Electron–positron pairs beaming in the Breit–Wheeler process

X Ribeyre, E d'Humières, O Jansen, S Jequier and V T Tikhonchuk, Plasma Phys. Control. Fusion **59** (2017) 014024
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Introduction

In this notebook we reproduce some results from the paper.

Here we reproduce figures 2, 3 and 7. All figures show two cases: $E\gamma1$=$E\gamma2$=4 or $E\gamma1$=4, $E\gamma2$=10.

Geometry

Primed values are from CM frame, unprimed from lab frame.

In[1165]:=
```
Clear[c, me, pγ1, pγ2, Eγ1, Eγ2, pγ1v, pγ2v,
 θcm, θp, Ecm, θpbeam, θe, Emax, Emin, Ee, θebeam]
c = me = 1;
(* photon momenta, by definition *)
pγ1 = Eγ1 / c;
pγ2 = Eγ2 / c;
(* photon momenta according to Figure 1 geometry *)
pγ1v = {pγ1 Cos[θp / 2], -pγ1 Sin[θp / 2], 0};
pγ2v = {pγ2 Cos[θp / 2], pγ2 Sin[θp / 2], 0};

(* equation 1 *)
βcm = (pγ1v + pγ2v) / (Eγ1 + Eγ2);

(* prove eq 2 *)
Refine[Norm[βcm]^2, {Eγ1 > 0, Eγ2 > 0, θp > 0}] // Simplify

(* prove eq 3 *)
(* θcm is angle between pγ1 and βcm (the latter being along x axis) *)
Refine[
```

```
Refine[Tan[Refine[ArcCos[pγ1v.βcm / (pγ1 Norm[βcm])] // Simplify, {Eγ1 > 0, Eγ2 > 0, θp > 0}] //
      Simplify], {Eγ1 > 0, Eγ2 > 0, θp > 0}]^2 // Simplify // Sqrt,
 {Eγ1 > 0, Eγ2 > 0, θp > 0, Eγ1 + Eγ2 Cos[θp] > 0, Sin[θp] > 0}]

(* prove eq 4 *)
(Eγ1^2 + 2 pγ1v.pγ2v + Eγ2^2) - (Eγ1 + Eγ2)^2 // Simplify // TrigReduce // Simplify
Ecm = Sqrt[-2 Eγ1 Eγ2 (-1 + Cos[θp])];

γcm = Refine[1 / Sqrt[1 - Norm[βcm]^2], {θp > 0}]
Eeprime = Ecm / 2;
peprime = Sqrt[Ecm^2 / 4 - 1];
(* equation 7 *)
pell = γcm (peprime Cos[θprime] + Norm[βcm] Eeprime) // Simplify;
θpbeam = ArcCos[1 - (Eγ1 + Eγ2)/(Eγ1 Eγ2)];

(* get θp @ pe|| has imaginary component *)
Solve[-2 + Eγ1 Eγ2 - Eγ1 Eγ2 Cos[θp] == 0, θp];

(* equation 7 *)
θebeam = Refine[ArcTan[peprime / (γcm (Norm[βcm]^2 Eeprime^2 - peprime^2))] // Simplify,
   {Eγ1 > 0, Eγ2 > 0, θp ∈ Reals}] // Simplify

(* don't use equation 14, use equation 6 *)
Ee = (γcm (Eeprime + Norm[βcm] peprime Cos[θprime])) // Simplify;
(*Emax=(Eγ1+Eγ2)/2 + Sqrt[(Eγ1-Eγ2)^2-Ecm^2]Sqrt[Ecm^2-4]/(2Ecm);*)
(*Emin=(Eγ1+Eγ2)/2 - Sqrt[(Eγ1-Eγ2)^2-Ecm^2]Sqrt[Ecm^2-4]/(2Ecm);*)
```

Out[1172]= $\dfrac{E\gamma 1^2 + E\gamma 2^2 + 2\, E\gamma 1\, E\gamma 2\, \mathrm{Cos}[\theta p]}{(E\gamma 1 + E\gamma 2)^2}$

Out[1173]= $\dfrac{E\gamma 2\, \mathrm{Sin}[\theta p]}{E\gamma 1 + E\gamma 2\, \mathrm{Cos}[\theta p]}$

Out[1174]= $2\, E\gamma 1\, E\gamma 2\, (-1 + \mathrm{Cos}[\theta p])$

Out[1176]= $\dfrac{1}{\sqrt{1 - \mathrm{Abs}\left[\frac{E\gamma 1\, \mathrm{Cos}\left[\frac{\theta p}{2}\right] + E\gamma 2\, \mathrm{Cos}\left[\frac{\theta p}{2}\right]}{E\gamma 1 + E\gamma 2}\right]^2 - \mathrm{Abs}\left[\frac{-E\gamma 1\, \mathrm{Sin}\left[\frac{\theta p}{2}\right] + E\gamma 2\, \mathrm{Sin}\left[\frac{\theta p}{2}\right]}{E\gamma 1 + E\gamma 2}\right]^2}}$

Out[1182]= $\mathrm{ArcTan}\left[\dfrac{2\, (E\gamma 1 + E\gamma 2)^2 \sqrt{-\frac{E\gamma 1\, E\gamma 2\, (-1 + \mathrm{Cos}[\theta p])}{(E\gamma 1 + E\gamma 2)^2}}\, \sqrt{-2 + E\gamma 1\, E\gamma 2 - E\gamma 1\, E\gamma 2\, \mathrm{Cos}[\theta p]}}{2\, E\gamma 1^2 + 4\, E\gamma 1\, E\gamma 2 + 2\, E\gamma 2^2 - 3\, E\gamma 1^2\, E\gamma 2^2 + 4\, E\gamma 1^2\, E\gamma 2^2\, \mathrm{Cos}[\theta p] - E\gamma 1^2\, E\gamma 2^2\, \mathrm{Cos}[2\, \theta p]}\right]$

Figure 2: pe|| in lab frame (eq.7)

In[1184]:=
```
Plot[
 {(HeavisideTheta[θp - ArcCos[(-2 + Eγ1 Eγ2)/(Eγ1 Eγ2)]]) pepll /. {Eγ1 → 4, Eγ2 → 4, θprime → π},
  (HeavisideTheta[θp - ArcCos[(-2 + Eγ1 Eγ2)/(Eγ1 Eγ2)]]) pepll /.
   {Eγ1 → 4, Eγ2 → 10, θprime → π},
  (20 HeavisideTheta[θpbeam - θp] - 10) /. {Eγ1 → 4, Eγ2 → 4, θprime → π},
  (20 HeavisideTheta[θpbeam - θp] - 10) /. {Eγ1 → 4, Eγ2 → 10, θprime → π}},
 {θp, 0, π}, PlotRange → {-4, 6}, GridLines → Automatic,
 Frame → True, FrameLabel → {"θp [rad]", "pe||"},
 PlotStyle → {Black, Directive[Blue, Dashed], Black, Directive[Blue, Dashed]},
 Filling → None]
```

Out[1184]=

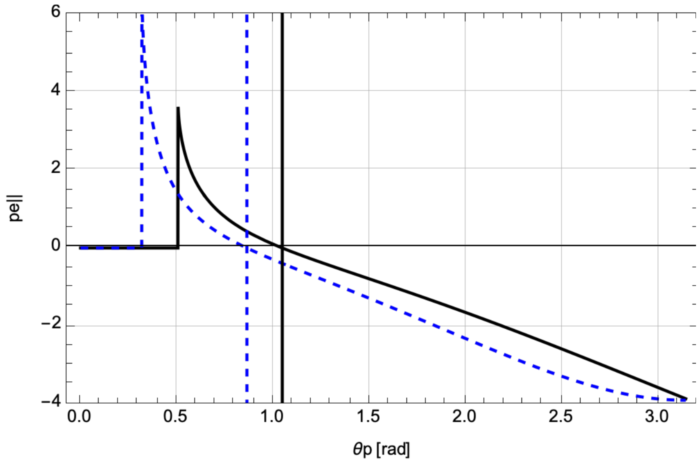

Figure 3: θe,beam (eq.11)

In[1185]:=
```
Plot[{θebeam /. {Eγ1 → 4, Eγ2 → 4, θprime → π
   }, θebeam /. {Eγ1 → 4, Eγ2 → 10, θprime → π}}, {θp, 0, π / 3},
 PlotRange → {{0, π / 3}, {0, π / 2}}, GridLines → Automatic,
 Frame → True, FrameLabel → {"θp [rad]", "θe beam [rad]"},
 PlotStyle → {Black, Directive[Blue, Dashed]}, Filling → None]
```

Out[1185]=

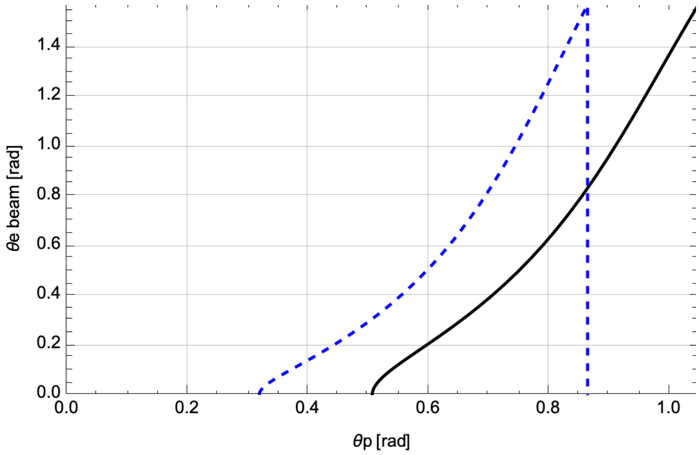

Figure 7: Maximum and minimum energies (eq.6)

bandwidth is maximum for θp=θp,beam
vertical line given by equation 10 θp,beam

In[1186]:=
```
Plot[{(HeavisideTheta[ArcCos[(-2 + Eγ1 Eγ2)/(Eγ1 Eγ2)] - θp] +
      HeavisideTheta[θp - ArcCos[(-2 + Eγ1 Eγ2)/(Eγ1 Eγ2)]] Ee) /.
    {Eγ1 → 4, Eγ2 → 4, θprime → π}, (HeavisideTheta[ArcCos[(-2 + Eγ1 Eγ2)/(Eγ1 Eγ2)] - θp] +
      HeavisideTheta[θp - ArcCos[(-2 + Eγ1 Eγ2)/(Eγ1 Eγ2)]] Ee) /.
    {Eγ1 → 4, Eγ2 → 4, θprime → 0}, (HeavisideTheta[ArcCos[(-2 + Eγ1 Eγ2)/(Eγ1 Eγ2)] - θp] +
      HeavisideTheta[θp - ArcCos[(-2 + Eγ1 Eγ2)/(Eγ1 Eγ2)]] Ee) /.
    {Eγ1 → 10, Eγ2 → 4, θprime → π}, (HeavisideTheta[ArcCos[(-2 + Eγ1 Eγ2)/(Eγ1 Eγ2)] - θp] +
      HeavisideTheta[θp - ArcCos[(-2 + Eγ1 Eγ2)/(Eγ1 Eγ2)]] Ee) /.
    {Eγ1 → 10, Eγ2 → 4, θprime → 0}, (20 HeavisideTheta[θp - ArcCos[1 - (Eγ1 + Eγ2)/(Eγ1 Eγ2)]]) /.
    {Eγ1 → 4, Eγ2 → 4, θprime → π},
   (20 HeavisideTheta[θp - ArcCos[1 - (Eγ1 + Eγ2)/(Eγ1 Eγ2)]]) /. {Eγ1 → 4, Eγ2 → 10, θprime → π}},
  {θp, 0, π}, PlotRange → {{0, π}, {0, 14}},
  GridLines → Automatic,
  Frame → True,
  FrameLabel → {"θp  [rad]", "Emax,Emin"},
  PlotStyle → {Black, Directive[Black, Dashed], Blue,
    Directive[Blue, Dashed], Black, Blue}, Filling → None]
```

Out[1186]=

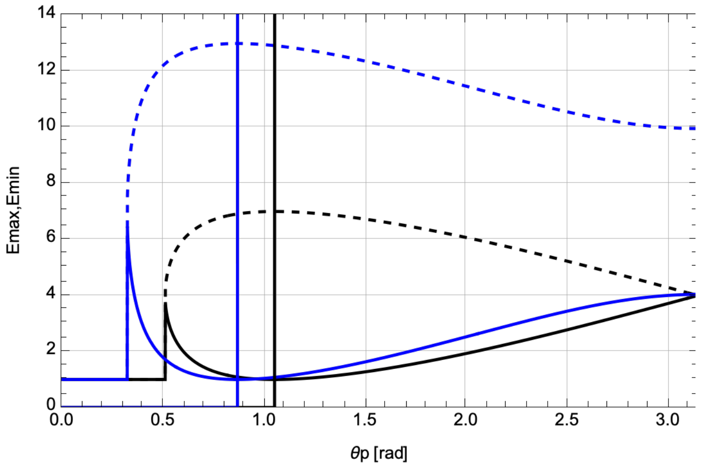

Figure 5

In[1187]:=
```
Clear[Nsmpl, glst, norms]
Nsmpl = 10^4;
glst = RandomVariate[NormalDistribution[], {Nsmpl, 3}];
norms = ParallelTable[Norm[glst[[i]]], {i, 1, Length[glst]}];
glst = glst / norms;
ListPointPlot3D[glst, AspectRatio → 1]
```

Out[1192]=