WELCOME TO

# TECH I.S.

## Inferential Statistics

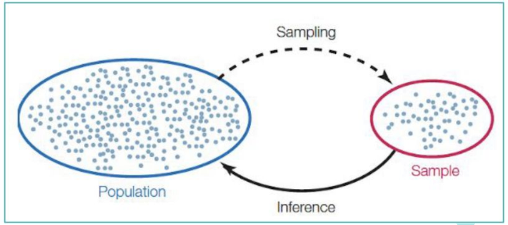
# Population and Sample

The population we wish to study are always so large that we are unable to gather information from every case thus we choose a sample – a carefully chosen subset of the population.

We use information gathered from the cases in the sample to generalize to the population

A large community college has about 25,000 students. In a study of 85 students from college, it was determined that about 60 of the students have moderate or high math anxiety.

- The population is all the students at this college.
- The sample is the 85 students whose math anxiety was measured.



# Statistical Hypothesis

**Statistic:**

Mathematical characteristics of samples.

**Parameter:**

Mathematical characteristics of populations.

**Statistics are used to estimate parameters.**

**Statistic**

$\bar{x}$ : sample mean

$s$ : sample standard deviation

**Sample Statistic**

**Parameter**

$\mu$ : population mean

$\sigma$ : population standard deviation

**Population Parameters**

# Statistical Hypothesis

<u>Statistical Hypothesis</u>: A claim about the value of a parameter or population characteristic.

<u>Example Hypothesis:</u>

- Average per student soda expenses in US high schools is 75 cents.
- $d < 10\%$ , where d is the percentage of defective helmets for a given manufacturer.

# Steps of a Hypothesis Test

**Steps for hypothesis testing:**

1. Formulate the hypothesis to be tested.
2. Determine the appropriate test statistic and calculate it using the sample data.
3. Comparison of test statistic to critical region to draw initial conclusions.
4. Calculation of p-value.
5. Conclusion, written in terms of the original problem.

# Statistical Hypothesis - STEP 1

STEP 1 - Formulate the hypothesis to be tested.

# Null vs Alternative Hypotheses

In any hypothesis-testing problem, there are always two competing hypotheses under consideration:

1. Null hypothesis - ($H_0$): A statement about the value of a population parameter that is assumed to be true for the purpose of testing.

1. Alternative hypothesis - ($H_1$): A statement about the value of a population parameter that is assumed to be true only if the Null Hypothesis is rejected during testing.

Example: A food company has a policy that stated weight match actual weight.

The quality control statistician decides to test the claim that a 16 ounce bottle of Soy sauce contains on average 16 ounces.

Ho: The mean amount of Soy Sauce is 16 ounces
Ha: The mean amount of Soy Sauce is not 16 ounces.

$Ho: \mu = 16 \quad Ha: \mu \neq 16$

# The Objective of Hypothesis Testing

**We check if there strong evidence for the alternative hypothesis?**

**We initially favour claim null hypothesis ($H_0$) will not be rejected unless the sample evidence provides significant support for the alternative assertion ($H_a$ or $H_1$)**

**The two possible conclusions:**

1. **Reject H0.**
2. **Fail to reject H0.**

# Example: Paint Product

**Example:**

Suppose a company is considering marketing a new type of paint that it produces.

The true average wear life with the current paint is known to be 1000 hours.
With μ denoting the true average life for the new paint, the company would not want to make any (costly) changes unless evidence strongly suggested that μ exceeds 1000.

An appropriate problem formulation would involve testing: → $H_0$: $\mu = 1000$ against $H_a$: $\mu > 1000$.

For switching to the new paint represented by $H_a$ -> It would take conclusive evidence to justify rejecting $H_0$

Alternative hypothesis is the hypothesis that we are trying to prove and which is accepted if we have sufficient evidence to reject the null hypothesis.

# Statistical Hypothesis - STEP 2

STEP 2 - Determine the appropriate test statistic and calculate it using the sample data

# Test Statistics

**A test statistic is a rule, based on sample data, for deciding whether to reject H0.**

**The test statistic is a function of the sample data that will be used to make a decision about whether the null hypothesis should be rejected or not.**

**Example:**

**Company A produces circuit boards, but 10% of them are defective. Company B claims that they produce fewer defective circuit boards. Given is a random sample of circuit boards n = 200 from Company B.**

$H_0$: $p$ = .10 versus $H_a$: $p$ < .10

**Which test statistic is "best"?**

**Choice of a particular test procedure must be based on the probability the test will produce incorrect results.**

# Errors in Hypothesis Testing

A **type I error** is when the null hypothesis is rejected despite of it being true.

A **type II error** is not rejecting H0 despite of H0 being false.

**Example**: Covid test kit

- **False Negative** = Test is Negative despite having Covid = **type I error**
- **False Positive** = Test is Positive despite not having Covid = type II error

| Decision | $H_0$ Is True | $H_0$ Is False |
|---|---|---|
| Fail to reject $H_0$ | No error | Type II error |
| Reject $H_0$ | Type I error | No error |

# Statistical Hypothesis - STEP 3

<u>STEP 3</u> -Comparison of test statistic to critical region to draw initial conclusions

# Level of Significance & Rejection Region

**Level of Significance:**

The probability of rejecting the null hypothesis when it is actually true - ( signified by α )

**Critical Region:**

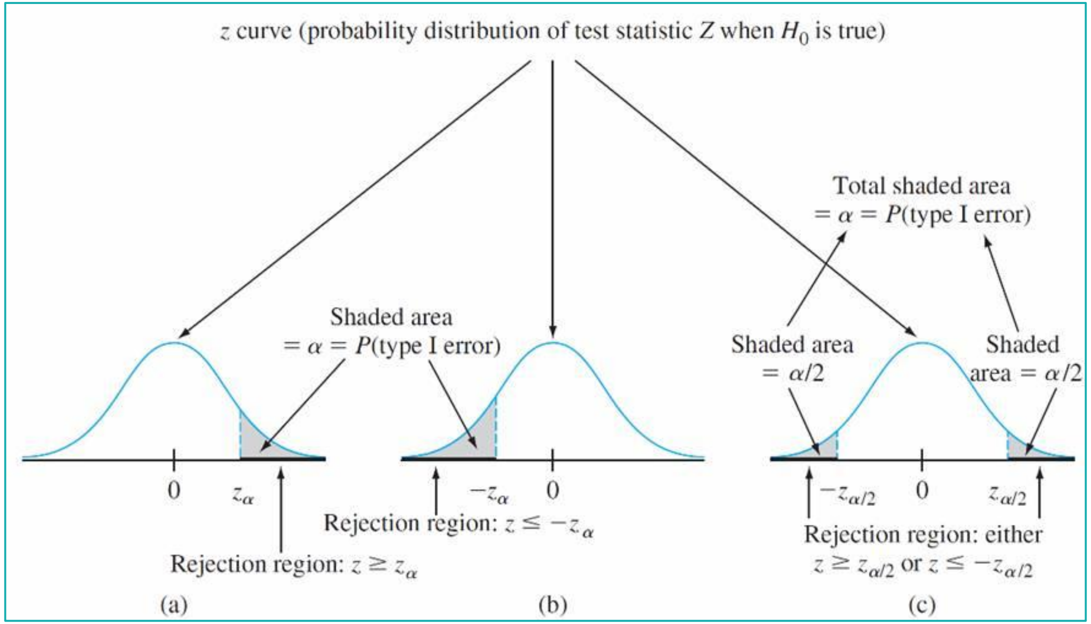
Region(s) of the Statistical Model which contain the values of the Test Statistic where the Null Hypothesis will be rejected.

We Specify the largest value of α that can be tolerated, and then find a rejection region with respect to α.

# Critical Regions

**Critical regions using z tests:**

**(a) upper-tailed test ; (b) lower-tailed test ; (c) two-tailed test**



# One-Tailed Tests of Significance

**A test is one-tailed when the alternate hypothesis, Ha , states a direction, such as:**

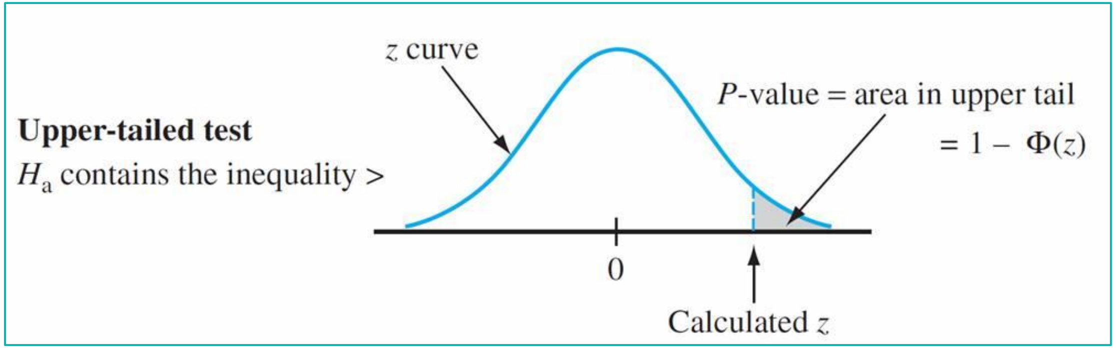
**H0 : The mean income of females is less than or equal to the mean income of males.**
**Ha : The mean income of females is greater than males.**

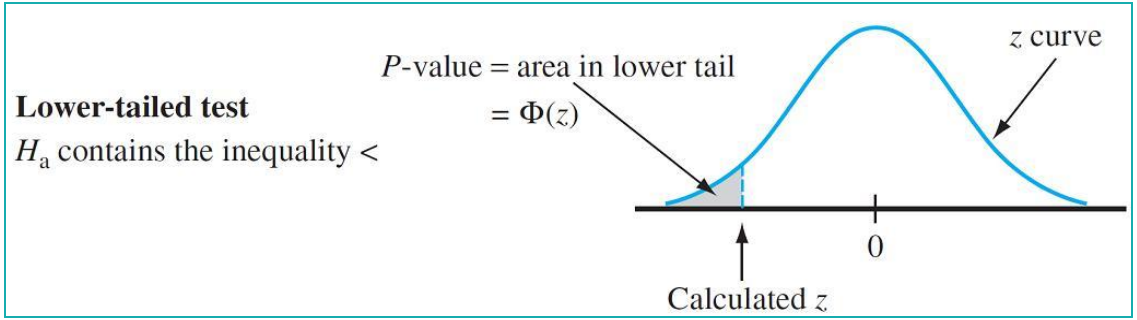
Equality is part of $H_0$

**Ha determines which tail to test:**

$H_a$: $\mu>\mu_0$ means test upper tail.
$H_a$: $\mu<\mu_0$ means test lower tail.

# Left-tailed Test & Right-tailed Test



# Two-Tailed Tests of Significance

A test is two-tailed when no direction is specified in the alternate hypothesis Ha , such as:

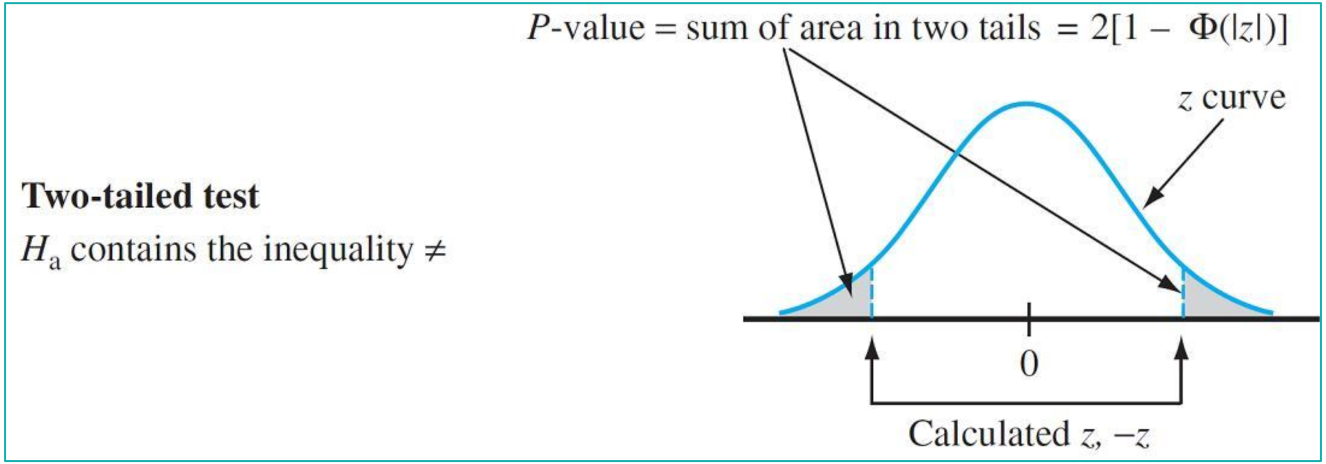
H0 : The mean income of females is equal to the mean income of males.
Ha : The mean income of females is not equal to the mean income of the males.

Equality is part of $H_0$

Ha determines which tail to test:

$H_a$: $\mu \neq \mu_0$ means test both tails.

# Two-tailed test



# Statistical Hypothesis - STEP 4

STEP 4 - Calculation of p-value

# p-value in Hypothesis Testing

The p-value is the the probability, assuming that the null hypothesis is true, of getting a value of the test statistic - at least as extreme as the computed value.

So, the smaller the P-value, the more evidence there is in the sample data against the null hypothesis and for the alternative hypothesis.

- If the p-value is smaller than the significance level, H0 is rejected.
- If the p-value is larger than the significance level, H0 is not rejected.

# Graphic where decision is to Reject Ho

Ho: $\mu = 10$
Ha: $\mu > 10$

Design: Critical Value is determined by significance level $\alpha$.

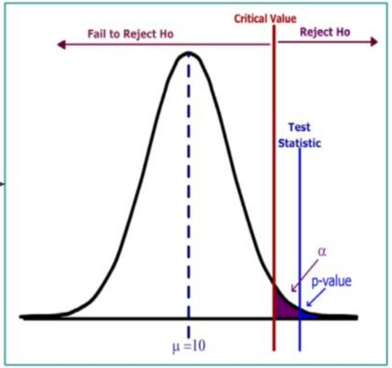
Data Analysis: p-value is determined by Test Statistic

Test Statistic falls in Rejection Region.

p-value (blue) < $\alpha$ (purple)

Reject Ho.

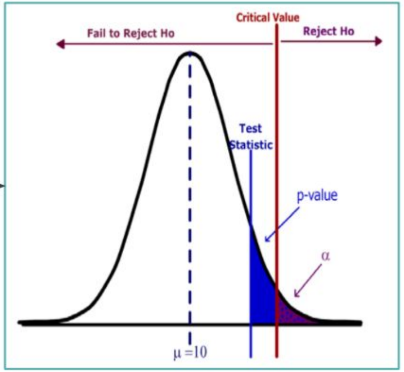
Strong statement: Data supports Alternative Hypothesis.




p-value (blue) > $\alpha$ (purple)

Fail to Reject Ho.

Weak statement: Data is inconclusive and does not support Alternative Hypothesis.



# EXAMPLE – Food Quantity

**Example:**

A food company has a policy that the stated contents of a product match the actual results.
A General Question might be "Does the stated net weight of a food product match the actual weight?"
The quality control statistician decides to test the 16 ounce bottle of Soy Sauce.

A sample of n=36 bottles will be selected hourly and the contents weighed. Assume α = 0.5

$H_o: \mu = 16 \quad H_a: \mu \neq 16$

The Statistical Model will be the one population test of mean using the Z Test Statistic and We will choose a significance level of 5%

# Conduct Experiment – Food Quantity

Last hour a sample of 36 bottles had a mean weight of 15.88 ounces.
From past data, assume the population standard deviation is 0.5 ounces.

Compute the Test Statistic:

$$Z = [15.88 - 16] / [.5 / \sqrt{36}] = -1.44$$

For a two tailed test, The Critical Values are at Z = ±1.96

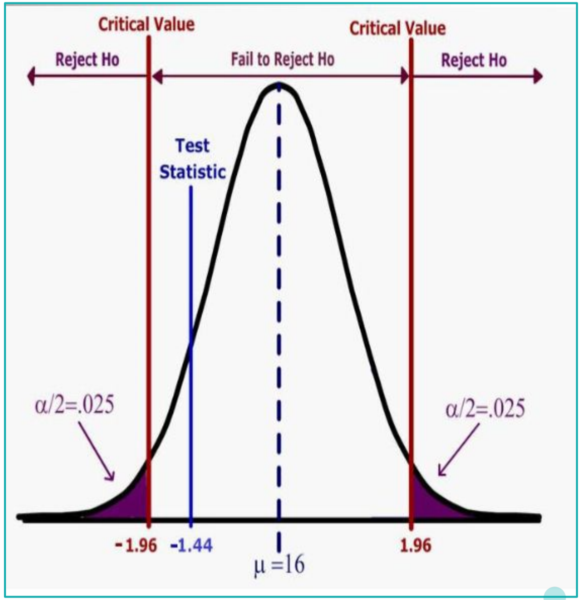
# Decision – Critical Value Method

This two-tailed test has two Critical Value and Two Rejection Regions.

The significance level (α) must be divided by 2 so that the sum of both purple areas is 0.05

The Test Statistic does not fall in the Rejection Regions.

Decision is Fail to Reject Ho.



# Statistical Hypothesis - STEP 5

**STEP 5** - Conclusion, written in terms of the original problem

# Converting Decision to Conclusion

Decision is Fail to Reject Ho.

- There is insufficient evidence to conclude that the mean amount of soy sauce being filled into bottles is not 16 ounces.
- There is insufficient evidence to conclude machine that fills 16 ounce soy sauce bottles is operating improperly.

# Much obliged.

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