Chapter 2

Calculus I: Topology Basics and Continuity

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This document may come across boring at the first sight, just like every one of those thick Math books with all the scary symbols you have ever read in your life, or ever dreamed in your nightmare. Well it is not (or at least I hope not). Although there will be a lot of quotations from orthogonal Math textbooks, especially when a new concept is introduced (because everyone who studies Math should know the language of Math as well as the rigorous definitions), the main propose of this document is translate symbolical, mathematical language into human readable English.

I'm sure there will be mistakes, in which case, feel free to open an issue and tell me.

This looks boring only because I don't have the extra time to design a fancy layout.

Keywords: Luke, I'm your auntie.

1 Open Ball & Closed Ball

Just ignore the very lousy-country-song-like title. Before we get into that. you should have known the rigorous definition of **ball** in Chapt 1: Linear Algebra, but if you can't find anything there, it's probably because I haven't finished it yet.

Anyhoo, we define a ball named B as:

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$$B_{\mathrm{p}}(x, \epsilon) = \{z \in \mathbb{R}^n | \|z - x\| < \epsilon\}.........(1)$$

So, a ball is just like, well, a ball. You could imagine it being round (although they might not visually look like so, which I can explain later, it is helpful to imagine them being that way), therefore there is a center, and a radius.

The definition above is actually quite straight forward:

- The letter B tells you, that you are looking at a thing named B. Not all balls are denoted as B, nor should they be. You can call your ball Taylor Swift if you want.
- The subscript p indicates what kind of norm is being used here, based on which the concept **distance** is defined.

  This is important and absolutely necessary, because in the crazy world of Math, you can define different ways in which *distance* is calculated. You can, for example, calculate it with the famous Manhattan Distance, a.k.a. L^1 norm. Later you'll see, it is crucial to know how the distance between the points in a ball and the center of the ball, is defined.
- We define x as the center, and ϵ as the radius of the ball.

As the equation (1) is essentially defining a set, we should here know that a ball is just another set. Imagine a concrete ball in real life, if you will. So the whole body, every bit of concrete that the ball is made of, is a subset of the ball. With this vision, we can turn to the definition of z in (1). z here, can be any single point in the space $\mathbb{R}^2$, which should not be difficult to understand, since it is basically a 2-D space made up with all real numbers. But the inequality after the bar sign tells you that although z can be any value in $\mathbb{R}$ space, it must full fill some conditions to be included in the set named B, which is:

$$\|z - x\| < \epsilon$$

It means, the distance between z and x must be smaller than ϵ, and only those z which satisfy this requirement, can be included in to the ball B.

Now here is the interesting part: You might have noticed, that for those z which are in $\mathbb{R}^2$, but stand right at the place where is exact ϵ away from the center x, should not be included in the ball B. So the ball B is without a surface, but imagine the surface being incredibly thin, no mater what number you have in mind to measure it, it's always thinner.

And that my friend, is a open ball.

Now you now the open ball, it is very easy to imagine a **closed ball**, right?

$$B_{\mathrm{p}}(x, \epsilon) = \{z \in \mathbb{R}^n | \|z - x\| \leq \epsilon\}.........(2)$$

2 Open Set & Closed Set

We have open and closed ball already, now we need to use the definition of ball to define what **open set** and **closed set** are.
A set can be either **open** or **closed**, and here is the mathematical definition:

An arbitrary set Ω is *open*, if for any $x\ \epsilon\ \in\ \Omega$, there exits $\epsilon > 0$, such that $B_{\mathrm{p}}(x, \epsilon) \subset \Omega$.

An arbitrary set Ω is *closed*, if the complement of Ω, Ω^{c} is open.

This should not be hard to understand, since the way an open set is defined is exact the same as *limit* is. Because you can always find a smaller ϵ as the radius so that subset B with center x wouldn't stick its nose outside of Ω, you can always find another x infinitely close to the *boundary*. You can imagine a smaller ball approaching the *boundary* of a big super set in motion as its radius getting smaller.(If the concept of *limit* is unfamiliar to you, maybe you can take an intro to Calculus course online first. It's usually the first thing being introduced during the first session.)

This is almost the standard way of defining the motion of infinite approximation. As a matter of fact, in some cases, the *boundary* of a set(no matter the set is open or closed) is exactly the limit to all the series/functions inside of that set.

3 Boundary and Closure

Now I have mentioned **boundary** couple of times already, and I've always made the font italic. I was not doing that because I didn't have better things to do, or in Chinese, 吃饱了撑的. I did that because when I said it, I meant it in a rigorous mathematical context:

The **boundary** $\partial\Omega$ of set $\Omega \subset \mathbb{R}^n$ are all points $x \in \mathbb{R}^n$, such that for every $\epsilon > 0$ the intersection $B(x, \epsilon) \cap \Omega \neq \emptyset$and $B(x, \epsilon) \cap \Omega^c \neq \emptyset$

So imagine a big super set, let's call it Ω, and there's a ball with its center as x and its radius of any positive value as ϵ. Then there is only one possible position that this ball could be in so that it could has non-empty intersection with both Ω and Ω^c. The position is the on the boarder of the set, and that's how we define a *boundary*, very intuitive. Maybe the Webster dictionary could consider defining the word this way.

=======the stuffs below are, for now, just nonsense from my template=======

Phasellus maximus erat ligula, accumsan rutrum augue facilisis in. Proin sit amet pharetra nunc, sed maximus erat. Duis egestas mi eget purus venenatis vulputate vel quis nunc. Nullam volutpat facilisis tortor, vitae semper ligula dapibus sit amet. Suspendisse fringilla, quam sed laoreet maximus, ex ex placerat ipsum, porta ultrices mi risus et lectus. Maecenas vitae mauris condimentum justo fringilla sollicitudin. Fusce nec interdum ante. Curabitur tempus dui et orci convallis molestie

(Chomsky 1957).

Table 1: Frequencies of word classes

	nouns	verbs	adjectives	adverbs
absolute	12	34	23	13
relative	3.1	8.9	5.7	3.2

Sed nisi urna, dignissim sit amet posuere ut, luctus ac lectus. Fusce vel ornare nibh. Nullam non sapien in tortor hendrerit suscipit. Etiam sollicitudin nibh ligula. Praesent dictum gravida est eget maximus. Integer in felis id diam sodales accumsan at at turpis. Maecenas dignissim purus non libero scelerisque porttitor. Integer porttitor mauris ac nisi iaculis molestie. Sed nec imperdiet orci. Suspendisse sed fringilla elit, non varius elit. Sed varius nisi magna, at efficitur orci consectetur a. Cras consequat mi dui, et cursus lacus vehicula vitae. Pellentesque sit amet justo sed lectus luctus vehicula. Suspendisse placerat augue eget felis sagittis placerat.

(1) cogito ergo sum
think.1SG.PRES therefore COP.1SG.PRES
'I think therefore I am.'

Sed cursus eros condimentum mi consectetur, ac consectetur sapien pulvinar. Sed consequat, magna eu scelerisque laoreet, ante erat tristique justo, nec cursus eros diam eu nisl. Vestibulum non arcu tellus. Nunc dignissim tristique massa ut gravida. Nullam auctor orci gravida tellus egestas, vitae pharetra nisl porttitor. Pellentesque turpis nulla, venenatis id porttitor non, volutpat ut leo. Etiam hendrerit scelerisque luctus. Nam sed egestas est. Suspendisse potenti. Nunc vestibulum nec odio non laoreet. Proin lacinia nulla lectus, eu vehicula erat vehicula sed.

4 Illustrating OSL commands and conventions

Below I illustrate the use of a number of commands defined in langsci-osl.tex (see the styles folder). In §4.4 I add a simple tree.

4.1 Typesetting semantics

Semantic interpretation brackets:

(2) $[\![\text{dog}]\!]^g = \textsc{dog} = \lambda x[\textsc{dog}(x)]$

Use noindent after example environments (but not after floats like tables or figures).

And here's a macro for semantic type brackets: The expression *dog* is of type $\langle e, t\rangle$. Don't forget to place the whole type formula into a math-environment. An example of a more complex type, such as the one of *every*: $\langle s, \langle\langle e, t\rangle, \langle e, t\rangle\rangle\rangle$. You can of course also use the type in a subscript: $\text{dog}_{\langle e,t\rangle}$

We distinguish between metalinguistic constants that are translations of object language, which are typeset using smallcaps, see (2), and logical constants. See the contrast in (3), where SPEAKER (= serif) in (3a) is the denotation of the word *speaker*, and SPEAKER (= sans-serif) in (3b) is the function that maps the context c to the speaker in that context.[1]

(3) a. $[\![\text{The speaker is drunk}]\!]^{g,c} = \textsc{drunk}\big(\iota x\ \textsc{speaker}(x)\big)$

b. $[\![\text{I am drunk}]\!]^{g,c} = \textsc{drunk}\big(\mathsf{SPEAKER}(c)\big)$

Notice that with more complex formulas, you can use bigger brackets indicating scope, cf. (vs. $\big($, as used in (3). Also notice the use of backslash plus comma, which produces additional space in math-environment.

4.2 Typesetting non-glossed elements in examples

Try to keep examples simple. But if you need to pack more information into an example or include more alternatives, you can resort to various brackets or slashes. For that, you will find the minsp-command useful. It works as follows:

(4) Hans {schläft / schlief / *schlafen}.
Hans sleeps slept sleep.INF
'Hans {sleeps / slept}.'

[1]Notice that both types of smallcaps are automatically turned into text-style, even if used in a math-environment. This enables you to use math throughout.

If you use the command, glosses will be aligned with the corresponding object language elements correctly. Notice also that brackets etc. do not receive their own gloss. Simply use the placeholder {} in the code.

The minsp-command is not needed for grammaticality judgments used for the whole sentence. For that, use the native langsci-gb4e method instead, as illustrated below:

(5) * Das sein ungrammatisch.
that be.INF ungrammatical
Intended: 'This is ungrammatical.'

Also notice that translations should never be ungrammatical. If the original is ungrammatical, provide the intended interpretation in idiomatic English.

4.3 Citation commands and macros

You can make your life easier if you use the following citation commands and macros (see code):

- Bailyn 2004: no brackets
- Bailyn (2004): year in brackets
- (Bailyn 2004): everything in brackets
- Bailyn's 2004: possessive
- Bailyn's (2004): possessive with year in brackets

4.4 A tree

Use the forest package for trees and place trees in a figure environment. Figure 1 shows a simple example.[2]

[2] See Vanden Wyngaerd (2017) for a simple and useful quickstart guide for the forest package.

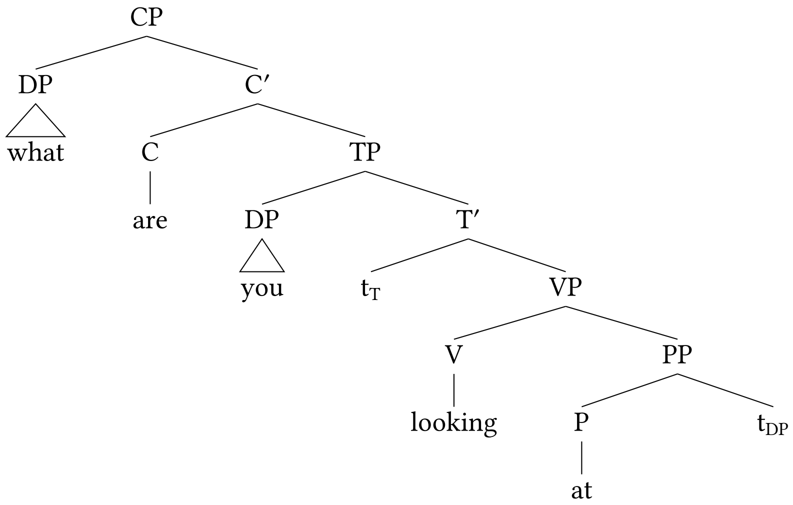

Figure 1: A normal CP

Abbreviations

1	first person	PRES	present tense
COP	copula	SG	singular

Acknowledgements

Place your acknowledgements here.

References

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