Difference Equasions

Linear Difference Equasion

The linear difference equation has the form:

$$y_{n+1} = ay_n + b$$

The general solution for y_n is:

$$y_n = \frac{b}{1-a} + (y_0 - \frac{b}{1-a})a^n$$

Long term behaviour

Type of behaviour	Indicator	Condition	Behavior
Vertical behavior	Sign of a	a > 0	Solution monotone
		a < 0	Solution oscillating
Long term behavior	Sign of a	lal < 1	Attracted to equilibrium
		lal > 1	Repelled by equilibrium

An equilibrium will be found at the point:

$$k = \frac{b}{1-a}$$

Special Cases

1. If $y_{n+1} = y_n + b$ then there is no equilibrium.
2. If $y_{n+1} = ay_n$ then the equilibrium is $y = 0$

Logistic Difference Equation

The logistic difference equasion has the form:

$$y_{n+1} = y_n + ry_n(M - y_n)$$

Where the initial value y_0 is given and M represents the carrying capacity.

Theorem 2

so…

$$\Delta y = ry_n(M - y_0)$$
$$\frac{\Delta y}{y_n} = r(M - y_0) = rM - ry_0$$

but…

$$\frac{\Delta y}{y_n} = ay_n + b$$

so…

$$a = -r, \;\; b = rm$$

Theorem 3

$$Let \;\frac{1}{r} < M < \frac{2}{r}$$

1. There exists 2 equilibria y = 0 and y = m.
2. If $y_0 \geq M + 1/r$,then $y_1 \leq 0$.
3. If $0 < y_0 < M + 1/r$, then y_n tends to equilibrium M as n increases.
4. The population will start oscillating as soon as y > 1/r

Constructing a Logistic Difference Equasion from Data

Recall that the relative growth for a logistic difference equasion is:

$$\frac{\Delta y_n}{y_n} = ay_n + b_n$$

where $a = -r$ and $b = rM$

This way when we construct a line through the data and reveive an *a* and *b* we can convert them back to r and M.

Other Difference Equasions

$$y_{n+1} = r(y_n)y_n$$

where $r(y_n)$ is the growth factor.
Equilibria are obtained by setting $r(y) = 0$

Systems of Difference Equasions