Math6120 - Nonlinear Optimisation
Coursework #1

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Contents

Chapter 1

The Problem

1.1 Formulation of Model and Assumptions Made

1.1.1 Deriving our Decision Variables

In this assignment, we seek to maximise the height of a rocket at the end of the first stage burn of launch on behalf of Elon Musk for his Falcon Heavy launch vehicle. We seek to do so by means of optimising the fuel flow into the combustion chamber of the rocket engine during the first stage burn interval.

To begin, we define the first stage burn interval as the time interval from launch time $t_0 = 0$ to the time at which all fuel has been used and the rocket finishes firing t_{final}. We control the flow of fuel into the chamber by means of considering five key variables:

- t_{final} = the total length of the first stage burn interval
- $h(t)$ = the height of the rocket at time t
- $v(t)$ = the speed of the rocket at time t
- $m(t)$ = the mass of the rocket at time t
- $\theta(t)$ = the thrust of the rocket at time t

On account of four of these being functions, we transform the first stage burn interval into a series of discrete points in time. Let $[t_0, t_{final}]$ be our interval. Let N be the number of sub-intervals we wish to define. We may then define a series of $N+1$ points in time during the first stage burn interval in the following way:

$$t_k = \frac{k}{N} t_{\text{final}}$$
$$k \in \{0, 1, 2, ..., N-1, N\}$$

This gives us a sequence of $N+1$ time points at which we may consider each of our four above functions. As a result, for a given parameter N, we have $1+4(N+1) = 1+4N+4 = 4N+5$ total variables to consider. Though the height, mass and velocity are all determined by the thrust produced by the engine. As per the problem specification, fuel flow into the engine is proportional to the thrust achieved by the rocket. As a result, since we are optimising our fuel flow, the thrust at every point in time is a decision variable which we may change to alter the height, mass and velocity. The total duration is also a decision we may make. Consequently, of our $4N+5$ variables, $(N+1)+1 = N+2$ are decision variables. Hence the following decision vector:

$$x = \begin{pmatrix} t_{\text{final}} \\ \\ \theta(t_0) \\ \theta(t_1) \\ \dots \\ \theta(t_{N-1}) \\ \theta(t_N) \\ \\ \end{pmatrix} = \begin{pmatrix} t_{\text{final}} \\ \\ \bar{\theta} \end{pmatrix} \in \mathbf{R}^{N+2}$$

For simplicity, we adopt the following notation:

$$\bar{\theta} = \begin{pmatrix} \theta(t_0) \\ \theta(t_1) \\ \dots \\ \theta(t_{N-1}) \\ \theta(t_N) \end{pmatrix}$$

1.1.2 Deriving our Objective Function

We seek to maximise the height of the rocket, $h(t)$, at the end of the first stage burn interval, t_{final}. This gives us the objective function:

$$\text{maximise} \quad h(t_{\text{final}})$$

1.1.3 Deriving our Constraints

We have three constraints describing the behaviour of the rocket itself during the first stage burn interval:

$$h'(t) = v(t) \tag{1.1}$$

$$v'(t) = \frac{\theta(t) - D(h(t), v(t))}{m(t)} - \left(\frac{h(0)}{h(t)}\right)^2 \tag{1.2}$$

$$m'(t) = -2\theta(t) \tag{1.3}$$

Where $D(h(t), v(t))$ is an auxiliary function with parameter D_0 defined in the following way:

$$D(h(t), v(t)) := D_0 \times (v(t))^2 \times exp\bigg(- h_0 \frac{h(t) - h(0)}{h(0)}\bigg)$$

To represent these constraints, we replace the left-hand side slopes f'(t) of the tangent of each function by the slope of a secant. We then replace each constraint with $N + 1$ corresponding constraints to follow this rule at every point in time.

We have 3 equations substituting for equation (1.1):

$$\frac{h(t_1) - h(t_0)}{(t_{\text{final}}/N)} = v(t_0) \tag{1.4}$$

$$\frac{h(t_{k+1}) - h(t_{k-1})}{(2t_{\text{final}}/N)} = v(t_k) \tag{1.5}$$

$$\frac{h(t_N) - h(t_{N-1})}{(t_{\text{final}}/N)} = v(t_N) \tag{1.6}$$

We have 3 equations substituting for equation (1.2):

$$\frac{v(t_1) - v(t_0)}{(t_{\text{final}}/N)} = \frac{\theta(t_0) - D(h(t_0), v(t_0))}{m(t_0)} - \left(\frac{h(0)}{h(t_0)}\right)^2 \tag{1.7}$$

$$\frac{v(t_{k+1}) - v(t_{k-1})}{(2t_{\text{final}}/N)} = \frac{\theta(t_k) - D(h(t_k), v(t_k))}{m(t_k)} - \left(\frac{h(0)}{h(t_k)}\right)^2 \tag{1.8}$$

$$\frac{v(t_N) - v(t_{N-1})}{(t_{\text{final}}/N)} = \frac{\theta(t_N) - D(h(t_N), v(t_N))}{m(t_N)} - \left(\frac{h(0)}{h(t_N)}\right)^2 \tag{1.9}$$

We have 3 equations substituting for equation (1.3):

$$\frac{m(t_1) - m(t_0)}{(t_{\text{final}}/N)} = -2\theta(t_0) \tag{1.10}$$

$$\frac{m(t_{k+1}) - m(t_{k-1})}{(2t_{\text{final}}/N)} = -2\theta(t_k) \tag{1.11}$$

$$\frac{m(t_N) - m(t_{N-1})}{(t_{\text{final}}/N)} = -2\theta(t_N) \tag{1.12}$$

In the middle equation of each of the above systems, $k \in \{1, ..., N-1\}$. The middle equation describes all points in time between time t_0 and t_N. Hence, we have a total $3(N+1) = 3N+3$ constraints describing the behaviour of the rocket during the first stage burn interval.

We also have the following boundary conditions from the problem specification:

$$v(0) = 0 \tag{1.13}$$

$$h(0) = 1 \tag{1.14}$$

$$m(0) = 1 \tag{1.15}$$

$$m(t_{\text{final}}) = 0.6 \tag{1.16}$$

Additionally, we have the following bounds for every point in time t:

$$v(t) \geq 0 \tag{1.17}$$

$$h(t) \geq h(0) \tag{1.18}$$

$$m(t_{\text{final}}) \leq m(t) \leq m(0) \tag{1.19}$$

$$0 \leq \theta(t) \leq \theta_{\max} \tag{1.20}$$

We may also adopt the notation $f_k = f(t_k)$ to represent the value of a given function f at time t_k. As a result, with rearrangement of the above, we have $10 + 3(N+1) = 3N+13$ total constraints for our problem. Hence, we have the following mathematical optimization model for our problem:

$$
\begin{array}{lll}
\max\limits_{x} & h(t_{\text{final}}) & \\
 & & \\
\text{subject to} & v(0) = 0 & \\
 & h(0) = 1 & \\
 & m(0) = 1 & \\
 & m(t_{\text{final}}) = 0.6 & \\
 & & \\
 & v(t) \geq 0 & \\
 & h(t) \geq h(0) & \\
 & m(t) \geq m(t_{\text{final}}) & \\
 & m(t) \leq m(0) & \\
 & \theta(t) \geq 0 & \\
 & \theta(t) \leq \theta_{\max} & \\
 & & \\
 & N(h(1) - h(0)) & = t_{\text{final}} \times v(0) \\
 & N(h(k+1) - h(k-1)) & = 2 \times t_{\text{final}} \times v(k) \\
 & N(h(N) - h(N-1)) & = t_{\text{final}} \times v(N) \\
 & & \\
 & N(v(1) - v(0)) & = t_{\text{final}} \times \left(\frac{\theta(t_0) - D(h(t_0), v(t_0))}{m(t_0)} - \left(\frac{h(0)}{h(t_0)} \right)^2 \right) \\
 & N(v(k+1) - v(k-1)) & = 2 \times t_{\text{final}} \times \left(\frac{\theta(t_k) - D(h(t_k), v(t_k))}{m(t_k)} - \left(\frac{h(0)}{h(t_k)} \right)^2 \right) \\
 & N(v(N) - v(N-1)) & = t_{\text{final}} \times \left(\frac{\theta(t_N) - D(h(t_N), v(t_N))}{m(t_N)} - \left(\frac{h(0)}{h(t_N)} \right)^2 \right) \\
 & & \\
 & N(m(1) - m(0)) & = t_{\text{final}} \times -2\theta(0) \\
 & N(m(k+1) - m(k-1)) & = 2 \times t_{\text{final}} \times -2\theta(k) = -4 t_{\text{final}} \times \theta(k) \\
 & N(m(N) - m(N-1)) & = t_{\text{final}} \times -2\theta(N) \\
 & & \\
\text{where} & t \in \{0, ..., N\} & \\
 & k \in \{1, ..., N-1\} & \\
 & x \in \mathbf{R}^{N+2} &
\end{array}
$$

Our problem formulation, as described above, has $N + 2$ decision variables (stored for readability in a vector x) and 19 total constraints:

- 4 Boundary Value Equality Conditions
- 6 Upper and Lower Bounds (after separating chained inequalities)
- 9 Constraints describing the motion of the rocket (after approximation)

1.2 Considering Convexity

A nonlinear programming model is convex if it satisfies two conditions:

- The objective function is convex.
- The solution space is a convex set.

A given function $f(x)$ is considered convex if is satisfies the following condition:

$$f''(x) > 0 \text{ everywhere}$$

Conversely, a given function $f(x)$ is considered non-convex if the following holds:

$$f''(x) \leq 0 \text{ at any given point x}$$

Now, considering the first condition, we seek to determine whether $h(t_{\text{final}})$ is a convex function. The height of the rocket is determined by the velocity and t_{final}. The velocity is, in turn, determined by the mass, height and velocity, as the velocity constraints invokes the function D which itself takes velocity as an argument. However, consider the right hand side of equation (1.8), re-arranged in the same way that we have in our model statement above:

$$\begin{aligned}
N(v(t_{k+1}) - v(t_{k-1})) &= 2t_{\text{final}} \times \frac{\theta(t_k) - D(h(t_k), v(t_k))}{m(t_k)} - \left(\frac{h(0)}{h(t_k)}\right)^2 \\
&= 2t_{\text{final}} \times \frac{\theta(t_k) - \left(D_0 \times (v(t_k))^2 \times exp\left(-h_0 \frac{h(t_k)-h(0)}{h(0)}\right)\right)}{m(t_k)} - \left(\frac{h(0)}{h(t_k)}\right)^2 \\
&= 2t_{\text{final}} \times \frac{\theta(t_k) - \left(D_0 \times exp\left(-h_0 \frac{h(t_k)-h(0)}{h(0)}\right)\right)(v(t_k))^2}{m(t_k)} - \left(\frac{h(0)}{h(t_k)}\right)^2
\end{aligned}$$

Clearly, this equation features a negative square of the velocity term $v(t)$ on the right-hand side, because $D_0 > 0$ and the exponential function is always positive. Consequently, this constraint is not convex, as per Section 1 page 29 of the lecture notes. As a result, we may conclude that the set of all feasible solutions is not a convex set as this constraint will satisfy $f''(x) \leq 0$ with respect to velocity.

1.3 Error with Model Constraint Substitution

In substituting three equations (1.1) - (1.3) for the nine equations (1.4) - (1.12), we introduce a degree of error. This is due to the use of an approximation for the derivatives of three key functions: $h(t)$, $v(t)$ and $m(t)$. It is natural that numerical differentiation will introduce error. We may examine this error by considering the approximation we are using in more detail:

$$h'(t_k) \approx \frac{h(t_{k+1}) - h(t_{k-1})}{2 \times (\frac{t_{\text{final}}}{N})}$$

The error introduced will be proportional to the width of the interval considered. The width of the interval is given by $2(\frac{t_{\text{final}}}{N})$. Consequently, we have:

$$\begin{aligned}
\lim_{N\to\infty} (\text{width}) &= \lim_{N\to\infty} 2\left(\frac{t_{\text{final}}}{N}\right) \\
&= 2t_{\text{final}} \times \lim_{N\to\infty} \left(\frac{1}{N}\right) \\
&= 2t_{\text{final}} \times 0 \\
&= 0
\end{aligned}$$

From the above, we see that the width of the interval approaches zero as N approaches infinity. The same holds true for the approximations used for $v'(t)$ and $m'(t)$. Hence, the error introduced by numerical differentiation will in turn also approach zero as N approaches infinity. As a result, in our investigation, we will seek to keep N as large as possible when computing solutions.

Chapter 2

Solution

2.1 Solution Method

2.1.1 Considering Different Values of N

We started by considering $N = 10$ for testing purposes. This produces values broadly similar to the below optimal solution, but, with significantly lower accuracy. We then moved to considering $N = 50$, 100, 200 and 400. $N = 400$ in particular produced a broad consensus of solutions across solvers to good accuracy (4 decimal places), which one may see by checking the $N = 400$ solution of Appendices (3.3.1)-(3.3.5).

Values of N greater than 400, such as $N = 1000$ and $N = 2000$, produced the same optimal solution as below. Though, as one may imagine, such takes significantly longer to compute, and solvers were more likely to declare the problem infeasible or fail to produce a solution.

2.1.2 Data Processing

A text-parser was written in Python to assist in interpreting the raw command-line output produced by AMPL. These results were then stored in Excel CSV files. Finally, this data was analysed and plotted using RStudio. The corresponding code is available in Appendices (3.1.1) and (3.1.2).

2.1.3 Considering Different Solvers

We considered five total solvers in our investigation: ConOpt, Knitro, Loqo, Minos and Snopt. Later on, we find a broad consensus of solutions across these five solvers.

2.1.4 Optimal Solution

The consensus across values of N and across solvers (with minor variation) is the following optimal solution:

$$h(t_{\text{final}}) = 1.0128 \text{ metres}$$

$$t_{\text{final}} = 0.1988 \text{ seconds}$$

As a result, the optimal first stage burn interval would achieve a height of 1.0128 metres and last for 0.1988 seconds. The particular solutions generated by each solver are:

Country List			
Solver	N	$h(t_{\text{final}})$	t_{final}
ConOpt	400	1.01284	0.198798
Knitro	400	1.01284	0.198848
Loqo	400	1.01284	0.198812
Minos	100	1.01283	0.19874
Snopt	400	1.01283	0.198603

Table 2.1: Table of Optimal Solutions

To minimise error, we seek to keep N as high as possible. Minos was the only solver to have an issue with optimising our model for large N, declaring infeasibility or throwing errors for larger values of N. Consequently, we have used $N = 400$ for every optimal solution provided, except that of Minos, which has $N = 100$.

2.2 Presentation of Results

Looking at the results produced for the decision variable thrust by each solver, we see the following values for each of our functions across the interval:

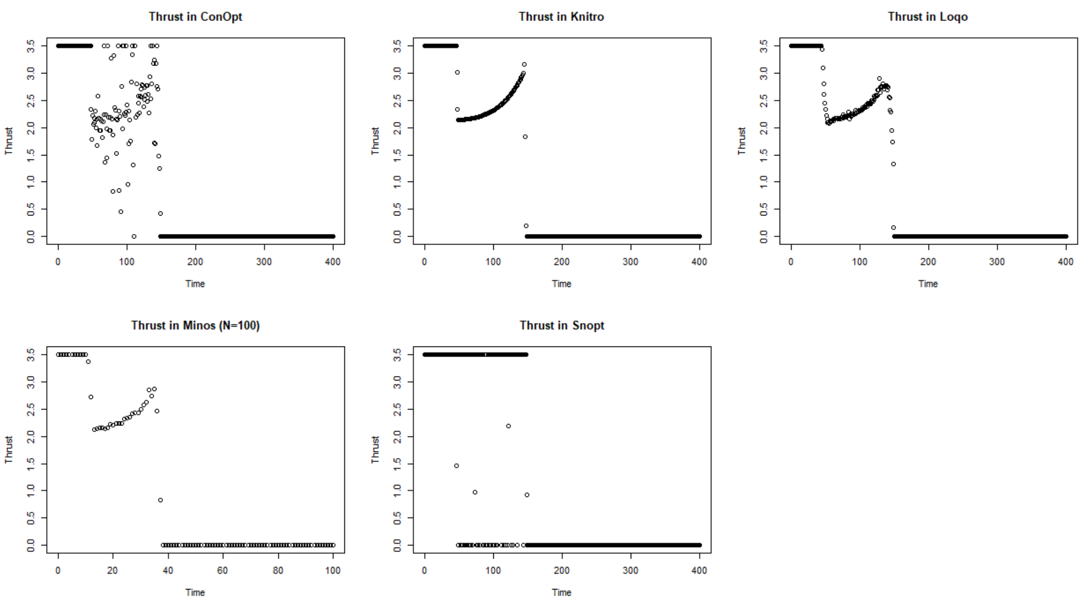

Figure 2.1: Optimal Decision Variable Values over Time for each Solver

This suggests a broad consensus of the optimal shape for a solution to take, starting at maximum thrust $\theta_{\max}$ until time $t_{47} = \frac{47}{400}t_{\text{final}} = 0.0234$. We then drop thrust to a slope of intermediate values until time $t_{147} = \frac{147}{400}t_{\text{final}} = 0.0731$. Finally, we drop the thrust to zero for the remainder of the first stage burn interval. The intermediate values also follow a similar shape between the solutions, with Knitro, Loqo and Minos most clearly showing the optimal thrust pattern.

Looking at the values of all our variables across the first stage burn interval for all solvers gives us the following:

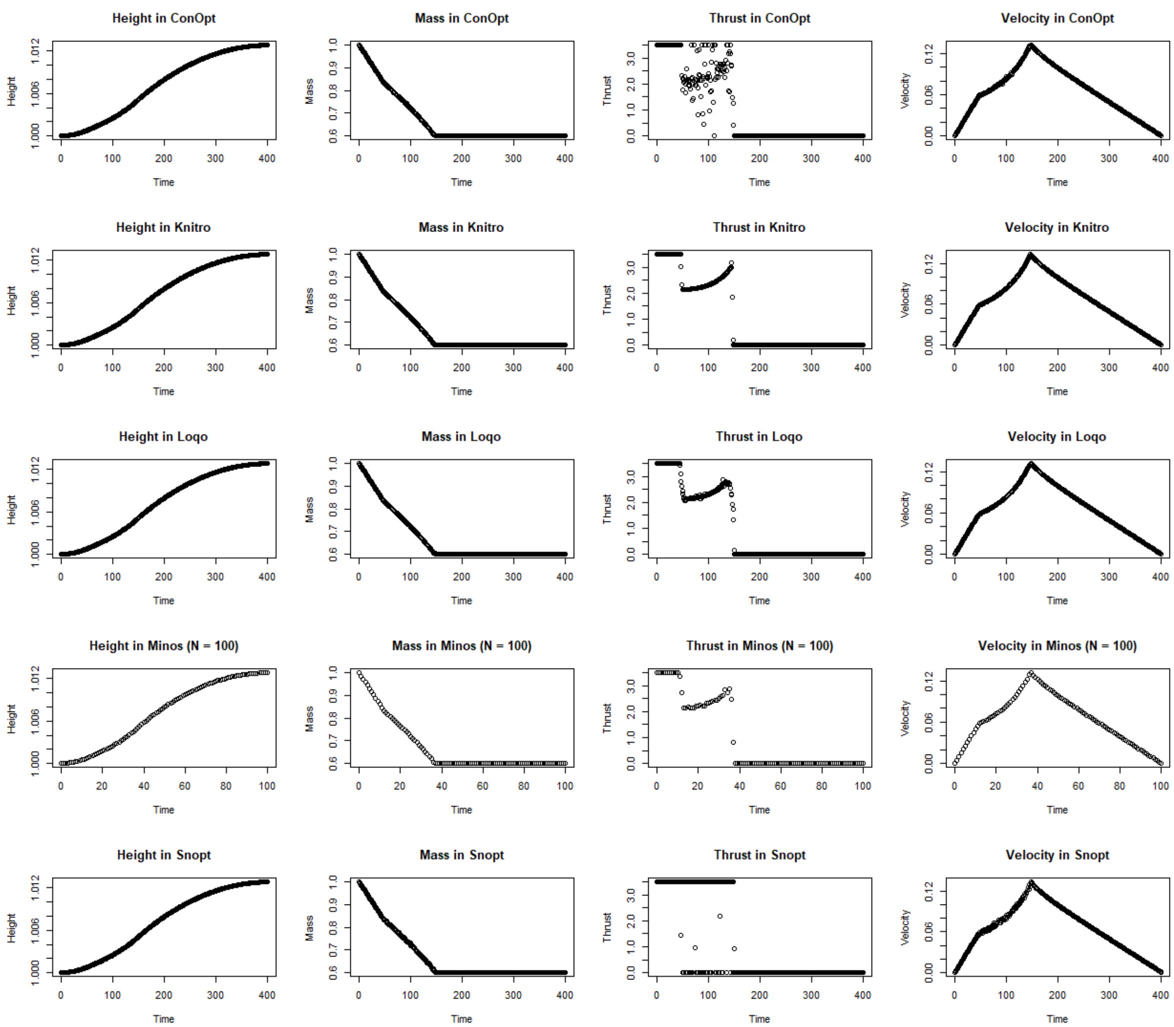

Figure 2.2: Optimal Function Values over Time for each Solver

This shows us several things. Firstly, we can safely sanity-check our solutions here. None of the values are oscillating, and all functions follow fairly smooth, reasonable patterns. We may hence conclude that our solution is sensible for a real-world rocket launch.

Secondly, we see the pattern we would expect from mass and thrust. The mass degraded from 1.0 to 0.6 over the course of active fuel injection. We also see that thrust follows a similar pattern, only slowing to zero when mass reaches 0.6. This tells us that the modelling of fuel injection into the combustion chamber has been represented faithfully in the constraints of our model.

Thirdly, we see the pattern we would expect from thrust and velocity. The increase in velocity slows when thrust moves from $\theta_{\max}$ to intermediate values. Moreover, the velocity is only increasing while thrust is active, decreasing when thrust is zero.

Finally, the broad consensus of optimal solutions is very clear from the above plots.

2.3 Considering Different Starting Points

For different starting points of t_{final}, AMPL always produces the same optimal solution (to within numerical error). This suggests we have found a stable maximum for our problem.

For different starting points of $\bar{\theta}$, AMPL again always produces the same optimal solution (to within numerical error). This too suggests we have found a stable maximum for our problem.

2.4 Summary of Recommended Actions

We are optimising the fuel flow into the combustion chamber of the rocket engine in order to maximise height, and the thrust is proportional to the fuel flow. Hence, we recommend following a pattern of fuel injection such that the following optimmal pattern of engine thrust is achieved:

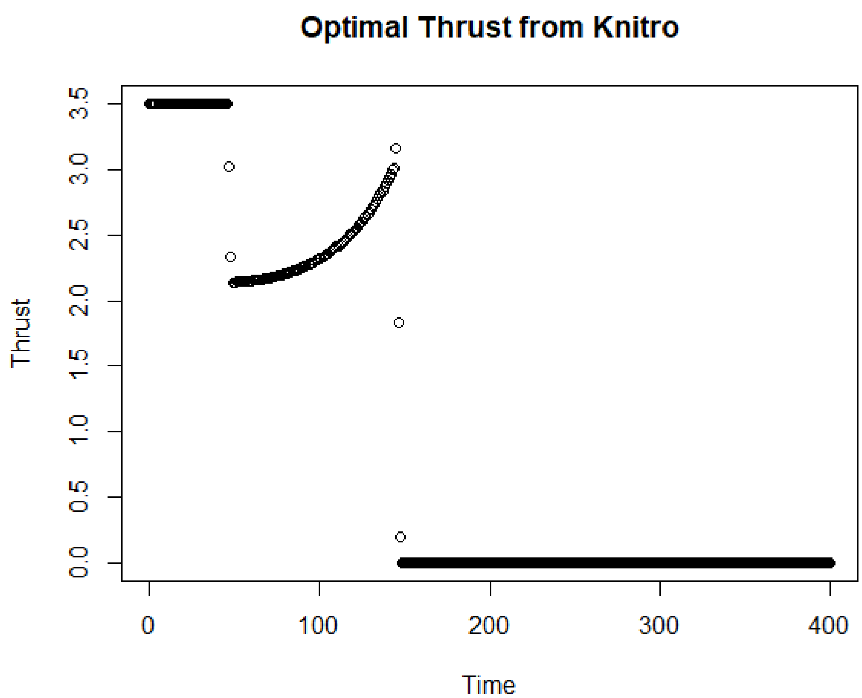

Figure 2.3: Optimal Thrust Values over Time from Knitro

By following this optimal thrust pattern during a first-stage burn interval of length 0.1988 seconds, Elon Musk will maximise the height achieved by the Falcon Heavy launch vehicle at the end of this interval, 1.0128 metres.

2.5 Suggestions for Further Investigation

It may be worth investigating different sizes of fuel tank, and considering the fuel tank size as a variable within the model. By carrying more fuel, we would enable more thrust to be achieved by the engine and also enable a greater value of $\theta_{\max}$. Conversely, we could also seek to decrease mass by carrying less fuel and thus increase the velocity achieved for a given thrust value.

Such an investigation could be carried out by means of more explicitly tracking current fuel amount as a variable within the model, and allowing $\theta_{\max}$ to be a variable. We would then need to introduce the impact that this fuel would have on thrust and mass through the constraints.

Chapter 3

Appendices

3.1 Data Processing Code

3.1.1 Python Text Parsing Code

```
## Math6120 - Nonlinear Optimisation
## Coursework 1 - AMPL Output Interpreter
## Emma Tarmey, 2940 4045


## This file translates the raw cmd output from AMPL into a format more usable

# import libraries
import csv
import numpy as np

output_conopt_height_400 = """
  0 1           81 1.00179    162 1.00592    243 1.00986    324 1.01214
  1 1           82 1.00182    163 1.00598    244 1.00989    325 1.01216
  2 1           83 1.00186    164 1.00604    245 1.00993    326 1.01217
  3 1           84 1.0019     165 1.0061     246 1.00997    327 1.01219
  4 1           85 1.00193    166 1.00616    247 1.01001    328 1.01221
  5 1.00001     86 1.00197    167 1.00622    248 1.01004    329 1.01223
  6 1.00001     87 1.00201    168 1.00627    249 1.01008    330 1.01224
  7 1.00001     88 1.00204    169 1.00633    250 1.01012    331 1.01226
  8 1.00002     89 1.00208    170 1.00639    251 1.01015    332 1.01228
  9 1.00003     90 1.00212    171 1.00645    252 1.01019    333 1.01229
 10 1.00003     91 1.00216    172 1.0065     253 1.01022    334 1.01231
 11 1.00004     92 1.0022     173 1.00656    254 1.01026    335 1.01233
 12 1.00005     93 1.00224    174 1.00662    255 1.01029    336 1.01234
 13 1.00005     94 1.00228    175 1.00667    256 1.01033    337 1.01236
 14 1.00006     95 1.00232    176 1.00673    257 1.01036    338 1.01237
```

15	1.00007	96	1.00236	177	1.00678	258	1.0104	339	1.01239
16	1.00008	97	1.0024	178	1.00684	259	1.01043	340	1.0124
17	1.00009	98	1.00244	179	1.0069	260	1.01047	341	1.01241
18	1.0001	99	1.00248	180	1.00695	261	1.0105	342	1.01243
19	1.00011	100	1.00252	181	1.007	262	1.01053	343	1.01244
20	1.00013	101	1.00256	182	1.00706	263	1.01057	344	1.01246
21	1.00014	102	1.0026	183	1.00711	264	1.0106	345	1.01247
22	1.00015	103	1.00265	184	1.00717	265	1.01063	346	1.01248
23	1.00017	104	1.00269	185	1.00722	266	1.01067	347	1.0125
24	1.00018	105	1.00273	186	1.00727	267	1.0107	348	1.01251
25	1.0002	106	1.00277	187	1.00732	268	1.01073	349	1.01252
26	1.00021	107	1.00282	188	1.00738	269	1.01076	350	1.01253
27	1.00023	108	1.00286	189	1.00743	270	1.01079	351	1.01255
28	1.00025	109	1.00291	190	1.00748	271	1.01082	352	1.01256
29	1.00026	110	1.00295	191	1.00753	272	1.01086	353	1.01257
30	1.00028	111	1.003	192	1.00758	273	1.01089	354	1.01258
31	1.0003	112	1.00304	193	1.00764	274	1.01092	355	1.01259
32	1.00032	113	1.00309	194	1.00769	275	1.01095	356	1.0126
33	1.00034	114	1.00314	195	1.00774	276	1.01098	357	1.01261
34	1.00036	115	1.00318	196	1.00779	277	1.01101	358	1.01262
35	1.00039	116	1.00323	197	1.00784	278	1.01104	359	1.01263
36	1.00041	117	1.00328	198	1.00789	279	1.01107	360	1.01264
37	1.00043	118	1.00333	199	1.00794	280	1.0111	361	1.01265
38	1.00045	119	1.00338	200	1.00798	281	1.01112	362	1.01266
39	1.00048	120	1.00343	201	1.00803	282	1.01115	363	1.01267
40	1.0005	121	1.00347	202	1.00808	283	1.01118	364	1.01268
41	1.00053	122	1.00353	203	1.00813	284	1.01121	365	1.01269
42	1.00055	123	1.00358	204	1.00818	285	1.01124	366	1.0127
43	1.00058	124	1.00363	205	1.00823	286	1.01127	367	1.0127
44	1.00061	125	1.00368	206	1.00827	287	1.01129	368	1.01271
45	1.00063	126	1.00373	207	1.00832	288	1.01132	369	1.01272
46	1.00066	127	1.00378	208	1.00837	289	1.01135	370	1.01273
47	1.00069	128	1.00384	209	1.00842	290	1.01137	371	1.01273
48	1.00072	129	1.00389	210	1.00846	291	1.0114	372	1.01274
49	1.00075	130	1.00395	211	1.00851	292	1.01143	373	1.01275
50	1.00078	131	1.004	212	1.00855	293	1.01145	374	1.01275
51	1.00081	132	1.00406	213	1.0086	294	1.01148	375	1.01276
52	1.00084	133	1.00411	214	1.00865	295	1.0115	376	1.01277
53	1.00087	134	1.00417	215	1.00869	296	1.01153	377	1.01277
54	1.0009	135	1.00423	216	1.00874	297	1.01155	378	1.01278
55	1.00093	136	1.00429	217	1.00878	298	1.01158	379	1.01278
56	1.00096	137	1.00435	218	1.00883	299	1.0116	380	1.01279
57	1.00099	138	1.00441	219	1.00887	300	1.01163	381	1.01279
58	1.00102	139	1.00447	220	1.00891	301	1.01165	382	1.0128
59	1.00105	140	1.00453	221	1.00896	302	1.01168	383	1.0128
60	1.00108	141	1.00459	222	1.009	303	1.0117	384	1.0128

```
 61 1.00111    142 1.00465   223 1.00904   304 1.01172   385 1.01281
 62 1.00115    143 1.00472   224 1.00909   305 1.01175   386 1.01281
 63 1.00118    144 1.00478   225 1.00913   306 1.01177   387 1.01282
 64 1.00121    145 1.00484   226 1.00917   307 1.01179   388 1.01282
 65 1.00124    146 1.00491   227 1.00921   308 1.01181   389 1.01282
 66 1.00127    147 1.00497   228 1.00926   309 1.01184   390 1.01282
 67 1.00131    148 1.00504   229 1.0093    310 1.01186   391 1.01283
 68 1.00134    149 1.00511   230 1.00934   311 1.01188   392 1.01283
 69 1.00137    150 1.00517   231 1.00938   312 1.0119    393 1.01283
 70 1.00141    151 1.00523   232 1.00942   313 1.01192   394 1.01283
 71 1.00144    152 1.0053    233 1.00946   314 1.01194   395 1.01283
 72 1.00147    153 1.00536   234 1.0095    315 1.01196   396 1.01283
 73 1.00151    154 1.00543   235 1.00954   316 1.01198   397 1.01284
 74 1.00154    155 1.00549   236 1.00958   317 1.012     398 1.01284
 75 1.00157    156 1.00555   237 1.00962   318 1.01202   399 1.01284
 76 1.00161    157 1.00561   238 1.00966   319 1.01204   400 1.01284
 77 1.00164    158 1.00567   239 1.0097    320 1.01206
 78 1.00168    159 1.00574   240 1.00974   321 1.01208
 79 1.00171    160 1.0058    241 1.00978   322 1.0121
 80 1.00175    161 1.00586   242 1.00982   323 1.01212
"""

output_conopt_velocity_400 = """
  0 0            101 0.0841806     202 0.0975864     303 0.0471458
  1 0.00124248   102 0.0853034     203 0.0970553     304 0.0466586
  2 0.00249664   103 0.0855483     204 0.0965254     305 0.0461715
  3 0.0037499    104 0.0858441     205 0.0959972     306 0.0456845
  4 0.00501393   105 0.0867122     206 0.0954702     307 0.0451975
  5 0.00627614   106 0.086484      207 0.0949447     308 0.0447106
  6 0.0075482    107 0.0888822     208 0.0944204     309 0.0442238
  7 0.00881747   108 0.0895954     209 0.0938974     310 0.0437371
  8 0.0100957    109 0.0917743     210 0.0933755     311 0.0432504
  9 0.0113701    110 0.0896034     211 0.092855      312 0.0427637
 10 0.0126525    111 0.0899776     212 0.0923354     313 0.0422772
 11 0.0139302    112 0.0928075     213 0.091817      314 0.0417907
 12 0.015215     113 0.0931807     214 0.0912995     315 0.0413042
 13 0.016494     114 0.0941474     215 0.0907832     316 0.0408178
 14 0.0177793    115 0.0954281     216 0.0902677     317 0.0403315
 15 0.019058     116 0.0955766     217 0.0897533     318 0.0398452
 16 0.020342     117 0.0973824     218 0.0892397     319 0.039359
 17 0.0216185    118 0.0973454     219 0.088727      320 0.0388728
 18 0.0228995    119 0.0989055     220 0.0882151     321 0.0383867
 19 0.0241722    120 0.0993245     221 0.0877042     322 0.0379006
 20 0.0254488    121 0.100883      222 0.0871939     323 0.0374146
 21 0.0267162    122 0.101536      223 0.0866845     324 0.0369286
 22 0.0279869    123 0.103227      224 0.0861757     325 0.0364427
```

23	0.0292477	124	0.10388	225	0.0856677	326	0.0359568
24	0.0305111	125	0.105004	226	0.0851604	327	0.0354709
25	0.0317641	126	0.106195	227	0.0846538	328	0.0349851
26	0.0330193	127	0.106993	228	0.0841477	329	0.0344994
27	0.0342635	128	0.108344	229	0.0836424	330	0.0340137
28	0.0355094	129	0.109433	230	0.0831375	331	0.033528
29	0.036744	130	0.110337	231	0.0826334	332	0.0330424
30	0.03798	131	0.111908	232	0.0821297	333	0.0325568
31	0.0392043	132	0.112573	233	0.0816267	334	0.0320712
32	0.0404298	133	0.113641	234	0.0811241	335	0.0315857
33	0.0416434	134	0.115345	235	0.0806221	336	0.0311002
34	0.0428581	135	0.1158	236	0.0801206	337	0.0306147
35	0.0440607	136	0.119061	237	0.0796196	338	0.0301293
36	0.0452645	137	0.118433	238	0.0791189	339	0.0296439
37	0.0464561	138	0.122829	239	0.0786189	340	0.0291586
38	0.0476488	139	0.121694	240	0.0781191	341	0.0286733
39	0.0488295	140	0.123815	241	0.0776199	342	0.028188
40	0.0500115	141	0.125122	242	0.077121	343	0.0277027
41	0.0511815	142	0.124792	243	0.0766226	344	0.0272175
42	0.0523531	143	0.128502	244	0.0761245	345	0.0267323
43	0.0535129	144	0.128727	245	0.0756268	346	0.0262471
44	0.0546746	145	0.131242	246	0.0751295	347	0.025762
45	0.0558249	146	0.131412	247	0.0746325	348	0.0252768
46	0.0569773	147	0.131928	248	0.0741358	349	0.0247917
47	0.0581187	148	0.13176	249	0.0736396	350	0.0243067
48	0.0592627	149	0.130918	250	0.0731436	351	0.0238216
49	0.0590057	150	0.130091	251	0.0726479	352	0.0233366
50	0.0595224	151	0.129279	252	0.0721525	353	0.0228516
51	0.05978	152	0.128481	253	0.0716575	354	0.0223667
52	0.0601259	153	0.127696	254	0.0711627	355	0.0218817
53	0.0605164	154	0.126924	255	0.0706682	356	0.0213968
54	0.0607729	155	0.126164	256	0.0701739	357	0.0209119
55	0.0614283	156	0.125415	257	0.06968	358	0.020427
56	0.0613178	157	0.124677	258	0.0691862	359	0.0199421
57	0.0615864	158	0.12395	259	0.0686928	360	0.0194573
58	0.0620788	159	0.123232	260	0.0681995	361	0.0189724
59	0.0628606	160	0.122524	261	0.0677065	362	0.0184876
60	0.0628713	161	0.121825	262	0.0672137	363	0.0180028
61	0.0633922	162	0.121134	263	0.0667212	364	0.0175181
62	0.0636618	163	0.120452	264	0.0662288	365	0.0170333
63	0.0639233	164	0.119777	265	0.0657367	366	0.0165486
64	0.0644396	165	0.11911	266	0.0652447	367	0.0160638
65	0.064324	166	0.118449	267	0.064753	368	0.0155791
66	0.0652199	167	0.117796	268	0.0642614	369	0.0150944
67	0.0668326	168	0.117149	269	0.0637701	370	0.0146098
68	0.0661272	169	0.116508	270	0.0632789	371	0.0141251

```
 69 0.0666788      170 0.115873      271 0.0627879     372 0.0136404
 70 0.0670788      171 0.115244      272 0.062297      373 0.0131558
 71 0.0666414      172 0.11462       273 0.0618064     374 0.0126712
 72 0.0677446      173 0.114002      274 0.0613159     375 0.0121865
 73 0.0692284      174 0.113388      275 0.0608255     376 0.0117019
 74 0.0686461      175 0.11278       276 0.0603353     377 0.0112173
 75 0.0698679      176 0.112175      277 0.0598453     378 0.0107327
 76 0.0695891      177 0.111576      278 0.0593554     379 0.0102482
 77 0.0705135      178 0.11098       279 0.0588656     380 0.00976359
 78 0.0719379      179 0.110388      280 0.058376      381 0.00927903
 79 0.0714154      180 0.1098        281 0.0578865     382 0.00879448
 80 0.0711426      181 0.109216      282 0.0573971     383 0.00830994
 81 0.0719888      182 0.108635      283 0.0569079     384 0.0078254
 82 0.0736161      183 0.108058      284 0.0564188     385 0.00734087
 83 0.0732187      184 0.107484      285 0.0559298     386 0.00685635
 84 0.0748051      185 0.106913      286 0.055441      387 0.00637183
 85 0.0741876      186 0.106345      287 0.0549522     388 0.00588732
 86 0.0749736      187 0.105781      288 0.0544636     389 0.00540282
 87 0.0751574      188 0.105218      289 0.0539751     390 0.00491832
 88 0.0777703      189 0.104659      290 0.0534866     391 0.00443382
 89 0.0763412      190 0.104102      291 0.0529983     392 0.00394933
 90 0.0770565      191 0.103548      292 0.0525101     393 0.00346484
 91 0.0774494      192 0.102995      293 0.052022      394 0.00298035
 92 0.0758412      193 0.102446      294 0.051534      395 0.00249587
 93 0.0793597      194 0.101898      295 0.0510461     396 0.00201139
 94 0.0787177      195 0.101352      296 0.0505582     397 0.00152691
 95 0.0802084      196 0.100809      297 0.0500705     398 0.00104243
 96 0.0816305      197 0.100267      298 0.0495828     399 0.000557954
 97 0.0814337      198 0.0997276     299 0.0490952     400 7.34773e-05
 98 0.082827       199 0.0991899     300 0.0486077
 99 0.0827189      200 0.0986537     301 0.0481204
100 0.085806       201 0.0981194     302 0.047633
"""

output_conopt_mass_400 = """
  0 1            81 0.763525   162 0.6         243 0.6         324 0.6
  1 0.996521     82 0.759593   163 0.6         244 0.6         325 0.6
  2 0.993042     83 0.75881    164 0.6         245 0.6         326 0.6
  3 0.989563     84 0.754981   165 0.6         246 0.6         327 0.6
  4 0.986084     85 0.754516   166 0.6         247 0.6         328 0.6
  5 0.982605     86 0.751942   167 0.6         248 0.6         329 0.6
  6 0.979126     87 0.750274   168 0.6         249 0.6         330 0.6
  7 0.975647     88 0.744984   169 0.6         250 0.6         331 0.6
  8 0.972168     89 0.745695   170 0.6         251 0.6         332 0.6
  9 0.968689     90 0.743306   171 0.6         252 0.6         333 0.6
 10 0.96521      91 0.741308   172 0.6         253 0.6         334 0.6
```

11	0.961731	92	0.742386	173	0.6	254	0.6	335	0.6
12	0.958253	93	0.735821	174	0.6	255	0.6	336	0.6
13	0.954774	94	0.735428	175	0.6	256	0.6	337	0.6
14	0.951295	95	0.731894	176	0.6	257	0.6	338	0.6
15	0.947816	96	0.72847	177	0.6	258	0.6	339	0.6
16	0.944337	97	0.727403	178	0.6	259	0.6	340	0.6
17	0.940858	98	0.724057	179	0.6	260	0.6	341	0.6
18	0.937379	99	0.722859	180	0.6	261	0.6	342	0.6
19	0.9339	100	0.717099	181	0.6	262	0.6	343	0.6
20	0.930421	101	0.718057	182	0.6	263	0.6	344	0.6
21	0.926942	102	0.715189	183	0.6	264	0.6	345	0.6
22	0.923463	103	0.713465	184	0.6	265	0.6	346	0.6
23	0.919984	104	0.711807	185	0.6	266	0.6	347	0.6
24	0.916505	105	0.709216	186	0.6	267	0.6	348	0.6
25	0.913026	106	0.708316	187	0.6	268	0.6	349	0.6
26	0.909547	107	0.70359	188	0.6	269	0.6	350	0.6
27	0.906068	108	0.701358	189	0.6	270	0.6	351	0.6
28	0.902589	109	0.696965	190	0.6	271	0.6	352	0.6
29	0.89911	110	0.698757	191	0.6	272	0.6	353	0.6
30	0.895631	111	0.696965	192	0.6	273	0.6	354	0.6
31	0.892152	112	0.691799	193	0.6	274	0.6	355	0.6
32	0.888673	113	0.690007	194	0.6	275	0.6	356	0.6
33	0.885194	114	0.687446	195	0.6	276	0.6	357	0.6
34	0.881715	115	0.68442	196	0.6	277	0.6	358	0.6
35	0.878236	116	0.682995	197	0.6	278	0.6	359	0.6
36	0.874758	117	0.679282	198	0.6	279	0.6	360	0.6
37	0.871279	118	0.678116	199	0.6	280	0.6	361	0.6
38	0.8678	119	0.674771	200	0.6	281	0.6	362	0.6
39	0.864321	120	0.672998	201	0.6	282	0.6	363	0.6
40	0.860842	121	0.669684	202	0.6	283	0.6	364	0.6
41	0.857363	122	0.66761	203	0.6	284	0.6	365	0.6
42	0.853884	123	0.664146	204	0.6	285	0.6	366	0.6
43	0.850405	124	0.662086	205	0.6	286	0.6	367	0.6
44	0.846926	125	0.659403	206	0.6	287	0.6	368	0.6
45	0.843447	126	0.656653	207	0.6	288	0.6	369	0.6
46	0.839968	127	0.65442	208	0.6	289	0.6	370	0.6
47	0.836489	128	0.651485	209	0.6	290	0.6	371	0.6
48	0.83301	129	0.648895	210	0.6	291	0.6	372	0.6
49	0.831847	130	0.646563	211	0.6	292	0.6	373	0.6
50	0.829452	131	0.643376	212	0.6	293	0.6	374	0.6
51	0.827437	132	0.641374	213	0.6	294	0.6	375	0.6
52	0.825341	133	0.638863	214	0.6	295	0.6	376	0.6
53	0.823118	134	0.635555	215	0.6	296	0.6	377	0.6
54	0.82118	135	0.633847	216	0.6	297	0.6	378	0.6
55	0.818539	136	0.628597	217	0.6	298	0.6	379	0.6
56	0.817202	137	0.628274	218	0.6	299	0.6	380	0.6

```
 57 0.815224   138 0.621639   219 0.6        300 0.6        381 0.6
 58 0.812919   139 0.621967   220 0.6        301 0.6        382 0.6
 59 0.810114   140 0.61821    221 0.6        302 0.6        383 0.6
 60 0.808588   141 0.615527   222 0.6        303 0.6        384 0.6
 61 0.806233   142 0.614824   223 0.6        304 0.6        385 0.6
 62 0.804295   143 0.609222   224 0.6        305 0.6        386 0.6
 63 0.802375   144 0.607866   225 0.6        306 0.6        387 0.6
 64 0.800058   145 0.603758   226 0.6        307 0.6        388 0.6
 65 0.79875    146 0.602494   227 0.6        308 0.6        389 0.6
 66 0.795857   147 0.600831   228 0.6        309 0.6        390 0.6
 67 0.791792   148 0.6        229 0.6        310 0.6        391 0.6
 68 0.791417   149 0.6        230 0.6        311 0.6        392 0.6
 69 0.789087   150 0.6        231 0.6        312 0.6        393 0.6
 70 0.786971   151 0.6        232 0.6        313 0.6        394 0.6
 71 0.786213   152 0.6        233 0.6        314 0.6        395 0.6
 72 0.783033   153 0.6        234 0.6        315 0.6        396 0.6
 73 0.779255   154 0.6        235 0.6        316 0.6        397 0.6
 74 0.778692   155 0.6        236 0.6        317 0.6        398 0.6
 75 0.775372   156 0.6        237 0.6        318 0.6        399 0.6
 76 0.774324   157 0.6        238 0.6        319 0.6        400 0.6
 77 0.771504   158 0.6        239 0.6        320 0.6
 78 0.767824   159 0.6        240 0.6        321 0.6
 79 0.767221   160 0.6        241 0.6        322 0.6
 80 0.76619    161 0.6        242 0.6        323 0.6
"""

output_conopt_thrust_400 = """
  0 3.5           101 0.96099       202 0              303 0
  1 3.5           102 2.30946       203 0              304 0
  2 3.5           103 1.70124       204 0              305 0
  3 3.5           104 2.13734       205 0              306 0
  4 3.5           105 1.75601       206 0              307 0
  5 3.5           106 2.82999       207 0              308 0
  6 3.5           107 3.5           208 0              309 0
  7 3.5           108 3.33283       209 0              310 0
  8 3.5           109 1.30826       210 0              311 0
  9 3.5           110 0             211 0              312 0
 10 3.5           111 3.5           212 0              313 0
 11 3.5           112 3.5           213 0              314 0
 12 3.5           113 2.18998       214 0              315 0
 13 3.5           114 2.81039       215 0              316 0
 14 3.5           115 2.23877       216 0              317 0
 15 3.5           116 2.58452       217 0              318 0
 16 3.5           117 2.45425       218 0              319 0
 17 3.5           118 2.26924       219 0              320 0
 18 3.5           119 2.57451       220 0              321 0
```

19 3.5	120 2.55892	221 0	322 0
20 3.5	121 2.7103	222 0	323 0
21 3.5	122 2.78573	223 0	324 0
22 3.5	123 2.77852	224 0	325 0
23 3.5	124 2.38598	225 0	326 0
24 3.5	125 2.73334	226 0	327 0
25 3.5	126 2.50646	227 0	328 0
26 3.5	127 2.5995	228 0	329 0
27 3.5	128 2.77932	229 0	330 0
28 3.5	129 2.47568	230 0	331 0
29 3.5	130 2.77588	231 0	332 0
30 3.5	131 2.61015	232 0	333 0
31 3.5	132 2.27045	233 0	334 0
32 3.5	133 2.92722	234 0	335 0
33 3.5	134 2.52293	235 0	336 0
34 3.5	135 3.5	236 0	337 0
35 3.5	136 2.80354	237 0	338 0
36 3.5	137 3.5	238 0	339 0
37 3.5	138 3.17233	239 0	340 0
38 3.5	139 1.72486	240 0	341 0
39 3.5	140 3.23945	241 0	342 0
40 3.5	141 1.70334	242 0	343 0
41 3.5	142 3.17172	243 0	344 0
42 3.5	143 3.5	244 0	345 0
43 3.5	144 2.74866	245 0	346 0
44 3.5	145 2.70241	246 0	347 0
45 3.5	146 1.4722	247 0	348 0
46 3.5	147 1.25443	248 0	349 0
47 3.5	148 0.417967	249 0	350 0
48 2.33479	149 1.02366e-14	250 0	351 0
49 1.78991	150 0	251 0	352 0
50 2.21842	151 0	252 0	353 0
51 2.06789	152 0	253 0	354 0
52 2.17247	153 0	254 0	355 0
53 2.09294	154 0	255 0	356 0
54 2.30375	155 2.30478e-22	256 0	357 0
55 2.00092	156 0	257 0	358 0
56 1.66719	157 0	258 0	359 0
57 2.15461	158 0	259 0	360 0
58 2.57055	159 0	260 0	361 0
59 2.17839	160 0	261 0	362 0
60 1.95235	161 0	262 0	363 0
61 2.15944	162 0	263 0	364 0
62 1.94058	163 0	264 0	365 0
63 2.13148	164 0	265 0	366 0
64 1.82374	165 0	266 0	367 0

```
 65 2.11332        166  0              267  0              368  0
 66 3.5            167  0              268  0              369  0
 67 2.23318        168  0              269  0              370  0
 68 1.36055        169  0              270  0              371  0
 69 2.23647        170  0              271  0              372  0
 70 1.44577        171  0              272  0              373  0
 71 1.98124        172  0              273  0              374  0
 72 3.5            173  0              274  0              375  0
 73 2.18338        174  0              275  0              376  0
 74 1.95332        175  0              276  0              377  0
 75 2.19735        176  0              277  0              378  0
 76 1.9457         177  0              278  0              379  0
 77 3.26987        178  0              279  0              380  0
 78 2.15431        179  0              280  0              381  0
 79 0.821521       180  0              281  0              382  0
 80 1.859          181  0              282  0              383  0
 81 3.31876        182  0              283  0              384  0
 82 2.37186        183  0              284  0              385  0
 83 2.31984        184  0              285  0              386  0
 84 2.16021        185  0              286  0              387  0
 85 1.52856        186  0              287  0              388  0
 86 2.13344        187  0              288  0              389  0
 87 3.5            188  0              289  0              390  0
 88 2.30376        189  0              290  0              391  0
 89 0.844174       190  0              291  0              392  0
 90 2.20653        191  0              292  0              393  0
 91 0.462763       192  0              293  0              394  0
 92 2.76018        193  0              294  0              395  0
 93 3.5            194  0              295  0              396  0
 94 1.97557        195  0              296  0              397  0
 95 3.5            196  0              297  0              398  0
 96 2.25902        197  0              298  0              399  0
 97 2.21988        198  0              299  0              400  0
 98 2.28561        199  0              300  0
 99 3.5            200  0              301  0
100 2.4157         201  0              302  0
"""


output_knitro_height_400 = """
  0 1           81 1.00179   162 1.00592   243 1.00986   324 1.01214
  1 1           82 1.00182   163 1.00598   244 1.0099    325 1.01216
  2 1           83 1.00186   164 1.00604   245 1.00994   326 1.01218
  3 1           84 1.0019    165 1.0061    246 1.00997   327 1.01219
  4 1           85 1.00193   166 1.00616   247 1.01001   328 1.01221
  5 1.00001     86 1.00197   167 1.00622   248 1.01005   329 1.01223
```

6	1.00001	87	1.00201	168	1.00628	249	1.01008	330	1.01225
7	1.00001	88	1.00205	169	1.00634	250	1.01012	331	1.01226
8	1.00002	89	1.00208	170	1.00639	251	1.01016	332	1.01228
9	1.00003	90	1.00212	171	1.00645	252	1.01019	333	1.0123
10	1.00003	91	1.00216	172	1.00651	253	1.01023	334	1.01231
11	1.00004	92	1.0022	173	1.00657	254	1.01026	335	1.01233
12	1.00005	93	1.00224	174	1.00662	255	1.0103	336	1.01234
13	1.00005	94	1.00228	175	1.00668	256	1.01033	337	1.01236
14	1.00006	95	1.00232	176	1.00673	257	1.01037	338	1.01237
15	1.00007	96	1.00236	177	1.00679	258	1.0104	339	1.01239
16	1.00008	97	1.0024	178	1.00684	259	1.01044	340	1.0124
17	1.00009	98	1.00244	179	1.0069	260	1.01047	341	1.01242
18	1.0001	99	1.00248	180	1.00695	261	1.0105	342	1.01243
19	1.00011	100	1.00252	181	1.00701	262	1.01054	343	1.01244
20	1.00013	101	1.00256	182	1.00706	263	1.01057	344	1.01246
21	1.00014	102	1.0026	183	1.00712	264	1.0106	345	1.01247
22	1.00015	103	1.00265	184	1.00717	265	1.01064	346	1.01248
23	1.00017	104	1.00269	185	1.00722	266	1.01067	347	1.0125
24	1.00018	105	1.00273	186	1.00728	267	1.0107	348	1.01251
25	1.0002	106	1.00278	187	1.00733	268	1.01073	349	1.01252
26	1.00021	107	1.00282	188	1.00738	269	1.01077	350	1.01254
27	1.00023	108	1.00286	189	1.00743	270	1.0108	351	1.01255
28	1.00025	109	1.00291	190	1.00749	271	1.01083	352	1.01256
29	1.00027	110	1.00295	191	1.00754	272	1.01086	353	1.01257
30	1.00028	111	1.003	192	1.00759	273	1.01089	354	1.01258
31	1.0003	112	1.00305	193	1.00764	274	1.01092	355	1.01259
32	1.00032	113	1.00309	194	1.00769	275	1.01095	356	1.0126
33	1.00034	114	1.00314	195	1.00774	276	1.01098	357	1.01261
34	1.00036	115	1.00319	196	1.00779	277	1.01101	358	1.01262
35	1.00039	116	1.00323	197	1.00784	278	1.01104	359	1.01263
36	1.00041	117	1.00328	198	1.00789	279	1.01107	360	1.01264
37	1.00043	118	1.00333	199	1.00794	280	1.0111	361	1.01265
38	1.00045	119	1.00338	200	1.00799	281	1.01113	362	1.01266
39	1.00048	120	1.00343	201	1.00804	282	1.01116	363	1.01267
40	1.0005	121	1.00348	202	1.00809	283	1.01119	364	1.01268
41	1.00053	122	1.00353	203	1.00814	284	1.01121	365	1.01269
42	1.00055	123	1.00358	204	1.00818	285	1.01124	366	1.0127
43	1.00058	124	1.00363	205	1.00823	286	1.01127	367	1.01271
44	1.00061	125	1.00368	206	1.00828	287	1.0113	368	1.01271
45	1.00063	126	1.00374	207	1.00833	288	1.01132	369	1.01272
46	1.00066	127	1.00379	208	1.00837	289	1.01135	370	1.01273
47	1.00069	128	1.00384	209	1.00842	290	1.01138	371	1.01274
48	1.00072	129	1.0039	210	1.00847	291	1.0114	372	1.01274
49	1.00075	130	1.00395	211	1.00851	292	1.01143	373	1.01275
50	1.00078	131	1.00401	212	1.00856	293	1.01146	374	1.01275
51	1.00081	132	1.00406	213	1.0086	294	1.01148	375	1.01276

```
 52 1.00084   133 1.00412   214 1.00865   295 1.01151   376 1.01277
 53 1.00087   134 1.00417   215 1.0087    296 1.01153   377 1.01277
 54 1.0009    135 1.00423   216 1.00874   297 1.01156   378 1.01278
 55 1.00093   136 1.00429   217 1.00879   298 1.01158   379 1.01278
 56 1.00096   137 1.00435   218 1.00883   299 1.01161   380 1.01279
 57 1.00099   138 1.00441   219 1.00887   300 1.01163   381 1.01279
 58 1.00102   139 1.00447   220 1.00892   301 1.01165   382 1.0128
 59 1.00105   140 1.00453   221 1.00896   302 1.01168   383 1.0128
 60 1.00108   141 1.00459   222 1.009     303 1.0117    384 1.01281
 61 1.00111   142 1.00466   223 1.00905   304 1.01173   385 1.01281
 62 1.00115   143 1.00472   224 1.00909   305 1.01175   386 1.01281
 63 1.00118   144 1.00478   225 1.00913   306 1.01177   387 1.01282
 64 1.00121   145 1.00485   226 1.00918   307 1.01179   388 1.01282
 65 1.00124   146 1.00491   227 1.00922   308 1.01182   389 1.01282
 66 1.00127   147 1.00498   228 1.00926   309 1.01184   390 1.01282
 67 1.00131   148 1.00504   229 1.0093    310 1.01186   391 1.01283
 68 1.00134   149 1.00511   230 1.00934   311 1.01188   392 1.01283
 69 1.00137   150 1.00518   231 1.00938   312 1.0119    393 1.01283
 70 1.00141   151 1.00524   232 1.00943   313 1.01192   394 1.01283
 71 1.00144   152 1.0053    233 1.00947   314 1.01194   395 1.01283
 72 1.00147   153 1.00537   234 1.00951   315 1.01197   396 1.01283
 73 1.00151   154 1.00543   235 1.00955   316 1.01199   397 1.01284
 74 1.00154   155 1.00549   236 1.00959   317 1.01201   398 1.01284
 75 1.00158   156 1.00556   237 1.00963   318 1.01203   399 1.01284
 76 1.00161   157 1.00562   238 1.00967   319 1.01205   400 1.01284
 77 1.00165   158 1.00568   239 1.00971   320 1.01206
 78 1.00168   159 1.00574   240 1.00974   321 1.01208
 79 1.00172   160 1.0058    241 1.00978   322 1.0121
 80 1.00175   161 1.00586   242 1.00982   323 1.01212
"""

output_knitro_mass_400 = """
  0 1          81 0.762914   162 0.6        243 0.6        324 0.6
  1 0.99652    82 0.760713   163 0.6        244 0.6        325 0.6
  2 0.99304    83 0.758512   164 0.6        245 0.6        326 0.6
  3 0.989561   84 0.756303   165 0.6        246 0.6        327 0.6
  4 0.986081   85 0.754093   166 0.6        247 0.6        328 0.6
  5 0.982601   86 0.751874   167 0.6        248 0.6        329 0.6
  6 0.979121   87 0.749654   168 0.6        249 0.6        330 0.6
  7 0.975641   88 0.747427   169 0.6        250 0.6        331 0.6
  8 0.972161   89 0.745196   170 0.6        251 0.6        332 0.6
  9 0.968682   90 0.742958   171 0.6        252 0.6        333 0.6
 10 0.965202   91 0.740716   172 0.6        253 0.6        334 0.6
 11 0.961722   92 0.738467   173 0.6        254 0.6        335 0.6
 12 0.958242   93 0.736213   174 0.6        255 0.6        336 0.6
 13 0.954762   94 0.733953   175 0.6        256 0.6        337 0.6
```

14	0.951282	95	0.731686	176	0.6	257	0.6	338	0.6
15	0.947803	96	0.729413	177	0.6	258	0.6	339	0.6
16	0.944323	97	0.727132	178	0.6	259	0.6	340	0.6
17	0.940843	98	0.724845	179	0.6	260	0.6	341	0.6
18	0.937363	99	0.72255	180	0.6	261	0.6	342	0.6
19	0.933883	100	0.72025	181	0.6	262	0.6	343	0.6
20	0.930403	101	0.717938	182	0.6	263	0.6	344	0.6
21	0.926924	102	0.715623	183	0.6	264	0.6	345	0.6
22	0.923444	103	0.713295	184	0.6	265	0.6	346	0.6
23	0.919964	104	0.710964	185	0.6	266	0.6	347	0.6
24	0.916484	105	0.708618	186	0.6	267	0.6	348	0.6
25	0.913004	106	0.70627	187	0.6	268	0.6	349	0.6
26	0.909524	107	0.703906	188	0.6	269	0.6	350	0.6
27	0.906045	108	0.701539	189	0.6	270	0.6	351	0.6
28	0.902565	109	0.699155	190	0.6	271	0.6	352	0.6
29	0.899085	110	0.696769	191	0.6	272	0.6	353	0.6
30	0.895605	111	0.694364	192	0.6	273	0.6	354	0.6
31	0.892125	112	0.691958	193	0.6	274	0.6	355	0.6
32	0.888645	113	0.68953	194	0.6	275	0.6	356	0.6
33	0.885166	114	0.687103	195	0.6	276	0.6	357	0.6
34	0.881686	115	0.684651	196	0.6	277	0.6	358	0.6
35	0.878206	116	0.6822	197	0.6	278	0.6	359	0.6
36	0.874726	117	0.679723	198	0.6	279	0.6	360	0.6
37	0.871246	118	0.677248	199	0.6	280	0.6	361	0.6
38	0.867766	119	0.674744	200	0.6	281	0.6	362	0.6
39	0.864287	120	0.672243	201	0.6	282	0.6	363	0.6
40	0.860807	121	0.66971	202	0.6	283	0.6	364	0.6
41	0.857327	122	0.667183	203	0.6	284	0.6	365	0.6
42	0.853847	123	0.664619	204	0.6	285	0.6	366	0.6
43	0.850367	124	0.662063	205	0.6	286	0.6	367	0.6
44	0.846887	125	0.659466	206	0.6	287	0.6	368	0.6
45	0.843408	126	0.65688	207	0.6	288	0.6	369	0.6
46	0.839928	127	0.654249	208	0.6	289	0.6	370	0.6
47	0.836449	128	0.65163	209	0.6	290	0.6	371	0.6
48	0.833916	129	0.648962	210	0.6	291	0.6	372	0.6
49	0.831803	130	0.646309	211	0.6	292	0.6	373	0.6
50	0.829668	131	0.643602	212	0.6	293	0.6	374	0.6
51	0.827548	132	0.640913	213	0.6	294	0.6	375	0.6
52	0.825412	133	0.638164	214	0.6	295	0.6	376	0.6
53	0.823289	134	0.635437	215	0.6	296	0.6	377	0.6
54	0.821152	135	0.632643	216	0.6	297	0.6	378	0.6
55	0.819027	136	0.629877	217	0.6	298	0.6	379	0.6
56	0.816887	137	0.627034	218	0.6	299	0.6	380	0.6
57	0.81476	138	0.624226	219	0.6	300	0.6	381	0.6
58	0.812618	139	0.621332	220	0.6	301	0.6	382	0.6
59	0.810488	140	0.61848	221	0.6	302	0.6	383	0.6

```
 60 0.808343   141 0.61553    222 0.6         303 0.6         384 0.6
 61 0.806209   142 0.612633   223 0.6         304 0.6         385 0.6
 62 0.804062   143 0.609624   224 0.6         305 0.6         386 0.6
 63 0.801924   144 0.606668   225 0.6         306 0.6         387 0.6
 64 0.799774   145 0.603646   226 0.6         307 0.6         388 0.6
 65 0.797632   146 0.600385   227 0.6         308 0.6         389 0.6
 66 0.795477   147 0.600001   228 0.6         309 0.6         390 0.6
 67 0.793331   148 0.6        229 0.6         310 0.6         391 0.6
 68 0.791172   149 0.6        230 0.6         311 0.6         392 0.6
 69 0.789021   150 0.6        231 0.6         312 0.6         393 0.6
 70 0.786857   151 0.6        232 0.6         313 0.6         394 0.6
 71 0.7847     152 0.6        233 0.6         314 0.6         395 0.6
 72 0.782532   153 0.6        234 0.6         315 0.6         396 0.6
 73 0.780369   154 0.6        235 0.6         316 0.6         397 0.6
 74 0.778195   155 0.6        236 0.6         317 0.6         398 0.6
 75 0.776026   156 0.6        237 0.6         318 0.6         399 0.6
 76 0.773846   157 0.6        238 0.6         319 0.6         400 0.6
 77 0.77167    158 0.6        239 0.6         320 0.6
 78 0.769483   159 0.6        240 0.6         321 0.6
 79 0.767299   160 0.6        241 0.6         322 0.6
 80 0.765106   161 0.6        242 0.6         323 0.6
"""

output_knitro_thrust_400 = """
  0 3.5           101 2.32669       202 4.32628e-07   303 1.35642e-07
  1 3.5           102 2.33505       203 4.22311e-07   304 1.34733e-07
  2 3.5           103 2.34309       204 4.11585e-07   305 1.34526e-07
  3 3.5           104 2.35198       205 4.0225e-07    306 1.33641e-07
  4 3.5           105 2.36053       206 3.92414e-07   307 1.33473e-07
  5 3.5           106 2.36998       207 3.83912e-07   308 1.3261e-07
  6 3.5           107 2.37906       208 3.74935e-07   309 1.32479e-07
  7 3.5           108 2.3891        209 3.67193e-07   310 1.31638e-07
  8 3.5           109 2.39874       210 3.58965e-07   311 1.31544e-07
  9 3.5           110 2.40941       211 3.51886e-07   312 1.30723e-07
 10 3.5           111 2.41964       212 3.44311e-07   313 1.30668e-07
 11 3.5           112 2.43098       213 3.37814e-07   314 1.29867e-07
 12 3.5           113 2.44182       214 3.30825e-07   315 1.2985e-07
 13 3.5           114 2.45388       215 3.2485e-07    316 1.29067e-07
 14 3.5           115 2.46537       216 3.18371e-07   317 1.29089e-07
 15 3.5           116 2.47818       217 3.12865e-07   318 1.28324e-07
 16 3.5           117 2.49036       218 3.06867e-07   319 1.28386e-07
 17 3.5           118 2.50398       219 3.01766e-07   320 1.27637e-07
 18 3.5           119 2.51688       220 2.96176e-07   321 1.27739e-07
 19 3.5           120 2.53135       221 2.91443e-07   322 1.27007e-07
 20 3.5           121 2.54501       222 2.86223e-07   323 1.2715e-07
 21 3.5           122 2.56041       223 2.81825e-07   324 1.26433e-07
```

22	3.5	123	2.57486	224	2.76935e-07	325	1.26619e-07
23	3.5	124	2.59124	225	2.72845e-07	326	1.25916e-07
24	3.5	125	2.60653	226	2.68264e-07	327	1.26146e-07
25	3.5	126	2.62397	227	2.64453e-07	328	1.25458e-07
26	3.5	127	2.64013	228	2.60154e-07	329	1.25732e-07
27	3.5	128	2.65871	229	2.566e-07	330	1.25057e-07
28	3.5	129	2.67578	230	2.5256e-07	331	1.25379e-07
29	3.5	130	2.6956	231	2.49234e-07	332	1.24717e-07
30	3.5	131	2.71362	232	2.45428e-07	333	1.25087e-07
31	3.5	132	2.73478	233	2.42317e-07	334	1.24437e-07
32	3.5	133	2.75377	234	2.38727e-07	335	1.24859e-07
33	3.5	134	2.77639	235	2.35814e-07	336	1.24221e-07
34	3.5	135	2.79639	236	2.32421e-07	337	1.24697e-07
35	3.5	136	2.82062	237	2.2969e-07	338	1.2407e-07
36	3.5	137	2.84162	238	2.2648e-07	339	1.24603e-07
37	3.5	138	2.86764	239	2.23917e-07	340	1.23987e-07
38	3.5	139	2.88963	240	2.20876e-07	341	1.24581e-07
39	3.49999	140	2.91764	241	2.18474e-07	342	1.23974e-07
40	3.49999	141	2.94071	242	2.15589e-07	343	1.24633e-07
41	3.49999	142	2.97	243	2.13329e-07	344	1.24035e-07
42	3.49998	143	2.99979	244	2.10588e-07	345	1.24763e-07
43	3.49998	144	3.00637	245	2.08461e-07	346	1.24175e-07
44	3.49996	145	3.15955	246	2.05853e-07	347	1.24977e-07
45	3.49992	146	1.83331	247	2.03851e-07	348	1.24397e-07
46	3.49952	147	0.193433	248	2.01367e-07	349	1.2528e-07
47	3.02328	148	0.000249842	249	1.99482e-07	350	1.24708e-07
48	2.33629	149	0.000106807	250	1.97113e-07	351	1.25679e-07
49	2.13627	150	5.59873e-05	251	1.95337e-07	352	1.25114e-07
50	2.14002	151	3.40878e-05	252	1.93076e-07	353	1.26181e-07
51	2.14059	152	2.27474e-05	253	1.91402e-07	354	1.25622e-07
52	2.14161	153	1.72709e-05	254	1.8924e-07	355	1.26794e-07
53	2.14239	154	1.31016e-05	255	1.87662e-07	356	1.26242e-07
54	2.14358	155	1.02509e-05	256	1.85595e-07	357	1.2753e-07
55	2.14453	156	8.31903e-06	257	1.84107e-07	358	1.26983e-07
56	2.14588	157	7.06644e-06	258	1.82127e-07	359	1.28401e-07
57	2.147	158	5.99341e-06	259	1.80724e-07	360	1.27859e-07
58	2.14854	159	5.09119e-06	260	1.78826e-07	361	1.29421e-07
59	2.14984	160	4.44262e-06	261	1.77503e-07	362	1.28884e-07
60	2.15155	161	3.90905e-06	262	1.75682e-07	363	1.30609e-07
61	2.15304	162	3.44866e-06	263	1.74435e-07	364	1.30075e-07
62	2.15494	163	3.10557e-06	264	1.72687e-07	365	1.31985e-07
63	2.15663	164	2.79769e-06	265	1.71511e-07	366	1.31455e-07
64	2.15873	165	2.54688e-06	266	1.69831e-07	367	1.33576e-07
65	2.16062	166	2.3087e-06	267	1.68724e-07	368	1.33048e-07
66	2.16293	167	2.11684e-06	268	1.67107e-07	369	1.35415e-07
67	2.16503	168	1.9505e-06	269	1.66065e-07	370	1.34888e-07

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 68 2.16756        169 1.80892e-06   270 1.64508e-07   371 1.3754e-07
 69 2.16987        170 1.67312e-06   271 1.63528e-07   372 1.37015e-07
 70 2.17263        171 1.56086e-06   272 1.62028e-07   373 1.40004e-07
 71 2.17518        172 1.45401e-06   273 1.61107e-07   374 1.39478e-07
 72 2.17817        173 1.36766e-06   274 1.59661e-07   375 1.42871e-07
 73 2.18096        174 1.28536e-06   275 1.58795e-07   376 1.42344e-07
 74 2.1842         175 1.21363e-06   276 1.574e-07     377 1.46227e-07
 75 2.18724        176 1.14315e-06   277 1.56589e-07   378 1.45697e-07
 76 2.19074        177 1.08257e-06   278 1.55241e-07   379 1.50186e-07
 77 2.19404        178 1.02652e-06   279 1.54482e-07   380 1.4965e-07
 78 2.19782        179 9.78896e-07   280 1.5318e-07    381 1.54903e-07
 79 2.20138        180 9.31911e-07   281 1.52471e-07   382 1.54362e-07
 80 2.20545        181 8.90228e-07   282 1.51211e-07   383 1.60602e-07
 81 2.2093         182 8.48635e-07   283 1.5055e-07    384 1.60051e-07
 82 2.21366        183 8.12901e-07   284 1.49332e-07   385 1.67606e-07
 83 2.21781        184 7.77198e-07   285 1.48717e-07   386 1.67042e-07
 84 2.22249        185 7.46662e-07   286 1.47537e-07   387 1.76417e-07
 85 2.22695        186 7.16911e-07   287 1.46967e-07   388 1.75836e-07
 86 2.23196        187 6.90944e-07   288 1.45824e-07   389 1.87857e-07
 87 2.23675        188 6.64668e-07   289 1.45298e-07   390 1.87251e-07
 88 2.2421         189 6.41461e-07   290 1.44189e-07   391 2.03386e-07
 89 2.24723        190 6.18174e-07   291 1.43706e-07   392 2.02745e-07
 90 2.25295        191 5.97874e-07   292 1.42631e-07   393 2.25929e-07
 91 2.25844        192 5.77476e-07   293 1.42189e-07   394 2.25236e-07
 92 2.26454        193 5.60188e-07   294 1.41145e-07   395 2.62504e-07
 93 2.2704         194 5.42369e-07   295 1.40744e-07   396 2.61723e-07
 94 2.27691        195 5.26915e-07   296 1.3973e-07    397 3.36449e-07
 95 2.28317        196 5.10918e-07   297 1.39369e-07   398 3.35477e-07
 96 2.2901         197 4.96816e-07   298 1.38383e-07   399 6.31134e-07
 97 2.29678        198 4.82215e-07   299 1.38061e-07   400 1.25882e-06
 98 2.30416        199 4.69383e-07   300 1.37102e-07
 99 2.31127        200 4.56109e-07   301 1.36819e-07
100 2.31912        201 4.44626e-07   302 1.35886e-07
"""

output_knitro_velocity_400 = """
  0 0              101 0.0842723     202 0.0975393     303 0.0470905
  1 0.0012428      102 0.0849716     203 0.0970082     304 0.0466032
  2 0.00249727     103 0.085682      204 0.0964783     305 0.046116
  3 0.00375085     104 0.0864042     205 0.0959501     306 0.0456289
  4 0.0050152      105 0.087139      206 0.0954231     307 0.0451418
  5 0.00627773     106 0.0878853     207 0.0948976     308 0.0446548
  6 0.00755012     107 0.0886457     208 0.0943733     309 0.0441679
  7 0.00881971     108 0.0894171     209 0.0938503     310 0.043681
  8 0.0100982      109 0.0902045     210 0.0933284     311 0.0431942
  9 0.011373       110 0.0910023     211 0.0928078     312 0.0427075
```

10	0.0126557	111	0.091818	212	0.0922882	313	0.0422208
11	0.0139337	112	0.0926436	213	0.0917698	314	0.0417342
12	0.0152188	113	0.0934891	214	0.0912523	315	0.0412476
13	0.0164982	114	0.0943438	215	0.0907359	316	0.0407611
14	0.0177838	115	0.0952207	216	0.0902204	317	0.0402747
15	0.0190628	116	0.0961061	217	0.0897059	318	0.0397883
16	0.0203471	117	0.0970161	218	0.0891923	319	0.0393019
17	0.0216239	118	0.0979337	219	0.0886796	320	0.0388156
18	0.0229053	119	0.0988786	220	0.0881676	321	0.0383294
19	0.0241783	120	0.0998301	221	0.0876566	322	0.0378432
20	0.0254552	121	0.100812	222	0.0871463	323	0.0373571
21	0.0267229	122	0.101799	223	0.0866368	324	0.036871
22	0.0279939	123	0.10282	224	0.086128	325	0.0363849
23	0.029255	124	0.103845	225	0.08562	326	0.0358989
24	0.0305187	125	0.104907	226	0.0851125	327	0.0354129
25	0.031772	126	0.105972	227	0.0846059	328	0.034927
26	0.0330275	127	0.107078	228	0.0840998	329	0.0344412
27	0.0342719	128	0.108184	229	0.0835944	330	0.0339553
28	0.0355182	129	0.109337	230	0.0830895	331	0.0334695
29	0.036753	130	0.110488	231	0.0825853	332	0.0329838
30	0.0379893	131	0.111689	232	0.0820815	333	0.0324981
31	0.0392138	132	0.112887	233	0.0815784	334	0.0320124
32	0.0404396	133	0.114141	234	0.0810758	335	0.0315267
33	0.0416535	134	0.115388	235	0.0805737	336	0.0310411
34	0.0428685	135	0.116699	236	0.0800721	337	0.0305556
35	0.0440713	136	0.117997	237	0.079571	338	0.03007
36	0.0452753	137	0.119368	238	0.0790703	339	0.0295845
37	0.0464671	138	0.120722	239	0.0785701	340	0.0290991
38	0.0476601	139	0.122156	240	0.0780703	341	0.0286136
39	0.048841	140	0.123568	241	0.077571	342	0.0281282
40	0.0500233	141	0.125071	242	0.077072	343	0.0276428
41	0.0511935	142	0.126544	243	0.0765735	344	0.0271575
42	0.0523653	143	0.128121	244	0.0760753	345	0.0266722
43	0.0535254	144	0.129668	245	0.0755776	346	0.0261869
44	0.0546873	145	0.131281	246	0.0750801	347	0.0257016
45	0.0558378	146	0.133107	247	0.0745831	348	0.0252164
46	0.0569904	147	0.132553	248	0.0740863	349	0.0247312
47	0.0581316	148	0.131695	249	0.07359	350	0.024246
48	0.0587097	149	0.130854	250	0.0730939	351	0.0237608
49	0.0590355	150	0.130028	251	0.0725982	352	0.0232757
50	0.0593825	151	0.129217	252	0.0721027	353	0.0227906
51	0.05972	152	0.128419	253	0.0716075	354	0.0223055
52	0.0600749	153	0.127636	254	0.0711126	355	0.0218204
53	0.0604209	154	0.126864	255	0.070618	356	0.0213354
54	0.0607841	155	0.126105	256	0.0701237	357	0.0208503
55	0.0611389	156	0.125357	257	0.0696296	358	0.0203653

```
56 0.0615108
57 0.0618747
58 0.0622555
59 0.0626288
60 0.0630188
61 0.0634019
62 0.0638016
63 0.0641948
64 0.0646045
65 0.0650082
66 0.0654282
67 0.0658429
68 0.0662737
69 0.0666996
70 0.0671416
71 0.0675793
72 0.0680329
73 0.0684828
74 0.0689484
75 0.0694112
76 0.0698893
77 0.0703652
78 0.0708564
79 0.0713462
80 0.071851
81 0.072355
82 0.072874
83 0.073393
84 0.0739267
85 0.0744613
86 0.0750104
87 0.0755612
88 0.0761263
89 0.0766942
90 0.077276
91 0.0778616
92 0.0784609
93 0.079065
94 0.0796825
95 0.0803059
96 0.0809426
97 0.0815863
98 0.0822429
99 0.0829077
100 0.0835852
157 0.12462
158 0.123893
159 0.123176
160 0.122468
161 0.12177
162 0.121079
163 0.120398
164 0.119723
165 0.119057
166 0.118396
167 0.117744
168 0.117097
169 0.116457
170 0.115822
171 0.115193
172 0.11457
173 0.113952
174 0.113338
175 0.11273
176 0.112125
177 0.111526
178 0.11093
179 0.110339
180 0.109751
181 0.109167
182 0.108587
183 0.10801
184 0.107436
185 0.106865
186 0.106297
187 0.105733
188 0.10517
189 0.104611
190 0.104054
191 0.1035
192 0.102948
193 0.102398
194 0.10185
195 0.101305
196 0.100762
197 0.10022
198 0.0996804
199 0.0991427
200 0.0986065
201 0.0980722
258 0.0691358
259 0.0686422
260 0.0681489
261 0.0676558
262 0.0671629
263 0.0666702
264 0.0661778
265 0.0656856
266 0.0651935
267 0.0647017
268 0.06421
269 0.0637186
270 0.0632272
271 0.0627362
272 0.0622452
273 0.0617544
274 0.0612638
275 0.0607734
276 0.060283
277 0.0597929
278 0.0593029
279 0.058813
280 0.0583233
281 0.0578337
282 0.0573442
283 0.0568549
284 0.0563657
285 0.0558766
286 0.0553876
287 0.0548988
288 0.05441
289 0.0539214
290 0.0534328
291 0.0529444
292 0.0524561
293 0.0519679
294 0.0514797
295 0.0509917
296 0.0505038
297 0.0500159
298 0.0495281
299 0.0490405
300 0.0485528
301 0.0480653
302 0.0475779
359 0.0198804
360 0.0193954
361 0.0189104
362 0.0184255
363 0.0179406
364 0.0174557
365 0.0169708
366 0.016486
367 0.0160011
368 0.0155163
369 0.0150315
370 0.0145467
371 0.0140619
372 0.0135771
373 0.0130923
374 0.0126076
375 0.0121229
376 0.0116381
377 0.0111534
378 0.0106687
379 0.010184
380 0.00969931
381 0.00921464
382 0.00872996
383 0.0082453
384 0.00776064
385 0.00727599
386 0.00679135
387 0.00630671
388 0.00582208
389 0.00533745
390 0.00485283
391 0.00436822
392 0.0038836
393 0.00339899
394 0.00291439
395 0.00242978
396 0.00194518
397 0.00146058
398 0.00097598
399 0.000491381
400 6.78423e-06
"""
```

```
output_loqo_height_400 = """
  0 1          81 1.00179   162 1.00591   243 1.00985   324 1.01214
  1 1          82 1.00182   163 1.00597   244 1.00989   325 1.01216
  2 1          83 1.00186   164 1.00603   245 1.00993   326 1.01217
  3 1          84 1.00189   165 1.00609   246 1.00997   327 1.01219
  4 1          85 1.00193   166 1.00615   247 1.01      328 1.01221
  5 1.00001    86 1.00197   167 1.00621   248 1.01004   329 1.01223
  6 1.00001    87 1.00201   168 1.00627   249 1.01008   330 1.01224
  7 1.00001    88 1.00204   169 1.00633   250 1.01011   331 1.01226
  8 1.00002    89 1.00208   170 1.00639   251 1.01015   332 1.01228
  9 1.00003    90 1.00212   171 1.00644   252 1.01019   333 1.01229
 10 1.00003    91 1.00216   172 1.0065    253 1.01022   334 1.01231
 11 1.00004    92 1.0022    173 1.00656   254 1.01026   335 1.01232
 12 1.00005    93 1.00224   174 1.00661   255 1.01029   336 1.01234
 13 1.00005    94 1.00228   175 1.00667   256 1.01033   337 1.01236
 14 1.00006    95 1.00232   176 1.00673   257 1.01036   338 1.01237
 15 1.00007    96 1.00236   177 1.00678   258 1.0104    339 1.01239
 16 1.00008    97 1.0024    178 1.00684   259 1.01043   340 1.0124
 17 1.00009    98 1.00244   179 1.00689   260 1.01047   341 1.01241
 18 1.0001     99 1.00248   180 1.00695   261 1.0105    342 1.01243
 19 1.00011   100 1.00252   181 1.007     262 1.01053   343 1.01244
 20 1.00013   101 1.00256   182 1.00706   263 1.01057   344 1.01246
 21 1.00014   102 1.0026    183 1.00711   264 1.0106    345 1.01247
 22 1.00015   103 1.00264   184 1.00716   265 1.01063   346 1.01248
 23 1.00017   104 1.00269   185 1.00722   266 1.01066   347 1.0125
 24 1.00018   105 1.00273   186 1.00727   267 1.0107    348 1.01251
 25 1.0002    106 1.00277   187 1.00732   268 1.01073   349 1.01252
 26 1.00021   107 1.00282   188 1.00737   269 1.01076   350 1.01253
 27 1.00023   108 1.00286   189 1.00743   270 1.01079   351 1.01254
 28 1.00025   109 1.00291   190 1.00748   271 1.01082   352 1.01256
 29 1.00026   110 1.00295   191 1.00753   272 1.01085   353 1.01257
 30 1.00028   111 1.003     192 1.00758   273 1.01089   354 1.01258
 31 1.0003    112 1.00304   193 1.00763   274 1.01092   355 1.01259
 32 1.00032   113 1.00309   194 1.00768   275 1.01095   356 1.0126
 33 1.00034   114 1.00314   195 1.00773   276 1.01098   357 1.01261
 34 1.00036   115 1.00318   196 1.00778   277 1.01101   358 1.01262
 35 1.00039   116 1.00323   197 1.00783   278 1.01104   359 1.01263
 36 1.00041   117 1.00328   198 1.00788   279 1.01107   360 1.01264
 37 1.00043   118 1.00333   199 1.00793   280 1.01109   361 1.01265
 38 1.00045   119 1.00337   200 1.00798   281 1.01112   362 1.01266
 39 1.00048   120 1.00342   201 1.00803   282 1.01115   363 1.01267
 40 1.0005    121 1.00347   202 1.00808   283 1.01118   364 1.01268
 41 1.00053   122 1.00352   203 1.00813   284 1.01121   365 1.01269
 42 1.00055   123 1.00358   204 1.00818   285 1.01124   366 1.0127
```

```
 43 1.00058   124 1.00363   205 1.00822   286 1.01126   367 1.0127
 44 1.00061   125 1.00368   206 1.00827   287 1.01129   368 1.01271
 45 1.00063   126 1.00373   207 1.00832   288 1.01132   369 1.01272
 46 1.00066   127 1.00378   208 1.00837   289 1.01135   370 1.01273
 47 1.00069   128 1.00384   209 1.00841   290 1.01137   371 1.01273
 48 1.00072   129 1.00389   210 1.00846   291 1.0114    372 1.01274
 49 1.00075   130 1.00395   211 1.00851   292 1.01143   373 1.01275
 50 1.00078   131 1.004     212 1.00855   293 1.01145   374 1.01275
 51 1.00081   132 1.00406   213 1.0086    294 1.01148   375 1.01276
 52 1.00084   133 1.00411   214 1.00864   295 1.0115    376 1.01277
 53 1.00087   134 1.00417   215 1.00869   296 1.01153   377 1.01277
 54 1.0009    135 1.00423   216 1.00873   297 1.01155   378 1.01278
 55 1.00093   136 1.00429   217 1.00878   298 1.01158   379 1.01278
 56 1.00096   137 1.00435   218 1.00882   299 1.0116    380 1.01279
 57 1.00099   138 1.00441   219 1.00887   300 1.01163   381 1.01279
 58 1.00102   139 1.00447   220 1.00891   301 1.01165   382 1.0128
 59 1.00105   140 1.00453   221 1.00895   302 1.01167   383 1.0128
 60 1.00108   141 1.00459   222 1.009     303 1.0117    384 1.0128
 61 1.00111   142 1.00465   223 1.00904   304 1.01172   385 1.01281
 62 1.00115   143 1.00471   224 1.00908   305 1.01174   386 1.01281
 63 1.00118   144 1.00478   225 1.00913   306 1.01177   387 1.01282
 64 1.00121   145 1.00484   226 1.00917   307 1.01179   388 1.01282
 65 1.00124   146 1.00491   227 1.00921   308 1.01181   389 1.01282
 66 1.00127   147 1.00497   228 1.00925   309 1.01183   390 1.01282
 67 1.00131   148 1.00504   229 1.0093    310 1.01186   391 1.01283
 68 1.00134   149 1.0051    230 1.00934   311 1.01188   392 1.01283
 69 1.00137   150 1.00517   231 1.00938   312 1.0119    393 1.01283
 70 1.00141   151 1.00523   232 1.00942   313 1.01192   394 1.01283
 71 1.00144   152 1.00529   233 1.00946   314 1.01194   395 1.01283
 72 1.00147   153 1.00536   234 1.0095    315 1.01196   396 1.01283
 73 1.00151   154 1.00542   235 1.00954   316 1.01198   397 1.01284
 74 1.00154   155 1.00548   236 1.00958   317 1.012     398 1.01284
 75 1.00157   156 1.00555   237 1.00962   318 1.01202   399 1.01284
 76 1.00161   157 1.00561   238 1.00966   319 1.01204   400 1.01284
 77 1.00164   158 1.00567   239 1.0097    320 1.01206
 78 1.00168   159 1.00573   240 1.00974   321 1.01208
 79 1.00171   160 1.00579   241 1.00978   322 1.0121
 80 1.00175   161 1.00585   242 1.00981   323 1.01212
"""

output_loqo_mass_400 = """
  0 1           81 0.762983    162 0.6          243 0.6          324 0.6
  1 0.996521    82 0.760867    163 0.6          244 0.6          325 0.6
  2 0.993042    83 0.758558    164 0.6          245 0.6          326 0.6
  3 0.989562    84 0.756331    165 0.6          246 0.6          327 0.6
  4 0.986083    85 0.754282    166 0.6          247 0.6          328 0.6
```

5	0.982604	86	0.752047	167	0.6	248	0.6	329	0.6
6	0.979125	87	0.749828	168	0.6	249	0.6	330	0.6
7	0.975646	88	0.747656	169	0.6	250	0.6	331	0.6
8	0.972166	89	0.745362	170	0.6	251	0.6	332	0.6
9	0.968687	90	0.743237	171	0.6	252	0.6	333	0.6
10	0.965208	91	0.740867	172	0.6	253	0.6	334	0.6
11	0.961729	92	0.738781	173	0.6	254	0.6	335	0.6
12	0.95825	93	0.736399	174	0.6	255	0.6	336	0.6
13	0.95477	94	0.734172	175	0.6	256	0.6	337	0.6
14	0.951291	95	0.731919	176	0.6	257	0.6	338	0.6
15	0.947812	96	0.729656	177	0.6	258	0.6	339	0.6
16	0.944333	97	0.727337	178	0.6	259	0.6	340	0.6
17	0.940853	98	0.725107	179	0.6	260	0.6	341	0.6
18	0.937374	99	0.722739	180	0.6	261	0.6	342	0.6
19	0.933895	100	0.7205	181	0.6	262	0.6	343	0.6
20	0.930416	101	0.718162	182	0.6	263	0.6	344	0.6
21	0.926937	102	0.715882	183	0.6	264	0.6	345	0.6
22	0.923457	103	0.713523	184	0.6	265	0.6	346	0.6
23	0.919978	104	0.711265	185	0.6	266	0.6	347	0.6
24	0.916499	105	0.708801	186	0.6	267	0.6	348	0.6
25	0.91302	106	0.706577	187	0.6	268	0.6	349	0.6
26	0.909541	107	0.704066	188	0.6	269	0.6	350	0.6
27	0.906061	108	0.701854	189	0.6	270	0.6	351	0.6
28	0.902582	109	0.699311	190	0.6	271	0.6	352	0.6
29	0.899103	110	0.697047	191	0.6	272	0.6	353	0.6
30	0.895624	111	0.694572	192	0.6	273	0.6	354	0.6
31	0.892145	112	0.692165	193	0.6	274	0.6	355	0.6
32	0.888665	113	0.689746	194	0.6	275	0.6	356	0.6
33	0.885186	114	0.687327	195	0.6	276	0.6	357	0.6
34	0.881707	115	0.684912	196	0.6	277	0.6	358	0.6
35	0.878228	116	0.682501	197	0.6	278	0.6	359	0.6
36	0.874749	117	0.679936	198	0.6	279	0.6	360	0.6
37	0.871269	118	0.677534	199	0.6	280	0.6	361	0.6
38	0.86779	119	0.674975	200	0.6	281	0.6	362	0.6
39	0.864311	120	0.672591	201	0.6	282	0.6	363	0.6
40	0.860832	121	0.669982	202	0.6	283	0.6	364	0.6
41	0.857352	122	0.667462	203	0.6	284	0.6	365	0.6
42	0.853873	123	0.664874	204	0.6	285	0.6	366	0.6
43	0.850394	124	0.662304	205	0.6	286	0.6	367	0.6
44	0.846915	125	0.659733	206	0.6	287	0.6	368	0.6
45	0.843436	126	0.657157	207	0.6	288	0.6	369	0.6
46	0.840086	127	0.654392	208	0.6	289	0.6	370	0.6
47	0.83728	128	0.65182	209	0.6	290	0.6	371	0.6
48	0.834497	129	0.648632	210	0.6	291	0.6	372	0.6
49	0.832083	130	0.646374	211	0.6	292	0.6	373	0.6
50	0.829629	131	0.643381	212	0.6	293	0.6	374	0.6

```
 51 0.82745    132 0.640891  213 0.6       294 0.6       375 0.6
 52 0.825217   133 0.637947  214 0.6       295 0.6       376 0.6
 53 0.82316    134 0.635301  215 0.6       296 0.6       377 0.6
 54 0.821048   135 0.632561  216 0.6       297 0.6       378 0.6
 55 0.819013   136 0.629831  217 0.6       298 0.6       379 0.6
 56 0.816925   137 0.627065  218 0.6       299 0.6       380 0.6
 57 0.81479    138 0.624326  219 0.6       300 0.6       381 0.6
 58 0.812726   139 0.621557  220 0.6       301 0.6       382 0.6
 59 0.810604   140 0.618842  221 0.6       302 0.6       383 0.6
 60 0.808485   141 0.616193  222 0.6       303 0.6       384 0.6
 61 0.806355   142 0.61338   223 0.6       304 0.6       385 0.6
 62 0.804256   143 0.611108  224 0.6       305 0.6       386 0.6
 63 0.802079   144 0.608317  225 0.6       306 0.6       387 0.6
 64 0.799959   145 0.606507  226 0.6       307 0.6       388 0.6
 65 0.797768   146 0.603771  227 0.6       308 0.6       389 0.6
 66 0.79563    147 0.602652  228 0.6       309 0.6       390 0.6
 67 0.793458   148 0.600322  229 0.6       310 0.6       391 0.6
 68 0.791317   149 0.6       230 0.6       311 0.6       392 0.6
 69 0.789179   150 0.6       231 0.6       312 0.6       393 0.6
 70 0.786982   151 0.6       232 0.6       313 0.6       394 0.6
 71 0.784845   152 0.6       233 0.6       314 0.6       395 0.6
 72 0.782664   153 0.6       234 0.6       315 0.6       396 0.6
 73 0.780546   154 0.6       235 0.6       316 0.6       397 0.6
 74 0.778317   155 0.6       236 0.6       317 0.6       398 0.6
 75 0.776203   156 0.6       237 0.6       318 0.6       399 0.6
 76 0.773989   157 0.6       238 0.6       319 0.6       400 0.6
 77 0.771746   158 0.6       239 0.6       320 0.6
 78 0.769614   159 0.6       240 0.6       321 0.6
 79 0.767383   160 0.6       241 0.6       322 0.6
 80 0.765266   161 0.6       242 0.6       323 0.6
"""

output_loqo_thrust_400 = """
  0 3.5          101 2.32311       202 7.10296e-10   303 2.24652e-10
  1 3.5          102 2.33347       203 7.01233e-10   304 2.20716e-10
  2 3.5          103 2.32224       204 6.75599e-10   305 2.22807e-10
  3 3.5          104 2.37498       205 6.68132e-10   306 2.18902e-10
  4 3.5          105 2.35782       206 6.44146e-10   307 2.21042e-10
  5 3.5          106 2.38193       207 6.37783e-10   308 2.17215e-10
  6 3.5          107 2.37588       208 6.15543e-10   309 2.19402e-10
  7 3.5          108 2.39155       209 6.10026e-10   310 2.15628e-10
  8 3.5          109 2.41792       210 5.89539e-10   311 2.17841e-10
  9 3.5          110 2.38357       211 5.84547e-10   312 2.14138e-10
 10 3.5          111 2.45527       212 5.65173e-10   313 2.16384e-10
 11 3.5          112 2.42744       213 5.6094e-10    314 2.12709e-10
 12 3.5          113 2.43378       214 5.42874e-10   315 2.15017e-10
```

13 3.5	114 2.43147	215 5.39633e-10	316 2.11387e-10
14 3.5	115 2.42706	216 5.22313e-10	317 2.13779e-10
15 3.5	116 2.50268	217 5.19473e-10	318 2.10147e-10
16 3.5	117 2.49828	218 5.03585e-10	319 2.1263e-10
17 3.5	118 2.49539	219 5.00867e-10	320 2.09013e-10
18 3.5	119 2.48624	220 4.85745e-10	321 2.11566e-10
19 3.5	120 2.51144	221 4.83651e-10	322 2.07962e-10
20 3.5	121 2.57995	222 4.69388e-10	323 2.10589e-10
21 3.5	122 2.56922	223 4.67478e-10	324 2.07018e-10
22 3.5	123 2.59451	224 4.54155e-10	325 2.097e-10
23 3.5	124 2.58587	225 4.52699e-10	326 2.06171e-10
24 3.5	125 2.58893	226 4.39897e-10	327 2.08896e-10
25 3.5	126 2.68638	227 4.38737e-10	328 2.05424e-10
26 3.5	127 2.68441	228 4.26504e-10	329 2.08212e-10
27 3.5	128 2.89766	229 4.25847e-10	330 2.04762e-10
28 3.5	129 2.73944	230 4.13975e-10	331 2.07629e-10
29 3.5	130 2.64088	231 4.13511e-10	332 2.04204e-10
30 3.5	131 2.75791	232 4.02218e-10	333 2.07146e-10
31 3.5	132 2.73316	233 4.02005e-10	334 2.03727e-10
32 3.5	133 2.81138	234 3.91284e-10	335 2.06778e-10
33 3.5	134 2.70941	235 3.9127e-10	336 2.03382e-10
34 3.5	135 2.75128	236 3.8096e-10	337 2.06491e-10
35 3.5	136 2.76426	237 3.81076e-10	338 2.0313e-10
36 3.5	137 2.76923	238 3.71375e-10	339 2.06338e-10
37 3.5	138 2.77029	239 3.71508e-10	340 2.02994e-10
38 3.5	139 2.75841	240 3.62129e-10	341 2.063e-10
39 3.5	140 2.69802	241 3.62466e-10	342 2.0297e-10
40 3.5	141 2.74725	242 3.53427e-10	343 2.06369e-10
41 3.5	142 2.55803	243 3.53797e-10	344 2.03073e-10
42 3.5	143 2.54652	244 3.45211e-10	345 2.06595e-10
43 3.5	144 2.31401	245 3.45725e-10	346 2.03301e-10
44 3.5	145 2.28683	246 3.37458e-10	347 2.06944e-10
45 3.43493	146 1.93893	247 3.38035e-10	348 2.03685e-10
46 3.09598	147 1.73473	248 3.3014e-10	349 2.07461e-10
47 2.81117	148 1.33408	249 3.30826e-10	350 2.04194e-10
48 2.61439	149 0.161852	250 3.23136e-10	351 2.08124e-10
49 2.44851	150 3.18278e-08	251 3.2392e-10	352 2.04882e-10
50 2.33022	151 4.07003e-08	252 3.16512e-10	353 2.0892e-10
51 2.21933	152 3.45843e-08	253 3.17353e-10	354 2.05699e-10
52 2.15792	153 2.59337e-08	254 3.10232e-10	355 2.09922e-10
53 2.09696	154 2.14445e-08	255 3.1113e-10	356 2.06713e-10
54 2.08602	155 1.67103e-08	256 3.04245e-10	357 2.11139e-10
55 2.07388	156 1.41507e-08	257 3.05257e-10	358 2.07921e-10
56 2.12367	157 1.15096e-08	258 2.98573e-10	359 2.12564e-10
57 2.11183	158 9.97229e-09	259 2.99664e-10	360 2.09337e-10
58 2.10581	159 8.50752e-09	260 2.9312e-10	361 2.14272e-10

```
 59 2.13319        160 7.42858e-09   261 2.94336e-10   362 2.11004e-10
 60 2.13696        161 6.53372e-09   262 2.8796e-10    363 2.16214e-10
 61 2.127          162 5.78616e-09   263 2.89165e-10   364 2.12939e-10
 62 2.15094        163 5.1726e-09    264 2.83074e-10   365 2.18484e-10
 63 2.16129        164 4.63958e-09   265 2.84307e-10   366 2.15202e-10
 64 2.1685         165 4.22543e-09   266 2.78377e-10   367 2.21126e-10
 65 2.17747        166 3.83075e-09   267 2.79718e-10   368 2.17805e-10
 66 2.16794        167 3.537e-09     268 2.73939e-10   369 2.24161e-10
 67 2.16935        168 3.22674e-09   269 2.75285e-10   370 2.2083e-10
 68 2.15213        169 3.00358e-09   270 2.69703e-10   371 2.27684e-10
 69 2.18053        170 2.76381e-09   271 2.71052e-10   372 2.24321e-10
 70 2.18011        171 2.5987e-09    272 2.65621e-10   373 2.31739e-10
 71 2.17201        172 2.40457e-09   273 2.66979e-10   374 2.28324e-10
 72 2.16195        173 2.27445e-09   274 2.61725e-10   375 2.36499e-10
 73 2.18668        174 2.1174e-09    275 2.63146e-10   376 2.33013e-10
 74 2.18454        175 2.01523e-09   276 2.58015e-10   377 2.42084e-10
 75 2.17684        176 1.88622e-09   277 2.59476e-10   378 2.38529e-10
 76 2.24219        177 1.80238e-09   278 2.54444e-10   379 2.48623e-10
 77 2.20032        178 1.692e-09     279 2.55966e-10   380 2.44982e-10
 78 2.19427        179 1.62461e-09   280 2.5105e-10    381 2.56432e-10
 79 2.18692        180 1.52984e-09   281 2.52615e-10   382 2.52732e-10
 80 2.21334        181 1.4756e-09    282 2.47828e-10   383 2.65861e-10
 81 2.21261        182 1.39229e-09   283 2.4944e-10    384 2.62023e-10
 82 2.22555        183 1.3482e-09    284 2.44752e-10   385 2.7739e-10
 83 2.28186        184 1.2769e-09    285 2.46405e-10   386 2.73451e-10
 84 2.15068        185 1.24075e-09   286 2.41787e-10   387 2.91966e-10
 85 2.15469        186 1.17773e-09   287 2.4347e-10    388 2.87843e-10
 86 2.2404         187 1.1479e-09    288 2.38972e-10   389 3.10886e-10
 87 2.20866        188 1.09201e-09   289 2.40716e-10   390 3.06527e-10
 88 2.24658        189 1.06638e-09   290 2.36278e-10   391 3.36584e-10
 89 2.2227         190 1.01631e-09   291 2.38027e-10   392 3.31873e-10
 90 2.26079        191 9.94799e-10   292 2.33713e-10   393 3.73892e-10
 91 2.24139        192 9.49498e-10   293 2.35533e-10   394 3.68703e-10
 92 2.24738        193 9.31286e-10   294 2.31265e-10   395 4.34446e-10
 93 2.31836        194 8.90391e-10   295 2.33108e-10   396 4.28378e-10
 94 2.25348        195 8.74488e-10   296 2.28906e-10   397 5.56743e-10
 95 2.27132        196 8.37801e-10   297 2.30835e-10   398 5.49112e-10
 96 2.30463        197 8.23921e-10   298 2.26688e-10   399 1.04472e-09
 97 2.2883         198 7.90623e-10   299 2.28666e-10   400 2.06046e-09
 98 2.31249        199 7.78854e-10   300 2.24604e-10
 99 2.31689        200 7.48246e-10   301 2.2659e-10
100 2.30227        201 7.37885e-10   302 2.22624e-10
"""

output_loqo_velocity_400 = """
  0 0            101 0.084166      202 0.0976193     303 0.0471725
```

1	0.00124257	102	0.0848481	203	0.097088	304	0.0466853
2	0.00249682	103	0.0855738	204	0.0965581	305	0.0461982
3	0.00375017	104	0.0862517	205	0.0960297	306	0.0457111
4	0.00501429	105	0.0870636	206	0.0955026	307	0.0452241
5	0.00627659	106	0.0877283	207	0.094977	308	0.0447372
6	0.00754874	107	0.0885876	208	0.0944525	309	0.0442503
7	0.00881811	108	0.0892547	209	0.0939295	310	0.0437635
8	0.0100964	109	0.0901503	210	0.0934075	311	0.0432768
9	0.0113709	110	0.090866	211	0.0928868	312	0.0427902
10	0.0126534	111	0.0917269	212	0.0923671	313	0.0423036
11	0.0139312	112	0.0925575	213	0.0918486	314	0.041817
12	0.0152161	113	0.0933921	214	0.0913311	315	0.0413305
13	0.0164952	114	0.0942453	215	0.0908146	316	0.0408441
14	0.0177806	115	0.0950906	216	0.0902991	317	0.0403577
15	0.0190593	116	0.0959515	217	0.0897846	318	0.0398714
16	0.0203435	117	0.0969214	218	0.0892708	319	0.0393851
17	0.02162	118	0.0977896	219	0.0887581	320	0.0388989
18	0.0229012	119	0.0987709	220	0.0882461	321	0.0384128
19	0.024174	120	0.0996398	221	0.0877351	322	0.0379266
20	0.0254506	121	0.100675	222	0.0872247	323	0.0374406
21	0.0267181	122	0.10166	223	0.0867152	324	0.0369546
22	0.0279889	123	0.102695	224	0.0862064	325	0.0364686
23	0.0292498	124	0.103734	225	0.0856984	326	0.0359827
24	0.0305133	125	0.104773	226	0.0851909	327	0.0354968
25	0.0317664	126	0.105833	227	0.0846842	328	0.035011
26	0.0330216	127	0.107037	228	0.0841781	329	0.0345252
27	0.0342659	128	0.10811	229	0.0836727	330	0.0340394
28	0.0355119	129	0.109658	230	0.0831678	331	0.0335537
29	0.0367466	130	0.110506	231	0.0826636	332	0.033068
30	0.0379827	131	0.111924	232	0.0821599	333	0.0325824
31	0.039207	132	0.112969	233	0.0816568	334	0.0320968
32	0.0404326	133	0.114371	234	0.0811541	335	0.0316112
33	0.0416463	134	0.115556	235	0.0806521	336	0.0311257
34	0.0428611	135	0.11682	236	0.0801504	337	0.0306402
35	0.0440638	136	0.118091	237	0.0796494	338	0.0301548
36	0.0452676	137	0.119397	238	0.0791487	339	0.0296694
37	0.0464592	138	0.120697	239	0.0786485	340	0.029184
38	0.047652	139	0.122029	240	0.0781487	341	0.0286986
39	0.0488328	140	0.123332	241	0.0776495	342	0.0282133
40	0.0500148	141	0.12459	242	0.0771505	343	0.027728
41	0.0511849	142	0.125996	243	0.076652	344	0.0272427
42	0.0523566	143	0.126971	244	0.0761539	345	0.0267575
43	0.0535165	144	0.128383	245	0.0756562	346	0.0262723
44	0.0546782	145	0.129002	246	0.0751587	347	0.0257871
45	0.0558286	146	0.130395	247	0.0746617	348	0.0253019
46	0.0569044	147	0.130463	248	0.074165	349	0.0248168

47	0.0576463	148	0.131547	249	0.0736687	350	0.0243317
48	0.0583833	149	0.130972	250	0.0731726	351	0.0238466
49	0.0588896	150	0.130144	251	0.0726769	352	0.0233616
50	0.0594325	151	0.129331	252	0.0721815	353	0.0228765
51	0.0598	152	0.128532	253	0.0716864	354	0.0223915
52	0.060217	153	0.127746	254	0.0711915	355	0.0219066
53	0.0605162	154	0.126973	255	0.070697	356	0.0214216
54	0.0608685	155	0.126212	256	0.0702027	357	0.0209367
55	0.0611619	156	0.125463	257	0.0697087	358	0.0204517
56	0.0615084	157	0.124724	258	0.0692149	359	0.0199668
57	0.0618701	158	0.123996	259	0.0687214	360	0.0194819
58	0.06221	159	0.123278	260	0.0682281	361	0.0189971
59	0.0625726	160	0.122569	261	0.067735	362	0.0185122
60	0.0629539	161	0.121869	262	0.0672422	363	0.0180274
61	0.0633288	162	0.121178	263	0.0667496	364	0.0175426
62	0.0637059	163	0.120495	264	0.0662572	365	0.0170578
63	0.0641185	164	0.11982	265	0.065765	366	0.016573
64	0.0645159	165	0.119152	266	0.065273	367	0.0160883
65	0.0649454	166	0.118491	267	0.0647813	368	0.0156035
66	0.0653615	167	0.117838	268	0.0642896	369	0.0151188
67	0.0657872	168	0.11719	269	0.0637982	370	0.0146341
68	0.066213	169	0.116549	270	0.063307	371	0.0141494
69	0.0666251	170	0.115914	271	0.062816	372	0.0136647
70	0.0670954	171	0.115284	272	0.0623251	373	0.01318
71	0.0675145	172	0.11466	273	0.0618344	374	0.0126953
72	0.0679834	173	0.114041	274	0.0613438	375	0.0122107
73	0.0683976	174	0.113427	275	0.0608534	376	0.011726
74	0.0689074	175	0.112818	276	0.0603632	377	0.0112414
75	0.0693263	176	0.112213	277	0.0598731	378	0.0107568
76	0.0698359	177	0.111613	278	0.0593831	379	0.0102722
77	0.0703467	178	0.111017	279	0.0588934	380	0.00978756
78	0.0708108	179	0.110425	280	0.0584037	381	0.00930297
79	0.0713229	180	0.109837	281	0.0579142	382	0.00881838
80	0.0717862	181	0.109253	282	0.0574247	383	0.0083338
81	0.0723424	182	0.108672	283	0.0569355	384	0.00784923
82	0.0728129	183	0.108094	284	0.0564463	385	0.00736467
83	0.073396	184	0.10752	285	0.0559573	386	0.00688011
84	0.0739486	185	0.106949	286	0.0554684	387	0.00639556
85	0.0743685	186	0.106381	287	0.0549796	388	0.00591101
86	0.0749379	187	0.105816	288	0.0544909	389	0.00542647
87	0.0754798	188	0.105253	289	0.0540024	390	0.00494193
88	0.0760172	189	0.104694	290	0.0535139	391	0.0044574
89	0.0766196	190	0.104137	291	0.0530256	392	0.00397288
90	0.0771345	191	0.103582	292	0.0525373	393	0.00348835
91	0.0777988	192	0.10303	293	0.0520492	394	0.00300383
92	0.0782964	193	0.10248	294	0.0515611	395	0.00251932

```
 93 0.0789807      194 0.101932       295 0.0510731      396 0.0020348
 94 0.079583       195 0.101386       296 0.0505852      397 0.00155029
 95 0.0801903      196 0.100843       297 0.0500975      398 0.00106577
 96 0.0808277      197 0.100301       298 0.0496098      399 0.000581261
 97 0.0814909      198 0.0997611      299 0.0491222      400 9.67503e-05
 98 0.0821165      199 0.0992232      300 0.0486346
 99 0.0828244      200 0.0986868      301 0.0481472
100 0.0834665      201 0.0981524      302 0.0476598
"""


output_minos_height_100 = """
  0 1            21 1.00189     42 1.00627     63 1.01018     84 1.01233
  1 1            22 1.00204     43 1.0065      64 1.01032     85 1.0124
  2 1.00002      23 1.00219     44 1.00672     65 1.01046     86 1.01245
  3 1.00004      24 1.00235     45 1.00695     66 1.01059     87 1.0125
  4 1.00008      25 1.00251     46 1.00716     67 1.01073     88 1.01255
  5 1.00012      26 1.00269     47 1.00737     68 1.01085     89 1.0126
  6 1.00018      27 1.00286     48 1.00758     69 1.01097     90 1.01264
  7 1.00024      28 1.00304     49 1.00778     70 1.01109     91 1.01267
  8 1.00032      29 1.00323     50 1.00798     71 1.01121     92 1.01271
  9 1.0004       30 1.00342     51 1.00817     72 1.01131     93 1.01274
 10 1.0005       31 1.00362     52 1.00836     73 1.01142     94 1.01276
 11 1.0006       32 1.00384     53 1.00855     74 1.01152     95 1.01278
 12 1.00072      33 1.00405     54 1.00873     75 1.01162     96 1.0128
 13 1.00084      34 1.00428     55 1.00891     76 1.01172     97 1.01281
 14 1.00096      35 1.00452     56 1.00908     77 1.01181     98 1.01282
 15 1.00108      36 1.00477     57 1.00925     78 1.01189     99 1.01283
 16 1.00121      37 1.00504     58 1.00941     79 1.01198    100 1.01283
 17 1.00134      38 1.00529     59 1.00958     80 1.01206
 18 1.00147      39 1.00555     60 1.00973     81 1.01213
 19 1.00161      40 1.00579     61 1.00989     82 1.0122
 20 1.00175      41 1.00603     62 1.01004     83 1.01227
"""

output_minos_mass_100 = """
  0 1            21 0.756421     42 0.6           63 0.6           84 0.6
  1 0.986088     22 0.747446     43 0.6           64 0.6           85 0.6
  2 0.972176     23 0.73868      44 0.6           65 0.6           86 0.6
  3 0.958265     24 0.729712     45 0.6           66 0.6           87 0.6
  4 0.944353     25 0.720302     46 0.6           67 0.6           88 0.6
  5 0.930441     26 0.711198     47 0.6           68 0.6           89 0.6
  6 0.916529     27 0.701574     48 0.6           69 0.6           90 0.6
  7 0.902618     28 0.691992     49 0.6           70 0.6           91 0.6
  8 0.888706     29 0.682281     50 0.6           71 0.6           92 0.6
  9 0.874794     30 0.67262      51 0.6           72 0.6           93 0.6
```

```
 10 0.860882     31 0.662372     52 0.6          73 0.6          94 0.6
 11 0.84697      32 0.652156     53 0.6          74 0.6          95 0.6
 12 0.834039     33 0.641445     54 0.6          75 0.6          96 0.6
 13 0.825293     34 0.629441     55 0.6          76 0.6          97 0.6
 14 0.81711      35 0.619602     56 0.6          77 0.6          98 0.6
 15 0.808209     36 0.606566     57 0.6          78 0.6          99 0.6
 16 0.799909     37 0.6          58 0.6          79 0.6         100 0.6
 17 0.791062     38 0.6          59 0.6          80 0.6
 18 0.7829       39 0.6          60 0.6          81 0.6
 19 0.77391      40 0.6          61 0.6          82 0.6
 20 0.765233     41 0.6          62 0.6          83 0.6

"""

output_minos_thrust_100 = """
  0 3.5         21 2.23748     42 0            63 0            84 0
  1 3.5         22 2.23164     43 0            64 0            85 0
  2 3.5         23 2.23085     44 0            65 0            86 0
  3 3.5         24 2.31191     45 0            66 0            87 0
  4 3.5         25 2.32883     46 0            67 0            88 0
  5 3.5         26 2.35583     47 0            68 0            89 0
  6 3.5         27 2.41604     48 0            69 0            90 0
  7 3.5         28 2.42684     49 0            70 0            91 0
  8 3.5         29 2.4368      50 0            71 0            92 0
  9 3.5         30 2.50441     51 0            72 0            93 0
 10 3.5         31 2.5743      52 0            73 0            94 0
 11 3.37666     32 2.63258     53 0            74 0            95 0
 12 2.72691     33 2.85739     54 0            75 0            96 0
 13 2.12959     34 2.74758     55 0            76 0            97 0
 14 2.14902     35 2.87746     56 0            77 0            98 0
 15 2.1637      36 2.46584     57 0            78 0            99 0
 16 2.15691     37 0.82596     58 0            79 0           100 0
 17 2.13964     38 0           59 0            80 0
 18 2.1576      39 0           60 0            81 0
 19 2.2224      40 0           61 0            82 0
 20 2.20002     41 0           62 0            83 0
"""

output_minos_velocity_100 = """
  0 0               26 0.0862919        52 0.0944221        78 0.0427805
  1 0.00496849      27 0.0893882        53 0.092339         79 0.0408353
  2 0.0101024       28 0.0926661        54 0.0902708        80 0.0388907
  3 0.0151757       29 0.096036         55 0.0882199        81 0.0369472
  4 0.0203554       30 0.099594         56 0.0861801        82 0.0350041
  5 0.0254137       31 0.103607         57 0.0841536        83 0.033062
  6 0.0305283       32 0.107831         58 0.0821354        84 0.0311202
```

```
  7 0.0354746       33 0.112479       59 0.0801275       85 0.0291793
  8 0.0404463       34 0.118388       60 0.0781259       86 0.0272386
  9 0.0452255       35 0.122664       61 0.0761325       87 0.0252987
 10 0.0500226       36 0.129788       62 0.0741438       88 0.0233589
 11 0.0546279       37 0.131734       63 0.0721616       89 0.0214197
 12 0.058686        38 0.128453       64 0.070183        90 0.0194806
 13 0.0601029       39 0.1254         65 0.0682096       91 0.017542
 14 0.0614224       40 0.122506       66 0.066239        92 0.0156036
 15 0.0630654       41 0.119768       67 0.0642726       93 0.0136655
 16 0.0645626       42 0.117138       68 0.0623083       94 0.0117275
 17 0.0663091       43 0.114615       69 0.0603475       95 0.00978975
 18 0.0678378       44 0.112168       70 0.0583884       96 0.00785207
 19 0.0698309       45 0.109798       71 0.0564321       97 0.00591457
 20 0.0718113       46 0.10748        72 0.0544771       98 0.0039771
 21 0.0738353       47 0.105218       73 0.0525244       99 0.00203972
 22 0.0761592       48 0.102994       74 0.0505728      100 0.000102361
 23 0.0783018       49 0.10081        75 0.0486231
 24 0.0807909       50 0.0986538      76 0.0466743
 25 0.0835425       51 0.0965279      77 0.0447271
"""


output_snopt_height_400 = """
  0 1           81 1.00177   162 1.00585   243 1.00981   324 1.01211
  1 1           82 1.00181   163 1.00591   244 1.00985   325 1.01213
  2 1           83 1.00184   164 1.00597   245 1.00988   326 1.01215
  3 1           84 1.00189   165 1.00603   246 1.00992   327 1.01217
  4 1           85 1.00191   166 1.00609   247 1.00996   328 1.01218
  5 1.00001     86 1.00197   167 1.00615   248 1.01      329 1.0122
  6 1.00001     87 1.00199   168 1.00621   249 1.01003   330 1.01222
  7 1.00001     88 1.00204   169 1.00626   250 1.01007   331 1.01224
  8 1.00002     89 1.00206   170 1.00632   251 1.01011   332 1.01225
  9 1.00002     90 1.00211   171 1.00638   252 1.01014   333 1.01227
 10 1.00003     91 1.00214   172 1.00644   253 1.01018   334 1.01229
 11 1.00004     92 1.00219   173 1.0065    254 1.01021   335 1.0123
 12 1.00005     93 1.00222   174 1.00655   255 1.01025   336 1.01232
 13 1.00005     94 1.00227   175 1.00661   256 1.01028   337 1.01233
 14 1.00006     95 1.0023    176 1.00666   257 1.01032   338 1.01235
 15 1.00007     96 1.00235   177 1.00672   258 1.01035   339 1.01236
 16 1.00008     97 1.00237   178 1.00678   259 1.01039   340 1.01238
 17 1.00009     98 1.00243   179 1.00683   260 1.01042   341 1.01239
 18 1.0001      99 1.00245   180 1.00689   261 1.01046   342 1.01241
 19 1.00011    100 1.0025    181 1.00694   262 1.01049   343 1.01242
 20 1.00013    101 1.00254   182 1.007     263 1.01053   344 1.01244
 21 1.00014    102 1.00258   183 1.00705   264 1.01056   345 1.01245
 22 1.00015    103 1.00262   184 1.0071    265 1.01059   346 1.01246
```

23	1.00017	104	1.00267	185	1.00716	266	1.01062	347	1.01248
24	1.00018	105	1.0027	186	1.00721	267	1.01066	348	1.01249
25	1.0002	106	1.00275	187	1.00726	268	1.01069	349	1.0125
26	1.00021	107	1.00279	188	1.00732	269	1.01072	350	1.01251
27	1.00023	108	1.00284	189	1.00737	270	1.01075	351	1.01253
28	1.00025	109	1.00287	190	1.00742	271	1.01078	352	1.01254
29	1.00026	110	1.00292	191	1.00747	272	1.01082	353	1.01255
30	1.00028	111	1.00296	192	1.00752	273	1.01085	354	1.01256
31	1.0003	112	1.00302	193	1.00758	274	1.01088	355	1.01257
32	1.00032	113	1.00306	194	1.00763	275	1.01091	356	1.01258
33	1.00034	114	1.00311	195	1.00768	276	1.01094	357	1.0126
34	1.00036	115	1.00315	196	1.00773	277	1.01097	358	1.01261
35	1.00038	116	1.0032	197	1.00778	278	1.011	359	1.01262
36	1.00041	117	1.00324	198	1.00783	279	1.01103	360	1.01263
37	1.00043	118	1.0033	199	1.00788	280	1.01106	361	1.01264
38	1.00045	119	1.00334	200	1.00793	281	1.01109	362	1.01265
39	1.00048	120	1.00339	201	1.00798	282	1.01112	363	1.01266
40	1.0005	121	1.00344	202	1.00802	283	1.01114	364	1.01266
41	1.00053	122	1.00349	203	1.00807	284	1.01117	365	1.01267
42	1.00055	123	1.00354	204	1.00812	285	1.0112	366	1.01268
43	1.00058	124	1.00359	205	1.00817	286	1.01123	367	1.01269
44	1.00061	125	1.00364	206	1.00822	287	1.01126	368	1.0127
45	1.00063	126	1.00369	207	1.00827	288	1.01128	369	1.01271
46	1.00066	127	1.00374	208	1.00831	289	1.01131	370	1.01271
47	1.00069	128	1.0038	209	1.00836	290	1.01134	371	1.01272
48	1.00072	129	1.00385	210	1.00841	291	1.01137	372	1.01273
49	1.00075	130	1.0039	211	1.00845	292	1.01139	373	1.01274
50	1.00077	131	1.00395	212	1.0085	293	1.01142	374	1.01274
51	1.00081	132	1.00401	213	1.00855	294	1.01144	375	1.01275
52	1.00083	133	1.00407	214	1.00859	295	1.01147	376	1.01276
53	1.00086	134	1.00412	215	1.00864	296	1.0115	377	1.01276
54	1.00089	135	1.00418	216	1.00868	297	1.01152	378	1.01277
55	1.00093	136	1.00423	217	1.00873	298	1.01155	379	1.01277
56	1.00095	137	1.00429	218	1.00877	299	1.01157	380	1.01278
57	1.00099	138	1.00434	219	1.00882	300	1.01159	381	1.01278
58	1.00101	139	1.00441	220	1.00886	301	1.01162	382	1.01279
59	1.00105	140	1.00446	221	1.0089	302	1.01164	383	1.01279
60	1.00108	141	1.00453	222	1.00895	303	1.01167	384	1.0128
61	1.00111	142	1.00458	223	1.00899	304	1.01169	385	1.0128
62	1.00114	143	1.00465	224	1.00903	305	1.01171	386	1.0128
63	1.00117	144	1.0047	225	1.00908	306	1.01174	387	1.01281
64	1.0012	145	1.00478	226	1.00912	307	1.01176	388	1.01281
65	1.00124	146	1.00483	227	1.00916	308	1.01178	389	1.01281
66	1.00126	147	1.0049	228	1.0092	309	1.0118	390	1.01282
67	1.0013	148	1.00497	229	1.00925	310	1.01183	391	1.01282
68	1.00133	149	1.00503	230	1.00929	311	1.01185	392	1.01282

```
 69 1.00137   150 1.0051    231 1.00933   312 1.01187   393 1.01282
 70 1.00139   151 1.00516   232 1.00937   313 1.01189   394 1.01283
 71 1.00143   152 1.00523   233 1.00941   314 1.01191   395 1.01283
 72 1.00146   153 1.00529   234 1.00945   315 1.01193   396 1.01283
 73 1.0015    154 1.00535   235 1.00949   316 1.01195   397 1.01283
 74 1.00153   155 1.00542   236 1.00953   317 1.01198   398 1.01283
 75 1.00156   156 1.00548   237 1.00957   318 1.012     399 1.01283
 76 1.0016    157 1.00554   238 1.00961   319 1.01202   400 1.01283
 77 1.00163   158 1.0056    239 1.00965   320 1.01204
 78 1.00167   159 1.00567   240 1.00969   321 1.01205
 79 1.0017    160 1.00573   241 1.00973   322 1.01207
 80 1.00174   161 1.00579   242 1.00977   323 1.01209
"""

output_snopt_mass_400 = """
  0 1          81 0.764239   162 0.6        243 0.6        324 0.6
  1 0.996524   82 0.761732   163 0.6        244 0.6        325 0.6
  2 0.993049   83 0.757287   164 0.6        245 0.6        326 0.6
  3 0.989573   84 0.754781   165 0.6        246 0.6        327 0.6
  4 0.986098   85 0.750336   166 0.6        247 0.6        328 0.6
  5 0.982622   86 0.754781   167 0.6        248 0.6        329 0.6
  6 0.979147   87 0.750336   168 0.6        249 0.6        330 0.6
  7 0.975671   88 0.74783    169 0.6        250 0.6        331 0.6
  8 0.972196   89 0.750336   170 0.6        251 0.6        332 0.6
  9 0.96872    90 0.740879   171 0.6        252 0.6        333 0.6
 10 0.965244   91 0.743385   172 0.6        253 0.6        334 0.6
 11 0.961769   92 0.740879   173 0.6        254 0.6        335 0.6
 12 0.958293   93 0.736434   174 0.6        255 0.6        336 0.6
 13 0.954818   94 0.733928   175 0.6        256 0.6        337 0.6
 14 0.951342   95 0.736434   176 0.6        257 0.6        338 0.6
 15 0.947867   96 0.733928   177 0.6        258 0.6        339 0.6
 16 0.944391   97 0.729483   178 0.6        259 0.6        340 0.6
 17 0.940916   98 0.726976   179 0.6        260 0.6        341 0.6
 18 0.93744    99 0.729483   180 0.6        261 0.6        342 0.6
 19 0.933964  100 0.720025   181 0.6        262 0.6        343 0.6
 20 0.930489  101 0.722532   182 0.6        263 0.6        344 0.6
 21 0.927013  102 0.720025   183 0.6        264 0.6        345 0.6
 22 0.923538  103 0.715581   184 0.6        265 0.6        346 0.6
 23 0.920062  104 0.713074   185 0.6        266 0.6        347 0.6
 24 0.916587  105 0.715581   186 0.6        267 0.6        348 0.6
 25 0.913111  106 0.713074   187 0.6        268 0.6        349 0.6
 26 0.909635  107 0.70863    188 0.6        269 0.6        350 0.6
 27 0.90616   108 0.706123   189 0.6        270 0.6        351 0.6
 28 0.902684  109 0.701679   190 0.6        271 0.6        352 0.6
 29 0.899209  110 0.699172   191 0.6        272 0.6        353 0.6
 30 0.895733  111 0.694727   192 0.6        273 0.6        354 0.6
```

31	0.892258	112	0.692221	193	0.6	274	0.6	355	0.6
32	0.888782	113	0.694727	194	0.6	275	0.6	356	0.6
33	0.885307	114	0.68527	195	0.6	276	0.6	357	0.6
34	0.881831	115	0.687776	196	0.6	277	0.6	358	0.6
35	0.878355	116	0.68527	197	0.6	278	0.6	359	0.6
36	0.87488	117	0.680825	198	0.6	279	0.6	360	0.6
37	0.871404	118	0.678319	199	0.6	280	0.6	361	0.6
38	0.867929	119	0.680825	200	0.6	281	0.6	362	0.6
39	0.864453	120	0.671368	201	0.6	282	0.6	363	0.6
40	0.860978	121	0.673874	202	0.6	283	0.6	364	0.6
41	0.857502	122	0.671368	203	0.6	284	0.6	365	0.6
42	0.854027	123	0.669511	204	0.6	285	0.6	366	0.6
43	0.850551	124	0.664416	205	0.6	286	0.6	367	0.6
44	0.847075	125	0.66256	206	0.6	287	0.6	368	0.6
45	0.8436	126	0.664416	207	0.6	288	0.6	369	0.6
46	0.840124	127	0.655609	208	0.6	289	0.6	370	0.6
47	0.840701	128	0.657465	209	0.6	290	0.6	371	0.6
48	0.833173	129	0.655609	210	0.6	291	0.6	372	0.6
49	0.83375	130	0.650514	211	0.6	292	0.6	373	0.6
50	0.833173	131	0.648658	212	0.6	293	0.6	374	0.6
51	0.826799	132	0.643563	213	0.6	294	0.6	375	0.6
52	0.826222	133	0.641707	214	0.6	295	0.6	376	0.6
53	0.826799	134	0.636612	215	0.6	296	0.6	377	0.6
54	0.819271	135	0.641707	216	0.6	297	0.6	378	0.6
55	0.819848	136	0.636612	217	0.6	298	0.6	379	0.6
56	0.819271	137	0.634756	218	0.6	299	0.6	380	0.6
57	0.812896	138	0.629661	219	0.6	300	0.6	381	0.6
58	0.81232	139	0.627804	220	0.6	301	0.6	382	0.6
59	0.812896	140	0.62271	221	0.6	302	0.6	383	0.6
60	0.81232	141	0.620853	222	0.6	303	0.6	384	0.6
61	0.805945	142	0.615759	223	0.6	304	0.6	385	0.6
62	0.805369	143	0.613902	224	0.6	305	0.6	386	0.6
63	0.805945	144	0.615759	225	0.6	306	0.6	387	0.6
64	0.798418	145	0.606951	226	0.6	307	0.6	388	0.6
65	0.798994	146	0.608808	227	0.6	308	0.6	389	0.6
66	0.798418	147	0.6	228	0.6	309	0.6	390	0.6
67	0.798994	148	0.601856	229	0.6	310	0.6	391	0.6
68	0.791467	149	0.6	230	0.6	311	0.6	392	0.6
69	0.792043	150	0.6	231	0.6	312	0.6	393	0.6
70	0.791467	151	0.6	232	0.6	313	0.6	394	0.6
71	0.785092	152	0.6	233	0.6	314	0.6	395	0.6
72	0.784515	153	0.6	234	0.6	315	0.6	396	0.6
73	0.778141	154	0.6	235	0.6	316	0.6	397	0.6
74	0.782585	155	0.6	236	0.6	317	0.6	398	0.6
75	0.77119	156	0.6	237	0.6	318	0.6	399	0.6
76	0.775634	157	0.6	238	0.6	319	0.6	400	0.6

```
 77 0.77119     158 0.6         239 0.6         320 0.6
 78 0.775634    159 0.6         240 0.6         321 0.6
 79 0.764239    160 0.6         241 0.6         322 0.6
 80 0.768683    161 0.6         242 0.6         323 0.6
"""

output_snopt_thrust_400 = """
  0 3.5          81 3.5         162 0           243 0           324 0
  1 3.5          82 3.5         163 0           244 0           325 0
  2 3.5          83 3.5         164 0           245 0           326 0
  3 3.5          84 3.5         165 0           246 0           327 0
  4 3.5          85 0           166 0           247 0           328 0
  5 3.5          86 0           167 0           248 0           329 0
  6 3.5          87 3.5         168 0           249 0           330 0
  7 3.5          88 0           169 0           250 0           331 0
  8 3.5          89 3.5         170 0           251 0           332 0
  9 3.5          90 3.5         171 0           252 0           333 0
 10 3.5          91 0           172 0           253 0           334 0
 11 3.5          92 3.5         173 0           254 0           335 0
 12 3.5          93 3.5         174 0           255 0           336 0
 13 3.5          94 0           175 0           256 0           337 0
 14 3.5          95 0           176 0           257 0           338 0
 15 3.5          96 3.5         177 0           258 0           339 0
 16 3.5          97 3.5         178 0           259 0           340 0
 17 3.5          98 0           179 0           260 0           341 0
 18 3.5          99 3.5         180 0           261 0           342 0
 19 3.5         100 3.5         181 0           262 0           343 0
 20 3.5         101 0           182 0           263 0           344 0
 21 3.5         102 3.5         183 0           264 0           345 0
 22 3.5         103 3.5         184 0           265 0           346 0
 23 3.5         104 0           185 0           266 0           347 0
 24 3.5         105 0           186 0           267 0           348 0
 25 3.5         106 3.5         187 0           268 0           349 0
 26 3.5         107 3.5         188 0           269 0           350 0
 27 3.5         108 3.5         189 0           270 0           351 0
 28 3.5         109 3.5         190 0           271 0           352 0
 29 3.5         110 3.5         191 0           272 0           353 0
 30 3.5         111 3.5         192 0           273 0           354 0
 31 3.5         112 0           193 0           274 0           355 0
 32 3.5         113 3.5         194 0           275 0           356 0
 33 3.5         114 3.5         195 0           276 0           357 0
 34 3.5         115 0           196 0           277 0           358 0
 35 3.5         116 3.5         197 0           278 0           359 0
 36 3.5         117 3.5         198 0           279 0           360 0
 37 3.5         118 0           199 0           280 0           361 0
 38 3.5         119 3.5         200 0           281 0           362 0
```

```
 39 3.5        120 3.5        201 0          282 0          363 0
 40 3.5        121 0          202 0          283 0          364 0
 41 3.5        122 2.19679    203 0          284 0          365 0
 42 3.5        123 3.5        204 0          285 0          366 0
 43 3.5        124 3.5        205 0          286 0          367 0
 44 3.5        125 0          206 0          287 0          368 0
 45 3.5        126 3.5        207 0          288 0          369 0
 46 1.4597     127 3.5        208 0          289 0          370 0
 47 3.5        128 0          209 0          290 0          371 0
 48 3.5        129 3.5        210 0          291 0          372 0
 49 0          130 3.5        211 0          292 0          373 0
 50 3.5        131 3.5        212 0          293 0          374 0
 51 3.5        132 3.5        213 0          294 0          375 0
 52 0          133 3.5        214 0          295 0          376 0
 53 3.5        134 0          215 0          296 0          377 0
 54 3.5        135 0          216 0          297 0          378 0
 55 0          136 3.5        217 0          298 0          379 0
 56 3.5        137 3.5        218 0          299 0          380 0
 57 3.5        138 3.5        219 0          300 0          381 0
 58 0          139 3.5        220 0          301 0          382 0
 59 0          140 3.5        221 0          302 0          383 0
 60 3.5        141 3.5        222 0          303 0          384 0
 61 3.5        142 3.5        223 0          304 0          385 0
 62 0          143 0          224 0          305 0          386 0
 63 3.5        144 3.5        225 0          306 0          387 0
 64 3.5        145 3.5        226 0          307 0          388 0
 65 0          146 3.5        227 0          308 0          389 0
 66 0          147 3.5        228 0          309 0          390 0
 67 3.5        148 0          229 0          310 0          391 0
 68 3.5        149 0.934727   230 0          311 0          392 0
 69 0          150 0          231 0          312 0          393 0
 70 3.5        151 0          232 0          313 0          394 0
 71 3.5        152 0          233 0          314 0          395 0
 72 3.5        153 0          234 0          315 0          396 0
 73 0.971764   154 0          235 0          316 0          397 0
 74 3.5        155 0          236 0          317 0          398 0
 75 3.5        156 0          237 0          318 0          399 0
 76 0          157 0          238 0          319 0          400 0
 77 0          158 0          239 0          320 0
 78 3.5        159 0          240 0          321 0
 79 3.5        160 0          241 0          322 0
 80 0          161 0          242 0          323 0
"""

output_snopt_velocity_400 = """
  0 0             101 0.0816095     202 0.0981876     303 0.0477414
```

1	0.00124127	102	0.0823234	203	0.0976551	304	0.0472546
2	0.00249419	103	0.0846528	204	0.0971241	305	0.0467679
3	0.00374622	104	0.0853607	205	0.0965947	306	0.0462812
4	0.00500899	105	0.0828366	206	0.0960665	307	0.0457946
5	0.00626995	106	0.0836097	207	0.09554	308	0.0453081
6	0.00754075	107	0.0859604	208	0.0950146	309	0.0448217
7	0.00880876	108	0.0867304	209	0.0944907	310	0.0443353
8	0.0100857	109	0.0891008	210	0.0939679	311	0.0438491
9	0.0113589	110	0.0898686	211	0.0934464	312	0.0433628
10	0.01264	111	0.0922607	212	0.092926	313	0.0428766
11	0.0139164	112	0.093027	213	0.0924069	314	0.0423905
12	0.0151999	113	0.0904218	214	0.0918887	315	0.0419045
13	0.0164778	114	0.0962566	215	0.0913716	316	0.0414185
14	0.0177618	115	0.093626	216	0.0908555	317	0.0409326
15	0.0190392	116	0.0944581	217	0.0903405	318	0.0404467
16	0.020322	117	0.0969015	218	0.0898263	319	0.0399609
17	0.0215972	118	0.0977383	219	0.0893131	320	0.0394751
18	0.0228771	119	0.0950805	220	0.0888007	321	0.0389894
19	0.0241486	120	0.101088	221	0.0882892	322	0.0385037
20	0.0254239	121	0.0984106	222	0.0877785	323	0.0380181
21	0.0266902	122	0.0993107	223	0.0872687	324	0.0375325
22	0.0279597	123	0.0998829	224	0.0867595	325	0.037047
23	0.0292193	124	0.102735	225	0.0862512	326	0.0365615
24	0.0304816	125	0.103314	226	0.0857435	327	0.0360761
25	0.0317335	126	0.100947	227	0.0852366	328	0.0355907
26	0.0329875	127	0.106813	228	0.0847302	329	0.0351054
27	0.0342306	128	0.104438	229	0.0842246	330	0.0346201
28	0.0354755	129	0.105062	230	0.0837196	331	0.0341348
29	0.036709	130	0.107997	231	0.0832152	332	0.0336496
30	0.0379439	131	0.108634	232	0.0827114	333	0.0331644
31	0.0391672	132	0.111594	233	0.0822082	334	0.0326793
32	0.0403917	133	0.112246	234	0.0817054	335	0.0321942
33	0.0416043	134	0.115234	235	0.0812033	336	0.0317091
34	0.042818	135	0.110442	236	0.0807016	337	0.0312241
35	0.0440196	136	0.113525	237	0.0802005	338	0.0307391
36	0.0452223	137	0.114165	238	0.0796998	339	0.0302542
37	0.0464129	138	0.117277	239	0.0791997	340	0.0297692
38	0.0476047	139	0.117933	240	0.0786999	341	0.0292843
39	0.0487845	140	0.121076	241	0.0782007	342	0.0287995
40	0.0499655	141	0.121751	242	0.0777018	343	0.0283147
41	0.0511346	142	0.124926	243	0.0772033	344	0.0278299
42	0.0523052	143	0.125621	244	0.0767053	345	0.0273451
43	0.0534642	144	0.12317	245	0.0762076	346	0.0268604
44	0.0546249	145	0.12956	246	0.0757103	347	0.0263757
45	0.0557742	146	0.127131	247	0.0752134	348	0.025891
46	0.0569257	147	0.133556	248	0.0747168	349	0.0254063

47	0.0556546	148	0.131151	249	0.0742206	350	0.0249217
48	0.0592646	149	0.131838	250	0.0737247	351	0.0244371
49	0.0579274	150	0.130994	251	0.0732292	352	0.0239525
50	0.0574208	151	0.130167	252	0.0727339	353	0.023468
51	0.06028	152	0.129353	253	0.0722389	354	0.0229834
52	0.0597284	153	0.128555	254	0.0717443	355	0.0224989
53	0.0584125	154	0.127769	255	0.0712499	356	0.0220144
54	0.0621162	155	0.126996	256	0.0707558	357	0.02153
55	0.0607351	156	0.126235	257	0.070262	358	0.0210455
56	0.0602533	157	0.125486	258	0.0697684	359	0.0205611
57	0.0631387	158	0.124747	259	0.0692752	360	0.0200767
58	0.0626156	159	0.12402	260	0.0687821	361	0.0195923
59	0.0612531	160	0.123301	261	0.0682893	362	0.019108
60	0.0607834	161	0.122593	262	0.0677967	363	0.0186236
61	0.063723	162	0.121893	263	0.0673044	364	0.0181393
62	0.0632139	163	0.121202	264	0.0668122	365	0.017655
63	0.0618679	164	0.120519	265	0.0663203	366	0.0171707
64	0.0657216	165	0.119844	266	0.0658286	367	0.0166864
65	0.0643164	166	0.119176	267	0.0653371	368	0.0162021
66	0.0638722	167	0.118516	268	0.0648458	369	0.0157179
67	0.0624898	168	0.117862	269	0.0643547	370	0.0152337
68	0.0664462	169	0.117215	270	0.0638637	371	0.0147495
69	0.0650058	170	0.116574	271	0.063373	372	0.0142653
70	0.0646263	171	0.115939	272	0.0628824	373	0.0137811
71	0.0675963	172	0.11531	273	0.062392	374	0.0132969
72	0.0671867	173	0.114686	274	0.0619018	375	0.0128127
73	0.0701818	174	0.114067	275	0.0614117	376	0.0123286
74	0.0665148	175	0.113454	276	0.0609218	377	0.0118444
75	0.0728221	176	0.112845	277	0.0604321	378	0.0113603
76	0.0690627	177	0.112241	278	0.0599424	379	0.0108762
77	0.0709816	178	0.111641	279	0.059453	380	0.0103921
78	0.0671833	179	0.111045	280	0.0589636	381	0.00990797
79	0.0736956	180	0.110453	281	0.0584745	382	0.00942388
80	0.0698055	181	0.109865	282	0.0579854	383	0.0089398
81	0.0718899	182	0.109281	283	0.0574965	384	0.00845572
82	0.0725034	183	0.108701	284	0.0570077	385	0.00797165
83	0.0746058	184	0.108123	285	0.0565191	386	0.00748759
84	0.0752016	185	0.10755	286	0.0560305	387	0.00700354
85	0.0773242	186	0.106979	287	0.0555421	388	0.00651949
86	0.0732704	187	0.106411	288	0.0550538	389	0.00603545
87	0.0755157	188	0.105846	289	0.0545656	390	0.00555141
88	0.076047	189	0.105284	290	0.0540775	391	0.00506738
89	0.0736685	190	0.104725	291	0.0535896	392	0.00458335
90	0.0788961	191	0.104168	292	0.0531017	393	0.00409933
91	0.076472	192	0.103614	293	0.052614	394	0.00361531
92	0.077077	193	0.103062	294	0.0521263	395	0.00313129

```
 93 0.0793485     194 0.102512      295 0.0516387     396 0.00264728
 94 0.0799393     195 0.101965      296 0.0511512     397 0.00216327
 95 0.0774978     196 0.101419      297 0.0506639     398 0.00167926
 96 0.0781546     197 0.100876      298 0.0501766     399 0.00119525
 97 0.080452      198 0.100335      299 0.0496894     400 0.000711243
 98 0.0810973     199 0.0997954     300 0.0492022
 99 0.0786356     200 0.0992577     301 0.0487152
100 0.0841089     201 0.0987219     302 0.0482282
"""


def convert_text(text):
    # dict to store intermediate results
    # key   = time index
    # value = variable value at time t_{key}
    values = {}

    text    = text.replace("\n", " ")
    numbers = text.split(" ")
    numbers = [x for x in numbers if ((x != '\n') and (len(x)>0))]

    n = 2 # batch size, see non-overlapping batch algorithm online
    for i in range(0, len(numbers)-n+1, n):
        pair = numbers[i:i+n]
        val = ((pair[1]).split("\n"))[0]
        values[int( pair[0] )] = float( pair[1] ) # append to dictionary

    # sort dictionary values by key (time index)
    values = dict(sorted(values.items()))

    # extract AMPL output from dict
    output = list(values.values())

    return output


# conopt results, N = 400

value_list = np.asarray( convert_text(output_conopt_height_400) )
value_list.tofile('output_conopt_height_400.csv', sep = ',\n')
print("conopt height complete")

value_list = np.asarray( convert_text(output_conopt_velocity_400) )
value_list.tofile('output_conopt_velocity_400.csv', sep = ',\n')
print("conopt velocity complete")
```

```python
value_list = np.asarray( convert_text(output_conopt_mass_400) )
value_list.tofile('output_conopt_mass_400.csv', sep = ',\n')
print("conopt mass complete")

value_list = np.asarray( convert_text(output_conopt_thrust_400) )
value_list.tofile('output_conopt_thrust_400.csv', sep = ',\n')
print("conopt thrust complete")


# knitro results, N = 400

value_list = np.asarray( convert_text(output_knitro_height_400) )
value_list.tofile('output_knitro_height_400.csv', sep = ',\n')
print("knitro height complete")

value_list = np.asarray( convert_text(output_knitro_velocity_400) )
value_list.tofile('output_knitro_velocity_400.csv', sep = ',\n')
print("knitro velocity complete")

value_list = np.asarray( convert_text(output_knitro_mass_400) )
value_list.tofile('output_knitro_mass_400.csv', sep = ',\n')
print("knitro mass complete")

value_list = np.asarray( convert_text(output_knitro_thrust_400) )
value_list.tofile('output_knitro_thrust_400.csv', sep = ',\n')
print("knitro thrust complete")


# loqo results, N = 400

value_list = np.asarray( convert_text(output_loqo_height_400) )
value_list.tofile('output_loqo_height_400.csv', sep = ',\n')
print("loqo height complete")

value_list = np.asarray( convert_text(output_loqo_velocity_400) )
value_list.tofile('output_loqo_velocity_400.csv', sep = ',\n')
print("loqo velocity complete")

value_list = np.asarray( convert_text(output_loqo_mass_400) )
value_list.tofile('output_loqo_mass_400.csv', sep = ',\n')
print("loqo mass complete")

value_list = np.asarray( convert_text(output_loqo_thrust_400) )
value_list.tofile('output_loqo_thrust_400.csv', sep = ',\n')
print("loqo thrust complete")
```

```python
# minos results, N = 100

value_list = np.asarray( convert_text(output_minos_height_100) )
value_list.tofile('output_minos_height_100.csv', sep = ',\n')
print("minos height complete")

value_list = np.asarray( convert_text(output_minos_velocity_100) )
value_list.tofile('output_minos_velocity_100.csv', sep = ',\n')
print("minos velocity complete")

value_list = np.asarray( convert_text(output_minos_mass_100) )
value_list.tofile('output_minos_mass_100.csv', sep = ',\n')
print("minos mass complete")

value_list = np.asarray( convert_text(output_minos_thrust_100) )
value_list.tofile('output_minos_thrust_100.csv', sep = ',\n')
print("minos thrust complete")


# snopt results, N = 400

value_list = np.asarray( convert_text(output_snopt_height_400) )
value_list.tofile('output_snopt_height_400.csv', sep = ',\n')
print("snopt height complete")

value_list = np.asarray( convert_text(output_snopt_velocity_400) )
value_list.tofile('output_snopt_velocity_400.csv', sep = ',\n')
print("snopt velocity complete")

value_list = np.asarray( convert_text(output_snopt_mass_400) )
value_list.tofile('output_snopt_mass_400.csv', sep = ',\n')
print("snopt mass complete")

value_list = np.asarray( convert_text(output_snopt_thrust_400) )
value_list.tofile('output_snopt_thrust_400.csv', sep = ',\n')
print("snopt thrust complete")


print("done")
```

3.1.2 RStudio Code

```
## Math6120 - Nonlinear Optimisation
## Coursework 1 - AMPL Model
## Emma Tarmey, 2940 4045


## This file graphs the functions height, mass, thrust and velocity over time
## We have chosen to put the output in CSV format


# Reset environment -----
rm(list = ls())


output_conopt_height   <- read.csv("output_conopt_height_400.csv",   header=FALSE)[,1]
output_conopt_mass     <- read.csv("output_conopt_mass_400.csv",     header=FALSE)[,1]
output_conopt_thrust   <- read.csv("output_conopt_thrust_400.csv",   header=FALSE)[,1]
output_conopt_velocity <- read.csv("output_conopt_velocity_400.csv", header=FALSE)[,1]

output_knitro_height   <- read.csv("output_knitro_height_400.csv",   header=FALSE)[,1]
output_knitro_mass     <- read.csv("output_knitro_mass_400.csv",     header=FALSE)[,1]
output_knitro_thrust   <- read.csv("output_knitro_thrust_400.csv",   header=FALSE)[,1]
output_knitro_velocity <- read.csv("output_knitro_velocity_400.csv", header=FALSE)[,1]

output_loqo_height   <- read.csv("output_loqo_height_400.csv",   header=FALSE)[,1]
output_loqo_mass     <- read.csv("output_loqo_mass_400.csv",     header=FALSE)[,1]
output_loqo_thrust   <- read.csv("output_loqo_thrust_400.csv",   header=FALSE)[,1]
output_loqo_velocity <- read.csv("output_loqo_velocity_400.csv", header=FALSE)[,1]

output_minos_height   <- read.csv("output_minos_height_100.csv",   header=FALSE)[,1]
output_minos_mass     <- read.csv("output_minos_mass_100.csv",     header=FALSE)[,1]
output_minos_thrust   <- read.csv("output_minos_thrust_100.csv",   header=FALSE)[,1]
output_minos_velocity <- read.csv("output_minos_velocity_100.csv", header=FALSE)[,1]

output_snopt_height   <- read.csv("output_snopt_height_400.csv",   header=FALSE)[,1]
output_snopt_mass     <- read.csv("output_snopt_mass_400.csv",     header=FALSE)[,1]
output_snopt_thrust   <- read.csv("output_snopt_thrust_400.csv",   header=FALSE)[,1]
output_snopt_velocity <- read.csv("output_snopt_velocity_400.csv", header=FALSE)[,1]



# arrange the plots in a grid
par(mfrow = c(5, 4))
```

```
# conopt plots

plot(seq(0, 400),
     output_conopt_height,
     xlab = "Time",
     ylab = "Height",
     main = "Height in ConOpt")

plot(seq(0, 400),
     output_conopt_mass,
     xlab = "Time",
     ylab = "Mass",
     main = "Mass in ConOpt")

plot(seq(0, 400),
     output_conopt_thrust,
     xlab = "Time",
     ylab = "Thrust",
     main = "Thrust in ConOpt")

plot(seq(0, 400),
     output_conopt_velocity,
     xlab = "Time",
     ylab = "Velocity",
     main = "Velocity in ConOpt")



# knitro plots

plot(seq(0, 400),
     output_knitro_height,
     xlab = "Time",
     ylab = "Height",
     main = "Height in Knitro")

plot(seq(0, 400),
     output_knitro_mass,
     xlab = "Time",
     ylab = "Mass",
     main = "Mass in Knitro")

plot(seq(0, 400),
     output_knitro_thrust,
     xlab = "Time",
     ylab = "Thrust",
```

```
     main = "Thrust in Knitro")

plot(seq(0, 400),
     output_knitro_velocity,
     xlab = "Time",
     ylab = "Velocity",
     main = "Velocity in Knitro")



# Loqo plots

plot(seq(0, 400),
     output_loqo_height,
     xlab = "Time",
     ylab = "Height",
     main = "Height in Loqo")

plot(seq(0, 400),
     output_loqo_mass,
     xlab = "Time",
     ylab = "Mass",
     main = "Mass in Loqo")

plot(seq(0, 400),
     output_loqo_thrust,
     xlab = "Time",
     ylab = "Thrust",
     main = "Thrust in Loqo")

plot(seq(0, 400),
     output_loqo_velocity,
     xlab = "Time",
     ylab = "Velocity",
     main = "Velocity in Loqo")



# Minos plots

plot(seq(0, 100),
     output_minos_height,
     xlab = "Time",
     ylab = "Height",
     main = "Height in Minos (N = 100)")
```

```
plot(seq(0, 100),
     output_minos_mass,
     xlab = "Time",
     ylab = "Mass",
     main = "Mass in Minos (N = 100)")

plot(seq(0, 100),
     output_minos_thrust,
     xlab = "Time",
     ylab = "Thrust",
     main = "Thrust in Minos (N = 100)")

plot(seq(0, 100),
     output_minos_velocity,
     xlab = "Time",
     ylab = "Velocity",
     main = "Velocity in Minos (N = 100)")



# Snopt plots

plot(seq(0, 400),
     output_snopt_height,
     xlab = "Time",
     ylab = "Height",
     main = "Height in Snopt")

plot(seq(0, 400),
     output_snopt_mass,
     xlab = "Time",
     ylab = "Mass",
     main = "Mass in Snopt")

plot(seq(0, 400),
     output_snopt_thrust,
     xlab = "Time",
     ylab = "Thrust",
     main = "Thrust in Snopt")

plot(seq(0, 400),
     output_snopt_velocity,
     xlab = "Time",
     ylab = "Velocity",
     main = "Velocity in Snopt")
```

3.2 AMPL Code

3.2.1 AMPL Model Code

```
## Math6120 - Nonlinear Optimisation
## Coursework 1 - AMPL Model
## Emma Tarmey, 2940 4045


## This file specifies the mathematical model used to solve our problem


# parameters
param N; # number of subintervals to be created
param theta_max;
param height_0;
param velocity_0;
param D_0 := (velocity_0 / 2);


# sets
set TIMEINDEX  := {0..N};     # moments in time
set KTIMEINDEX := {1..(N-1)}; # moments in time except for time 0 and time N


# variables

# decision variables
var t_final >= 0;
var height   {t in TIMEINDEX} >= 0;
var velocity {t in TIMEINDEX} >= 0;
var mass     {t in TIMEINDEX} >= 0;
var thrust   {t in TIMEINDEX} >= 0;

# define time passing in the interval [t_0, t_final], points in time are indexed by the set
var REALTIME {t in TIMEINDEX}          = ((t / N) * t_final);

# defines the starting points of the velocity and mass functions
# these starting points are used via the 'let' command in the corresponding 'cw1.run' file
var velocity_function {t in TIMEINDEX} = ((REALTIME[t] / t_final) * (1 - (REALTIME[t] / t_fi
var mass_function {t in TIMEINDEX}     = (1 + ((REALTIME[t] / t_final) * (0.6 - 1)));

# D(height, velocity) function from spec
var D {t in TIMEINDEX}   = D_0 * (velocity[t] * velocity[t]) * exp(-1 * height_0 * ( (height

# Right-hand-side of below constraints, seperated out for readability
```

```
var RHS {t in TIMEINDEX} = (((thrust[t] - D[t]) / mass[t]) - ((height[0] / height[t]) * (hei


# objective function
maximize final_height: height[ N ] ;


# equality constraints
subject to velocity_initial:
        velocity[0] = 0;

subject to height_initial:
        height[0]   = 1;

subject to mass_initial:
        mass[0]     = 1;

subject to velocity_final:
        mass[N] = 0.6;


# lower and upper bound inequality constraints
subject to velocity_lower_bound {t in TIMEINDEX}:
        velocity[t] >= 0;

subject to height_lower_bound {t in TIMEINDEX}:
        height[t]   >= height[0];

subject to mass_lower_bound {t in TIMEINDEX}:
        mass[t]     >= mass[N]; # final mass as lower bound

subject to mass_upper_bound {t in TIMEINDEX}:
        mass[t]     <= mass[0]; # initial mass as upper bound

subject to thrust_upper_bound {t in TIMEINDEX}:
        thrust[t]   <= theta_max;

subject to thrust_lower_bound {t in TIMEINDEX}:
        thrust[t]   >= 0;


# rocket motion height constraints
subject to height_change_initial:
        N * (height[1] - height[0])     =     (t_final * velocity[0]);

subject to height_change {k in KTIMEINDEX}:
```

```
        N * (height[k+1] - height[k-1]) = (2 * t_final * velocity[k]);

subject to height_change_final:
        N * (height[N] - height[N-1])   =     (t_final * velocity[N]);


# rocket motion velocity constraints
subject to velocity_change_initial:
        N * (velocity[1] - velocity[0])     =     (t_final * RHS[0]);

subject to velocity_change {k in KTIMEINDEX}:
        N * (velocity[k+1] - velocity[k-1]) = (2 * t_final * RHS[k]);

subject to velocity_change_final:
        N * (velocity[N] - velocity[N-1])   =     (t_final * RHS[N]);


# rocket motion mass constraints
subject to mass_change_initial:
        N * (mass[1] - mass[0])     = ((-2) * t_final * thrust[0]);

subject to mass_change {k in KTIMEINDEX}:
        N * (mass[k+1] - mass[k-1]) = ((-4) * t_final * thrust[k]);

subject to mass_change_final:
        N * (mass[N] - mass[N-1])   = ((-2) * t_final * thrust[N]);
```

3.2.2 AMPL Data File

```
## Math6120 - Nonlinear Optimization
## Coursework 1 - AMPL Model - Data File
## Emma Tarmey, 2940 4045

## This file specifies all parameters required by the corresponding cw1 model

param theta_max  := 3.5;
param height_0   := 500;
param velocity_0 := 620;
```

3.2.3 AMPL Run File

```
reset;
option solver snopt;
option display_1col 1;
model cw1.mod;
data  cw1.dat;

print "";
print "***** Math6120 - Nonlinear Optimization *****";
print "***** Coursework 1 - AMPL Model *****";
print "***** Emma Tarmey, 2940 4045 *****";
print "";

for {value in {50, 100, 200, 400}} {
        let N       := value;
        let t_final := 1;
        let {t in TIMEINDEX} height[t]   := 1;
        let {t in TIMEINDEX} velocity[t] := velocity_function[t];
        let {t in TIMEINDEX} mass[t]     := mass_function[t];
        let {t in TIMEINDEX} thrust[t]   := (theta_max / 2);

        solve;
        print "";
        print "***** Optimal Solution: *****";
        print "";

        display N;
        display final_height;
        display t_final;
        display thrust;
        print "";
        print "";
}
```

3.3 AMPL Output

3.3.1 AMPL Output using ConOpt Solver

```
ampl: include cw1.run;

***** Math6120 - Nonlinear Optimization *****
***** Coursework 1 - AMPL Model *****
***** Emma Tarmey, 2940 4045 *****

CONOPT 3.17A: Locally optimal; objective 1.012809644
99 iterations; evals: nf = 42, ng = 0, nc = 335, nJ = 44, nH = 2, nHv = 15

***** Optimal Solution: *****

N = 50

final_height = 1.01281

t_final = 0.198439

thrust [*] :=
 0 3.5        11 2.25013    22 0         33 0         44 0
 1 3.5        12 2.27625    23 0         34 0         45 0
 2 3.5        13 2.35546    24 0         35 0         46 0
 3 3.5        14 2.40737    25 0         36 0         47 0
 4 3.5        15 2.53698    26 0         37 0         48 0
 5 3.41738    16 2.60997    27 0         38 0         49 0
 6 2.82524    17 2.8978     28 0         39 0         50 0
 7 2.14056    18 1.98821    29 0         40 0
 8 2.14807    19 0.411128   30 0         41 0
 9 2.18724    20 0          31 0         42 0
10 2.19157    21 0          32 0         43 0
;


CONOPT 3.17A: Locally optimal; objective 1.01283016
470 iterations; evals: nf = 53, ng = 0, nc = 911, nJ = 112, nH = 10, nHv = 20

***** Optimal Solution: *****

N = 100

final_height = 1.01283
```

```
t_final = 0.198758

thrust [*] :=
  0 3.5           26 2.30705        52 0              78 0
  1 3.5           27 2.40034        53 0              79 0
  2 3.5           28 2.43156        54 0              80 0
  3 3.5           29 2.48022        55 0              81 0
  4 3.5           30 2.53084        56 0              82 0
  5 3.5           31 2.59156        57 0              83 0
  6 3.5           32 2.649          58 0              84 0
  7 3.5           33 2.73741        59 0              85 0
  8 3.5           34 2.76991        60 0              86 0
  9 3.5           35 3.0357         61 0              87 0
 10 3.5           36 2.41842        62 0              88 0
 11 3.37484       37 0.739438       63 0              89 0
 12 2.72436       38 0              64 0              90 0
 13 2.13893       39 0              65 0              91 0
 14 2.15001       40 0              66 0              92 0
 15 2.14546       41 0              67 0              93 0
 16 2.1564        42 0              68 0              94 0
 17 2.16764       43 0              69 0              95 0
 18 2.15607       44 0              70 0              96 0
 19 2.20225       45 0              71 0              97 0
 20 2.22232       46 0              72 0              98 0
 21 2.21788       47 9.66473e-16    73 0              99 0
 22 2.21905       48 0              74 0             100 0
 23 2.25961       49 0              75 0
 24 2.32741       50 0              76 0
 25 2.32114       51 0              77 0
;


CONOPT 3.17A: Locally optimal; objective 1.01283521
800 iterations; evals: nf = 97, ng = 0, nc = 1402, nJ = 144, nH = 4, nHv = 22

***** Optimal Solution: *****

N = 200

final_height = 1.01284

t_final = 0.198842

thrust [*] :=
  0 3.5       41 2.24758    82 0        123 0        164 0
```

```
  1 3.5        42 2.20004    83 0         124 0         165 0
  2 3.5        43 2.22919    84 0         125 0         166 0
  3 3.5        44 2.20494    85 0         126 0         167 0
  4 3.5        45 2.2223     86 0         127 0         168 0
  5 3.5        46 2.314      87 0         128 0         169 0
  6 3.5        47 2.28388    88 0         129 0         170 0
  7 3.5        48 2.29685    89 0         130 0         171 0
  8 3.5        49 2.32075    90 0         131 0         172 0
  9 3.5        50 2.31311    91 0         132 0         173 0
 10 3.5        51 2.3222     92 0         133 0         174 0
 11 3.5        52 2.32616    93 0         134 0         175 0
 12 3.5        53 2.3634     94 0         135 0         176 0
 13 3.5        54 2.3512     95 0         136 0         177 0
 14 3.5        55 2.43924    96 0         137 0         178 0
 15 3.5        56 2.42046    97 0         138 0         179 0
 16 3.5        57 2.47858    98 0         139 0         180 0
 17 3.5        58 2.54903    99 0         140 0         181 0
 18 3.5        59 2.4989    100 0         141 0         182 0
 19 3.5        60 2.52528   101 0         142 0         183 0
 20 3.5        61 2.52286   102 0         143 0         184 0
 21 3.5        62 2.54509   103 0         144 0         185 0
 22 3.5        63 2.6178    104 0         145 0         186 0
 23 3.27914    64 2.62425   105 0         146 0         187 0
 24 2.59522    65 2.78002   106 0         147 0         188 0
 25 2.11934    66 2.5725    107 0         148 0         189 0
 26 2.19851    67 2.72269   108 0         149 0         190 0
 27 2.14544    68 3.30981   109 0         150 0         191 0
 28 2.06877    69 2.88623   110 0         151 0         192 0
 29 2.15995    70 2.37303   111 0         152 0         193 0
 30 2.10307    71 2.99694   112 0         153 0         194 0
 31 2.15897    72 3.45326   113 0         154 0         195 0
 32 2.1654     73 1.59623   114 0         155 0         196 0
 33 2.08059    74 0         115 0         156 0         197 0
 34 2.27558    75 0         116 0         157 0         198 0
 35 2.29413    76 0         117 0         158 0         199 0
 36 2.15068    77 0         118 0         159 0         200 0
 37 2.10391    78 0         119 0         160 0
 38 2.19875    79 0         120 0         161 0
 39 2.2119     80 0         121 0         162 0
 40 2.19717    81 0         122 0         163 0
;


CONOPT 3.17A: Locally optimal; objective 1.012836073
1433 iterations; evals: nf = 148, ng = 0, nc = 2261, nJ = 217, nH = 10, nHv = 49
```

```
***** Optimal Solution: *****

N = 400

final_height = 1.01284

t_final = 0.198798

thrust [*] :=
  0 3.5          101 0.96099       202 0             303 0
  1 3.5          102 2.30946       203 0             304 0
  2 3.5          103 1.70124       204 0             305 0
  3 3.5          104 2.13734       205 0             306 0
  4 3.5          105 1.75601       206 0             307 0
  5 3.5          106 2.82999       207 0             308 0
  6 3.5          107 3.5           208 0             309 0
  7 3.5          108 3.33283       209 0             310 0
  8 3.5          109 1.30826       210 0             311 0
  9 3.5          110 0             211 0             312 0
 10 3.5          111 3.5           212 0             313 0
 11 3.5          112 3.5           213 0             314 0
 12 3.5          113 2.18998       214 0             315 0
 13 3.5          114 2.81039       215 0             316 0
 14 3.5          115 2.23877       216 0             317 0
 15 3.5          116 2.58452       217 0             318 0
 16 3.5          117 2.45425       218 0             319 0
 17 3.5          118 2.26924       219 0             320 0
 18 3.5          119 2.57451       220 0             321 0
 19 3.5          120 2.55892       221 0             322 0
 20 3.5          121 2.7103        222 0             323 0
 21 3.5          122 2.78573       223 0             324 0
 22 3.5          123 2.77852       224 0             325 0
 23 3.5          124 2.38598       225 0             326 0
 24 3.5          125 2.73334       226 0             327 0
 25 3.5          126 2.50646       227 0             328 0
 26 3.5          127 2.5995        228 0             329 0
 27 3.5          128 2.77932       229 0             330 0
 28 3.5          129 2.47568       230 0             331 0
 29 3.5          130 2.77588       231 0             332 0
 30 3.5          131 2.61015       232 0             333 0
 31 3.5          132 2.27045       233 0             334 0
 32 3.5          133 2.92722       234 0             335 0
 33 3.5          134 2.52293       235 0             336 0
 34 3.5          135 3.5           236 0             337 0
 35 3.5          136 2.80354       237 0             338 0
```

36 3.5	137 3.5	238 0	339 0
37 3.5	138 3.17233	239 0	340 0
38 3.5	139 1.72486	240 0	341 0
39 3.5	140 3.23945	241 0	342 0
40 3.5	141 1.70334	242 0	343 0
41 3.5	142 3.17172	243 0	344 0
42 3.5	143 3.5	244 0	345 0
43 3.5	144 2.74866	245 0	346 0
44 3.5	145 2.70241	246 0	347 0
45 3.5	146 1.4722	247 0	348 0
46 3.5	147 1.25443	248 0	349 0
47 3.5	148 0.417967	249 0	350 0
48 2.33479	149 1.02366e-14	250 0	351 0
49 1.78991	150 0	251 0	352 0
50 2.21842	151 0	252 0	353 0
51 2.06789	152 0	253 0	354 0
52 2.17247	153 0	254 0	355 0
53 2.09294	154 0	255 0	356 0
54 2.30375	155 2.30478e-22	256 0	357 0
55 2.00092	156 0	257 0	358 0
56 1.66719	157 0	258 0	359 0
57 2.15461	158 0	259 0	360 0
58 2.57055	159 0	260 0	361 0
59 2.17839	160 0	261 0	362 0
60 1.95235	161 0	262 0	363 0
61 2.15944	162 0	263 0	364 0
62 1.94058	163 0	264 0	365 0
63 2.13148	164 0	265 0	366 0
64 1.82374	165 0	266 0	367 0
65 2.11332	166 0	267 0	368 0
66 3.5	167 0	268 0	369 0
67 2.23318	168 0	269 0	370 0
68 1.36055	169 0	270 0	371 0
69 2.23647	170 0	271 0	372 0
70 1.44577	171 0	272 0	373 0
71 1.98124	172 0	273 0	374 0
72 3.5	173 0	274 0	375 0
73 2.18338	174 0	275 0	376 0
74 1.95332	175 0	276 0	377 0
75 2.19735	176 0	277 0	378 0
76 1.9457	177 0	278 0	379 0
77 3.26987	178 0	279 0	380 0
78 2.15431	179 0	280 0	381 0
79 0.821521	180 0	281 0	382 0
80 1.859	181 0	282 0	383 0
81 3.31876	182 0	283 0	384 0

```
 82 2.37186        183 0             284 0             385 0
 83 2.31984        184 0             285 0             386 0
 84 2.16021        185 0             286 0             387 0
 85 1.52856        186 0             287 0             388 0
 86 2.13344        187 0             288 0             389 0
 87 3.5            188 0             289 0             390 0
 88 2.30376        189 0             290 0             391 0
 89 0.844174       190 0             291 0             392 0
 90 2.20653        191 0             292 0             393 0
 91 0.462763       192 0             293 0             394 0
 92 2.76018        193 0             294 0             395 0
 93 3.5            194 0             295 0             396 0
 94 1.97557        195 0             296 0             397 0
 95 3.5            196 0             297 0             398 0
 96 2.25902        197 0             298 0             399 0
 97 2.21988        198 0             299 0             400 0
 98 2.28561        199 0             300 0
 99 3.5            200 0             301 0
100 2.4157         201 0             302 0
;


ampl:
```

3.3.2 AMPL Output using Knitro Solver

```
ampl: include cw1.run;

***** Math6120 - Nonlinear Optimization *****
***** Coursework 1 - AMPL Model *****
***** Emma Tarmey, 2940 4045 *****

Artelys Knitro 13.0.1:                    Knitro 13.0.1: Locally optimal or satisfactory
objective 1.012809643; feasibility error 2.12e-06
16 iterations; 22 function evaluations

suffix feaserror OUT;
suffix opterror OUT;
suffix numfcevals OUT;
suffix numiters OUT;

***** Optimal Solution: *****

N = 50

final_height = 1.01281

t_final = 0.198439

thrust [*] :=
 0 3.5          13 2.35681       26 6.85344e-07   39 2.3311e-07
 1 3.5          14 2.40978       27 5.74627e-07   40 2.26981e-07
 2 3.5          15 2.53493       28 5.0297e-07    41 2.24461e-07
 3 3.5          16 2.60637       29 4.40356e-07   42 2.20048e-07
 4 3.5          17 2.90008       30 3.98939e-07   43 2.23331e-07
 5 3.41748      18 1.98966       31 3.60069e-07   44 2.20073e-07
 6 2.82538      19 0.410231      32 3.33777e-07   45 2.32945e-07
 7 2.14045      20 9.30656e-06   33 3.08197e-07   46 2.3052e-07
 8 2.14705      21 3.76958e-06   34 2.90393e-07   47 2.66939e-07
 9 2.18742      22 2.06478e-06   35 2.73127e-07   48 2.64783e-07
10 2.19304      23 1.42667e-06   36 2.6048e-07    49 4.30297e-07
11 2.2493       24 1.05494e-06   37 2.49049e-07   50 8.58556e-07
12 2.2754       25 8.28531e-07   38 2.40202e-07
;


Artelys Knitro 13.0.1:                    Knitro 13.0.1: Locally optimal or satisfactory
objective 1.012830127; feasibility error 1.61e-05
23 iterations; 28 function evaluations
```

```
***** Optimal Solution: *****

N = 100

final_height = 1.01283

t_final = 0.198736

thrust [*] :=
  0 3.49999       26 2.34621        52 6.2984e-06     78 2.18372e-06
  1 3.5           27 2.39097        53 5.73128e-06    79 2.15845e-06
  2 3.5           28 2.42348        54 5.34246e-06    80 2.13044e-06
  3 3.49999       29 2.48051        55 4.92747e-06    81 2.11752e-06
  4 3.49999       30 2.52086        56 4.64385e-06    82 2.09218e-06
  5 3.49999       31 2.59486        57 4.32912e-06    83 2.09105e-06
  6 3.49999       32 2.64243        58 4.11302e-06    84 2.06932e-06
  7 3.49998       33 2.74323        59 3.86706e-06    85 2.08261e-06
  8 3.49997       34 2.78155        60 3.70092e-06    86 2.06344e-06
  9 3.49994       35 2.99373        61 3.50532e-06    87 2.09469e-06
 10 3.49987       36 2.42762        62 3.37325e-06    88 2.07764e-06
 11 3.37399       37 0.779993       63 3.21416e-06    89 2.133e-06
 12 2.72712       38 0.000473056    64 3.10761e-06    90 2.11803e-06
 13 2.13246       39 0.000146912    65 2.9783e-06     91 2.20855e-06
 14 2.14283       40 7.78395e-05    66 2.89213e-06    92 2.19525e-06
 15 2.15392       41 4.71192e-05    67 2.78464e-06    93 2.34345e-06
 16 2.15545       42 3.32441e-05    68 2.71191e-06    94 2.33223e-06
 17 2.16976       43 2.43835e-05    69 2.62372e-06    95 2.59515e-06
 18 2.17476       44 1.93335e-05    70 2.56387e-06    96 2.58514e-06
 19 2.19278       45 1.55153e-05    71 2.4911e-06     97 3.15236e-06
 20 2.20179       46 1.31001e-05    72 2.43961e-06    98 3.14138e-06
 21 2.22438       47 1.10697e-05    73 2.38007e-06    99 5.46161e-06
 22 2.23804       48 9.7212e-06     74 2.3368e-06    100 1.0861e-05
 23 2.26633       49 8.4945e-06     75 2.28992e-06
 24 2.28539       50 7.66026e-06    76 2.25221e-06
 25 2.32088       51 6.8538e-06     77 2.21602e-06
;


Artelys Knitro 13.0.1:                       Knitro 13.0.1: Locally optimal or satisfactory
objective 1.01283521; feasibility error 2.69e-05
17 iterations; 32 function evaluations

***** Optimal Solution: *****
```

```
N = 200

final_height = 1.01284

t_final = 0.198824

thrust [*] :=
  0 3.5          51 2.3366         102 3.83268e-06   153 1.24936e-06
  1 3.5          52 2.34953        103 3.62781e-06   154 1.2423e-06
  2 3.5          53 2.37172        104 3.49188e-06   155 1.23199e-06
  3 3.5          54 2.38628        105 3.31933e-06   156 1.22551e-06
  4 3.5          55 2.4114         106 3.20593e-06   157 1.21681e-06
  5 3.5          56 2.42766        107 3.05892e-06   158 1.21086e-06
  6 3.5          57 2.4562         108 2.96331e-06   159 1.2038e-06
  7 3.5          58 2.4742         109 2.83678e-06   160 1.19833e-06
  8 3.5          59 2.50678        110 2.7554e-06    161 1.19293e-06
  9 3.5          60 2.52647        111 2.64554e-06   162 1.18791e-06
 10 3.49999      61 2.56389        112 2.57573e-06   163 1.18425e-06
 11 3.49999      62 2.58511        113 2.47959e-06   164 1.17964e-06
 12 3.49999      63 2.62844        114 2.41927e-06   165 1.17782e-06
 13 3.49999      64 2.65079        115 2.33456e-06   166 1.17362e-06
 14 3.49999      65 2.70156        116 2.28212e-06   167 1.17378e-06
 15 3.49999      66 2.72417        117 2.207e-06     168 1.16996e-06
 16 3.49998      67 2.78472        118 2.16116e-06   169 1.1723e-06
 17 3.49998      68 2.80585        119 2.0942e-06    170 1.16886e-06
 18 3.49997      69 2.87921        120 2.05391e-06   171 1.17364e-06
 19 3.49995      70 2.90103        121 1.99393e-06   172 1.17058e-06
 20 3.49993      71 2.96871        122 1.95837e-06   173 1.17811e-06
 21 3.49985      72 3.12234        123 1.90442e-06   174 1.17542e-06
 22 3.49963      73 1.63271        124 1.87289e-06   175 1.18611e-06
 23 3.25738      74 0.135499       125 1.82418e-06   176 1.18377e-06
 24 2.59358      75 0.00047842     126 1.79612e-06   177 1.1982e-06
 25 2.13288      76 0.000237247    127 1.75201e-06   178 1.19636e-06
 26 2.14055      77 0.000122476    128 1.72695e-06   179 1.21558e-06
 27 2.14474      78 8.11622e-05    129 1.6869e-06    180 1.21449e-06
 28 2.14465      79 5.50413e-05    130 1.66445e-06   181 1.23981e-06
 29 2.14968      80 4.20647e-05    131 1.62801e-06   182 1.23749e-06
 30 2.15031      81 3.19986e-05    132 1.60784e-06   183 1.26914e-06
 31 2.15608      82 2.62602e-05    133 1.57464e-06   184 1.26709e-06
 32 2.15747      83 2.13058e-05    134 1.55647e-06   185 1.30717e-06
 33 2.16406      84 1.82589e-05    135 1.52619e-06   186 1.30477e-06
 34 2.16628      85 1.54346e-05    136 1.50979e-06   187 1.356e-06
 35 2.17376      86 1.36162e-05    137 1.48216e-06   188 1.35243e-06
 36 2.17686      87 1.18414e-05    138 1.46732e-06   189 1.41889e-06
 37 2.18534      88 1.06641e-05    139 1.44214e-06   190 1.4132e-06
 38 2.18938      89 9.46923e-06    140 1.42868e-06   191 1.50213e-06
```

```
 39 2.19896        90 8.66219e-06   141 1.40574e-06   192 1.49405e-06
 40 2.20402        91 7.81489e-06   142 1.39352e-06   193 1.62125e-06
 41 2.21483        92 7.23737e-06   143 1.37267e-06   194 1.61371e-06
 42 2.22098        93 6.61136e-06   144 1.36156e-06   195 1.82028e-06
 43 2.23316        94 6.18357e-06   145 1.34267e-06   196 1.82147e-06
 44 2.24048        95 5.70537e-06   146 1.33255e-06   197 2.24029e-06
 45 2.2542         96 5.37918e-06   147 1.31551e-06   198 2.26428e-06
 46 2.26279        97 5.00379e-06   148 1.30628e-06   199 3.72636e-06
 47 2.27824        98 4.74892e-06   149 1.29101e-06   200 7.50959e-06
 48 2.28818        99 4.44753e-06   150 1.28258e-06
 49 2.30558       100 4.24423e-06   151 1.26901e-06
 50 2.31697       101 3.99769e-06   152 1.26129e-06
;


Artelys Knitro 13.0.1:                        Knitro 13.0.1: Locally optimal or satisfactory
objective 1.012836495; feasibility error 4.7e-06
23 iterations; 39 function evaluations

***** Optimal Solution: *****

N = 400

final_height = 1.01284

t_final = 0.198848

thrust [*] :=
  0 3.5          101 2.32669       202 4.32628e-07   303 1.35642e-07
  1 3.5          102 2.33505       203 4.22311e-07   304 1.34733e-07
  2 3.5          103 2.34309       204 4.11585e-07   305 1.34526e-07
  3 3.5          104 2.35198       205 4.0225e-07    306 1.33641e-07
  4 3.5          105 2.36053       206 3.92414e-07   307 1.33473e-07
  5 3.5          106 2.36998       207 3.83912e-07   308 1.3261e-07
  6 3.5          107 2.37906       208 3.74935e-07   309 1.32479e-07
  7 3.5          108 2.3891        209 3.67193e-07   310 1.31638e-07
  8 3.5          109 2.39874       210 3.58965e-07   311 1.31544e-07
  9 3.5          110 2.40941       211 3.51886e-07   312 1.30723e-07
 10 3.5          111 2.41964       212 3.44311e-07   313 1.30668e-07
 11 3.5          112 2.43098       213 3.37814e-07   314 1.29867e-07
 12 3.5          113 2.44182       214 3.30825e-07   315 1.2985e-07
 13 3.5          114 2.45388       215 3.2485e-07    316 1.29067e-07
 14 3.5          115 2.46537       216 3.18371e-07   317 1.29089e-07
 15 3.5          116 2.47818       217 3.12865e-07   318 1.28324e-07
 16 3.5          117 2.49036       218 3.06867e-07   319 1.28386e-07
```

17 3.5	118 2.50398	219 3.01766e-07	320 1.27637e-07
18 3.5	119 2.51688	220 2.96176e-07	321 1.27739e-07
19 3.5	120 2.53135	221 2.91443e-07	322 1.27007e-07
20 3.5	121 2.54501	222 2.86223e-07	323 1.2715e-07
21 3.5	122 2.56041	223 2.81825e-07	324 1.26433e-07
22 3.5	123 2.57486	224 2.76935e-07	325 1.26619e-07
23 3.5	124 2.59124	225 2.72845e-07	326 1.25916e-07
24 3.5	125 2.60653	226 2.68264e-07	327 1.26146e-07
25 3.5	126 2.62397	227 2.64453e-07	328 1.25458e-07
26 3.5	127 2.64013	228 2.60154e-07	329 1.25732e-07
27 3.5	128 2.65871	229 2.566e-07	330 1.25057e-07
28 3.5	129 2.67578	230 2.5256e-07	331 1.25379e-07
29 3.5	130 2.6956	231 2.49234e-07	332 1.24717e-07
30 3.5	131 2.71362	232 2.45428e-07	333 1.25087e-07
31 3.5	132 2.73478	233 2.42317e-07	334 1.24437e-07
32 3.5	133 2.75377	234 2.38727e-07	335 1.24859e-07
33 3.5	134 2.77639	235 2.35814e-07	336 1.24221e-07
34 3.5	135 2.79639	236 2.32421e-07	337 1.24697e-07
35 3.5	136 2.82062	237 2.2969e-07	338 1.2407e-07
36 3.5	137 2.84162	238 2.2648e-07	339 1.24603e-07
37 3.5	138 2.86764	239 2.23917e-07	340 1.23987e-07
38 3.5	139 2.88963	240 2.20876e-07	341 1.24581e-07
39 3.49999	140 2.91764	241 2.18474e-07	342 1.23974e-07
40 3.49999	141 2.94071	242 2.15589e-07	343 1.24633e-07
41 3.49999	142 2.97	243 2.13329e-07	344 1.24035e-07
42 3.49998	143 2.99979	244 2.10588e-07	345 1.24763e-07
43 3.49998	144 3.00637	245 2.08461e-07	346 1.24175e-07
44 3.49996	145 3.15955	246 2.05853e-07	347 1.24977e-07
45 3.49992	146 1.83331	247 2.03851e-07	348 1.24397e-07
46 3.49952	147 0.193433	248 2.01367e-07	349 1.2528e-07
47 3.02328	148 0.000249842	249 1.99482e-07	350 1.24708e-07
48 2.33629	149 0.000106807	250 1.97113e-07	351 1.25679e-07
49 2.13627	150 5.59873e-05	251 1.95337e-07	352 1.25114e-07
50 2.14002	151 3.40878e-05	252 1.93076e-07	353 1.26181e-07
51 2.14059	152 2.27474e-05	253 1.91402e-07	354 1.25622e-07
52 2.14161	153 1.72709e-05	254 1.8924e-07	355 1.26794e-07
53 2.14239	154 1.31016e-05	255 1.87662e-07	356 1.26242e-07
54 2.14358	155 1.02509e-05	256 1.85595e-07	357 1.2753e-07
55 2.14453	156 8.31903e-06	257 1.84107e-07	358 1.26983e-07
56 2.14588	157 7.06644e-06	258 1.82127e-07	359 1.28401e-07
57 2.147	158 5.99341e-06	259 1.80724e-07	360 1.27859e-07
58 2.14854	159 5.09119e-06	260 1.78826e-07	361 1.29421e-07
59 2.14984	160 4.44262e-06	261 1.77503e-07	362 1.28884e-07
60 2.15155	161 3.90905e-06	262 1.75682e-07	363 1.30609e-07
61 2.15304	162 3.44866e-06	263 1.74435e-07	364 1.30075e-07
62 2.15494	163 3.10557e-06	264 1.72687e-07	365 1.31985e-07

```
 63 2.15663        164 2.79769e-06   265 1.71511e-07   366 1.31455e-07
 64 2.15873        165 2.54688e-06   266 1.69831e-07   367 1.33576e-07
 65 2.16062        166 2.3087e-06    267 1.68724e-07   368 1.33048e-07
 66 2.16293        167 2.11684e-06   268 1.67107e-07   369 1.35415e-07
 67 2.16503        168 1.9505e-06    269 1.66065e-07   370 1.34888e-07
 68 2.16756        169 1.80892e-06   270 1.64508e-07   371 1.3754e-07
 69 2.16987        170 1.67312e-06   271 1.63528e-07   372 1.37015e-07
 70 2.17263        171 1.56086e-06   272 1.62028e-07   373 1.40004e-07
 71 2.17518        172 1.45401e-06   273 1.61107e-07   374 1.39478e-07
 72 2.17817        173 1.36766e-06   274 1.59661e-07   375 1.42871e-07
 73 2.18096        174 1.28536e-06   275 1.58795e-07   376 1.42344e-07
 74 2.1842         175 1.21363e-06   276 1.574e-07     377 1.46227e-07
 75 2.18724        176 1.14315e-06   277 1.56589e-07   378 1.45697e-07
 76 2.19074        177 1.08257e-06   278 1.55241e-07   379 1.50186e-07
 77 2.19404        178 1.02652e-06   279 1.54482e-07   380 1.4965e-07
 78 2.19782        179 9.78896e-07   280 1.5318e-07    381 1.54903e-07
 79 2.20138        180 9.31911e-07   281 1.52471e-07   382 1.54362e-07
 80 2.20545        181 8.90228e-07   282 1.51211e-07   383 1.60602e-07
 81 2.2093         182 8.48635e-07   283 1.5055e-07    384 1.60051e-07
 82 2.21366        183 8.12901e-07   284 1.49332e-07   385 1.67606e-07
 83 2.21781        184 7.77198e-07   285 1.48717e-07   386 1.67042e-07
 84 2.22249        185 7.46662e-07   286 1.47537e-07   387 1.76417e-07
 85 2.22695        186 7.16911e-07   287 1.46967e-07   388 1.75836e-07
 86 2.23196        187 6.90944e-07   288 1.45824e-07   389 1.87857e-07
 87 2.23675        188 6.64668e-07   289 1.45298e-07   390 1.87251e-07
 88 2.2421         189 6.41461e-07   290 1.44189e-07   391 2.03386e-07
 89 2.24723        190 6.18174e-07   291 1.43706e-07   392 2.02745e-07
 90 2.25295        191 5.97874e-07   292 1.42631e-07   393 2.25929e-07
 91 2.25844        192 5.77476e-07   293 1.42189e-07   394 2.25236e-07
 92 2.26454        193 5.60188e-07   294 1.41145e-07   395 2.62504e-07
 93 2.2704         194 5.42369e-07   295 1.40744e-07   396 2.61723e-07
 94 2.27691        195 5.26915e-07   296 1.3973e-07    397 3.36449e-07
 95 2.28317        196 5.10918e-07   297 1.39369e-07   398 3.35477e-07
 96 2.2901         197 4.96816e-07   298 1.38383e-07   399 6.31134e-07
 97 2.29678        198 4.82215e-07   299 1.38061e-07   400 1.25882e-06
 98 2.30416        199 4.69383e-07   300 1.37102e-07
 99 2.31127        200 4.56109e-07   301 1.36819e-07
100 2.31912        201 4.44626e-07   302 1.35886e-07
;


ampl:
```

3.3.3 AMPL Output using Loqo Solver

```
ampl: include cw1.run;

***** Math6120 - Nonlinear Optimization *****
***** Coursework 1 - AMPL Model *****
***** Emma Tarmey, 2940 4045 *****

LOQO 7.03: optimal solution (26 iterations, 27 evaluations)
primal objective 1.012809641
  dual objective 1.012809666

***** Optimal Solution: *****

N = 50

final_height = 1.01281

t_final = 0.198431

thrust [*] :=
 0 3.5          13 2.35678       26 5.09388e-07   39 2.17528e-07
 1 3.5          14 2.40968       27 4.95475e-07   40 2.05589e-07
 2 3.5          15 2.53492       28 3.82628e-07   41 2.11724e-07
 3 3.5          16 2.60617       29 3.81101e-07   42 2.07798e-07
 4 3.49999      17 2.89979       30 3.09561e-07   43 2.11144e-07
 5 3.41737      18 1.99103       31 3.14031e-07   44 2.18761e-07
 6 2.82544      19 0.411464      32 2.6586e-07    45 2.20008e-07
 7 2.1407       20 3.29712e-06   33 2.72724e-07   46 2.42536e-07
 8 2.14703      21 3.08431e-06   34 2.3792e-07    47 2.45594e-07
 9 2.18741      22 1.32351e-06   35 2.46172e-07   48 2.88532e-07
10 2.19301      23 1.19847e-06   36 2.20091e-07   49 3.64762e-07
11 2.24928      24 7.4668e-07    37 2.28785e-07   50 8.69612e-07
12 2.27535      25 7.05246e-07   38 2.09522e-07
;



LOQO 7.03: optimal solution (28 iterations, 30 evaluations)
primal objective 1.012830162
  dual objective 1.012830162

***** Optimal Solution: *****

N = 100
```

```
final_height = 1.01283

t_final = 0.198748

thrust [*] :=
  0 3.5          26 2.34012        52 8.05332e-10    78 3.34084e-10
  1 3.5          27 2.39397        53 7.27061e-10    79 3.29018e-10
  2 3.5          28 2.43145        54 6.81513e-10    80 3.3048e-10
  3 3.5          29 2.47505        55 6.25627e-10    81 3.26642e-10
  4 3.5          30 2.52458        56 5.94753e-10    82 3.28851e-10
  5 3.5          31 2.60325        57 5.53681e-10    83 3.26323e-10
  6 3.5          32 2.63237        58 5.32921e-10    84 3.29335e-10
  7 3.5          33 2.746          59 5.01585e-10    85 3.28424e-10
  8 3.5          34 2.77803        60 4.87047e-10    86 3.32608e-10
  9 3.5          35 2.94787        61 4.62605e-10    87 3.33842e-10
 10 3.5          36 2.43867        62 4.52199e-10    88 3.3939e-10
 11 3.36838      37 0.816203       63 4.32301e-10    89 3.43523e-10
 12 2.72464      38 2.51913e-08    64 4.24603e-10    90 3.5038e-10
 13 2.13648      39 2.41381e-08    65 4.08102e-10    91 3.58798e-10
 14 2.14207      40 1.0045e-08     66 4.02435e-10    92 3.66781e-10
 15 2.15163      41 5.88084e-09    67 3.88402e-10    93 3.82373e-10
 16 2.15642      42 4.15343e-09    68 3.84295e-10    94 3.9187e-10
 17 2.17228      43 2.99098e-09    69 3.72374e-10    95 4.21581e-10
 18 2.17398      44 2.39958e-09    70 3.6949e-10     96 4.3237e-10
 19 2.191        45 1.90927e-09    71 3.59158e-10    97 4.98951e-10
 20 2.20178      46 1.65085e-09    72 3.57276e-10    98 5.07691e-10
 21 2.22458      47 1.3933e-09     73 3.48427e-10    99 7.79323e-10
 22 2.23857      48 1.24583e-09    74 3.47536e-10   100 1.54546e-09
 23 2.26632      49 1.08178e-09    75 3.39877e-10
 24 2.28225      50 9.84305e-10    76 3.39787e-10
 25 2.32188      51 8.72398e-10    77 3.3342e-10
;


LOQO 7.03: optimal solution (34 iterations, 35 evaluations)
primal objective 1.012834874
  dual objective 1.012834875

***** Optimal Solution: *****

N = 200

final_height = 1.01283

t_final = 0.198749
```

```
thrust [*] :=
  0 3.5            51 2.32241        102 2.90358e-10   153 1.28378e-10
  1 3.5            52 2.29732        103 2.93633e-10   154 1.22237e-10
  2 3.5            53 2.32962        104 2.60605e-10   155 1.2806e-10
  3 3.5            54 2.28733        105 2.65396e-10   156 1.21753e-10
  4 3.5            55 2.33613        106 2.36941e-10   157 1.28062e-10
  5 3.5            56 2.58763        107 2.40742e-10   158 1.21804e-10
  6 3.5            57 2.45221        108 2.17314e-10   159 1.28303e-10
  7 3.5            58 2.54804        109 2.20377e-10   160 1.22444e-10
  8 3.5            59 2.53708        110 2.01964e-10   161 1.2895e-10
  9 3.5            60 2.20232        111 2.02076e-10   162 1.22979e-10
 10 3.5            61 2.62805        112 1.89014e-10   163 1.29843e-10
 11 3.5            62 2.48404        113 1.89483e-10   164 1.24393e-10
 12 3.5            63 2.38882        114 1.77879e-10   165 1.3036e-10
 13 3.5            64 2.80656        115 1.79264e-10   166 1.24703e-10
 14 3.5            65 2.86617        116 1.68995e-10   167 1.31153e-10
 15 3.5            66 2.64319        117 1.70063e-10   168 1.25596e-10
 16 3.5            67 2.89704        118 1.6131e-10    169 1.31619e-10
 17 3.5            68 2.66873        119 1.62084e-10   170 1.27212e-10
 18 3.5            69 2.45966        120 1.54926e-10   171 1.32996e-10
 19 3.5            70 2.66937        121 1.56873e-10   172 1.28566e-10
 20 3.5            71 3.25602        122 1.49881e-10   173 1.34437e-10
 21 3.5            72 2.47581        123 1.51837e-10   174 1.30339e-10
 22 3.45326        73 1.84463        124 1.45208e-10   175 1.36758e-10
 23 3.18383        74 1.32048        125 1.47628e-10   176 1.32293e-10
 24 2.61277        75 0.0439674      126 1.41392e-10   177 1.40419e-10
 25 2.26655        76 3.54366e-09    127 1.44585e-10   178 1.35255e-10
 26 2.17093        77 5.65454e-09    128 1.38082e-10   179 1.43317e-10
 27 2.10789        78 4.0395e-09     129 1.41577e-10   180 1.38796e-10
 28 1.95697        79 5.10503e-09    130 1.35291e-10   181 1.46289e-10
 29 2.13759        80 2.83628e-09    131 1.3894e-10    182 1.43537e-10
 30 2.2995         81 3.28931e-09    132 1.33035e-10   183 1.50651e-10
 31 2.16374        82 1.90535e-09    133 1.36351e-10   184 1.47935e-10
 32 2.18262        83 2.14642e-09    134 1.30844e-10   185 1.56018e-10
 33 2.03096        84 1.40681e-09    135 1.34263e-10   186 1.5314e-10
 34 2.23354        85 1.48378e-09    136 1.29316e-10   187 1.62394e-10
 35 1.92811        86 1.04155e-09    137 1.33122e-10   188 1.58989e-10
 36 1.94582        87 1.04961e-09    138 1.27613e-10   189 1.7045e-10
 37 3.12972        88 7.81414e-10    139 1.31588e-10   190 1.67073e-10
 38 2.29718        89 7.83172e-10    140 1.26468e-10   191 1.8043e-10
 39 1.3912         90 6.29524e-10    141 1.30284e-10   192 1.7865e-10
 40 1.81293        91 6.31298e-10    142 1.25557e-10   193 1.95406e-10
 41 2.21746        92 5.33222e-10    143 1.29625e-10   194 1.93359e-10
 42 2.87422        93 5.4345e-10     144 1.24947e-10   195 2.17368e-10
 43 2.47975        94 4.63762e-10    145 1.29315e-10   196 2.13178e-10
```

```
 44 2.15647        95 4.72986e-10   146 1.23965e-10   197 2.60659e-10
 45 2.21486        96 4.08114e-10   147 1.28469e-10   198 2.41456e-10
 46 2.19615        97 4.17099e-10   148 1.23339e-10   199 4.14164e-10
 47 2.27554        98 3.63159e-10   149 1.28519e-10   200 8.28179e-10
 48 2.33338        99 3.69869e-10   150 1.22703e-10
 49 2.24063       100 3.26154e-10   151 1.28395e-10
 50 2.3631        101 3.30018e-10   152 1.22837e-10
;


LOQO 7.03: optimal solution (88 iterations, 161 evaluations)
primal objective 1.012836359
  dual objective 1.012836359

***** Optimal Solution: *****

N = 400

final_height = 1.01284

t_final = 0.198812

thrust [*] :=
  0 3.5          101 2.32311       202 7.10296e-10   303 2.24652e-10
  1 3.5          102 2.33347       203 7.01233e-10   304 2.20716e-10
  2 3.5          103 2.32224       204 6.75599e-10   305 2.22807e-10
  3 3.5          104 2.37498       205 6.68132e-10   306 2.18902e-10
  4 3.5          105 2.35782       206 6.44146e-10   307 2.21042e-10
  5 3.5          106 2.38193       207 6.37783e-10   308 2.17215e-10
  6 3.5          107 2.37588       208 6.15543e-10   309 2.19402e-10
  7 3.5          108 2.39155       209 6.10026e-10   310 2.15628e-10
  8 3.5          109 2.41792       210 5.89539e-10   311 2.17841e-10
  9 3.5          110 2.38357       211 5.84547e-10   312 2.14138e-10
 10 3.5          111 2.45527       212 5.65173e-10   313 2.16384e-10
 11 3.5          112 2.42744       213 5.6094e-10    314 2.12709e-10
 12 3.5          113 2.43378       214 5.42874e-10   315 2.15017e-10
 13 3.5          114 2.43147       215 5.39633e-10   316 2.11387e-10
 14 3.5          115 2.42706       216 5.22313e-10   317 2.13779e-10
 15 3.5          116 2.50268       217 5.19473e-10   318 2.10147e-10
 16 3.5          117 2.49828       218 5.03585e-10   319 2.1263e-10
 17 3.5          118 2.49539       219 5.00867e-10   320 2.09013e-10
 18 3.5          119 2.48624       220 4.85745e-10   321 2.11566e-10
 19 3.5          120 2.51144       221 4.83651e-10   322 2.07962e-10
 20 3.5          121 2.57995       222 4.69388e-10   323 2.10589e-10
 21 3.5          122 2.56922       223 4.67478e-10   324 2.07018e-10
```

22	3.5	123	2.59451	224	4.54155e-10	325	2.097e-10
23	3.5	124	2.58587	225	4.52699e-10	326	2.06171e-10
24	3.5	125	2.58893	226	4.39897e-10	327	2.08896e-10
25	3.5	126	2.68638	227	4.38737e-10	328	2.05424e-10
26	3.5	127	2.68441	228	4.26504e-10	329	2.08212e-10
27	3.5	128	2.89766	229	4.25847e-10	330	2.04762e-10
28	3.5	129	2.73944	230	4.13975e-10	331	2.07629e-10
29	3.5	130	2.64088	231	4.13511e-10	332	2.04204e-10
30	3.5	131	2.75791	232	4.02218e-10	333	2.07146e-10
31	3.5	132	2.73316	233	4.02005e-10	334	2.03727e-10
32	3.5	133	2.81138	234	3.91284e-10	335	2.06778e-10
33	3.5	134	2.70941	235	3.9127e-10	336	2.03382e-10
34	3.5	135	2.75128	236	3.8096e-10	337	2.06491e-10
35	3.5	136	2.76426	237	3.81076e-10	338	2.0313e-10
36	3.5	137	2.76923	238	3.71375e-10	339	2.06338e-10
37	3.5	138	2.77029	239	3.71508e-10	340	2.02994e-10
38	3.5	139	2.75841	240	3.62129e-10	341	2.063e-10
39	3.5	140	2.69802	241	3.62466e-10	342	2.0297e-10
40	3.5	141	2.74725	242	3.53427e-10	343	2.06369e-10
41	3.5	142	2.55803	243	3.53797e-10	344	2.03073e-10
42	3.5	143	2.54652	244	3.45211e-10	345	2.06595e-10
43	3.5	144	2.31401	245	3.45725e-10	346	2.03301e-10
44	3.5	145	2.28683	246	3.37458e-10	347	2.06944e-10
45	3.43493	146	1.93893	247	3.38035e-10	348	2.03685e-10
46	3.09598	147	1.73473	248	3.3014e-10	349	2.07461e-10
47	2.81117	148	1.33408	249	3.30826e-10	350	2.04194e-10
48	2.61439	149	0.161852	250	3.23136e-10	351	2.08124e-10
49	2.44851	150	3.18278e-08	251	3.2392e-10	352	2.04882e-10
50	2.33022	151	4.07003e-08	252	3.16512e-10	353	2.0892e-10
51	2.21933	152	3.45843e-08	253	3.17353e-10	354	2.05699e-10
52	2.15792	153	2.59337e-08	254	3.10232e-10	355	2.09922e-10
53	2.09696	154	2.14445e-08	255	3.1113e-10	356	2.06713e-10
54	2.08602	155	1.67103e-08	256	3.04245e-10	357	2.11139e-10
55	2.07388	156	1.41507e-08	257	3.05257e-10	358	2.07921e-10
56	2.12367	157	1.15096e-08	258	2.98573e-10	359	2.12564e-10
57	2.11183	158	9.97229e-09	259	2.99664e-10	360	2.09337e-10
58	2.10581	159	8.50752e-09	260	2.9312e-10	361	2.14272e-10
59	2.13319	160	7.42858e-09	261	2.94336e-10	362	2.11004e-10
60	2.13696	161	6.53372e-09	262	2.8796e-10	363	2.16214e-10
61	2.127	162	5.78616e-09	263	2.89165e-10	364	2.12939e-10
62	2.15094	163	5.1726e-09	264	2.83074e-10	365	2.18484e-10
63	2.16129	164	4.63958e-09	265	2.84307e-10	366	2.15202e-10
64	2.1685	165	4.22543e-09	266	2.78377e-10	367	2.21126e-10
65	2.17747	166	3.83075e-09	267	2.79718e-10	368	2.17805e-10
66	2.16794	167	3.537e-09	268	2.73939e-10	369	2.24161e-10
67	2.16935	168	3.22674e-09	269	2.75285e-10	370	2.2083e-10

```
 68 2.15213        169 3.00358e-09   270 2.69703e-10   371 2.27684e-10
 69 2.18053        170 2.76381e-09   271 2.71052e-10   372 2.24321e-10
 70 2.18011        171 2.5987e-09    272 2.65621e-10   373 2.31739e-10
 71 2.17201        172 2.40457e-09   273 2.66979e-10   374 2.28324e-10
 72 2.16195        173 2.27445e-09   274 2.61725e-10   375 2.36499e-10
 73 2.18668        174 2.1174e-09    275 2.63146e-10   376 2.33013e-10
 74 2.18454        175 2.01523e-09   276 2.58015e-10   377 2.42084e-10
 75 2.17684        176 1.88622e-09   277 2.59476e-10   378 2.38529e-10
 76 2.24219        177 1.80238e-09   278 2.54444e-10   379 2.48623e-10
 77 2.20032        178 1.692e-09     279 2.55966e-10   380 2.44982e-10
 78 2.19427        179 1.62461e-09   280 2.5105e-10    381 2.56432e-10
 79 2.18692        180 1.52984e-09   281 2.52615e-10   382 2.52732e-10
 80 2.21334        181 1.4756e-09    282 2.47828e-10   383 2.65861e-10
 81 2.21261        182 1.39229e-09   283 2.4944e-10    384 2.62023e-10
 82 2.22555        183 1.3482e-09    284 2.44752e-10   385 2.7739e-10
 83 2.28186        184 1.2769e-09    285 2.46405e-10   386 2.73451e-10
 84 2.15068        185 1.24075e-09   286 2.41787e-10   387 2.91966e-10
 85 2.15469        186 1.17773e-09   287 2.4347e-10    388 2.87843e-10
 86 2.2404         187 1.1479e-09    288 2.38972e-10   389 3.10886e-10
 87 2.20866        188 1.09201e-09   289 2.40716e-10   390 3.06527e-10
 88 2.24658        189 1.06638e-09   290 2.36278e-10   391 3.36584e-10
 89 2.2227         190 1.01631e-09   291 2.38027e-10   392 3.31873e-10
 90 2.26079        191 9.94799e-10   292 2.33713e-10   393 3.73892e-10
 91 2.24139        192 9.49498e-10   293 2.35533e-10   394 3.68703e-10
 92 2.24738        193 9.31286e-10   294 2.31265e-10   395 4.34446e-10
 93 2.31836        194 8.90391e-10   295 2.33108e-10   396 4.28378e-10
 94 2.25348        195 8.74488e-10   296 2.28906e-10   397 5.56743e-10
 95 2.27132        196 8.37801e-10   297 2.30835e-10   398 5.49112e-10
 96 2.30463        197 8.23921e-10   298 2.26688e-10   399 1.04472e-09
 97 2.2883         198 7.90623e-10   299 2.28666e-10   400 2.06046e-09
 98 2.31249        199 7.78854e-10   300 2.24604e-10
 99 2.31689        200 7.48246e-10   301 2.2659e-10
100 2.30227        201 7.37885e-10   302 2.22624e-10
;


ampl:
```

3.3.4 AMPL Output using Minos Solver

```
ampl: include cw1.run;

***** Math6120 - Nonlinear Optimization *****
***** Coursework 1 - AMPL Model *****
***** Emma Tarmey, 2940 4045 *****

MINOS 5.51: optimal solution found.
1334 iterations, objective 1.012809644
Nonlin evals: constrs = 2156, Jac = 2155.

***** Optimal Solution: *****

N = 50

final_height = 1.01281

t_final = 0.198439

thrust [*] :=
 0 3.5        9 2.18883   18 1.99132   27 0         36 0         45 0
 1 3.5       10 2.19564   19 0.41078   28 0         37 0         46 0
 2 3.5       11 2.25212   20 0         29 0         38 0         47 0
 3 3.5       12 2.27005   21 0         30 0         39 0         48 0
 4 3.5       13 2.34489   22 0         31 0         40 0         49 0
 5 3.41861   14 2.41127   23 0         32 0         41 0         50 0
 6 2.82456   15 2.54617   24 0         33 0         42 0
 7 2.13764   16 2.60626   25 0         34 0         43 0
 8 2.14753   17 2.89759   26 0         35 0         44 0
;



MINOS 5.51: optimal solution found.
1605 iterations, objective 1.012830166
Nonlin evals: constrs = 2528, Jac = 2527.

***** Optimal Solution: *****

N = 100

final_height = 1.01283

t_final = 0.19874
```

```
thrust [*] :=
  0 3.5        21 2.23748    42 0          63 0          84 0
  1 3.5        22 2.23164    43 0          64 0          85 0
  2 3.5        23 2.23085    44 0          65 0          86 0
  3 3.5        24 2.31191    45 0          66 0          87 0
  4 3.5        25 2.32883    46 0          67 0          88 0
  5 3.5        26 2.35583    47 0          68 0          89 0
  6 3.5        27 2.41604    48 0          69 0          90 0
  7 3.5        28 2.42684    49 0          70 0          91 0
  8 3.5        29 2.4368     50 0          71 0          92 0
  9 3.5        30 2.50441    51 0          72 0          93 0
 10 3.5        31 2.5743     52 0          73 0          94 0
 11 3.37666    32 2.63258    53 0          74 0          95 0
 12 2.72691    33 2.85739    54 0          75 0          96 0
 13 2.12959    34 2.74758    55 0          76 0          97 0
 14 2.14902    35 2.87746    56 0          77 0          98 0
 15 2.1637     36 2.46584    57 0          78 0          99 0
 16 2.15691    37 0.82596    58 0          79 0         100 0
 17 2.13964    38 0          59 0          80 0
 18 2.1576     39 0          60 0          81 0
 19 2.2224     40 0          61 0          82 0
 20 2.20002    41 0          62 0          83 0
;


MINOS 5.51: infeasible problem (or bad starting guess).
4368 iterations
Nonlin evals: constrs = 847, Jac = 847.

***** Optimal Solution: *****

N = 200

final_height = 5.49472

t_final = 0.0711429

thrust [*] :=
  0 3.5          41 1.3349       82 0           123 1.45622     164 1.43965
  1 3.5          42 2.28168      83 0           124 3.5         165 3.5
  2 3.5          43 3.5          84 0           125 1.45581     166 1.43965
  3 3.5          44 1.34228      85 0           126 3.5         167 3.5
  4 3.5          45 1.69236      86 0           127 1.43965     168 1.43965
  5 3.5          46 1.49691      87 0           128 3.5         169 3.5
  6 3.5          47 2.2407       88 0           129 1.42348     170 1.43965
```

```
  7 3.5         48 3.20778      89 0          130 3.5         171 3.5
  8 3.5         49 1.26301      90 0          131 2.87976     172 1.43965
  9 3.5         50 1.39813      91 0          132 3.5         173 3.5
 10 3.5         51 1.5272       92 0          133 0           174 1.43965
 11 3.5         52 1.64639      93 0          134 3.5         175 3.5
 12 3.5         53 1.30021      94 0          135 2.25094     176 1.43965
 13 3.5         54 1.93132      95 0          136 3.5         177 3.5
 14 3.5         55 2.75661      96 0          137 1.4452      178 1.44119
 15 3.5         56 0.838163     97 0          138 3.5         179 3.5
 16 3.5         57 1.08458      98 0          139 1.4772      180 1.41404
 17 3.28788     58 0.85419      99 1.3784     140 3.5         181 3.5
 18 2.97229     59 1.55477     100 1.47463    141 1.50307     182 1.46526
 19 2.94042     60 2.37929     101 0          142 3.5         183 3.5
 20 3.08066     61 0.488127    102 3.5        143 1.48408     184 1.41404
 21 2.98236     62 0.720145    103 0          144 3.5         185 3.5
 22 2.96104     63 0.799147    104 3.5        145 0.617861    186 1.46526
 23 2.91711     64 0.613538    105 1.51489    146 3.5         187 3.5
 24 3.09537     65 0.862301    106 3.5        147 1.37581     188 1.41404
 25 2.98926     66 0.776481    107 1.43574    148 3.5         189 3.5
 26 2.82386     67 0.0693569   108 3.5        149 2.27264     190 1.46526
 27 2.89116     68 0           109 1.44562    150 1.42432     191 3.5
 28 2.6908      69 0           110 3.5        151 3.5         192 1.41404
 29 2.63966     70 0           111 1.43775    152 1.43965     193 3.5
 30 2.76135     71 0           112 3.5        153 3.5         194 1.46526
 31 2.57877     72 0           113 1.43896    154 1.43965     195 3.5
 32 2.43336     73 0           114 3.5        155 3.5         196 1.41706
 33 2.39808     74 0           115 1.45601    156 1.43965     197 3.5
 34 2.34049     75 0           116 3.5        157 3.5         198 1.50238
 35 2.51734     76 0           117 1.43965    158 1.42145     199 3.5
 36 2.41113     77 0           118 3.5        159 3.5         200 1.43965
 37 2.29425     78 0           119 1.43965    160 1.45784
 38 2.10179     79 0           120 3.5        161 3.5
 39 2.70279     80 0           121 1.43965    162 1.43965
 40 3.5         81 0           122 3.5        163 3.5
;


MINOS 5.51: numerical error: the general constraints
cannot be satisfied accurately.
1 iterations, objective 1
Nonlin evals: constrs = 1, Jac = 1.

***** Optimal Solution: *****

N = 400
```

```
final_height = 1

t_final = 1

thrust [*] :=
  0           1.75   134           1.75   268           1.75
  1           1.75   135           0.2    269           1.75
  2           1.75   136           1.75   270           1.75
  3           1.75   137           0.2    271           1.75
  4           1.75   138           1.75   272           1.75
  5           1.75   139           0.2    273           1.75
  6           1.75   140           1.75   274           1.75
  7           1.75   141           0.2    275           1.75
  8           1.75   142           1.75   276           1.75
  9           1.75   143           0.2    277           1.75
 10           1.75   144           1.75   278           1.75
 11           1.75   145           0.2    279           1.75
 12           1.75   146           1.75   280           1.75
 13           1.75   147           0.2    281           1.75
 14           1.75   148           1.75   282           1.75
 15           1.75   149         -38.55   283           1.75
 16           1.75   150           0.2    284           1.75
 17    -67108900     151           1.75   285           1.75
 18    134218000     152           0.2    286           1.75
 19   -145752000     153           1.75   287           1.75
 20   -495976000     154           0.2    288           1.75
 21   -115343000     155           1.75   289           1.75
 22    365429000     156           0.2    290           1.75
 23    144703000     157           1.75   291           1.75
 24   -498074000     158           0.2    292           1.75
 25     46137300     159           1.75   293           1.75
 26     70254600     160           0.2    294           1.75
 27   3251630000     161           1.75   295           1.75
 28   1354500000     162           0.2    296           1.75
 29  -3070750000     163           1.75   297           1.75
 30   -524026000     164           0.2    298           1.75
 31    -80216100     165           1.75   299           1.75
 32   -156107000     166           0.2    300           1.75
 33    101253000     167           1.75   301           1.75
 34    118620000     168           0.2    302           1.75
 35   -131596000     169           1.75   303           1.75
 36   -247726000     170           0.2    304           1.75
 37   -128451000     171           1.75   305           1.75
 38    221250000     172           0.2    306           1.75
 39    197132000     173           1.75   307           1.75
```

40	1.75	174	0.2	308	1.75
41	-54526000	175	1.75	309	1.75
42	-268960000	176	0.2	310	1.75
43	1.75	177	1.75	311	1.75
44	134218000	178	0.2	312	1.75
45	46137300	179	1.75	313	1.75
46	-861929000	180	0.2	314	1.75
47	1149240000	181	1.75	315	1.75
48	390070000	182	0.2	316	1.75
49	-1174410000	183	1.75	317	1.75
50	143655000	184	0.2	318	1.75
51	-100663000	185	1.75	319	1.75
52	44040200	186	0.2	320	1.75
53	35651600	187	1.75	321	1.75
54	48234500	188	0.2	322	1.75
55	-12582900	189	1.75	323	1.75
56	-37748700	190	0.2	324	1.75
57	-33554400	191	1.75	325	1.75
58	60817400	192	0.2	326	1.75
59	25165800	193	1.75	327	1.75
60	12582900	194	0.2	328	1.75
61	-50331600	195	1.75	329	1.75
62	-154141000	196	0.2	330	1.75
63	-12582900	197	1.75	331	1.75
64	97517600	198	0.2	332	1.75
65	-301990000	199	1.75	333	1.75
66	-33554400	200	0.2	334	1.75
67	402653000	201	1.75	335	1.75
68	1.75	202	1.75	336	1.75
69	1.75	203	1.75	337	1.75
70	1.75	204	1.75	338	1.75
71	1.75	205	1.75	339	1.75
72	1.75	206	1.75	340	1.75
73	1.75	207	1.75	341	1.75
74	1.75	208	1.75	342	1.75
75	1.75	209	1.75	343	1.75
76	1.75	210	1.75	344	1.75
77	1.75	211	1.75	345	1.75
78	1.75	212	1.75	346	1.75
79	1.75	213	1.75	347	1.75
80	1.75	214	1.75	348	1.75
81	1.75	215	1.75	349	1.75
82	1.75	216	1.75	350	1.75
83	1.75	217	1.75	351	1.75
84	1.75	218	1.75	352	1.75
85	1.75	219	1.75	353	1.75

86	1.75	220	1.75	354	1.75
87	1.75	221	1.75	355	1.75
88	1.75	222	1.75	356	1.75
89	1.75	223	1.75	357	1.75
90	1.75	224	1.75	358	1.75
91	1.75	225	1.75	359	1.75
92	1.75	226	1.75	360	1.75
93	1.75	227	1.75	361	1.75
94	1.75	228	1.75	362	1.75
95	1.75	229	1.75	363	1.75
96	1.75	230	1.75	364	1.75
97	1.75	231	1.75	365	1.75
98	1.75	232	1.75	366	1.75
99	-2617250000	233	1.75	367	1.75
100	0	234	1.75	368	1.75
101	1.75	235	1.75	369	1.75
102	1.75	236	1.75	370	1.75
103	1.75	237	1.75	371	1.75
104	1.75	238	1.75	372	1.75
105	0.2	239	1.75	373	1.75
106	1.75	240	1.75	374	1.75
107	0.2	241	1.75	375	1.75
108	1.75	242	1.75	376	1.75
109	0.2	243	1.75	377	1.75
110	1.75	244	1.75	378	1.75
111	0.2	245	1.75	379	1.75
112	1.75	246	1.75	380	1.75
113	0.2	247	1.75	381	1.75
114	1.75	248	1.75	382	1.75
115	0.2	249	1.75	383	1.75
116	1.75	250	1.75	384	1.75
117	0.2	251	1.75	385	1.75
118	1.75	252	1.75	386	1.75
119	0.2	253	1.75	387	1.75
120	1.75	254	1.75	388	1.75
121	0.2	255	1.75	389	1.75
122	1.75	256	1.75	390	1.75
123	0.2	257	1.75	391	1.75
124	1.75	258	1.75	392	1.75
125	0.2	259	1.75	393	1.75
126	1.75	260	1.75	394	1.75
127	0.2	261	1.75	395	1.75
128	1.75	262	1.75	396	1.75
129	0.2	263	1.75	397	1.75
130	1.75	264	1.75	398	1.75
131	-1.35	265	1.75	399	1.75

```
132             1.75   266             1.75   400             1.75
133             1.75   267             1.75
;


ampl:
```

3.3.5 AMPL Output using Snopt Solver

```
ampl: include cw1.run;

***** Math6120 - Nonlinear Optimization *****
***** Coursework 1 - AMPL Model *****
***** Emma Tarmey, 2940 4045 *****

SNOPT 7.5-1.2 : Optimal solution found.
3794 iterations, objective 1.012809646
Nonlin evals: constrs = 1268, Jac = 1267.

***** Optimal Solution: *****

N = 50

final_height = 1.01281

t_final = 0.198447

thrust [*] :=
 0 3.5        11 2.24978    22 0         33 0         44 0
 1 3.5        12 2.27553    23 0         34 0         45 0
 2 3.5        13 2.35551    24 0         35 0         46 0
 3 3.5        14 2.40926    25 0         36 0         47 0
 4 3.5        15 2.53666    26 0         37 0         48 0
 5 3.4177     16 2.60463    27 0         38 0         49 0
 6 2.82504    17 2.89572    28 0         39 0         50 0
 7 2.13891    18 1.99252    29 0         40 0
 8 2.1463     19 0.414082   30 0         41 0
 9 2.18723    20 0          31 0         42 0
10 2.19231    21 0          32 0         43 0
;


SNOPT 7.5-1.2 : Optimal solution found.
12774 iterations, objective 1.012830134
Nonlin evals: constrs = 2075, Jac = 2074.

***** Optimal Solution: *****

N = 100

final_height = 1.01283
```

```
t_final = 0.198711

thrust [*] :=
  0 3.5         21 2.22675     42 0          63 0          84 0
  1 3.5         22 2.23635     43 0          64 0          85 0
  2 3.5         23 2.26864     44 0          65 0          86 0
  3 3.5         24 2.28387     45 0          66 0          87 0
  4 3.5         25 2.32318     46 0          67 0          88 0
  5 3.5         26 2.34454     47 0          68 0          89 0
  6 3.5         27 2.39434     48 0          69 0          90 0
  7 3.5         28 2.42123     49 0          70 0          91 0
  8 3.5         29 2.48319     50 0          71 0          92 0
  9 3.5         30 2.51938     51 0          72 0          93 0
 10 3.5         31 2.59947     52 0          73 0          94 0
 11 3.5         32 2.6373      53 0          74 0          95 0
 12 2.75944     33 2.74947     54 0          75 0          96 0
 13 2.00095     34 2.77644     55 0          76 0          97 0
 14 2.14186     35 3.00034     56 0          77 0          98 0
 15 2.15593     36 2.42686     57 0          78 0          99 0
 16 2.15366     37 0.754754    58 0          79 0         100 0
 17 2.17234     38 0           59 0          80 0
 18 2.17329     39 0           60 0          81 0
 19 2.19506     40 0           61 0          82 0
 20 2.20023     41 0           62 0          83 0
;


SNOPT 7.5-1.2 : Optimal solution found.
6482 iterations, objective 1.01283523
Nonlin evals: constrs = 60, Jac = 59.

***** Optimal Solution: *****

N = 200

final_height = 1.01284

t_final = 0.198836

thrust [*] :=
  0 3.5            51 2.34124       102 0             153 0
  1 3.5            52 2.34998       103 0             154 0
  2 3.5            53 2.36167       104 0             155 0
  3 3.5            54 2.38867       105 0             156 0
  4 3.5            55 2.4058        106 0             157 0
```

5	3.5
6	3.5
7	3.5
8	3.5
9	3.5
10	3.5
11	3.5
12	3.5
13	3.5
14	3.5
15	3.5
16	3.5
17	3.5
18	3.5
19	3.5
20	3.5
21	3.5
22	3.5
23	3.26558
24	2.59375
25	2.11924
26	2.13997
27	2.14131
28	2.14352
29	2.15712
30	2.15006
31	2.15959
32	2.15661
33	2.16051
34	2.16553
35	2.17099
36	2.17755
37	2.18728
38	2.18999
39	2.20205
40	2.20428
41	2.21185
42	2.22146
43	2.22717
44	2.23995
45	2.25454
46	2.26164
47	2.28701
48	2.28747
49	2.3027
50	2.31532
56	2.4315
57	2.47809
58	2.47033
59	2.49768
60	2.5193
61	2.57333
62	2.58089
63	2.61321
64	2.6689
65	2.70271
66	2.74345
67	2.80133
68	2.74992
69	2.87216
70	2.93535
71	2.92572
72	3.10831
73	1.6655
74	0.141673
75	0
76	3.86795e-14
77	0
78	0
79	0
80	0
81	0
82	0
83	0
84	0
85	0
86	0
87	0
88	0
89	0
90	0
91	0
92	0
93	0
94	0
95	0
96	0
97	0
98	0
99	0
100	0
101	0
107	0
108	0
109	0
110	0
111	0
112	0
113	0
114	0
115	0
116	0
117	0
118	0
119	0
120	0
121	0
122	0
123	0
124	0
125	0
126	0
127	0
128	0
129	0
130	0
131	0
132	0
133	0
134	0
135	0
136	0
137	0
138	0
139	0
140	0
141	0
142	0
143	0
144	0
145	0
146	0
147	0
148	0
149	0
150	0
151	0
152	0
158	0
159	0
160	0
161	0
162	0
163	0
164	0
165	0
166	0
167	0
168	0
169	0
170	0
171	0
172	0
173	0
174	0
175	0
176	0
177	0
178	0
179	0
180	0
181	0
182	0
183	0
184	0
185	0
186	0
187	0
188	0
189	0
190	0
191	0
192	0
193	0
194	0
195	0
196	0
197	0
198	0
199	0
200	0

```
;



SNOPT 7.5-1.2 : Optimal solution found.
78851 iterations, objective 1.012832699
Nonlin evals: constrs = 16710, Jac = 16709.

***** Optimal Solution: *****

N = 400

final_height = 1.01283

t_final = 0.198603

thrust [*] :=
  0 3.5        81 3.5       162 0        243 0        324 0
  1 3.5        82 3.5       163 0        244 0        325 0
  2 3.5        83 3.5       164 0        245 0        326 0
  3 3.5        84 3.5       165 0        246 0        327 0
  4 3.5        85 0         166 0        247 0        328 0
  5 3.5        86 0         167 0        248 0        329 0
  6 3.5        87 3.5       168 0        249 0        330 0
  7 3.5        88 0         169 0        250 0        331 0
  8 3.5        89 3.5       170 0        251 0        332 0
  9 3.5        90 3.5       171 0        252 0        333 0
 10 3.5        91 0         172 0        253 0        334 0
 11 3.5        92 3.5       173 0        254 0        335 0
 12 3.5        93 3.5       174 0        255 0        336 0
 13 3.5        94 0         175 0        256 0        337 0
 14 3.5        95 0         176 0        257 0        338 0
 15 3.5        96 3.5       177 0        258 0        339 0
 16 3.5        97 3.5       178 0        259 0        340 0
 17 3.5        98 0         179 0        260 0        341 0
 18 3.5        99 3.5       180 0        261 0        342 0
 19 3.5       100 3.5       181 0        262 0        343 0
 20 3.5       101 0         182 0        263 0        344 0
 21 3.5       102 3.5       183 0        264 0        345 0
 22 3.5       103 3.5       184 0        265 0        346 0
 23 3.5       104 0         185 0        266 0        347 0
 24 3.5       105 0         186 0        267 0        348 0
 25 3.5       106 3.5       187 0        268 0        349 0
 26 3.5       107 3.5       188 0        269 0        350 0
 27 3.5       108 3.5       189 0        270 0        351 0
 28 3.5       109 3.5       190 0        271 0        352 0
```

29	3.5	110	3.5	191	0	272	0	353	0
30	3.5	111	3.5	192	0	273	0	354	0
31	3.5	112	0	193	0	274	0	355	0
32	3.5	113	3.5	194	0	275	0	356	0
33	3.5	114	3.5	195	0	276	0	357	0
34	3.5	115	0	196	0	277	0	358	0
35	3.5	116	3.5	197	0	278	0	359	0
36	3.5	117	3.5	198	0	279	0	360	0
37	3.5	118	0	199	0	280	0	361	0
38	3.5	119	3.5	200	0	281	0	362	0
39	3.5	120	3.5	201	0	282	0	363	0
40	3.5	121	0	202	0	283	0	364	0
41	3.5	122	2.19679	203	0	284	0	365	0
42	3.5	123	3.5	204	0	285	0	366	0
43	3.5	124	3.5	205	0	286	0	367	0
44	3.5	125	0	206	0	287	0	368	0
45	3.5	126	3.5	207	0	288	0	369	0
46	1.4597	127	3.5	208	0	289	0	370	0
47	3.5	128	0	209	0	290	0	371	0
48	3.5	129	3.5	210	0	291	0	372	0
49	0	130	3.5	211	0	292	0	373	0
50	3.5	131	3.5	212	0	293	0	374	0
51	3.5	132	3.5	213	0	294	0	375	0
52	0	133	3.5	214	0	295	0	376	0
53	3.5	134	0	215	0	296	0	377	0
54	3.5	135	0	216	0	297	0	378	0
55	0	136	3.5	217	0	298	0	379	0
56	3.5	137	3.5	218	0	299	0	380	0
57	3.5	138	3.5	219	0	300	0	381	0
58	0	139	3.5	220	0	301	0	382	0
59	0	140	3.5	221	0	302	0	383	0
60	3.5	141	3.5	222	0	303	0	384	0
61	3.5	142	3.5	223	0	304	0	385	0
62	0	143	0	224	0	305	0	386	0
63	3.5	144	3.5	225	0	306	0	387	0
64	3.5	145	3.5	226	0	307	0	388	0
65	0	146	3.5	227	0	308	0	389	0
66	0	147	3.5	228	0	309	0	390	0
67	3.5	148	0	229	0	310	0	391	0
68	3.5	149	0.934727	230	0	311	0	392	0
69	0	150	0	231	0	312	0	393	0
70	3.5	151	0	232	0	313	0	394	0
71	3.5	152	0	233	0	314	0	395	0
72	3.5	153	0	234	0	315	0	396	0
73	0.971764	154	0	235	0	316	0	397	0
74	3.5	155	0	236	0	317	0	398	0

```
 75 3.5         156 0          237 0          318 0          399 0
 76 0           157 0          238 0          319 0          400 0
 77 0           158 0          239 0          320 0
 78 3.5         159 0          240 0          321 0
 79 3.5         160 0          241 0          322 0
 80 0           161 0          242 0          323 0
;


ampl:
```

3.4 Acknowledgements

- The Python tutorial available at:
`https://blog.finxter.com/how-to-loop-through-a-python-list-in-batches/`
was consulted towards the goal of implementing a non-overlapping window algorithm for our text-parser.

- The AMPL Guide available at:
`https://ampl.com/resources/the-ampl-book/` was consulted to help learning the AMPL language and the features available in the AMPLIDE environment.

- The AMPL forum available at:
`https://groups.google.com/g/ampl` was consulted to help with understanding AMPL syntax, implementing our model and de-bugging our code.

- The advice available at:
`https://stackoverflow.com/questions/tagged/ampl` was consulted to help with de-bugging our code.