

WELCOME TO



**Counting - Permutations &  
Combinations**



# Counting

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If all outcomes are equally likely, the probability of an event  $E$  is give  $P(E)$ , where  $|E|$  and  $|S|$  denotes the number of elements.


$$\frac{|E|}{|S|}$$

To apply this rule, we need to be able to count the number of elements in events.

- Multiplication Rules
- Permutations of distinct objects
- Permutations where some objects are identical
- Combinations



# Multiplication Rule

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If one operation can be done in  $n_1$  ways and a second operation can be done in  $n_2$  ways then the number of different ways of doing both is  $n_1 \times n_2$ .



## Examples:

- If we roll a fair 6-sided die and toss a coin, the total number of possible outcomes is  $6 \times 2 = 12$ .
- If we roll a fair 4-sided die 3 times, the total number of possible outcomes is  $4 \times 4 \times 4 = 64$ .
- A simple survey consists of three multiple choice questions. The first question has 3 possible answers, the second has 4 possible answers and the third has 3 possible answers. What is the total number of different ways in which this survey could be completed?
  - $3 \times 4 \times 3 = 36$ .



# Permutations

A permutation is an arrangement of a collection/set of objects in a definite order.



How many different arrangements/permutations of  $n$  distinct objects are possible?:

1. The first object can be chosen in  $n$  ways;
2. The second object can then be chosen in  $n - 1$  ways and so on;
3. The number of ways of permuting (arranging in order)  $n$  distinguishable objects is  $n!$



$$n! \text{ (} n \text{ factorial)} = n \cdot (n - 1) \cdot (n - 2) \cdots 3 \cdot 2 \cdot 1$$



# Permutations - Examples

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## Examples of Pure Permutations:

- The total number of different ways in which the letters of the word “count” can be arranged is  
 $5! = (5)(4)(3)(2)(1) = 120.$
- 6 horses run a race. The total number of possible results of this race (assuming no ties) is  $6! = (6)(5)(4)(3)(2)(1) = 720.$



# What if not all the objects are distinct?

What is the total number of different arrangements of the letters in the word "stat"?

1. In the word stat - Suppose the two "t"s can be distinguished -  $t_1$  and  $t_2$ .
2. Then we would have  $4!$  arrangements.
3. Also we would generate  $2!$  arrangements of  $t_1$  and  $t_2$ .
4. So the number of arrangements of the word stat is  $(4! / 2!) = 12$ .

In general if we have  $n$  items and  $k$  of which are identical, the total number of distinct permutations is:

$$\frac{n!}{k!}$$

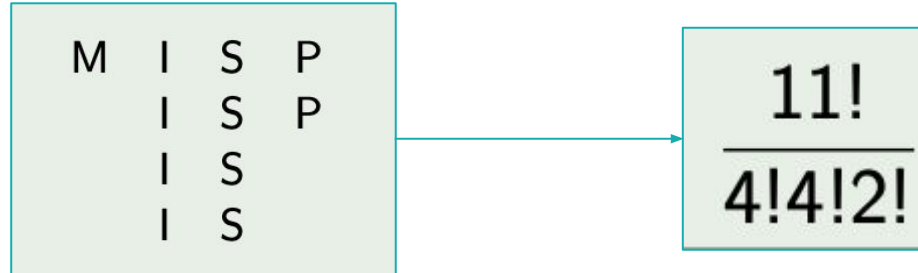


# Permutations with some Objects Identical

How many different ways can we rearrange the letters of MISSISSIPPI?



We have 11 letters in total, of which 4 are 'I', 4 are 'S' and '2' are 'P'.



# r - Permutations

How many permutations of n distinct objects, taken r at a time are possible?

1. Again, we have n ways of choosing the first object.
2. We then have n - 1 ways of choosing the second object and so on.
3. When choosing the r<sup>th</sup> object, we have already chosen r - 1 objects, so there are still [ n - (r - 1) = n - r + 1 ] possible choices.
4. The total number of r permutations of a set of n distinguishable objects is:

$$n \cdot (n - 1) \cdots (n - r + 1).$$

$${}^n P_r = \frac{n!}{(n - r)!}.$$





## r -Permutations - Example

What is the total number of different 3-letter words that can be formed from the letters 'spring' if no letters are repeated? (A word is any arrangement of letters in order - we're not playing scrabble!)

Answer :

$${}^6P_3$$



Continuing with the previous example, if letters can be repeated, what is the total number of 3-letter words repeating at least 1 letter that can be formed from 'spring'



1. Total number of words when we allow letter to be repeated is  $6^3$ .
2. Total number of words with no repetition is  ${}^6P_3$ .
3. Hence our answer is  $6^3 - {}^6P_3$ .



# Combinations

How many different ways can we select a set of size  $r$  from a larger set of  $n$  distinguishable objects?

The order of selection does not matter.

We are asking for the number of combinations of  $n$  objects taken  $r$  at a time.

$$\binom{n}{r}$$

Each combination/set of  $r$ -objects can be permuted in exactly  $r!$  distinct ways.

$$\binom{n}{r} = \frac{{}^nP_r}{r!} = \frac{n!}{r!(n-r)!}$$



# Combinations - Examples

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How many ways can a company select 3 candidates to interview from a short list of 15?

$$\binom{15}{3} = \frac{(15)(14)(13)}{(3)(2)(1)} = 455$$

In how many ways can a subcommittee of 5 be chosen from a panel of 20?

$$\binom{20}{5} = \frac{(20)(19)(18)(17)(16)}{(5)(4)(3)(2)(1)} = 15504.$$



**Much obliged.**

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**TECH I.S.**

