Assignment 2, AI1110

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Abstract—**This document provides a solution to Q18 from ICSE Class 12 maths paper, 2018.**

Question 18: Find the image of a point having position vector: $3\hat{i} - 2\hat{j} + \hat{k}$ in the Plane $\mathbf{r}.\left(3\hat{i} - \hat{j} + 4\hat{k}\right) = 2$

Solution: Let the given point be $\mathbf{A}$.

$$\mathbf{A} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \tag{1}$$

The equation of the plane in vector form is

$$\mathbf{n}^\top \mathbf{x} = c \tag{2}$$

Where,

$$\mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} \tag{3}$$

$$c = 2 \tag{4}$$

Let the image of $\mathbf{A}$ in the plane be $\mathbf{R}$. Let the foot of the perpendicular from $\mathbf{A}$ onto the plane be $\mathbf{F}$.

$$\mathbf{F} = \lambda \mathbf{n} + \mathbf{A} \tag{5}$$

$$\mathbf{n}^\top \mathbf{F} = c \tag{6}$$

By property of reflection in plane mirror,

$$(\mathbf{A} - \mathbf{F}) = -(\mathbf{R} - \mathbf{F}) \tag{7}$$

Solving (5), (6) and (5), (7) we obtain,

$$\lambda = \frac{c - \mathbf{n}^\top \mathbf{A}}{\|\mathbf{n}\|^2} \tag{8}$$

$$\mathbf{R} = 2\lambda \mathbf{n} + \mathbf{A} \tag{9}$$

From (8) and (9) we obtain a formula to find $\mathbf{R}$,

$$\mathbf{R} = \mathbf{A} + 2\left(\frac{c - \mathbf{n}^\top \mathbf{A}}{\|\mathbf{n}\|^2}\right)\mathbf{n} \tag{10}$$

$$\implies \mathbf{R} = \mathbf{A} + (-1)\mathbf{n} \tag{11}$$

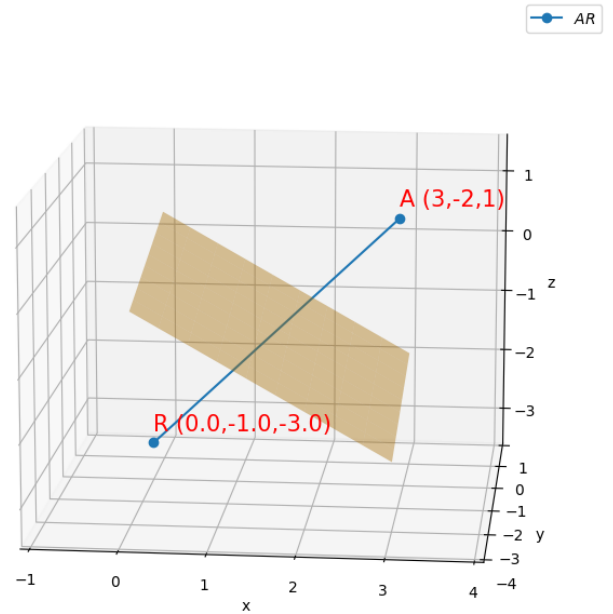$$\implies \mathbf{R} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix} \tag{12}$$

Fig. 1. Point A and its image R about the given plane