Assignment 1, AI1110

Rajiv Chitale (cs21btech11051)

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Question 5 (a)

Given matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$. Find the matrix X if, $X = B^2 - 4B$. Hence solve for a and b given $X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

Solution:

$$B^2 = \begin{bmatrix} 1^2 + 1 \cdot 8 & 1^2 + 1 \cdot 3 \\ 8 \cdot 1 + 3 \cdot 8 & 1 \cdot 8 + 3^2 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} \tag{1}$$

$$4B = \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix} \tag{2}$$

Substituting (1) and (2) in $X = B^2 - 4B$

$$\Rightarrow X = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

Thus we obtain $X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5a \\ 5b \end{bmatrix}$ (3)

Also, given that $X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$ (4)

From (3) and (4),

$$\Rightarrow \begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

On equating elements $a = 1, b = 10$