# Project #12 : Smallest Enclosing Ellipsoid

This final project serves as the final competency review for the course. It can be completed as a report or as a presentation/discussion with the instructor. This project does not require any coding, but should be carefully considered

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Consider the problem of finding the boundary of a point cloud in $\mathbb{R}^n$. We seek the ellipsoid of smallest volume that contains the set of points $\{x_1, x_2, \ldots, x_m\} \subset \mathbb{R}^n$. One possible representation of an ellipsoid is

$$E = \left\{ x \in \mathbb{R}^n \ \mid \ (x - x_0)^\mathsf{T} U^\mathsf{T} D U (x - x_0) \leq 1 \right\}.$$

In this representation, $D$ is a diagonal matrix of strictly positive entries, $U$ is a orthogonal matrix representing a coordinate rotation, and $x_0$ is the center of the ellipsoid. Other equivalent representations are acceptable. Your report or presentation should address the following tasks/ideas.

1. Formulate the problem of finding the minimum volume ellipse as a constrained optimization problem.

2. Analyze your formulation in terms of differentiability, continuity, etc.

3. Discuss any necessary relationships between the number of decision variables and the number of data points.

4. Suggest and justify a solution method/algorithm.

5. Suggest a modification to the problem formulation that allows outlier points to lie outside of the ellipsoid. How do the above analyses change?

It is acceptable to discuss some of your ideas and work with others. Be sure and give proper credit to the ideas of others.