Figure 3

```
In[•]:= Clear[Id, RY, θ]
        Id = IdentityMatrix[2];
        RY[θ_] := {{Cos[θ / 2], -Sin[θ / 2]}, {Sin[θ / 2], Cos[θ / 2]}}

        RY[θ1].{1, 0}
        KroneckerProduct[RY[θ1], Id].Flatten[KroneckerProduct[{1, 0}, {1, 0}]]

        KroneckerProduct[RY[θ1], RY[θ2]].Flatten[KroneckerProduct[{1, 0}, {1, 0}]]
```

$$\text{Out[•]=}\ \left\{\text{Cos}\left[\frac{\theta 1}{2}\right], \text{Sin}\left[\frac{\theta 1}{2}\right]\right\}$$

$$\text{Out[•]=}\ \left\{\text{Cos}\left[\frac{\theta 1}{2}\right], 0, \text{Sin}\left[\frac{\theta 1}{2}\right], 0\right\}$$

$$\text{Out[•]=}\ \left\{\text{Cos}\left[\frac{\theta 1}{2}\right]\text{Cos}\left[\frac{\theta 2}{2}\right], \text{Cos}\left[\frac{\theta 1}{2}\right]\text{Sin}\left[\frac{\theta 2}{2}\right], \text{Cos}\left[\frac{\theta 2}{2}\right]\text{Sin}\left[\frac{\theta 1}{2}\right], \text{Sin}\left[\frac{\theta 1}{2}\right]\text{Sin}\left[\frac{\theta 2}{2}\right]\right\}$$

Figure 4

```
In[•]:= Clear[Id, RY, θ, res]
        Id = IdentityMatrix[2];
        RY[θ_] := {{Cos[θ / 2], -Sin[θ / 2]}, {Sin[θ / 2], Cos[θ / 2]}}

        res = KroneckerProduct[RY[θ1], RY[θ2]].Flatten[KroneckerProduct[{1, 0}, {1, 0}]]
        GraphicsRow[{BarChart[res /. {θ1 → π / 2, θ2 → π / 2}, ImageSize → Small],
          BarChart[res /. {θ1 → π / 2, θ2 → 5 π / 2}, ImageSize → Small],
          BarChart[res /. {θ1 → 3 π / 2, θ2 → 3 π / 2}, ImageSize → Small],
          BarChart[res /. {θ1 → -π / 2, θ2 → 3 π / 2}, ImageSize → Small]}, Spacings → 20]
```

$$\text{Out[•]=}\ \left\{\text{Cos}\left[\frac{\theta 1}{2}\right]\text{Cos}\left[\frac{\theta 2}{2}\right], \text{Cos}\left[\frac{\theta 1}{2}\right]\text{Sin}\left[\frac{\theta 2}{2}\right], \text{Cos}\left[\frac{\theta 2}{2}\right]\text{Sin}\left[\frac{\theta 1}{2}\right], \text{Sin}\left[\frac{\theta 1}{2}\right]\text{Sin}\left[\frac{\theta 2}{2}\right]\right\}$$

Out[•]=

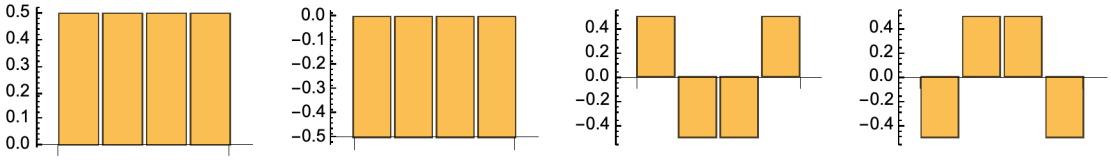
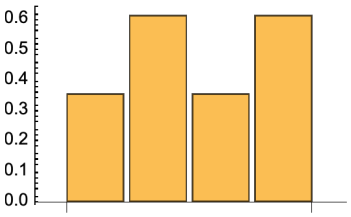


Figure 5

```
In[•]:= Clear[Id, RY, θ, res, CNOT]
        Id = IdentityMatrix[2];
        RY[θ_] := {{Cos[θ / 2], -Sin[θ / 2]}, {Sin[θ / 2], Cos[θ / 2]}}

        CNOT = IdentityMatrix[4];
        u = CNOT[[3, ;;]]; CNOT[[3, ;;]] = CNOT[[4, ;;]]; CNOT[[4, ;;]] = u;
        CNOT // MatrixForm;

        res = KroneckerProduct[RY[θ1], RY[θ2]].Flatten[KroneckerProduct[{1, 0}, {1, 0}]]
        BarChart[res /. {θ1 → π / 2, θ2 → 2 π / 3}, ImageSize → Small]

        res = CNOT.KroneckerProduct[RY[θ1], RY[θ2]].
          Flatten[KroneckerProduct[{1, 0}, {1, 0}]]
        BarChart[res /. {θ1 → π / 2, θ2 → 2 π / 3}, ImageSize → Small]
```

$$\text{Out[•]=}\ \left\{\text{Cos}\left[\frac{\theta 1}{2}\right]\text{Cos}\left[\frac{\theta 2}{2}\right], \text{Cos}\left[\frac{\theta 1}{2}\right]\text{Sin}\left[\frac{\theta 2}{2}\right], \text{Cos}\left[\frac{\theta 2}{2}\right]\text{Sin}\left[\frac{\theta 1}{2}\right], \text{Sin}\left[\frac{\theta 1}{2}\right]\text{Sin}\left[\frac{\theta 2}{2}\right]\right\}$$

Out[•]=



$$\text{Out[•]=}\ \left\{\text{Cos}\left[\frac{\theta 1}{2}\right]\text{Cos}\left[\frac{\theta 2}{2}\right], \text{Cos}\left[\frac{\theta 1}{2}\right]\text{Sin}\left[\frac{\theta 2}{2}\right], \text{Sin}\left[\frac{\theta 1}{2}\right]\text{Sin}\left[\frac{\theta 2}{2}\right], \text{Cos}\left[\frac{\theta 2}{2}\right]\text{Sin}\left[\frac{\theta 1}{2}\right]\right\}$$

Out[•]=

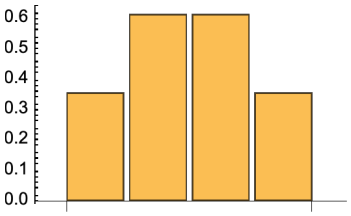


Figure 6

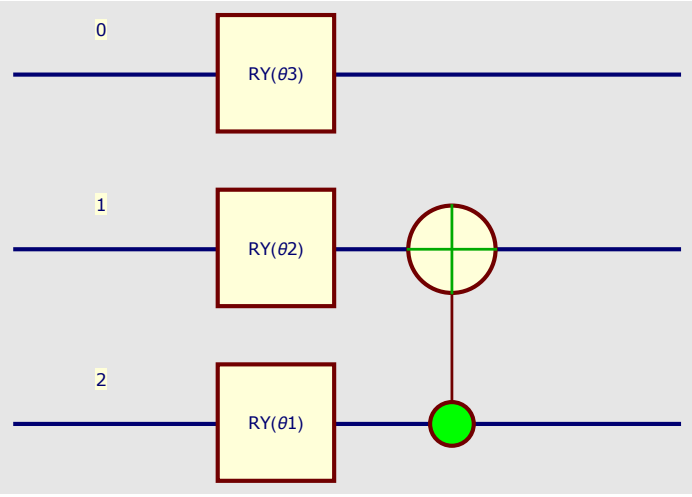
```
        (* import package to facilitate operations on >2 qubits *)
        Needs["Quantum`Computing`"];
```

$$\text{In[•]:=}\ \text{MatrixToDirac}\left[\left\{\left\{\text{Cos}\left[\frac{\theta}{2}\right], -\text{Sin}\left[\frac{\theta}{2}\right]\right\}, \left\{\text{Sin}\left[\frac{\theta}{2}\right], \text{Cos}\left[\frac{\theta}{2}\right]\right\}\right\}, \{2\}\right]$$

$$\text{Out[•]=}\ \text{Cos}\left[\frac{\theta}{2}\right]\ |0_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}}| + \text{Sin}\left[\frac{\theta}{2}\right]\ |1_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}}| - \text{Sin}\left[\frac{\theta}{2}\right]\ |0_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}}| + \text{Cos}\left[\frac{\theta}{2}\right]\ |1_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}}|$$

In[•]:=
```
Clear[Δt, I2, Z, RZ, U1, U2, rot, psi, θ1, θ2, θ3, circ, circ1, circ2]
I2 = PauliMatrix[0];
Z = PauliMatrix[3];

(* define RY rotation *)
SetQuantumGate[RY, 1,
  Function[{q1},
   Function[{θ},
    Cos[θ/2] |0_q1^⟩ · ⟨0_q1^| + Sin[θ/2] |1_q1^⟩ · ⟨0_q1^| -
     Sin[θ/2] |0_q1^⟩ · ⟨1_q1^| + Cos[θ/2] |1_q1^⟩ · ⟨1_q1^| ] ] ];
QuantumTableForm[RY_1^[Δt]];
QuantumMatrixForm[RY_1^[Δt]];
QuantumMatrixForm[C^{3^}[NOT_4^]];

(* build circuit *)
circ1 = C^{2^}[NOT_1^] · RY_0^[θ3] · RY_1^[θ2] · RY_2^[θ1];
QuantumPlot[circ1]
U1 = QuantumMatrix[circ1];

(* build circuit *)
circ2 = C^{2^}[NOT_0^] · C^{2^}[NOT_1^] · RY_0^[θ3] · RY_1^[θ2] · RY_2^[θ1];
QuantumPlot[circ2]
U2 = QuantumMatrix[circ2];
```

Out[•]=




```
BarChart[(U1.{1, 0, 0, 0, 0, 0, 0}) /. {θ1 → π / 2, θ2 → 2 π / 3, θ3 → 100 / 180 π},
 ImageSize → Small]
BarChart[(U2.{1, 0, 0, 0, 0, 0, 0}) /. {θ1 → π / 2, θ2 → 2 π / 3, θ3 → 100 / 180 π},
 ImageSize → Small]
```