In `Regression and Other Stories`, mcmc is *just* a tool. Hence whether one uses Stan or Turing is not the main focus of the book. This notebook uses `ElectionsEconomy: hibbs.csv` to illustrate how Turing and other computations are used in the Julia *project* ROSTuringPluto.jl.

Over time I plan to expand below the list of topics:

1. Turing (see the Turing playground)
2. Using median and mad to summarize a posterior distribution.
3. ...
4. Model comparisons (TBD)
5. DAGs (TBD)
6. Graphs (TBD)
7. ...

Widen the cells.

```
html"""
<style>
    main {
        margin: 0 auto;
        max-width: 2000px;
        padding-left: max(160px, 10%);
        padding-right: max(160px, 10%);
    }
</style>
"""
```

A typical set of Julia packages to include in notebooks.

```
using Pkg
```

```
begin
    # Specific to this notebook
    using GLM

    # Specific to ROSTuringPluto
    using Optim
    using Logging
    using Turing

    # Graphics related
    using GLMakie

    # Common data files and functions
    using RegressionAndOtherStories
    import RegressionAndOtherStories: link

    Logging.disable_logging(Logging.Warn)
end;
```

Note

All data files are available (as .csv files) in the data subdirectory of package RegressionAndOtherStories.jl.

```
"/Users/rob/.julia/packages/RegressionAndOtherSt
ros_datadir()
```

Note

After evaluating above cell, use `ros_datadir("ElectionsEconomy", "hibbs.dat")` to obtain data.

hibbs =

	year	growth	vote	inc_party_candidat
1	1952	2.4	44.6	"Stevenson"
2	1956	2.89	57.76	"Eisenhower"
3	1960	0.85	49.91	"Nixon"
4	1964	4.21	61.34	"Johnson"
5	1968	3.02	49.6	"Humphrey"
6	1972	3.62	61.79	"Nixon"
7	1976	1.08	48.95	"Ford"
8	1980	-0.39	44.7	"Carter"
9	1984	3.86	59.17	"Reagan"
10	1988	2.27	53.94	"Bush, Sr."
⋮ more				
16	2012	0.95	52.0	"Obama"

```
hibbs =
CSV.read(ros_datadir("ElectionsEconomy",
"hibbs.csv"), DataFrame)
```

```
hibbs_lm =
StatsModels.TableRegressionModel{LinearModel{GLM

vote ~ 1 + growth

Coefficients:
──────────────────────────────────────────────────
                Coef.  Std. Error      t  Pr(>|t
──────────────────────────────────────────────────
(Intercept)  46.2476     1.62193    28.51    <1e-1
growth        3.06053    0.696274    4.40    0.000
──────────────────────────────────────────────────
```

```
hibbs_lm = lm(@formula(vote ~ growth),
hibbs)
```

```
▸ [-8.99292, 2.66743, 1.0609, 2.20753, -5.89044, 4
```

```
residuals(hibbs_lm)
```

```
2.2744434224582912
```

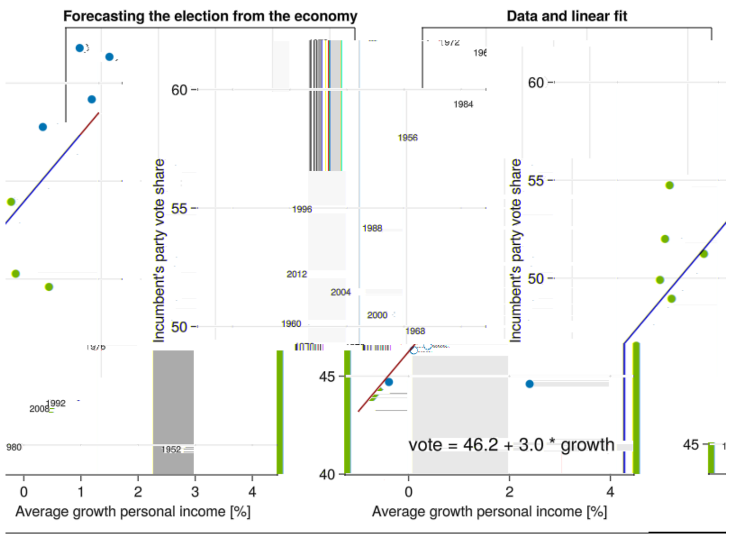
```
mad(residuals(hibbs_lm))
```

3.635681268522063

```
std(residuals(hibbs_lm))
```

[46.2476, 3.06053]

```
coef(hibbs_lm)
```




```
let
    fig = Figure()
    hibbs.label = string.(hibbs.year)
    xlabel = "Average growth personal income [%]"
    ylabel = "Incumbent's party vote share"
    let
        title = "Forecasting the election from the economy"
        ax = Axis(fig[1, 1]; title, xlabel, ylabel)
        for (ind, yr) in enumerate(hibbs.year)
            annotations!("$(yr)"; position=(hibbs.growth[ind], hibbs.vote[ind]), textsize=10)
        end
    end
    let
        x = LinRange(-1, 4, 100)
        title = "Data and linear fit"
        ax = Axis(fig[1, 2]; title, xlabel, ylabel)
        scatter!(hibbs.growth, hibbs.vote)
        lines!(x, coef(hibbs_lm)[1] .+ coef(hibbs_lm)[2] .* x; color=:darkred)
        annotations!("vote = 46.2 + 3.0 * growth"; position=(0, 41))
    end
    fig
end
```

Below some additional cells demonstrating the use of Turing.

```
ppl1_1 (generic function with 2 methods)
```

```
@model function ppl1_1(growth, vote)
    a ~ Normal(50, 20)
    b ~ Normal(0, 5)
    σ ~ Exponential(1)
    μ = a .+ b .* growth
    for i in eachindex(vote)
        vote[i] ~ Normal(μ[i], σ)
    end
end
```

Note

The sequence of the statements matter in Turing models!

	parameters	mean	std	naive_se
1	:a	46.3462	1.56997	0.0248234
2	:b	3.00368	0.669857	0.0105914
3	:σ	3.59603	0.627185	0.0099166

```
begin
    m1_1t = ppl1_1(hibbs.growth, hibbs.vote)
    chns1_1t = sample(m1_1t, NUTS(),
    MCMCThreads(), 1000, 4)
    describe(chns1_1t)
end
```

Note

Mostly I disable logging early on in notebooks using Turing. But it is also possible to do this `by cell`. Click on the little circle with 3 dots at the top of the selected cell and select `Hide logs`.

post1_1t =

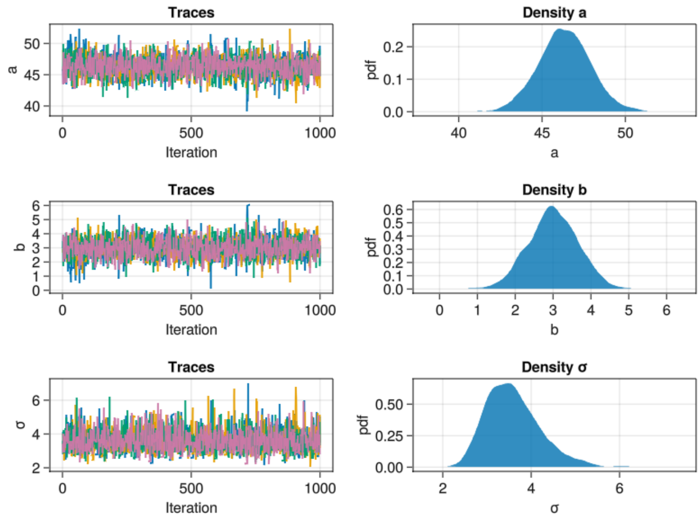
	a	b	σ
1	45.8543	2.97746	2.8181
2	44.6516	3.5725	4.1422
3	43.4512	3.90471	3.7756
4	49.171	2.07733	3.85881
5	48.2288	2.02114	3.61326
6	45.2312	3.51025	2.8635
7	44.071	3.05668	3.16067
8	46.8092	2.00857	3.99798
9	47.9348	3.09165	4.13494
10	47.0714	2.54041	3.46668
⋮ more			
4000	47.6985	2.63067	2.81791

```
post1_1t = DataFrame(chns1_1t)[:, [:a, :b,
:σ]]
```

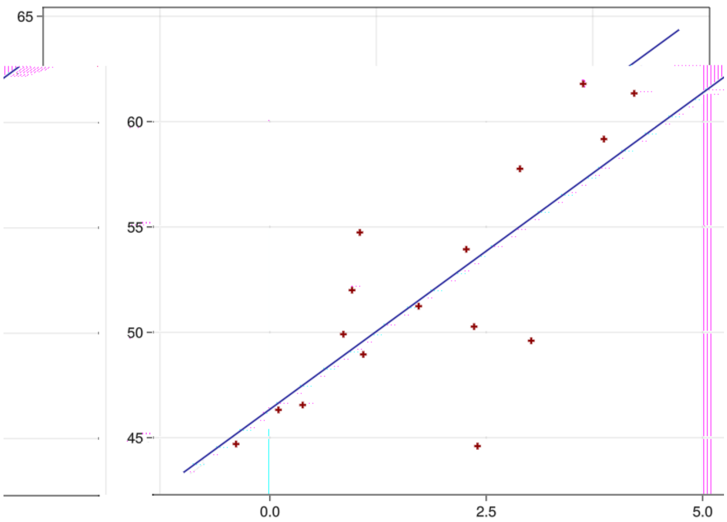
ms1_1t =

	parameters	median	mad_sd	mean	st
1	"a"	46.341	1.52	46.346	1.57
2	"b"	3.002	0.652	3.004	0.67
3	"σ"	3.52	0.6	3.596	0.62

```
ms1_1t = model_summary(post1_1t, [:a, :b,
:σ])
```




```
plot_chains(post1_1t, [:a, :b, :σ])
```

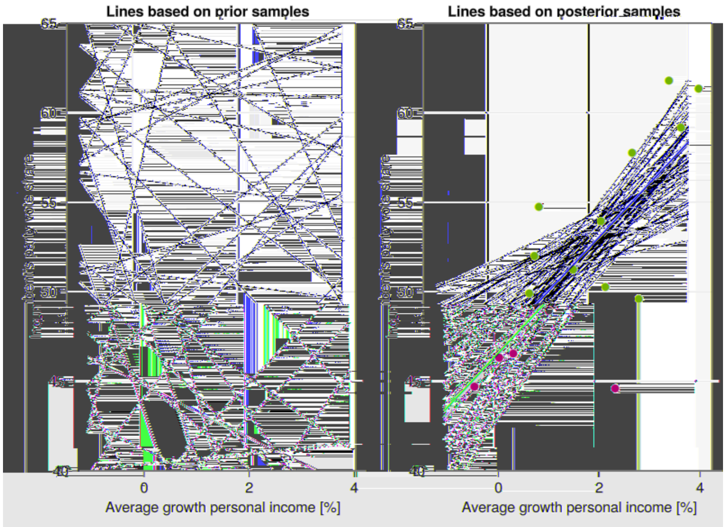


```
let
    x = -1.0:0.1:6.0
    preds = mean(post1_1t.a) .+
    mean(post1_1t.b) .* x
    lines(x, preds, color=:darkblue,
    label="Regression line")
    scatter!(hibbs.growth, hibbs.vote,
    marker=:cross, markersize=10,
        color=:darkred,
        label="Observations")
    current_figure()
end
```

Priors of Turing models.

	parameters	mean	std	naive_se
1	:a	49.6665	21.0826	0.666692
2	:b	0.133146	4.94452	0.15636
3	:σ	1.0248	1.04171	0.0329417

```
begin
    prior_chns1_1t = sample(m1_1t, Prior(),
    1000)
    describe(prior_chns1_1t)
end
```

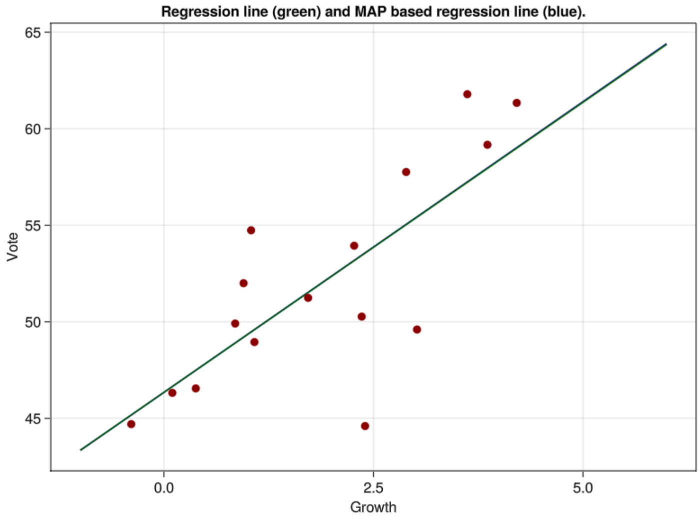



```
let
    N = 100
    x = LinRange(-1, 4, N)
    priors4_1t = DataFrame(prior_chns1_1t)

    mat1 = zeros(50, 100)
    for i in 1:50
        mat1[i, :] = priors4_1t.a[i] .+
        priors4_1t.b[i] .* x
    end
    ā = mean(post1_1t.a)
    b̄ = mean(post1_1t.b)

    # Maybe could use a `link` function here
    mat2 = zeros(50, 100)
    for i in 1:50
        mat2[i, :] = post1_1t.a[i] .+
        post1_1t.b[i] .* x
    end

    fig = Figure()
    xlabel = "Average growth personal
    income [%]"
    ylabel="Incumbent's party vote share"
    ax = Axis(fig[1, 1]; title="Lines based
    on prior samples",
        xlabel, ylabel)
    ylims!(ax, 40, 65)
    series!(fig[1, 1], x, mat1,
    solid_color=:lightgrey)
    ax = Axis(fig[1, 2]; title="Lines based
    on posterior samples",
        xlabel, ylabel)
    ylims!(ax, 40, 65)
    series!(fig[1, 2], x, mat2,
    solid_color=:lightgrey)
```




```
let
    x = -1:0.1:6
    preds = mean(post1_1t.a) .+
    mean(post1_1t.b) .* x
    f = Figure()
    ax = Axis(f[1, 1], title = "Regression
    line (green) and MAP based regression
    line (blue).",
        xlabel = "Growth", ylabel = "Vote")

    lines!(f[1, 1], x, â .+ b̂ .* x,
    color=:darkblue)
    lines!(x, preds, color=:darkgreen,
    label="Regression line")
    scatter!(hibbs.growth, hibbs.vote,
    color=:darkred, leg=false)
    current_figure()
end
```

Prediction

	iteration	chain	vote[1]	vote[2]	vote[3]
1	1	1	45.8542	46.7915	47.1881
2	2	1	44.3899	47.6689	53.9884
3	3	1	46.4277	43.9124	44.7674
4	4	1	50.9526	46.8558	52.6576
5	5	1	50.1537	48.2343	47.8607
6	6	1	47.384	49.6843	49.374
7	7	1	43.7402	54.2323	45.0889
8	8	1	45.2788	49.6192	39.3787
9	9	1	46.6127	48.007	52.2737
10	10	1	47.6908	41.2327	50.7031

⋮ more

```
begin
    x_test = [0, 1, 2, 3, 4, 5]
    m_test = ppl1_1(x_test, fill(missing,
    length(x_test)))
    pred_chns1_1t = predict(m_test,
    chns1_1t)
    pred_chns1_1t
end
```

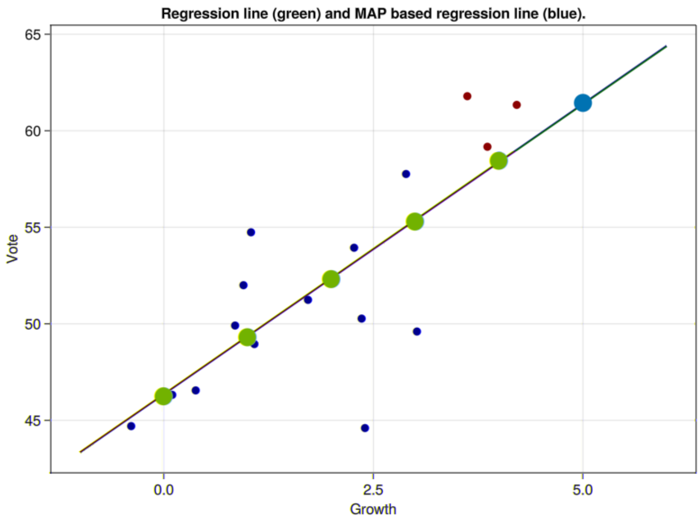
▸ [

	parameters	mean	std	nai
1	Symbol("vote[1]")	46.2443	3.97209	0.06
2	Symbol("vote[2]")	49.3039	3.8636	0.06
3	Symbol("vote[3]")	52.3164	3.75129	0.05
4	Symbol("vote[4]")	55.2959	3.87343	0.06
5	Symbol("vote[5]")	58.4459	3.99468	0.06
6	Symbol("vote[6]")	61.4424	4.30721	0.06

```
describe(pred_chns1_1t)
```

	parameters	median	mad_sd	mean	st
1	"vote[1]"	46.34	3.861	46.244	3.97
2	"vote[2]"	49.367	3.762	49.304	3.86
3	"vote[3]"	52.299	3.606	52.316	3.75
4	"vote[4]"	55.277	3.704	55.296	3.87
5	"vote[5]"	58.395	3.815	58.446	3.99
6	"vote[6]"	61.491	4.171	61.442	4.30

	vote[1]	vote[2]	vote[3]	vote[4]	vot
1	45.8542	46.7915	47.1881	58.3118	56.
2	44.3899	47.6689	53.9884	56.5369	63.
3	46.4277	43.9124	44.7674	56.2496	58.
4	50.9526	46.8558	52.6576	55.6043	56.
5	50.1537	48.2343	47.8607	56.0421	57.
6	47.384	49.6843	49.374	48.2467	60.
7	43.7402	54.2323	45.0889	52.272	52.
8	45.2788	49.6192	39.3787	56.9915	51.
9	46.6127	48.007	52.2737	57.2739	58.
10	47.6908	41.2327	50.7031	55.7591	57.
⋮ more					
4000	52.632	52.0344	53.4976	61.7088	59.




```
let
    x = -1:0.1:6
    preds = mean(post1_1t.a) .+
    mean(post1_1t.b) .* x
    f = Figure()
    ax = Axis(f[1, 1], title = "Regression
    line (green) and MAP based regression
    line (blue).",
        xlabel = "Growth", ylabel = "Vote")

    lines!(f[1, 1], x, â .+ b̂ .* x,
    color=:darkblue)
    lines!(x, preds, color=:darkgreen,
    label="Regression line")
    scatter!(hibbs.growth, hibbs.vote,
    color=:darkred, leg=false)
    scatter!(x_test,
    reshape(mean(Matrix(pred1_1t); dims=1),
    ncol(pred1_1t)), markersize=20)
    current_figure()
end
```

```
4000×6 Matrix{Float64}:
 45.8542  46.7915  47.1881  58.3118  56.8116  62.
 44.3899  47.6689  53.9884  56.5369  63.4704  55.
 46.4277  43.9124  44.7674  56.2496  58.8367  65.
 50.9526  46.8558  52.6576  55.6043  56.4084  63.
 50.1537  48.2343  47.8607  56.0421  57.0635  56.
 47.384   49.6843  49.374   48.2467  60.3191  62.
 43.7402  54.2323  45.0889  52.272   52.5568  62.
  ⋮                                            ⋮
 40.8571  52.016   47.0977  51.3776  60.1531  60.
 46.7978  44.8882  49.5469  53.4672  55.0919  65.
 48.2217  52.5758  51.6126  55.7905  54.4956  62.
 51.8283  54.5989  48.9244  60.7026  58.49    60.
 42.6466  48.4912  49.5714  56.0471  46.793   68.
 52.632   52.0344  53.4976  61.7088  59.1616  64.
```

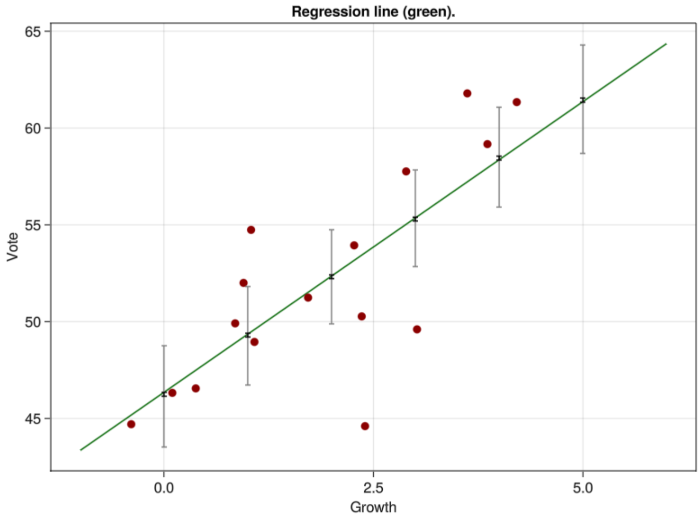
```
Array(group(pred_chns1_1t, :vote))
```

	parameters	estimate	se	p
1	"vote[1]"	46.2443	0.0628042	[0.055, (
2	"vote[2]"	49.3039	0.0610889	[0.055, (
3	"vote[3]"	52.3164	0.059313	[0.055, (
4	"vote[4]"	55.2959	0.0612443	[0.055, (
5	"vote[5]"	58.4459	0.0631614	[0.055, (
6	"vote[6]"	61.4424	0.068103	[0.055, (

```
errorbars_mean(pred1_1t)
```

	parameters	median	mad_sd	p
1	"vote[1]"	46.3402	3.86139	[0.055, 0.9
2	"vote[2]"	49.3666	3.76239	[0.055, 0.9
3	"vote[3]"	52.2987	3.60625	[0.055, 0.9
4	"vote[4]"	55.2769	3.70398	[0.055, 0.9
5	"vote[5]"	58.395	3.81504	[0.055, 0.9
6	"vote[6]"	61.4909	4.17119	[0.055, 0.9

```
errorbars_draws(pred1_1t, [0.055, 0.945])
```




```
let
    x = -1:0.1:6
    preds = mean(post1_1t.a) .+
    mean(post1_1t.b) .* x
    pred_values =
    reshape(mean(Matrix(pred1_1t); dims=1),
    ncol(pred1_1t))

    f = Figure()
    ax = Axis(f[1, 1], title = "Regression
    line (green).",
        xlabel = "Growth", ylabel = "Vote")

    lines!(x, preds, color=:darkgreen,
    label="Regression line")
    scatter!(hibbs.growth, hibbs.vote,
    color=:darkred, leg=false)

    # 50% interval predictions
    error_bars =
    nested_column_to_array(errorbars_draws(pred1_1t, [0.25, 0.75]), "q")
    errorbars!(x_test, pred_values,
    error_bars[:, 1], error_bars[:, 2],
    whiskerwidth = 6, color=:grey)

    # 89% s.e. of the mean
    error_bars =
    nested_column_to_array(errorbars_mean(pred1_1t, [0.055, 0.945]), :q)
    errorbars!(x_test, pred_values,
    error_bars[:, 1], error_bars[:, 2],
    whiskerwidth = 6, color=:black)
    current_figure()
end
```

```
6×2 Matrix{Float64}:
 2.72267  2.51099
 2.58164  2.50705
 2.43288  2.42853
 2.45352  2.53978
 2.52928  2.62923
 2.75313  2.84876
```

```
nested_column_to_array(errorbars_draws(pred1_1t, [0.25, 0.75]), "q")
```

A quick look at broadcasting and vectorization. See also more dots

```
f (generic function with 1 method)
```

```
f(x) = 3x^2 + 5x + 2
```

```
nobcst (generic function with 1 method)
```

```
function nobcst(f, x)
    f.(2 .* x.^2 .+ 6 .* x.^3 .- sqrt.(x))
end
```

```
bcst (generic function with 1 method)
```

```
function bcst(f, x)
    @. f(2 * x^2 + 6 * x^3 - sqrt(x))
end
```

```
▸[2.0, 1.99293, 1.99001, 1.98777, 1.98588, 1.984
```

```
let
    n = 10^6
    x = LinRange(0, 2, n)
    @time nobcst(f, x)
end
```

```
▸[2.0, 1.99293, 1.99001, 1.98777, 1.98588, 1.984
```

```
let
    n = 10^6
    x = LinRange(0, 2, n)
    @time bcst(f, x)
end
```

Compute median and mad.

```
▸[1.52, 0.652, 0.6]
```

```
[ms1_1t[v, "mad_sd"] for v in [:a, :b, :σ]]
```

Alternative computation of mad().

```
▸ [1.51994, 0.652132, 0.599873]
```

```julia
let
    1.483 .* [median(abs.(post1_1t.a .-
    median(post1_1t.a))),
    median(abs.(post1_1t.b .-
    median(post1_1t.b))),
    median(abs.(post1_1t.σ .-
    median(post1_1t.σ)))]
end
```

Quick simulation with median, mad, mean and std of Normal observations.

```
nt =
▸ (x = [0.897963, 2.87332, 4.87619, 6.41182, 5.49
```

```julia
nt = (x=rand(Normal(5, 2), 10000),)
```

```
▸ [5.02562, 2.02818, 5.00777, 2.02935]
```

```julia
[median(nt.x), mad(nt.x), mean(nt.x),
std(nt.x)]
```

```
sd_mean = 0.02
```

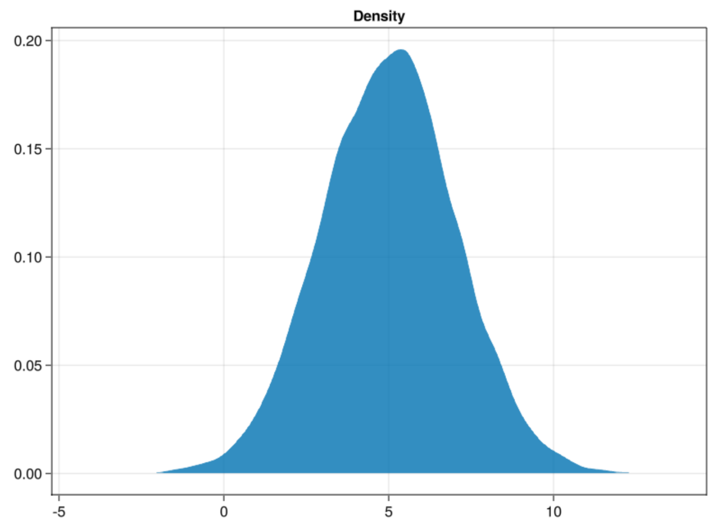
```julia
sd_mean = round(mad(nt.x)/√10000; digits=2)
```

```
1.3679885818444575
```

```julia
median(abs.(nt.x .- median(nt.x)))
```

```
2.0287270668753306
```

```julia
1.483 * median(abs.(nt.x .- median(nt.x)))
```




```
let
    fig = Figure()
    ax = Axis(fig[1, 1]; title = "Density")
    den = density!(nt.x)
    fig
end
```

[1.04308, 8.98945]

```
quantile(nt.x, [0.025, 0.975])
```

[3.62681, 6.35746]

```
quantile(nt.x, [0.25, 0.75])
```

Note

Click on "Live docs" and place cursor on link to see more help.

Click little down arrow to the right to remove live docs again.