

DG Solver for 1D- Advection equation:

AdvectDriver1D.m is the main file to be run. Here one could specify the order of the polynomial and (N) , the number of elements (K) , whether periodic (**periodic -1**) or not (**periodic -0**) with the variable.

To choose between the quadrature rule ,use
X_intgr = 'GL' for Gauss lobatto quadrature.
X_intgr = 'GQ' for Gaussian quadrature.

And to choose between consistent and diagonal lumped mass matrix, we have **Mass_mat_type** = 'C' or **Mass_mat_type** = 'D' respectively.

For the construction of the flux matrix , where we could choose between the central or the upwind flux by the parameter **alpha**. (= 0 upwind and = 1 central flux).

In section , **transport simulation** the motion of the wave will be simulated. The rest of the section plots eigen values and the dispersion and dissipation plots.

Advect1D.m consists mainly of the explicit time integration scheme for loop. Within which we calculate the derivative matrix (**d_L**) and the global stiffness matrix (**sti1**) using the chosen quadrature rule in every time step and then calculate the jacobian and calculate the solution I each time step using the runge kutta scheme.

consistent mass.m determines the local consistent mass matrix using the quadrature points and corresponding weights **X_quad** and **w_quad** where the **Lagrange polynomials** (the coefficients of the polynomials are stored in each row of matrix **L**) are evaluated at the **X_quad** using the **polyval** function.

Diag Mass matrix.m calculates the local diagonal mass matrix. The kind of evaluation of entries for consistent mass matrix also holds for the diagonal mass matrix.

JacobiP.m **JacobiGL.m** **JacobiGQ.m** are the files from Hesthaven implementation for jacobi polynomial , gauss lobatto and gauss points.

JacobiPoly.m and **gam.m** are jacobi polynomial implementation and the gamma functions which are used to determine the weights of gauss lobatto quadrature in **weights LGL.m**.

Globals1D.m consists of all global variables and the value of the advection speed.

Startup1D.m initializes the mass matrices , the grid and the quadrature rules.

Dispersion and Dissipation analysis:

The dissipation and dispersion properties for the diagonal mass matrices are quiet damped as compared to that of the consistent mass matrices.

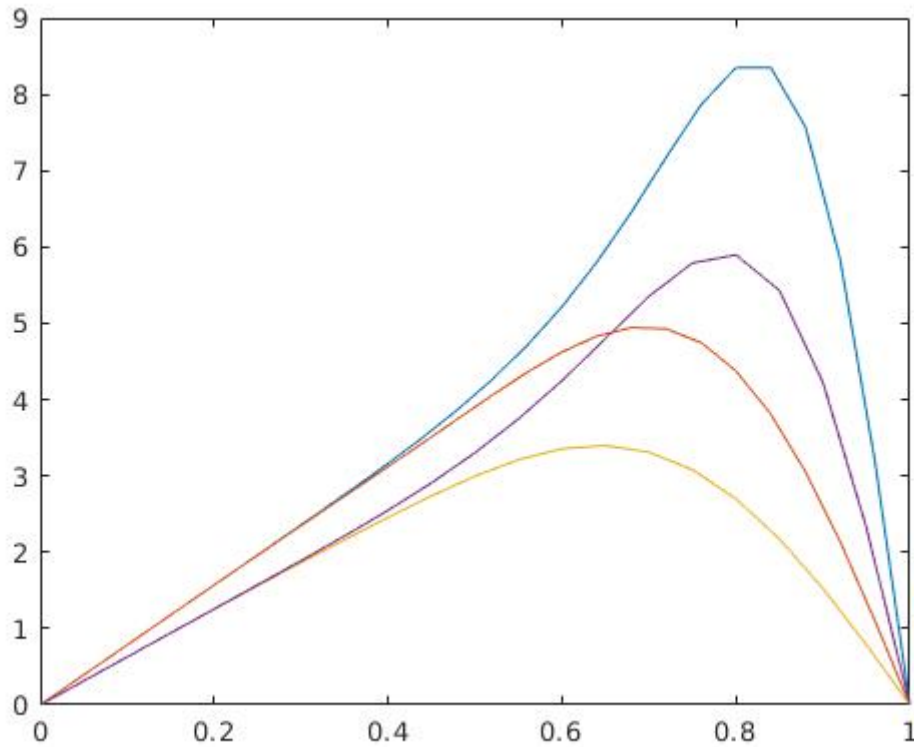


Fig 1: showing the dispersion curves for $N=3,4$ the steeper ones are one with the consistent matrices.

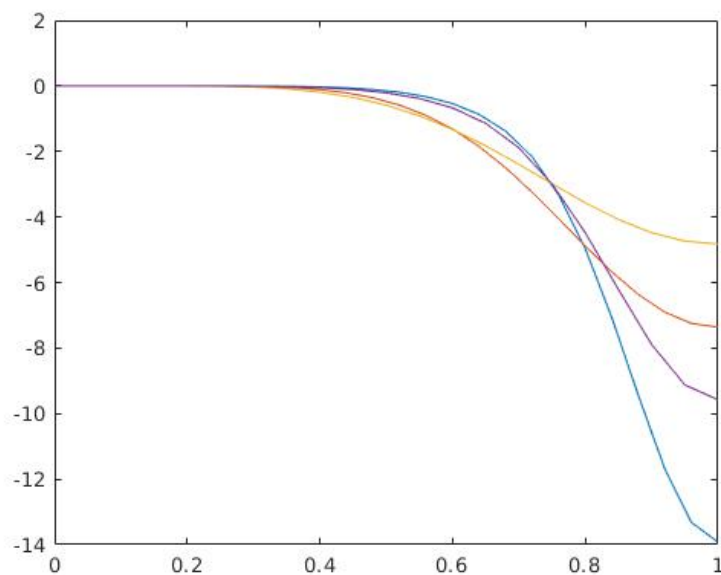


Fig 2: showing the dissipation curves for $N=3,4$ the steeper ones are one with the consistent matrices.

Simulation results for the DG Methods with GL and Guass uadrature.

The mass matrices for both the Guass and Guass Lobatto quadratures are the same . But the stiffness matrices differ, this indeed causes the simulation results to oscillate . The simulation of the transport could be seen by uncommenting the transport simulation section and running the code.

The local mass and stiffness matrcies for N=4 is shown below for both GL and guass quadrature.

Note : the lagrange polynomials and the derivative matrices are evaluated on the guass lobatto grid points.

Mass_mat_GL =

0.0889	0.0259	-0.0296	0.0259	-0.0111
0.0259	0.4840	0.0691	-0.0605	0.0259
-0.0296	0.0691	0.6321	0.0691	-0.0296
0.0259	-0.0605	0.0691	0.4840	0.0259
-0.0111	0.0259	-0.0296	0.0259	0.0889

Mass_mat1_GQ =

0.0889	0.0259	-0.0296	0.0259	-0.0111
0.0259	0.4840	0.0691	-0.0605	0.0259
-0.0296	0.0691	0.6321	0.0691	-0.0296
0.0259	-0.0605	0.0691	0.4840	0.0259
-0.0111	0.0259	-0.0296	0.0259	0.0889

s_GL =

-0.5000	0.6757	-0.2667	0.1410	-0.0500
-0.6757	0.0000	0.9505	-0.4158	0.1410
0.2667	-0.9505	0.0000	0.9505	-0.2667
-0.1410	0.4158	-0.9505	-0.0000	0.6757
0.0500	-0.1410	0.2667	-0.6757	0.5000

s_GQ =

-0.4171	0.5636	-0.2225	0.1176	-0.0417
-0.8783	0.4905	0.5312	-0.2147	0.0712
0.2014	-0.9160	0.2401	0.6497	-0.1753
-0.1159	0.3962	-0.8266	0.0474	0.4989
0.0572	-0.1796	0.3351	-0.5797	0.3670