Assignment 8, AI1110

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Outline

Question

Papoulis Problem 15-3

Find the stationary distribution q_1, q_2, $\ldots$ for the Markov chain whose only nonzero stationary probabilities are

$$p_{i,1} = \frac{i}{i+1} \tag{1}$$

$$p_{i,i+1} = \frac{1}{i+1} \tag{2}$$

for $i = 1, 2, \ldots$

Diagram

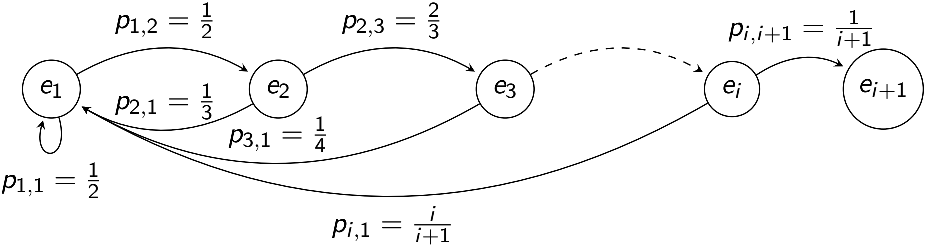

Figure: Graph of Markov Chain

State Transition

- The transition probability from (2) can be written as,

$$p_{i-1,i} = \frac{1}{i} \tag{3}$$

- For $i = 2, 3\ldots$ we can describe state probabilities by

$$P_i^{(t+1)} = \frac{1}{i} \times P_{i-1}^{(t)} \tag{4}$$

Limiting Case

- The stationary state probabilities are given by

$$\lim_{i \to \infty} P_i^{(t)} = q_i \tag{5}$$

- In the limiting case, equation (4) gives

$$q_i = \frac{1}{i} \times q_{i-1} \tag{6}$$

- Applying the same formula recursively yields

$$q_i = \frac{q_1}{i!} \tag{7}$$

Calculations

- The states in a Markov chain are mutually exclusive and exhaustive.

$$\sum_{i=1}^{\infty} q_i = 1 \tag{8}$$

$$\sum_{i=1}^{\infty} \frac{q_1}{i!} = 1 \tag{9}$$

- We can use the Taylor series expansion,

$$\sum_{n=1}^{\infty} \frac{x^n}{n!} = e^x \tag{10}$$

$$\implies \sum_{i=1}^{\infty} \frac{1}{i!} = e \tag{11}$$

Stationary Distribution

Equation (9) reduces to

$$q_1 \times e = 1 \tag{12}$$

$$q_1 = \frac{1}{e} \tag{13}$$

Stationary Distribution

Substituting in equation (7), we obtain required terms,

$$q_i = \frac{1}{i!e} \tag{14}$$

for $i = 1, 2 \ldots$