Assignment 6, AI1110

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LaTeX

Outline

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4. Transition Probability Matrix
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Question

Question 15, NCERT class 12 Probability Ex 13.1

Consider the experiment of throwing a die.

If a multiple of 3 comes up, throw the die again

If any other number comes, toss a coin.

Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Markov Chain

Let us construct a Markov chain X_t with discrete time t. The states e_0, e_1 and e_2 describe the outcomes from the latest dice throw. The states e_3 and e_4 describe the outcomes of the latest coin toss.

States

Let $Y \in \{1, 2, 3, 4, 5, 6\}$ denote the number obtained from a die throw.

i	**State** (e_i)
0	$Y = 3$
1	$Y = 6$
2	$\sum (Y = k)\,; k \in \{1, 2, 4, 5\}$
3	Obtaining heads from coin toss
4	Obtaining tails from coin toss

Table: States in Markov Chain

Graph of Markov Chain

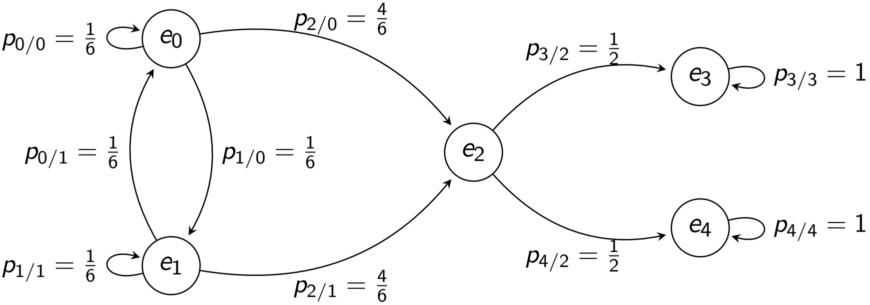

Figure: Graph of Markov Chain

Transition Probability Matrix

$p_{j/i}$ is the probability of moving from state e_i to e_j.

$$p_{j/i} = \Pr\left(\frac{X_{t+1} = e_j}{X_t = e_i}\right) \tag{1}$$

These probabilities are contained in the transition probability matrix.

$$\mathbf{P_{ij}} = \begin{pmatrix} 1/6 & 1/6 & 4/6 & 0 & 0 \\ 1/6 & 1/6 & 4/6 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

Limiting state vector

$\mathbf{Q_t}$ is the state vector at a given t. The given condition is that 3 occurs at least once. Let the first occurrence of 3 be the initial state $\mathbf{Q_0}$.

$$\mathbf{Q_0} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3}$$

$$\mathbf{Q_t} = \mathbf{Q_0} \mathbf{P}^t \tag{4}$$

The limiting probabilities of states, calculated from solve.py,

$$\lim_{t \to \infty} \mathbf{Q_t} = \begin{pmatrix} 0 & 0 & 0 & 0.5 & 0.5 \end{pmatrix} \tag{5}$$

Required conditional probability is,

$$\lim_{t \to \infty} \Pr\left(X_t = e_4\right) = 0.5 \tag{6}$$