

Assignment 1, AI1110

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Question 5(a)

Given matrix $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 8 & 3 \end{pmatrix}$. Find the matrix \mathbf{X} if, $\mathbf{X} = \mathbf{B}^2 - 4\mathbf{B}$. Hence solve for a and b given $\mathbf{X} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 50 \end{pmatrix}$

Solution:

First, obtain the characteristic equation of B,

$$|\mathbf{B} - \lambda\mathbf{I}| = 0 \quad (1)$$

$$\begin{vmatrix} 1 - \lambda & 1 \\ 8 & 3 - \lambda \end{vmatrix} = 0 \quad (2)$$

$$(1 - \lambda)(3 - \lambda) - (8)(1) = 0 \quad (3)$$

$$\lambda^2 - 4\lambda - 5 = 0 \quad (4)$$

From Cayley-Hamilton theorem,

$$\mathbf{B}^2 - 4\mathbf{B} - 5\mathbf{I} = 0 \quad (5)$$

$$\mathbf{B}^2 - 4\mathbf{B} = 5\mathbf{I} \quad (6)$$

$$\Rightarrow \mathbf{X} = 5\mathbf{I} \quad (7)$$

$$\text{Thus we obtain } \mathbf{X} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5a \\ 5b \end{pmatrix} \quad (8)$$

$$\text{It is given that } \mathbf{X} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 50 \end{pmatrix} \quad (9)$$

From (8) and (9),

$$\Rightarrow \begin{pmatrix} 5a \\ 5b \end{pmatrix} = \begin{pmatrix} 5 \\ 50 \end{pmatrix}$$

On equating elements $a = 1, b = 10$