

Assignment 1, AI1110

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From (8) and (9),

Question 5 (a)

$$\Rightarrow \begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

Given matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$. Find the matrix X if, $X = B^2 - 4B$. Hence solve for a and b given $X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

On equating elements $a = 1, b = 10$

Solution:

First, obtain the characteristic equation of B,

$$|B - \lambda I| = 0 \quad (1)$$

$$\begin{vmatrix} 1 - \lambda & 1 \\ 8 & 3 - \lambda \end{vmatrix} = 0 \quad (2)$$

$$(1 - \lambda)(3 - \lambda) - (8)(1) = 0 \quad (3)$$

$$\lambda^2 - 4\lambda - 5 = 0 \quad (4)$$

From Cayley-Hamilton theorem,

$$B^2 - 4B - 5I = 0 \quad (5)$$

$$B^2 - 4B = 5I \quad (6)$$

$$\Rightarrow X = 5I \quad (7)$$

$$\text{Thus we obtain } X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5a \\ 5b \end{bmatrix} \quad (8)$$

$$\text{It is given that } X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix} \quad (9)$$