TD 02 - Logical aspect of databases

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Exercise 1. We suggest the following changes:

- Add a unique identifier to `Movie`
- Replace the `Title` field in `Cinema`, `Produced`, `Seen` and `Likes` by a `Movie` identifier

Exercise 2. Consider the following queries:

1. PSJR algebra:
$$\Pi_{\text{Name,Time}}\left(\sigma_{\text{Title=“Mad Max”}}(\text{Cinema})\right)$$
Conjunctive calculus:
$$\text{Cinema}(x_{\text{Title}}, x_{\text{Name}}, \text{“Mad Max”})$$

2. PSJR algebra:
$$\Pi_{\text{Title}}\left(\sigma_{\text{Director=“Orson Welles”}}(\text{Movie})\right)$$
Conjunctive calculus:
$$\exists x, \text{Movie}(x_{\text{Title}}, \text{“Orson Welles”}, x)$$

3. PSJR algebra:
$$\Pi_{\text{Actor}}\left(\sigma_{\text{Title=“Ran”}}\right)$$
Conjunctive calculus:
$$\exists x, \text{Movie}(\text{“Ran”}, x, x_{\text{Actor}})$$

4. PSJR algebra:
$$\Pi_{\text{Name}}\left(\Pi_{\text{Title}}(\sigma_{\text{Actor=“Signoret”}}(\text{Movie})) \underset{\text{Title=Cinema.Title}}{\bowtie} \text{Cinema}\right)$$
Conjunctive calculus:
$$\exists x_{\text{Title}}\left(\exists x \text{ Movie}(x_{\text{Title}}, x, \text{“Signoret”})\right) \wedge \left(\exists x \text{ Cinema}(x_{\text{Name}}, x, x_{\text{Title}})\right)$$

5. PSJR algebra:
$$\Pi_{\text{Actor}}\left(\text{Movie} \underset{\text{Movie.Actor=Produced.Produced}}{\bowtie} \text{Produced}\right)$$
Conjunctive calculus:
$$(\exists x, \exists y, \text{Movie}(x, y, x_{\text{Actor}})) \wedge (\exists x, \text{Produced}(x_{\text{Actor}}, x))$$

6. PSJR algebra:

$$\Pi_{\text{Actor}}\left(\text{Movie} \underset{\substack{\text{Movie.Actor=Produced.Producer}\\ \text{Movie.Title=Produced.Title}}}{\bowtie} \text{Produced}\right)$$

Conjunctive calculus:

$$\exists x_{\text{Title}},\ (\exists d,\ \text{Movie}(x_{\text{Title}}, d, x_{\text{Actor}}) \wedge \text{Produced}(x_{\text{Actor}}, x_{\text{Title}}))$$

7. PSJR algebra:

$$\Pi_{\text{Actor}}\left(\Pi_{\text{Title}}(\sigma_{\text{Actor=“Orson Welles”}}(\text{Movie})) \underset{\text{Title=Movie.Title}}{\bowtie} \text{Movie}\right)$$

Conjunctive calculus:

$$\exists x_{\text{Title}},\ \Big((\exists d\ \text{Movie}(x_{\text{Title}}, d, \text{“Orson Welles”})) \wedge (\exists d\ \text{Movie}(x_{\text{Title}}, d, x_{\text{Actor}}))\Big)$$

8. This query is impossible in PSJR algebra since it is not monotone. Indeed, consider D_0 a database on the given schema, containing a movie

$$\text{Movie}(\text{“Citizen Kane”}, \text{“Orson Welles”}, \text{“Orson Welles”})$$

and a producer

$$\text{Produced}(\text{“Orson Welles”}, \text{“Citizen Kane”})$$

In this database, the query “Which producers produce all the movies directed by Akira Kurosawa?” returns the set {“Orson Welles”}. Now, consider D_1 the database D_0 to which we added the following movie:

$$\text{Movie}(\text{“七人の侍”}, \text{“Akira Kurosawa”}, \text{“Toshiro Mifune”})$$

In this database, the query “Which producers produce all the movies directed by Akira Kurosawa?” returns the empty set. Therefore, this query is not monotone, and cannot be expressed in PSJR algebra and conjunctive calculus.

Exercise 3. Consider the following queries:

1. PSJRU algebra:

$$\Pi_{\text{Viewer}}(\text{Seen}) \setminus \Pi_{\text{Viewer}}\ \underbrace{\Big((\Pi_{\text{Viewer}}(\text{Seen}) \times \Pi_{\text{Title}}(\text{Movie})) \setminus \text{Seen}\Big)}_{\text{set of pairs (Viewer,Movie) such that } \neg\text{Seen(Viewer,Movie)}}$$

Conjunctive calculus:

$$\forall m,\ \text{Seen}(x_{\text{Viewer}}, m)$$

2. PSJRU algebra:

$$\Pi_{\text{Viewer}}(\text{Seen}) \setminus \Pi_{\text{Viewer}}(\text{Seen} \setminus \text{Likes})$$

Conjunctive calculus:

$$\underbrace{(\exists m,\ \text{Seen}(x_{\text{Viewer}}, m))}_{x_{\text{Viewer}} \text{ is a viewer}} \wedge \underbrace{(\forall m,\ \neg\text{Seen}(x_{\text{Viewer}}, m) \vee \text{Likes}(x_{\text{Viewer}}, m))}_{\text{for each film that they saw, they liked it (Seen} \implies \text{Likes)}}$$

3. PSJRU algebra:

$$\Pi_{\text{Producer}}\Big(\text{Produced} \underset{\text{Produced.Title=Title}}{\bowtie} (\Pi_{\text{Title}}(\text{Movie}) \setminus \Pi_{\text{Title}}(\text{Cinema}))\Big)$$

Conjunctive calculus:

$$\exists m,\ \underbrace{\text{Produced}(x_{\text{Producer}}, m)}_{x_{\text{Producer}} \text{ produces a movie } m} \wedge \underbrace{\neg(\exists n,\ \exists t,\ \ \text{Cinema}(n,t,m))}_{\text{and this movie does not play in theater}}$$

4. PSJRU algebra:

$$\Pi_{\text{Producer}}(\text{Produced}) \setminus \Pi_{\text{Producer}}(\text{Produced} \setminus \text{Seen})$$

Conjunctive calculus:

$$(\exists m,\ \text{Produced}(x,m)) \wedge (\forall m,\ \underbrace{\neg\text{Produced}(x,m) \vee \text{Seen}(x,m)}_{\text{Produced} \implies \text{Seen}})$$

5. Impossible in RSJRU algebra.

Exercise 4. Division can be expressed using the other operators of PSJRUC algebra:

$$\Pi_1(I) \setminus \underbrace{\Pi_1(\overbrace{\Pi_1(I) \times J}^{\text{every possible } (i,j) \text{ couple}} \setminus I)}_{\text{every } x \text{ such that } \exists y \text{ with } (x,y) \notin I}$$

Exercise 7. 1. An equivalent relational calculus expression is:

$$\Pi_{R.A}(R) \setminus \Pi_{R.A}(\sigma_{S.C \vee R.B \leqslant 1}(R \times S))$$

An equivalent relational algebra expression is:

$$\exists x_B,\ R(r_A, r_b) \wedge r_b > 1 \wedge \neg(\exists s_c,\ \exists s_B, S(s_C, s_B) \wedge s_C = r_A)$$

2. An equivalent relational calculus expression is:

$$\Pi_A(R) \setminus \Pi_{A_1}(\sigma_{A_1 < A_2}(\rho_{A \to A_1}(R) \times \rho_{A \to A_2}(R)))$$

An equivalent relational algebra expression is:

$$\underbrace{\Big(\exists b,\ R(x_{\max}, b)\Big)}_{x_{\max} \text{ is in } A} \wedge \underbrace{\Big(\forall a,\ (\exists b,\ R(a,b)) \wedge x_{\max} \geqslant a\Big)}_{\text{and } x_{\max} \text{ is the maximum of } A}$$