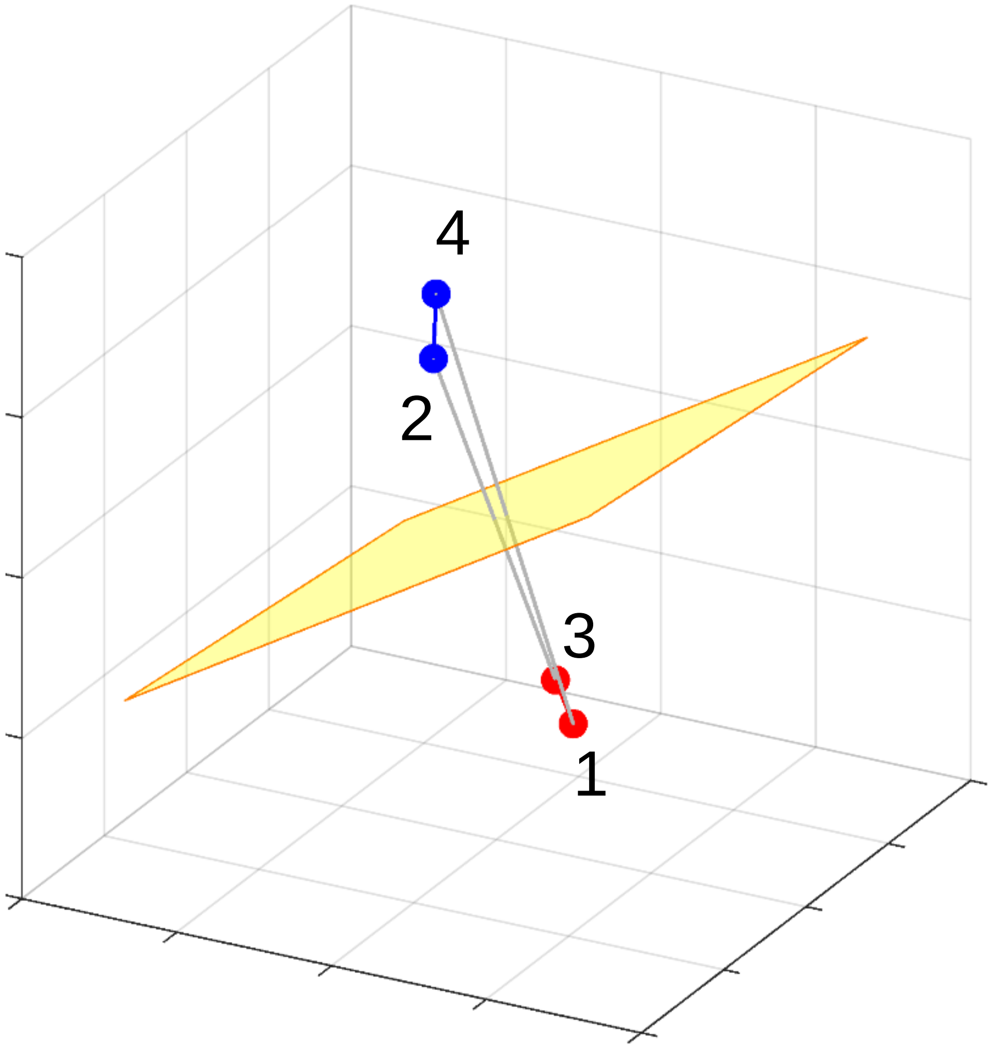
Train 2
3
Train 1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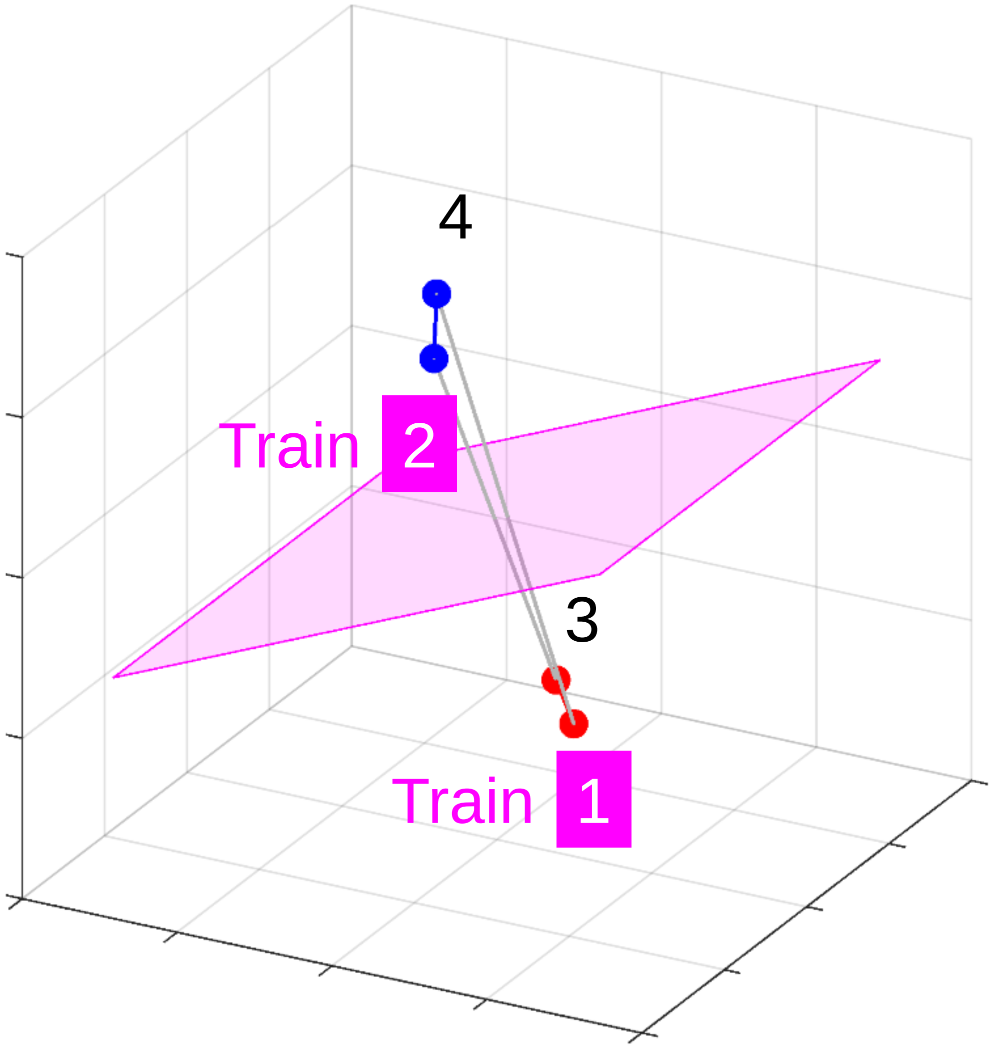
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Train 2
3
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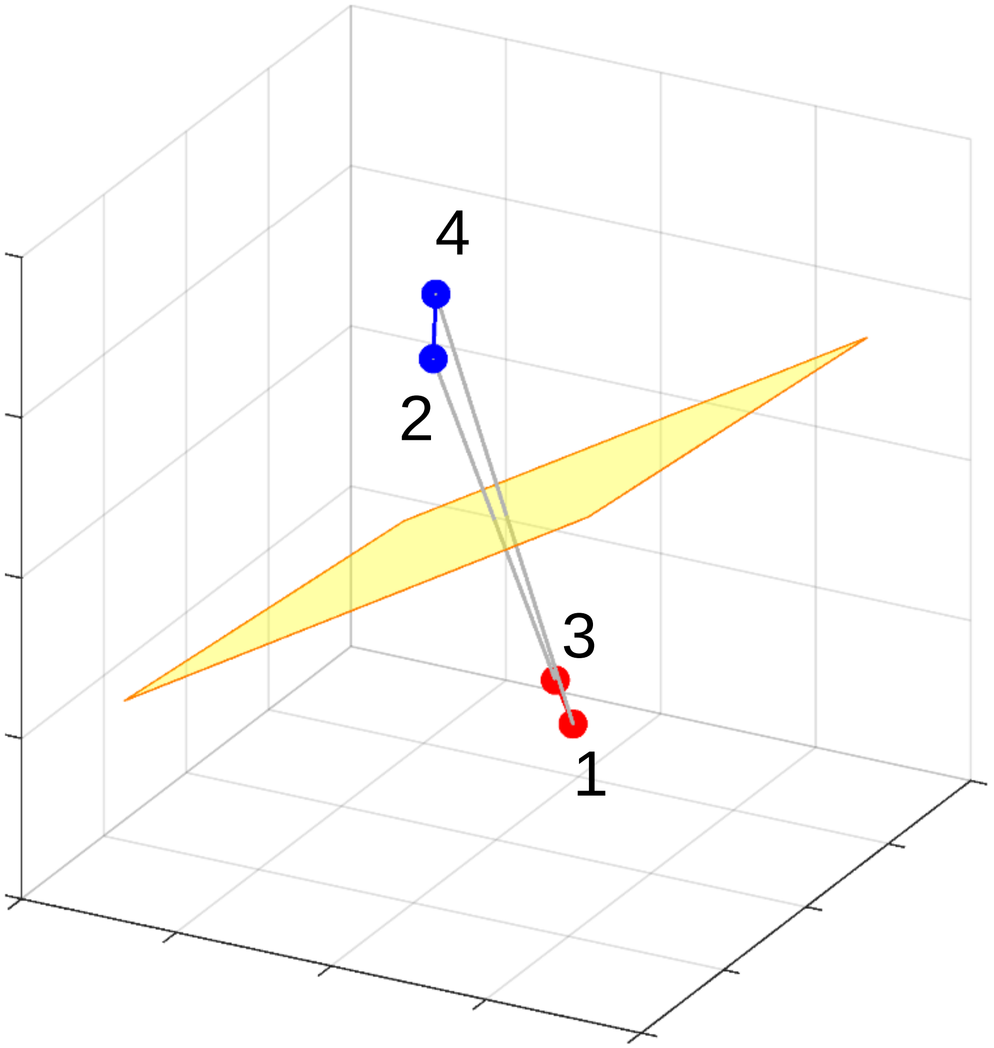
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2
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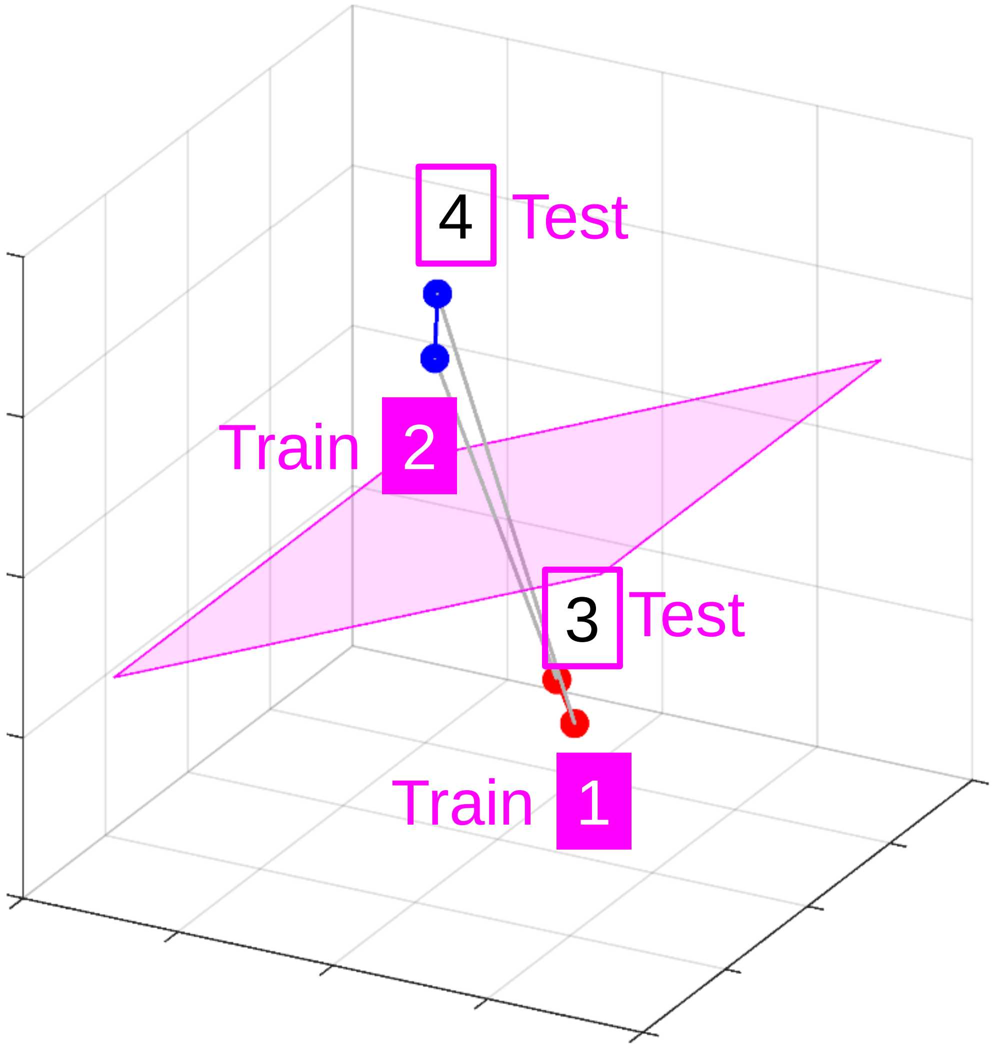
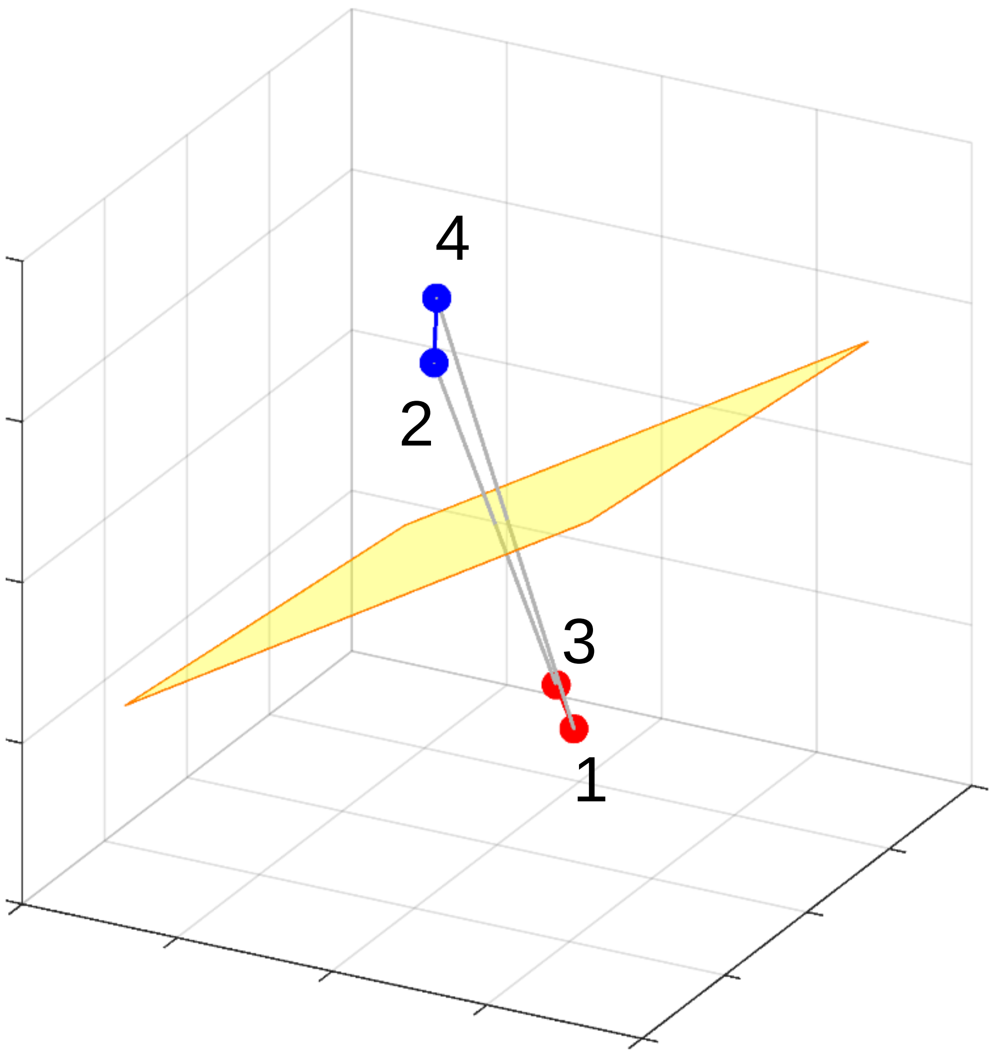
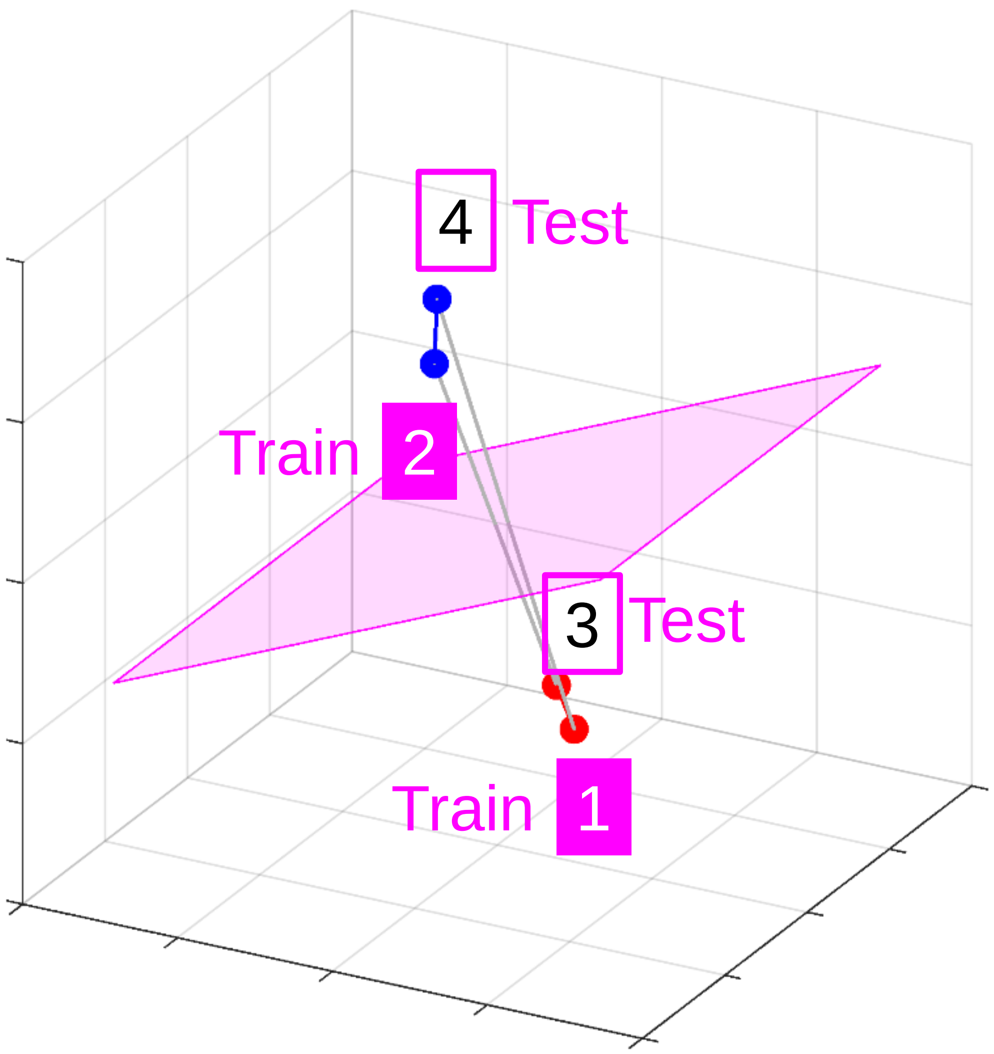
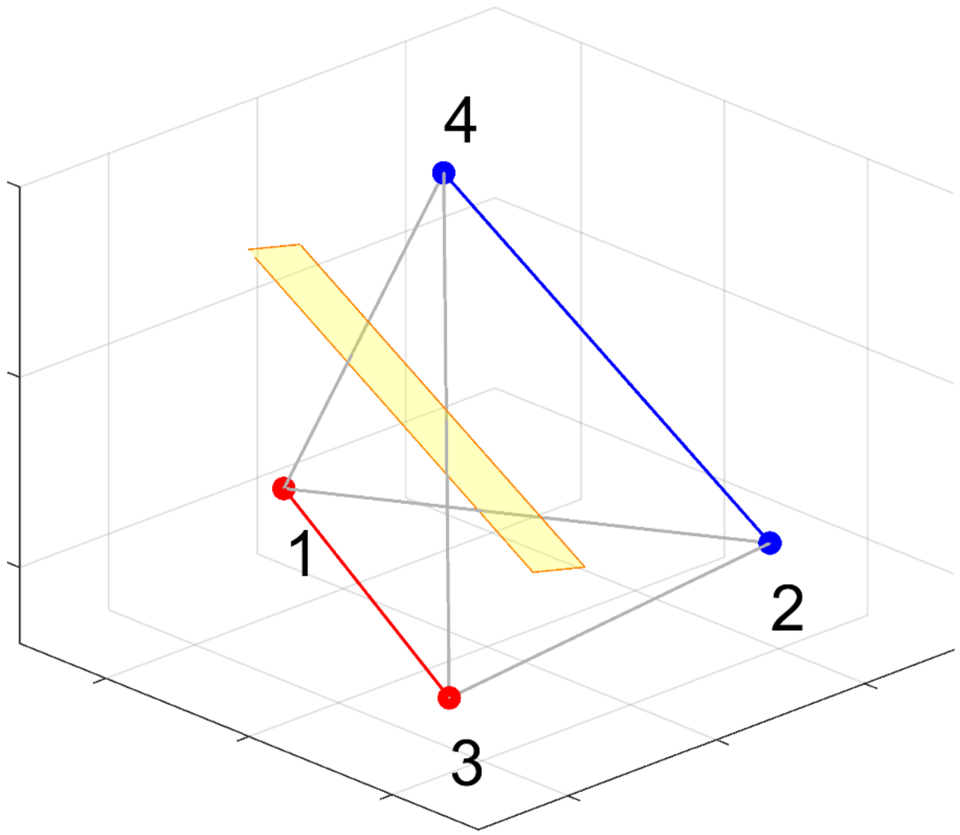

4 Test
Train 2
3 Test
Train 1

CCG: Cross Condition Generalization





Variable is in an **abstract format** when a linear neural decoder trained to report the value of the variable can **generalize** to situations not experienced by the decoder during training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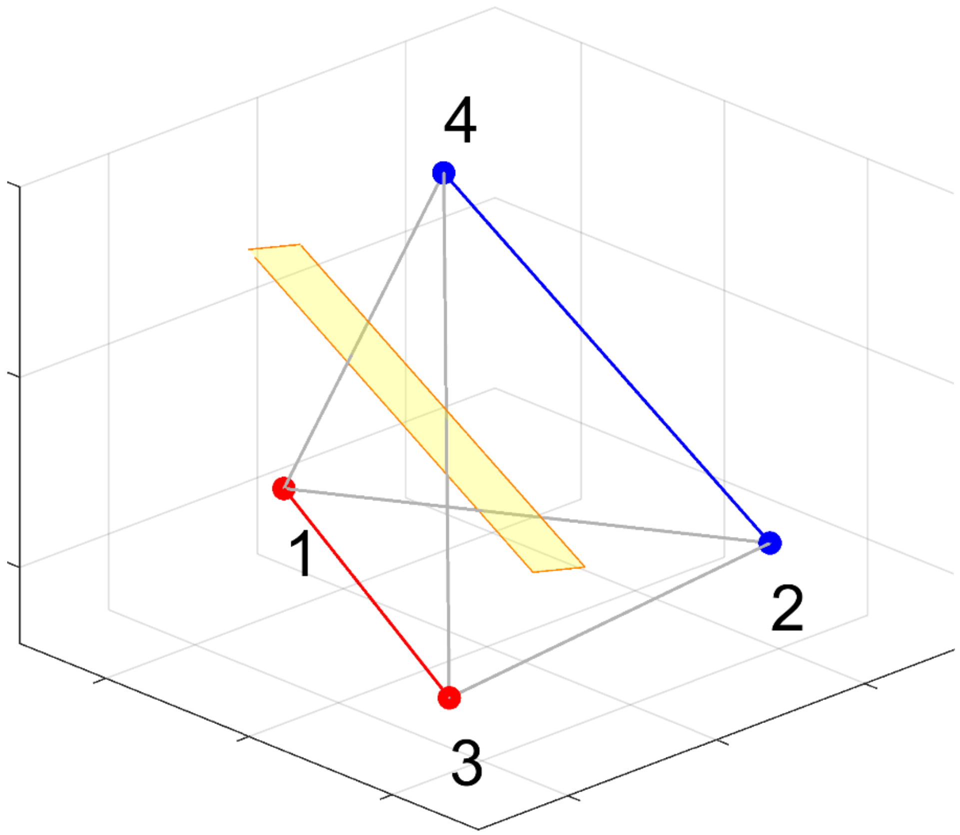
4
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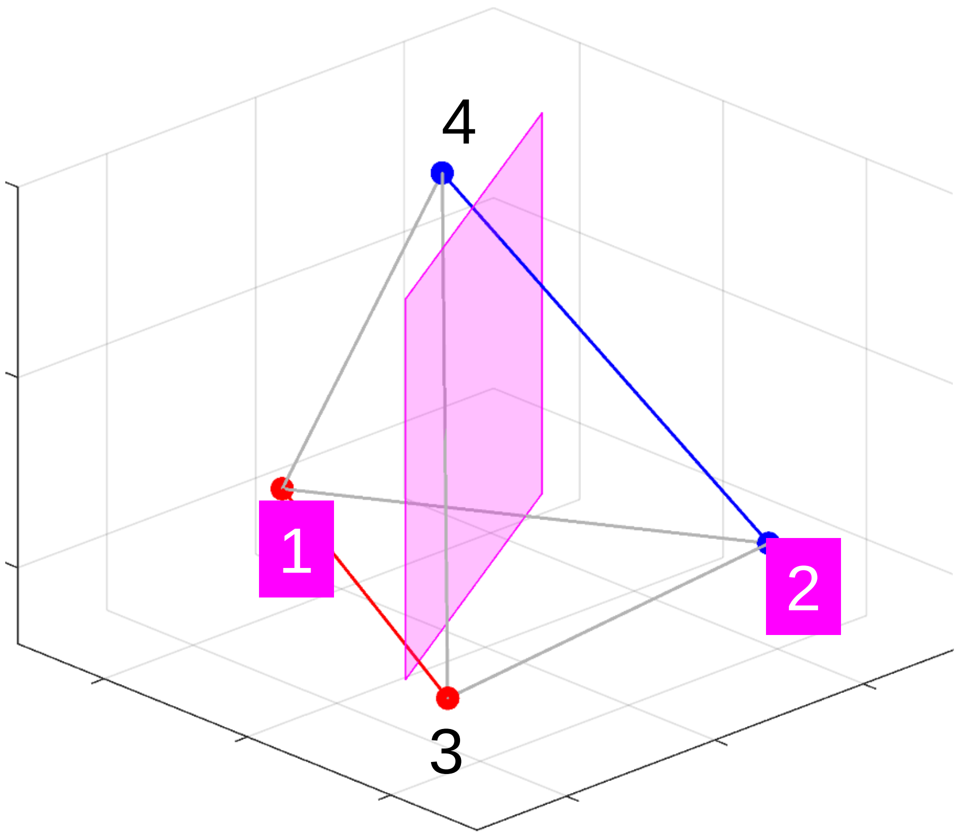
3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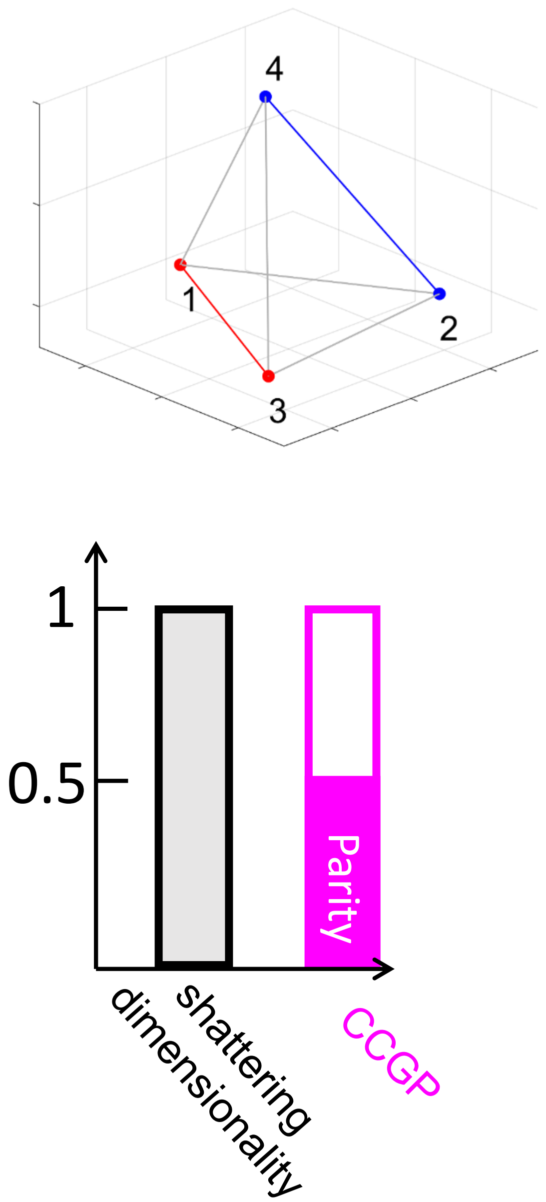
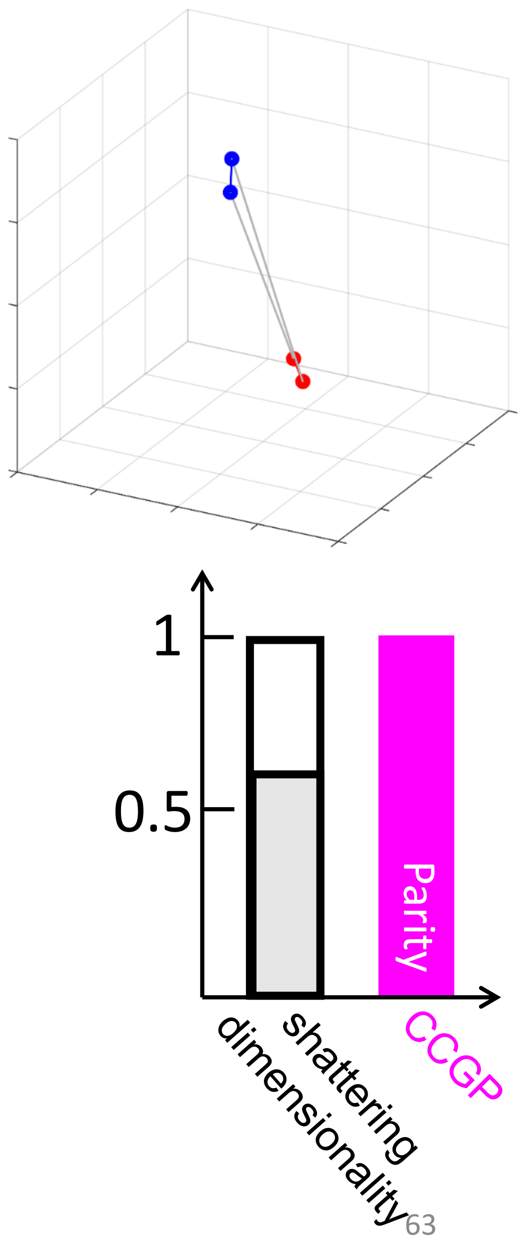
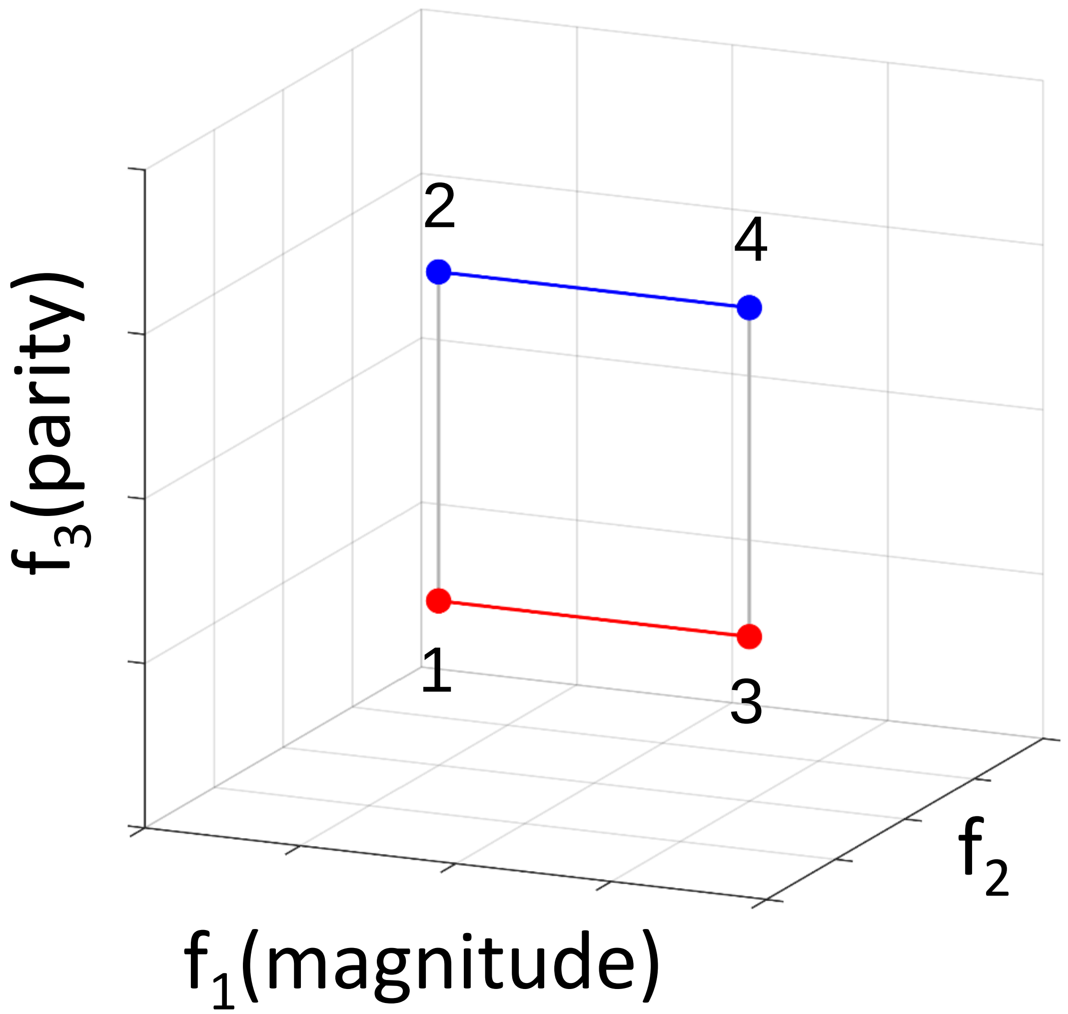
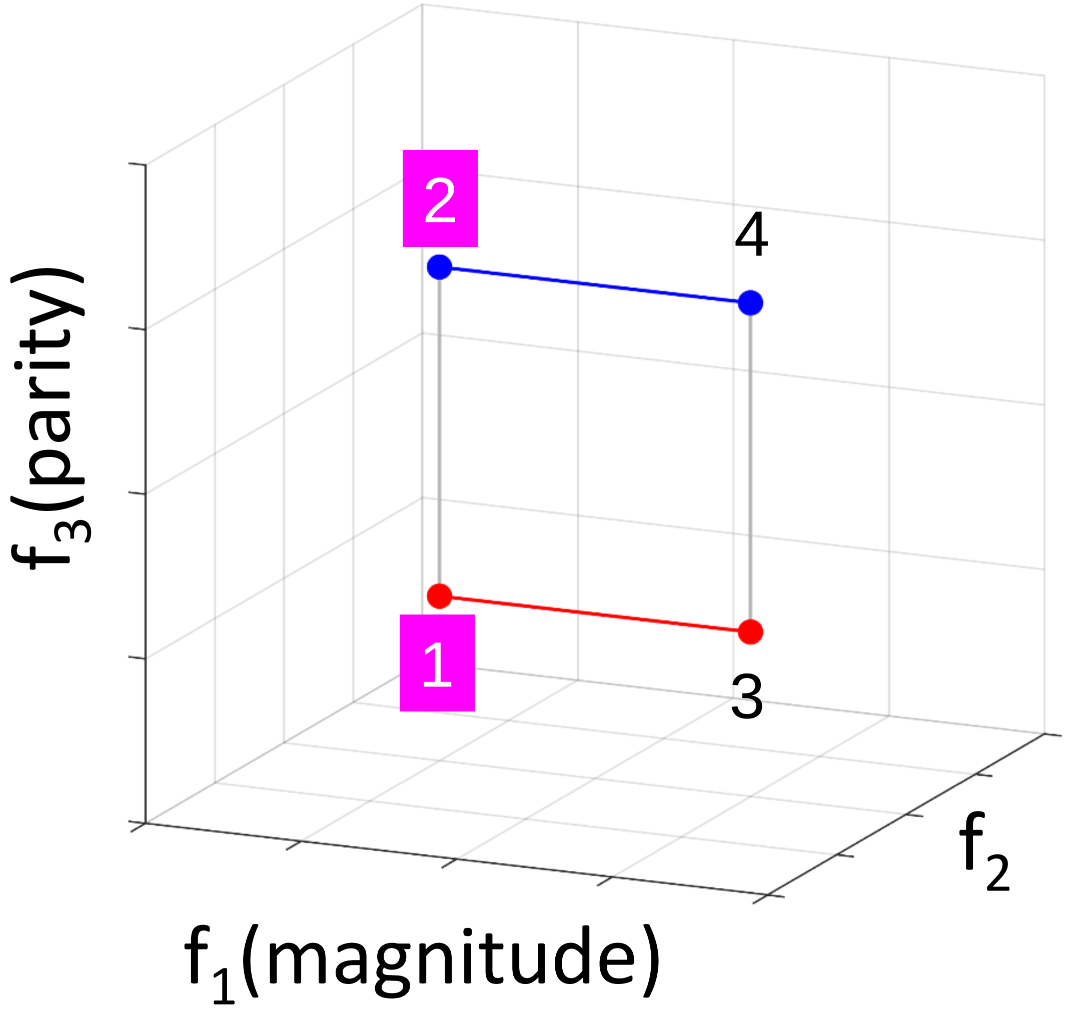
Unstructured: high dimensionality



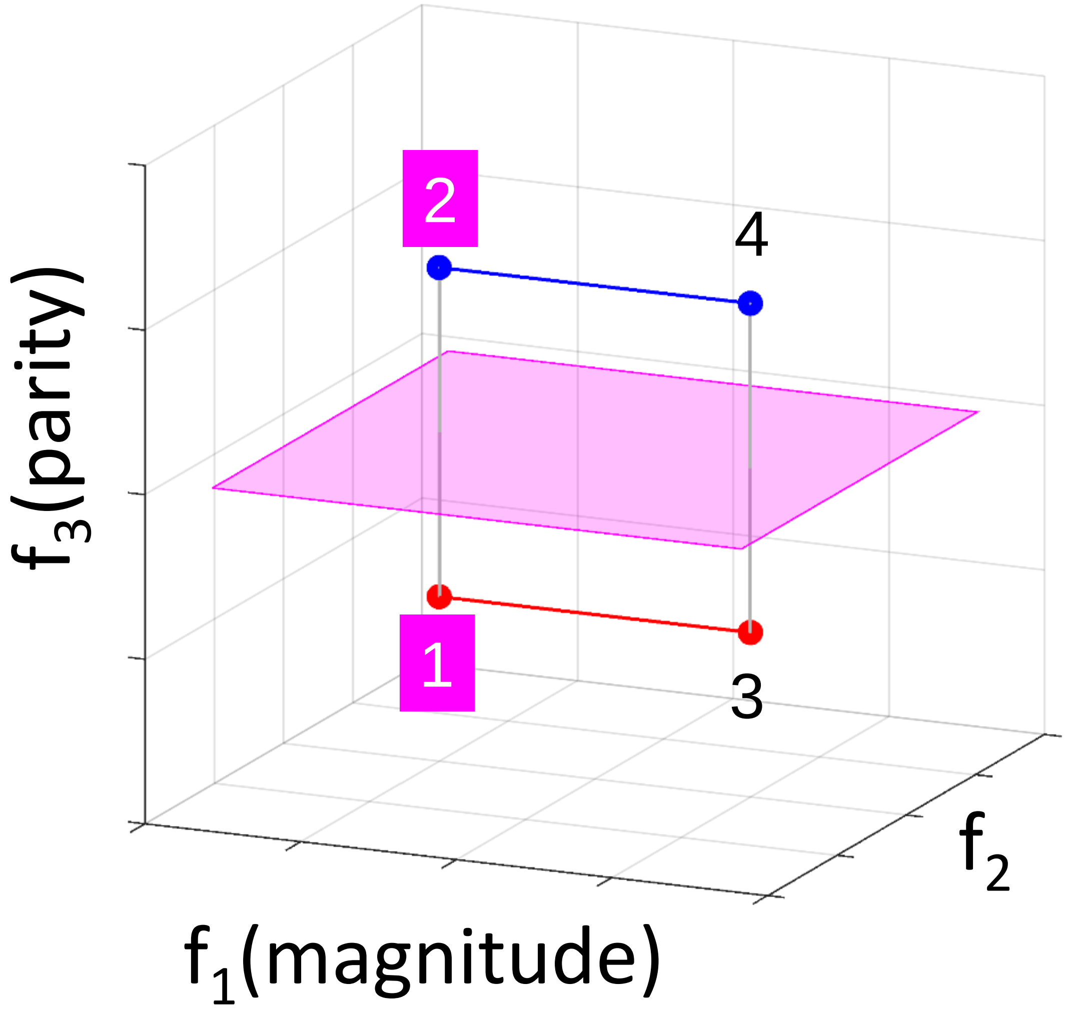
Abstract: low dimensionality

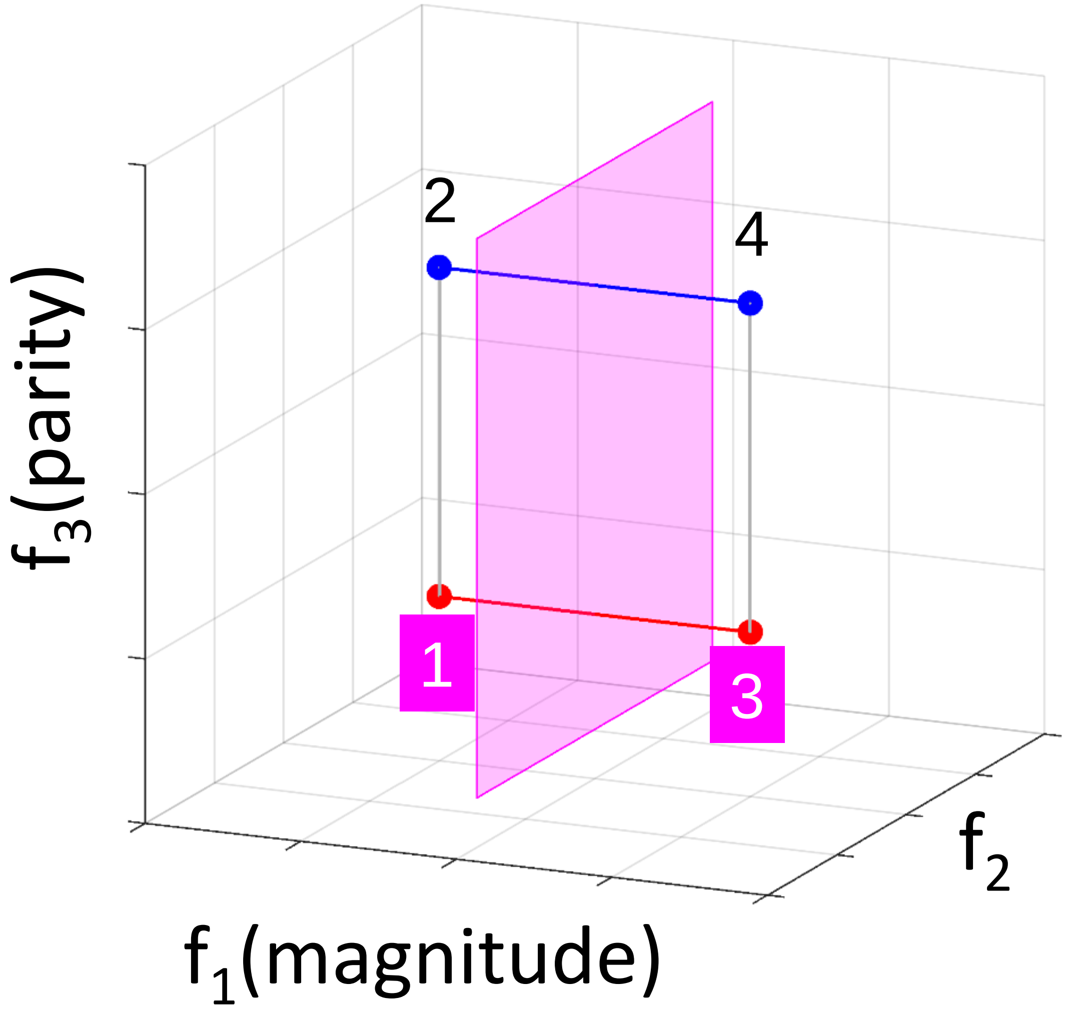


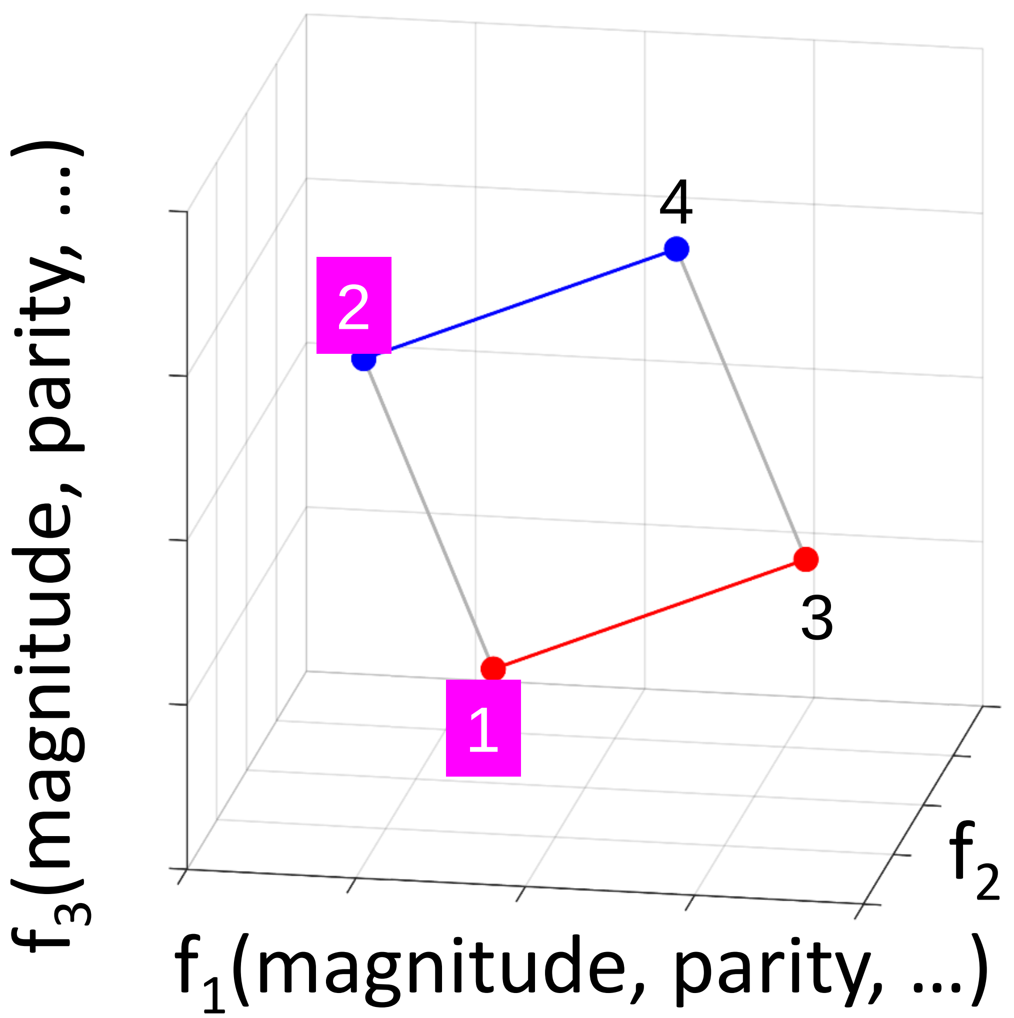

2
4
1
3
f3(parity)
f1(magnitude)
f2



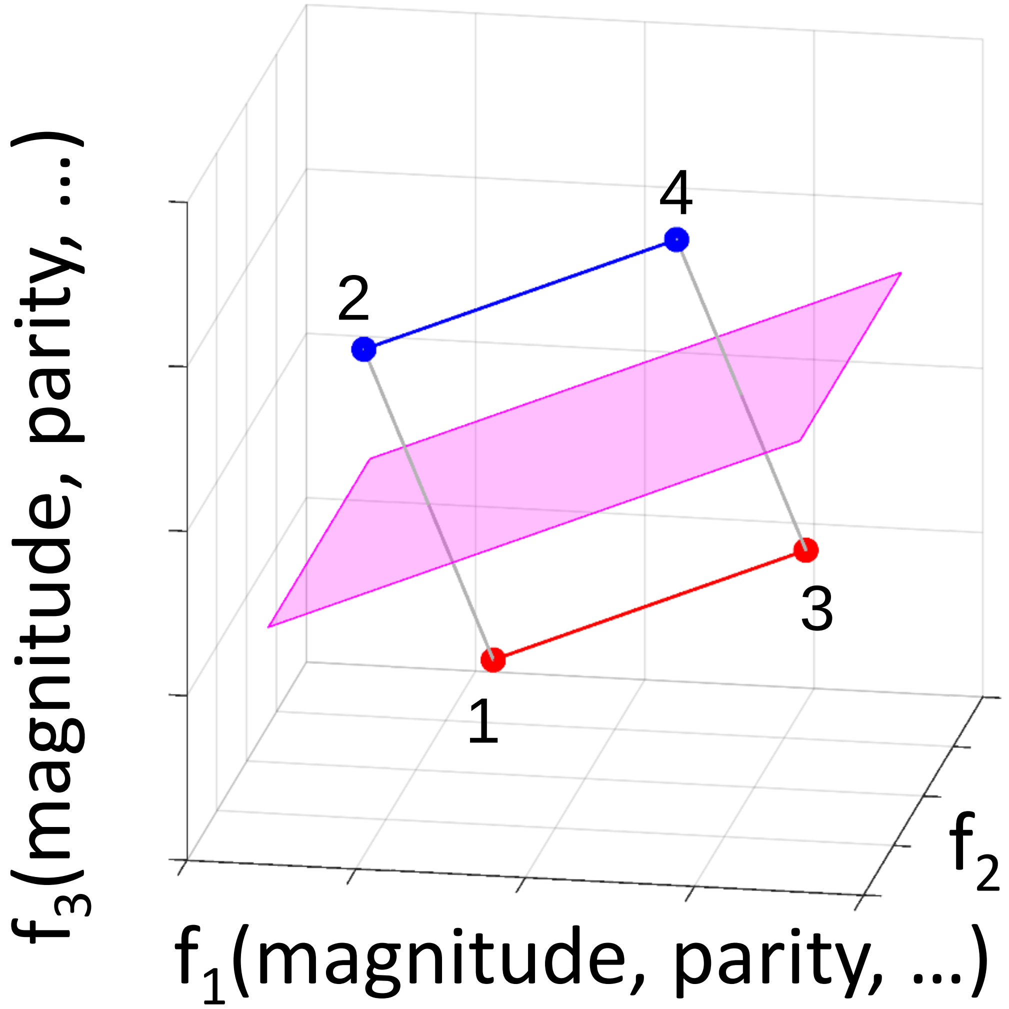
2
4
1
3
f_3(parity)
f_2
f_1(magnitude)


2
4
1
3
f_3(parity)
f_2
f_1(magnitude)

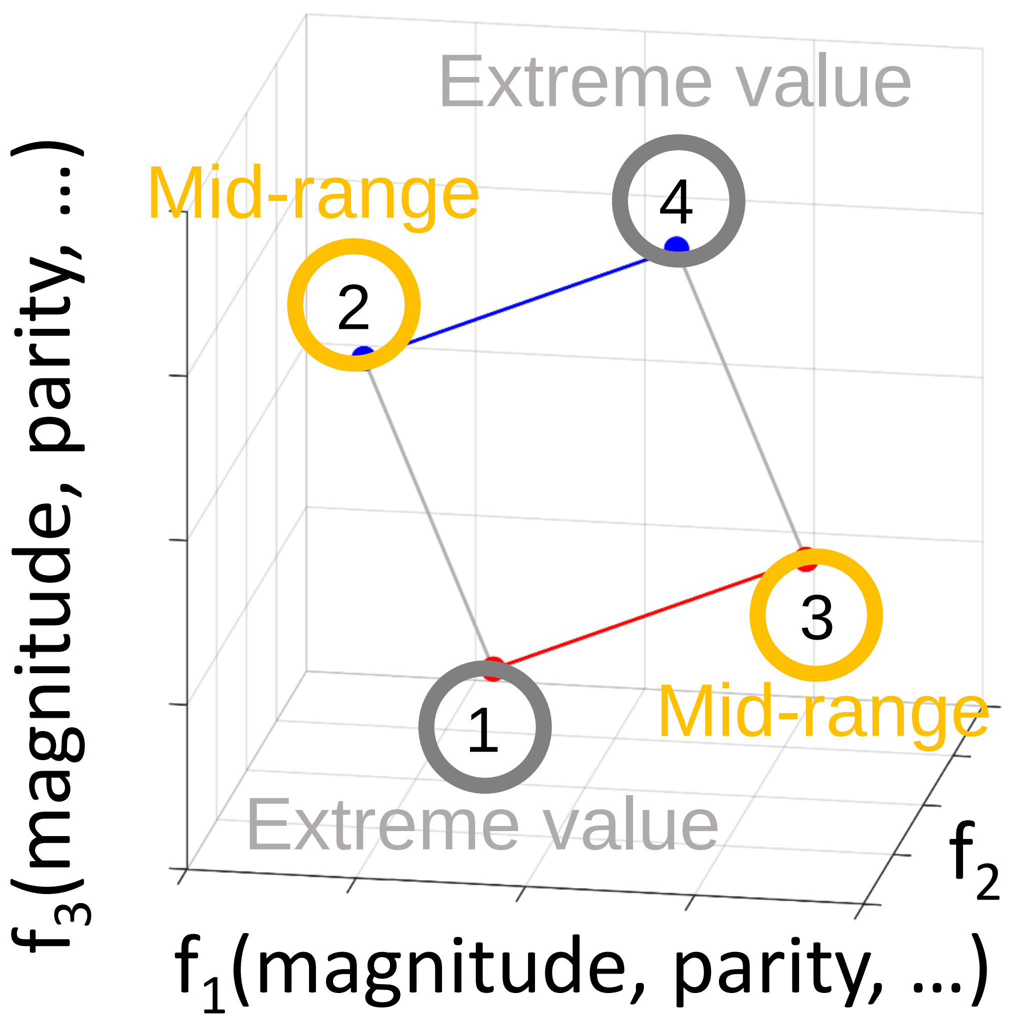

2
4
1
3
f3(parity)
f2
f1(magnitude)



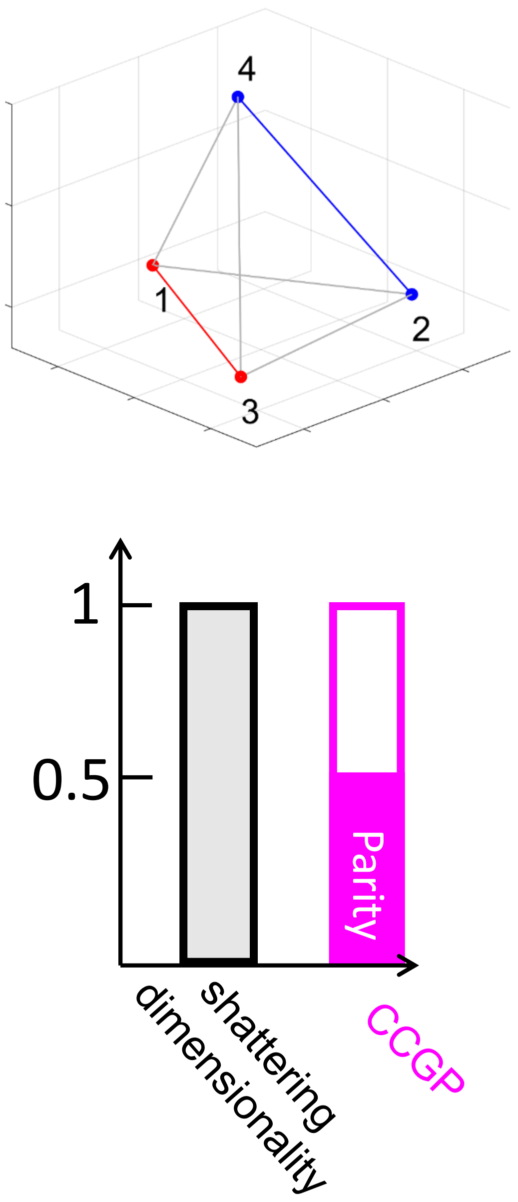
4
2
3
1
f_2
f_3(magnitude, parity, …)
f_1(magnitude, parity, …)



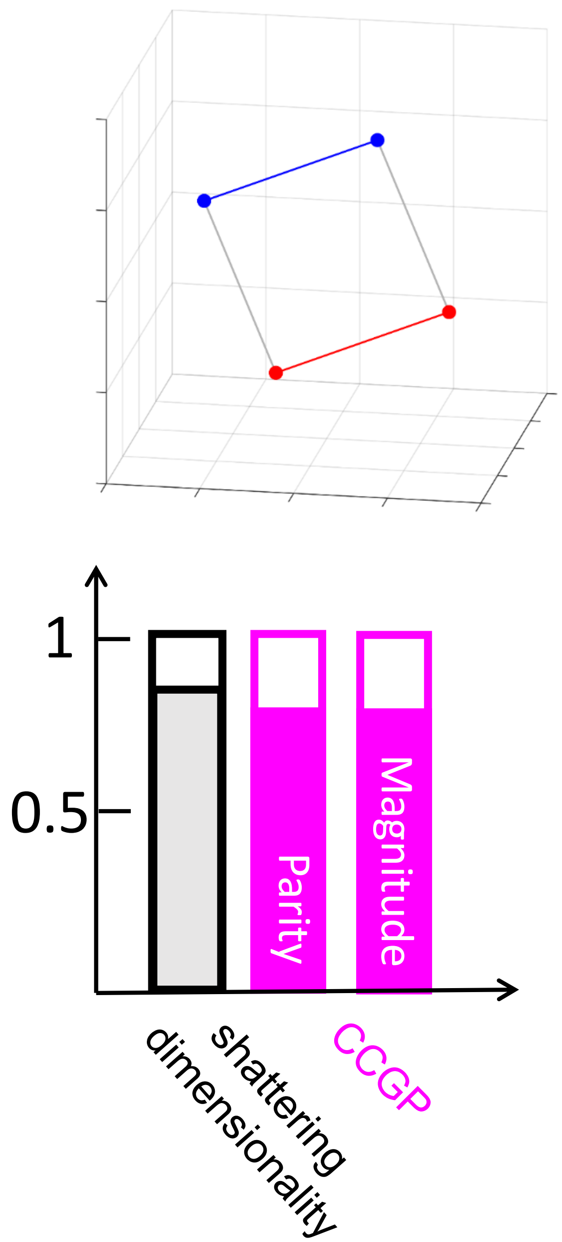
4
2
3
1
f_2
f_3(magnitude, parity, …)
f_1(magnitude, parity, …)

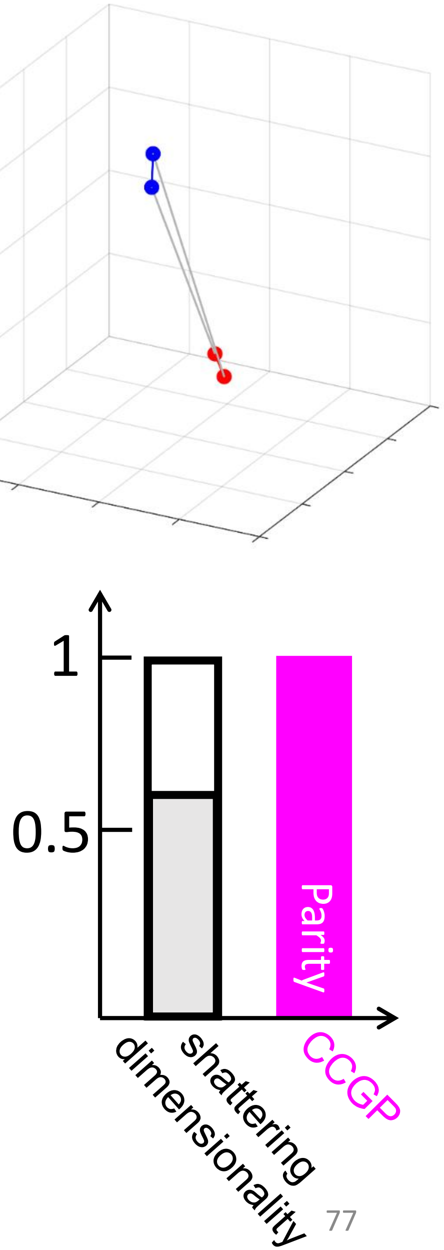

Extreme value
Mid-range
4
2
3
1
Mid-range
Extreme value
f_3(magnitude, parity, …)
f_2
f_1(magnitude, parity, …)

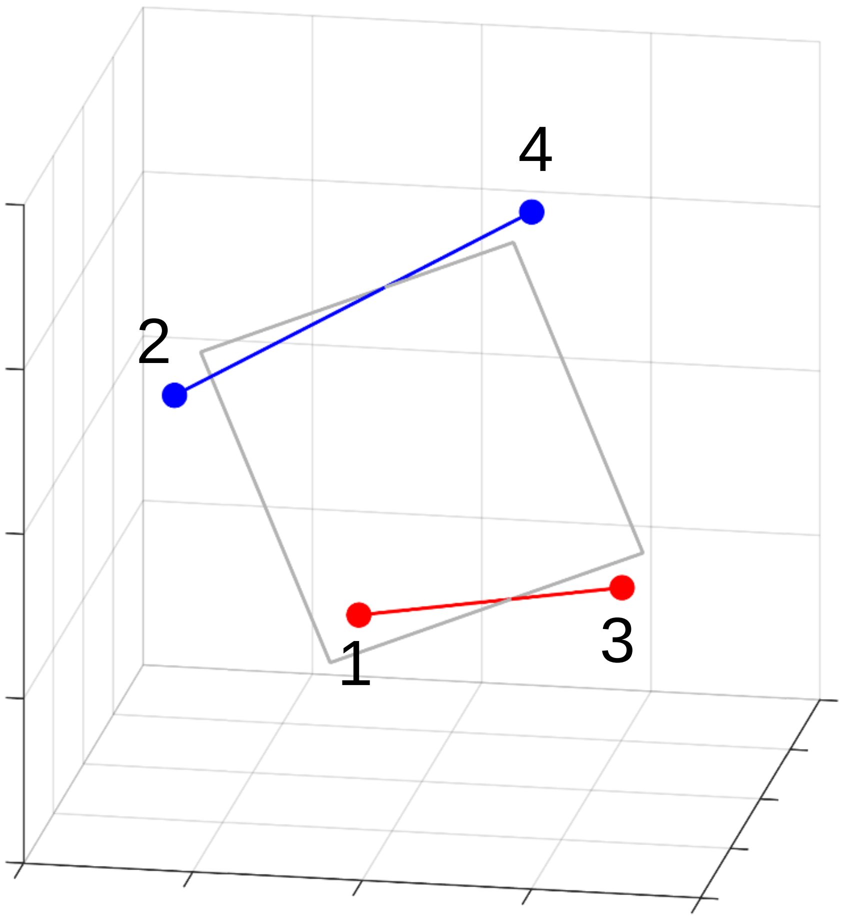
Unstructured

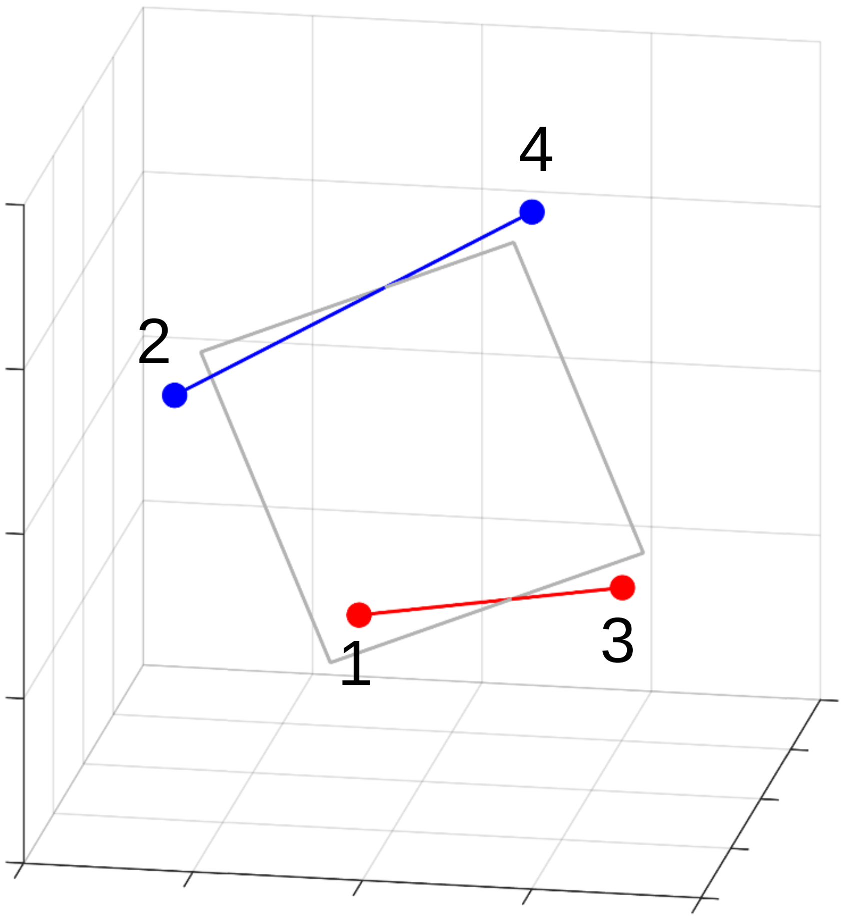


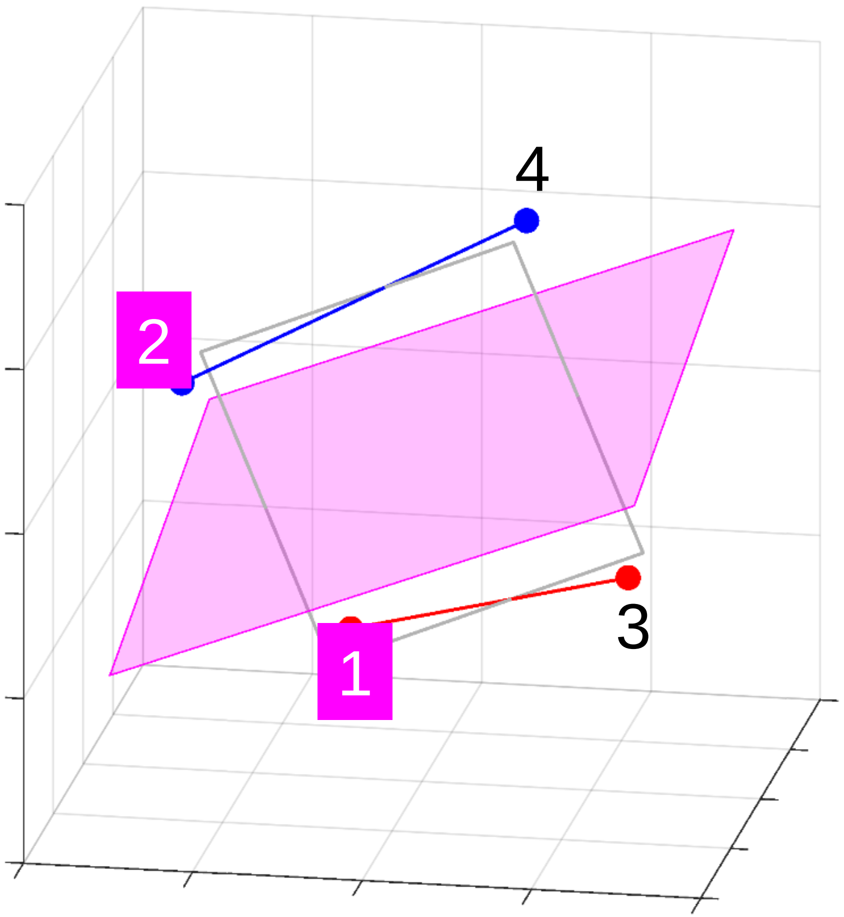
Abstract

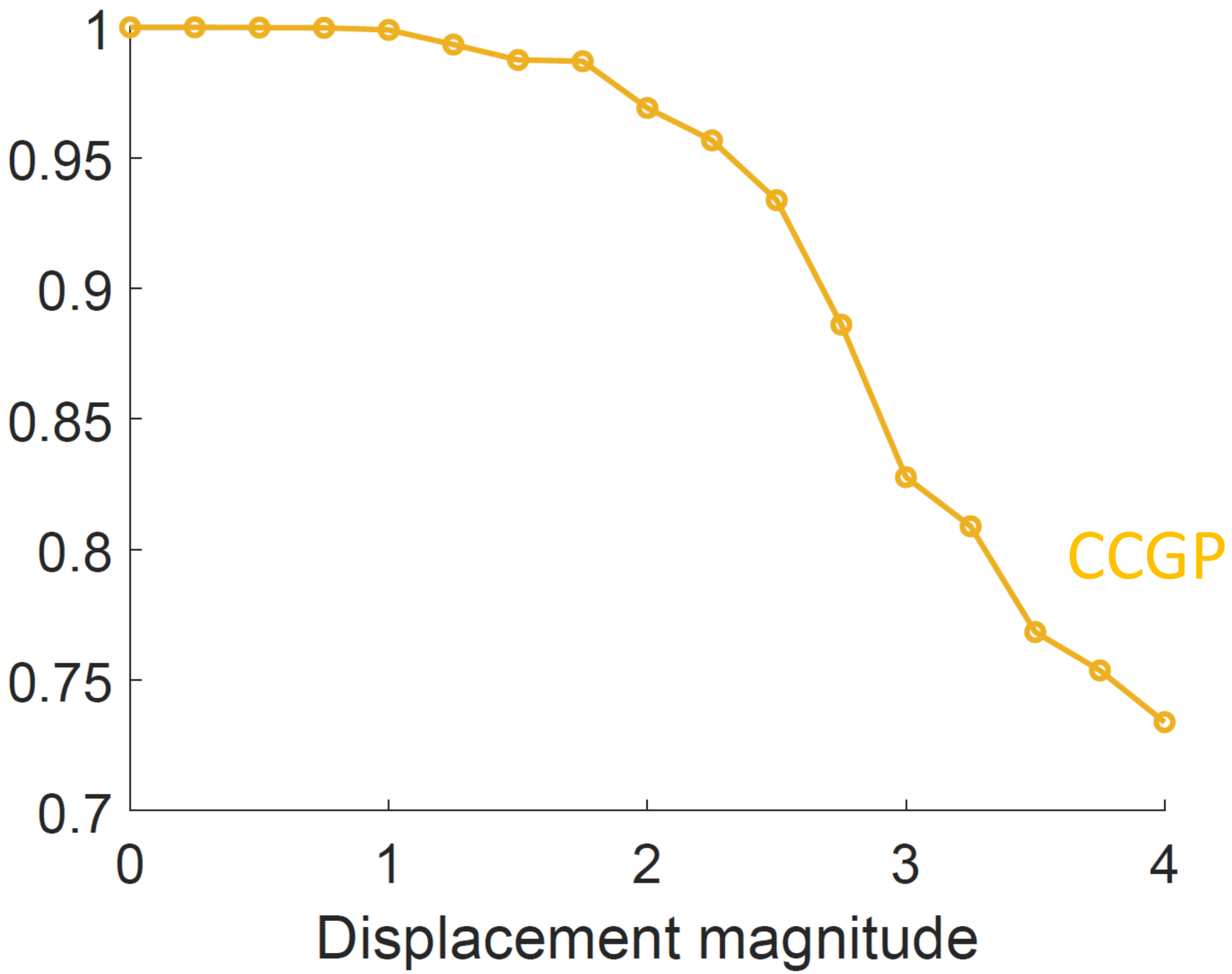




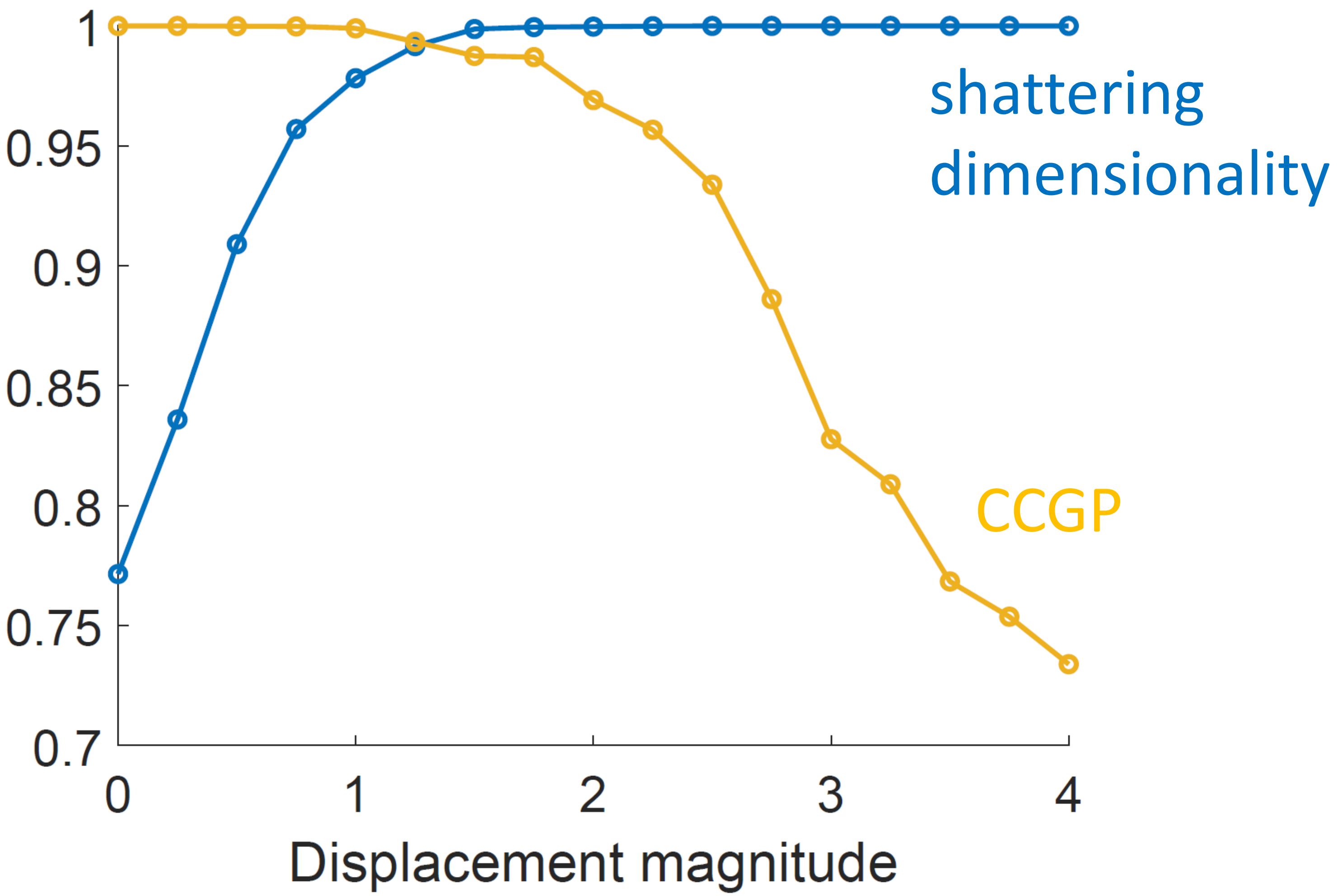

4
2
1
3


4
2
1
3

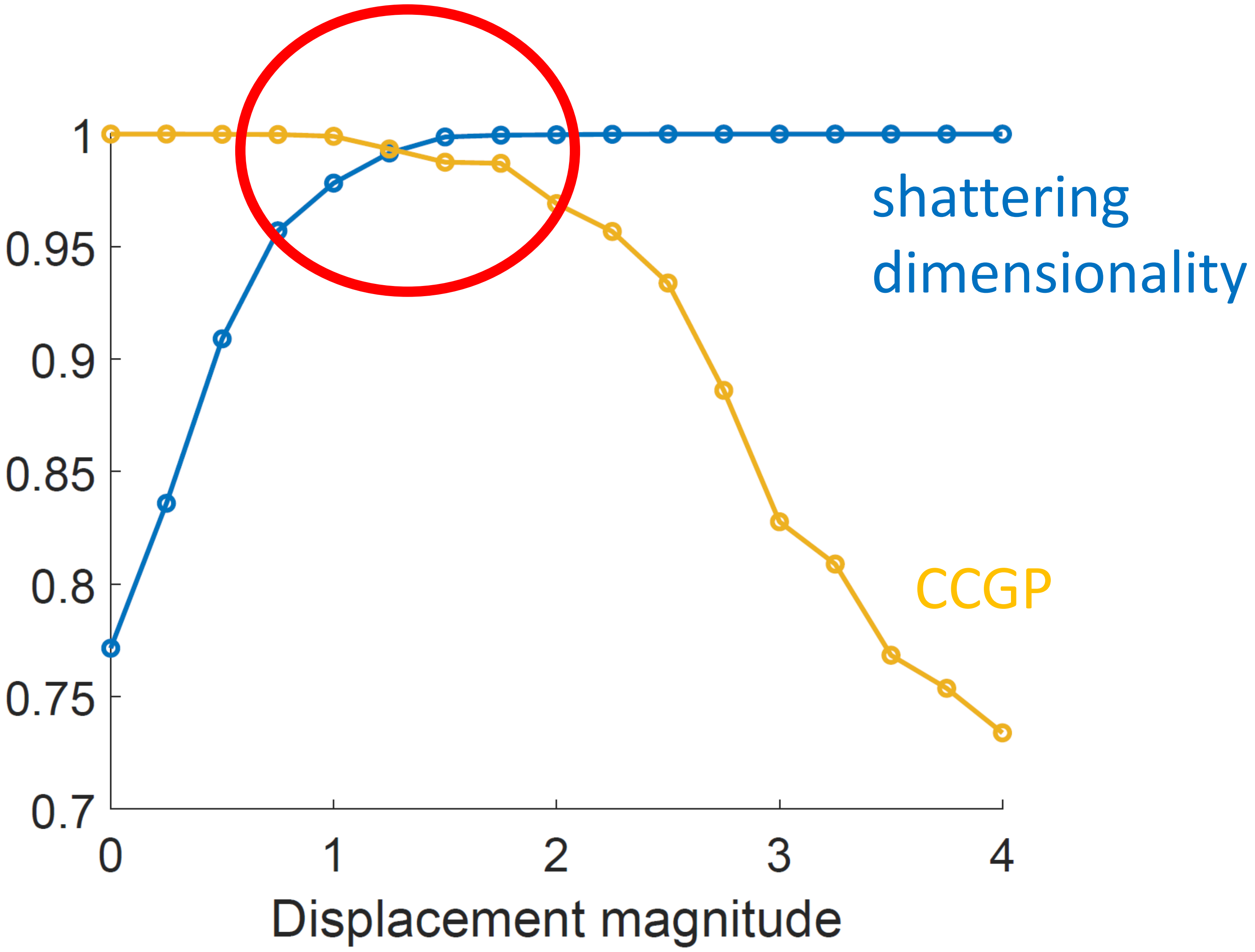

4
2
1
3



Bernardi et al., 2020



Bernardi et al., 2020

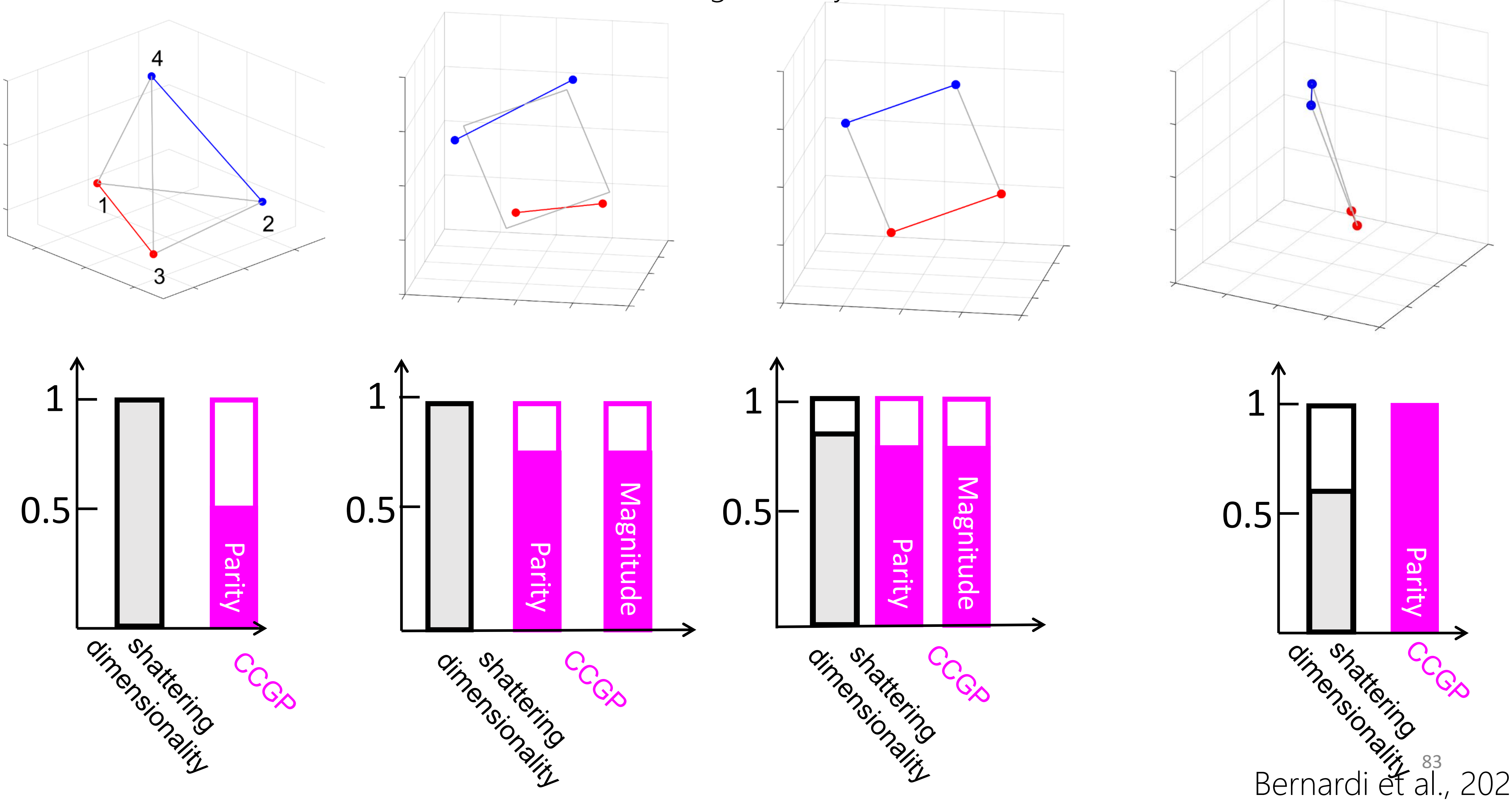


Bernardi et al., 2020

Unstructured

Abstract

(i.e. when CCGP is significantly different from the one in the unstructured case)

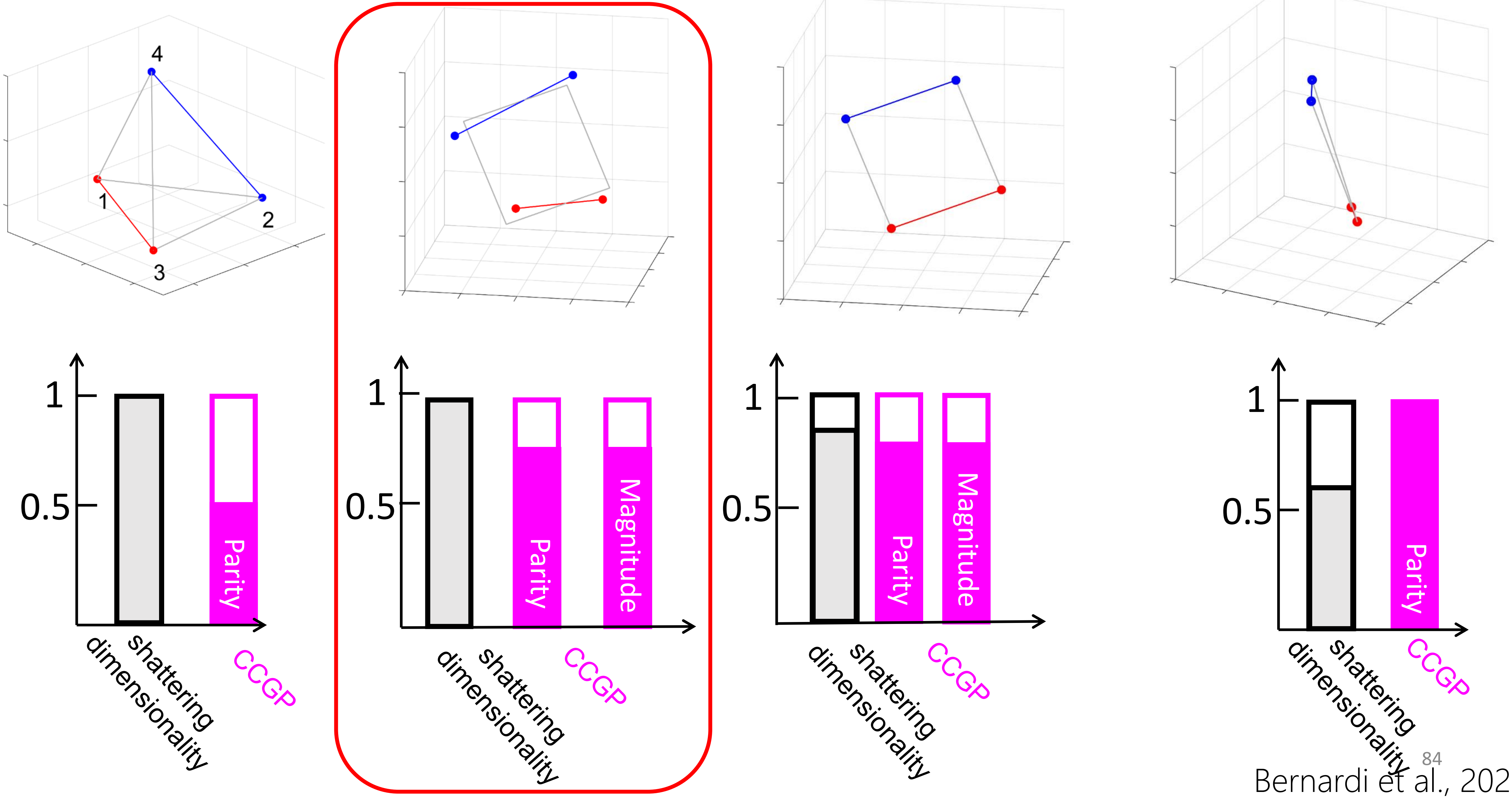


Bernardi et al., 2020

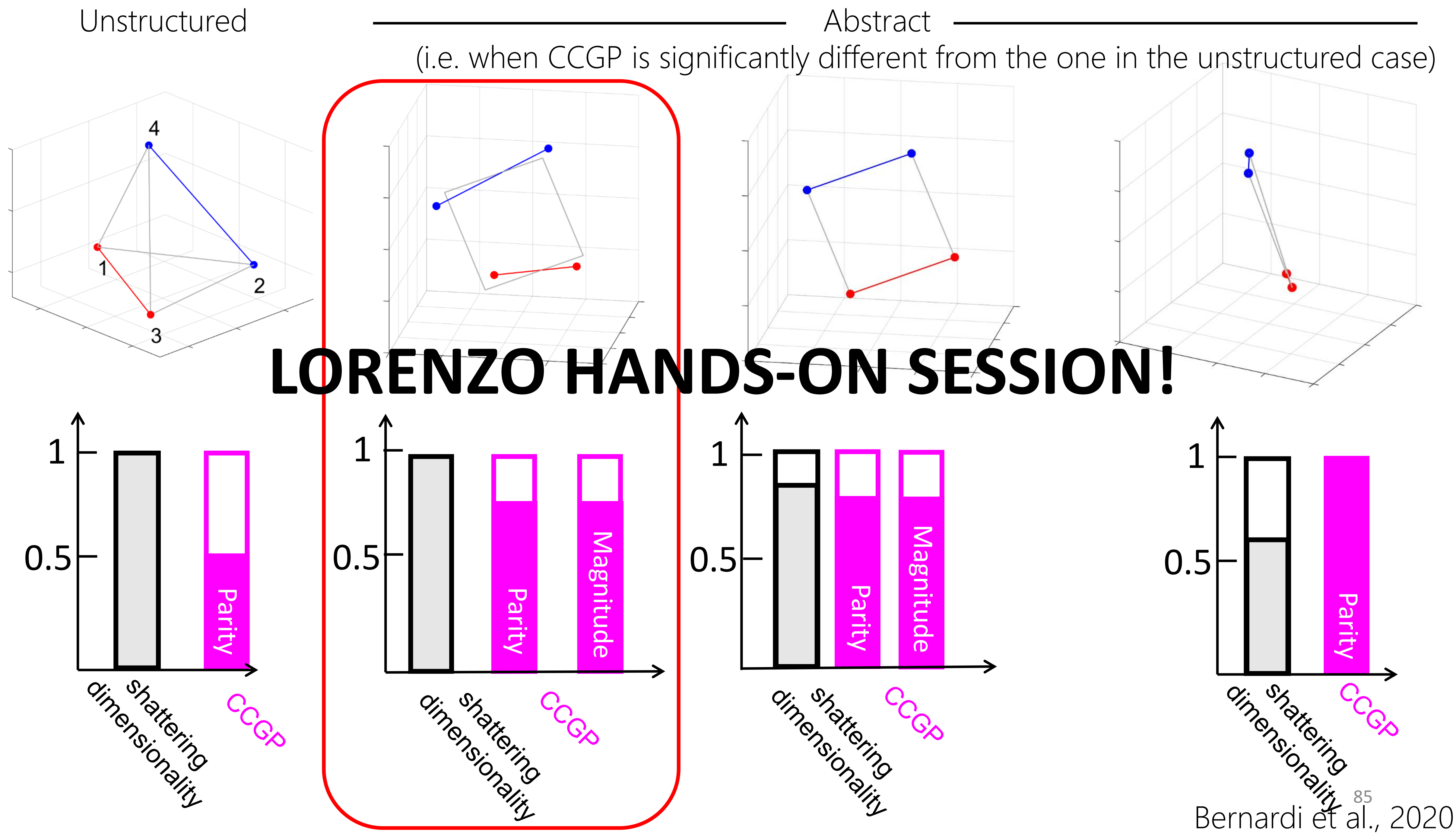
Unstructured

Abstract

(i.e. when CCGP is significantly different from the one in the unstructured case)

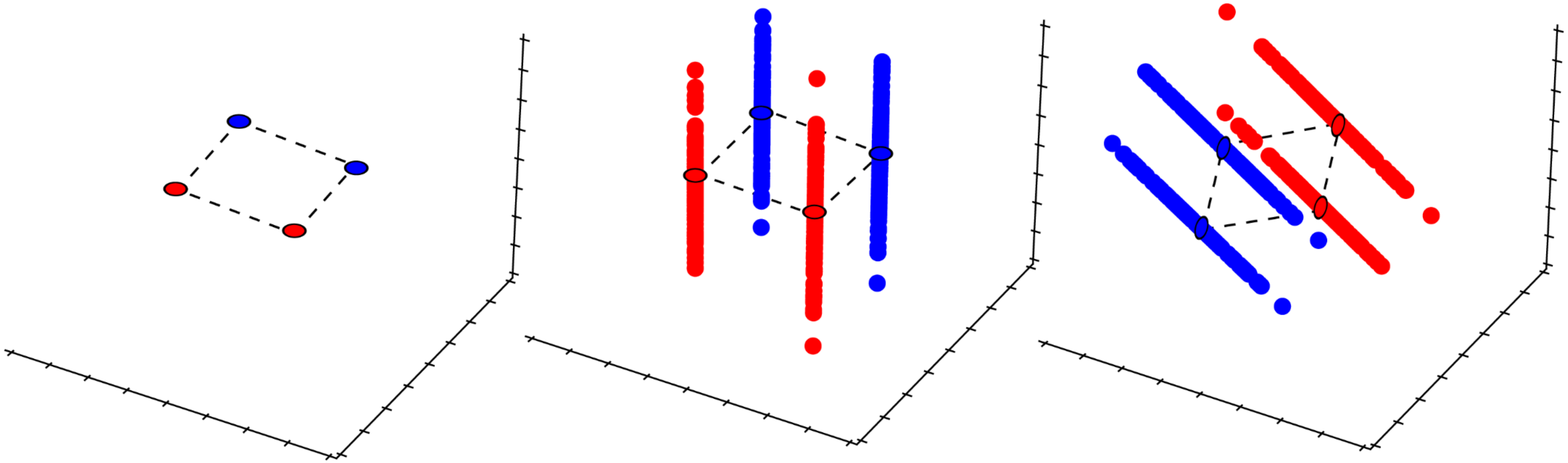


Bernardi et al., 2020

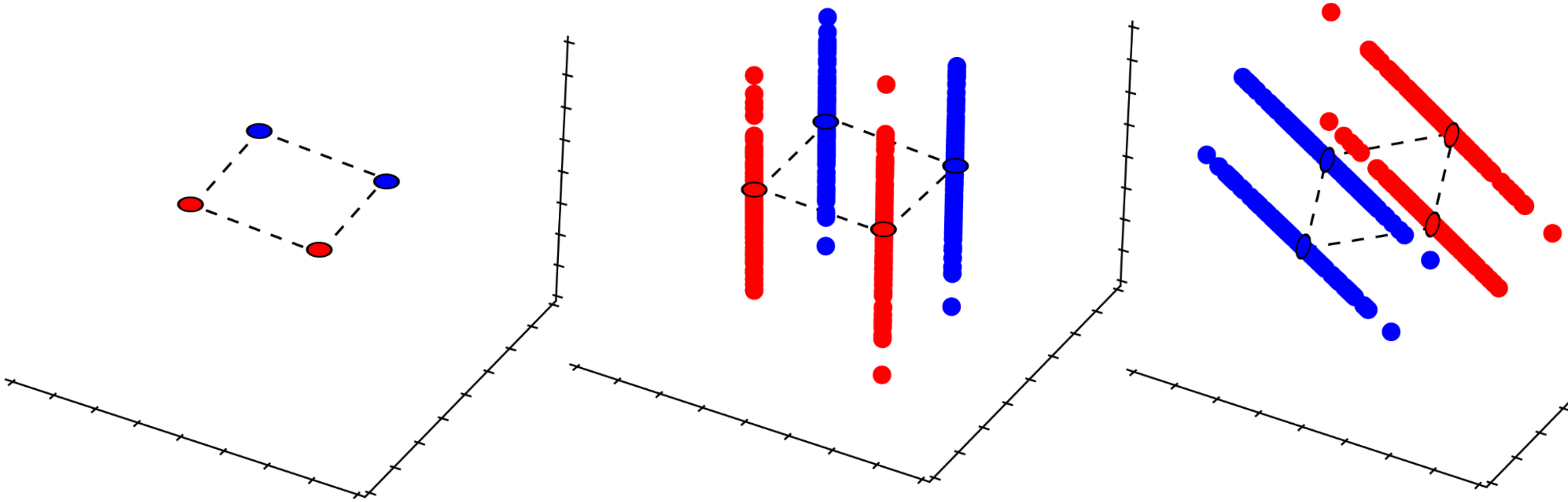


LORENZO HANDS-ON SESSION!

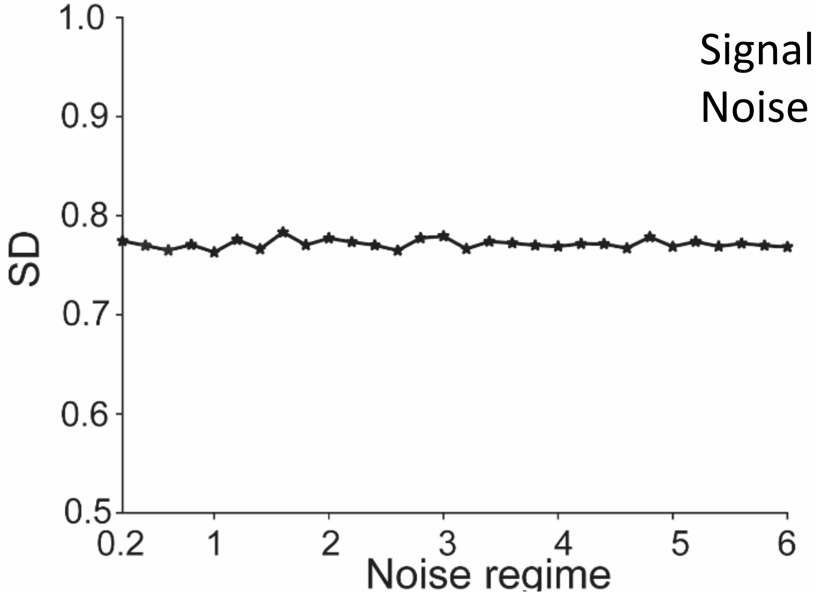
Bernardi et al., 2020



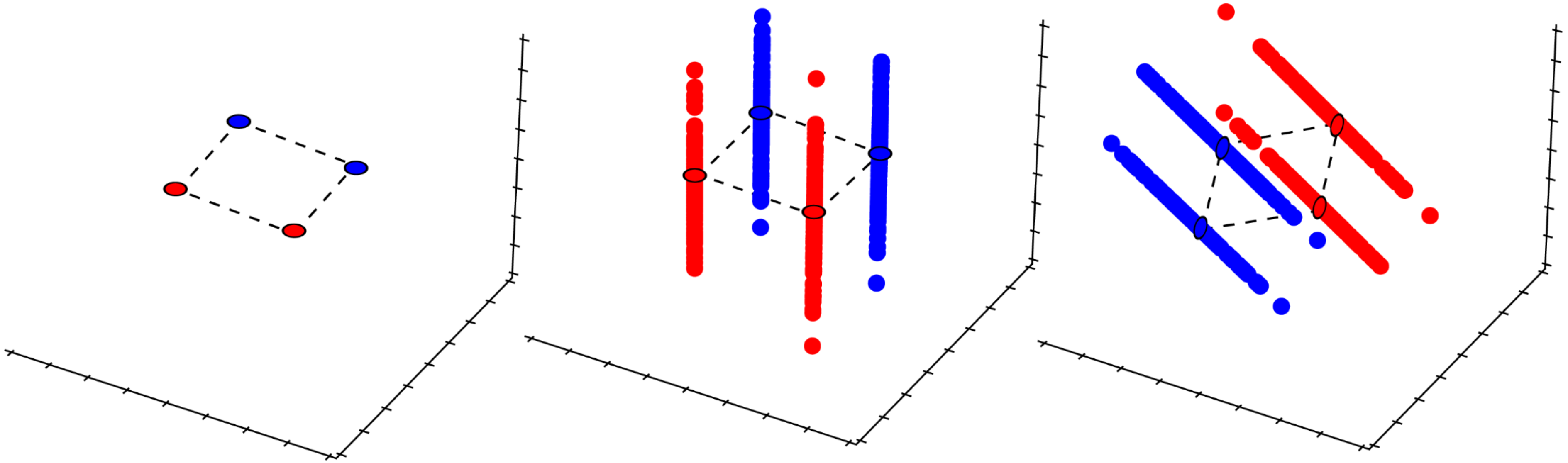
Signal = 2D
Noise = 3D



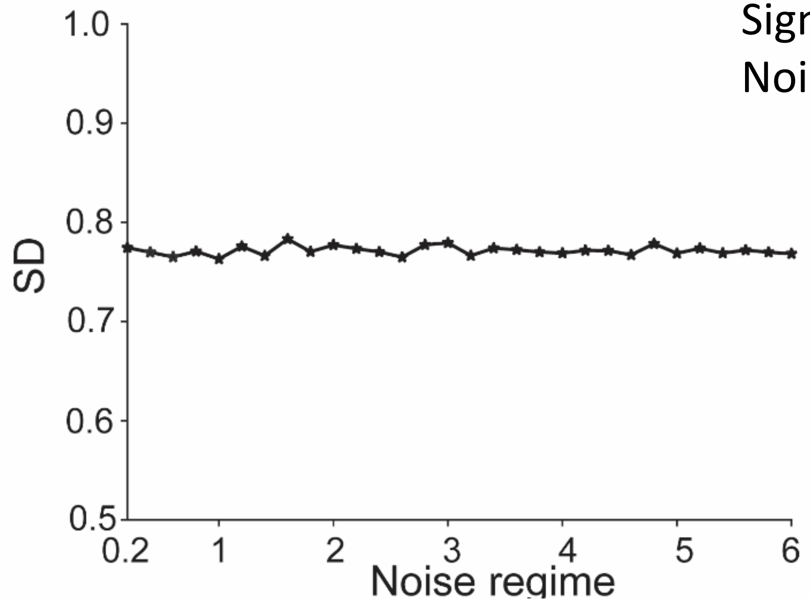
Signal = 2D
Noise = 3D



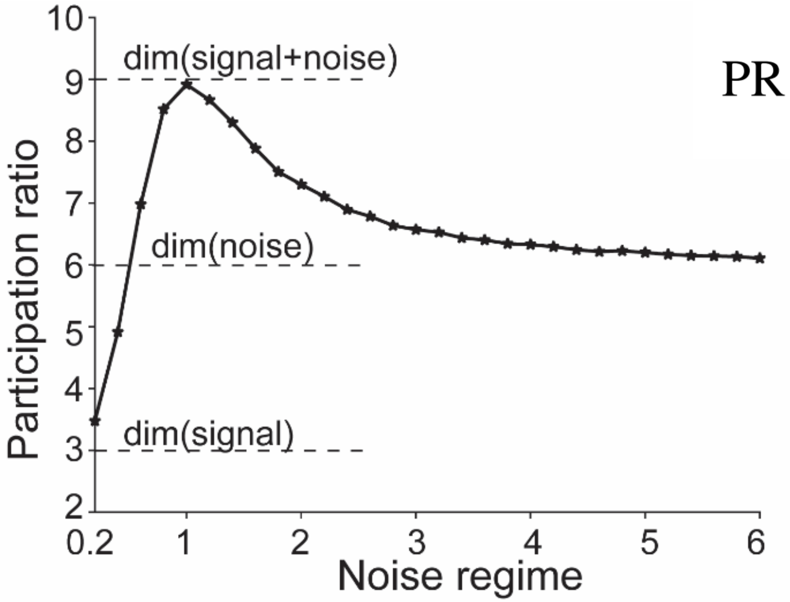
Signal = 3D
Noise = 6D



Signal = 2D
Noise = 3D

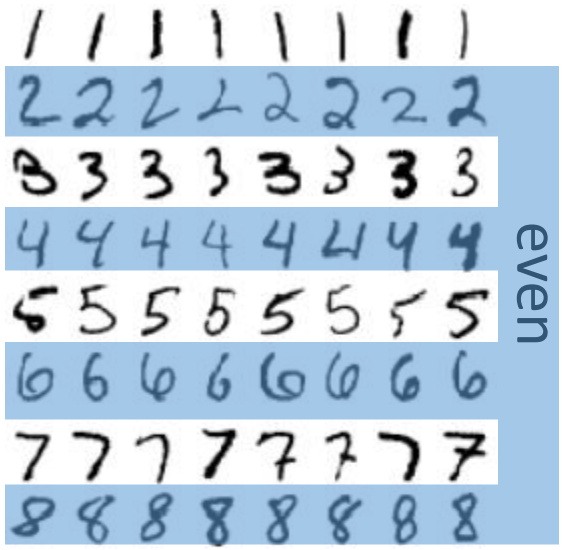


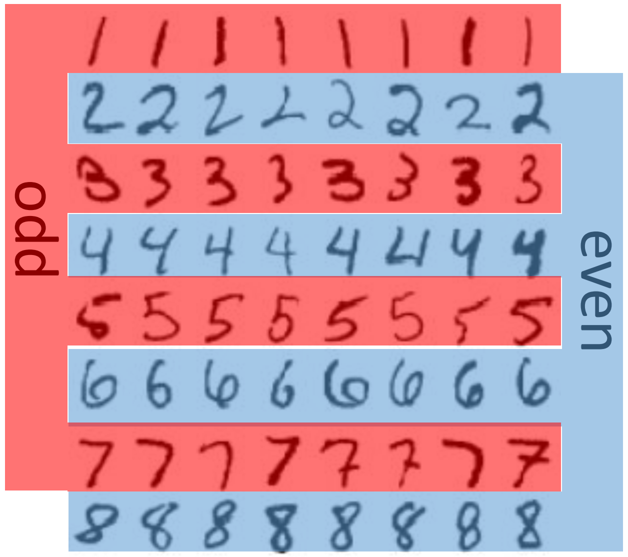
Signal = 3D
Noise = 6D

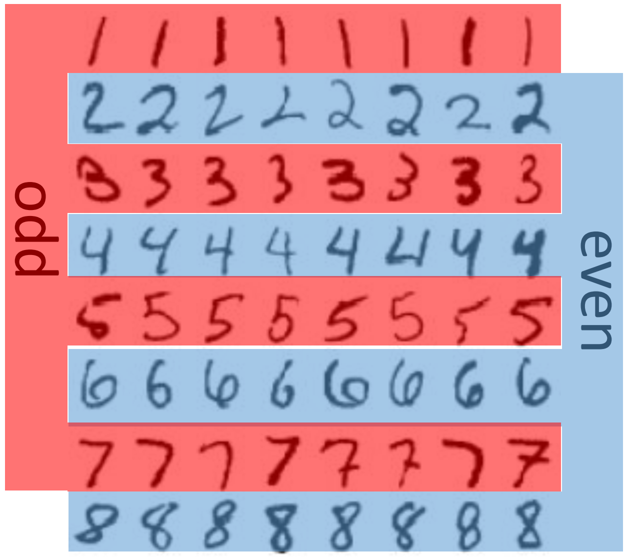


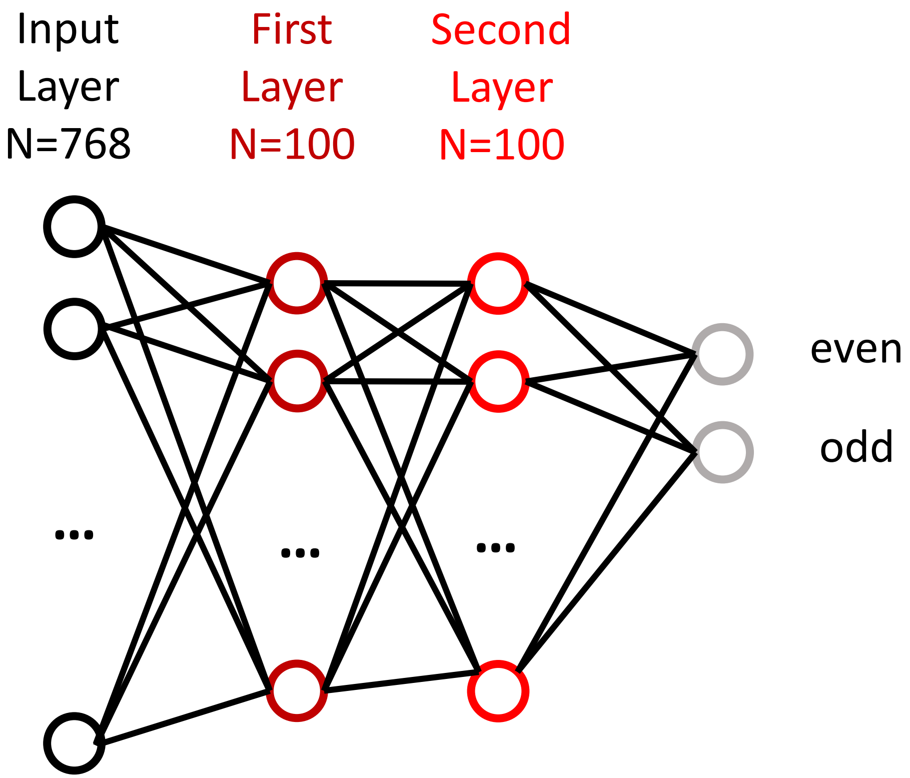
$$\mathrm{PR} = \frac{\left(\sum_i \mu_i\right)^2}{\sum_i \mu_i^2}$$

Abstraction in shallow deep networks

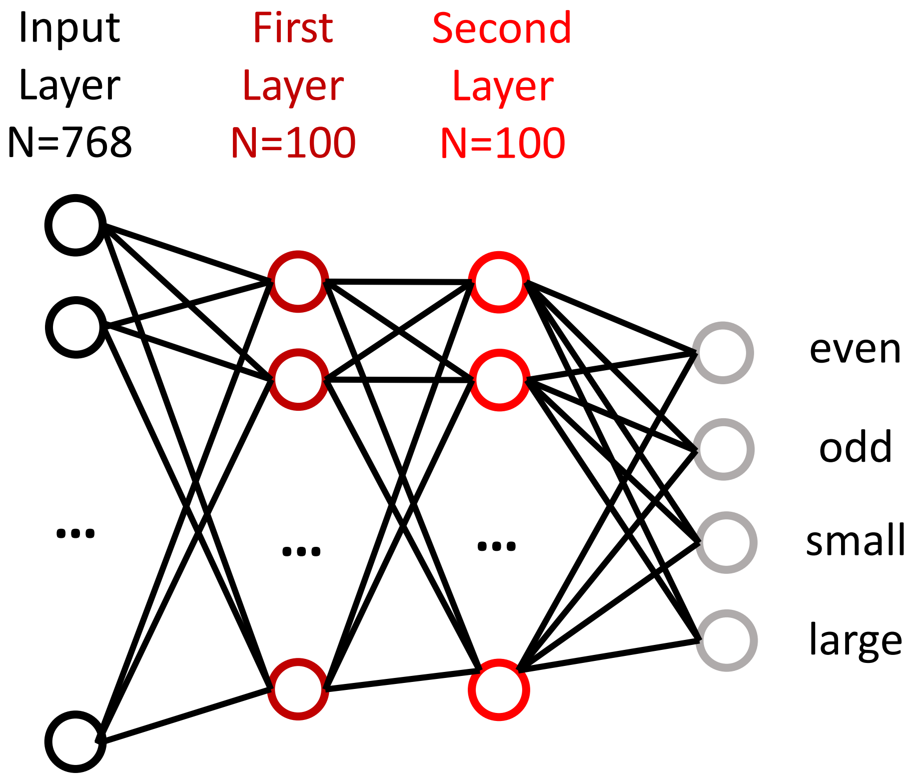

even

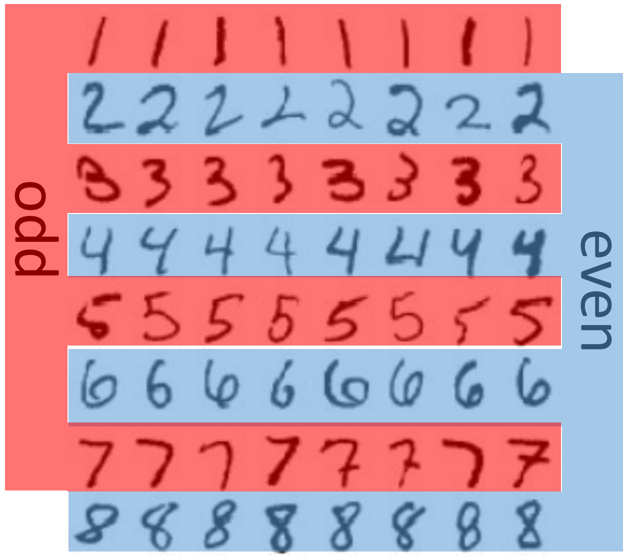

odd
even


odd
even

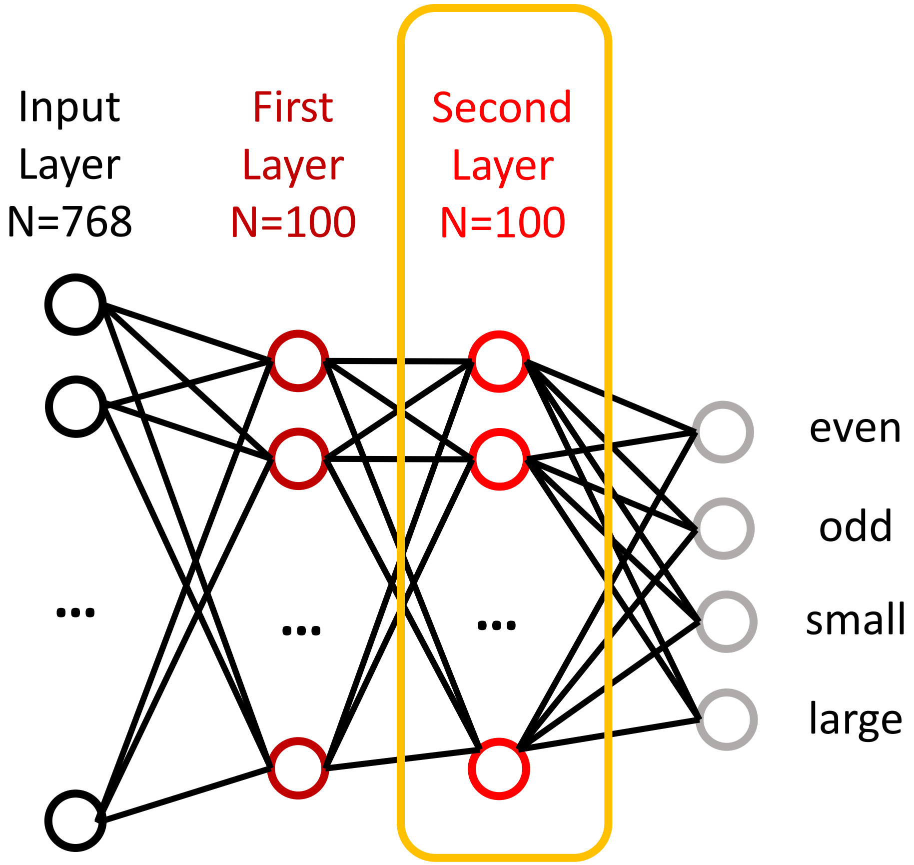

Input
Layer
N=768
First
Layer
N=100
Second
Layer
N=100
...
...
...
even
odd

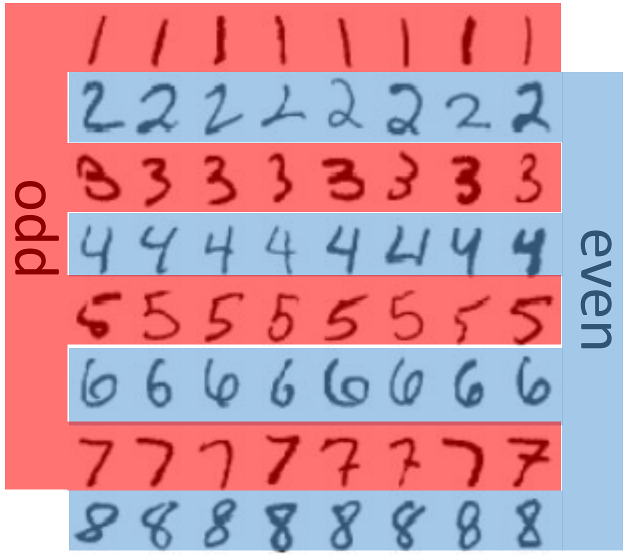

odd
even
small
large

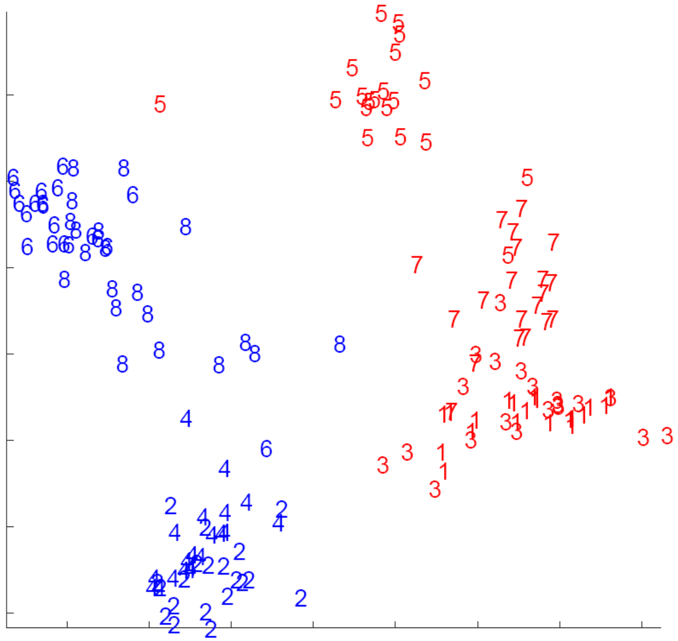

Input
Layer
N=768
First
Layer
N=100
Second
Layer
N=100
...
even
odd
small
large

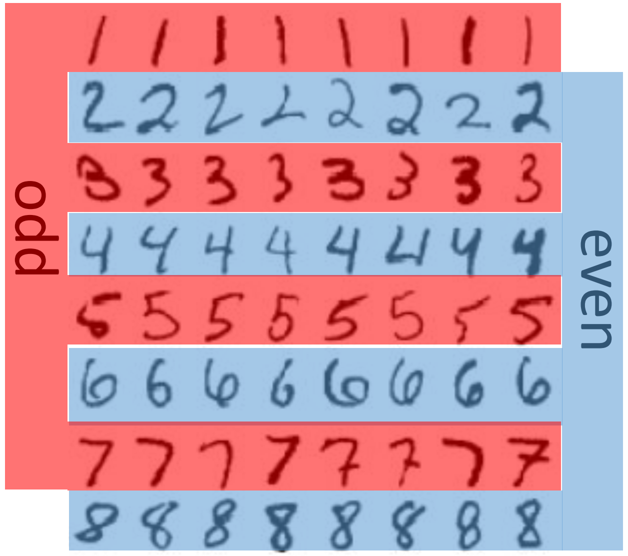

odd
even

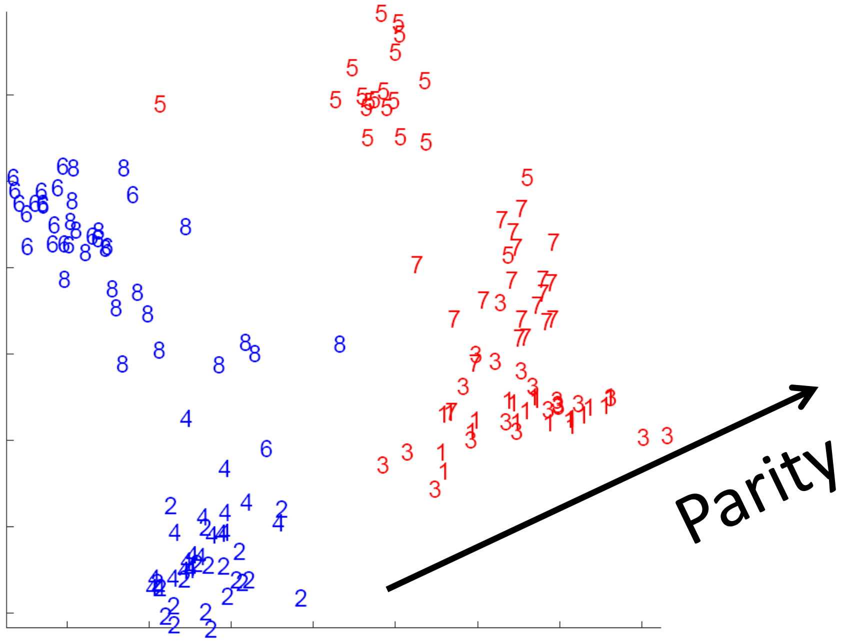

small
large

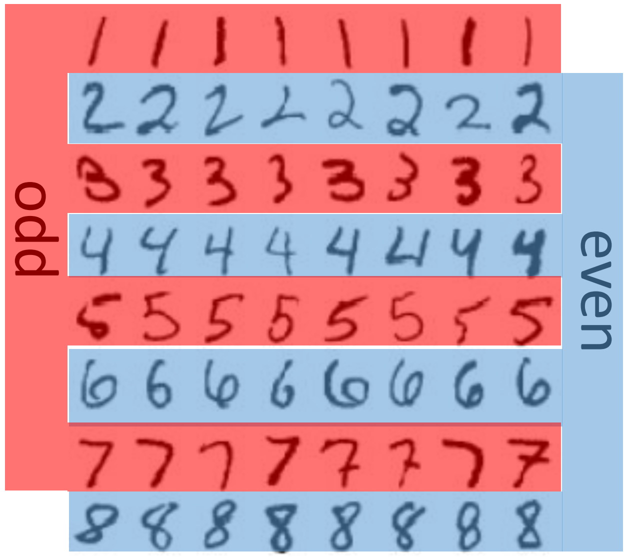

Input
Layer
N=768
First
Layer
N=100
Second
Layer
N=100
...
even
odd
small
large

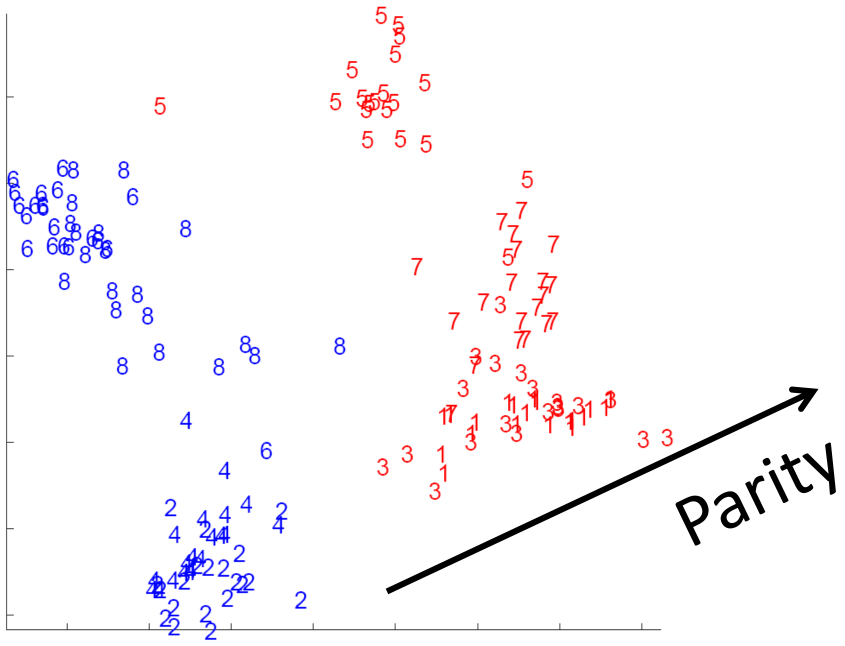

odd
even



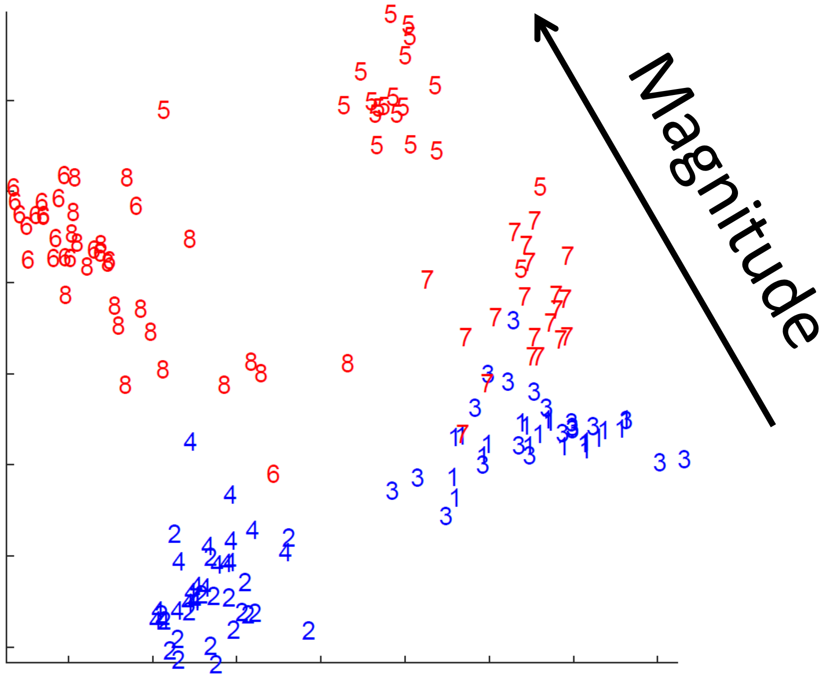

odd
even

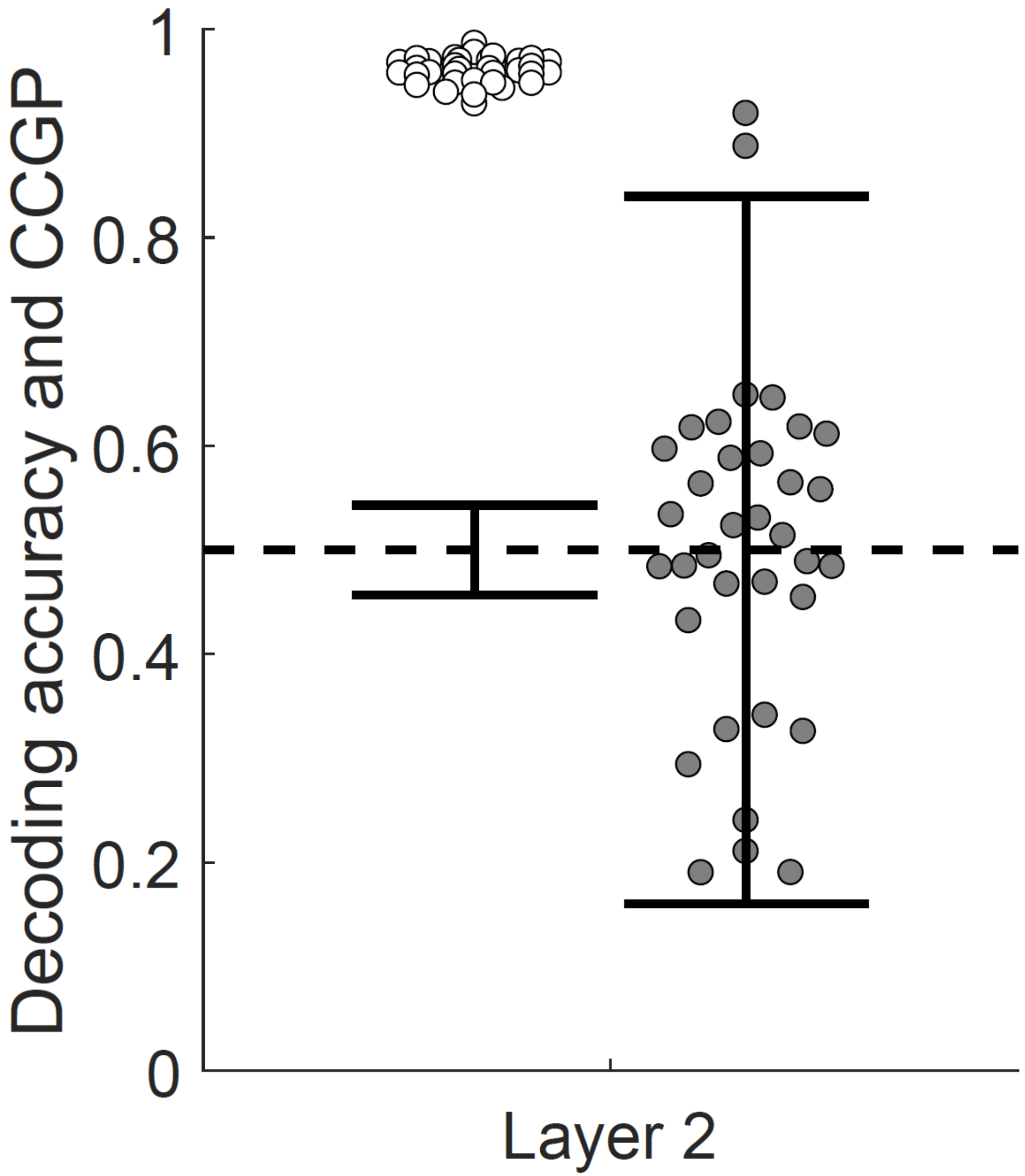

Parity


odd
even


Parity

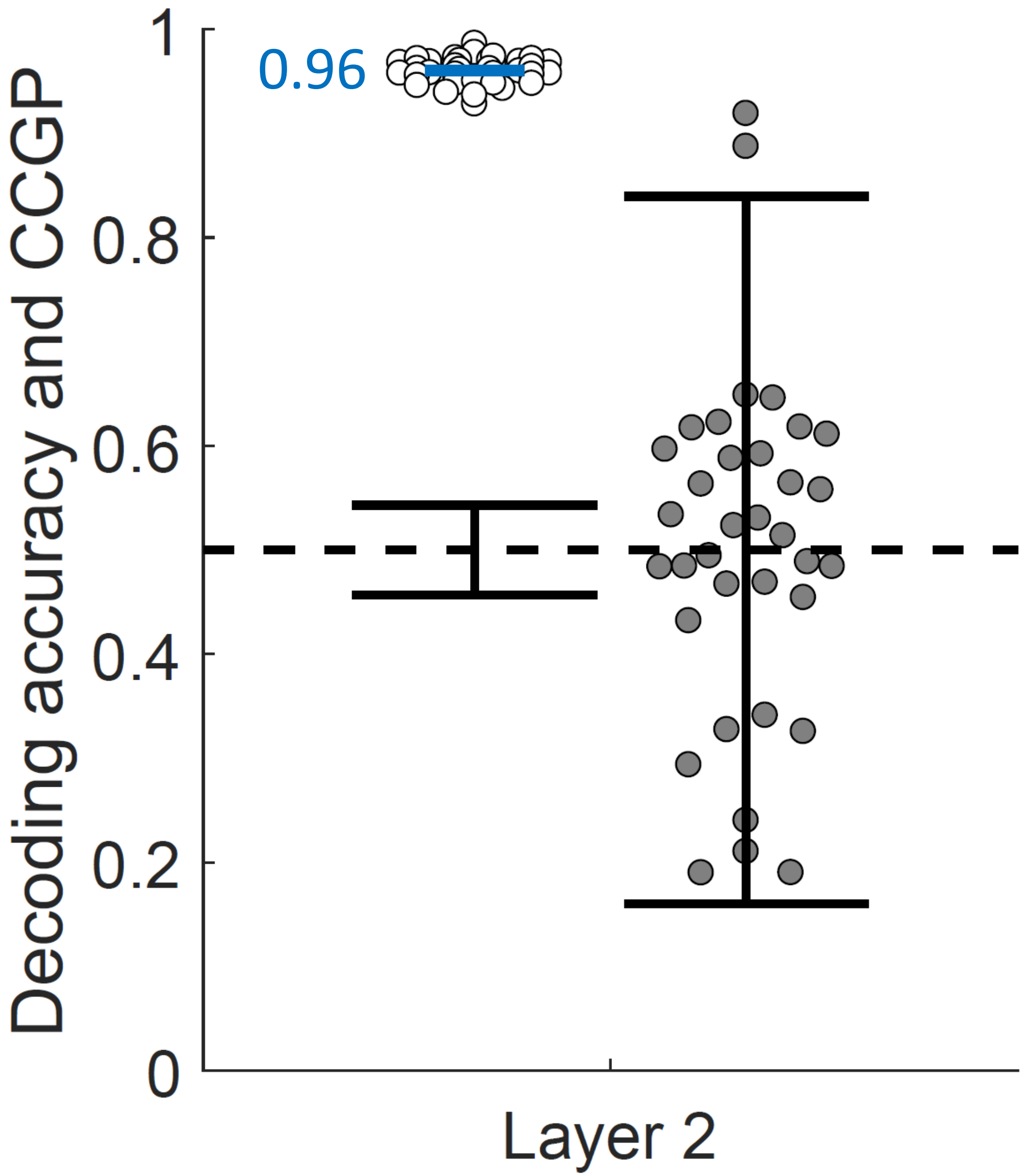

small
large

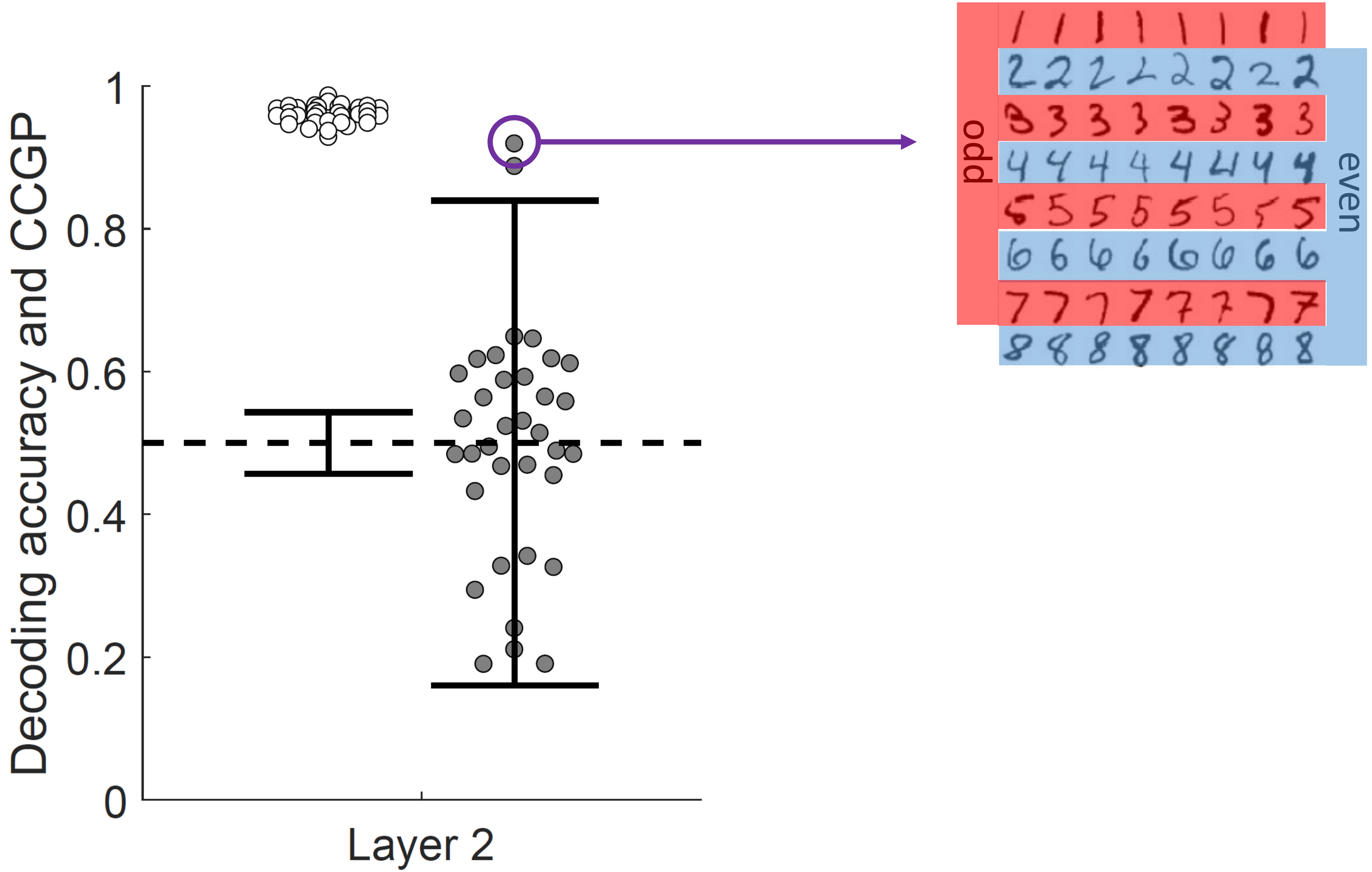

Magnitude

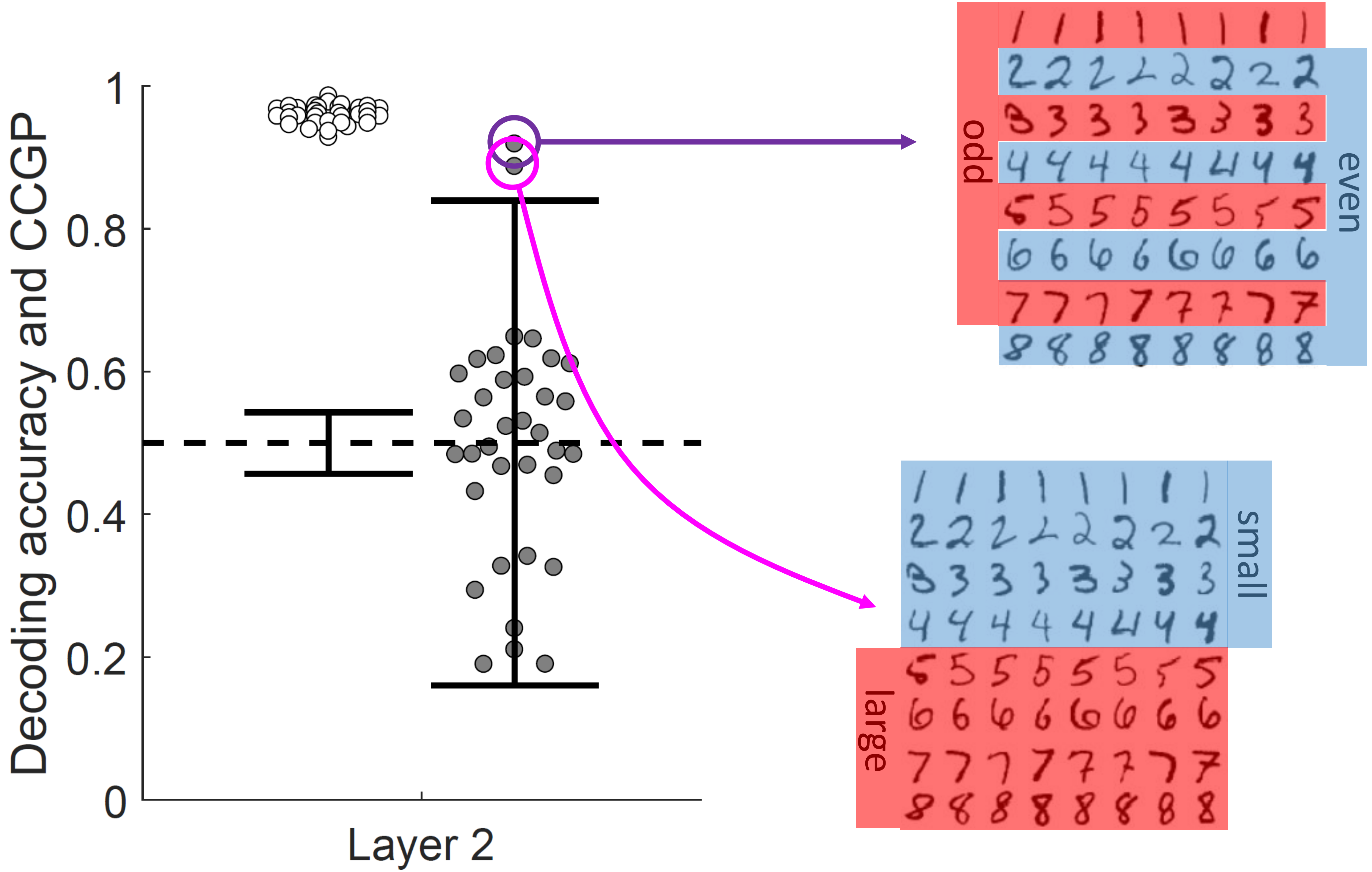


Decoding accuracy and CCGP
1
0.8
0.6
0.4
0.2
0
Layer 2

Shattering dimensionality




Decoding accuracy and CCGP
1
0.8
0.6
0.4
0.2
0
Layer 2
odd
even
1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6
7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8


Decoding accuracy and CCGP
1
0.8
0.6
0.4
0.2
0
Layer 2
odd
even
small
large