

Assignment 6, AI1110

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Question

Question 15, NCERT class 12 Probability Ex 13.1

Consider the experiment of throwing a die.

If a multiple of 3 comes up, throw the die again

If any other number comes, toss a coin.

Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Random Variables

Solution:

Some useful random variables are defined below. Y_n describes whether 3 has occurred before n^{th} throw.

Event	Description
$X_n \in \{1, 2, 3, 4, 5, 6\}$	Number obtained from n^{th} die throw
$Y_n = 0$	$\nexists k < n : X_k = 3$
$Y_n = 1$	$\exists k < n : X_k = 3$
$Z = 0$	Getting heads from a coin toss
$Z = 1$	Getting tails from a coin toss

Table: Random variables

Markov Chain

Let us construct a Markov chain with discrete time t . The states $e_1 \dots e_5$ describe the outcomes from the latest dice throw or coin toss. States $e_6 \dots e_{10}$ are similar but they denote that at least one die showed 3 before t .

States

i	State e_i
1	$X_t = 3 \wedge Y_t = 0$
2	$X_t = 6 \wedge Y_t = 0$
3	$X_t = i : i \in \{1, 2, 4, 5\} \wedge Y_t = 0$
4	$Z = 0 \wedge Y_t = 0$
5	$Z = 1 \wedge Y_t = 0$
6	$X_t = 3 \wedge Y_t = 1$
7	$X_t = 6 \wedge Y_t = 1$
8	$X_t = i : i \in \{1, 2, 4, 5\} \wedge Y_t = 1$
9	$Z = 0 \wedge Y_t = 1$
10	$Z = 1 \wedge Y_t = 1$

Table: Event states

Transition Probability Matrix

The transition probability matrix $\mathbf{P}_{ij} =$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 4/6 & 0 & 0 \\ 1/6 & 1/6 & 4/6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 4/6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 4/6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

State vector

An element in \mathbf{P}_{ij} represents the probability of moving from state e_i to e_j . \mathbf{Q}_t is the state vector at a given t . Initial state vector is provided for $t = 1$

$$\mathbf{Q}_1 = (1/6 \quad 1/6 \quad 4/6 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0) \quad (2)$$

$$\mathbf{Q}_t = \mathbf{Q}_1 \mathbf{P}^{t-1} \quad (3)$$

The limiting probabilities of states are given by,

$$\lim_{t \rightarrow \infty} \mathbf{Q}_t = (0 \quad 0 \quad 0 \quad 0.4 \quad 0.4 \quad 0 \quad 0 \quad 0 \quad 0.1 \quad 0.1) \quad (4)$$

$$(5)$$

Conditional Probability

Required conditional probability is,

$$\lim_{t \rightarrow \infty} \Pr((Z = 1) | \Pr(Y_t = 1)) \quad (6)$$

$$= \lim_{t \rightarrow \infty} \frac{\Pr((Z = 1)(Y_t = 1))}{\Pr(Y_t = 1)} \quad (7)$$

$$= \lim_{t \rightarrow \infty} \frac{\Pr(e_{10})}{\sum_{i=6}^{10} \Pr(e_i)} \quad (8)$$

$$= \frac{0.1}{0.2} \quad (9)$$

$$= 0.5 \quad (10)$$