Hydrodynamical model of QED cascade expansion in an extremely strong laser pulse

Paper: Samsonov et al, Matter Radiat. Extremes 6, 034401 (2021); https://doi.org/10.1063/5.0035347
Notebook: Óscar Amaro, March 2023 @ GoLP-EPP

Introduction

We compare this notebook's implementation with data retrieved from the paper (with WebPlotDigitizer).

In[1]:=
```
SetDirectory[NotebookDirectory[]];
```

Figure 10

In[27]:=
```
Clear[a0, p0, θ, μ, ctλ, ϵ0, Δϵacc, dataJoined]
a0 = 2500;
p0 = 500;
θ = π;
μ = ctλ / 3;
ϵ0 = Sqrt[1 + p0^2];
Δϵacc = (55 μ^2)^(1/3) a0^(2/3) ϵ0^(1/3) (1 - Cos[θ])^(1/3);
dataJoined = False;
Show[{
  ListPlot[Import["data/fig10_Deacc.csv"], Joined → dataJoined,
   PlotStyle → {Red, Opacity[0.3]}, GridLines → Automatic,
   Frame → True, FrameLabel → {"ct/λ", "Δϵ"}, PlotRange → {0, 10 000}],
  ListPlot[Import["data/fig10_DeaccApprox.csv"], Joined → dataJoined,
   PlotStyle → {Dashed, Purple, Opacity[0.3]}],
  ListPlot[Import["data/fig10_Derad.csv"], Joined → dataJoined,
   PlotStyle → {Darker[Darker[Green]], Opacity[0.3]}],
  ListPlot[Import["data/fig10_DeradApprox.csv"], Joined → dataJoined,
   PlotStyle → {Dashed, Cyan, Opacity[0.3]}],
  Plot[Δϵacc, {ctλ, 0, 5}, PlotStyle → Directive[Purple]]}]
```

Out[35]=

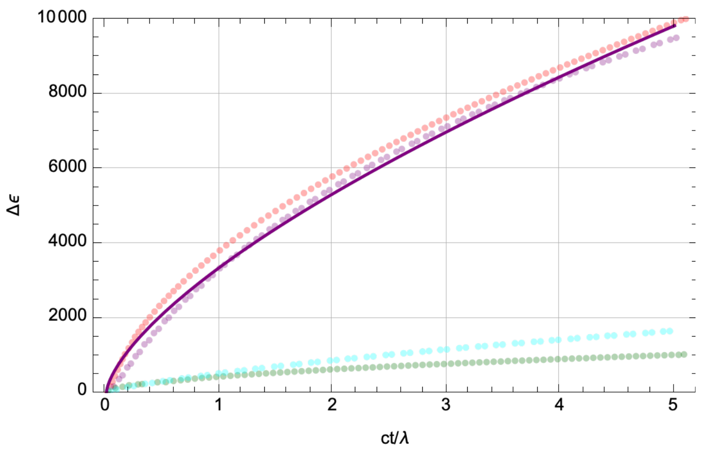


Figure 11

In[11]:=
```
Clear[vx, np, ν, E, sols, S]
sols = Solve[1/vx == Sqrt[1 + (2 np ν)/E Sqrt[1 - vx^2]], vx] // Simplify
S = 4 np ν / E; (sols⟦5, 1, 2⟧^2 - (2/(1 + Sqrt[1 + S^2]))) // FullSimplify
(* they are the same expression *)
```

Solve: Solutions may not be valid for all values of parameters.

Out[12]=
$$\left\{\{vx \to 1\}, \left\{vx \to -\frac{\sqrt{-\frac{E^2+\sqrt{E^4+16\,np^2\,E^2\,\nu^2}}{np^2\,\nu^2}}}{2\sqrt{2}}\right\}, \left\{vx \to \frac{\sqrt{-\frac{E^2+\sqrt{E^4+16\,np^2\,E^2\,\nu^2}}{np^2\,\nu^2}}}{2\sqrt{2}}\right\}, \left\{vx \to -\frac{\sqrt{\frac{-E^2+\sqrt{E^4+16\,np^2\,E^2\,\nu^2}}{np^2\,\nu^2}}}{2\sqrt{2}}\right\}, \left\{vx \to \frac{\sqrt{\frac{-E^2+\sqrt{E^4+16\,np^2\,E^2\,\nu^2}}{np^2\,\nu^2}}}{2\sqrt{2}}\right\}\right\}$$

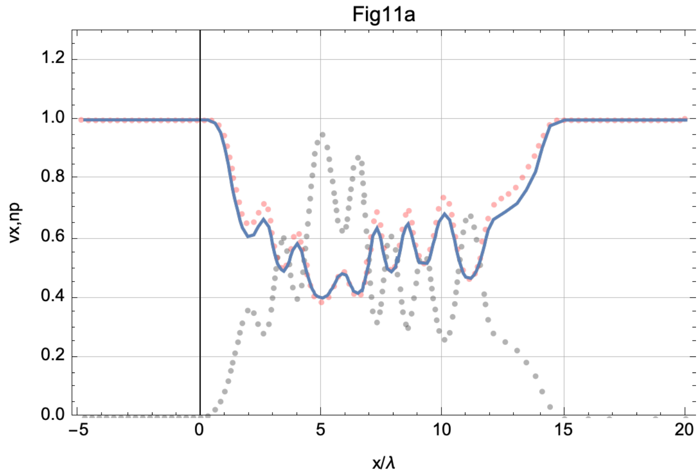
Out[13]=
$$\frac{-E^2\sqrt{1+\frac{16\,np^2\,\nu^2}{E^2}}+\sqrt{E^4+16\,np^2\,E^2\,\nu^2}}{8\,np^2\,\nu^2}$$

```
In[62]:= Clear[dataJoined, ν, S, np, E]
ν = 1;(* ν^2=1 *)
vx = Sqrt[2/(1 + Sqrt[1 + S^2])];(* (B17) *)
S = 4 np ν / E;(* (B18) *)
E = 1 / 3;(* value of E is not explicit in text *)

dataJoined = False;
npX = Import["data/fig11a_np.csv"][[All, 1]];
np = Import["data/fig11a_np.csv"][[All, 2]];
Show[{ListPlot[Import["data/fig11a_np.csv"], Joined → dataJoined,
    PlotStyle → {Black, Opacity[0.3]}, GridLines → Automatic, Frame → True,
    FrameLabel → {"x/λ", "vx,np"}, PlotRange → {0, 1.3}, PlotLabel → "Fig11a"],
   ListPlot[Import["data/fig11a_approx.csv"],
    Joined → dataJoined, PlotStyle → {Red, Opacity[0.3]}],
   ListPlot[Transpose[{npX, vx}], Joined → True]}]

Clear[np, npX]
npX = Import["data/fig11b_np.csv"][[All, 1]];
np = Import["data/fig11b_np.csv"][[All, 2]];
Show[{ListPlot[Import["data/fig11b_np.csv"], Joined → dataJoined,
    PlotStyle → {Black, Opacity[0.3]}, GridLines → Automatic, Frame → True,
    FrameLabel → {"x/λ", "vx,np"}, PlotRange → {0, 1.3}, PlotLabel → "Fig11b"],
   ListPlot[Import["data/fig11b_approx.csv"], Joined → dataJoined,
    PlotStyle → {Red, Opacity[0.3]}], ListPlot[Import["data/fig11b_vx.csv"],
    Joined → dataJoined, PlotStyle → {Darker[Darker[Green]], Opacity[0.3]}],
   ListPlot[Transpose[{npX, vx}], Joined → True]}]
```

Out[70]=



Out[74]=