THE KINEMATIC ANALYSIS OF RAVIGNEAUX PLANETARY GEAR SET

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KINEMATSKA ANALIZA RAVINJONOVOG PLANETARNOG PRENOSNIKA




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Resume: *The power transfer is an irreplaceable part of every industry. Due to technology improvement, the mechanisms are working with less and less losses, but their constructions are becoming more complex. This essay is dedicated to the planetary gear sets, with a focus on the Ravigneaux planetary gear set which represents, in fact, improved version of Simpson's planetary gear set. The detailed kinematic analysis of the gear set will be explained in a form of the calculation of the number of revolutions for every gear set, independently to the axis around which they revolve.*

Key words: *Ravigneaux planetary gear set, planetary mechanism, kinematic analysis of Ravigneaux planetary gear set.*

1. INTRODUCTION

The planetary gear sets belong to the group of mechanical gear sets whose the most common use is in military and automotive industries. The reason of the more and more intense use of these gear sets lies

in the advantages which they provide, like achieving the high transmission ratios, the compactness, achieving the different degrees of transmission and much more

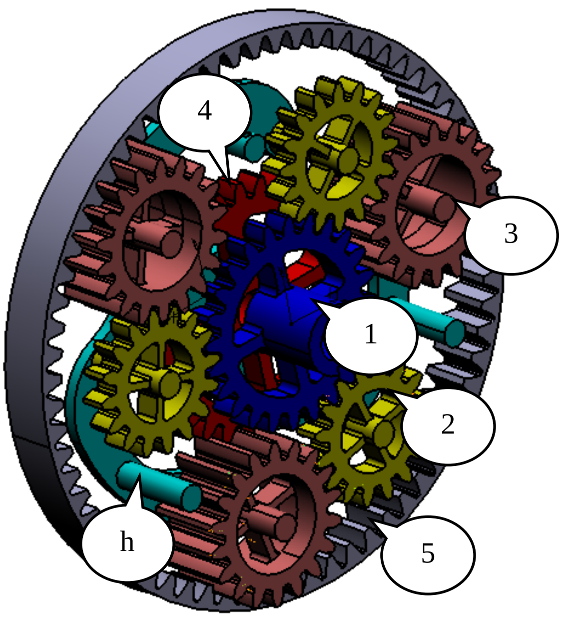


Figure 1. Ravigneaux planetary gear set (1,4,5 – central gears; 2,3 - satellites; h – satellite carrier)

The part which differ the planetary gear sets from the others is the existence of units which revolve around their axes and at the same time around the central unit and the central axis. Their movement is, by its characteristics, similar to the movement of the planets, so that's the reason why they've got their name.[1] The gears which are spinning only around their own axes are called *central* or *sun gears*, while the ones moving around them are called *satellites (see the Figure 1).*

Ravigneaux planetary gear set mechanism belongs to the mechanical power gear sets with gradually changeable transmission ratio which can act either as reductor or multiplier. That's double mechanical gear set which was invented by Paul Ravigneaux. It was patented on 28th of July 1949. in France. It is composed of two sun gears and one central gear with internal teeth and between them two satellites are placed, bonded with a carrier, which can be seen in Figure 1. [2, 3].

2. THE APPLICATION OF RAVIGNEAUX PLANETARY GEAR SET

The application of the planetary gear sets is made more difficult due to the use of the following elements, which allow the turning on/turning off or the blocking of certain elements of the gear set, which makes the construction more complex and more expensive.

These gear sets can accomplish huge transmission ratios, but it should be mentioned that the degree of efficiently is reduced that way. The possibility that the space between central gears can be filled by bigger number of satellites enables the better use of interior space and makes possible the compact construction of the planetary gear set. The application of bigger number of satellites enables the transfer of the load simultaneously by larger number of gear teeth, which leads to a reduction of the load and enables the choice of smaller modules. Thanks to these characteristics, it is used in different branches of industry, but the most frequent commercial application of Ravigneaux planetary gear set is in the automotive industry, in construction of automatic gearboxes.[4.5] The cars which use this kind of power transfer are: Hyundai Solaris, Mercedes-Benz W7A 700 *(Figure 2)*, Mustang C4, C6 and many others.[5]

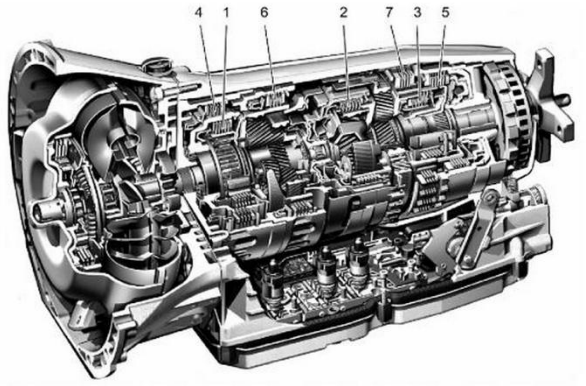


Figure 2. The seven-speed automatic gearbox Mercedes- Benz W7A 700 ($1 \div 3$ couplings, $4 \div 7$ brakes) [6]

3. METHODOLOGY OF THE ANALYSIS OF THE PLANETARY GEAR SET

The design of the planetary gear sets implies previous kinematic analysis of the adopted scheme of the gear set. The main tasks of the kinematic analysis are:

- determining transmission unit's position,
- determining velocity and acceleration of transmission elements,
- defining equations of motion of transmission elements,
- determining number of revolutions of transmission elements and
- determining transmission ratios.

From the point of view of rational construction, determining number of revolutions of certain elements and defining of the transmission ratios are especially important. An exact defining of these characteristics can be made difficult, especially in the cases of complex planetary gear sets.

For kinematic examinations and assignation of kinematic characteristics of planetary gear sets, today in the technical literature are most commonly used:

- method of plan velocity and number of revolutions,
- method of blocking the satellite carrier – method of Willis,
- determining the transmission ratio by using of the rotation angle,
- methods of the current pole and
- method of decomposition of complex motion – method of Swamp [1]

In this essay, for the kinematic analysis, it was used combined method because other methods, which are based on graphic methods, are making the situation more complicated because the construction of the acceleration plan and the plan of number of revolutions is pretty long and inconvenient process.

Ravigneaux planetary gear set

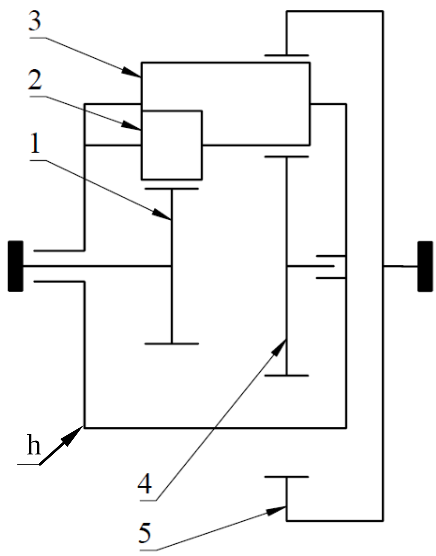


Figure 3. Scheme of Ravigneaux planetary mechanism (1- first central gear; 2,3- satellites; 4- second central gear; 5 - third central gear; h- satellite carrier)

In Figure number 2 is shown the scheme of Ravigneaux gear set, on which we can see the marks used in the general equations of motion *(1)*:

$$\begin{aligned} n_{cz1} - n_h &= (-1)^m \cdot i_{o1}(n_{cz2} - n_h) \\ n_{cz2} - n_h &= (-1)^m \cdot i_{o2}(n_{cz3} - n_h) \end{aligned} \tag{1}$$

Where are:

- n_{cz1} – number of revolutions of first central gear*(gear 1, Figure 3),*
- n_{cz2} – number of revolutions of second central gear *(gear 4, Figure 3),*
- n_{cz3} – number of revolutions of third central gear *(gear 5, Figure 3),*
- n_h – number of revolutions of satellite carrier,
- m – number of external couplings between gears in the gearbox from one central gear to another and
- i_{o1}, i_{o2} – kinematic transmission ratios of corresponding classical mechanical gear set.

Following equation is being used to determine self-rotating speeds of satellites:

$$n_s = (-1)^m \cdot i_k \cdot (n_{cz1} - n_h) \qquad (2)$$

The proof for the above mentioned equations can be seen in the literature [1].

4. KINEMATIC ANALYSIS

Using the different coupling, the brake, etc. it is possible to block one of the transmission elements and reach certain transmission ratio, or even change the spinning direction of the output gear. If it's supposed that the gear number 5 is connected to the output shaft, there can be seven possible gear ratios which can be reached by this gear set. In this essay it will be explained in details the process of obtaining the formulas of velocities and trans-mission ratios of only one concept while the rest of them can be seen in the literature [5]. System of labeling is being necessary to create due to characteristic of planetary gear sets to easily change gear ratios depending on which element is blocked i.e. its application as gearbox. Due to characteristic of planetary gear sets that it is easily possible to change the transmission ratio depending on which element is blocked: due to the application of planetary gear sets as gearboxes, the need for the system of labeling is created. To remove doubts a-bout labeling and types of concepts in this essay, following system from the literature will be used [1] *(Figure 4).*

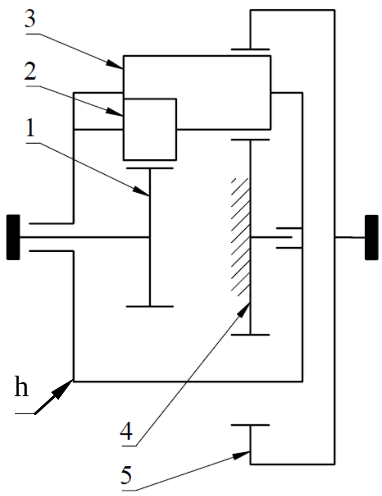
$$Concept_{input\_output}^{immobile\ element}$$

Figure 4. Labeling of the planetary gear sets

So, Ravigneaux planetary gear set will be marked with Latin letter R and the corresponding concepts will follow the marks of the Figure 4.

4.1. Concept R_{15}^{4}

This concept represents Ravigneaux gear set in case where the central gear 4 is blocked *(Figure 5).* The starting point is the general equation *(1)* where $n_{cz1} = n_1;\ n_{cz3} = n_5;\ n_{cz2} = n_4$. The number of necessary general equations of motion are always one less than the number of central gears, so in this case there are two of them. While their defining, it is recommended to follow the power flow, i.e. the longest path from the input to the exit, from the gear number 1 through the gear 4 and then up to the output gear 5. Marks $z_1 \div z_5$ represent the number of gear teeth of the corresponding gears.



The starting point are the general equations *(1)*:

$$n_1 - n_h = (-1)^m\, i_{o1}\,(n_4 - n_h)$$

Where:

$$m = 3;\ i_{o1} = \frac{z_2}{z_1} \cdot \frac{z_3}{z_2} \cdot \frac{z_4}{z_3} = \frac{z_4}{z_1};\ n_4 = 0$$

- m – number of external couplings between gears, from 1st to 4th gear and
- i_{o1} – kinematic transmission ratio, from 1st to 4th gear.

After commutation of the given values from the last equations following expression is obtained:

$$n_1 - n_h = -i_{o1}(-n_h)$$

$$n_1 = (i_{o1} + 1)\,n_h \Rightarrow n_h = \frac{n_1}{i_{o1} + 1}$$

The number of revolutions for the satellite carriers:

$$\boxed{n_h = n_1 \frac{z_1}{z_4 + z_1}} \qquad (3)$$

The second general equation is:

$$n_4 - n_h = (-1)^m i_{o2} (n_5 - n_h)$$

Where:

$$m = 1;\ \mathrm{i}_{o2} = \frac{z_3}{z_4} \cdot \frac{z_5}{z_3} \cdot = \frac{z_5}{z_4};\ n_4 = 0$$

- m – number of external couplings between gears, from 4th to 5th gear and
- i_{o2} – kinematic transmission from 4th to 5th gear.

After ordering the expression:

$$-n_h = -i_{o2} (n_5 - n_h)$$

$$\Rightarrow n_5 = n_h \frac{i_{o2} + 1}{i_{o2}} = n_1 \frac{z_1}{z_4 + z_1} \cdot \frac{\frac{z_5}{z_4} + 1}{\frac{z_5}{z_4}}$$

The number of revolutions output gear:

$$\boxed{n_5 = n_1 \frac{z_1 (z_5 + z_4)}{z_5 (z_4 + z_1)}} \qquad (4)$$

It is recommended to always express the characteristic through the same one, relatively speaking, it is recommended to express the depended values from their dependent ones.

Transmission ratio of the planetary gear set:

$$\boxed{i_{pp} = \frac{n_1}{n_5} = \frac{n_1}{n_1 \frac{z_1 (z_5 + z_4)}{z_5 (z_4 + z_1)}} = \frac{z_5 (z_4 + z_1)}{z_1 (z_5 + z_4)}} \qquad (5)$$

To determine the number of self-rotation of satellites, equation *(2)*, is used:

$$n_2 = (-1)^m i_1 (n_1 - n_h)$$

Where is:

$$m = 1;\ i_1 = \frac{z_1}{z_2}$$

- m – number of external couplings between gears, from 2nd to 1st gear and
- i_1 – kinematic transmission ratio from 2nd to 1st gear.

After commutation of the given values from the last equations:

$$n_2 = -i_1 (n_1 - n_h)$$

$$n_2 = -i_1 \left(n_1 - n_1 \frac{z_1}{z_4 + z_1} \right) = -n_1 \frac{z_1}{z_2} \cdot \frac{z_4}{z_4 + z_1}$$

The number of revolutions of satellite number 2:

$$\boxed{n_2 = -n_1 \frac{z_1 z_4}{z_2 (z_4 + z_1)}} \qquad (6)$$

The negative value of number of revolutions of satellite 2 shows that it revolves in the opposite direction of direction of revolutions of gear 1, i.e. input.

Verification:

To successfully determine equation for verification the number of revolutions calculated by expression *(6)*, it is necessary to observe the system in the opposite direction:

$$n_{2s}^p = (-1)^m i_2 (n_5 - n_h)$$

Where:

$$m = 1;\ i_2 = \frac{z_3}{z_2} \cdot \frac{z_5}{z_3} = \frac{z_5}{z_2}$$

- m – number of external couplings between gears, from 2nd to 5th gear and
- i_2 – kinematic transmission from 2nd to 5th gear.

After the arranging of earlier equation:

$$n_{2s}^p = -i_2 (n_5 - n_h) = -i_2 \left(n_1 \frac{z_1 (z_5 + z_4)}{z_5 (z_4 + z_1)} - n_1 \frac{z_1}{z_4 + z_1} \right)$$

$$\boxed{n_{2s}^p = -n_1 \frac{z_1 z_4}{z_2 (z_4 + z_1)}} \qquad (7)$$

This verification proves the veracity.

By making analogy with the process used for calculation of the number of revolutions of satellite 2, same approach is followed for calculating the number of revolutions of other satellite i.e. gear 3.

$$n_{3s} = (-1)^m i_1 (n_1 - n_h)$$

Where:

$$m = 2;\ i_1 = \frac{z_2}{z_3} \cdot \frac{z_1}{z_2} = \frac{z_1}{z_3}$$

- m – number of external couplings between gears, from 3rd to 1st gear and

- i_1 – kinematic transmission ratio from 3rd to 1st gear.

After commutation of given values in the previous line:

$$n_{3s} = i_1(n_1 - n_h) = n_1 \frac{z_1}{z_3} \cdot \frac{z_4}{z_4 + z_1}$$

Number of revolutions of the satellite 3:

$$\boxed{n_{3s} = n_1 \frac{z_1 z_4}{z_3(z_4 + z_1)}} \tag{8}$$

Verification:

To successfully determine equation for verification the number of revolutions calculated by expression (8) it is necessary to observe the system in the opposite direction:

$$n_{3s}^p = (-1)^m i_2 (n_5 - n_h)$$

Where:

$$m = 0;\ i_2 = \frac{z_5}{z_3}$$

- m – number of external couplings between gears, from 3rd to 5th gear and
- i_2 – kinematic transmission ratio from 3rd to 5th gear.

After the arranging of earlier equation:

$$n_{3s}^p = i_2(n_5 - n_h) = n_1 \frac{z_5}{z_3} \cdot \frac{z_1 z_4}{z_5(z_4 + z_1)}$$

$$\boxed{n_{3s}^p = n_1 \frac{z_1 z_4}{z_3(z_4 + z_1)}} \tag{9}$$

This verification proves the veracity.

Equations that determine the number of self-revolution of the satellites can be verified by checking their interrelation, wherein its well-known that they should have opposite directions of revolution as well as that they should have kinematic gear ratio as in the following expression:

$$\boxed{\frac{n_2}{n_3} = \frac{-n_1 \dfrac{z_1 z_4}{z_2(z_4 + z_1)}}{n_1 \dfrac{z_1 z_4}{z_3(z_4 + z_1)}} = -\frac{z_3}{z_2}} \tag{10}$$

5. CONCLUSION

In analogy to calculation from the previous chapter, depending on the input, detailed analysis of velocity of elements of Ravigneaux gear set can be done.

Number of revolutions in correspondence to number of teeth of every element of Ravigneaux gear set is shown in table 1. It is clear that, while choosing the adequate number of gear teeth depending on the need, it is necessary to pay attention on getting the highest possible number of varieties of transmission ratios. It should be highlighted that, in case of having two blocked gears, the whole system is acting like a coupling, so the transmission ratio is 1. The detailed derivation of kinematic equations of each of mentioned concepts in the table 1 can be seen in literature [5].

6. LITERATURE

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Table 1. Specific values of Ravigneaux planetary gear set [5]

		Conceptions					
		R_{15}^{4}	R_{15}^{h}	R_{h5}^{1}	R_{h5}^{4}	R_{45}^{h}	R_{45}^{1}
Number of revolutions	n_1	input	input	immobile	$n_h \frac{z_4 + z_1}{z_1}$	$n_h \frac{z_4 + z_1}{z_1}$	immobile
	n_2	$-n_1 \frac{z_1 z_4}{z_2 \left(z_4 + z_1 \right)}$	$-\frac{z_1}{z_2} n_1$	$\frac{z_1}{z_2} n_h$	$-\frac{z_4}{z_2} n_h$	$\frac{z_4}{z_2} n_4$	$\frac{z_4 z_1}{z_2 \left(z_1 + z_4 \right)} n_4$
	n_3	$n_1 \frac{z_1 z_4}{z_3 \left(z_4 + z_1 \right)}$	$\frac{z_1}{z_3} n_1$	$-\frac{z_1}{z_3} n_h$	$\frac{z_4}{z_3} n_h$	$-\frac{z_4}{z_3} n_4$	$-\frac{z_4 z_1}{z_3 \left(z_1 + z_4 \right)} n_4$
	n_4	immobile	$-n_1 \frac{z_1}{z_4}$	$n_h \frac{z_4 + z_1}{z_4}$	immobile	input	input
	n_5	$n_1 \frac{z_1 \left(z_5 + z_4 \right)}{z_5 \left(z_4 + z_1 \right)}$	$n_1 \frac{z_1}{z_5}$	$\frac{z_5 - z_1}{z_5} n_h$	$\frac{z_5 + z_4}{z_5} n_h$	$-n_4 \frac{z_4}{z_5}$	$n_4 \frac{z_4 \left(z_5 - z_1 \right)}{z_5 \left(z_1 + z_4 \right)}$
	n_h	$n_1 \frac{z_1}{z_4 + z_1}$	immobile	input	input	immobile	$\frac{z_4}{z_1 + z_4} n_4$
i_{pp}		$\frac{z_5 \left(z_4 + z_1 \right)}{z_1 \left(z_5 + z_4 \right)}$	$\frac{z_5}{z_1}$	$\frac{z_5}{z_5 - z_1}$	$\frac{z_5}{z_5 + z_4}$	$-\frac{z_5}{z_4}$	$\frac{z_5 \left(z_1 + z_4 \right)}{z_4 \left(z_5 - z_1 \right)}$

Key:

- n_1, n_2, n_3, n_4, n_5 – number of revolutions around their own axes,
- n_h – number of revolutions of satellite around central axis,
- i_{pp} – transmission ratio of Ravigneaux planetary gear set
- z_1, z_2, z_3, z_4, z_5 – number of gear teeth.