

# Abstract Algebra Definitions Checklist

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## 1 Basic

### 1.1 Properties for operations

1. Closure.
2. Associative.
3. Commutative.
4. Unital (existence of identity)
5. Distributive.
6. Invertible.

## 2 Group-related

### 2.1 Group

### 2.2 Semi-group

### 2.3 Monoid

## 3 Ring-related

### 3.1 Ring

#### 3.1.1 Definition 1

A nonempty set  $R$  with two operations  $+$  (usually written as addition and multiplication) that satisfy the following axioms. For  $\forall a, b, c \in R$ :

1. If  $a \in R$  and  $b \in R$ , then  $a + b \in R$  (closure addition).
2.  $a + (b + c) = (a + b) + c$  (Associative addition).
3.  $a + b = b + a$  (Commutative addition).
4. There is an element  $0_R$  in  $R$  such that  $a + 0_R = a = 0_R + a$  for every  $a \in R$ . (Additive identity or zero element).
5. For each  $a \in R$ , the equation  $a + x = 0_R$  has a solution in  $R$ .
- 6.

### 3.1.2 Definition 2

A nonempty set  $R$  with addition and multiplication such that:

1.  $(R, +)$  is an abelian group.
2.  $(R, \cdot)$  is a semigroup.
3.  $(R, +, \cdot)$  is distributive for addition and multiplication.

### 3.1.3 Relative Extension

1. Commutative Ring: ring  $R$  satisfies  $ab = ba$  for  $\forall a, b \in R$  (Commutative multiplication).
2. Ring with identity: ring  $R$  that contains an element  $1_R$  satisfying  $a1_R = a = 1_Ra$  for  $\forall a \in R$  (multiplicative identity).

## 4 Field-related

## 5 Others

- 1.