Ultra-intense laser pulse characterization using ponderomotive electron scattering

Felix Mackenroth, Amol R Holkundkar and Hans-Peter Schlenvoigt, New J. Phys. **21** (2019) 123028
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Introduction

In this notebook we reproduce some results from the paper.

Focus dominated ponderomotive scattering: $W0/zR=\lambda/(\pi W0) > \sqrt{2}\, a0/ <\gamma>$, “low laser power” -> harmonic oscillator

Amplitude dominated ponderomotive scattering: $W0/zR=\lambda/(\pi W0) < \sqrt{2}\, a0/ <\gamma>$ -> exponentially accelerated 2nd order ODE

Frequency parameter Ω is defined in equation 15

Equation14: harmonic oscillator or exponential acceleration

```
(* this would be the 1st derivative in xprp *)
Clear[ξprp, τ, c, t, lR, t, xprp, eq]
ξprp[t_] := xprp[t] / Sqrt[1 + (c t / lR)^2];
τ = lR (ArcTan[c t / lR] + π / 2);

(* equation 13 *)
(*D[ξprp,τ]=D[ξprp,t] D[t,τ]*)
D[ξprp[t], t] / D[τ, t] - ((Sqrt[1 + (c t / lR)^2] - c t ξprp[t] / lR^2)) // FullSimplify
```

Out[184]= $\dfrac{\sqrt{1+\frac{c^2\,t^2}{lR^2}}\ \left(-c+xprp'[t]\right)}{c}$

```
In[233]:= (* get 2nd term of LHS of eq 14 *)
Clear[ξprp, τ, c, t, lR, t, xprp, eq, d2ξd2τ]
t = lR/c Tan[τ / lR - π / 2];
(* differentiate equation 13 *)
d2ξd2τ = D[Sqrt[1 + (c t / lR)^2] - c t ξprp[τ] / lR^2, τ];
(* replace again with eq 13 *)
d2ξd2τ /. {ξprp'[τ] → (Sqrt[1 + (c t / lR)^2] - c t ξprp / lR^2), ξprp[τ] → ξprp} //
 FullSimplify
```

Out[236]= $-\dfrac{\xi prp}{lR^2}$

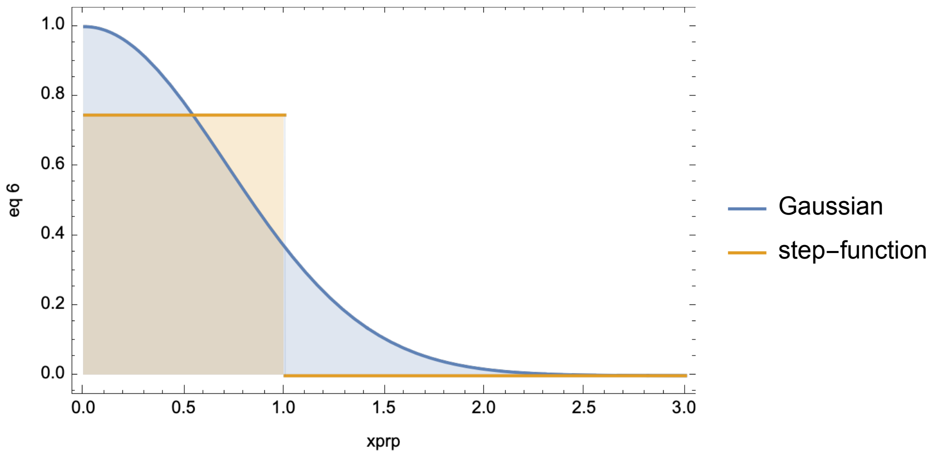
Transverse profile

```
Clear[xprp, W]
(* equation 6 *)
1/W Integrate[Exp[-(xprp/W)^2], {xprp, 0, W}]

W = 1;
Plot[{Exp[-(xprp/W)^2], Sqrt[π / 4] Erf[1] HeavisideTheta[W - xprp]},
 {xprp, 0, 3 W}, Frame → True, FrameLabel → {"xprp", "eq 6"},
 PlotLegends → {"Gaussian", "step-function"}, Filling → Bottom]
```

Out[∘]= $\frac{1}{2}\sqrt{\pi}$ Erf[1]

Out[∘]=



Equation 23

In[∘]:=
```
Clear[ν]
FindRoot[{Tan[ν] - ν/(1 - ν^2) == 0}, {ν, 2.5}]
Plot[Tan[ν] - ν/(1 - ν^2), {ν, 0, 3}];
```

Out[∘]= {ν → 2.74371}

Figure 2

```
Clear[xW0, Ω, nu]
(*
(*start root find at nu=2 which is the first root*)
Ω=0.45;
Plot[Tan[ArcSin[Ω nu]/Ω]-(nu Sqrt[1-(Ω nu)^2])/(1-nu^2),{nu,0,10}]
*)

(* equation 38 *)
getnu[Ω_] := 1/FindRoot[Tan[ArcSin[Ω nu]/Ω] - (nu Sqrt[1 - (Ω nu) ^2])/(1 - nu^2), {nu, 2}][[1, 2]]
Plot[{getnu[Ω], 0.37 - 8 × 10^-2 Ω + 0.64 Ω^2, 0.5, 1/2.74}, {Ω, 0.01, 0.49},
 PlotRange → {0, 1}, PlotStyle → {Default, Dashed, Dashed, Dashed},
 Frame → True, FrameLabel → {"Ω", "x⊥[W0]"},
 PlotLegends → {"Numerics eq 38", "Analytical fit", "0.5", "1/2.74"},
 PlotLabel → "Fig2 Focus dominated regime"]
```

Out[•]=

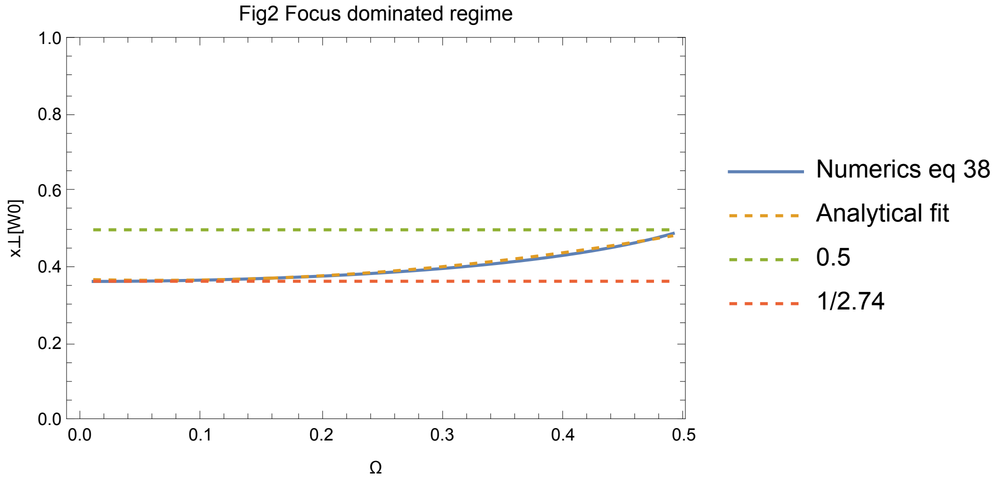


Figure 3

In[∘]:=
```
Clear[xW0, Ω, nu, getnu]
(*
(*start root search at analytical fit *)
Ω=5;
Plot[Tan[ArcSinh[Ω nu]/Ω]-nu Sqrt[1+(Ω nu)^2]/(1-nu^2),
 {nu,0.9/(1/2.74 1/Cosh[3.8 10^-2+1.12Ω+2.2 10^-2 Ω^2]),1000}]
*)
(* in eq 49 the minus sign in nominator RHS should be + and not - *)
getnu[Ω_] := 1/FindRoot[Tan[ArcSinh[Ω nu]/Ω] - nu Sqrt[1 + (Ω nu)^2]/(1 - nu^2),
    {nu, 0.9/(1/2.74 × (1/Cosh[3.8 × 10^-2 + 1.12 Ω + 2.2 × 10^-2 Ω^2]))}][[1, 2]]
LogPlot[{getnu[Ω], 1/2.74 1/Cosh[3.8 × 10^-2 + 1.12 Ω + 2.2 × 10^-2 Ω^2], 1/2.74},
 {Ω, 0.01, 9.99}, PlotRange → {10^-6, 10^0},
 PlotStyle → {Default, Dashed, Dashed, Dashed},
 Frame → True, FrameLabel → {"Ω", "x⊥[W0]"},
 PlotLegends → {"Numerics eq 49", "Analytical fit", "0.5", "1/2.74"},
 PlotLabel → "Fig2 Focus dominated regime"]
```

Out[∘]=

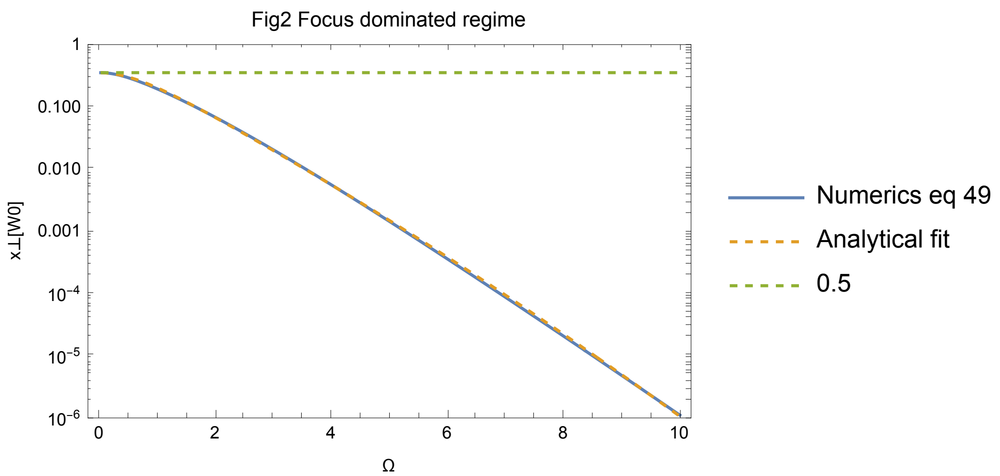


Figure 4

from focus 4(a) to detector 4(b)

In[∘]:=
```
Clear[x, θtmax, d]
θtmax = 4.4 × 10^-2;(* [], as it appears in the text *)
d = 0.2;(*[m] detector is 20 cm from laser focus *)
x = d Tan[θtmax](* transverse distance at the detector ~ 22mm *)
```

Out[∘]= 0.00880568

Plot

```
Clear[a0, ϵi, W0, λ, lR]
a0 = 10; (*[]*)
ϵi = 200 m; (*[?]*)
W0 = 2.5; (*[μm]*)
λ = 0.8; (*[μm]*)
lR = π W0 ^ 2 / λ;
(*ArcTan[W0/lR]*)
xprp0 = 10 W0;
```

Out[∘]= 0.101509

Equation 47...

```
In[∘]:= Clear[W0, Ω, xprp0, eq47, tmax, lR, gtil, tmax, θtmax, diff, λ, a0, γ0, γavg]
λ = 0.8;
lR = π W0 ^ 2 / λ;
γavg = Sqrt[1 + a0 ^ 2 + γ0 ^ 2] // N;
(* equation 15 *)
```

$$\Omega = \text{Sqrt}\left[\text{Abs}\left[1 - 2\left(\frac{\text{a0 lR}}{\gamma\text{avg W0}}\right)^2\right]\right];$$

```
(* equation 47 *)
```

$$\text{tmax} = \text{lR Tan}\left[\frac{\text{ArcSinh}\left[\frac{\text{W0}\,\Omega}{\text{xprp0}}\right]}{\Omega} - \frac{\pi}{2}\right];$$

```
(* in-line expression *)
```

$$\theta\text{tmax} = \left(\text{tmax W0 / lR} + \text{Sqrt}\left[\text{xprp0}^2 + (\text{W0}\,\Omega)^2\right]\right) \Big/ \text{Sqrt}\left[\text{lR}^2 + \text{tmax}^2\right];$$

```
(* defined after equation 47 *)
gtil[xprp_] := ArcSinh[W0 Ω / xprp0] / Ω
(* equation 47 *)
```

$$\text{eq47} = \text{ArcTan}\left[\frac{\text{W0}}{\text{lR}}\left(\text{Sqrt}\left[\left(\frac{\text{xprp0}}{\text{W0}}\right)^2 + \Omega^2\right] \text{Sin[gtil[xprp0]]} - \text{Cos[gtil[xprp0]]}\right)\right];$$

```
(* assign random values *)
W0 = RandomReal[{0, 10}];
a0 = RandomReal[{0, 15}];
xprp0 = W0 RandomReal[{0, 3}];
```

$$\gamma 0 = \text{RandomReal}\left[\left\{10^3, 10^4\right\}\right];$$

```
(* difference *)
diff = θtmax - eq47 // N
```

Out[∘]= 1.0458×10^{-7}

Equation 41 -> 42

In[408]:=
```
Clear[xprp, vprp, t, Ω, lR, c, xprp0]
(* eq 41 *)
xprp = xprp0/Ω Sqrt[1 + (t/lR)^2] Sinh[Ω (ArcTan[t/lR] + π/2)];
(* eq 42 *)
vprp = D[xprp, t] // Simplify

Limit[(xprp0 (lR Ω Cosh[Ω (π/2 + ArcTan[t/lR])] + t Sinh[Ω (π/2 + ArcTan[t/lR])]))/(lR^2 Sqrt[1 + t^2/lR^2] Ω), t → ∞]
```

Out[410]=
$$\frac{\text{xprp0}\left(\text{lR}\,\Omega\,\text{Cosh}\left[\Omega\left(\frac{\pi}{2}+\text{ArcTan}\left[\frac{t}{\text{lR}}\right]\right)\right]+t\,\text{Sinh}\left[\Omega\left(\frac{\pi}{2}+\text{ArcTan}\left[\frac{t}{\text{lR}}\right]\right)\right]\right)}{\text{lR}^2\sqrt{1+\frac{t^2}{\text{lR}^2}}\;\Omega}$$

Out[411]=
$$\frac{\sqrt{\frac{1}{\text{lR}^2}}\;\text{xprp0}\;\text{Sinh}\left[\frac{1}{2}\left(\pi+\sqrt{\frac{1}{\text{lR}^2}}\;\text{lR}\,\pi\right)\Omega\right]}{\Omega}$$

Figure 5

$\sqrt{2}$ a0 lR/W0 <γ> ~2.08 ~2.1 , assuming <γ> ~γ0

```
Clear[lR, a0, W0, λ, lR, γ0]
a0 = 15;(*[]*)
γ0 = 200;(*[]*)
W0 = 5;(*[μm]*)
λ = 0.8;(*[μm]*)
lR = π W0^2 / λ;
√2 a0 lR / (W0 γ0)
```

Out[∘]= 2.0826

Plot...

In[460]:=
```
Clear[γavg, a0, γ0, lR, ξ, gtil, Ω, xprp, xprp0, C14, c, x0, θtmax]
lR = π W0^2 / λ;
a0 = 10 ; (*√2*)
λ = 0.8;
W0 = 3;
γ0 = 1000;
γavg = Sqrt[1 + a0^2 + γ0^2] // N;
C14 = (2 a0^2/(γavg^2 W0^2) - 1/(lR^2));

(* equation 15 *)
Ω = Sqrt[Abs[1 - 2 (a0 lR/(γavg W0))^2]];

(* defined after equation 47 *)
gtil[xprp_] := ArcSinh[W0 Ω / xprp0] / Ω

(* equation 47 *)
θtmax = ArcTan[W0/lR (Sqrt[(xprp0/W0)^2 + Ω^2] Sin[gtil[xprp0]] - Cos[gtil[xprp0]])];

Plot[{10 θtmax /. {xprp0 → xprp0W0 W0}, 1.1, 2 HeavisideTheta[xprp0W0 - Ω/Sinh[π Ω]],
  (Sqrt[1/lR^2] xprp0 Sinh[1/2 (π + Sqrt[1/lR^2] lR π) Ω])/Ω /. {xprp0 → xprp0W0 W0}},
 {xprp0W0, 0, 1}, PlotRange → {0, 2}, Filling → Bottom,
 PlotStyle → {Blue, Directive[Gray, Dashed], Directive[Gray, Dashed], Black},
 Frame → True, FrameLabel → {"xprp/W0", "θf[10^-1rad]"},
 PlotLegends → {"eq 47", "", "", "eq 42"}]
```

Out[471]=

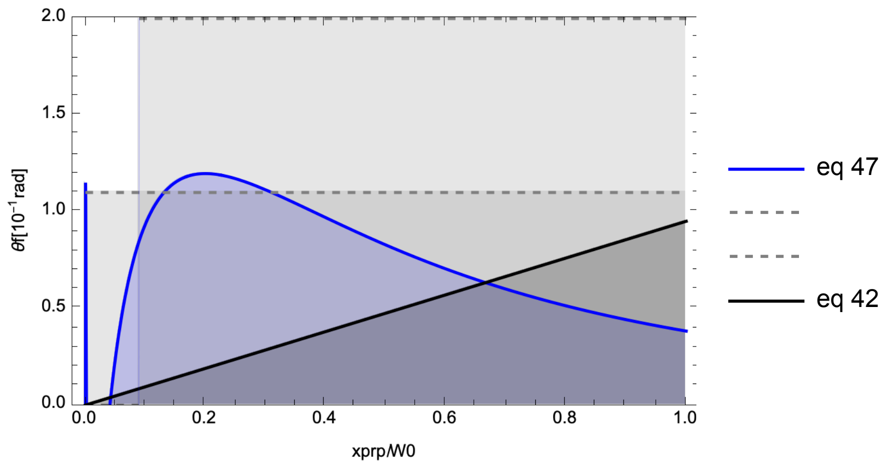


Figure 6

```
Clear[γavg, a0, γ0, lR, ξ, gtil, Ω, xprp, xprp0]
lR = π W0^2 / λ;
γavg = Sqrt[1 + a0^2 + γ0^2];
C14 = (2 a0^2/(γavg^2 W0^2) - 1/(lR^2));

a0 = 10;
c = 1;
λ = 0.8;
W0 = 3;
γ0 = 1000;
x0 = 1;
gtil[xprp_] := ArcSinh[W0 Ω / xprp0]

(* equation 47 *)
θtmax = ArcTan[W0/lR (Sqrt[(xprp0/W0)^2 + Ω^2] Sin[gtil[xprp0]] - Cos[gtil[xprp0]])]
```

from focus 6(a) to detector 6(b)

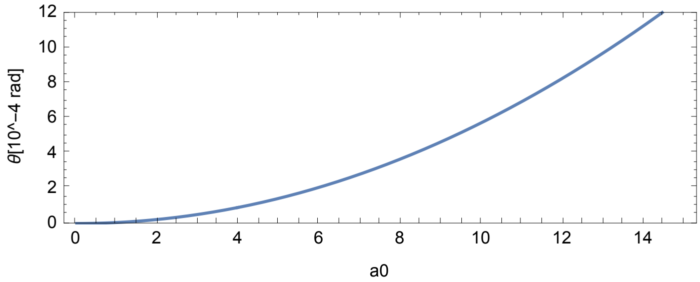
```
In[∘]:= Clear[x, θtmax, d]
θtmax = ArcTan[0.11];(* [], as it appears in the text *)
d = 0.2;(*[m] detector is 20 cm from laser focus *)
x = d Tan[θtmax](* transverse distance at the detector ~ 22mm *)
Out[∘]= 0.022
```

Figure 7

0.23 $(a0 / \gamma)^2$ instead of 0.33 $(a0 / \gamma)^2$?

In[∘]:=
```
Clear[a0, m, γ]
γ = 200 ;
Plot[{0.23 × 10^4 (a0 / γ) ^2}, {a0, 0, 15}, PlotRange → {0, 12},
 AspectRatio → 1 / 3, Frame → True, FrameLabel → {"a0", "θ[10^-4 rad]"}]
Clear[a0, m, γ]
a0 = 10;
Plot[{10^3/3 (a0 / γ) ^2}, {γ, 0, 250}, PlotRange → {0, 12},
 AspectRatio → 1 / 3, Frame → True, FrameLabel → {"a0", "θ[10^-4 rad]"}]
```

Out[∘]=



Out[∘]=