WELCOME TO

TECH I.S.

Continuous Random Variables

Continuous Random Variable

Random Variable: A random variable is one whose value is not known ahead of time.

Continuous Random Variable:

- Data that you Measure.
- "Uncountable" Number of possibilities
- Probability at a point makes no sense thus Probability is measured over intervals.
- Example: Temperature, Height, Time

A probability distribution for continuous random variables is described using a probability density function (PDF).

Probability Density Function - PDF

A probability distribution for continuous random variables is described using a <u>probability density function (PDF).</u>

$$P(X \in [a,b]) = \int_a^b P(X)dX$$

F(x)	$\sum_{i=1}^{n} p_i = 1$	$\int_{-\infty}^{\infty} f(x)dx = 1$
Mean μ	$\sum_{i=1}^{n} x_i p_i$	$\int_{-\infty}^{\infty} xf(x)dx$
Variance σ^2	$\sum_{i=1}^{n} (x_i - \mu)^2 p_i$	$\int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Exponential Distribution

The likelihood of when the event occurs next does not depend on when it happened previously - Memoryless Distribution

$$P(X > x) = e^{-x\lambda} \quad \text{where } x \text{ is} > 0$$

The exponential distribution is closely related to the Poisson distribution. If the Poisson distribution represents the number of successful events per unit time, then the exponential distribution models the time period between 2 consecutive successful events.

Exponential Distribution - Contd

<u>Example</u>: The time until a screen is cracked on a smartphone has exponential distribution with mu = 500 hours of use.

A: Find the probability screen will not crack for at least 600 hours.

B: Assuming that screen has already lasted 500 hours without cracking, find the chance the display will last an additional 600 hours.

$$P(x>600) = e^{-600/500} = e^{-1.2} = .3012$$

$$P(x>1100|x>500) = P(x>600) = .3012$$

Uniform Distribution

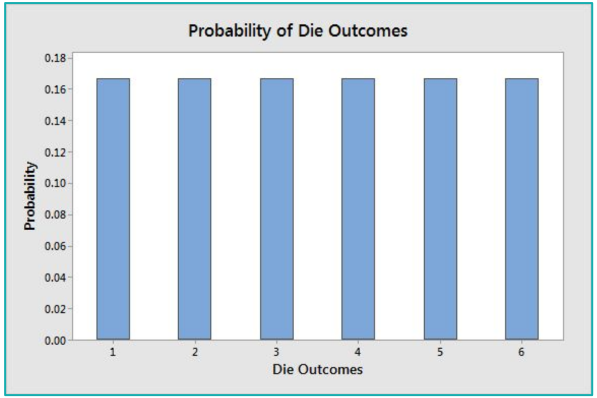
The probability of each value is same thus, Rectangular distribution.

$$f(x) = \frac{1}{b-a} \qquad a \le x \le b$$

$$\mu = E(X) = \frac{b+a}{2}$$

$$\sigma^2 = Var(X) = \frac{(b-a)^2}{12}$$

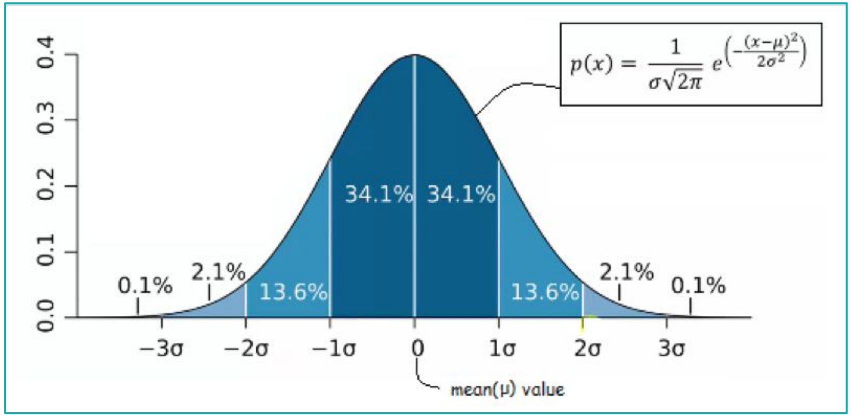
<u>Example</u>: Probability of a dice throw.



Normal Distribution

Normal Distribution exhibit "centrality".
The concentration is maximum near the central or mean value and it reduces symmetrically as we move away from it.

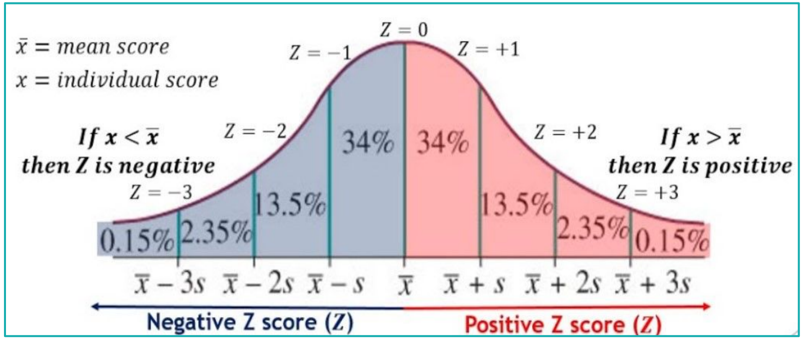
$$P_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Normal Distribution - Contd

A normal distribution with a mean of 0 and a standard deviation of 1 is called the standard normal distribution.

Z value: The distance between a selected value, designated x, and the population mean μ , divided by the population standard deviation σ.



$$Z = \frac{X - \mu}{\sigma}$$

Normal Distribution - Example

The daily water usage per person in a town is normally distributed with a mean of 20 gallons and a standard deviation of 5 gallons.

Given: Interval in terms of X → Convert to Z-value → Look up probability in table

Q A: What is probability that a person from the town selected at random will use less than 18 gallons per day.

$P(X<18)=P(Z<-0.40)=\mathbf{0.3446}$

Q B: What proportion of the people uses between 18 and 24 gallons?

$P(18<X<24)= P(-0.40<Z<0.80)$
$= .7881. - 3446$
$= \mathbf{0.4435}$

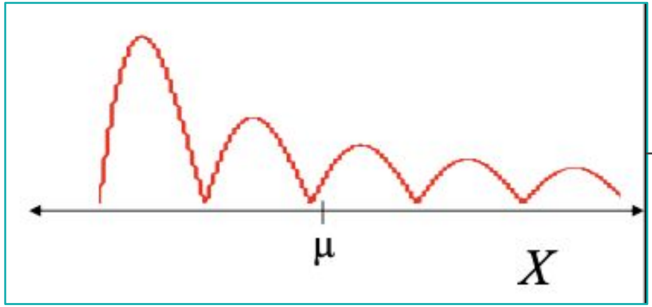
Q C: What proportion of the people uses between 18 and 24 gallons?

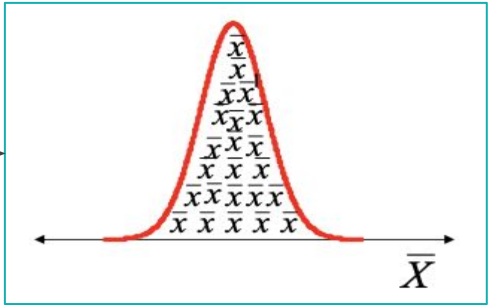
$P(X>26.2)=P(Z>1.24)$
$=1-.8925= \mathbf{0.1075}$

Central Limit Theorem

If a random sample of sufficiently large size is taken from a population with any Distribution with mean = μ and standard deviation = σ .

Then the distribution of the sample mean has approximately a Normal Distribution.





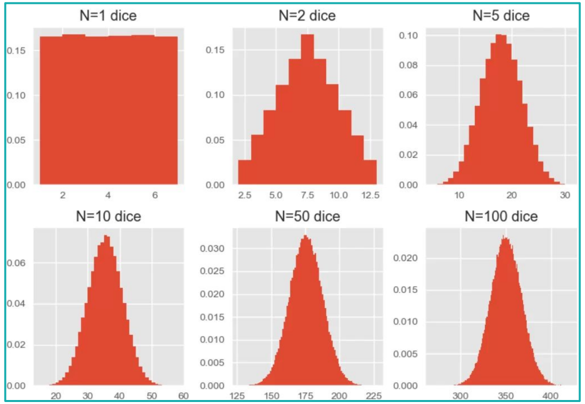
Central Limit Theorem - Properties

Important results for Central Limit Theorem:

Mean Stays the same → $\mu_{\overline{X}} = \mu$

Standard Deviation Gets Smaller → $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}}$

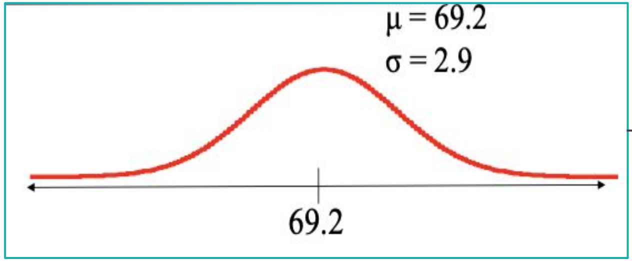
If n is sufficiently large, X has a Normal Distribution

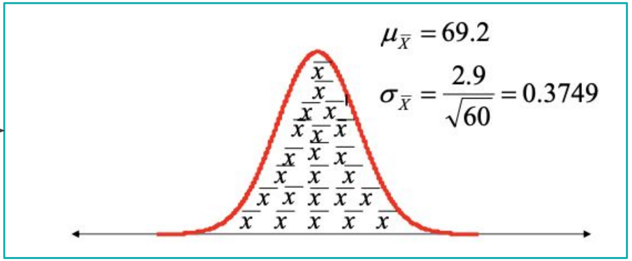


Central Limit Theorem - Example

The mean height of American men (ages 20-29) is = 69.2 inches. If a random sample of 60 men in this age group is selected, what is the probability the mean height for the sample is greater than 70 inches? Assume σ = 2.9”.

$$P(\overline{X} > 70) = P\left(Z > \frac{(70 - 69.2)}{2.9/\sqrt{60}}\right) = P(Z > 2.14) = 0.0162$$





Much obliged.

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