Axicon Gaussian Laser Beams

Kirk T. McDonald, arXiv:0003056v2 (2000)

Notebook: Óscar Amaro, December 2022 @ GoLP-EPP

Introduction

In this notebook we reproduce some results from the paper.

axicon solution for a Gaussian laser beam in vacuum, i.e., a beam with radial polarization of the electric field.

Figure 2

The electric field Ex(x, 0, z) of a linearly polarized Gaussian beam with diffraction angle θ0 = 0.45, according to eq. (27).

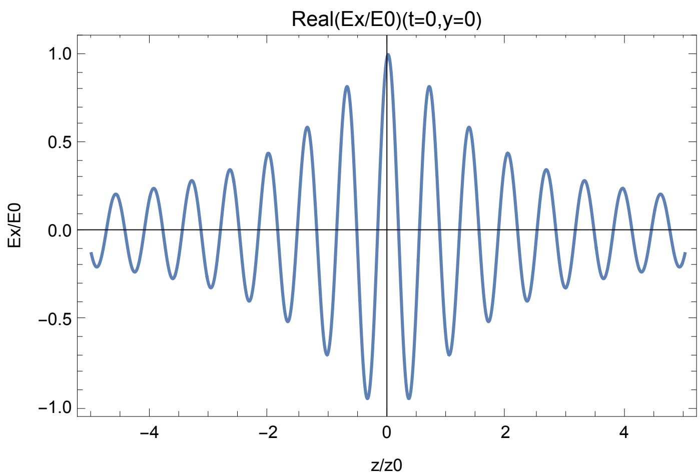
Equation 27 is the definition of W(z) spotsize. Instead, eq 25 is being plotted.

In[313]:=
```
Clear[Ex, E0, rprp, W, W0, g, ϕ, λ, W0, zR, k, z0, x, y, z, ω, t, θ0]
```

In[314]:=
```
rprp = Sqrt[x^2 + y^2]; (**)
W = W0 Sqrt[1 + z^2 / z0^2]; (**)
z0 = π W0^2 / λ; (* Rayleigh range *)
k = 2 π / λ;
W0 = 2 / (k θ0); (* θ0=2/(k W0) equation 11 *)

(* parameters *)
θ0 = 0.45; (**)
λ = 1; (*[μm]*)
g = 1; (* envelope equation...*)
t = 0;

(* equation 25 *)
Ex = E0 Exp[- rprp^2/W^2] / Sqrt[1 + z^2 / z0^2] g Exp[I (k z (1 + rprp^2/(2 (z^2 + z0^2))) - ω t - ArcTan[z/z0])]

(* lineout *)
Plot[Re[Ex/E0] //. {y → 0, z → zz0 z0, x → 0}, {zz0, -5, +5}, Frame → True,
 FrameLabel → {"z/z0", "Ex/E0"}, PlotLabel → "Real(Ex/E0)(t=0,y=0)"]

(* figure 2 *)
Plot3D[Re[Ex/E0] //. {y → 0, z → zz0 z0, x → xW0 W0}, {zz0, -5, +5},
 {xW0, -10, +10}, PlotRange → All, PlotPoints → 50, Mesh → None,
 AxesLabel → {"z/z0", "x/W0", "Ex/E0"}, PlotLabel → "Real(Ex/E0)(t=0,y=0)"]
```

Out[316]=
$$\frac{e^{-\frac{1.99859\,(x^2+y^2)}{1+0.404715\,z^2}+i\left(2\pi z\left(1+\frac{x^2+y^2}{2\times(2.47087+z^2)}\right)-\mathrm{ArcTan}[0.636173\,z]\right)}\,E0}{\sqrt{1+0.404715\,z^2}}$$

Out[317]=



Out[318]=

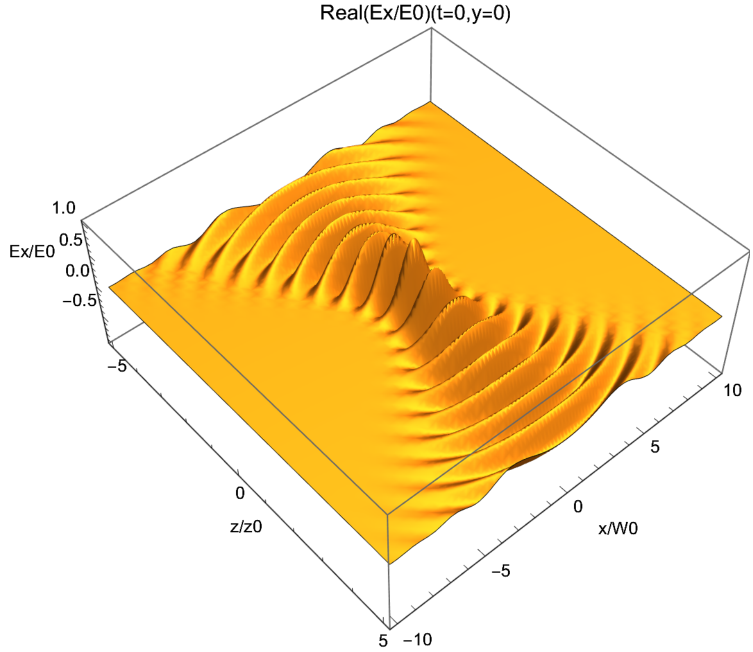


Figure 3

Same as previous figure, but for Ez component

In[319]:=
```
Clear[Ex, E0, rprp, W, W0, g, ϕ, λ, W0, zR, k, z0, x, y, z, ω, t, θ0, Ez, ξ, ξ, f]
```

In[320]:=
```
rprp = Sqrt[x^2 + y^2]; (**)
W = W0 Sqrt[1 + z^2 / z0^2]; (**)
z0 = π W0^2 / λ; (* Rayleigh range *)
k = 2 π / λ;
W0 = 2 / (k θ0); (* θ0=2/(k W0) equation 11 *)

(* parameters *)
θ0 = 0.45; (**)
λ = 1; (*[μm]*)
g = 1; (* envelope equation...*)
t = 0;

(* eq 12 *)
ξ = x / W0;
ζ = z / z0;

(* eq 19 *)
f = 1/(1 + I ζ);

(* equation 25 *)
Ex = E0 Exp[-rprp^2/W^2]/Sqrt[1 + z^2 / z0^2] g Exp[I (k z (1 + rprp^2/(2 (z^2 + z0^2))) - ω t - ArcTan[z/z0])];

Ez = -I θ0 f ξ Ex;

(* figure 3 *)
Plot3D[Re[Ez/E0] //. {y → 0, z → zz0 z0, x → xW0 W0}, {zz0, -5, +5},
 {xW0, -10, +10}, PlotRange → All, PlotPoints → 50, Mesh → None,
 AxesLabel → {"z/z0", "x/W0", "Ez/E0"}, PlotLabel → "Real(Ez/E0)(t=0,y=0)"]
```

Out[327]=

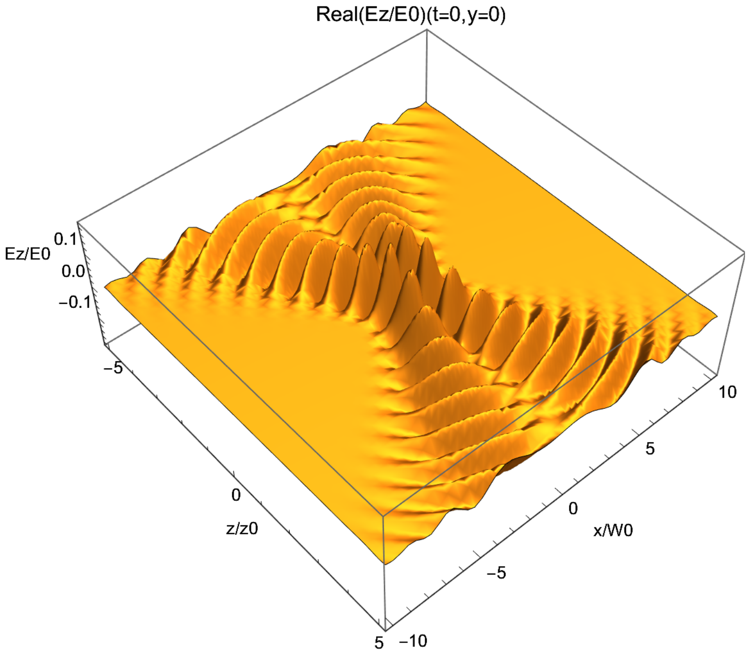


Figure 4

Lowest-Order Axicon Beam: Er

In[338]:=
```
Clear[Ex, E0, rprp, W, W0, g, ϕ, λ, W0,
 zR, k, z0, x, y, z, ω, t, θ0, Ez, ξ, ξ, f, ρ, Eprp]
```

In[339]:=
```
rprp = Sqrt[x^2 + y^2]; (**)
ρ = rprp / W0; (* eq 12 *)
W = W0 Sqrt[1 + z^2 / z0^2]; (**)
z0 = π W0^2 / λ; (* Rayleigh range *)
k = 2 π / λ;
W0 = 2 / (k θ0); (* θ0=2/(k W0) equation 11 *)

(* parameters *)
θ0 = 0.45; (**)
λ = 1; (*[μm]*)
g = 1; (* envelope equation...*)
t = 0;

(* eq 12 *)
ξ = x / W0;
ς = z / z0;

ϕ = k z + ω t;

(* eq 19 *)
f = 1 / (1 + I ς);

(* eq 32 *)
Eprp = E0 ρ f^2 Exp[-f ρ^2] g Exp[I ϕ];

(* figure 4 *)
Plot3D[Re[Eprp / E0] //. {y → 0, z → zz0 z0, x → xW0 W0}, {zz0, -5, +5},
 {xW0, -10, +10}, PlotRange → All, PlotPoints → 50, Mesh → None,
 AxesLabel → {"z/z0", "x/W0", "E⊥/E0"}, PlotLabel → "Real(E⊥/E0)(t=0,y=0)"]
```

Out[346]=

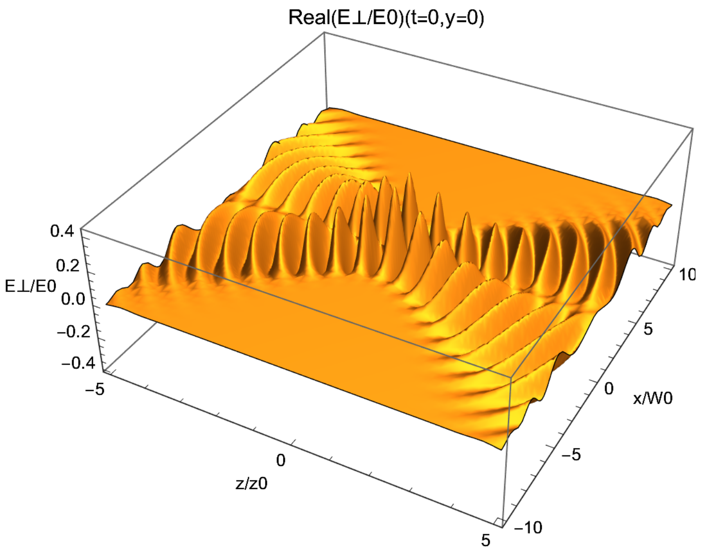


Figure 5

Lowest-Order Axicon Beam: Ez

In[348]:=
```
Clear[Ex, E0, rprp, W, W0, g, ϕ, λ, W0,
  zR, k, z0, x, y, z, ω, t, θ0, Ez, ξ, ξ, f, ρ, Eprp]
```

In[349]:=
```
rprp = Sqrt[x^2 + y^2]; (**)
ρ = rprp / W0; (* eq 12 *)
W = W0 Sqrt[1 + z^2 / z0^2]; (**)
z0 = π W0^2 / λ; (* Rayleigh range *)
k = 2 π / λ;
W0 = 2 / (k θ0); (* θ0=2/(k W0) equation 11 *)

(* parameters *)
θ0 = 0.45; (**)
λ = 1; (*[μm]*)
g = 1; (* envelope equation...*)
t = 0;

(* eq 12 *)
ξ = x / W0;
ς = z / z0;

ϕ = k z + ω t;

(* eq 19 *)
f = 1/(1 + I ς);

(* eq 32 *)
Ez = I θ0 E0 f^2 (1 - f ρ^2) Exp[-f ρ^2] g Exp[I ϕ];

(* figure 5 *)
Plot3D[Re[Ez/E0] //. {y → 0, z → zz0 z0, x → xW0 W0}, {zz0, -5, +5},
 {xW0, -10, +10}, PlotRange → All, PlotPoints → 50, Mesh → None,
 AxesLabel → {"z/z0", "x/W0", "Ez/E0"}, PlotLabel → "Real(Ez/E0)(t=0,y=0)"]
```

Out[356]=