Assignment 1, AI1110

Rajiv Shailesh Chitale (cs21btech11051)

ICSE 10th, 2017 Paper

Question 5(a)

Given matrix $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 8 & 3 \end{pmatrix}$. Find the matrix $\mathbf{X}$ if, $\mathbf{X} = \mathbf{B}^2 - 4\mathbf{B}$. Hence solve for a and b given $\mathbf{X} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 50 \end{pmatrix}$

Solution:

First, obtain the characteristic equation of B,

$$|\mathbf{B} - \lambda\mathbf{I}| = 0 \tag{1}$$

$$\begin{vmatrix} 1 - \lambda & 1 \\ 8 & 3 - \lambda \end{vmatrix} = 0 \tag{2}$$

$$(1 - \lambda)(3 - \lambda) - (8)(1) = 0 \tag{3}$$

$$\lambda^2 - 4\lambda - 5 = 0 \tag{4}$$

From Cayley-Hamilton theorem,

$$\mathbf{B}^2 - 4\mathbf{B} - 5\mathbf{I} = 0 \tag{5}$$

$$\mathbf{B}^2 - 4\mathbf{B} = 5\mathbf{I} \tag{6}$$

$$\Rightarrow \mathbf{X} = 5\mathbf{I} \tag{7}$$

Thus we obtain

$$\mathbf{X} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5a \\ 5b \end{pmatrix} \tag{8}$$

It is also given that

$$\mathbf{X} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 50 \end{pmatrix} \tag{9}$$

From (8) and (9),

$$\Rightarrow \begin{pmatrix} 5a \\ 5b \end{pmatrix} = \begin{pmatrix} 5 \\ 50 \end{pmatrix} \tag{10}$$

On equating elements,

$$a = 1, b = 10 \tag{11}$$