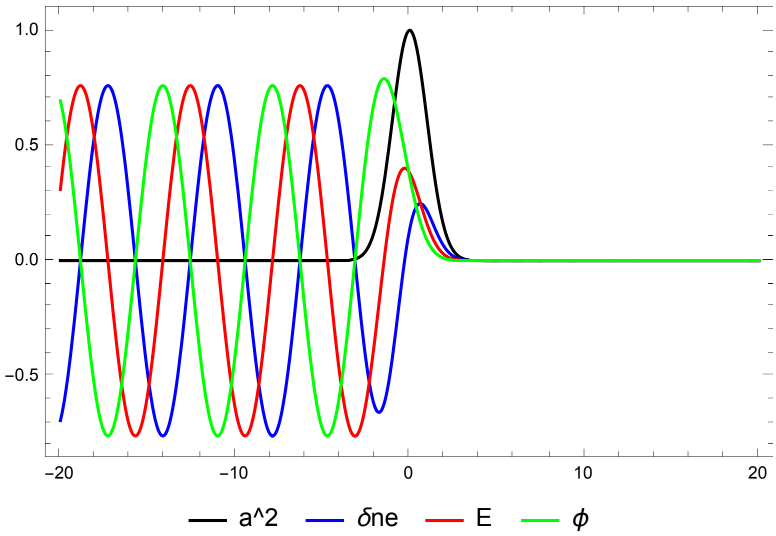
https://www.cockcroft.ac.uk/wp-content/uploads/2018/04/LWFA_comp.pdf

# Plasma wave generation by a laser pulse

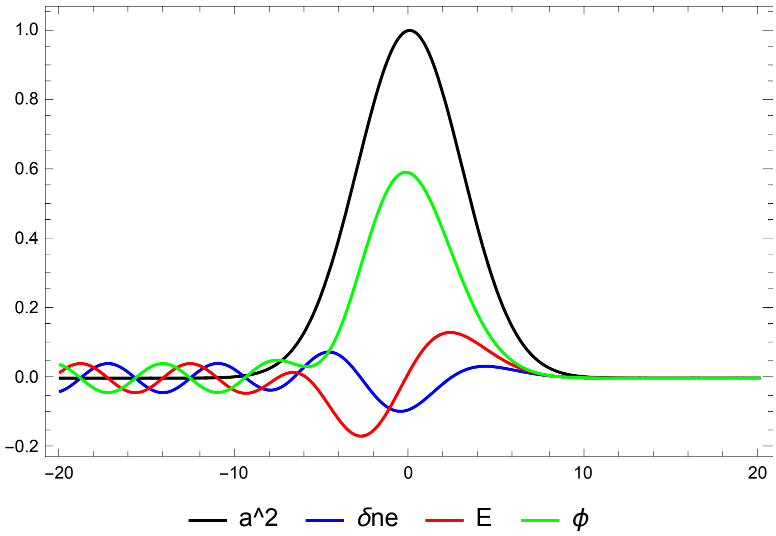
In[49]:=
```
(* clear variables *)
Clear[a, kp, σξ, ξ, Ex, ϕ, eps, pltpnt, asp, imgsz]

(** plot style **)
(* contour precision *)
eps = 0.0001;
(* point density *)
pltpnt = 60;
(* aspect ratio *)
asp = 0.62;
(* image size *)
imgsz = 400;

(* normalizing wavevector *)
kp = 1;

(* laser function *)
a[ξ_] := Exp[-0.5 ξ^2 / σξ^2]
(* width *)
σξ = √2;

(* plot laser *)
plt1 = Plot[a[ξ]^2, {ξ, -20, +20}, PlotStyle → Black, Axes → False, Frame → True,
   PlotRange → All, PlotLegends → {"a^2"}, AspectRatio → asp, ImageSize → imgsz];

(* to get electron density perturbation, solve differential equation *)
solδne = NDSolve[{D[δne[ξ], {ξ, 2}] + kp^2 δne[ξ] == 1/2 D[a[ξ]^2, {ξ, 2}],
    δne[20] == 0, δne'[20] == 0}, δne[ξ], {ξ, -20, +20}];
plt2 = Plot[Evaluate[δne[ξ] /. solδne], {ξ, -20, 20}, PlotStyle → Blue,
   Axes → False, Frame → True, PlotLegends → {"δne"},
   AspectRatio → asp, ImageSize → imgsz];

(* to get field, integrate density *)
solEx =
  NDSolve[{D[Ex[ξ], ξ] == -δne[ξ] /. solδne, Ex[20] == 0}, Ex[ξ], {ξ, -20, +20}];
plt3 = Plot[Evaluate[Ex[ξ] /. solEx], {ξ, -20, 20}, PlotStyle → Red, Axes → False,
   Frame → True, PlotLegends → {"E"}, AspectRatio → asp, ImageSize → imgsz];

(* to get potential, integrate field *)
solϕ = NDSolve[{D[ϕ[ξ], ξ] == -Ex[ξ] /. solEx, ϕ[20] == 0}, ϕ[ξ], {ξ, -20, +20}];
plt4 = Plot[Evaluate[ϕ[ξ] /. solϕ], {ξ, -20, 20}, PlotStyle → Green, Axes → False,
   Frame → True, PlotLegends → {"ϕ"}, AspectRatio → asp, ImageSize → imgsz];

(* overlay all plots *)
Show[{plt1, plt2, plt3, plt4}, PlotRange → All, PlotLegends → Automatic]
```

Out[64]=



In[33]:=
```
(* clear variables *)
Clear[a, kp, σξ, ξ, Ex, ϕ, eps, pltpnt, asp, imgsz]

(** plot style **)
(* contour precision *)
eps = 0.0001;
(* point density *)
pltpnt = 60;
(* aspect ratio *)
asp = 0.62;
(* image size *)
imgsz = 400;

(* normalizing wavevector *)
kp = 1;

(* laser function *)
a[ξ_] := Exp[-0.5 ξ^2 / σξ^2]
(* width *)
σξ = 3 × √2;

(* plot laser *)
plt1 = Plot[a[ξ]^2, {ξ, -20, +20}, PlotStyle → Black, Axes → False, Frame → True,
   PlotRange → All, PlotLegends → {"a^2"}, AspectRatio → asp, ImageSize → imgsz];

(* to get electron density perturbation, solve differential equation *)
solδne = NDSolve[{D[δne[ξ], {ξ, 2}] + kp^2 δne[ξ] == 1/2 D[a[ξ]^2, {ξ, 2}],
    δne[20] == 0, δne'[20] == 0}, δne[ξ], {ξ, -20, +20}];
plt2 = Plot[Evaluate[δne[ξ] /. solδne], {ξ, -20, 20}, PlotStyle → Blue,
   Axes → False, Frame → True, PlotLegends → {"δne"},
   AspectRatio → asp, ImageSize → imgsz];

(* to get field, integrate density *)
solEx =
  NDSolve[{D[Ex[ξ], ξ] == -δne[ξ] /. solδne, Ex[20] == 0}, Ex[ξ], {ξ, -20, +20}];
plt3 = Plot[Evaluate[Ex[ξ] /. solEx], {ξ, -20, 20}, PlotStyle → Red, Axes → False,
   Frame → True, PlotLegends → {"E"}, AspectRatio → asp, ImageSize → imgsz];

(* to get potential, integrate field *)
solϕ = NDSolve[{D[ϕ[ξ], ξ] == -Ex[ξ] /. solEx, ϕ[20] == 0}, ϕ[ξ], {ξ, -20, +20}];
plt4 = Plot[Evaluate[ϕ[ξ] /. solϕ], {ξ, -20, 20}, PlotStyle → Green, Axes → False,
   Frame → True, PlotLegends → {"ϕ"}, AspectRatio → asp, ImageSize → imgsz];

(* overlay all plots *)
Show[{plt1, plt2, plt3, plt4}, PlotRange → All, PlotLegends → Automatic]
```

Out[48]=



# Particle trapping in 1D plasma wave: Phase-space trajectories

In[65]:=
```
(* we now move on to phase-space trajectories *)

(* wave gamma *)
γϕ = 10; βϕ = Sqrt[1 - 1 / γϕ ^ 2];
(* initial momentum *)
γ0 = {10, 16, 20, 30, 40, 50}; β0 = Sqrt[1 - 1 / γ0 ^ 2];

(* scan different initial momenta to get trajectories in phase-space *)
plt = {};
For[i = 1, i ≤ Length[γ0], i++,
  AppendTo[plt, ContourPlot[(γ γϕ - βϕ Sqrt[(γ γϕ) ^ 2 - 1] - 1 / 50 ϕ[ξ] /. solϕ) ==
     eps + (γ0[[i]] - βϕ Sqrt[γ0[[i]] ^ 2 - 1]), {ξ, -20, +10}, {γ, 0, 2}, ContourStyle →
     Gray, AspectRatio → asp, PlotPoints → pltpnt, ImageSize → imgsz]]
 ];

(* this value is close to separatrix *)
γ0 = 17.15;
AppendTo[plt, ContourPlot[(γ γϕ - βϕ Sqrt[(γ γϕ) ^ 2 - 1] - 1 / 50 ϕ[ξ] /. solϕ) ==
    eps + (γ0 - βϕ Sqrt[γ0 ^ 2 - 1]), {ξ, -20, +10}, {γ, 0, 2}, ContourStyle → Red,
   AspectRatio → asp, PlotPoints → pltpnt, ImageSize → imgsz]];

(* symmetry line *)
line = Graphics[{Dashed, Black, Line[{{-20, 1}, {10, 1}}]}];

(* overlay plots *)
Show[{plt, line}]
```

Out[72]=