Numerical Simulation of Quantum Field Fluctuations

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Notebook: Óscar Amaro, December 2023

Introduction

In this notebook we reproduce some results from the paper.
Main idea: imposition of correlation function on RNG, with application in quantum field fluctuations.

Figure 1

In[670]:=
```
Clear[f, a, b, t0, x, seq, Ct0lst, npoints, nsteps, i, x1, x2]

npoints = 30;(*801;*)
nsteps = 30 000; (*20000;*)
seq = Table[0, {i, 1, nsteps}];
Ct0lst = Table[0, {i, 1, npoints}];

f = 1/b ArcTan[(4 - t0^2)/(4 π^2 a (4 + t0^2)^2)]; (*eq 3.4 typo, should π^2*)
a = 0.01404;
b = 1.58;
t0lst = Table[7 / npoints * j, {j, 0, npoints - 1}];
For[j = 1, j < npoints, j++,
 t0 = t0lst[[j]];
 x = RandomVariate[NormalDistribution[0, 1]];
 seq[[1]] = x;
 For[i = 1, i < nsteps, i++,
  x = RandomVariate[NormalDistribution[0, 1]] + x f;
  (* if you take out the -x f term in the mean,
  an uncorrelated bi-gaussian could of points will be produced *)
  seq[[i + 1]] = x;
  (*Print[i];*)
 ];
 x1 = seq[[ ;; -2]];
 x2 = seq[[2 ;;]];
 Ct0lst[[j]] = Correlation[x1, x2];
 (*ListPlot[Transpose[{x1,x2}],AspectRatio→1];
 Mean[x1 x2];
 CorrelationFunction[seq,1];
 *)
]
```

```
Clear[eq210, t0, f]
eq210 = (4 - t0^2)/(4 π^2 (t0^2 + 4)^2)
f = 1/b ArcTan[(4 - t0^2)/(4 π^2 a (4 + t0^2)^2)];
a = 0.01404;
b = 1.58;
a Tan[b f] /. {t0 → 0}
eq210 /. {t0 → 0}
Show[{
  Plot[{eq210, a Tan[b f]}, {t0, 0, 7},
   PlotStyle → {Red, {Dashed, Black}}, AxesLabel → {"t0", "C(t0)"}],
  ListPlot[Transpose[{t0lst, Ct0lst / Ct0lst[[1]] * 1/(16 π^2)}], Joined → False]}]
(* red - eq 2.10, dashed black - C(f(t0)), dots - sampled *)
```

Out[679]= $\frac{4 - \text{t0}^2}{4\,\pi^2\,\left(4 + \text{t0}^2\right)^2}$

Out[683]= 0.00633257

Out[684]= $\frac{1}{16\,\pi^2}$

Out[685]=

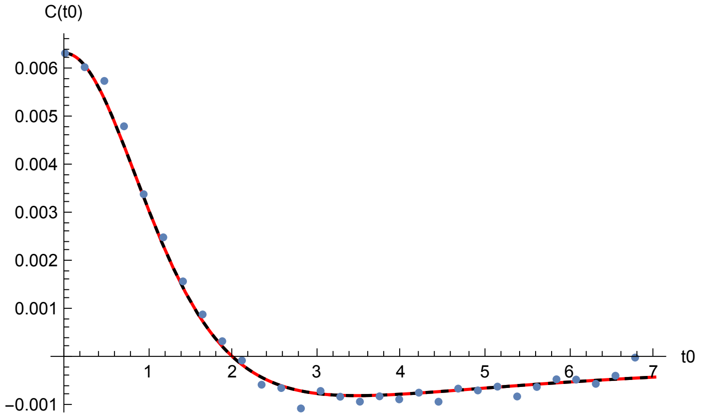


Figure 2

In[806]:=
```
Clear[f, a, b, t0, x, seq, Kt0lst, npoints, nsteps, i, x1, x2]

npoints = 70; (*801;*)
nsteps = 30 000; (*20000;*)
seq = Table[0, {i, 1, nsteps}];
Kt0lst = Table[0, {i, 1, npoints}];

f = 1/b ArcTan[(1 - 6 t0^2 + t0^4)/(a (1 + t0^2)^4)];
a = 0.672;
b = 1.59;
t0lst = Table[5 / npoints * j, {j, 0, npoints - 1}];
For[j = 1, j < npoints, j++,
 t0 = t0lst[[j]];
 x = RandomVariate[NormalDistribution[0, 1]];
 seq[[1]] = x;
 For[i = 1, i < nsteps, i++,
  x = RandomVariate[NormalDistribution[0, 1]] + x f;
  seq[[i + 1]] = x;
 ];
 x1 = seq[[ ;; -2]];
 x2 = seq[[2 ;;]];
 Kt0lst[[j]] = Correlation[x1, x2];
]
```

In[814]:=
```
Clear[eq213, t0, a, b]
eq213 = (1 - 6 t0^2 + t0^4)/((1 + t0^2)^4)
f = 1/b ArcTan[(1 - 6 t0^2 + t0^4)/(a (1 + t0^2)^4)];
a = 0.672;
b = 1.59;
Show[{
  Plot[{eq213, a Tan[b f]}, {t0, 0, 5}, PlotStyle → {Red, {Dashed, Black}},
   AxesLabel → {"t0", "K(t0)"}, PlotRange → {-0.4, 0.2}],
  ListPlot[Transpose[{t0lst, Kt0lst}], Joined → False]}]
(* red - eq 2.13, dashed black - K(f(t0)), dots - sampled *)
```

Out[815]=
$$\frac{1 - 6\,\text{t0}^2 + \text{t0}^4}{\left(1 + \text{t0}^2\right)^4}$$

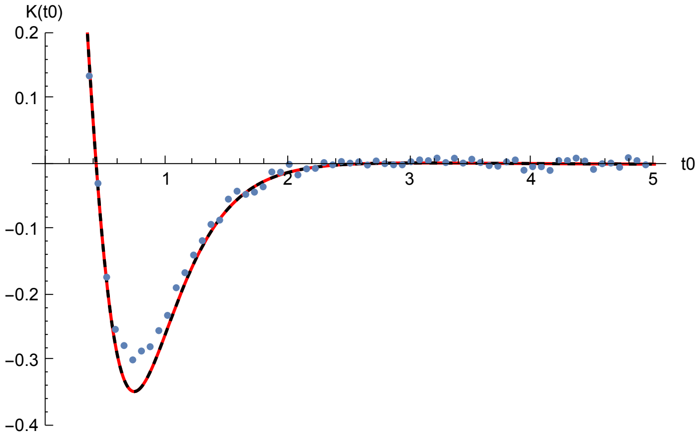
Out[819]=


Figure 3

In[893]:=

```
Clear[f, a, b, t0, x, seq, C1t0lst, npoints, nsteps, i, x1, x2]

npoints = 70; (*801;*)
nsteps = 30 000; (*20000;*)
seq = Table[0, {i, 1, nsteps}];
C1t0lst = Table[0, {i, 1, npoints}];

f = 1/b ArcTan[Cos[t0]/a];
a = 0.672;
b = 1.59;
t0lst  = Table[4 π / npoints * j, {j, 0, npoints - 1}];
For[j = 1, j < npoints, j++,
 t0 = t0lst[[j]];
 x = RandomVariate[NormalDistribution[0, 1]];
 seq[[1]] = x;
 For[i = 1, i < nsteps, i++,
  x = RandomVariate[NormalDistribution[0, 1]] + x f;
  seq[[i + 1]] = x;
 ];
 x1  = seq[[ ;; -2]];
 x2  =  seq[[2 ;;]];
 C1t0lst[[j]] = Correlation[x1, x2];
]
```

In[916]:=
```
Clear[t, k, t0]
f = 1/b ArcTan[Cos[t0]/a];
a = 0.672;
b = 1.59;

Show[{Plot[{Cos[ t0], a Tan[b f]}, {t0, 0, 4 π}, AxesLabel → {"t0", "C1(t0)"},
    PlotStyle → {Red, {Dashed, Black}}, PlotRange → {-1.05, +1.05}]
  , ListPlot[Transpose[{t0lst, C1t0lst / C1t0lst[[1]]}], Joined → False]}]
(* red - eq 2.13, dashed black - C1(f(t0)), dots - sampled *)
```

Out[920]=