Abstract Algebra Definitions Checklist

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1 Basic

1.1 Properties for operations

1. Closure.
2. Associative.
3. Commutative.
4. Unital (existence of identity)
5. Distributive.
6. Invertible.

2 Group-related

2.1 Group

2.2 Semi-group

2.3 Monoid

3 Ring-related

3.1 Ring

1. Definition 1
   A nonempty set R with two operations *(usually written as addition and multiplication) that satisfy the following axioms. For $\forall a, b, c \in R$:
   (a) If $a \in R$ and $b \in R$, then $a + b \in R$ (closure for addition).
   (b) $a + (b + c) = (a + b) + c$ (Associative addition).
   (c) $a + b = b + a$ (Commutative addition).
   (d) There is an element 0_R in R such that $a + 0_R = a = 0_R + a$ for every $a \in R$. (Additive identity or zero element).
   (e) For each $a \in R$, the equation $a + x = 0_R$ has a solution in R.
   (f) If $a \in R$ and $b \in R$, then $ab \in R$ (closure for multiplication).
   (g) $a(bc) = (ab)c$ (associative multiplication).
   (h) $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ (distributivity).
2. Definition 2
   A nonempty set R with addition and multiplication such that:

(a) $(R, +)$ is an abelian group.
(b) $(R, \cdot)$ is a semigroup.
(c) $(R, +, \cdot)$ is distributive for addition and multiplication.

3. Relative Extension

   (a) Commutative Ring: ring R satisfies $ab = ba$ for $\forall a, b \in R$ (Commutative multiplication).
   (b) Ring with identity: ring R that contains an element 1_R satisfying $a1_R = a = 1_R a$ for $\forall a \in R$ (multiplicative identity).

3.2 Subring

1. When a subset S of a ring R is itself a ring under the addition and multiplication in R, then we say that S is a subring of R.

2. R is a ring and S is a subset of R such that

   (a) S is closure under addition and multiplication.
   (b) $0_R \in S$.
   (c) If $a \in S$ then the solution of the equation $a + x = 0_R$ in S (addition inverse exists).

   Then S is a subring of R.

3.3 Integral Domain

An integral domain is a commutative ring R with identity $1_R \neq 0_R$ such that:
whenever $a, b \in R$ and $ab = 0_R$ then $a = 0_R$ or $b = 0_R$.

4 Field-related

4.1 Field

A field is a commutative ring R with identity $1_R \neq 0_R$ such that:
For each $a \neq 0_R$ in R, the equation $ax = 1_R$ has a solution in R.

5 Others

1.