WELCOME TO

TECH I.S.

Counting - Permutations & Combinations

Counting

If all outcomes are equally likely, the probability of an event E is give P(E), where |E| and |S| denotes the number of elements.

$$\frac{|E|}{|S|}$$

<u>To apply this rule, we need to be able to count the number of elements in events.</u>

- Multiplication Rules
- Permutations of distinct objects
- Permutations where some objects are identical
- Combinations

Multiplication Rule

If one operation can be done in n1 ways and a second operation can be done in n2 ways then the number of different ways of doing both is n1*n2.

Examples:

- If we roll a fair 6-sided die and toss a coin, the total number of possible outcomes is 6 × 2 = 12.
- If we roll a fair 4-sided die 3 times, the total number of possible outcomes is 4 × 4 × 4 = 64.
- A simple survey consists of three multiple choice questions. The first question has 3 possible answers, the second has 4 possible answers and the third has 3 possible answers. What is the total number of different ways in which this survey could be completed?
    - 3×4×3 = 36.

Permutations

A permutation is an arrangement of a collection/set of objects in a definite order.

<u>How many different arrangements/permutations of n distinct objects are possible?</u>:

1. The first object can be chosen in n ways;
2. The second object can then be chosen in n – 1 ways and so on;
3. The number of ways of permuting (arranging in order) n distinguishable objects is n!

$$n!\ (n \text{ factorial}) = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Permutations - Examples

Examples of Pure Permutations:

- The total number of different ways in which the letters of the word “count” can be arranged is 5! = (5)(4)(3)(2)(1) = 120.

- 6 horses run a race. The total number of possible results of this race (assuming no ties) is 6! = (6)(5)(4)(3)(2)(1) = 720.

What if not all the objects are distinct?

<u>What is the total number of different arrangements of the letters in the word "stat"?</u>

1. In the word stat - Suppose the two "t"s can be distinguished - t_1 and t_2.
2. Then we would have 4! arrangements.
3. Also we would generate 2! arrangements of t_1 and t_2.
4. So the number of arrangements of the word stat is (4! / 2!) = 12.

In general if we have n items and k of which are identical, the total number of distinct permutations is:

$$\boxed{\frac{n!}{k!}}$$

Permutations with some Objects Identical

How many different ways can we rearrange the letters of MISSISSIPPI?

We have 11 letters in total, of which 4 are ‘I’, 4 are ‘S’ and ‘2’ are ‘P’.

M	I	S	P
	I	S	P
	I	S	
	I	S	

$$\frac{11!}{4!4!2!}$$

r - Permutations

How many permutations of n distinct objects, taken r at a time are possible?

1. Again, we have n ways of choosing the first object.
2. We then have n – 1 ways of choosing the second object and so on.
3. When choosing the r th object, we have already chosen r – 1 objects, so there are still

   [n – (r – 1) = n – r + 1] possible choices.
4. The total number of r permutations of a set of n distinguishable objects is:

$$\boxed{n \cdot (n-1) \cdots (n-r+1).}$$

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}.}$$

r -Permutations - Example

What is the total number of different 3-letter words that can be formed from the letters ‘spring’ if no letters are repeated? (A word is any arrangement of letters in order - we’re not playing scrabble!)

Answer :

$^{6}P_{3}$

Continuing with the previous example, if letters can be repeated, what is the total number of 3-letter words repeating at least 1 letter that can be formed from ‘spring’

1. Total number of words when we allow letter to be repeated is 6^3.
2. Total number of words with no repetition is $^{6}P_{3}$.
3. Hence our answer is $6^3 - {}^{6}P_{3}$.

Combinations

How many different ways can we select a set of size r from a larger set of n distinguishable objects?

<u>The order of selection does not matter.</u>

We are asking for the number of combinations of n objects taken r at a time.

$$\binom{n}{r}$$

Each combination/set of r-objects can be permuted in exactly r! distinct ways.

$$\binom{n}{r} = \frac{{}^{n}P_{r}}{r!} = \frac{n!}{r!(n-r)!}$$

Combinations - Examples

How many ways can a company select 3 candidates to interview from a short list of 15?

$$\binom{15}{3} = \frac{(15)(14)(13)}{(3)(2)(1)} = 455$$

In how many ways can a subcommittee of 5 be chosen from a panel of 20?

$$\binom{20}{5} = \frac{(20)(19)(18)(17)(16)}{(5)(4)(3)(2)(1)} = 15504.$$

Much obliged.

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