Assignment 7, AI1110

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Example 7.3 Papoulis

In this example, the following are discussed

1. Probability of Components Being Good
2. Number of Good Components
3. Expected Number of Good Components
4. Failure Rate
5. Relative Expected Failure Rate

Probability of Components Being Good

- A system consists of m components. The time to failure of the i^{th} component is a random variable X_i.
- Its cumulative distribution $F_i\left(X_i\right)$ denotes the probability that i^{th} component has fails at or before time t.
- The probability that the i^{th} component is good at time t is

$$1 - F_i\left(t\right) = P\left(X_i > t\right) \tag{1}$$

Number of Good Components

Let $n(t)$ denote the number of components that are good at time t. Then,

$$n(t) = n_1 + \cdots + n_m \tag{2}$$

$$n_i = \begin{cases} 1 & X_i > t \\ 0 & X_i < t \end{cases} \tag{3}$$

Expected Number of Good Components

- The expectation value of a component being good at time t is

$$E\{n_i\} = 0 \times p(n_i = 0) + 1 \times p(n_i = 1) \tag{4}$$
$$E\{n_i\} = 0 + 1 \times P(X_i > t) \tag{5}$$
$$E\{n_i\} = 1 - F_i(t) \tag{6}$$

- We obtain the expectation value of $n(t)$,

$$\eta(t) = E\{n(t)\} \tag{7}$$
$$\eta(t) = 1 - F_1(t) + \cdots + 1 - F_m(t) \tag{8}$$

- If we assume the case that each X_i has the same distribution $F(t)$, then

$$\eta(t) = m[1 - F(t)] \tag{9}$$

Failure Rate

- The difference $\eta\left(t\right) - \eta\left(t + dt\right)$ is the expected number of failures in the interval $\left(t, t + dt\right)$.
- The rate of failure is given by $-\eta'\left(t\right)$

$$-\eta'\left(t\right) = \frac{\eta\left(t\right) - \eta\left(t + dt\right)}{dt} \tag{10}$$

- Equation (9) can be differentiated to obtain,

$$-\eta'\left(t\right) = m \times f\left(t\right) \tag{11}$$

Relative Expected Failure Rate

The relative expected failure rate is calculated with respect to the number of components that are good at time t. It is given by the ratio,

$$\beta(t) = \frac{-\eta'(t)}{\eta(t)} \tag{12}$$

$$\beta(t) = \frac{f(t)}{1 - F(t)} \tag{13}$$

On integrating (12) we obtain,

$$-\int_0^t \beta(\tau)\, d\tau = \ln \eta(t) - \ln \eta(0) \tag{14}$$

- Let us assume operations start at $t = 0$ with $n(0) = m$.
- Then we have $\eta(0) = E\{n(0)\} = m$

$$-\int_0^t \beta(\tau)\, d\tau = \ln \eta(t) - \ln m \tag{15}$$

$$\implies \eta(t) = m \exp\left\{-\int_0^t \beta(\tau)\, d\tau\right\} \tag{16}$$