Electron–positron cascades in multiple-laser optical traps

Marija Vranic, Thomas Grismayer, Ricardo A Fonseca and Luis O Silva, Plasma Phys. Control. Fusion 59 014040 (2017)
Notebook: Óscar Amaro, November 2022 @ GoLP-EPP
Contact: oscar.amaro@tecnico.ulisboa.pt

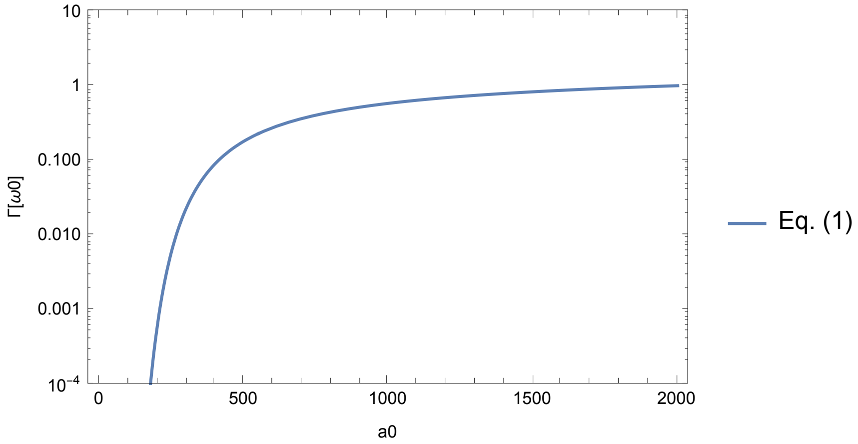
Introduction

In this notebook we reproduce some results from the paper.

Figure 2

In[453]:=
```
Clear[γavg, χavg, a0, aS, τc, ℏ, m, c, α, e, Γ, ω0]

(* physical constants and derived quantities*)
e = 1.6 × 10^-19;
m = 9.1 × 10^-31;
c = 3 × 10^8;
ℏ = 1.054571817 × 10^-34;
ϵ0 = 8.854 × 10^-12;
(**)
α = e^2 / (2 ϵ0 2 π ℏ c);
(*~1/137 fine-structure constant,
the paper uses units in which ϵ0 is not explicit *)
τc = ℏ / (m c^2); (* Compton wavelength *)

(* parameters *)
λ = 0.8 × 10^-6;
ω0 = 2 π c / λ;
aS = m c^2 / (ℏ ω0);
(* normalized vector potential of the Schwinger-Sauter field *)
γavg = 6 a0; (* see text before eq 2 *)
χavg = 12 a0^2 / (π aS); (* see text right after eq 2 *)


(* equation 1 *)
Γ = 8/(15 π) (2 π/3)^0.25 α/(τc γavg) BesselK[1 / 3, 4/(3 χavg)]^2;

(* main plot *)
LogPlot[Γ/ω0 // Quiet, {a0, 0, 2000}, Frame → True, FrameLabel → {"a0", "Γ[ω0]"},
 PlotRange → {10^-4, 10^1}, PlotLegends → {"Eq.  (1)"}]
(* inset *)
Plot[Γ/ω0 // Quiet, {a0, 400, 2000}, Frame → True,
 FrameLabel → {"a0", "Γ[ω0]"}, PlotRange → {10^-4, 1.5}, ImageSize → Small]
```

Out[463]=



Out[464]=

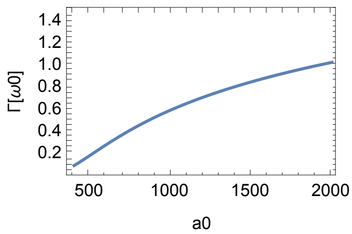


Figure 4

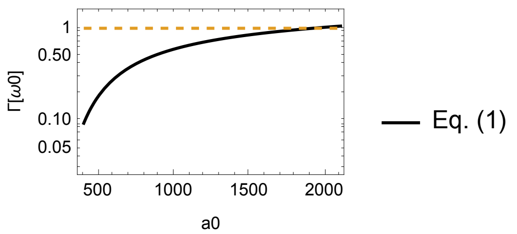
```
Clear[γavg, χavg, a0, aS, τc, ħ, m, c, α, e, Γ, ω0]

(* physical constants and derived quantities*)
e = 1.6 × 10^-19;
m = 9.1 × 10^-31;
c = 3 × 10^8;
ħ = 1.054571817 × 10^-34;
ϵ0 = 8.854 × 10^-12;
(**)
α = e^2 / (2 ϵ0 2 π ħ c);
(*~1/137 fine-structure constant,
the paper uses units in which ϵ0 is not explicit *)
τc = ħ / (m c^2); (* Compton wavelength *)

(* parameters *)
λ = 0.8 × 10^-6;
ω0 = 2 π c / λ;
aS = m c^2 / (ħ ω0);
(* normalized vector potential of the Schwinger-Sauter field *)
γavg = 6 a0; (* see text before eq 2 *)
χavg = 12 a0^2 / (π aS); (* see text right after eq 2 *)

(* equation 1 *)
```

$$\Gamma = \frac{8}{15\,\pi}\left(\frac{2\,\pi}{3}\right)\verb|^|0.25\,\frac{\alpha}{\tau c\,\gamma avg}\,\mathrm{BesselK}\left[1/3,\ \frac{4}{3\,\chi avg}\right]\verb|^|2;$$

```
(* main plot *)
```

$$\mathrm{LogPlot}\left[\left\{\frac{\Gamma}{\omega 0}\ \text{// Quiet, 1}\right\},\ \{a0,\ 400,\ 2100\},\ \mathrm{Frame}\to\mathrm{True},\ \mathrm{FrameLabel}\to\{\text{"a0"},\ \text{"}\Gamma[\omega 0]\text{"}\},\ \mathrm{PlotRange}\to\left\{10^{-1.6},\ 10^{0.2}\right\},\ \mathrm{PlotLegends}\to\{\text{"Eq. (1)"}\},\ \mathrm{ImageSize}\to\mathrm{Small},\ \mathrm{PlotStyle}\to\{\mathrm{Black},\ \mathrm{Dashed}\}\right]$$

Out[607]=



Appendix: Standing wave: 2 pulses, propagating in x-direction, polarized in z-

direction

```
(* confirm with text after (A.1) and before (A.2) *)
Clear[a0, k0, ω0, x, t, ap, am]
ap = {0, 0, a0 Cos[k0 x - ω0 t]};
am = {0, 0, a0 Cos[k0 x + ω0 t]};

(* E: only z component *)
E = -D[ap + am, t] // Simplify
(* B: only y component *)
B = Curl[ap + am, {x, y, z}] // Simplify
```

```
Out[•]= {0, 0, 2 a0 ω0 Cos[k0 x] Sin[t ω0]}

Out[•]= {0, 2 a0 k0 Cos[t ω0] Sin[k0 x], 0}
```

Setup A

In[795]:=
```
Clear[a0, k0, ω0, x, y, z, t, a1, a2, a3, a4, ExB2, fxy, E, B, xmax]
a1 = {0, 0, a0 Cos[k0 x - ω0 t]};
a2 = {0, 0, a0 Cos[k0 x + ω0 t]};
a3 = {0, 0, a0 Cos[k0 y - ω0 t]};
a4 = {0, 0, a0 Cos[k0 y + ω0 t]};

(* E: only z component *)
E = -D[a1 + a2 + a3 + a4, t] // Simplify
(* B: x and y components *)
B = Curl[a1 + a2 + a3 + a4, {x, y, z}] // Simplify

ExB2 = Refine[Norm[Cross[E, B]] ^2 // Simplify,
    {ω0 > 0, k0 > 0, a0 > 0, (Cos[k0 x] + Cos[k0 y]) Sin[k0 y] Sin[2 t ω0] > 0,
     (Cos[k0 x] + Cos[k0 y]) Sin[k0 x] Sin[2 t ω0] > 0}] // Simplify;
fxy = (Refine[Sqrt[ExB2] / (4 a0^2 ω0 k0 Sin[ω0 t] Cos[ω0 t]) // Simplify,
      {ω0 > 0, k0 > 0, a0 > 0, (Cos[k0 x] + Cos[k0 y]) Sin[k0 y] Sin[2 t ω0] > 0,
       (Cos[k0 x] + Cos[k0 y]) Sin[k0 x] Sin[2 t ω0] > 0, t ω0 > 0,
       Sin[2 t ω0] > 0, Cos[k0 x] + Cos[k0 y] > 0}] // Simplify) ^2 // Simplify
(* confirm with equation 4 *)
fxy / ((Cos[k0 x] + Cos[k0 y])^2 (Sin[k0 x] ^2 + Sin[k0 y] ^2)) // Simplify

(* plotting parameters *)
t = 0; k0 = 1; a0 = 1;
xmax = π;

ContourPlot[Sqrt[fxy], {x, -xmax, +xmax}, {y, -xmax, +xmax},
 ColorFunction → "DeepSeaColors", PlotPoints → 20,
 Frame → True, FrameLabel → {"x", "y"}, PlotLabel → "√f"]
ContourPlot[(Norm[E] ^2 + Norm[B] ^2), {x, -xmax, +xmax},
 {y, -xmax, +xmax}, ColorFunction → "DeepSeaColors", PlotPoints → 20,
 Frame → True, FrameLabel → {"x", "y"}, PlotLabel → "E^2+B^2"]
```

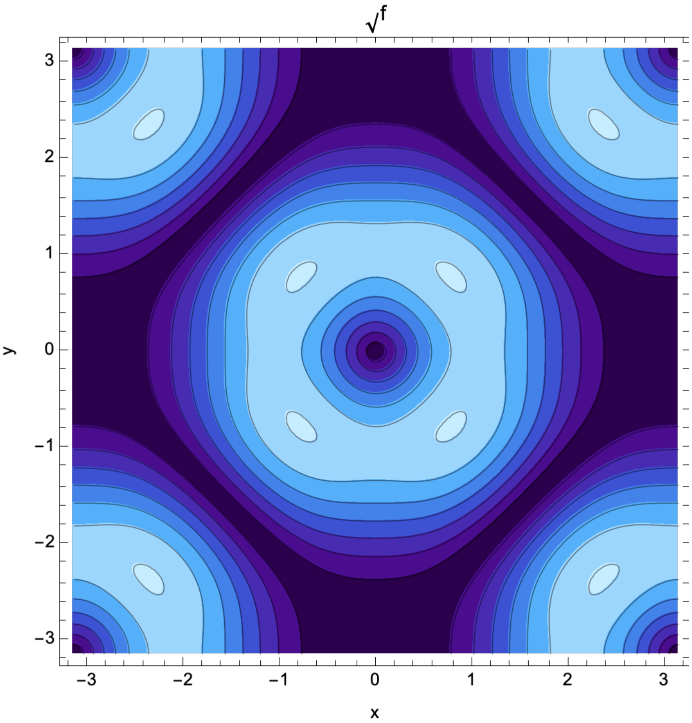
Out[800]= {0, 0, 2 a0 ω0 (Cos[k0 x] + Cos[k0 y]) Sin[t ω0]}

Out[801]= {-2 a0 k0 Cos[t ω0] Sin[k0 y], 2 a0 k0 Cos[t ω0] Sin[k0 x], 0}

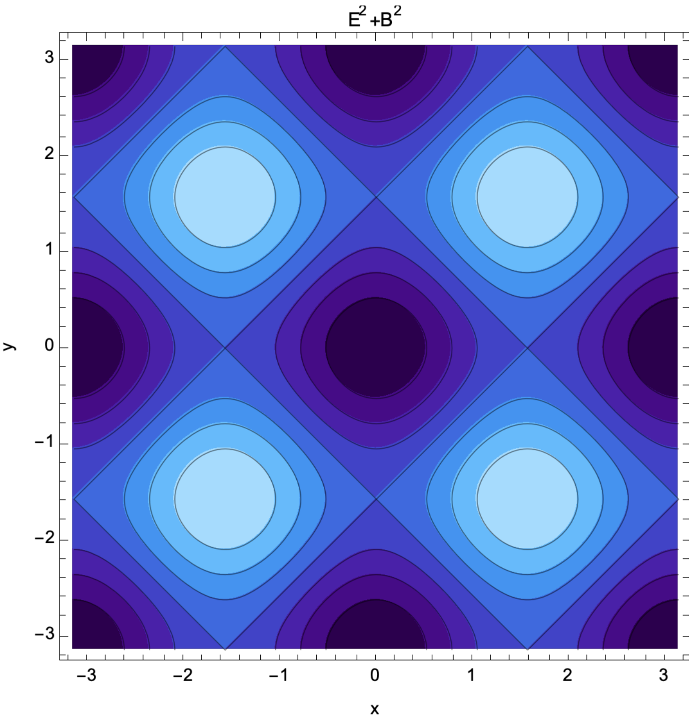
Out[803]= $-\frac{1}{2}\left(\text{Cos}[\text{k0 x}]+\text{Cos}[\text{k0 y}]\right)^2\left(-2+\text{Cos}[2\text{ k0 x}]+\text{Cos}[2\text{ k0 y}]\right)$

Out[804]= 1

Out[807]=



Out[808]=



Setup B

In[809]:=
```
Clear[a0, k0, ω0, x, y, z, t, a1, a2, a3, a4, ExB2, fxy, E, B, xmax]
a1 = {0, a0 Cos[k0 x - ω0 t], 0};
a2 = {0, a0 Cos[k0 x + ω0 t], 0};
a3 = {a0 Cos[k0 y - ω0 t], 0, 0};
a4 = {a0 Cos[k0 y + ω0 t], 0, 0};

(* E: x and y components *)
E = -D[a1 + a2 + a3 + a4, t] // Simplify
(* B: only z component *)
B = Curl[a1 + a2 + a3 + a4, {x, y, z}] // Simplify

ExB2 = Refine[Norm[Cross[E, B]] ^2 // Simplify,
    {ω0 > 0, k0 > 0, a0 > 0, (Cos[k0 x] + Cos[k0 y]) Sin[k0 y] Sin[2 t ω0] > 0,
     (Cos[k0 x] + Cos[k0 y]) Sin[k0 x] Sin[2 t ω0] > 0}] // Simplify;
fxy = (Refine[Sqrt[ExB2] / (4 a0^2 ω0 k0 Sin[ω0 t] Cos[ω0 t]) // Simplify,
      {ω0 > 0, k0 > 0, a0 > 0, (Cos[k0 x] + Cos[k0 y]) Sin[k0 y] Sin[2 t ω0] > 0,
       (Cos[k0 x] + Cos[k0 y]) Sin[k0 x] Sin[2 t ω0] > 0, t ω0 > 0,
       Sin[2 t ω0] > 0, Cos[k0 x] + Cos[k0 y] > 0}] // Simplify) ^2 // Simplify
(* confirm with equation 5 *)
Refine[fxy / ((Cos[k0 x] ^2 + Cos[k0 y] ^2) (Sin[k0 x] - Sin[k0 y]) ^2 ) // Simplify,
 {Sin[k0 x] - Sin[k0 y] > 0}]

(* plotting parameters *)
t = 0; k0 = 1; a0 = 1;
xmax = π;

ContourPlot[Sqrt[fxy], {x, -xmax, +xmax}, {y, -xmax, +xmax},
 ColorFunction → "DeepSeaColors", PlotPoints → 20,
 Frame → True, FrameLabel → {"x", "y"}, PlotLabel → "√f"]
ContourPlot[(Norm[E] ^2 + Norm[B] ^2), {x, -xmax, +xmax},
 {y, -xmax, +xmax}, ColorFunction → "DeepSeaColors", PlotPoints → 20,
 Frame → True, FrameLabel → {"x", "y"}, PlotLabel → "E^2+B^2"]
```

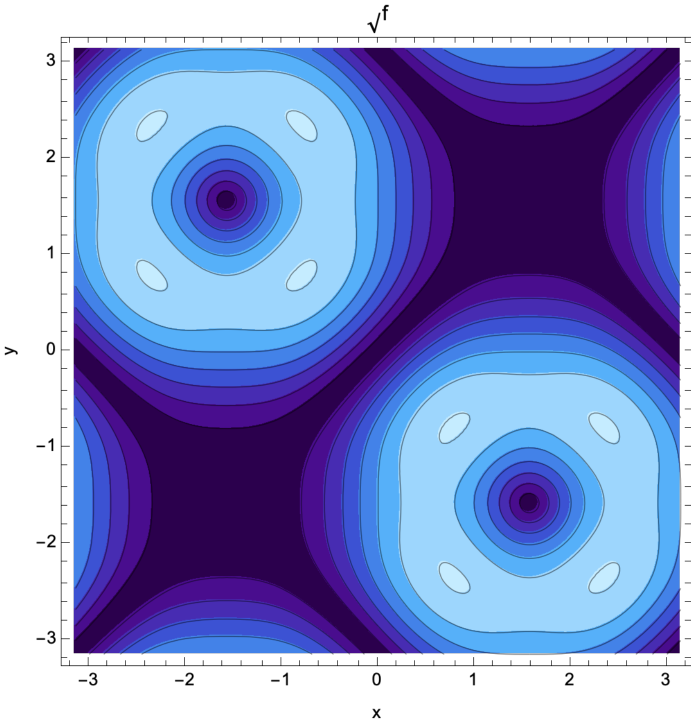
Out[814]= {2 a0 ω0 Cos[k0 y] Sin[t ω0], 2 a0 ω0 Cos[k0 x] Sin[t ω0], 0}

Out[815]= {0, 0, 2 a0 k0 Cos[t ω0] (-Sin[k0 x] + Sin[k0 y])}

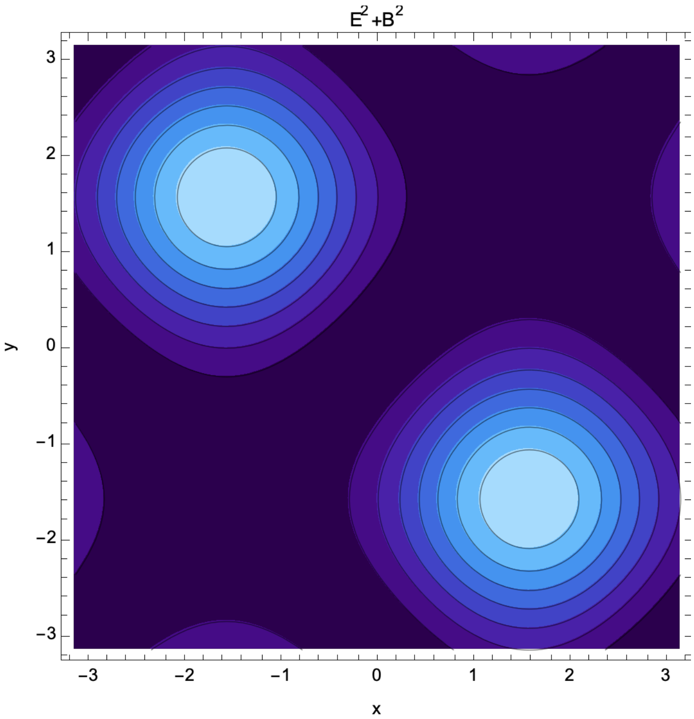
Out[817]= $\frac{1}{2}$ Abs[Sin[k0 x] - Sin[k0 y]]^2 (2 + Cos[2 k0 x] + Cos[2 k0 y])

Out[818]= 1

Out[821]=



Out[822]=



Setup C

In[823]:=
```
Clear[a0, k0, ω0, x, y, z, t, a1, a2, a3, a4, ExB2, fxy, E, B, xmax]
a1 = {0, a0 Cos[k0 x - ω0 t], 0};
a2 = {0, a0 Cos[k0 x + ω0 t], 0};
a3 = {0, 0, a0 Cos[k0 y - ω0 t]};
a4 = {0, 0, a0 Cos[k0 y + ω0 t]};

(* E: y and z components *)
E = -D[a1 + a2 + a3 + a4, t] // Simplify
(* B: x and z components *)
B = Curl[a1 + a2 + a3 + a4, {x, y, z}] // Simplify

ExB2 = Refine[Norm[Cross[E, B]] ^2 // Simplify,
    {ω0 > 0, k0 > 0, a0 > 0, (Cos[k0 x] + Cos[k0 y]) Sin[k0 y] Sin[2 t ω0] > 0,
     (Cos[k0 x] + Cos[k0 y]) Sin[k0 x] Sin[2 t ω0] > 0}] // Simplify;
fxy = (Refine[Sqrt[ExB2] / (4 a0^2 ω0 k0 Sin[ω0 t] Cos[ω0 t]) // Simplify,
       {ω0 > 0, k0 > 0, a0 > 0, (Cos[k0 x] + Cos[k0 y]) Sin[k0 y] Sin[2 t ω0] > 0,
        (Cos[k0 x] + Cos[k0 y]) Sin[k0 x] Sin[2 t ω0] > 0, t ω0 > 0,
        Sin[2 t ω0] > 0, Cos[k0 x] + Cos[k0 y] > 0, Sin[2 k0 x] > 0,
        Sin[2 k0 y] > 0}] // Simplify) ^2 // FullSimplify

(* confirm with equation 6 *)
Refine[fxy /
    (Sin[k0 x] ^2 Cos[k0 x] ^2 + Sin[k0 y] ^2 Cos[k0 y] ^2 + Cos[k0 x] ^2 Sin[k0 y] ^2) //
   Simplify, {Sin[2 k0 x] > 0, Sin[2 k0 y] > 0}] // FullSimplify

(* plotting parameters *)
t = 0; k0 = 1; a0 = 1;
xmax = π;

ContourPlot[Sqrt[fxy], {x, -xmax, +xmax}, {y, -xmax, +xmax},
 ColorFunction → "DeepSeaColors", PlotPoints → 20,
 Frame → True, FrameLabel → {"x", "y"}, PlotLabel → "√f"]
ContourPlot[(Norm[E] ^2 + Norm[B] ^2), {x, -xmax, +xmax},
 {y, -xmax, +xmax}, ColorFunction → "DeepSeaColors", PlotPoints → 20,
 Frame → True, FrameLabel → {"x", "y"}, PlotLabel → "E²+B²"]
```

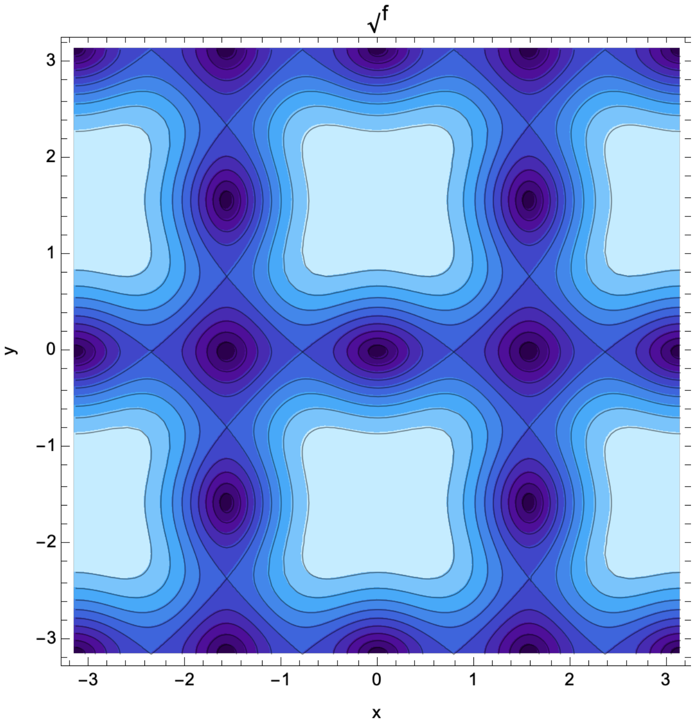
Out[828]= {0, 2 a0 ω0 Cos[k0 x] Sin[t ω0], 2 a0 ω0 Cos[k0 y] Sin[t ω0]}

Out[829]= {-2 a0 k0 Cos[t ω0] Sin[k0 y], 0, -2 a0 k0 Cos[t ω0] Sin[k0 x]}

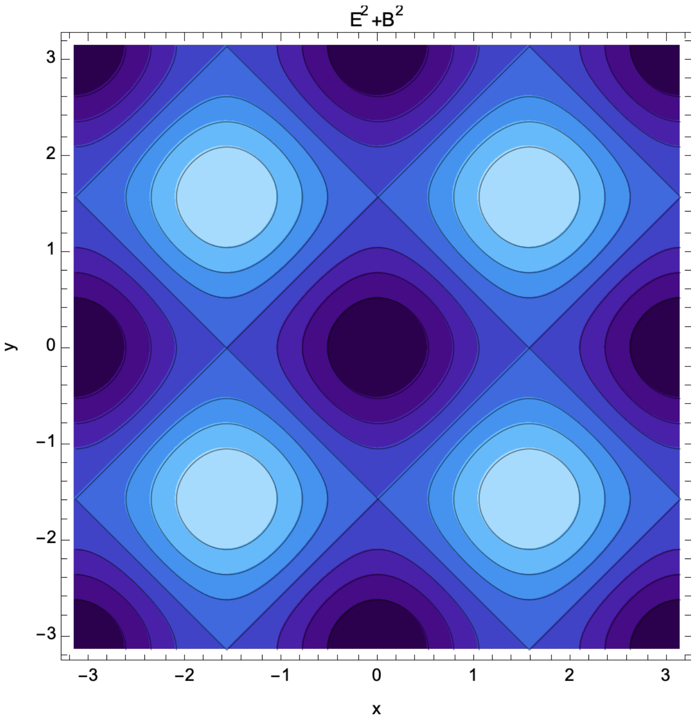
Out[831]= $\frac{1}{4}\left(\mathrm{Sin}[2\,k0\,x]^2 + 2\times(2+\mathrm{Cos}[2\,k0\,x]+\mathrm{Cos}[2\,k0\,y])\,\mathrm{Sin}[k0\,y]^2\right)$

Out[832]= 1

Out[835]=



Out[836]=



In[837]:=
```
(* spatial averages *)
Clear[t, k0, a0, x, y, xmax, fA, fB, fC, fAavg, fBavg, fCavg]
t = 0; k0 = 1; a0 = 1;
xmax = π;

fA = (Cos[k0 x] + Cos[k0 y])^2 (Sin[k0 x]^2 + Sin[k0 y]^2);
fB = (Cos[k0 x]^2 + Cos[k0 y]^2) (Sin[k0 x] - Sin[k0 y])^2 ;
fC = Sin[k0 x]^2 Cos[k0 x]^2 + Sin[k0 y]^2 Cos[k0 y]^2 + Cos[k0 x]^2 Sin[k0 y]^2;

fAavg = NIntegrate[Sqrt[fA], {x, -xmax, +xmax}, {y, -xmax, +xmax}] /
   NIntegrate[1, {x, -xmax, +xmax}, {y, -xmax, +xmax}] // Quiet

fBavg = NIntegrate[Sqrt[fB], {x, -xmax, +xmax}, {y, -xmax, +xmax}] /
   NIntegrate[1, {x, -xmax, +xmax}, {y, -xmax, +xmax}] // Quiet

fCavg = NIntegrate[Sqrt[fC], {x, -xmax, +xmax}, {y, -xmax, +xmax}] /
   NIntegrate[1, {x, -xmax, +xmax}, {y, -xmax, +xmax}] // Quiet

fAavg / fCavg
0.9 / 0.8

(* "It is, therefore, expected that χe⁻ is
  on the same order for all the configurations A-C" *)
```

Out[843]= 0.730708

Out[844]= 0.730708

Out[845]= 0.665408

Out[846]= 1.09813

Out[847]= 1.125