Unscented Kalman Filter Design for Curvilinear Motion Models Suitable for Automotive Safety Applications

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Abstract – *Research in automotive safety leads to the conclusion that modern vehicles should utilize active and passive sensors for the recognition of the environment surrounding them. Thus, the development of tracking systems utilizing efficient state estimators is very important. In this case problems such as moving platform carrying the sensor and maneuvering targets could introduce large errors in the state estimation and in some cases can lead to the divergence of the filter. In order to avoid sub-optimal performance, the unscented Kalman filter is chosen, while a new curvilinear model is applied which takes into account both the turn rate of the detected object and its tangential acceleration, leading to a more accurate modeling of its movement. The performance of the unscented filter using the proposed model in the case of automotive applications is proven to be superior compared to the performance of the extended and linear Kalman filter.*

Keywords: Tracking, filtering, estimation, unscented, automotive applications, curvilinear.

1 Introduction

In applications such as advanced driver assistant systems the estimation of the state of detected objects is crucial for the immediate warning of the driver. The sensor that is used in such systems is mainly a long range microwave-radar that provides information about moving and stationary obstacles located in the road. More specifically the distance, the angle and the relative radial velocity of all objects located inside the field of view of the sensor are measured. The main scope of the system is to identify all the threats in the road environment and assign a level of danger to each one of them. Then, according to some predefined warning strategies the situation analysis is performed and the necessary warnings are sent to the driver using visual, auditory or haptic messages. In order for this to be achieved the full dynamic state of each potential threat must be known exactly. Because the measured quantities are not enough to fully identify the dynamic state of the detected objects an efficient estimator must be used. Usually, to find if there is going to be a collision with a moving object near the ego vehicle the paths of both objects are computed and then are compared. In order to extract a reliable conclusion the accurate position, velocity, tangential acceleration and turn rate of the object must be estimated [4]. Usually, this is accomplished using Kalman filters. Though, the performance of these estimators depends very much on the motion model that will be selected for the state update.

The motion of the vehicles located in the road cannot be modeled efficiently using the current dynamic models found in the literature [5]. Typical scenarios in the road environment involve lateral maneuvers such as lane changes or overtaking of other vehicles moving in front. It is obvious that in such cases the turn rate of the detected vehicles is not remaining constant due to their complex movement. Also the speed of the vehicle is changing with a non constant rate when the driver is accelerating or when he is slowing down by pushing the brake. Also due to the motion of the ego vehicle the relative values of turn rate and velocity of the detected objects are changing even if their motion is constant. For example if the ego vehicle is going to overtake a preceding car then during the whole process the corresponding track given by the tracking system will be observed to be performing a maneuver.

This is why a more complicated model should be chosen to describe the complex motions that are performed by the moving objects located in the road environment. This model should combine the turn rate of an object with its tangential acceleration. The first variable is rating how fast the heading of the vehicle is changing while the second one is measuring how fast is changing the velocity of the vehicle. Existing models like the constant turn rate, take into consideration only the one of the two variables, simplifying the modeling of the real motion.

In this paper is proposed a filter utilizing the constant turn rate and constant tangential acceleration (CTRA) model [3] and is compared with the constant acceleration (CA), constant turn rate (CTR) filters. Also, in order to avoid large errors and possible divergence of the filter the unscented transform is applied [1], [2]. The technique of Monte Carlo simulations is used for consistent filter performance evaluation and the RMS estimation errors are calculated for each filter [6]. It will be seen that the proposed model combined with the unscented transform improves significantly the state estimator's performance and is making it more stable and more reliable.

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In the first section is done an introduction to the existing models and then is presented the proposed dynamic model (CTRA). In the second section are presented the three basic variations of the Kalman filter (linear, extended and unscented). Then, using the three models, 4 filters are formulated. These are a linear using the CA model, two extended using the CTR and CTRA models and finally a filter using the CTRA model and the unscented transform. The performance of all these filters is evaluated by presenting the rms distance and velocity estimation errors of each one of them.

Due to the fact that the platform carrying the sensor (ego vehicle) is constantly moving the analysis will be performed in the local coordinate system. This system is defined as the one with its center located in the center of the ego vehicle and the x axis parallel to the heading of the vehicle. Also, simulated data are used for creating the measurements of the radar and testing the performance of the filters. The refresh rate of the sensor is supposed to be $T = 100ms$.

2 Motion Modeling

The dynamic state of a moving object can be modeled as a discrete-time Markov process:

$$x_{k+1} = A_k \cdot x_k + v_k \tag{1}$$

where A_k is the transition matrix, x_k is the state vector and v_k is the process noise. In order to achieve a successful estimation of the true dynamic state of the tracked vehicle the motion model that will be applied must be chosen carefully. The most common models are the Constant Acceleration (*CA*) and the Constant Turn Rate (*CTR*). In the first one it is assumed that the tracked object is moving with a constant acceleration in both axes:

$$a_{k+1}^x = a_k^x, \quad a_{k+1}^y = a_k^y \tag{2}$$

while in the second one it is assumed that the object is moving with a constant turn rate keeping constant velocity:

$$\omega_{k+1} = \omega_k, \quad u_{k+1} = u_k \tag{3}$$

It is obvious that the true motion of a vehicle cannot be approximated by either of these two models. Therefore, a combination of them can be applied using the proposed non linear Constant Turn Rate and constant tangential Acceleration (*CTRA*) model.

2.1 CTRA Model

The main assumption of this model is that the tracked object is making a turn with a constant rate (ω) and a constant velocity rate which is the tangential acceleration (a^T).

$$\omega_{k+1} = \omega_k, \quad a_{k+1}^T = a_{k+1}^T \tag{4}$$

Based on the two basic assumptions listed above, the exact solution of the following state equation is used in order to compute the remaining state vector elements ([5]):

$$\underline{x}_{k+1} = \underline{x}_k + \int_{t_k}^{t_{k+1}} \underline{\dot{x}}_{\tau|t_k} \cdot d\tau \tag{5}$$

Thus, the extrapolated heading (φ), velocity (u), tangential and centripetal acceleration (a^T, a^c) components are:

$$\begin{gathered}
\varphi_{k+1} = \varphi_k + \omega_k \cdot T, \quad u_{k+1} = u_k + a_k^T \cdot T \\
u_{k+1}^x = u_{k+1} \cdot \cos(\varphi_{k+1}), \quad u_{k+1}^x = u_{k+1} \cdot \sin(\varphi_{k+1}) \\
a_{k+1}^{T,x} = a_{k+1}^{T,x} \cdot \cos(\varphi_{k+1}), \quad a_{k+1}^{T,y} = a_{k+1}^{T,x} \cdot \sin(\varphi_{k+1}) \\
a_{k+1}^{C,x} = -\omega_k \cdot U_{k+1} \cdot \sin(\varphi_{k+1}) \\
a_{k+1}^{C,y} = \omega_k \cdot U_{k+1} \cdot \cos(\varphi_{k+1})
\end{gathered} \tag{6}$$

where T is the time interval between two sequential scans. Also, it must be noted that the centripetal acceleration a^C is normal to the velocity vector as shown in figure 1. Finally, the displacement of the object in the two axes is computed using the following equations:

$$\begin{gathered}
x_{k+1} = x_k + \frac{u_k \cdot (\sin(\varphi_{k+1}) - \sin(\varphi_k))}{\omega_k} + \\
+ \frac{a_k^T \cdot (\cos(\varphi_{k+1}) - \cos(\varphi_k)) + a_k^T \cdot \omega_k \cdot T \cdot \sin(\varphi_{k+1})}{\omega_k^2} \\
y_{k+1} = y_k - \frac{u_k (\cos(\varphi_{k+1}) - \cos(\varphi_k))}{\omega_k} + \\
+ \frac{a_k^T \cdot (\sin(\varphi_{k+1}) - \sin(\varphi_k)) - a_k^T \cdot \omega_k \cdot T \cdot \cos(\varphi_{k+1})}{\omega_k^2}
\end{gathered} \tag{7}$$

This model can efficiently describe the true motion of a vehicle in the road, since the true trajectory can be considered that is consisted of segments where the turn rate and the tangential acceleration remain constant. It is obvious that in the case where the tangential acceleration is zero the constant turn rate (CTR) model is depicted. In the next figure is shown how it is defined the local coordinate system of the ego vehicle and a case of two tracked objects moving in a road with non-zero curvature. Also, it is shown how the velocity and acceleration vectors and the turn rate are defined in the case of a tracked object.

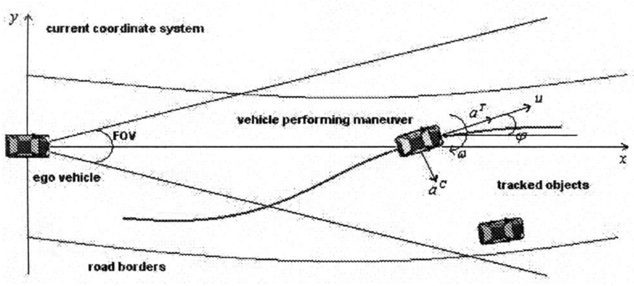

Figure 1. Motion analysis using CTRA dynamic model

3 Filtering Techniques

Filtering is one of the most fundamental tasks related with target tracking. Despite the fact that the sensor provides to the system only measurements of distance, angle and radial velocity, the state estimator is capable of providing not only the position but also the velocity and the acceleration of an object, even its turn rate. Therefore, in an active safety automotive application it is feasible to predict if there is going to be a collision with a preceding vehicle and act respectively by warning the driver or even activating the brakes. The filters that are mostly used for estimating the state of an object are those based on the Kalman algorithm [4]. This algorithm provides a general solution to the recursive minimized mean square estimation problem, assuming that the target dynamics and the measurement noise are modeled accurately. Also, it is considered that the estimation error (estimated-true) is a random variable with Gaussian distribution and zero-mean.

3.1 Linear and Extended Kalman Filters

Considering that the dynamic state of a moving object can be modeled as a discrete-time Markov process and that there is a mapping from the state to the measurement space, the equations describing the Kalman filter are the following:

$$\begin{aligned} \hat{x}_{k+1|k} &= A_k \cdot \hat{x}_{k|k-1} + K_k \cdot \left(y_k - C \cdot \hat{x}_{k|k-1}\right) \\ K_k &= A_k \cdot P_{k|k-1} \cdot C' \cdot \left[C \cdot P_{k|k-1} \cdot C' + R\right]^{-1} \\ P_{k+1|k} &= \left[A_k - K_k \cdot C\right] \cdot P_{k|k-1} \cdot A_k' + Q \end{aligned} \tag{8}$$

where Q is the covariance matrix of the zero-mean, white, Gaussian process noise and R is the covariance matrix of the measurement noise which is also considered as zero-mean, white, Gaussian. Finally, K is the gain of the filter and P is the estimation error covariance matrix.

In the case of nonlinear target dynamics or nonlinear measurement process an extension of the linear filter is used for estimating the state of the target. In this case the state update and the mapping from the state-space to the measurement-space are performed using the following relationships:

$$x_{k+1} = f(x_k), \quad y_k = g(x_k) \tag{9}$$

This is the extended Kalman filter in which the state distribution is approximated by a Gaussian random variable which is propagated through the first-order linearization of the nonlinear system. This means that the transition and the measurement matrices are computed according to the following equations:

$$A_{k+1|k} = \left.\frac{\partial f(x_k)}{\partial x_k}\right|_{x=\hat{x}}, \quad C_k = \left.\frac{\partial g(x_k)}{\partial x_k}\right|_{x=\hat{x}_{k|k-1}} \tag{10}$$

In the case of the constant acceleration model the linear filter is used as there are not nonlinearities. The state vector is formed using the position, velocity and acceleration components:

$$\underline{x}_{ca} = \begin{bmatrix} x & u_x & a_x & y & u_y & a_y \end{bmatrix}^T \tag{11}$$

Contrarily, in the case of the constant turn rate model the extended filter must be used due to the computation nonlinearities of the position and velocity as presented in [5]. The state vector in that case will have the following form:

$$\underline{x}_{ctr} = \begin{bmatrix} x & u_x & y & u_y & \omega \end{bmatrix}^T \tag{12}$$

The transition matrices for the CA and CTR kalman filters can be computed easily as shown in [5]. In the case of the linear CA filter this matrix will be constant over time. Contrarily, in the case of the CTR filter this matrix will change over time as it will depends from the current estimate.

Finally, the CTRA model can be used with the extended Kalman filter. In this case the state vector will be formed as in CTR model but it will include additionally the two components of the tangential acceleration. So the state vector will be:

$$\underline{x}_{ctra} = \begin{bmatrix} x & u_x & a_x^T & y & u_y & a_y^T & \omega \end{bmatrix}^T \tag{13}$$

In order for the transition matrix to be formed equations (6), (7) and (10) must be used. After the computation of the partial derivatives that are required due to the nonlinear state transition the following matrix is derived:

$$A=\begin{bmatrix} 1 & c_1 & f_{13} & 0 & c_2 & f_{16} & f_{17} \\ 0 & c_3 & T\cdot c_3 & 0 & -c_4 & -T\cdot c_4 & f_{27} \\ 0 & 0 & c_3 & 0 & 0 & -c_4 & f_{37} \\ 0 & -c_2 & f_{43} & 1 & c_1 & f_{46} & f_{47} \\ 0 & c_4 & T\cdot c_4 & 0 & c_3 & T\cdot c_3 & f_{57} \\ 0 & 0 & c_4 & 0 & 0 & c_3 & f_{67} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

where the parameters c_1, c_2, c_3, c_4 are defined below:

$$c_1 = \frac{\sin(\hat{\omega}_k \cdot T)}{\hat{\omega}_k \cdot T}, \quad c_2 = \frac{\cos(\hat{\omega}_k \cdot T)-1}{\hat{\omega}_k \cdot T}$$
$$c_3 = \cos(\hat{\omega}_k \cdot T), \quad c_4 = \sin(\hat{\omega}_k \cdot T) \quad (15)$$

The rest of the parameters are defined according to the following equations:

$$\begin{gathered} f_{13} = \partial x / \partial a_x^T = T \cdot (c_2 + c_4)/\hat{\omega}_k \\ f_{43} = \partial y / \partial a_x^T = (c_4 - \hat{\omega}_k \cdot T \cdot c_3)/\hat{\omega}_k^2 \\ f_{16} = \partial x / \partial a_y^T = -f_{43} \\ f_{46} = \partial y / \partial a_y^T = f_{13} \\ f_{17} = \partial x / \partial \omega = \hat{u}_k^x \cdot d_1 + \hat{u}_y^k \cdot d_2 + \hat{a}_k^{T,x} \cdot d_3 + \hat{a}_k^{T,y} \cdot d_4 \\ f_{47} = \partial y / \partial \omega = \hat{u}_k^x \cdot d_5 + \hat{u}_k^y \cdot d_6 + \hat{a}_k^{T,x} \cdot d_7 + \hat{a}_k^{T,y} \cdot d_8 \\ f_{27} = \partial u_x / \partial \omega = -T \cdot \left(\hat{u}_x^{k+1} \cdot c_4 + \hat{u}_y^{k+1} \cdot c_3\right) \\ f_{57} = \partial u_y / \partial \omega = T \cdot \left(\hat{u}_x^{k+1} \cdot c_3 - \hat{u}_y^{k+1} \cdot c_4\right) \\ f_{37} = \partial a_x^T / \partial \omega = -T \cdot \left(\hat{a}_k^{T,x} \cdot c_4 - \hat{a}_k^{T,y} \cdot c_3\right) \\ f_{67} = \partial a_y^T / \partial \omega = T \cdot \left(\hat{a}_k^{T,x} \cdot c_3 - \hat{a}_k^{T,y} \cdot c_4\right) \quad (16) \end{gathered}$$

where the parameters $d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8$ are calculated using the following relationships:

$$\begin{gathered} d_1 = (\hat{\omega}_k \cdot T \cdot c_3 - c_4)/\omega_k^2 \\ d_2 = (T \cdot c_4 - c_2)/\hat{\omega}_k \\ d_3 = \left(\hat{\omega}_k \cdot T^2 \cdot c_3 - 2 \cdot T \cdot c_4 - 2 \cdot c_2\right)/\hat{\omega}_k^2 \\ d_4 = \left(\left[2 - (\hat{\omega}_k \cdot T)^2\right] \cdot c_4 - 2 \cdot \hat{\omega}_k \cdot T \cdot c_3\right)/\hat{\omega}_k^3 \\ d_5 = -d_2, \quad d_6 = d_1 \\ d_7 = -d_4, \quad d_8 = d_3 \quad (17) \end{gathered}$$

In all the previous mentioned approaches the measurement vector will be consisted by the position components x and y which will be extracted using the measured distance and angle of the detected object. So there will be no nonlinearities regarding the transformation from the state to the measurement space for all the filters.

3.2 Unscented Kalman Filter

The state distribution in the unscented Kalman filter is represented by a Gaussian random variable and is specified using a minimal set of carefully chosen sample points. The true mean and covariance of the Gaussian random variable is completely captured by these sample points. The propagation through the true nonlinear system is leading to the capture of the posterior mean and covariance accurately to the third order for any kind of nonlinearity. This is accomplished using the unscented transform.

It is considered the case of a random variable x of dimension L, which is propagating through a nonlinear function $y = f(x)$. The mean and the covariance of this random variable is $\bar{x}$ and P_x respectively. In order to calculate the statistics of y, a matrix X is formed which is consisted of $2 \cdot L + 1$ sigma vectors X_i. Each of these vectors is calculated using a corresponding weight W_i as shown in the following equations:

$$\begin{gathered} X_i = \bar{x} + \left(\sqrt{(L+\lambda)\cdot P_X}\right)_i \quad i = 1,\ldots,L \\ X_i = \bar{x} - \left(\sqrt{(L+\lambda)\cdot P_X}\right) \quad i = L+1,\ldots,2L \\ X_0 = \bar{x} \quad (18) \end{gathered}$$

The corresponding weights are calculated using the next relationships:

$$\begin{gathered} W_0^{(m)} = \lambda/(L+\lambda) \\ W_0^{(c)} = \lambda/(L+\lambda) + \left(1 - a^2 + \beta\right) \\ W_i^{(m)} = W_i^{(c)} = 1/\{2(L+\lambda)\} \\ \lambda = a^2 \cdot (L+\kappa) - L \quad (19) \end{gathered}$$

for $i = 1,\ldots,2L$. Parameter λ is used for scaling purposes. The spread of the sigma points around $\bar{x}$ is expressed by the factor a which is usually set to a small positive value, such as 10^{-3}. κ is a secondary scaling factor which is usually set to zero and β is used for the incorporation of prior knowledge of the distribution of x. For Gaussian distributions an optimal value is $\beta = 2$. Finally $\sqrt{(L+\lambda)\cdot P_X}$ is the $i-th$ row of the matrix square root.

The unscented Kalman filter starts with the initialization and includes the calculation of the sigma points, the time update and the measurement update equations in every scan k. The initialization is shown below:

$$\hat{x}_0 = E[x_0]$$
$$P_0 = E\left[(x_0 - \hat{x}_0)\cdot(x_0 - \hat{x}_0)^T\right] \quad (20)$$

The time update equations are the following ones:

$$X_{k-1} = \left[\hat{x}_{k-1} \quad \hat{x}_{k-1} + \xi\cdot\sqrt{P_{k-1}} \quad \hat{x}_{k-1} - \xi\cdot\sqrt{P_{k-1}}\right]$$
$$X_{k|k-1} = f(X_{k-1}), \quad \hat{x}_k^- = \sum_{i=0}^{2\cdot L} W_i^{(m)}\cdot X_{i,k|k-1}$$
$$P_k^- = \sum_{i=0}^{2\cdot L} W_i^{(c)}\cdot\left(X_{i,k|k-1} - \hat{x}_k^-\right)\cdot\left(X_{i,k|k-1} - \hat{x}_k^-\right)^T + Q$$
$$Y_{k|k-1} = g(X_{k|k-1}), \quad \hat{y}_k^- = \sum_{i=0}^{2\cdot L} W_i^{(m)}\cdot Y_{i,k|k-1} \quad (21)$$

where $\xi = \sqrt{L+\lambda}$. Also, Q and R are the covariance matrices of the process and the measurement noise respectively. Next, the measurement update equations are presented below:

$$P_{\tilde{y}_k\tilde{y}_k} = \sum_{i=0}^{2\cdot L} W_i^{(c)}\cdot\left(Y_{i,k|k-1} - \hat{y}_k^-\right)\cdot\left(Y_{i,k|k-1} - \hat{y}_k^-\right)^T + R$$
$$P_{x_k y_k} = \sum_{i=0}^{2\cdot L} W_i^{(c)}\cdot\left(X_{i,k|k-1} - \hat{x}_k^-\right)\cdot\left(Y_{i,k|k-1} - \hat{y}_k^-\right)^T \quad (22)$$

Finally, the gain computation and the state estimation with its covariance error matrix are listed below:

$$K_k = P_{x_k y_k}\cdot P_{\tilde{y}_k\tilde{y}_k}^{-1}$$
$$\hat{x}_k = \hat{x}_k^- + K_k\cdot\left(y_k - \hat{y}_k^-\right)$$
$$P_k = P_k^- + K_k\cdot P_{\tilde{y}_k\tilde{y}_k}\cdot K_k^T \quad (23)$$

Unlike general Monte-Carlo sampling methods, the unscented transform doesn't require too many sampling points in the attempt to propagate an accurate distribution of the state. The state vector that is used in the case of the unscented filter is exactly the same with the one used in the extended CTRA filter.

4 Simulation and Results

In order to evaluate the performance of the four different filters, simulated data are used. The sensor is located in the ego vehicle, inspecting the front area of the road and is considered to have a field of view of 11^0 and a maximum detection distance of 150m. The measurements gathered from the sensor are used as input to a tracking system. Those raw data include not only the moving vehicles but also detections from the road borders in order for a more realistic modeling of the scenario to be achieved. All the steps of a tracking system are performed (gating-data association-filtering) and the output is the full state vector of the confirmed tracks.

The criteria that are used to evaluate the performance of each filter are based in the calculation of the Rms estimation errors of the distance and the velocity of a selected target. The best filter is the one that will minimize those errors. The definition of the Rms estimation error is done using the following relationships:

$$Rms_k^D = \sqrt{(\hat{x}_k - x_k)^2 + (\hat{y}_k - y_k)^2}$$
$$Rms_k^U = \sqrt{(\hat{u}_k^x - u_k^x)^2 + (\hat{u}_k^y - u_k^y)^2}$$
$$Rms_k^A = \sqrt{(\hat{a}_k^x - a_k^x)^2 + (\hat{a}_k^y - a_k^y)^2} \quad (24)$$

where $(\hat{x}_k, \hat{y}_k)$, (x_k, y_k) are the estimated and true position of the target respectively at scan k. $\hat{u}_k^{x,y}$, $u_k^{x,y}$ are the estimated and true velocity components of the target respectively and $\hat{a}_k^{x,y}$, $a_k^{x,y}$ are the estimated and true acceleration components respectively. Finally, Rms_k^D, Rms_k^U are the rms estimation errors for the distance and the velocity respectively. In order to extract smoother values (lower variability) for these errors the method of the Monte Carlo simulations is used. The total number of iterations is selected to be $N_{MC} = 50$. In this case the respective rms errors become ([6]):

$$\overline{Rms_k}^{D,U} = (1/N_{MC})\cdot\sum_{i=1}^{N_{MC}} Rms_{k,i}^{D,U} \quad (25)$$

Two basic scenarios are examined. In the first one the motion is done in a straight road and the ego vehicle is following two other preceding vehicles. The first one during the process is overtaking the second, while the ego vehicle is changing lane. In the current coordinate system of the ego vehicle this will be observed as a very composite maneuver performed by the detected objects. The object that will be used to calculate the rms estimation errors is the first preceding vehicle. The trajectories in the global coordinate system are shown in the next figure:

Global Position

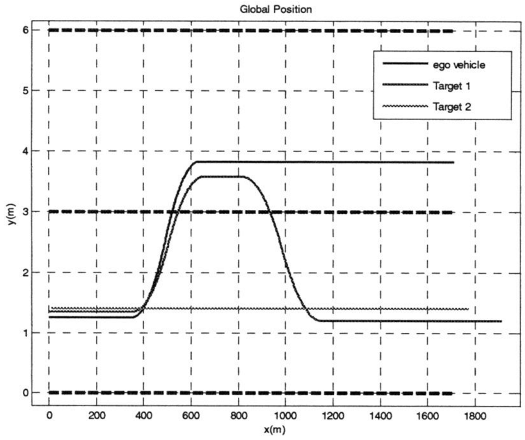

Figure 2. Scenario 1 in global coordinate system

In the next figure is shown the distance rms errors for the 4 filters. During the beginning of the overtaking the errors in all the filters are raising, but right after only the UKF CTRA filter maintains a very small error in estimating the actual position of the tracked object.

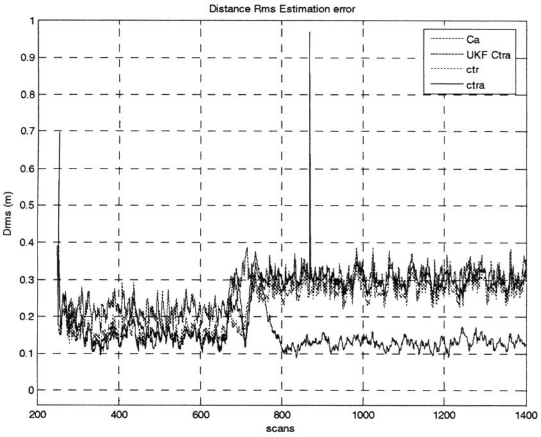

Figure 3. Rms estimation error for distance

Also, it can be seen that the extended filter utilizing the CTRA curvilinear model is unstable at scan 830. Contrarily, the filter using the same model with the unscented transform doesn't have the same performance. On the contrary it is stable and has a very smaller error than any other filter.

In the next figure is shown the velocity rms error for the same vehicle. As it can be seen the largest error has the linear CA model, which proves that linear filters are not suitable for use in cases where maneuvering targets are being tracked. On the other hand it can be observed that the rms error of the estimated velocity using the proposed filter (UKF-CTRA) is the smallest. It follows the extended CTRA filter with a little bigger error and then the CTR filter. Also, it can be seen that while the tracked object starts to change lane, during the beginning of the overtaking, the velocity rms error remains almost constant in the case of the CA filter, while the filters using the curvilinear models temporarily estimate the velocity with a significant error.

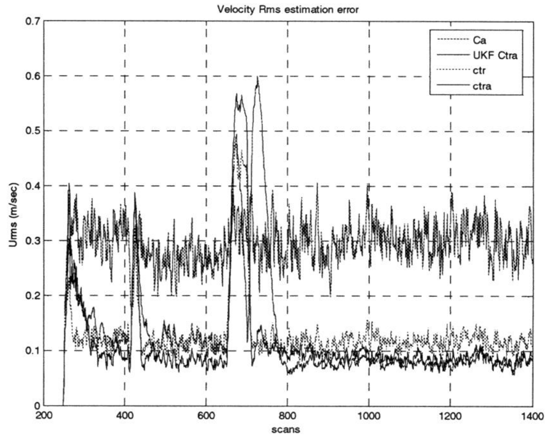

Figure 4. Rms estimation error for velocity

Finally, in the next figure the tangential acceleration rms estimation error is presented:

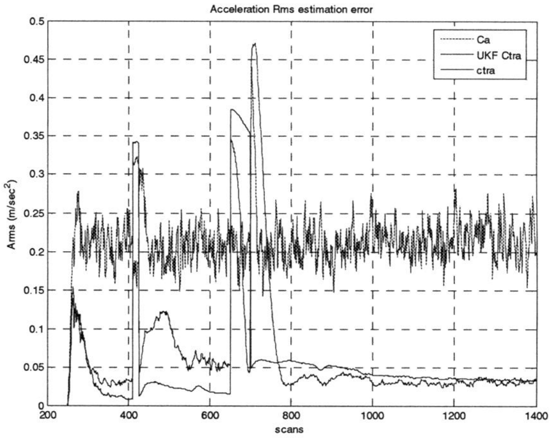

Figure 5.

It is obvious that the CTR filter is not making an estimate of the object's tangential acceleration, so the respective rms error is omitted. As it can be seen the two curvilinear models when they are used in the kalman filters has as a result a better estimation of the vehicle's acceleration. Their errors are almost the same and very much smaller than the respective error of the CA-filter. Also, the error is temporarily maximized in the start of the maneuver, but is falling much faster in the case of the UKF-CTRA model.

In this point it must be noted that the phenomenon of the rapid growth of the rms error in the case of the velocity and the acceleration is caused by the fact that are used simulated data where the created dynamic state transitions of the vehicles are not too smooth as in real data. Nevertheless, these problems can easily be eliminated by using in the filters rules for adjusting the levels of the process noise. For example when a high level of turn rate is detected by the estimated values then a higher level for the process noise will be selected.

In the following section the second scenario is examined. In this case the road is consisted from a straight segment in the start followed by a turn with a constant curvature and then again a straight segment. The trajectories in the global coordinate system in this case are shown below:

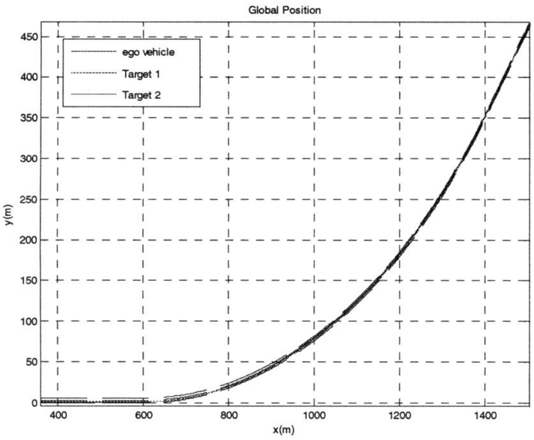

Figure 6. Scenario 2 in global coordinate system

The second scenario is more or less the same with the first one. The main difference is that the lane change of the ego vehicle and the overtaking of the third from the second vehicle are happening while they are turning and not in the straight segments of the road.

The distance rms error is shown in the next diagram (figure 7) where it is obvious that the CTRA-UKF filter makes better estimates of the true position. Once again it can be noticed the divergence of the extended curvilinear models. On the contrary, the filter using the unscented transform remains always stable. Not only that, but also keeps having the smallest rms estimation error regarding the distance of the detected object. In this case because the vehicles are already turning, the maneuver that is performed by the preceding vehicle doesn't affect too much the resulting rms error and its levels remain constant. In the previous scenario when the maneuver was performed the rms error has been raising. Also it can be observed that the variance of the error is very much higher in the case of the linear and extended filters compared to the variance of the UKF-CTRA filter.

In figure 8 is shown the velocity rms estimation error of the same target. Once again the UKF-CTRA filter is making a better estimation of the velocity that the tracked object has. In this case the CTR filter performs better during the whole scenario except an important interval. When the tracked preceding vehicle starts to accelerate in order to overtake the second car in the road, the extended CTRA filter and the CA filter make better estimates of the velocity than the CTR filter. This is logical as these filters take into consideration the object's acceleration, something that is not done in the case of the constant turn rate model.

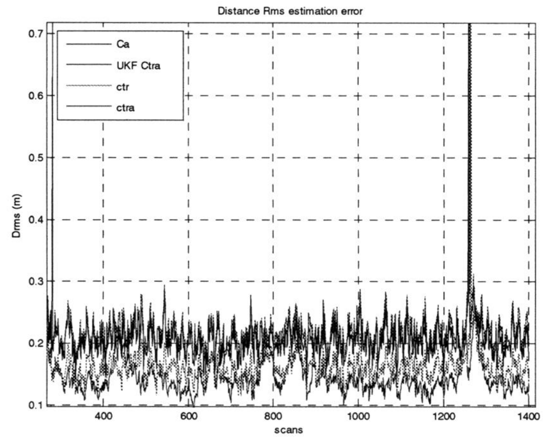

Figure 7. Scenario 2 distance rms error

In the next figure is shown the velocity rms error.

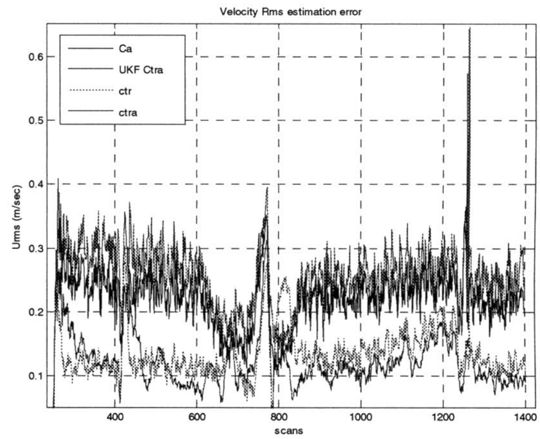

Figure 8. Scenario 2 velocity rms error

5 Conclusions

As presented in this article, the unscented Kalman filter using the proposed model improves extensively the performance of the tracking system used in automotive applications. The dynamic behavior of an object can be described more accurately using the constant turn rate and constant tangential acceleration model (CTRA), but there is the problem of the filter divergence. This is eliminated by using the unscented transform.

It was shown that the CTRA-UKF filter is estimating the position, the velocity and the acceleration of the tracked object with a very small error compared with the other three models. It appears to be the best solution in cases where the tracked object is performing a maneuver, is starting to accelerate or when it is turning, like in an overtaking scenario.

The problem that arises during a sharp maneuver in the UKF CTRA filter can be eliminated by using a filter with adaptive process noise. When the systems detects a sharp maneuver by checking the turn rate then it can apply higher levels of process noise in order to avoid the peaks in the rms errors in the start of the maneuver. Also, future steps include the design of an interactive multiple model filter utilizing the unscented transform and the curvilinear model that was presented in this paper.

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