Assignment 6, AI1110

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Outline

Question

Source

Question 15, NCERT class 12 Probability Ex 13.1

Consider the experiment of throwing a die.

If a multiple of 3 comes up, throw the die again

If any other number comes, toss a coin.

Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Solution

Let us define some random variables.

Event	Description
$X_1 \in \{1, 2, 3, 4, 5, 6\}$	Number obtained from a die throw
$X_2 = 1$	Coin shows tails after experiment
$X_2 = 0$	Coin shows heads after experiment
$X_3 = 1$	At least one die shows a 3
$X_3 = 0$	No die shows a 3
$X_4 = 1$	Getting tails from a coin toss
$X_4 = 0$	Getting heads from a coin toss

Table: Random variables

Recursion

Equations are obtained using recursive cases. Solving them yields probabilities.

$$\Pr(X_3 = 1) = \Pr(X_1 = 3) + \Pr(X_1 = 6) \times \Pr(X_3 = 1) \tag{1}$$

$$\Pr(X_3 = 1) = \frac{1}{6} + \frac{1}{6} \times \Pr(X_3 = 1) \tag{2}$$

$$\Pr(X_3 = 1) = \frac{1}{5} \tag{3}$$

Recursion (contd.)

$$\Pr(X_2 = 1) = \Pr(X_1 \in \{1,2,4,5\}) \times \Pr(X_4 = 1) + \Pr(X_1 \in \{3,6\}) \times \Pr(X_2 = 1) \quad (4)$$

$$\Pr(X_2 = 1) = \frac{4}{6} \times \frac{1}{2} + \frac{2}{6} \times \Pr(X_2 = 1) \quad (5)$$

$$\Pr(X_2 = 1) = \frac{1}{2} \quad (6)$$

Conditional Probability

Required probability:

$$\Pr\left(X_2 = 1 | X_3 = 1\right) = \frac{\Pr\left((X_2 = 1)(X_3 = 1)\right)}{\Pr\left(X_3 = 1\right)} \tag{7}$$

$$\Pr\left((X_2 = 1)(X_3 = 1)\right) = \Pr\left(X_1 = 3\right) \times \Pr\left(X_2 = 1\right) + \Pr\left(X_1 = 6\right) \times \Pr\left((X_2 = 1)(X_3 = 1)\right) \tag{8}$$

$$\Pr\left((X_2 = 1)(X_3 = 1)\right) = \frac{1}{6} \times \frac{1}{2} + \frac{1}{6} \times \Pr\left((X_2 = 1)(X_3 = 1)\right) \tag{9}$$

$$\implies \Pr\left((X_2 = 1)(X_3 = 1)\right) = \frac{1}{10} \tag{10}$$

Answer

$$\therefore \Pr\left(X_2 = 1 | X_3 = 1\right) = \frac{1}{10} \div \frac{1}{5} \tag{11}$$

$$= \frac{1}{2} \tag{12}$$