https://journals.aps.org/pr/abstract/10.1103/PhysRev.155.1404

Equation 12: L'[w0] as series

```
Clear[Lpint, Lpser]
π^2/12 - Sum[(-1)^(n-1)/(n^2 w0^n), {n, 1, ∞}]
Lpint[w0_] := NIntegrate[1/w Log[1 + 1/w], {w, 1, w0}]
Lpser[w0_, nmax_] := π^2/12 - Sum[(-1)^(n-1)/(n^2 w0^n), {n, 1, nmax}]
LogLogPlot[{Lpint[w0], Lpser[w0, 2], Lpser[w0, 7]},
 {w0, 1, 2}, PlotPoints → 2, PlotStyle → {Default, Dashed},
 AspectRatio → 0.5, Frame → True, FrameLabel → {"w0", "L'[w0]"},
 PlotLegends → {"∫ form", "nmax=2", "nmax=7"}, PlotLabel → "Equation 12"]
```

Out[46]= $\frac{\pi^2}{12} + \text{PolyLog}\left[2, -\frac{1}{w0}\right]$

Out[49]=

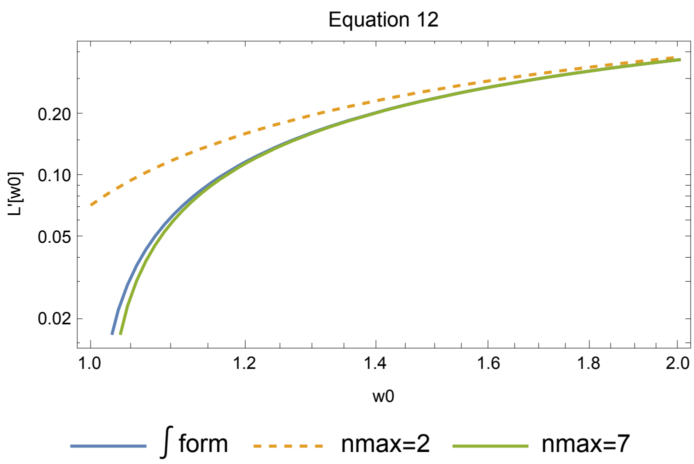


Figure 1

In[∘]:=

```
Clear[σ, β, r0, ϕ9, s0]

(* equation 4 defines β as function of s *)
```

$$(1-\beta^2)\times\left((3-\beta^4)\,\mathrm{Log}\left[\frac{1+\beta}{1-\beta}\right]-2\beta\,(2-\beta^2)\right)\ /.\ \{\beta\to\mathrm{Sqrt}[1-1/s]\};$$

```
(* total cross section *)
```

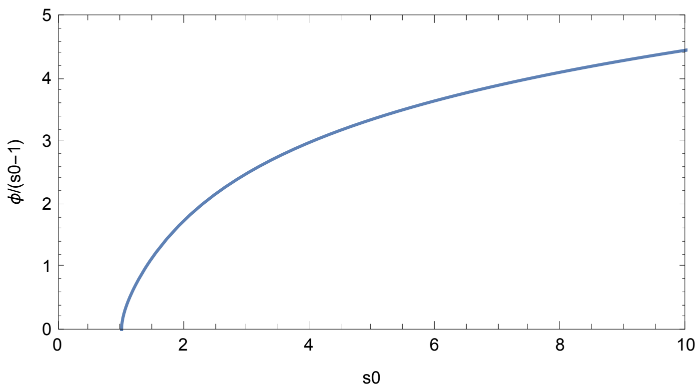
$$\sigma[s\_]:=\frac{-2\sqrt{1-\frac{1}{s}}\left(1+\frac{1}{s}\right)+\left(3-\left(1-\frac{1}{s}\right)^2\right)\mathrm{Log}\left[\frac{1+\sqrt{1-\frac{1}{s}}}{1-\sqrt{1-\frac{1}{s}}}\right]}{s}\ (*\ \text{multiplied by}\ \frac{\pi\ r0\hat{}2}{2}\ *)$$

```
(* function to plot in figure 1 *)
```

$$\phi 9[s0\_]:=\mathrm{NIntegrate}[s\,\sigma[s],\{s,1,s0\}]\ (*\ \text{divided by}\ \frac{2}{\pi\ r0\hat{}2}\ *)$$

$$\mathrm{Plot}\left[\frac{\phi 9[s0]}{s0-1},\{s0,1,10\},\mathrm{PlotRange}\to\{\{0,10\},\{0,5\}\},\right.$$
$$\left.\mathrm{AspectRatio}\to 0.5,\mathrm{Frame}\to\mathrm{True},\mathrm{FrameLabel}\to\{\text{"s0"},\text{"}\phi/(s0-1)\text{"}\}\right]$$

Out[∘]=



Equation 13: asymptotic formulas

In[84]:= Clear[σ, β, r0, ϕ9, s0, ϕ13a, ϕ13b]

$$\phi9[s0\_] := \text{NIntegrate}\left[s\,\frac{-2\sqrt{1-\frac{1}{s}}\left(1+\frac{1}{s}\right)+\left(3-\left(1-\frac{1}{s}\right)^2\right)\text{Log}\left[\frac{1+\sqrt{1-\frac{1}{s}}}{1-\sqrt{1-\frac{1}{s}}}\right]}{s},\ \{s, 1, s0\}\right]$$

ϕ13a[s0_] :=

$$2\,s0\,(\text{Log}[4\,s0]-2)+\text{Log}[4\,s0]\,(\text{Log}[4\,s0]-2)-(\pi\wedge 2-9)/3+\frac{1}{s0}\,(\text{Log}[4\,s0]+9/8)$$

$$\phi13b[s0\_] := \frac{2}{3}\,(s0-1)\wedge 1.5+\frac{5}{3}\,(s0-1)\wedge 2.5-\frac{1507}{420}\,(s0-1)\wedge 3.5$$

```
LogLogPlot[{ϕ9[s0], ϕ13a[s0], ϕ13b[s0]}, {s0, 1, 5},
 PlotRange → {{0, 5}, {0, 10}}, AspectRatio → 0.5, Frame → True,
 FrameLabel → {"s0", "ϕ[s0])"}, PlotLegends → {"(9)", "13a", "13b"}]
```

Out[88]=

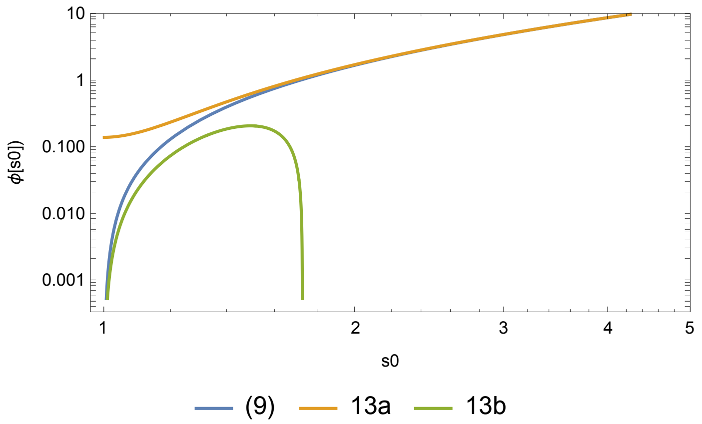


Figure 2: graph of function eq. (18)

In[184]:=

```
Clear[ν, f]
Clear[σ, β, r0, ϕ9, s0, Fβ, σm, s]
ϕ9[s0_?NumericQ] :=
```

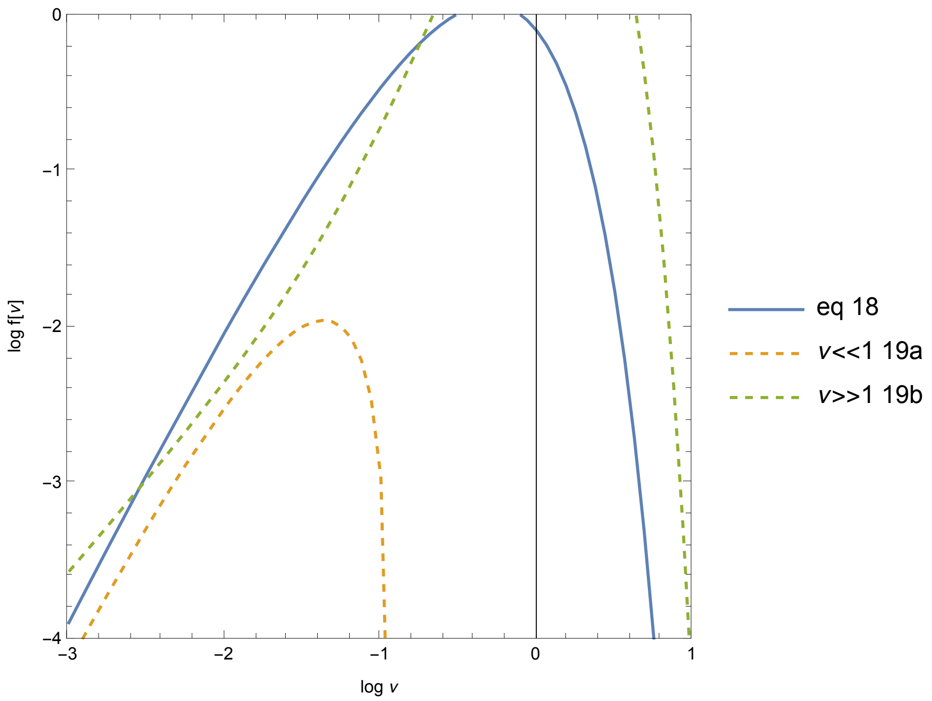
$$\phi9[s0] = \text{NIntegrate}\left[s\,\frac{-2\sqrt{1-\frac{1}{s}}\left(1+\frac{1}{s}\right)+\left(3-\left(1-\frac{1}{s}\right)^{2}\right)\text{Log}\left[\frac{1+\sqrt{1-\frac{1}{s}}}{1-\sqrt{1-\frac{1}{s}}}\right]}{s},\ \{s, 1, s0\}\right]$$

```
f[ν_?NumericQ] := f[ν] = ν^2 NIntegrate[(Exp[ϵ] - 1)^(-1) ϕ9[ϵ / ν], {ϵ, ν, ∞}]
```

$$\text{Plot}\left[\left\{\text{Log}[f[10\text{^}\log\nu]],\ \text{Log}\left[\left(\frac{\pi\text{^}2}{3}\right)10\text{^}\log\nu\ \text{Log}\left[\frac{0.117}{10\text{^}\log\nu}\right]\right],\right.\right.$$
$$\left.\text{Log}\left[\left(\frac{\pi\,10\text{^}\log\nu}{4}\right)\text{^}0.5\ \text{Exp}[-(10\text{^}\log\nu)]\left(1+\frac{75}{8}\times 10\text{^}\log\nu\right)\right]\right\},\ \{\log\nu, -3, 1\},$$

```
AspectRatio → 1, Frame → True, FrameLabel → {"log ν", "log f[ν]"}, PlotPoints → 2,
PlotRange → {{-3, 1}, {-4, 0}}, PlotLegends → {"eq 18", "ν<<1 19a", "ν>>1 19b"},
PlotStyle → {Default, Dashed, Dashed}]
```

Out[188]=




```
(* maximum value ~1 at ν ~1, but to be more precise *)
FindMaximum[f[ν], {ν, 0.5}]
```

Out[173]= {1.07603, {ν → 0.503615}}

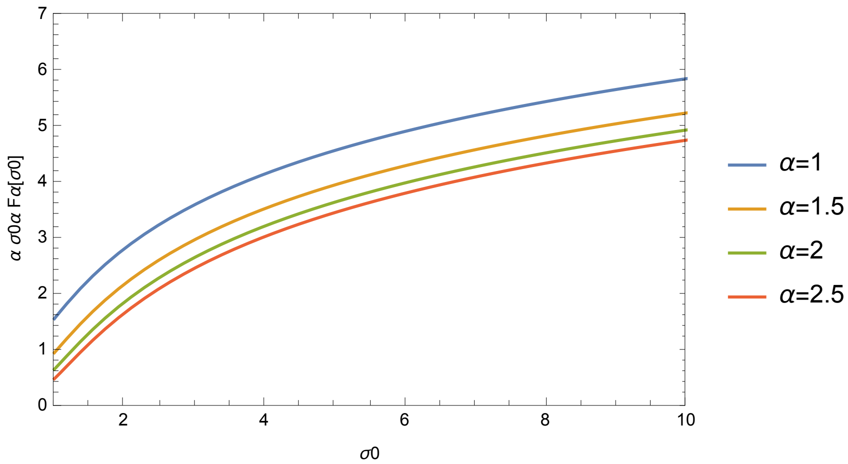
Figure 3: graph of function eq. (23)

In[202]:=
```
Clear[σ, β, r0, ϕ9, s0, Fα, σm, s, eqFig3]
ϕ9[s0_?NumericQ] :=
```

$$\phi9[s0] = \text{NIntegrate}\left[s\,\frac{-2\sqrt{1-\frac{1}{s}}\left(1+\frac{1}{s}\right)+\left(3-\left(1-\frac{1}{s}\right)^2\right)\text{Log}\left[\frac{1+\sqrt{1-\frac{1}{s}}}{1-\sqrt{1-\frac{1}{s}}}\right]}{s},\ \{s,1,s0\}\right]$$

```
Fα[σ0_?NumericQ, α_?NumericQ] :=
 Fα[σ0, α] = NIntegrate[s0^(-α-2) ϕ9[s0], {s0, σ0, ∞}]
eqFig3[σ0_?NumericQ, α_?NumericQ] := eqFig3[σ0, α] = α σ0^α Fα[σ0, α]
Plot[{eqFig3[σ0, 1], eqFig3[σ0, 1.5], eqFig3[σ0, 2], eqFig3[σ0, 2.5]}, {σ0, 1, 10},
 PlotRange → {{1, 10}, {0, 7}}, Frame → True, FrameLabel → {"σ0", "α σ0α Fα[σ0]"},
 PlotLegends → {"α=1", "α=1.5", "α=2", "α=2.5"}]
```

Out[206]=



In[208]:=
```
LogLogPlot[{Fα[σ0, 1], (2/(α σ0^α) (Log[4 σ0] + 1/α - 2)) /. {α → 1},
  (Fα1 - 4/15 (σ0 - 1)^2.5 + (2/21 × (2 α - 1)) (σ0 - 1)^3.5) /. {Fα1 → 1.579, α → 1.5},
  Fα[σ0, 1.5], (2/(α σ0^α) (Log[4 σ0] + 1/α - 2)) /. {α → 1.5},
  (Fα1 - 4/15 (σ0 - 1)^2.5 + (2/21 × (2 α - 1)) (σ0 - 1)^3.5) /. {Fα1 → 0.6373, α → 1.5},
  Fα[σ0, 2], (2/(α σ0^α) (Log[4 σ0] + 1/α - 2)) /. {α → 2},
  (Fα1 - 4/15 (σ0 - 1)^2.5 + (2/21 × (2 α - 1)) (σ0 - 1)^3.5) /. {Fα1 → 0.3275, α → 2},
  Fα[σ0, 2.5], (2/(α σ0^α) (Log[4 σ0] + 1/α - 2)) /. {α → 2.5},
  (Fα1 - 4/15 (σ0 - 1)^2.5 + (2/21 × (2 α - 1)) (σ0 - 1)^3.5) /. {Fα1 → 0.1932, α → 2.5}},
 {σ0, 1, 20}, Frame → True, FrameLabel → {"σ0", "Fα[σ0]"},
 PlotStyle → {Default, Dashed, Dotted, Default, Dashed, Dotted,
   Default, Dashed, Dotted, Default, Dashed, Dotted}, PlotPoints → 2]
```

Out[208]=

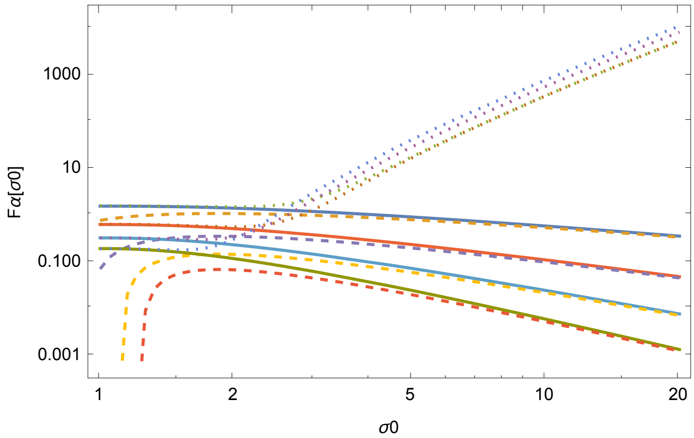


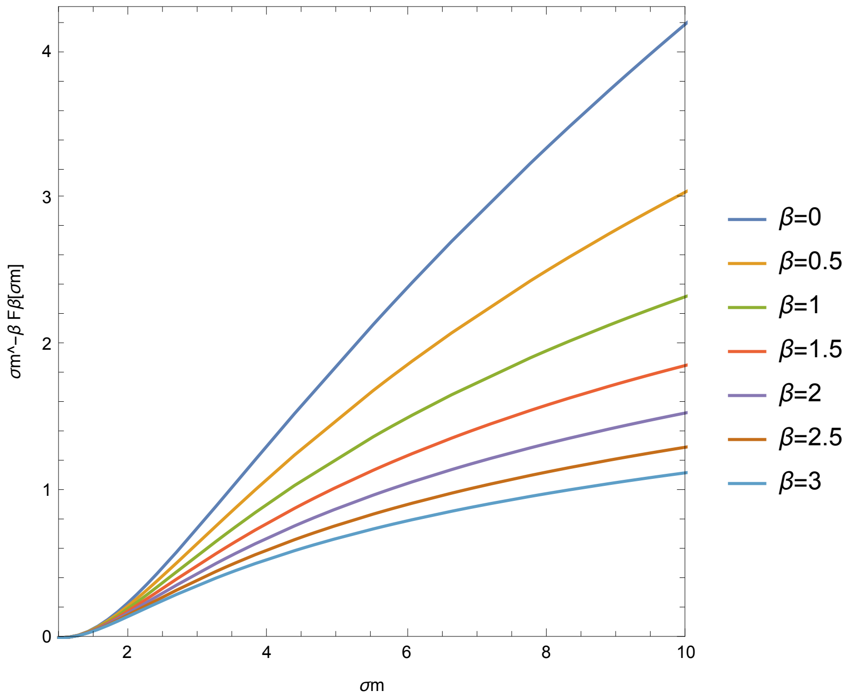
Figure 4: graph of function eq. (28)

In[89]:=
```
Clear[σ, β, r0, ϕ9, s0, Fβ, σm, s]
ϕ9[s0_?NumericQ] :=
```

$$\phi9[s0] = \text{NIntegrate}\left[s\,\frac{-2\sqrt{1-\frac{1}{s}}\left(1+\frac{1}{s}\right)+\left(3-\left(1-\frac{1}{s}\right)^2\right)\text{Log}\left[\frac{1+\sqrt{1-\frac{1}{s}}}{1-\sqrt{1-\frac{1}{s}}}\right]}{s},\ \{s,1,s0\}\right]$$

```
Fβ[σm_?NumericQ, β_?NumericQ] :=
 Fβ[σm, β] = NIntegrate[s0^(β - 2) ϕ9[s0], {s0, 1, σm}]
eq28[σm_, β_] := σm^(-β) Fβ[σm, β]
Plot[{eq28[σm, 0], eq28[σm, 0.5], eq28[σm, 1],
   eq28[σm, 1.5], eq28[σm, 2], eq28[σm, 2.5], eq28[σm, 3]},
  {σm, 1, 10}, PlotPoints → 2, PlotRange → {{1, 10}, {0, 4.3}},
  Frame → True, FrameLabel → {"σm", "σm^-β  Fβ[σm]"}, PlotLegends →
   {"β=0", "β=0.5", "β=1", "β=1.5", "β=2", "β=2.5", "β=3"}, AspectRatio → 1]
```

Out[93]=



In[155]:=
```
Clear[σ, β, r0, ϕ9, s0, Fβ, σm, s, A, eq29a, eq29b]
ϕ9[s0_?NumericQ] :=
 ϕ9[s0] = NIntegrate[s (-2 Sqrt[1 - 1/s] (1 + 1/s) + (3 - (1 - 1/s)^2) Log[(1 + Sqrt[1 - 1/s])/(1 - Sqrt[1 - 1/s])])/s, {s, 1, s0}]
Fβ[σm_?NumericQ, β_?NumericQ] :=
 Fβ[σm, β] = NIntegrate[s0^(β - 2) ϕ9[s0], {s0, 1, σm}]
eq28[σm_, β_] := σm^(-β) Fβ[σm, β]
A[β_] := Piecewise[{{8.111, β == 0}, {13.53, β == 0.5}, {9.489, β == 1},
    {15.675, β == 1.5}, {34.54, β == 2}, {85.29, β == 2.5}, {222.9, β == 3}}]
eq29a[σm_, β_] := Piecewise[{{A[β] + Log[σm]^2 - 4 Log[σm], β == 0},
   {A[β] + 2/β σm^β (Log[4 σm] - 1/β - 2), β ≠ 0}}]
eq29b[σm_, β_] := 4/15 (σm - 1)^2.5 + (2/21 × (2 β + 1)) (σm - 1)^2.5
LogLogPlot[{Fβ[σm, 0], eq29a[σm, 0], eq29b[σm, 0],
  Fβ[σm, 0.5], eq29a[σm, 0.5], eq29b[σm, 0.5], Fβ[σm, 1], eq29a[σm, 1],
  eq29b[σm, 1], Fβ[σm, 1.5], eq29a[σm, 1.5], eq29b[σm, 1.5],
  Fβ[σm, 2], eq29a[σm, 2], eq29b[σm, 2], Fβ[σm, 2.5], eq29a[σm, 2.5],
  eq29b[σm, 2.5], Fβ[σm, 3], eq29a[σm, 3], eq29b[σm, 3]}, {σm, 1, 20},
 PlotPoints → 2, Frame → True, FrameLabel → {"σm", "Fβ[σm]"},
 PlotLegends → {"β=0", "β=0.5", "β=1", "β=1.5", "β=2", "β=2.5", "β=3"},
 AspectRatio → 0.5, PlotStyle → {Default, Dashed, Dotted, Default, Dashed,
   Dotted, Default, Dashed, Dotted, Default, Dashed, Dotted, Default,
   Dashed, Dotted, Default, Dashed, Dotted, Default, Dashed, Dotted}]
```

Out[162]=