ENHANCING QED EFFECTS BY TEMPORAL PULSE SHAPING IN LASER - ELECTRON-BEAM COLLISIONS + Effect of laser temporal intensity skew on enhancing pair production in laser-electron-beam collisions

Laurence E. Bradley, Master of Science by Research, University of York, Physics, June 2019

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Notebook: Óscar Amaro, December 2022 @ GoLP-EPP

Introduction

In this notebook we (try to) reproduce some results from the paper and thesis.

Figure 3 (paper)

In[1087]:=
```
Clear[It, t, ℛ, τp, τf, τr, I0, int, int1, int2, dataℛ1, dataℛ052, dataℛ072]

dataℛ1 = {{-30.357554955318683, 0.06162322941545817},
   {-27.208610937678245, 0.13702164016137974},
   {-24.057382844049172, 0.2919058953119622}, {-21.27831416856715,
    0.5322916136047295}, {-19.17260747260036, 0.8056490311544353},
   {-17.51644964696007, 1.0786848189341134}, {-16.159985387524152,
    1.3517038745292238}, {-14.953199814706572, 1.630564321670163},
   {-13.820995298698278, 1.921333834779778}, {-12.763972737397445,
    2.2031011670016096}, {-11.782001548940386, 2.480410567192125},
   {-10.875272072992814, 2.7466382149842685}, {-9.968449640464314,
```

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3.0161007517928815}, {-9.061565236881847, 3.28771988127914},
{-8.079386002743647, 3.5722692711731803}, {-7.021965056095979,
 3.867900413465592}, {-5.965101848933884, 4.144122221659191},
{-4.758212622151049, 4.4265898267907176}, {-3.1764840980129208,
 4.716815765227067}, {-0.697234701444728, 4.952809949046327},
{2.222382480031399, 4.869623990070146}, {4.311744826942913,
 4.574198060496403}, {5.652043936954584, 4.284669916127591},
{6.767798458015633, 4.002979605072819}, {7.808407543026007,
 3.7135579623867327}, {8.774416758585701, 3.435390705749321},
{9.665808398679332, 3.1678616658241148}, {10.55701412561109,
 2.893862847865971}, {11.448374780177716, 2.6252555116019427},
{12.414246774117956, 2.342312942606887}, {13.380551680135113,
 2.074435713888529}, {14.347026563900258, 1.812473710800285},
{15.462844162641218, 1.5329785030066887}, {16.72832923419074,
 1.2472516630874964}, {18.218774395852662, 0.968146430283432},
{20.008917930448135, 0.6892252038395741}, {22.473415696472756,
 0.4118626447427758}, {25.53918482139666, 0.20010463188483385},
{28.682728504335273, 0.08757139500733047}, {31.82799852377694,
 0.0351146684356749}, {34.97415384398937, 0.013466408687533793},
{38.12059246044848, 0.0016768583229147538},
{40.068590570480666, 0.0014351712124893723}};

dataℛ052 = {{-21.81781092515711, 0.009113384797555746},
{-18.671044747412815, 0.008722967157637207},
{-15.524278569668525, 0.008332549517718668}, {-12.377353037785483,
 0.01348765590603307}, {-9.228639198345448, 0.08087586527784119},
{-6.600128091821972, 0.29651786967739646}, {-4.793501005975017,
 0.5912242267846404}, {-3.3614883510876012, 0.8859976011602324},
{-1.704538090457362, 1.1866101265897049}, {0.846915755975914,
 1.3279893227820558}, {3.9935845506354326, 1.324209973791552},
{7.1398372539326544, 1.3059506453939953}, {10.285744689929288,
 1.2756760149352697}, {13.431439653740924, 1.2380073524389},
{16.57689558634445, 1.1920204039001838}, {19.722103634732136,
 1.1374070846508832}, {22.867161181988806, 1.0775563260415861},
{26.012068228114444, 1.0124681280722925}, {29.156860185139877,
 0.9433748294159416}, {32.30158131810364, 0.8718168534137103},
{35.446267039036584, 0.799026538738536}, {38.5909350539541,
 0.7256200547268934}, {41.735647333910165, 0.6537539940564265},
{44.88046584995871, 0.5855849494047813}, {48.02535519006892,
 0.5198805820990167}, {49.897401133723065, 0.4838333479643513}};

dataℛ072 = {{-23.465293688544428, 0.03796976342147573},
{-20.31731464874415, 0.0797869453297686}, {-17.166901031824203,
 0.20632741100271978}, {-14.313087128316418, 0.4405172216598192},
{-12.132184482078529, 0.7233638657125345}, {-10.475936798410004,
 0.9995267128747987}, {-9.04460582511608, 1.2705775677962858},
{-7.687749598763901, 1.5572370720775064}, {-6.330976000483645,
```

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     1.8410211194551986}, {-4.8993413690253504, 2.1226392784971515},
    {-3.1680751026915885, 2.4021189594772294}, {-0.6146184335145506,
     2.6131963871526187}, {2.5323248043839044, 2.618967662877402},
    {5.676727229070195, 2.536318638818706}, {8.819615789438416,
     2.4009871364918034}, {11.9611055745888, 2.2169782565837517},
    {15.101541851821935, 1.9963073011557197}, {18.24125218242304,
     1.750373402932409}, {21.38069692279288, 1.4951969646620435},
    {24.520451518432523, 1.2508034897799085}, {27.615773753686952,
     1.024561779251676}, {30.801439161999014, 0.8134666796109018},
    {33.94300862421875, 0.6322305617169661}, {37.08537485713222,
     0.4787220639641907}, {40.22856441976252, 0.35386544035728296},
    {43.37246222300945, 0.2536555902091848}, {46.51706826687301,
     0.1780925135198963}, {49.203334625081325, 0.14393141267740006}};

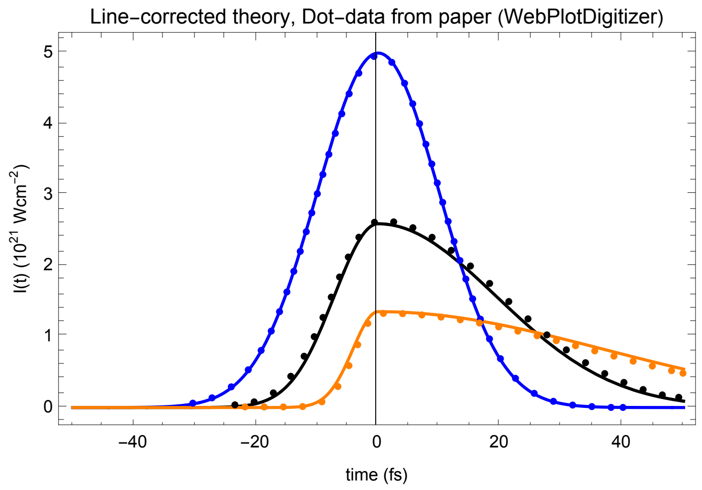
(* from the figure in the paper it seems that
 insensity is scaled with ℛ^2 instead of ℛ . also,
τf and τr don't seem to follow eq 1 and eq 2. It would seem that τr→ℛ 20,
τf→20/ℛ^2, but for the orange curve τr does not follow this *)
Show[{Plot[{(5 Exp[-2 t^2 / τp^2] /. {τp → 20}),
     (5 ℛ^2 Piecewise[{{Exp[-2 t^2 / τr^2], t < 0}, {Exp[-2 t^2 / τf^2], t > 0}}] //.
       {ℛ → 0.72, τp → 20, τr → ℛ τp, τf → τp / ℛ^2}),
     (5 ℛ^2 Piecewise[{{Exp[-2 t^2 / τr^2], t < 0}, {Exp[-2 t^2 / τf^2], t > 0}}] //.
       {ℛ → 0.52, τp → 20, τr → 0.8 ℛ τp, τf → τp / ℛ^2})}, {t, -50, +50},
    PlotRange → All, PlotStyle → {Blue, Black, Orange}, Frame → True,
    FrameLabel → {"time (fs)", "I(t) (10^21 Wcm^-2)"}, PlotLabel →
     "Line-corrected theory, Dot-data from paper (WebPlotDigitizer)"],
  ListPlot[{dataℛ1, dataℛ072, dataℛ052}, PlotStyle → {Blue, Black, Orange}]}]

(* energy does not seem to be kept constant between pulses *)
τp = 20;
NIntegrate[5 Exp[-2 t^2 / τp^2], {t, -∞, +∞}]

ℛ = 0.72; τr = ℛ τp; τf = τp / ℛ^2;
5 ℛ^2
 (NIntegrate[Exp[-2 t^2 / τr^2], {t, -∞, 0}] + NIntegrate[Exp[-2 t^2 / τf^2], {t, 0, ∞}])

ℛ = 0.52; τr = 0.8 ℛ τp; τf = τp / ℛ^2;
5 ℛ^2
 (NIntegrate[Exp[-2 t^2 / τr^2], {t, -∞, 0}] + NIntegrate[Exp[-2 t^2 / τf^2], {t, 0, ∞}])
```

Out[1091]=



Out[1093]= 125.331

Out[1095]= 86.0556

Out[1097]= 69.7147

```
Clear[me, c, α, λc, ϵ0]
Clear[δ, τrf, ɣ0, t, τp, τr, τf, τrf, ɣavg,
 Ecrit, ɣ, E0, I0, I1, ℛ, dataℛ1, dataℛ072, dataℛ052]

(* data from the paper, taken with WebPlotDigitizer *)
dataℛ1 = {{-48.54359279470809, 2.848792281131093},
   {-46.16018892127147, 2.789944717581904}, {-43.775324907144004,
    2.6942468533477477}, {-41.559073613651464, 2.561604833829775},
   {-39.73908070876641, 2.4141492814086374}, {-38.25930437039602,
    2.2682939417859966}, {-36.94960492471627, 2.1224464891354207},
   {-35.75326652509319, 1.9760099748019755}, {-34.61364034387569,
    1.8300788417096414}, {-33.47387788286036, 1.6807083472200435},
   {-32.333992770027, 1.5282424274729083}, {-31.194284820930804,
    1.380247677542216}, {-30.054794919511103, 1.2377559058471457},
   {-28.801690821986057, 1.0894643214818451}, {-27.43524735928125,
    0.9423089699307972}, {-25.89875623619742, 0.7958834818719018},
   {-24.078825579750358, 0.6499989244045845}, {-21.918894234931464,
    0.507962378768581}, {-19.533089241247687, 0.3885165429818911},
   {-17.14847182865912, 0.29904228508564046}, {-14.76479539562687,
    0.23331599874192444}, {-12.381819830126219, 0.18527785672699215},
   {-9.999337467703363, 0.14968692738787093}, {-7.617186070503763,
    0.12244873287067559}, {-5.235326701442332, 0.10258059849047374},
   {-2.853713933919792, 0.08893607044817742}, {-0.47219201959579493,
    0.07758445000405656}, {1.625752320809255, 0.06917569749291852}};
dataℛ072 = {{-38.79466819944804, 2.9026155005631162},
   {-36.41275691427315, 2.881437133269671}, {-34.02968400605978,
    2.8309423045424076}, {-31.644819991932316, 2.735244440308252},
   {-29.5984711018597, 2.5982066347929473}, {-28.062077645964308,
    2.4542460224020908}, {-26.80903260301831, 2.3074448279756052},
   {-25.726147272729534, 2.1627416894280493}, {-24.69964240901529,
```

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    2.0101713452858414}, {-23.72993025826394, 1.8601378637757036},
   {-22.817025962675253, 1.7130233961639987}, {-21.90400242226349,
    1.5628994873296882}, {-20.991183301548432, 1.4179346205912733},
   {-20.02155064734582, 1.2699074332295393}, {-18.93823376436397,
    1.1143129835906485}, {-17.741574487762414, 0.9597783365127377},
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    0.6612155333691803}, {-12.961711204569504, 0.5125941115471835},
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    0.28809074319080885}, {-5.9203792869831915, 0.22236445684709238},
   {-3.5375010641952542, 0.17678300154449111}, {-1.1550602346631678,
    0.14224025853596478}, {1.2268666253457496, 0.12066882136854673},
   {3.6086156726661045, 0.10358493192898655}, {5.990189503103537,
    0.09092307857162307}, {7.577849147605605, 0.08390126198939374}};
dataℛ052 = {{-41.970214621448505, 2.922391402723014}, {-39.58914697311721,
    2.9225043202697716}, {-37.20804038770084, 2.921634563131597},
   {-34.82697273936955, 2.921747480678355}, {-32.44589211200989,
    2.9215328399968024}, {-30.064811484650242, 2.9213181993152495},
   {-27.683730857290584, 2.9211035586336966}, {-25.302650229930933,
    2.920888917952144}, {-22.92156960257128, 2.920674277270591},
   {-20.540385142984718, 2.9178391707625515}, {-18.15873343837709,
    2.9032119680353228}, {-15.775238711741927, 2.8420714968879577},
   {-13.899222371720896, 2.711493730991922}, {-12.75976426892069,
    2.569804476956213}, {-11.903386886609887, 2.418518752425963},
   {-11.160100645561414, 2.259832243965017}, {-10.530656058090344,
    2.1126860061254473}, {-9.958008347687112, 1.9681817225161977},
   {-9.385115333647818, 1.817486588391874}, {-8.812086039810715,
    1.6633520928702863}, {-8.239165769811862, 1.5119690864665094},
   {-7.666490803449065, 1.3667769305778075}, {-7.037000789378723,
    1.21848423893915}, {-6.350832007398651, 1.0705303729478008},
   {-5.6082141863108745, 0.928713113244862}, {-4.69516117268887,
    0.7778453742670957}, {-3.5553678766485746, 0.6276966808350868},
   {-2.018812382008747, 0.4796466156313479}, {0.08392689818717258,
    0.35022557014863276}, {2.468414520492118, 0.26402689453549044},
   {4.8515393448189315, 0.21222183289498364}, {7.2340152177276025,
    0.1767946826700184}, {9.616082251242844, 0.15168561663684343},
   {11.997922151761742, 0.13230881959910734}, {14.379560877341007,
    0.11800917510018971}, {16.76109577069336, 0.1063299964277582},
   {19.142552789875538, 0.0966161671251915}, {21.5239384244 0172,
    0.08870390807833495}, {23.848580395876354, 0.08209061452572275}};

(* the derivative of γavg at t→
 0 is discontinuous (has τr and τf as pre-factors) *)
D[γ0 / (1 + δ (1 + Erf[t / τr])), t] /. {t → 0}

me = 9.1 × 10^-31;(*[Kg]*)
c = 3 × 10^8;(*[m/s]*)
```

```
α = 1 / 137 // N; (*[]*)
λc = 3.86 × 10^-13; (*[m] defined after eq 4 in paper *)
τrf = 0.5 τp;
ϵ0 = 8.854 × 10^-12; (*[F/m]*)
Ecrit = 1.38 × 10^18; (*[V/m] defined in §1 Introduction *)
δ = 2 Sqrt[π] τrf γ0 α c E0^2 / (3 λc Ecrit^2) 10^-15 × 4;
(* defined after eq 7 in paper. Got to multiply by 10^-15 because τrf in fs *)

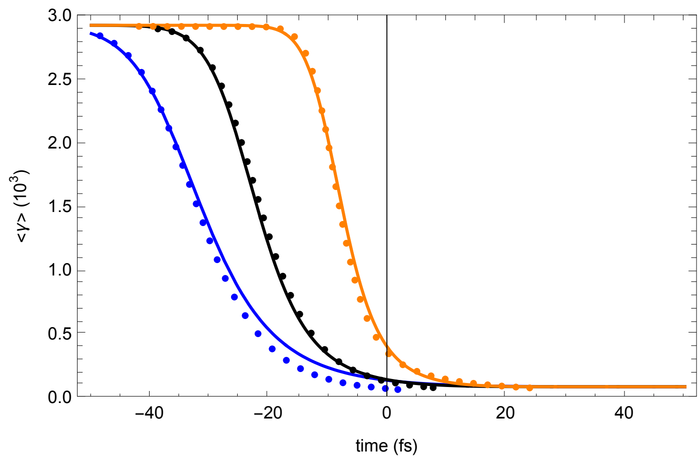
I0 = 5 × 10^21 × 10^4; (*[W/m^2]*)
E0 = Sqrt[2 I0 / (c ϵ0)]; (*[V/m] *)

γ0 = 2931; (*[] in text *)
γavg = Piecewise[{{γ0 / (1 + δ (1 + Erf[t / τr])), t < 0}, {γ0 / (1 + δ (1 + Erf[t / τf])), t ≥ 0}}]
(* also, the 3 curves don't have the same t0... so t→tt+X. Is τr→ℛ τp,
τf→τr? Derivative of <γ> seems continuous in the plot, so τr=τf must hold  *)
Show[{Plot[{(10^-3 γavg //. {ℛ → 1, I1 → I0 ℛ^2, τp → 20, τr → ℛ τp, τf → τr, t → tt + 5}),
      (10^-3 γavg //. {ℛ → 0.72, I1 → I0 ℛ^2, τp → 20, τr → ℛ τp, τf → τr, t → tt + 3}),
      (10^-3 γavg //. {ℛ → 0.52, I1 → I0 ℛ^2, τp → 20, τr → ℛ τp, τf → τr, t → tt - 6})},
    {tt, -50, +50}, Frame → True, FrameLabel → {"time (fs)", "<γ> (10^3)"},
    PlotRange → {0, 3}, PlotStyle → {Blue, Black, Orange}],
  ListPlot[{dataℛ1, dataℛ072, dataℛ052}, PlotStyle → {Blue, Black, Orange}]}]
```

Out[924]= $$-\frac{2\,\gamma 0\,\delta}{\sqrt{\pi}\,(1+\delta)^2\,\tau r}$$

Out[927]= $$\begin{cases} \dfrac{2931}{1+0.776826\,\tau p\left(1+\mathrm{Erf}\left[\frac{t}{\tau r}\right]\right)} & t < 0 \\ \dfrac{2931}{1+0.776826\,\tau p\left(1+\mathrm{Erf}\left[\frac{t}{\tau f}\right]\right)} & t \geq 0 \\ 0 & \text{True} \end{cases}$$

Out[928]=



```
Clear[me, c, α, λc, g, χe, P]
Clear[γ0]
```

```
g[χe_] := (1 + 4.8 × (1 + χe) Log[1 + 1.7 χe] + 2.44 χe^2)^(-2/3)
P[χe_] := 10^-15 × (2/3) (α c/λc) (me c^2/(me c^2)) χe^2 g[χe]; (*[] adimensional,
multiplied by fs/s and mc^2, equation 4 *)

γ0 = 2931;
```

Figure 8 (paper): Gaunt factor

§2.2 Modified-classical emission equations

```
In[∘]:= Clear[χe, χγ, t, y, g, F, χelst, glst, lst1, lst2, lst3]
```

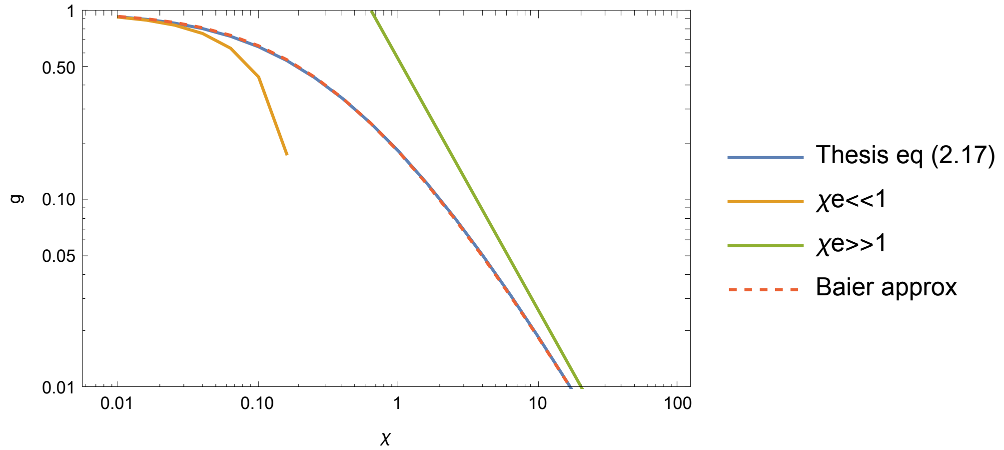
```
F[χe_?NumericQ, χγ_?NumericQ] :=
 F[χe, χγ] = (4 χγ^2/(χe^2)) (4 χγ/(3 χe (χe - 2 χγ))) BesselK[2/3, 4 χγ/(3 χe (χe - 2 χγ))] +
   (1 - 2 χγ/χe) (4 χγ/(3 χe (χe - 2 χγ))) NIntegrate[BesselK[5/3, t], {t, 4 χγ/(3 χe (χe - 2 χγ)), ∞}]
g[χe_?NumericQ] := g[χe] = (3 × √3/(2 π χe^2)) NIntegrate[F[χe, χγ], {χγ, 0, χe/2}]
```

```
χe = 10^ParallelTable[x, {x, -2, 2, 0.2}];
glst = ParallelTable[g[χe[[i]]], {i, 1, Length[χe]}];
lst1 = 1 - (55 × √3/16) χe + 4.8 χe^2;
lst2 = 0.5564 χe^(-4/3);
lst3 = (1 + 4.8 × (1 + χe) Log[1 + 1.7 χe] + 2.44 χe^2)^(-2/3);
```

In[∘]:=
```
ListLogLogPlot[{Transpose[{χe, glst}], Transpose[{χe, lst1}],
   Transpose[{χe, lst2}], Transpose[{χe, lst3}]}, PlotRange → {10^-2, 10^0},
  PlotLegends → {"Thesis eq (2.17)", "χe<<1", "χe>>1", "Baier approx"},
  PlotStyle → {Default, Default, Default, Dashed},
  Joined → True, Frame → True, FrameLabel → {"χ", "g"}]

ListLogLogPlot[Transpose[{χe, Abs[glst - lst3] / Abs[glst]}],
  PlotLabel → "Baier approx relative error to Thesis eq (2.17) < 1%",
  PlotStyle → {Default, Default, Default, Dashed}, Joined → True]
```

Out[∘]=



Out[∘]=

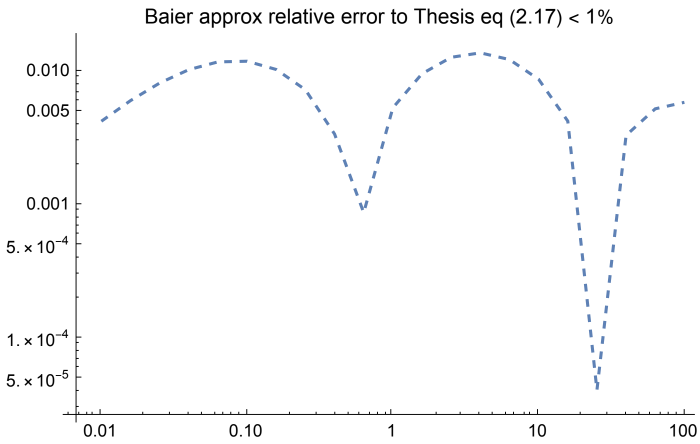


Figure 1.4 (thesis) Rate of photon emission

```
Clear[χe, h, α, c, λc, γ, dNγdt, λγ, f]
h = 5.298 χe^(-1/3);(*eq 1.5 *)
α = 1 / 137;
λγ = (√3 α c / λc) (χe / γ) h;
f = (γ λγ / c) 10^(-6.75) (*[μm]???*)
λc = 3.86 × 10^-13;(*[m] defined after eq 4 in paper *)
LogLogPlot[f, {χe, 1, 100},
 PlotRange → {3 × 10^4, 10^6}, AspectRatio → 1, Frame → True]
```

Out[1085]= $\frac{1.19111 \times 10^{-8}\, \chi e^{2/3}}{\lambda c}$

Out[1086]=

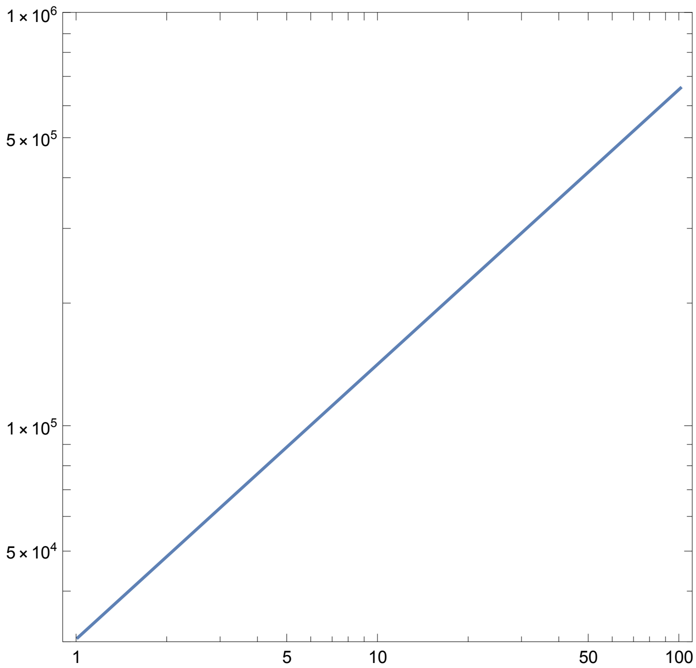


Figure 3.4 (thesis)

In[∘]:=

```
Clear[χγ, Tm, Tp, T, x, Wpm, εγ]
Clear[x, me, m, ħ, α, c, τc, e]

(* see table 1.1 definition of fundamental constants*)
c = 2.998 × 10^8; (*[m/s]*)
me = m = 9.11 × 10^-31; (*[Kg]*)
e = 1.6 × 10^-19; (*[C]*)
ħ = 1.06 × 10^-34; (*[Js]*)
α = 1 / 137; (*[]*)
τc = 1.29 × 10^-21; (*[s]*)

Wpm = α/τc (me c^2)/(ħ ω) χγ T[χγ]; (*[1/s]  eq 3.5*)
εγ = ħ ω;

χγ = 10^ParallelTable[x, {x, -2, 2, 0.1}];
(* aux function T *)
Tm = (3 × √3)/(8 × √2) Exp[-4/(3 χγ)];
Tp = 0.6 χγ^(-1/3);

(* function to plot does not depend on ω directly *)
fm = εγ/(m c^3) α/τc (me c^2)/(ħ ω) χγ Tm 10^-6; (*[μm]*)
fp = εγ/(m c^3) α/τc (me c^2)/(ħ ω) χγ Tp 10^-6; (*[μm]*)

ListLogLogPlot[{Transpose[{χγ, fm}], Transpose[{χγ, fp}]},
 Joined → True, AspectRatio → 1, Frame → True, PlotRange → {10^-7, 10^6},
 FrameLabel → {"χγ", "εγ W/m c^3(μm^-1)"}, PlotStyle → {Dashed, Dashed}]
```

Out[∘]=

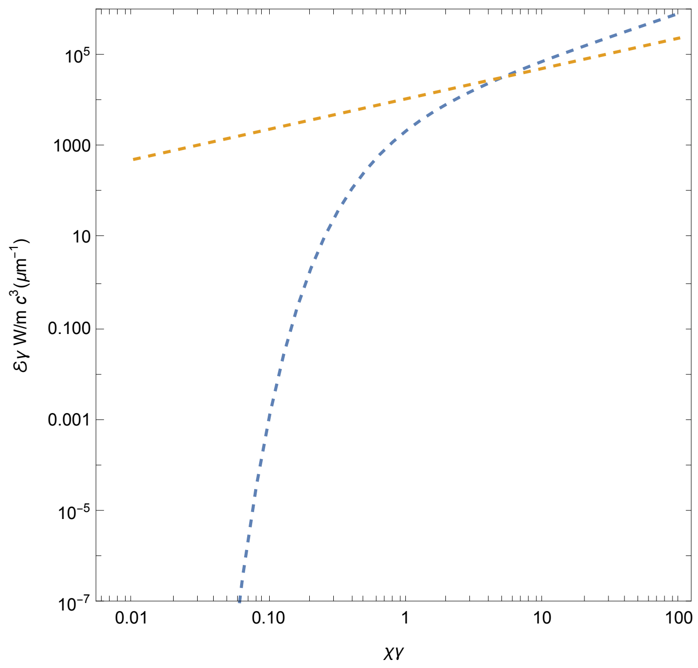


Figure 3.8 (thesis)

The theoretical model of eq 3.11 has a factor of 2 missing in its argument...

In[∘]:=

```
Clear[It, t, ℛ, τp, τf, I0, int, int1, int2, c, I1, datasym, dataskew, datasym]
τp = 40;
τr = 10;
τf = 160;
ℛ = 2 τp / (τr + τf) // N(*[], = 0.47058823529411764`*)

c = 3 × 10^8; (*[m/s]*)
I0 = 5 ; (*[10^21 W cm^-2]*)
I1 = I0 ℛ;

(* data extracted from figure 3.8 using WebPlotDigitizer *)
dataskew = {{-38.327826580639375, 0.6238761301945832},
   {-39.98781373434649, 0.5467647130081081}, {-35.067963228651884,
    0.7673988839843311}, {-32.432329029172635, 0.9023131820712571},
   {-29.972403776325336, 1.037849038835752}, {-27.512478523478038,
    1.1784058274380769}, {-25.14040774394671, 1.320159958518385},
   {-22.85619143773137, 1.4566509323692536}, {-20.571975131516012,
    1.5921587866994278}, {-18.287758825300656, 1.7252088422278664},
   {-15.827833572453358, 1.8609764343079522}, {-13.104344899658145,
    1.999026138090243}, {-9.941583860283032, 2.134997029577387},
   {-6.387471076136777, 2.245327460160323}, {-4.523891339131254,
    2.3363551801571143}, {-1.4196999486334647, 2.3430490045855836},
   {0.6888074109499343, 2.3133764808086057}, {1.1280797775298055,
    2.1303463280669908}, {1.3330735486004244, 1.9429718389413368},
```

```
    {1.567352144109691, 1.7822340224754507}, {1.7430610907416337,
     1.625735352650648}, {1.9187700373735908, 1.4596512674990763},
    {2.0944789840055336, 1.2935671823475037}, {2.2701879306374906,
     1.1274830971959329}, {2.4458968772694334, 0.9677892889288735},
    {2.6216058239013904, 0.8176808959885848}, {2.797314770533333,
     0.6803530568173226}, {3.031593366042614, 0.5206802864988553},
    {3.4122960837451615, 0.3386941419560676}, {3.8515684503250327,
     0.17802995831025026}, {6.135784756540389, -0.0064675913455856104},
    {9.82567263581133, -0.013662569763704724}, {13.515560515082285,
     -0.012337179002471999}, {17.20544839435324, 0.016375112692387894},
    {20.89533627362418, 0.000761198061339563}, {24.585224152895137,
     -0.00836100671877471}, {28.275112032166092, -0.007035615957542873},
    {31.964999911437033, -0.005710225196310148}, {35.65488779070799,
     -0.004384834435077423}, {38.90550330339906, -0.0008208744565862958}};
datasym = {{-18.22626069397947, 0.02131275679445821},
    {-15.476415679189458, 0.13119661593553467}, {-13.895035159501916,
     0.2717697977445539}, {-12.840781479710216, 0.4165687384091923},
    {-12.050091219866431, 0.5583973837071392}, {-11.347255433338631,
     0.7136140535396782}, {-10.732274120126817, 0.8693316895230394},
    {-10.20514728023096, 1.0250177685835142}, {-9.678020440335118,
     1.195614493707855}, {-9.150893600439261, 1.3853820494857327},
    {-8.71162123385939, 1.5548821715397674}, {-7.042386240855862,
     2.224736465519349}, {-8.36020334059549, 1.698789629132862},
    {-8.00878544733159, 1.8522825020527263}, {-7.65736755406769,
     2.0057753749725906}, {-6.427404927644034, 2.594122645268222},
    {-6.075987034380134, 2.773176625726139}, {-5.724569141116234,
     2.9522306061840564}, {-5.373151247852334, 3.131284586641974},
    {-5.021733354588434, 3.303948290215378}, {-4.670315461324535,
     3.4766119937887825}, {-4.318897568060635, 3.6556659742466997},
    {-3.967479674796735, 3.818744262493334}, {-3.616061781532835,
     3.9818225507399685}, {-3.2646438882689353, 4.141705700544346},
    {-2.9132259950050354, 4.282418019695184}, {-2.5618081017411356,
     4.419935200403765}, {-2.1225357351612644, 4.572394584099097},
    {-1.5954088952654075, 4.73021075545441}, {0.5130984643179914,
     4.995293210917644}, {2.6216058239013904, 4.706164380214908},
    {3.148732663797233, 4.544466707344558}, {3.5880050303771185,
     4.386997662140996}, {3.9394229236410183, 4.24653779837325},
    {4.290840816904918, 4.102882796163248}, {4.642258710168818,
     3.9464472401842188}, {4.993676603432718, 3.7804262688784194},
    {5.345094496696618, 3.608015020688107}, {5.6965123899605175,
     3.4419940493823082}, {6.047930283224417, 3.2631925243074824},
    {6.399348176488317, 3.084390999232657}, {6.750766069752217,
     2.905589474157831}, {7.102183963016117, 2.726787949083006},
    {7.453601856280017, 2.5543767008926936}, {7.805019749543916,
     2.381965452702381}, {8.156437642807816, 2.2127493429543255},
    {8.507855536071716, 2.0499235100907836}, {8.859273429335616,
     1.896683092554011}, {9.210691322599516, 1.7434426750172385},
```

```
{9.562109215863416, 1.5933973959227226}, {9.913527109127315,
 1.456132670597233}, {10.352799475707187, 1.2922733485091582},
{10.879926315603043, 1.1156650295749433}, {11.407053155498886,
 0.9518372644097557}, {12.022034468710714, 0.7859109638726158},
{12.724870255238514, 0.6200162202583632}, {13.515560515082285,
 0.469489698089828}, {14.481959721558013, 0.325629575881063},
{15.887631294613612, 0.1795841367624087}, {18.435411020776883,
 0.02067136233679001}, {20.89533627362418, -0.0023832238977075093}};

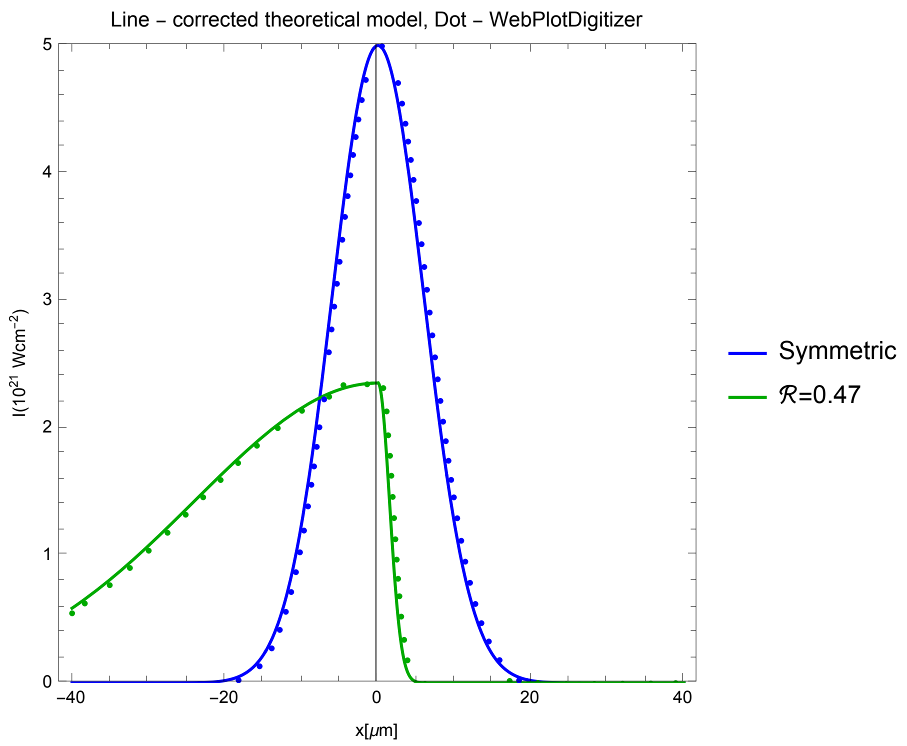
(* extra 2 in argument of exponential *)
It[t_] := Module[{},
  Return[I1 Piecewise[{{Exp[-2 t^2 / τf^2], t < 0}, {Exp[-2 t^2 / τr^2], t > 0}}]]
 ]

Show[{Plot[{I0 Exp[-2 ((x 10^-6)/c 10^15)^2 / τp^2], It[(x 10^-6)/c 10^15]},
   {x, -40, +40}, PlotRange → {0, 5}, PlotStyle → {Blue, Green // Darker},
   Frame → True, FrameLabel → {"x[μm]", "I(10^21 Wcm^-2)"}, AspectRatio → 1,
   PlotLabel → "Line - corrected theoretical model, Dot - WebPlotDigitizer",
   PlotLegends → {"Symmetric", "ℛ=0.47"}],
  ListPlot[{datasym, dataskew}, PlotStyle → {Blue, Green // Darker}]}]

(* confirm they have the same energy *)
I0 τp √π - 1/2 √π (τr + τf) I1 // N
```

Out[∘]= 0.470588

Out[∘]=



Out[∘]= 0.

Figure 3.9 (thesis): Conserving the energy in the modified pulse

In[∘]:=
```
(* since there is only one explicit equation/constraint
  (conservation of energy between the symmetric and skewed pulse),
there is ambiguity regarding ℛ vs τf (you have to fix one to get the other)*)
Clear[It, t, ℛ, τp, τf, τr, I0, int, int1, int2, c, I1]
I0 = 5 ;(*[10^21 W cm^-2]*)
τp = 40;(*[fs]*)
τr = {5, 10, 15, 20, 25, 30, 35, 40};(*[fs]*)
(* retrieved with WebPlotDigitizer *)
I1 = {2.1783490443656484, 2.3625789155063557,
    2.591304159820116, 2.8685462250281066, 3.218563521577445,
    3.657506702412869, 4.23391420911528, 5.020582893712704};
ℛ = I1 / I0
τf = 2 τp / ℛ - τr
```

Out[∘]= {0.43567, 0.472516, 0.518261, 0.573709, 0.643713, 0.731501, 0.846783, 1.00412}

Out[∘]= {178.625, 159.307, 139.362, 119.443, 99.279, 79.3641, 59.4752, 39.672}