Temporal laser-pulse-shape effects in nonlinear Thomson scattering

V. Yu. Kharin, D. Seipt, and S. G. Rykovanov

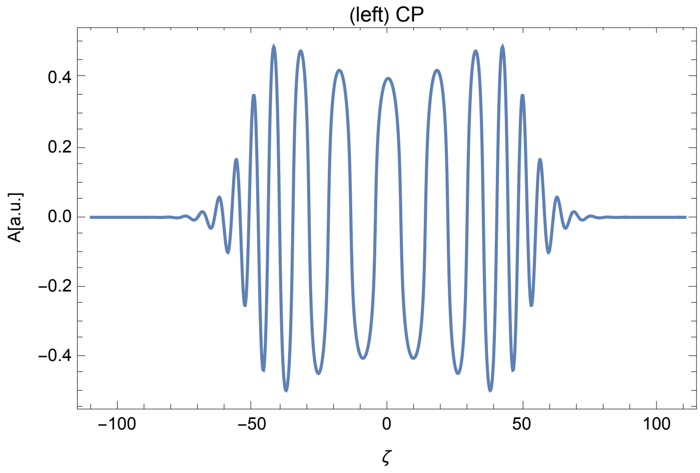
PHYSICAL REVIEW A 93, 063801 (2016)

Notebook: Óscar Amaro, September 2022 @ GoLP-EPP

Figure 2

```
In[•]:= Clear[ξ, ξ, ALP, ACP, R, a, a0, τ, ϕ, int]
R = 1;
a[ϕ_] := a0 Exp[-ϕ^2 / τ^2];
a0 = 2;
τ = 20;
(* the integral has a full analytical solution *)
int = Integrate[(a0 Exp[-ξ^2 / τ^2])^2 Cos[ξ]^2, {ξ, 0, ϕ}];

(* eq 29 implicit relation, solve numerically*)
ϕ[ξ_] := (FindRoot[ϕ + int - ξ, {ϕ, 0}] // Quiet)〚1, 2〛 // Re // Quiet

(* eq 13 *)
ACP[ξ_] := 1/R a[ϕ[ξ]]/(1 + a[ϕ[ξ]]^2) Cos[ϕ[ξ]]
(* eq 28 *)
ALP[ξ_] := 1/R a[ϕ[ξ]] Cos[ϕ[ξ]]/(1 + a[ϕ[ξ]]^2 Cos[ϕ[ξ]]^2)

(* lists *)
ACPlst = ParallelTable[{ξ, ACP[ξ]}, {ξ, -110, +110, 0.2}];
ALPlst = ParallelTable[{ξ, ALP[ξ]}, {ξ, -110, +110, 0.2}];

(* plots *)
ListPlot[ACPlst, Joined → True, Axes → False, Frame → True,
 FrameLabel → {"ξ", "A[a.u.]"}, PlotLabel → "(left) CP"]
ListPlot[ALPlst, Joined → True, Axes → False, Frame → True,
 FrameLabel → {"ξ", "A[a.u.]"}, PlotLabel → "(right) LP"]
```

Out[∘]=



Out[∘]=

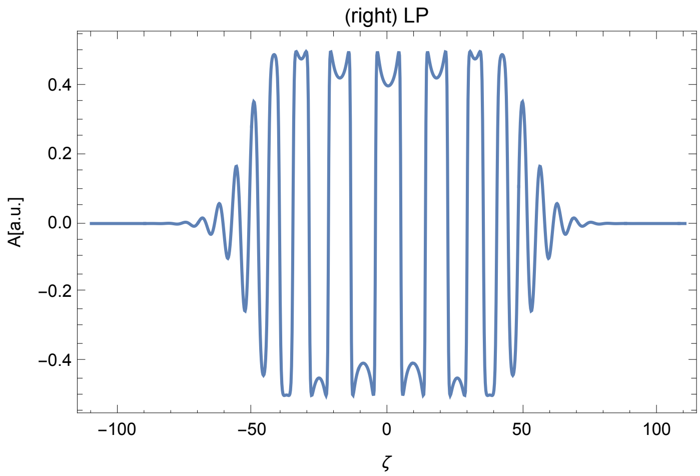


Table 1 / Figure 3 Gaussian

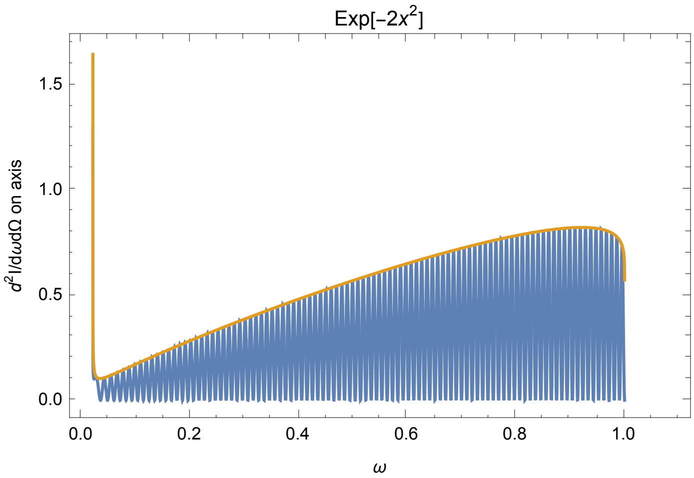
```
Clear[f, ϕ, a, τ, ω, d2I, χ, y, ff, fm1ξ, dfmi, a0]
f = Exp[-2 x^2];
fm1 = (Solve[f == ff, x][[2, 1, 2]] /. {c1 → 0}) // Normal // Simplify;
fm1ξ = fm1 /. {ff → ξ};
dfmi = D[fm1, ff];
y = (1 - ω)/(a0^2 ω);
χ = Refine[ω τ a0^2 Integrate[fm1ξ, {ξ, y, 1}] - π / 4 // Normal, {ω > 0}];
d2I = (2 ω τ)/π Abs[y dfmi /. {ff → y}] Cos[χ]^2;
d2I // Simplify
```

Out[∘]=

$$\frac{\tau\,\omega\,\mathrm{Sin}\left[\frac{1}{4}\left(\pi+\sqrt{2}\,\tau\left(\mathrm{a0}^2\sqrt{\pi}\,\omega\,\mathrm{Erf}\left[\sqrt{\mathrm{Log}\left[-\frac{\mathrm{a0}^2\,\omega}{-1+\omega}\right]}\right]+2\times(-1+\omega)\sqrt{\mathrm{Log}\left[-\frac{\mathrm{a0}^2\,\omega}{-1+\omega}\right]}\right)\right)\right]^2}{\sqrt{2}\,\pi\sqrt{\mathrm{Abs}\left[\mathrm{Log}\left[-\frac{\mathrm{a0}^2\,\omega}{-1+\omega}\right]\right]}}$$

In[•]:=
```
τ = 10; (* choose 10 instead of 200 to see the oscillations,
factor of 2 missing in table? *)
a0 = 7;
Plot[
```
$$\left\{\frac{\tau\,\omega\,\mathrm{Sin}\left[\frac{1}{4}\left(\pi+\sqrt{2}\,\tau\left(a0^2\sqrt{\pi}\,\omega\,\mathrm{Erf}\left[\sqrt{\mathrm{Log}\left[-\frac{a0^2\,\omega}{-1+\omega}\right]}\right]+2\times(-1+\omega)\sqrt{\mathrm{Log}\left[-\frac{a0^2\,\omega}{-1+\omega}\right]}\right)\right)\right]^2}{\sqrt{2}\,\pi\sqrt{\mathrm{Abs}\left[\mathrm{Log}\left[-\frac{a0^2\,\omega}{-1+\omega}\right]\right]}},\right.$$
$$\left.2\,\frac{\omega\,\tau}{4\,\pi}\left(\frac{1}{2}\,\mathrm{Log}\left[\frac{a0\,\hat{}\,2\,\omega}{1-\omega}\right]\right)\hat{}-0.5\right\},\ \{\omega,0,1.1\},\ \mathrm{Frame}\to\mathrm{True},$$
FrameLabel → {"ω", "$d^2I/d\omega d\Omega$ on axis"}, PlotLabel → "$Exp[-2x^2]$"]

Out[•]=

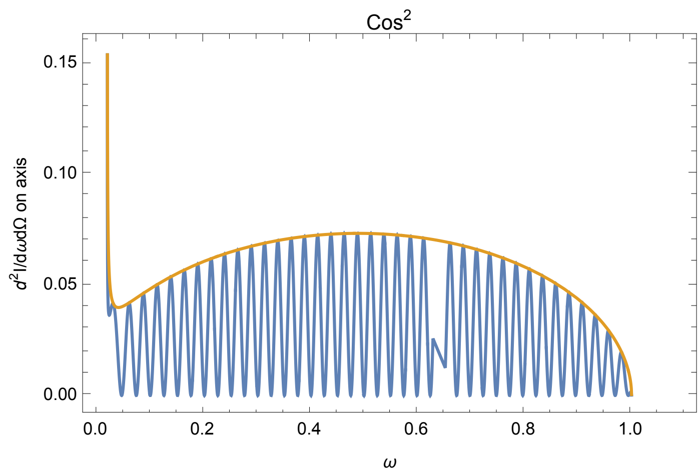


$\cos^2$

In[•]:=
```
Clear[f, ϕ, a, τ, ω, d2I, χ, y, ff, fm1ξ, dfmi, a0]
f = Cos[π x]^2;
fm1 = (Solve[f == ff, x][[4, 1, 2]] /. {c1 → 0}) // Normal // Simplify;
fm1ξ = fm1 /. {ff → ξ};
dfmi = D[fm1, ff];
```
$$y = \frac{1-\omega}{a0\,\hat{}\,2\,\omega};$$
```
χ = Refine[ω τ a0^2 Integrate[fm1ξ, {ξ, y, 1}] - π / 4 // Normal, {ω > 0}];
```
$$d2I = \frac{2\,\omega\,\tau}{\pi}\,\mathrm{Abs}[y\,\mathrm{dfmi}\ /.\ \{ff\to y\}]\ \mathrm{Cos}[\chi]\,\hat{}\,2;$$
```
d2I // Simplify
```

Out[•]=
$$\frac{\tau\,\omega\,\mathrm{Abs}\left[\frac{1-\omega}{a0^2\,\omega\sqrt{-\frac{(-1+\omega)\times(-1+\omega+a0^2\,\omega)}{a0^4\,\omega^2}}}\right]\mathrm{Sin}\left[\frac{\pi^2+2\,a0^2\,\tau\sqrt{-\frac{(-1+\omega)\times(-1+\omega+a0^2\,\omega)}{a0^4}}+2\,\tau\left(-2+(2+a0^2)\,\omega\right)\mathrm{ArcCos}\left[\frac{\sqrt{\frac{1-\omega}{a0^2}}}{\sqrt{\omega}}\right]}{4\,\pi}\right]^2}{\pi^2}$$

In[•]:=
$$\tau = 10;$$
$$a0 = 7;$$
$$\mathrm{Plot}\Big[\Big\{\frac{1}{\pi^2}\,\tau\,\omega\,\mathrm{Abs}\Big[\frac{1-\omega}{a0^2\,\omega\,\sqrt{-\frac{(-1+\omega)\times(-1+\omega+a0^2\,\omega)}{a0^4\,\omega^2}}}\Big]$$
$$\mathrm{Sin}\Bigg[\frac{\pi^2+2\,a0^2\,\tau\,\sqrt{-\frac{(-1+\omega)\times(-1+\omega+a0^2\,\omega)}{a0^4}}+2\,\tau\,\left(-2+\left(2+a0^2\right)\omega\right)\mathrm{ArcCos}\Big[\frac{\sqrt{\frac{1-\omega}{a0^2}}}{\sqrt{\omega}}\Big]}{4\,\pi}\Bigg]^2,$$
$$\frac{\omega\,\tau}{\pi\wedge 2}\left(\frac{a0\wedge 2\,\omega}{1-\omega}-1\right)\wedge -0.5\Big\},\ \{\omega,\ 0,\ 1.1\},\ \mathrm{Frame}\to\mathrm{True},$$
$$\mathrm{FrameLabel}\to\{\text{"}\omega\text{"},\ \text{"}d^2I/d\omega d\Omega\text{ on axis"}\},\ \mathrm{PlotLabel}\to\text{"}\mathrm{Cos}^2\text{"}\Big]$$

Out[•]=


cos^4

In[•]:=
```
Clear[f, ϕ, a, τ, ω, d2I, χ, y, ff, fm1ξ, dfmi, a0]
f = Cos[π x]^4;
fm1 = (Solve[f == ff, x][[8, 1, 2]] /. {ℂ1 → 0}) // Normal // Simplify;
fm1ξ = fm1 /. {ff → ξ};
dfmi = D[fm1, ff];
```
$$y = \frac{1-\omega}{a0\wedge 2\,\omega};$$
```
χ = Refine[ω τ a0^2 Integrate[fm1ξ, {ξ, y, 1}] - π / 4 // Normal, {ω > 0}];
```
$$d2I = \frac{2\,\omega\,\tau}{\pi}\,\mathrm{Abs}[y\,\mathrm{dfmi}\ /.\ \{ff\to y\}]\,\mathrm{Cos}[\chi]\wedge 2;$$
```
d2I // Simplify
```

Out[•]=
$$\frac{\tau\,\omega\,\mathrm{Abs}\Bigg[\frac{\left(\frac{1-\omega}{a0^2\,\omega}\right)^{1/4}}{\sqrt{1-\sqrt{\frac{1-\omega}{a0^2\,\omega}}}}\Bigg]\,\mathrm{Sin}\Bigg[\frac{\pi}{4}+\frac{\tau\left(a0^2\sqrt{1-\frac{\sqrt{\frac{1-\omega}{a0^2}}}{\sqrt{\omega}}}\left(2\sqrt{\frac{1-\omega}{a0^2}}+3\sqrt{\omega}\right)\left(\frac{1-\omega}{a0^2}\right)^{1/4}\omega^{1/4}+\left(-8+\left(8+3\,a0^2\right)\omega\right)\mathrm{ArcCos}\Big[\frac{\left(\frac{1-\omega}{a0^2}\right)^{1/4}}{\omega^{1/4}}\Big]\right)}{8\,\pi}\Bigg]^2}{2\,\pi^2}$$

In[•]:= $\tau = 10;$

$a0 = 5;$

$$\text{Plot}\left[\left\{\frac{1}{2\,\pi^{2}}\;\tau\,\omega\;\text{Abs}\left[\frac{\left(\frac{1-\omega}{a0^{2}\,\omega}\right)^{1/4}}{\sqrt{1-\sqrt{\frac{1-\omega}{a0^{2}\,\omega}}}}\right]\right.\right.$$

$$\text{Sin}\left[\frac{\pi}{4}+\frac{1}{8\,\pi}\;\tau\left(a0^{2}\sqrt{1-\frac{\sqrt{\frac{1-\omega}{a0^{2}}}}{\sqrt{\omega}}}\;\left(2\sqrt{\frac{1-\omega}{a0^{2}}}+3\sqrt{\omega}\right)\left(\frac{1-\omega}{a0^{2}}\right)^{1/4}\omega^{1/4}+\right.\right.$$

$$\left.\left.\left(-8+\left(8+3\,a0^{2}\right)\omega\right)\;\text{ArcCos}\left[\frac{\left(\frac{1-\omega}{a0^{2}}\right)^{1/4}}{\omega^{1/4}}\right]\right)\right]^{2},$$

$$\frac{\omega\,\tau}{2\,\pi\,\hat{}\,2}\left(\text{Sqrt}\left[\frac{a0\,\hat{}\,2\,\omega}{1-\omega}\right]-1\right)\hat{}-0.5\Big\},\;\{\omega,\,0,\,1.1\},\;\text{Frame}\rightarrow\text{True},$$

FrameLabel → {"ω", "d^2I/dωdΩ on axis"},

PlotLabel → "Cos^4"]

Out[•]=

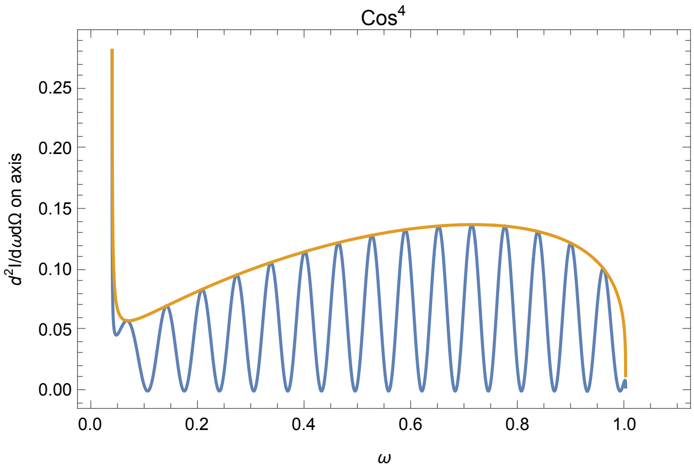


Figure 4

Fig.4 left

In[•]:=
```
Clear[ω, τ, a0]
a0 = 5;
τ = 100;
Plot[{2 (ω τ)/(4 π) (1/2 Log[(a0^2 ω)/(1 - ω)])^-0.5, (ω τ)/(π^2) ((a0^2 ω)/(1 - ω) - 1)^-0.5,
  (ω τ)/(2 π^2) (Sqrt[(a0^2 ω)/(1 - ω)] - 1)^-0.5}, {ω, 0, 1.1}, Frame → True,
 FrameLabel → {"ω", "d²I/dωdΩ on axis"}, PlotLabel → "a0=5,τ=100",
 PlotRange → {0, 12}, PlotLegends → {"Exp[-2x²]", "Cos²", "Cos⁴"}]
```

Out[•]=

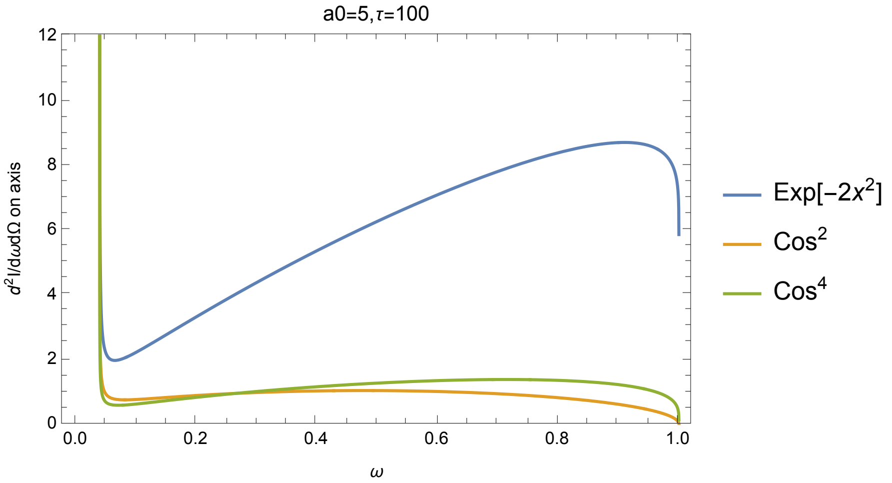
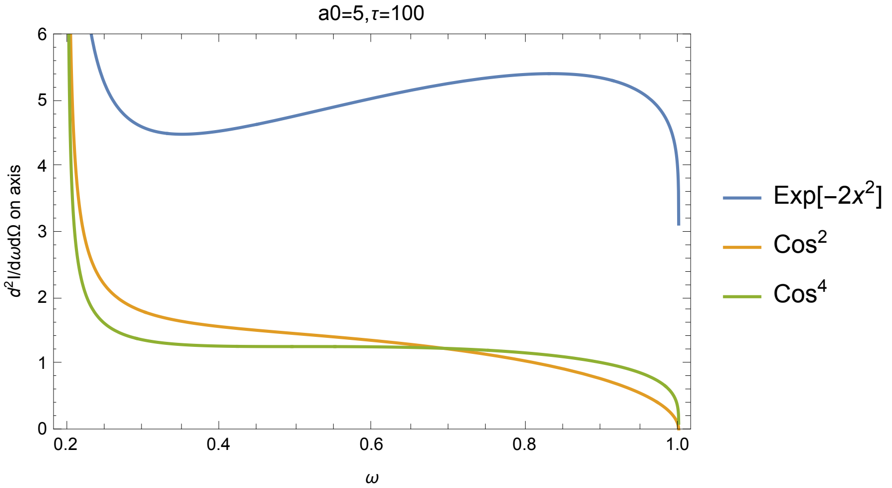


Fig.4 right (a0=2 seems to match the figure, though one would have to normalize it with int |A|^2 fixed)

In[∘]:=
```
Clear[ω, τ, a0]
a0 = 2;
τ = 50;
Plot[{2 (ω τ)/(4 π) (1/2 Log[(a0^2 ω)/(1 - ω)])^-0.5, (ω τ)/(π^2) ((a0^2 ω)/(1 - ω) - 1)^-0.5,
   (ω τ)/(2 π^2) (Sqrt[(a0^2 ω)/(1 - ω)] - 1)^-0.5}, {ω, 0, 1.1}, Frame → True,
  FrameLabel → {"ω", "d²I/dωdΩ on axis"}, PlotLabel → "a0=5,τ=100",
  PlotRange → {0, 6}, PlotLegends → {"Exp[-2x²]", "Cos²", "Cos⁴"}]
```

Out[∘]=

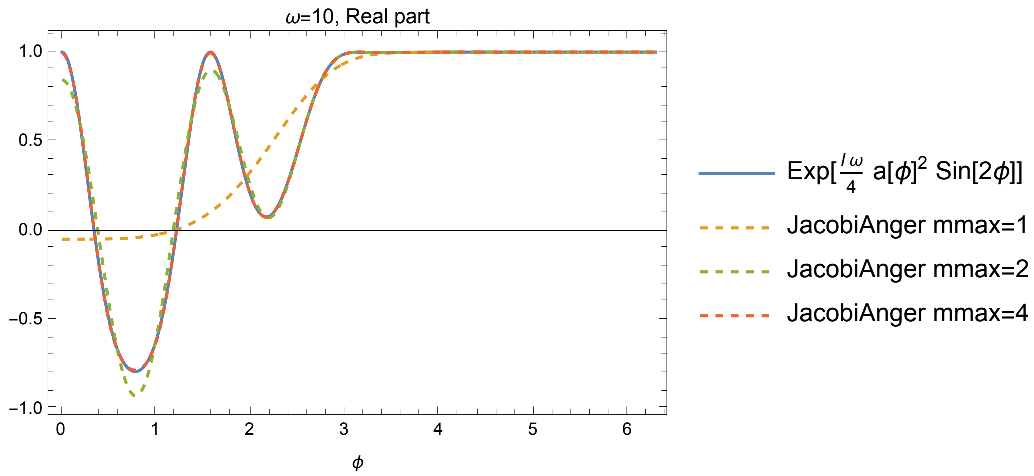


Jacobi-Anger expansion (example)

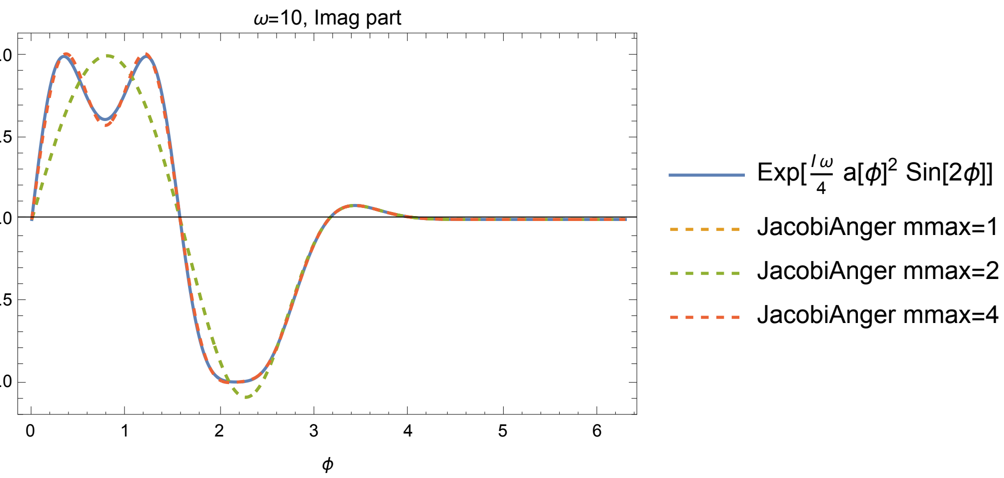
Exp[-ϕ^4/100]

In[∘]:=
```
Clear[ϕ, a, ω, A2L, A2R, m, mmax]
a[ϕ_] := Exp[-ϕ^4 / 100]
ω = 10;
A2L[ϕ_] := Exp[(I ω)/4 a[ϕ]^2 Sin[2 ϕ]]
mmax = 2;
A2R[ϕ_, mmax_] := Sum[BesselJ[m, (ω a[ϕ]^2)/4] Exp[2 I m ϕ], {m, -mmax, mmax}]

Plot[{Re[A2L[ϕ]], Re[A2R[ϕ, 1]], Re[A2R[ϕ, 2]], Re[A2R[ϕ, 4]]},
 {ϕ, 0, 2 π}, PlotStyle → {Default, Dashed, Dashed, Dashed},
 Frame → True, FrameLabel → {"ϕ", ""},
 PlotLegends → {"Exp[(I ω)/4 a[ϕ]^2 Sin[2ϕ]]", "JacobiAnger mmax=1",
   "JacobiAnger mmax=2", "JacobiAnger mmax=4"}, PlotLabel → "ω=10, Real part"]
Plot[{Im[A2L[ϕ]], Im[A2R[ϕ, 1]], Im[A2R[ϕ, 2]], Im[A2R[ϕ, 4]]},
 {ϕ, 0, 2 π}, PlotStyle → {Default, Dashed, Dashed, Dashed},
 Frame → True, FrameLabel → {"ϕ", ""},
 PlotLegends → {"Exp[(I ω)/4 a[ϕ]^2 Sin[2ϕ]]", "JacobiAnger mmax=1",
   "JacobiAnger mmax=2", "JacobiAnger mmax=4"}, PlotLabel → "ω=10, Imag part"]
```

Out[∘]=



Out[]=


ω=10, Imag part
1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
4
5
6
ϕ
Exp[Iω/4 a[ϕ]^2 Sin[2ϕ]]
JacobiAnger mmax=1
JacobiAnger mmax=2
JacobiAnger mmax=4