

# Assignment 6, AI1110

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**Abstract—**This document provides a solution to Question 15 from NCERT class 12 Probability Ex 13.1

**Question 15:** Consider the experiment of throwing a die. If a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event ‘the coin shows a tail’, given that ‘at least one die shows a 3’.

**Solution:** Let us define some random variables.

Event	Description
$X_1 \in \{1, 2, 3, 4, 5, 6\}$	Number obtained from a die throw
$X_2 = 1$	Coin shows tails after experiment
$X_2 = 0$	Coin shows heads after experiment
$X_3 = 1$	At least one die shows a 3
$X_3 = 0$	No die shows a 3
$X_4 = 1$	Getting tails from a coin toss
$X_4 = 0$	Getting heads from a coin toss

TABLE I  
RANDOM VARIABLES

Equations are obtained using recursive cases. Solving them yields probabilities.

$$\begin{aligned} & \Pr(X_3 = 1) \\ &= \Pr(X_1 = 3) + \Pr(X_1 = 6) \times \Pr(X_3 = 1) \end{aligned} \quad (1)$$

$$\Pr(X_3 = 1) = \frac{1}{6} + \frac{1}{6} \times \Pr(X_3 = 1) \quad (2)$$

$$\Pr(X_3 = 1) = \frac{1}{5} \quad (3)$$

$$\begin{aligned} & \Pr(X_2 = 1) \\ &= \Pr(X_1 \in \{1, 2, 4, 5\}) \times \Pr(X_4 = 1) \\ &+ \Pr(X_1 \in \{3, 6\}) \times \Pr(X_2 = 1) \end{aligned} \quad (4)$$

$$\Pr(X_2 = 1) = \frac{4}{6} \times \frac{1}{2} + \frac{2}{6} \times \Pr(X_2 = 1) \quad (5)$$

$$\Pr(X_2 = 1) = \frac{1}{2} \quad (6)$$

Required probability is,

$$\Pr(X_2 = 1 | X_3 = 1) = \frac{\Pr((X_2 = 1)(X_3 = 1))}{\Pr(X_3 = 1)} \quad (7)$$

$$\begin{aligned} & \Pr((X_2 = 1)(X_3 = 1)) \\ &= \Pr(X_1 = 3) \times \Pr(X_2 = 1) \\ &+ \Pr(X_1 = 6) \times \Pr((X_2 = 1)(X_3 = 1)) \end{aligned} \quad (8)$$

$$\begin{aligned} & \Pr((X_2 = 1)(X_3 = 1)) \\ &= \frac{1}{6} \times \frac{1}{2} + \frac{1}{6} \times \Pr((X_2 = 1)(X_3 = 1)) \end{aligned} \quad (9)$$

$$\implies \Pr((X_2 = 1)(X_3 = 1)) = \frac{1}{10} \quad (10)$$

$$\begin{aligned} \therefore \Pr(X_2 = 1 | X_3 = 1) &= \frac{1}{10} \div \frac{1}{5} \quad (11) \\ &= \frac{1}{2} \quad (12) \end{aligned}$$