WELCOME TO

TECH I.S.

Matrices

Matrices

Matrix is a rectangular array of real-valued scalars arranged in *m* horizontal rows and *n* vertical column.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Each element (a_{ij}) belongs to the (i^{th}) row and (j^{th}) column

Matrices Algebra I

Matrix Addition or Subtraction

$$(\mathbf{A} \pm \mathbf{B})_{i,j} = \mathbf{A}_{i,j} \pm \mathbf{B}_{i,j}$$

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 5 \\ 7 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 & 1+5 \\ 1+7 & 0+5 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 8 & 5 & 0 \end{bmatrix}$$

Scalar multiplication

$$(c\mathbf{A})_{i,j} = c \cdot \mathbf{A}_{i,j}$$

$$2 \cdot \begin{bmatrix} 1 & 8 & -3 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 & 2 \cdot 8 & 2 \cdot -3 \\ 2 \cdot 4 & 2 \cdot -2 & 2 \cdot 5 \end{bmatrix} = \begin{bmatrix} 2 & 16 & -6 \\ 8 & -4 & 10 \end{bmatrix}$$

Matrix Multiplication

$$(\mathbf{AB})_{i,j} = \mathbf{A}_{i,1}\mathbf{B}_{1,j} + \mathbf{A}_{i,2}\mathbf{B}_{2,j} + \cdots + \mathbf{A}_{i,n}\mathbf{B}_{n,j}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Matrices Algebra II

Transpose of the matrix

$$\left(\mathbf{A}^T\right)_{i,j} = \mathbf{A}_{j,i}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -6 & 7 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 0 \\ 2 & -6 \\ 3 & 7 \end{bmatrix}$$

Properties of Transpose

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$
$$(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \boldsymbol{B}^T$$
$$(\mathbf{A}^T)^T = \mathbf{A}$$
$$\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$$
$$\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$$
$$(\mathbf{AB})^T = \boldsymbol{B}^T\mathbf{A}^T$$

Identity matrix

$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrices Algebra III

Determinant of the matrix

$\det(\mathbf{A})$ or $|\mathbf{A}|$

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

Trace of Matrix

sum of all diagonal elements

$$\mathrm{Tr}(\mathbf{A}) = \sum_i a_{ii}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Trace =1+5+9
=15

Linear Independence & Matrix Rank

Linear Independence:

$$\sum_{i=1}^{k} a_i \mathbf{v_i} = 0$$

For a column vector $v = (v_1, v_2, \ldots, v_k)$,

v_i are linearly independent if given condition is <u>not true</u>.

Rank of Matrix:

Rank of the matrix is the <u>largest number of linearly independent columns</u>.

$$\mathbf{C} = \begin{bmatrix} 1 & 3 & 0 & -1 & 0 \\ -1 & 0 & 1 & 1 & -1 \\ 0 & -3 & 1 & 0 & -1 \\ 2 & 3 & -1 & -2 & 1 \end{bmatrix}$$

$$\mathbf{c}_4 = -1 \cdot \mathbf{c}_1, \ \mathbf{c}_5 = -1 \cdot \mathbf{c}_3, \ \mathbf{c}_2 = 3 \cdot \mathbf{c}_1 + 3 \cdot \mathbf{c}_3$$

$$rank(\mathbf{C}) = 2$$

Inverse of a Matrix

For a square matrix *A*, its inverse matrix A^{-1} is given by:

$$\mathbf{A}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example:

$$F = \begin{bmatrix} 4 & -10 \\ 3 & 2 \end{bmatrix}$$

$$F^{-1} = \frac{1}{4(2) - 3(-10)} \begin{bmatrix} 2 & 10 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{19} & \frac{5}{19} \\ -\frac{3}{38} & \frac{2}{19} \end{bmatrix}$$

Eigen Decomposition

Eigen decomposition is decomposing a matrix *A* into a set of eigenvalues and eigenvectors.

Eigenvalues of a matrix A are Scalars λ

Eigenvectors of a matrix A are non-zero vectors v

Such that they satisfy the following equation:

Eigenvector of Matrix **A**

$$\mathbf{Ax} = \lambda \mathbf{x}$$

Eigenvalue of Matrix **A**

$$\underbrace{\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}}_{A} \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix} = \underbrace{4}_{\textit{eigenvalue}} \underbrace{\begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}}_{\textit{eigenvector}}$$

Example - Eigen Decomposition

Scalar λ and vector v are eigenvalues and eigenvectors of Matrix A:

Eigenvector of Matrix **A**

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$$

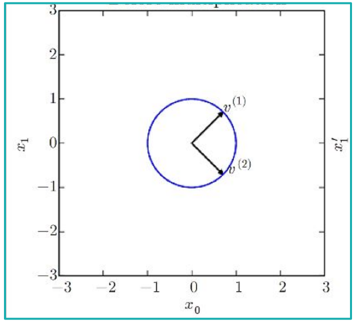
Eigenvalue of Matrix **A**

$$\underbrace{\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}}_{A} \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix} = \underbrace{4}_{\textit{eigenvalue}} \underbrace{\begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}}_{\textit{eigenvector}}$$

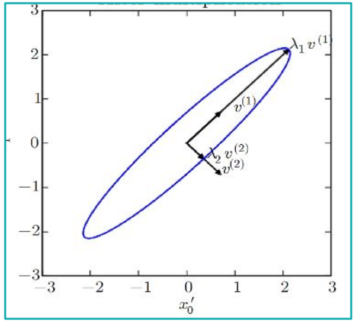
Use Case: Eigen Decomposition

Eigenvalues and Eigenvectors allow us stretch the linear space in specific directions.

BEFORE
v^1, v^2



AFTER
$(v^1, v^2) * (\lambda^1, \lambda^2)$



Eigenvalues & Eigenvectors used with PCA (principal component analysis) Technique for extracting the most important data.

Used in Data Compression.

Much obliged.

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