

High quality beam produced by tightly focused laser driven wake-field accelerators

Original authors of paper: Jia Wang, Ming Zeng, Dazhang Li, Xiaoning Wang, and Jie Gao

e-print: <https://arxiv.org/abs/2304.10730> (2023)

Notebook: Óscar Amaro, September 2023 @ [GoLP-EPP](#)

Equation 5: bubble radius


```
In[ ]:= Clear[Fpr, F, α, Ω, γavg, ωp, k, c, κ2, res, r, W, W0]
```

```
α = Ω γavg;
```

```
ωp = k c;
```

```
κ2 = ωp^2 me / α;
```

```
res = Solve[-κ2 r ==  $\frac{me c^2 a_0^2 W_0^2 r}{W^4 \gamma_{avg}} \text{Exp}[-2 r^2 / W^2]$ , r][[3, 1, 2]]
```

 **Solve:** Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

```
Out[ ]:= 
$$\frac{i W \sqrt{\text{Log}\left[-\frac{k^2 W^4}{a_0^2 W_0^2 \Omega}\right]}}{\sqrt{2}}$$

```

Figure 7 and 8

```

In[ ]:= Clear[Linj, zRe,  $\Omega$ , a0, kp, W0,  $\Gamma$ , Q, C,  $\lambda$ , kp,  $\omega$ p, c, e,  $\epsilon$ 0, me, npcmlst, a0lst, npcmlst]

me = 9.1  $\times 10^{-31}$ ; (* [Kg] *)
c = 3  $\times 10^8$ ; (* [m/s] *)
 $\omega$ p = Sqrt[ $\frac{np e^2}{me \epsilon_0}$ ]; (* [1/s] *)
kp =  $\omega$ p / c; (* [1/m] *)
e = 1.6  $\times 10^{-19}$ ; (* [C] *)
 $\epsilon$ 0 = 8.854  $\times 10^{-12}$ ; (* [F/m] *)

 $\lambda$  = 0.8; (* [ $\mu$ m] see §IV *)
 $\Omega$  = 2; (* see §IV *)
zRe =  $\pi W_0^2 \Gamma^2 / \lambda$ ; (* after eq 2 *)
Linj = zRe Sqrt[Exp[-1]  $\frac{\sqrt{\Omega} a_0}{\Gamma^2 kp W_0} - 1$ ]; (*eq 7*)
 $\Gamma = \frac{-np}{20.16} + \frac{a_0}{100} - \frac{W_0}{46.42} + 1.029$ ; (*eq 9 *)
Q = C Linj np a0 (*eq 8 *)

C = 8  $\times 10^{-8}$ ; (* this value, although not explicit in the text,
seems to reproduce results of figures 7 and 8 *)
np = 10-12 npcmlst;

(* Figure 7 *)
a0lst = {4.9, 8.45, 12, 16.97};
GraphicsRow[
  Table[LogPlot[109 Q /. {W0 → 5, a0 → a0lst[[i]]}, {npcmlst, 2, 8.5}, AspectRatio → 1.5,
    ImageSize → 150, Frame → True, PlotRange → {10-2, 101}, PlotStyle → Blue,
    FrameLabel → {"np[1018 cm-3", "Q[nC]"}], {i, 1, 4}]]

npcmlst = {2, 4, 6, 8};
GraphicsRow[Table[LogPlot[109 Q /. {W0 → 5, npcmlst → npcmlst[[i]]}, {a0, 5, 35},
  AspectRatio → 1.5, ImageSize → 150, Frame → True, PlotRange → {10-2, 101},
  PlotStyle → Blue, FrameLabel → {"a0", "Q[nC]"}], {i, 1, 4}]]

(* Figure 8 *)
a0lst = {12, 10, 8, 6};
npcmlst = {2, 4, 6};
GraphicsGrid[Table[Plot[109 Q /. {a0 → a0lst[[i]], npcmlst → npcmlst[[j]]}, {W0, 3, 9},
  AspectRatio → 1, ImageSize → 150, Frame → True, PlotRange → {0, 2},
  PlotStyle → Blue, FrameLabel → {"a0", "Q[nC]"}], {j, 1, 3}, {i, 1, 4}]
]

```

Out[]= $3.92699 \times 10^{-12} a_0 \text{ npcm}$

$$\sqrt{-1 + \frac{2.76893 \times 10^{12} a_0}{\sqrt{\text{npcm}} \left(1.029 + \frac{a_0}{100} - 4.96032 \times 10^{-14} \text{npcm} - 0.0215424 W_0 \right)^2 W_0}} \left(1.029 + \frac{a_0}{100} - 4.96032 \times 10^{-14} \text{npcm} - 0.0215424 W_0 \right)^2 W_0^2 C$$

