Mean-field models of network dynamics

Advanced theory course

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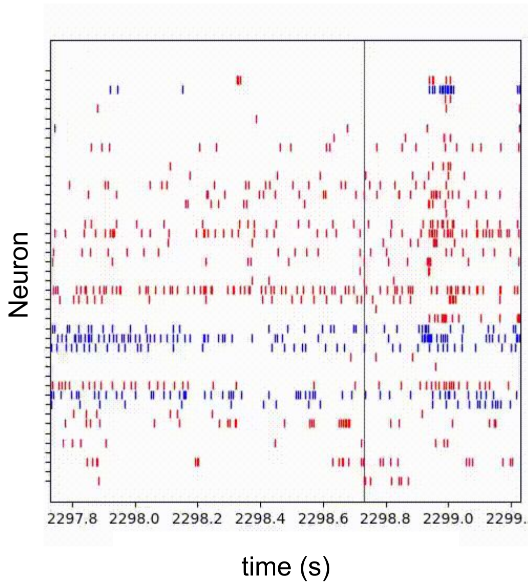
Outline

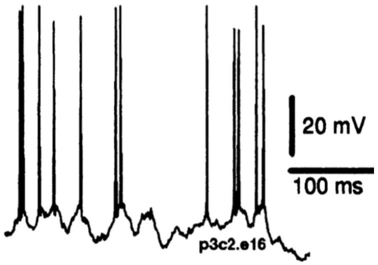
- Irregular activity in cortical circuits
- Spiking models of cortical circuits
- Computation of input statistics that each neuron receive
- Analysis of single neuron and network dynamics with different effective descriptions
- References using mean field models to understand network dynamics and computations

Neurons in cortical circuits fire irregularly

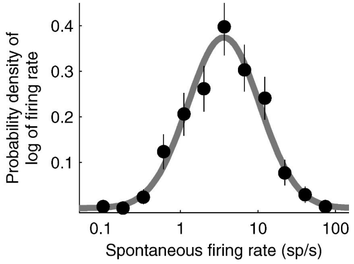


Rodgers et al. 2021

- Temporarlly irregular single cell firing pattern (CV of interspike interval~1) [Holt 1996]



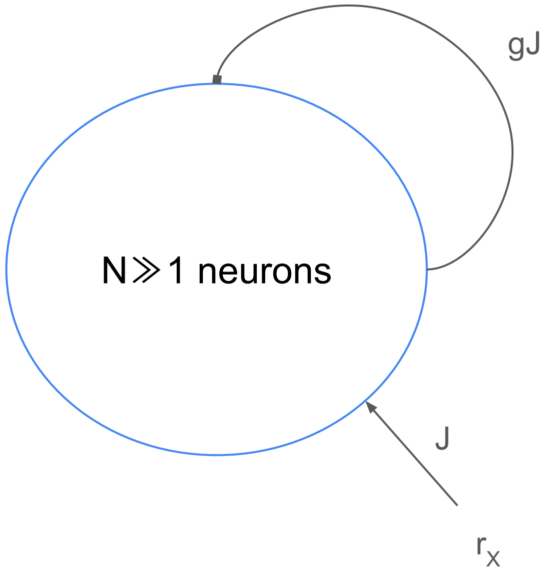
- Broad distribution of rates (~ lognormal) [Hromádka 2010]



<u>What are the underlying mechanisms?</u>

Network model

- Large randomly-connected networks of neurons
  - gJ synaptic efficacy;
  - the number of projections per neuron has mean K ($\gg 1$) and variance ΔK^2.
- External drive from an excitatory population firing with poisson statistics
  - firing rate r_X
  - J synaptic efficacy;
  - K projection per neuron.
- Leaky integrate-and-fire (LIF) single neuron model



Leaky integrate-and-fire (LIF) single neuron model

- Single neuron activity described by its membrane potential V

- Membrane potential evolves in time according to $\tau \frac{dV_i}{dt} = -V + R\, I_i$

- Spiking dynamics of single cell:
    - When V reaches firing threshold θ(=20mV), the neuron emits a spike and V to a reset value V_r (=10mV)

Statistical description of input current

- In the network, input current into a neuron is given by the sum of the presynaptic spikes

$$\tau \frac{dV_i}{dt} = -V + R\, I_i(t) = -V + R\, \left(I_i^{rec} + I^{ff}\right)$$

$$R\, I_i^{rec}(t) = \tau g J \sum_j C_{ij} \sum_n \delta(t - t_j^n)\,, \quad R\, I_i^{ff}(t) = \tau J \sum_j C_{ij} \sum_n \delta(t - t_j^n)$$

- Under the (biologically motivated) assumptions:
    - large K,
    - small J,
    - sparse connectivity
    - asynchronous irregular activity

  the central limit theorem gives:

$$R\, I_i(t) = \mu + \sigma \sqrt{\tau}\, \eta_i(t) + \Delta\, z_i\,,$$

$$\mu = K J \tau (r_X + g \bar{r})\,, \quad \sigma^2 = \tau J^2 K \left(r_x + g^2 \bar{r}\right)\,, \quad \Delta^2 = \tau^2 g^2 J^2 \Delta_K^2 \left(\bar{r}^2 + \Delta r^2\right)$$

Derivation of statistical description of input current

Amit and Brunel, *Dynamics of a recurrent network of spiking neurons before and following learning*. 1997

Mean-field descriptions of input current

$$R\,I_i(t) = \mu + \sigma\sqrt{\tau}\,\eta_i(t) + \Delta\, z_i\,,$$

$$\mu = KJ\tau(r_X + g\bar{r})\,, \quad \sigma^2 = \tau J^2 K\left(r_x + g^2\bar{r}\right)\,, \quad \Delta^2 = \tau^2 g^2 J^2 \Delta_K^2\left(\bar{r}^2 + \Delta r^2\right)$$

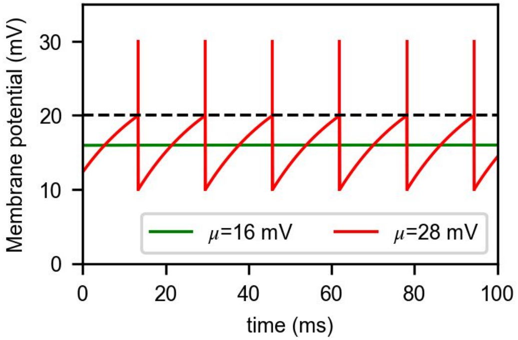
<u>What is a good effective description of the input into a single cell?</u>

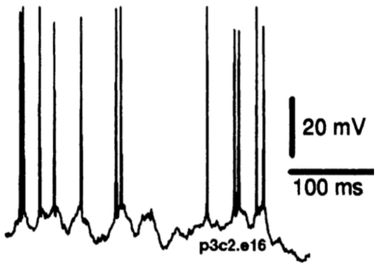
- In cortex, the mean μ is expected to dominate
- let’s neglect the other contributions (σ and Δ) and see what happens
  - *“Traditional mean-field”*

μ mean-field: single neuron response

- Single neuron dynamics

$$\tau \frac{dV_i}{dt} = -V + \mu$$

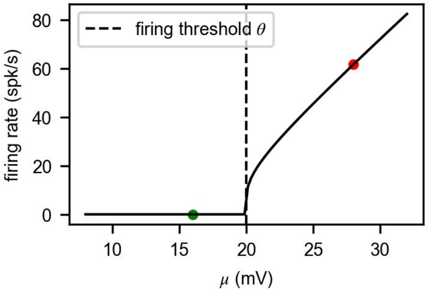




Looks nothing like the data!

- Single neuron firing rate

$$r = \frac{1}{\tau \log\left(\frac{\mu - V_r}{\mu - \theta}\right)}$$



μ mean-field: network response

- Single neuron dynamics determined by activity of other neurons in the network

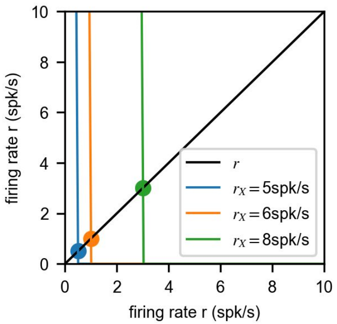
$$\tau \frac{dV_i}{dt} = -V + \mu \,, \quad \mu = KJ\tau(r_X + gr)\,.$$

- Implicit equation defining network response

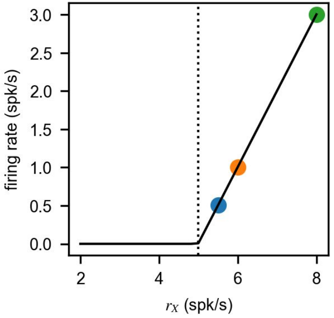
$$r = \frac{1}{\tau \log\left(\frac{\mu - V_r}{\mu - \theta}\right)}$$

$$\mu = KJ\tau(r_X + gr)\,.$$

Solutions only for g<0



Network transfer function



Mean-field descriptions of input current

$$R\,I_i(t) = \mu + \sigma\sqrt{\tau}\,\eta_i(t) + \Delta\, z_i\,,$$

$$\mu = KJ\tau(r_X + g\bar{r})\,, \quad \sigma^2 = \tau J^2 K\left(r_x + g^2\bar{r}\right)\,, \quad \Delta^2 = \tau^2 g^2 J^2 \Delta_K^2\left(\bar{r}^2 + \Delta r^2\right)$$

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- In cortex, the mean μ is expected to dominate
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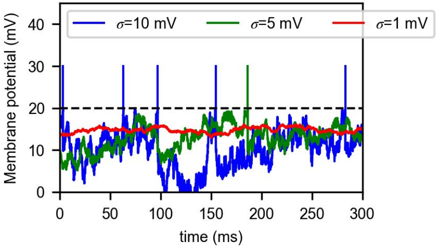
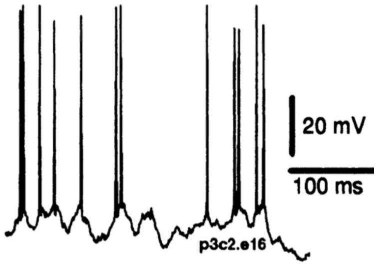
μ-σ mean-field: single neuron response

- Single neuron dynamics

$$\tau \frac{dV_i}{dt} = -V + \mu + \sigma\,\eta_i(t)$$

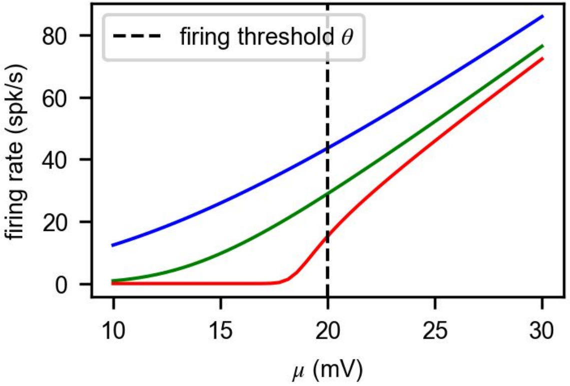




Dynamics resembles data for θ-μ~σ but <u>requires fine tuning</u>

- Single neuron firing rate (first passage time OU process)

$$r = \left[\tau\sqrt{\pi}\int_{\frac{V_r-\mu}{\sigma}}^{\frac{\theta-\mu}{\sigma}} e^{u^2}\left(1+\mathrm{erf}(u)\right)\,du\right]^{-1}$$



μ-σ mean-field: network response

- Single neuron dynamics determined by activity of other neurons in the network

$$\tau \frac{dV_i}{dt} = -V + \mu + \sigma\sqrt{\tau}\eta_i(t)\,, \quad \mu = KJ\tau(r_X + gr)\,, \quad \sigma^2 = \tau J^2 K\left(r_x + g^2\bar{r}\right)$$

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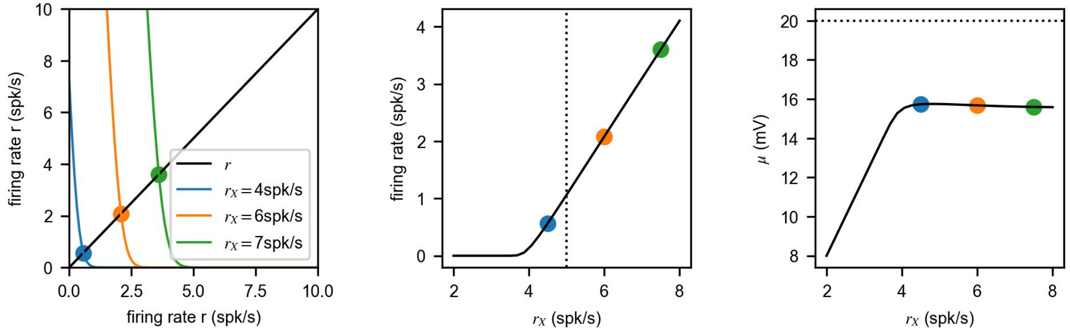
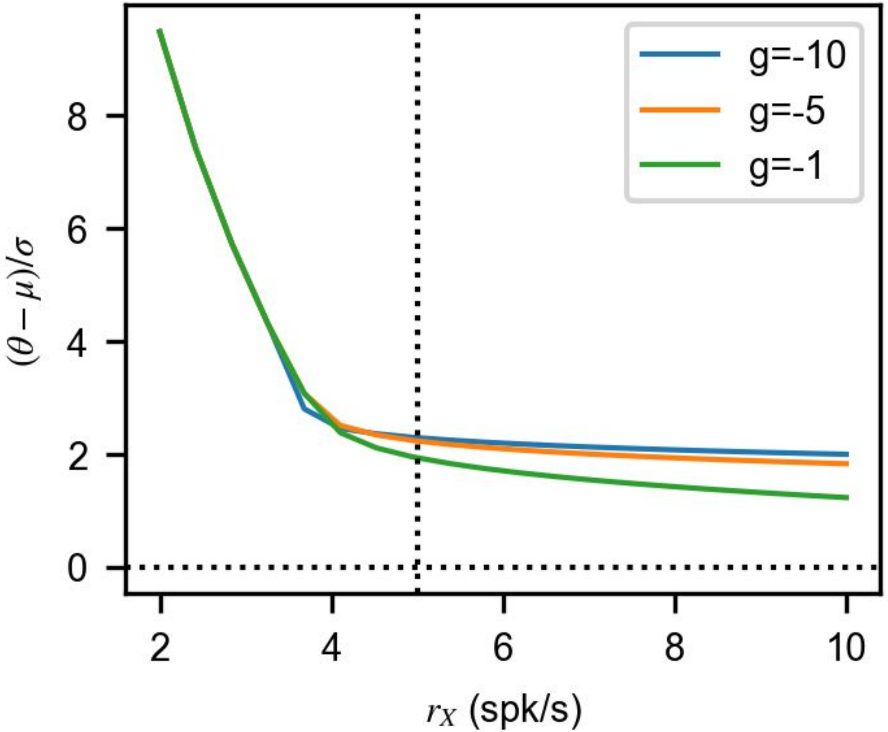
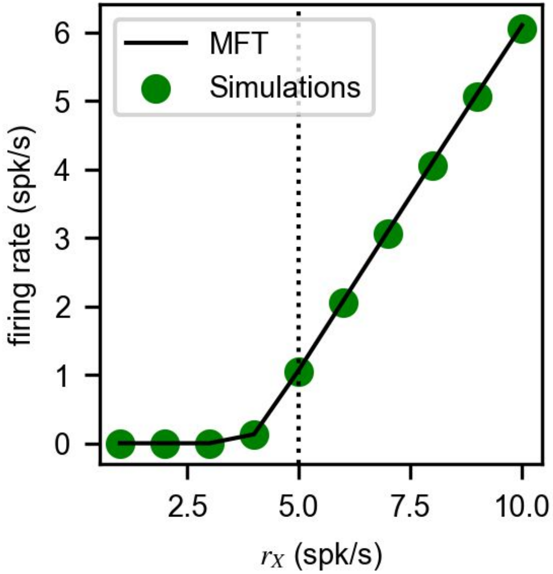
Solutions only for g<0

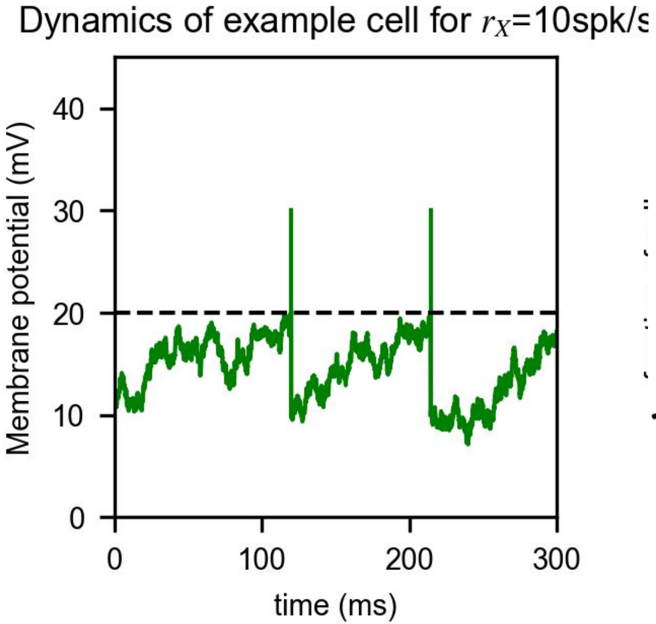


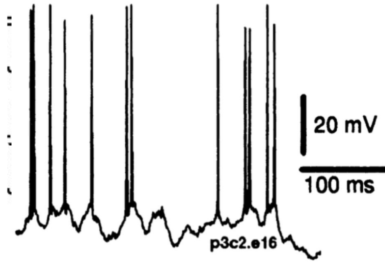
Robustness of fluctuation-driven regime



Agreement with numerical simulations

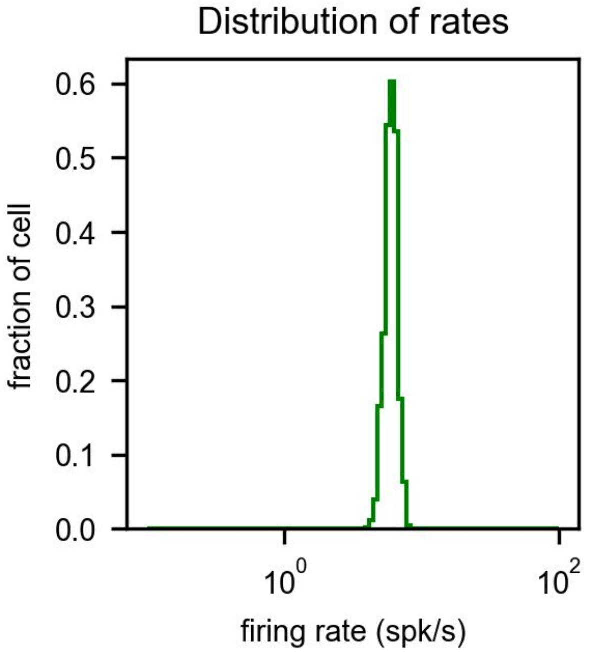


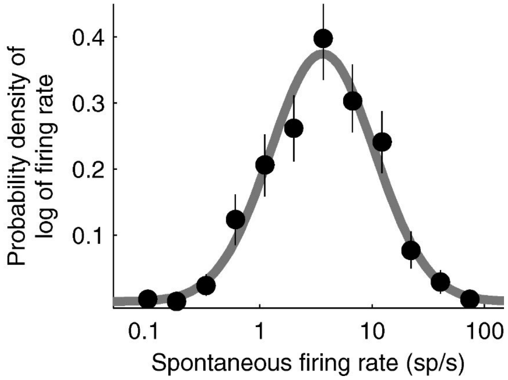




<u>E-I balance implements fine tuning of inputs;
signature in linearity of network transfer function</u>

Network model fails to capture cortical distribution of rates





μ-σ-Δ mean-field

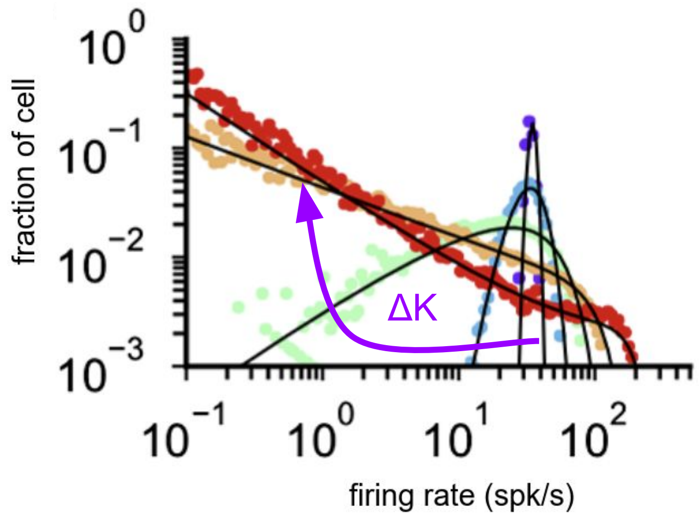
- Single neuron dynamics determined by activity of other neurons in the network

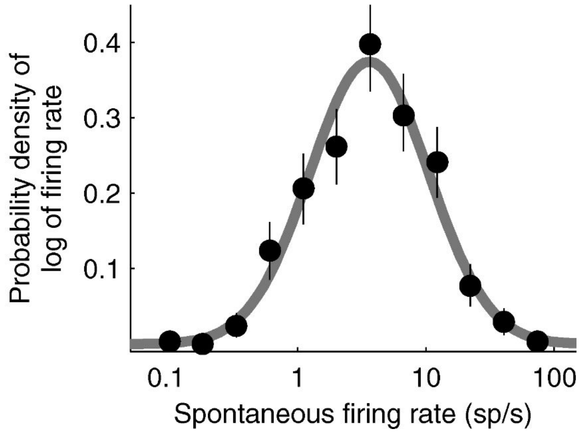
- MFT predicts the size of quenched disorder at which broad distribution of rates emerge

$$R\,I_i(t) = \mu + \sigma\sqrt{\tau}\,\eta_i(t) + \Delta\,z_i\,,$$

$$\mu = KJ\tau(r_X + g\bar{r})\,, \quad \sigma^2 = \tau J^2 K\left(r_x + g^2\bar{r}\right)\,, \quad \Delta^2 = \tau^2 g^2 J^2 \Delta_K^2\left(\bar{r}^2 + \Delta r^2\right)$$

Increasing ΔK generates broad (~lognormal) distribution of rates consistent with data

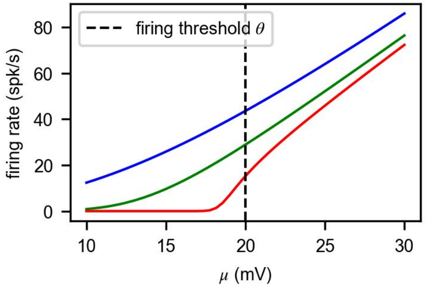


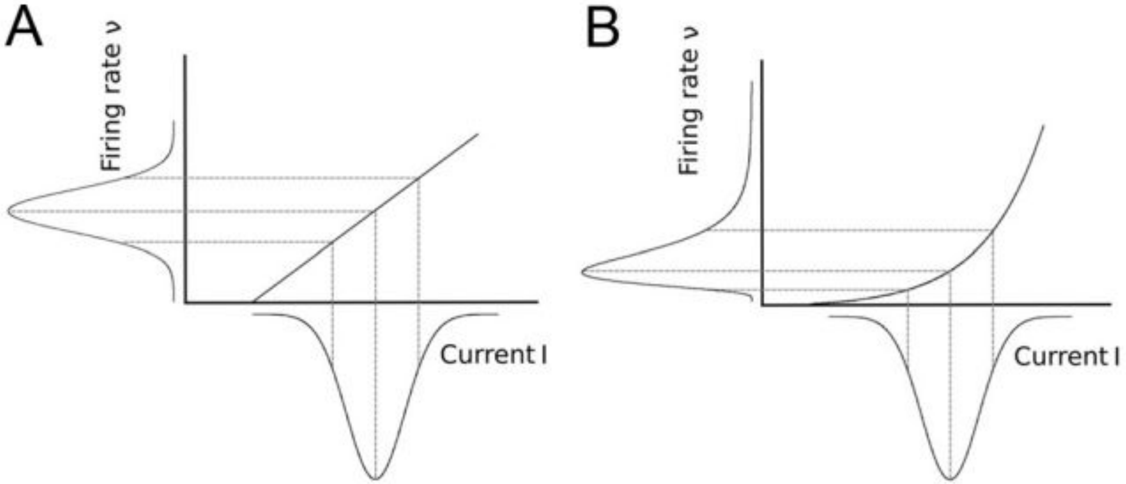


Fluctuation driven regime gives rise to skewed distribution

$$R\,I_i(t) = \mu + \sigma\sqrt{\tau}\,\eta_i(t) + \Delta\,z_i\,,$$

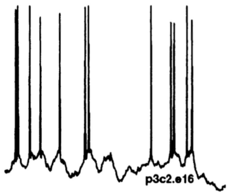
$$\mu = KJ\tau(r_X + g\bar{r})\,, \quad \sigma^2 = \tau J^2 K\left(r_x + g^2\bar{r}\right)\,, \quad \Delta^2 = \tau^2 g^2 J^2 \Delta_K^2\left(\bar{r}^2 + \Delta r^2\right)$$

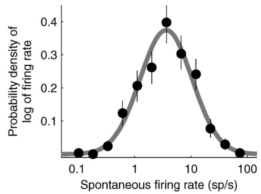




Summary

- Irregular activity observed in cortical circuits
- Mean field models of networks of spiking neurons
  - Derived a statistical description of inputs to cells in the network
  - Network models with only mean input fail to capture irregular activity
  - Network models with mean and temporal variance capture CV of interspike interval~1
  - Including quenched disorder of the correct size allows to capture distribution of rates in the network





References and other notable examples

- Dynamics of networks of spiking neurons
    - Amit and Brunel, Dynamics of a recurrent network of spiking neurons before and following learning. 1997
    - Brunel, Dynamics of Sparsely Connected Networks of Excitatory and Inhibitory Spiking Neurons. 2000
- Models with more biophysical details
    - Mongillo et al, Bistability and Spatiotemporal Irregularity in Neuronal Networks with Nonlinear Synaptic Transmission. 2012
    - Ostojic and Brunel. From Spiking Neuron Models to Linear-Nonlinear Models. 2011
- Effects of disorder
    - Roxin et al, On the Distribution of Firing Rates in Networks of Cortical Neurons. 2011
    - Landau et al, The Impact of Structural Heterogeneity on Excitation-Inhibition Balance in Cortical Networks. 2016
- Mean-field for computations
    - Babadi and Sompolinsky. Sparseness and Expansion in Sensory Representations. 2014