

SOLUTIONS MANUAL

**INTRODUCTION TO
ENVIRONMENTAL
ENGINEERING
and SCIENCE**

SECOND EDITION

GILBERT M. MASTERS

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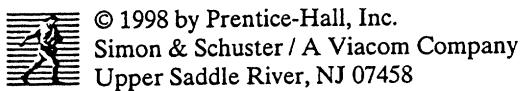
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SOLUTIONS FOR CHAPTER 1

1.1 Ozone at 0.08 ppm; $\text{mg/m}^3 = \frac{\text{ppm} \times \text{mol wt}}{24.414} \times \frac{273.15}{\text{T(K)}} \times \frac{\text{P(atm)}}{1 \text{ atm}}$ (1.9)

a. $\text{mg/m}^3 \text{ (at } 25^\circ\text{C, 1 atm)} = \frac{0.08 \times (3 \times 16)}{22.414} \times \frac{273.15}{(273.15+25)} = 0.157 \text{ mg/m}^3 = 157 \mu\text{g/m}^3$

b. In Denver, at 15°C and 0.82 atm:

$$\text{mg/m}^3 = \frac{0.08 \times (3 \times 16)}{22.414} \times \frac{273.15}{(273.15+15)} \times \frac{0.82}{1} = 0.133 \text{ mg/m}^3 = 133 \mu\text{g/m}^3$$

1.2 Exhaust at 1% CO, 25°C , 1 atm: $1\% \text{ CO} = \frac{1 \text{ part CO}}{100 \text{ parts air}} = \frac{10^4 \text{ parts CO}}{10^6 \text{ parts air}} = 1 \times 10^4 \text{ ppm}$

$$\text{mg/m}^3 = \frac{1 \times 10^4 \times (12 + 16)}{22.414} \times \frac{273.15}{(273.15+25)} = 11,445 \text{ mg/m}^3$$

1.3 400 $\mu\text{g/m}^3$ of SO₂ at 25°C , 1 atm:

$$\text{ppm} = \frac{24.465 \times \text{SO}_2(\text{mg/m}^3)}{\text{mol wt}} = \frac{24.465 \times 0.400}{(32 + 2 \times 16)} = 0.15 \text{ ppm}$$

YES, exceeds the air quality standard of 0.14 ppm.

1.4 Motorcycle emitting 20 g/mi of CO:

a. $V = 20 \text{ g CO/mi} \times 5 \text{ mi} \times \frac{\text{mol}}{28 \text{ g}} \times 24.465 \text{ L/mol} \times \frac{1 \text{ m}^3}{10^3 \text{ L}} = 0.087 \text{ m}^3$

b. 9 ppm CO: $\frac{9 \text{ m}^3 \text{ CO}}{10^6 \text{ m}^3 \text{ air}} = \frac{0.087 \text{ m}^3 \text{ CO}}{V \text{ m}^3 \text{ air}} \quad V = 9666 \text{ m}^3$

$$\frac{\text{m}^3 \text{ polluted}}{\text{mile}} = \frac{9666 \text{ m}^3}{5 \text{ mi} \times 1609 \text{ m/mi}} = 1.2 \text{ m}^3 / \text{mi}$$

1.5 Air density with 79% N₂ and 21% O₂:

$$N_2 : \frac{0.79 \text{ m}^3 \text{ N}_2}{1 \text{ m}^3 \text{ air}} \times \frac{28 \text{ g}}{\text{mol}} \times \frac{1 \text{ mol N}_2}{22.414 \times 10^{-3} \text{ m}^3 \text{ N}_2} = 987 \text{ g N}_2 / \text{m}^3 \text{ air}$$

$$O_2 : \frac{0.21 \text{ m}^3 \text{ N}_2}{1 \text{ m}^3 \text{ air}} \times \frac{32 \text{ g}}{\text{mol}} \times \frac{1 \text{ mol N}_2}{22.414 \times 10^{-3} \text{ m}^3 \text{ N}_2} = 300 \text{ g O}_2 / \text{m}^3 \text{ air}$$

$$\text{Total} = 987 + 300 = 1287 \text{ g/m}^3 = 1.287 \text{ kg/m}^3$$

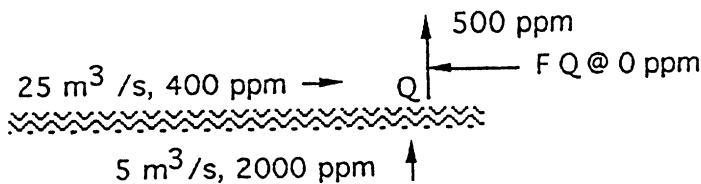
1.6 Mixing 10 MGD, 3.0 mg/L, with 5 MGD, 10.0 mg/L:

$$\text{a. } 10 \text{ MGD} \times 3.0 \text{ mg/L} + 5 \text{ MGD} \times 10.0 \text{ mg/L} = (10 + 5) \text{ MGD} \times C \text{ mg/L}$$

$$C = 80 / 15 = 5.33 \text{ mg/L}$$

$$\text{b. } 5.33 \frac{\text{mg}}{\text{L}} \times 15 \times 10^6 \frac{\text{gal}}{\text{day}} \times 3.785 \frac{\text{L}}{\text{gal}} \times \frac{1 \text{ kg}}{10^6 \text{ mg}} \times 2.2 \frac{\text{lb}}{\text{kg}} = 666 \text{ lb/day}$$

1.7



$$\text{Upstream of take-out: } C = \frac{25 \text{ m}^3/\text{s} \times 400 \text{ mg/L} + 5 \text{ m}^3/\text{s} \times 2000 \text{ mg/L}}{(25 + 5) \text{ m}^3/\text{s}} = 667 \text{ mg/L}$$

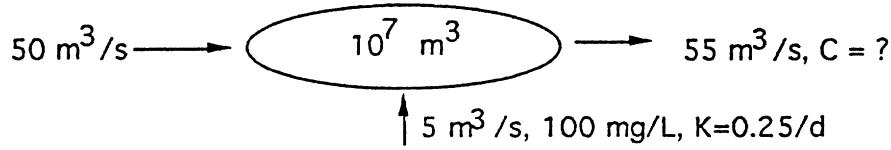
Drinking water @ 500 ppm:

$$500 \text{ mg/L} \times (Q + FQ) \text{ m}^3/\text{s} = 667 \text{ mg/L} \times Q \text{ m}^3/\text{s}$$

$$500(1 + F) = 667$$

$$F = \frac{667}{500} - 1 = 0.333 \quad (\text{that is, } 1/3 \text{ pure water})$$

1.8

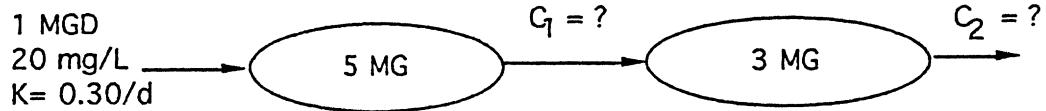


$$\text{Input} = \text{Output} + \text{Decay}, \text{ where decay} = KCV$$

$$5 \text{ m}^3/\text{s} \times 100 \text{ mg/L} = 55 \text{ m}^3/\text{s} \times C (\text{mg/L}) + \frac{0.25/\text{day}}{24 \text{ hr/d} \times 3600 \text{ s/hr}} \times C (\text{mg/L}) \times 10^7 \text{ m}^3$$

$$500 = 55C + 28.9C \quad C = \frac{500}{83.9} = 5.96 \text{ mg/L}$$

1.9



$$\text{Lake 1: Input} = \text{Output} + \text{KCV}$$

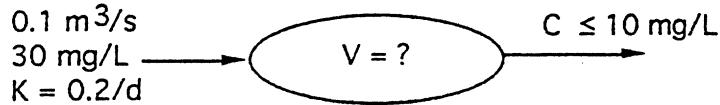
$$1 \text{ MGD} \times 20 \text{ mg/L} = 1 \text{ MGD} \times C_1 + 0.3/\text{day} \times 5 \text{ MG} \times C_1$$

$$C_1 = \frac{20}{1+1.5} = 8.0 \text{ mg/L}$$

$$\text{Lake 2: } 1 \text{ MGD} \times 8.0 \text{ mg/L} = 1 \text{ MGD} \times C_2 + 0.3/\text{d} \times 3 \text{ MG} \times C_2$$

$$C_2 = \frac{8.0}{1+0.9} = 4.2 \text{ mg/L}$$

1.10

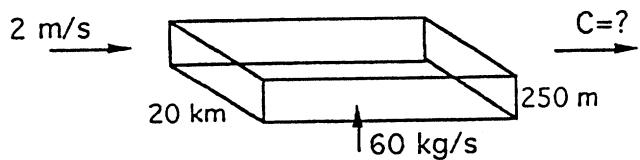


$$\text{Input} = \text{Output} + \text{KCV}$$

$$0.1 \text{ m}^3/\text{s} \times 30 \text{ mg/L} = 0.1 \text{ m}^3/\text{s} \times 10 \text{ mg/L} + \left(\frac{0.2/\text{day}}{24 \text{ hr/d} \times 3600 \text{ s/hr}} \right) \times 10 \text{ mg/L} \times V \text{ m}^3$$

$$V = \frac{3.0 - 1.0}{2.31 \times 10^{-5}} = 86,400 \text{ m}^3$$

1.11

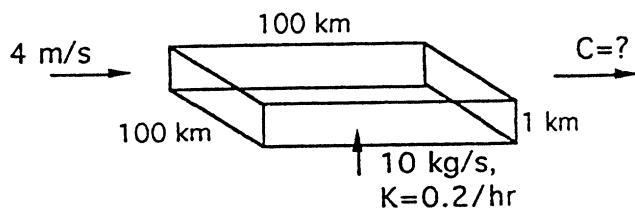


CO Input rate = Output rate

$$60 \text{ kg/s} \times 10^6 \text{ mg/kg} = 20 \times 10^3 \text{ m} \times 250 \text{ m} \times 2 \text{ m/s} \times (\text{CO}) \text{ mg/m}^3$$

$$\text{CO} = \frac{60 \times 10^6}{20,000 \times 250 \times 2} = 6 \text{ mg/m}^3$$

1.12



Input rate = Output rate + KCV

$$\text{Input rate} = 10 \text{ kg/s} \times 10^9 \mu\text{g/kg} = 10 \times 10^9 \mu\text{g/s}$$

$$\text{Output rate} = 100 \times 10^3 \text{ m} \times 10^3 \text{ m} \times 4 \text{ m/s} \times C (\mu\text{g/m}^3) = 4 \times 10^8 C \mu\text{g/s}$$

$$\text{Decay rate} = \frac{0.2/\text{hr}}{3600 \text{ s/hr}} \times C (\mu\text{g/m}^3) \times 10^5 \text{ m} \times 10^5 \text{ m} \times 10^3 \text{ m} = 5.55 \times 10^8 C \mu\text{g/s}$$

$$C_{\infty} = \frac{10 \times 10^9}{4 \times 10^8 + 5.55 \times 10^8} = 10.5 \mu\text{g/m}^3$$

1.13 $C_{\infty} = \frac{S}{Q + KV} \quad (1.20)$

$$S = 10 \times 10^9 \mu\text{g/s}$$

$$Q = 1 \text{ m/s} \times 10^5 \text{ m} \times 10^3 \text{ m} = 1 \times 10^8 \text{ m}^3/\text{s} \text{ (new value at 1 m/s wind)}$$

$$KV = \frac{0.2/\text{hr}}{3600 \text{ s/hr}} \times 10^5 \text{ m} \times 10^5 \text{ m} \times 10^3 \text{ m} = 5.55 \times 10^8 \text{ m}^3/\text{s}$$

$$C_{\infty} = \frac{10 \times 10^9 \mu\text{g/s}}{(1 \times 10^8 + 5.55 \times 10^8) \text{ m}^3/\text{s}} = 15.26 \mu\text{g/m}^3$$

$$C(t) = [C_0 - C_{\infty}] \exp[-(K + Q/V)t] + C_{\infty} \quad (1.29)$$

$$C(2\text{hr}) = [10.5 - 15.26 \mu\text{g/m}^3] \exp \left[- \left(\frac{0.2}{\text{hr}} + \frac{10^8 \text{ m}^3/\text{s} \times 3600 \text{ s/hr}}{10^{13} \text{ m}^3} \right) 2\text{hr} \right] + 15.26 \mu\text{g/m}^3$$

$$= 12.3 \mu\text{g/m}^3$$

1.14



a. Input rate = Output rate (conservative)

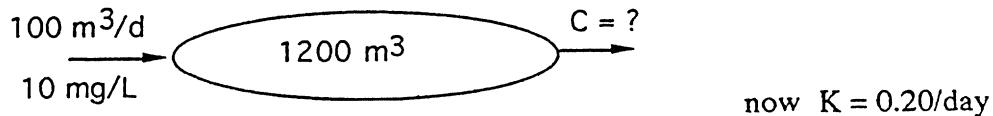
$$100 \text{ m}^3/\text{d} \times 10 \text{ mg/L} = 100 \text{ m}^3/\text{d} \times C \quad C = 10 \text{ mg/L}$$

b. Change input concentration suddenly to 100 mg/L, $C_\infty = 100 \text{ mg/L}$, $C(7\text{days}) = ?$

$$C(t) = [C_0 - C_\infty] \exp[-(K + Q/V)t] + C_\infty \quad (1.29)$$

$$C(7 \text{ hr}) = [10 - 100 \text{ mg/L}] \exp \left[- \left(0 + \frac{100 \text{ m}^3/\text{d}}{1200 \text{ m}^3} \right) 7 \text{ d} \right] + 100 \text{ mg/L} = 49.8 \text{ mg/L}$$

1.15



a. Input rate = Output rate + KCV

$$100 \text{ m}^3/\text{d} \times 10 \text{ mg/L} = 100 \text{ m}^3/\text{d} \times C \text{ mg/L} + 0.20/\text{d} \times C \text{ mg/L} \times 1200 \text{ m}^3$$

$$C = \frac{100 \times 10}{100 + 0.20 \times 1200} = 2.94 \text{ mg/L}$$

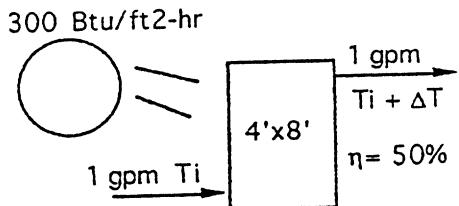
b. Change input suddenly to 100 mg/L, $C(7 \text{ days}) = ?$

$$C_\infty = \frac{S}{Q + KV} = \frac{100 \text{ m}^3/\text{d} \times 100 \text{ mg/L}}{100 \text{ m}^3/\text{d} + 0.20/\text{d} \times 1200 \text{ m}^3} = 29.4 \text{ mg/L}$$

$$C(t) = [C_0 - C_\infty] \exp[-(K + Q/V)t] + C_\infty$$

$$\begin{aligned} C(7 \text{ d}) &= [2.94 - 29.4 \text{ mg/L}] \exp \left[- \left(\frac{0.20}{\text{d}} + \frac{100 \text{ m}^3/\text{d}}{1200 \text{ m}^3} \right) \times 7 \text{ d} \right] + 29.4 \text{ mg/L} \\ &= 25.8 \text{ mg/L} \end{aligned}$$

1.16

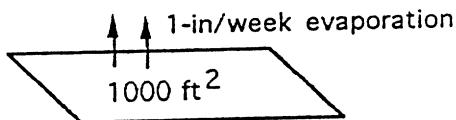


Rate of energy absorbed = Rate of change of stored energy

$$0.50 \times 300 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2} \times 4 \text{ ft} \times 8 \text{ ft} = 1.0 \frac{\text{gal}}{\text{min}} \times 8.34 \frac{\text{lb}}{\text{gal}} \times 60 \frac{\text{min}}{\text{hr}} \times \frac{1 \text{ Btu}}{\text{lb} \cdot {}^\circ\text{F}} \times \Delta T({}^\circ\text{F})$$

$$\Delta T = \frac{0.50 \times 300 \times 32}{1 \times 8.34 \times 60} = 9.6 \text{ } {}^\circ\text{F}$$

1.17

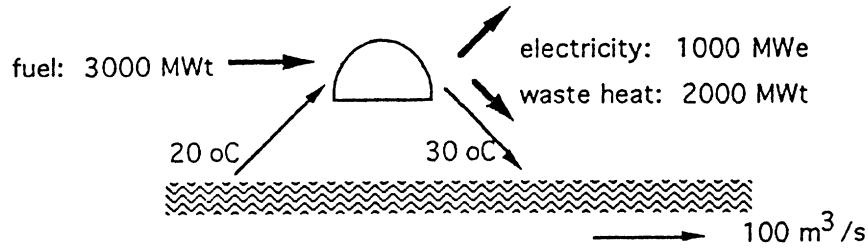


$$1.0 \frac{\text{in}}{\text{wk}} \times 15 \text{ wks} \times \frac{1 \text{ ft}}{12 \text{ in}} \times 1000 \text{ ft}^2 \times 62.4 \frac{\text{lb}}{\text{ft}^3} \times 1050 \frac{\text{Btu}}{\text{lb}} = 81.9 \times 10^6 \text{ Btu/season}$$

$$\text{Cover saves: } \frac{2}{3} \times 81.9 \times 10^6 \frac{\text{Btu}}{\text{season}} \times \frac{\$10.00}{10^6 \text{ Btu}} = \$546 / \text{yr}$$

YES a \$500 cover does pay for itself in less than one season.

1.18



a. Rate of heat added to cooling water = rate of change of stored energy = $\dot{m} c \Delta T$

$$2000 \times 10^6 \text{ W} \times \frac{1 \text{ J/s}}{\text{W}} = \dot{m} \left(\frac{\text{kg}}{\text{s}} \right) \times 4184 \frac{\text{J}}{\text{kg}^\circ\text{C}} \times 10^\circ\text{C}$$

$$\dot{m} = \frac{2000 \times 10^6}{4184 \times 10} = 47.8 \times 10^3 \text{ kg/s} \times 10^{-3} \text{ m}^3/\text{kg} = 47.8 \text{ m}^3/\text{s}$$

$$\frac{\text{nuclear plant cooling water}}{\text{coal plant cooling water (Ex 1.10)}} = \frac{47.8 \text{ m}^3/\text{s}}{40.6 \text{ m}^3/\text{s}} = 1.177 \quad (\approx 18\% \text{ more})$$

b. River temperature rises by ΔT as it receives 2000 MWt of heat:

$$\Delta T = \frac{\text{cooling water heat gain rate}}{\dot{m} c} = \frac{2000 \times 10^6 \text{ J/s}}{100 \text{ m}^3/\text{s} \times 4184 \text{ J/kg}^\circ\text{C} \times 10^3 \text{ kg/m}^3} = 4.8^\circ\text{C}$$

1.19 Moisture condensing releases latent heat = $5 \text{ mL} \times 1 \text{ g/mL} \times 2.5 \text{ kJ/g} = 12.5 \text{ kJ}$

Sensible heating = $\dot{m} c \Delta T = 0.354 \text{ kg} \times 4.18 \text{ kJ/kg}^\circ\text{C} \times \Delta T = 12.5 \text{ kJ}$

$$\Delta T = \frac{12.5}{0.354 \times 4.18} = 8.4^\circ\text{C}$$

1.20 Energy needed to vaporize 1 kg of water at 15°C (Table 1.4) = 2465 kJ

To raise 1 kg 3000 m requires:

$$3000 \text{ m} \times 1 \text{ kg} \times \frac{9.8 \text{ N}}{\text{kg}} \times \frac{1 \text{ J}}{\text{N} \cdot \text{m}} = 29,400 \text{ J} = 29.4 \text{ kJ}$$

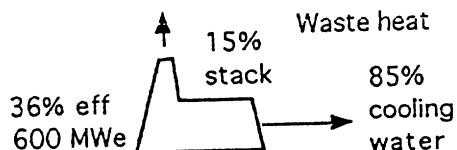
$$\frac{\text{Energy to vaporize water}}{\text{Energy to raise it 3 km}} = \frac{2465 \text{ kJ}}{29.4 \text{ kJ}} = 83:1$$

1.21 Condensation of 1 lb of water releases 1060 Btu (Table 1.4)

$$\text{Potential energy of 1 lb at 5000 ft} = 5000 \text{ ft} \times 1 \text{ lb} \times \frac{\text{Btu}}{778 \text{ ft} \cdot \text{lb}} = 6.43 \text{ Btu}$$

$$\frac{\text{Potential energy}}{\text{Latent heat}} = \frac{6.43 \text{ Btu}}{1060 \text{ Btu}} = 0.006 = 0.06\%$$

1.22

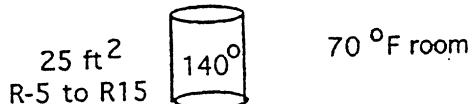


$$\text{Heat input} = \frac{600 \text{ MWe}}{0.36} = 1667 \text{ MWt}$$

$$\text{Heat to cooling water} = 0.85 \times (1667 - 600) \text{ MW} = 907 \text{ MWt}$$

$$\text{Evaporation rate} = 907 \times 10^3 \text{ kW} \times \frac{1 \text{ kJ/s}}{\text{kW}} \times \frac{\text{kg}}{2465 \text{ kJ}} \times \frac{\text{m}^3}{10^3 \text{ kg}} = 0.37 \text{ m}^3/\text{s}$$

1.23 Water heater jacket:



$$\text{Heat loss before} = \frac{A \Delta T}{R} = \frac{25 \text{ ft}^2 (140 - 70^\circ \text{F})}{5 \text{ hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F/Btu}} = 350 \text{ Btu/hr}$$

$$\text{Heat loss after} = \frac{25 \text{ ft}^2 (140 - 70^\circ \text{F})}{15 \text{ hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F/Btu}} = 117 \text{ Btu/hr}$$

$$\text{Savings} = (350 - 117) \frac{\text{Btu}}{\text{hr}} \times \frac{\text{kWhr}}{3412 \text{ Btu}} \times 24 \frac{\text{hr}}{\text{day}} \times 365 \frac{\text{day}}{\text{yr}} = 599 \text{ kWh/yr}$$

$$\text{worth } 599 \text{ kWh/yr} \times 0.08 \text{ \$/kWhr} = \$47.92 / \text{yr}$$

1.24 60-watt incandescents vs 15-watt CFL:

$$\text{Energy savings} = (60 - 15 \text{ watts}) \times 9 \text{ khr} = 405 \text{ kWh per CFL}$$

$$\text{Carbon savings} = 405 \text{ kWh} \times 280 \text{ gC/kWh} = 113,400 \text{ gC} = 113.4 \text{ kg C}$$

$$\text{SO}_2 \text{ savings} = 405 \text{ kWh} \times 2.8 \text{ g SO}_2/\text{kWh} = 1134 \text{ g SO}_2 = 1.134 \text{ kg SO}_2$$

$$\text{Particulate savings} = 405 \text{ kWh} \times 0.14 \text{ g/kWh} = 56.7 \text{ g particulate matter}$$

1.25 75-watt incandescents vs 18-watt CFL:

a. Electricity savings = (75 - 18 watts) x 10 khr = 570 kWh per CFL

b. Heat into plant saved = $\frac{570 \text{ kWh}}{0.36} \times \frac{3412 \text{ Btu}}{\text{kWh}} = 5.4 \times 10^6 \text{ Btu}$

$$\text{SO}_2 \text{ saved} = \frac{0.6 \text{ lbs SO}_2}{10^6 \text{ Btu into plant}} \times 5.4 \times 10^6 \text{ Btu} \times \frac{\text{ton}}{2000 \text{ lbs}} = 0.00162 \text{ tons}$$

c. Allowances = \$800 / ton x 0.00162 ton / CFL = \$1.30 per CFL

1.26 At 40% efficiency, the power plant needs 2.5 kWh of heat input for each 1 kWhr of electricity delivered.

$$\text{Carbon : } \frac{2.5 \text{ kWh heat}}{1 \text{ kWh electricity}} \times \frac{10^3 \text{ kJ/s}}{\text{kW}} \times \frac{3600 \text{ s}}{\text{hr}} \times \frac{20 \text{ kg C}}{10^9 \text{ J}} = 0.18 \text{ kgC/kWh}$$

$$\text{SO}_2 : \frac{2.5 \text{ kWh heat}}{1 \text{ kWh electricity}} \times \frac{10^3 \text{ kJ/s}}{\text{kW}} \times \frac{3600 \text{ s}}{\text{hr}} \times \frac{86 \text{ g SO}_2}{10^9 \text{ J}} = 0.77 \text{ g SO}_2 / \text{kWh}$$

$$\text{NO}_x : \frac{2.5 \text{ kWh heat}}{1 \text{ kWh electricity}} \times \frac{10^3 \text{ kJ/s}}{\text{kW}} \times \frac{3600 \text{ s}}{\text{hr}} \times \frac{130 \text{ g SO}_2}{10^9 \text{ J}} = 1.17 \text{ g NO}_x / \text{kWh}$$

1.27 Mars with a peak wavelength of 13.2 μm:

a. $T(K) = \frac{2898}{\lambda_{\max}(\mu\text{m})} = \frac{2898}{13.2} = 219 \text{ K}$

b. $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{13.2 \times 10^{-6} \text{ m}} = 2.27 \times 10^{13} \text{ Hz}$

$$E = h\nu = 6.6 \times 10^{-34} \text{ J-s} \times 2.27 \times 10^{13} / \text{s} = 1.5 \times 10^{-20} \text{ J/photon}$$

1.28

$$\frac{\text{Solar constant Earth}}{\text{Solar constant Mars}} = \left(\frac{R_{\text{mars}}}{R_{\text{Earth}}} \right)^2$$

$$S_{\text{for Mars}} = S_{\text{for Earth}} \times \left(\frac{R_{\text{earth}}}{R_{\text{mars}}} \right)^2 = 1370 \text{ W/m}^2 \times \left(\frac{150 \times 10^6 \text{ km}}{228 \times 10^6 \text{ km}} \right)^2 = 593 \text{ W/m}^2$$

$$S_{\text{for Venus}} = 1370 \text{ W/m}^2 \times \left(\frac{150}{108} \right)^2 = 2643 \text{ W/m}^2$$

1.29

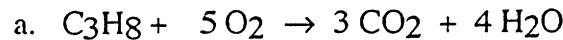
$$\begin{aligned} E_{\text{net}} &= \sigma A \left[(T_1)^4 - (T_2)^4 \right] \\ &= 5.67 \times 10^{-8} \text{ W/m}^2 \times 1.35 \text{ m}^2 \left[(32 + 273)^4 - (15 + 273)^4 \right] = 136 \text{ W} \end{aligned}$$

1.30

$$\begin{aligned} E_{\text{net}} &= \sigma A \left[(T_1)^4 - (T_2)^4 \right] \\ &= 5.67 \times 10^{-8} \text{ W/m}^2 \times 2 \text{ m}^2 \left[(80 + 273)^4 - (20 + 273)^4 \right] = 925 \text{ W} \end{aligned}$$

SOLUTIONS FOR CHAPTER 2

2.1 Combustion of propane:



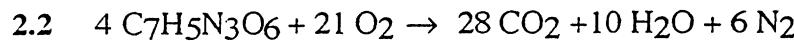
b. 5 moles of O_2 needed per mole of propane.

$$\text{c. } 100 \text{ g C}_3\text{H}_8 \times \frac{1 \text{ mol C}_3\text{H}_8}{44 \text{ g C}_3\text{H}_8} \times \frac{5 \text{ mol O}_2}{\text{mol C}_3\text{H}_8} \times \frac{32 \text{ g O}_2}{\text{mol O}_2} = 363.6 \text{ g O}_2$$

$$\text{d. } 363.6 \text{ g O}_2 \times \frac{\text{mol}}{32 \text{ g}} \times 22.414 \times 10^3 \frac{\text{m}^3}{\text{mol}} = 0.255 \text{ m}^3 \text{ O}_2$$

$$0.255 \text{ m}^3 \text{ O}_2 \times \frac{\text{m}^3 \text{ air}}{0.21 \text{ m}^3 \text{ O}_2} = 1.213 \text{ m}^3 \text{ air}$$

$$\text{e. } \frac{3 \text{ mol CO}_2}{\text{mol C}_3\text{H}_8} \times \frac{100 \text{ g C}_3\text{H}_8}{44 \text{ g/mol C}_3\text{H}_8} \times \frac{22.414 \times 10^{-3} \text{ m}^3}{\text{mol}} = 0.153 \text{ m}^3 \text{ CO}_2$$



$$\text{mol wt of TNT} = 7 \times 12 + 5 \times 1 + 3 \times 14 + 6 \times 16 = 227 \text{ g/mol}$$

$$\frac{100 \text{ g TNT}}{227 \text{ g/mol}} \times \frac{21 \text{ mol O}_2}{4 \text{ mol TNT}} \times \frac{32 \text{ g O}_2}{\text{mol O}_2} = 74.0 \text{ g O}_2$$

2.3

$$\frac{40.00 \text{ g C}}{12 \text{ g/mol}} = 3\frac{1}{3} \text{ mol C} \quad \frac{6.67 \text{ g H}}{1 \text{ g/mol}} = 6\frac{2}{3} \text{ mol H} \quad \frac{53.33 \text{ g O}}{16 \text{ g/mol}} = 3\frac{1}{3} \text{ mol O}$$

C: H: O = 1 : 2 : 1 so could be CH_2O (mol wt = 30, too low)

could be $\text{C}_2\text{H}_4\text{O}_2$ (mol wt = 60 g/mol) YES

2.4 mol wt of glucose ($\text{C}_6\text{H}_{12}\text{O}_6$) = $6 \times 12 + 12 \times 1 + 6 \times 16 = 180 \text{ g/mol}$

$$\frac{10 \text{ g glucose / L}}{180 \text{ g/mol}} = 0.0555 \text{ mol/L} = 0.0555 \text{ M}$$

2.5 mol wt of ethyl alcohol ($\text{CH}_3\text{CH}_2\text{OH}$) = $2 \times 12 + 6 \times 1 + 1 \times 16 = 46 \text{ g/mol}$

$$1 \text{ L whiskey} \times \frac{0.43 \text{ L alcohol}}{\text{L whiskey}} \times \frac{790 \text{ g}}{\text{L alcohol}} \times \frac{\text{mol}}{46 \text{ g}} = 7.38 \text{ mol/L}$$

2.6 a. percentage chlorine:

$$\text{CFC-11} (\text{CCl}_3\text{F}): \text{Cl} = \frac{35.45 \times 3}{12 + 3 \times 35.45 + 19} = 0.774 = 77.4\%$$

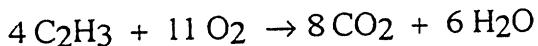
$$\text{CFC-12} (\text{CCl}_2\text{F}_2): \text{Cl} = \frac{35.45 \times 2}{12 + 2 \times 35.45 + 2 \times 19} = 0.586 = 58.6\%$$

$$\text{HCFC-22} (\text{CHF}_2\text{Cl}): \text{Cl} = \frac{35.45}{12 + 1 + 35.45 + 2 \times 19} = 0.410 = 41.0\%$$

b. HCFC-22 substituting for CFC-11:

$$\frac{774 \text{ g} - 410 \text{ g}}{774 \text{ g}} = 0.47 = 47\% \text{ reduction}$$

2.7 Combustion of petroleum:



a. world energy, oil equivalents = $\frac{3 \times 10^{17} \text{ kJ/yr}}{43 \times 10^3 \text{ kJ/kg}} = 6.98 \times 10^{12} \text{ kg oil}$

$$\text{at } 27 \text{ g/mol} = 6.98 \times 10^{12} \text{ kg} \times \frac{10^3 \text{ g}}{\text{kg}} \times \frac{\text{mol}}{27 \text{ g}} = 2.58 \times 10^{14} \text{ mol oil}$$

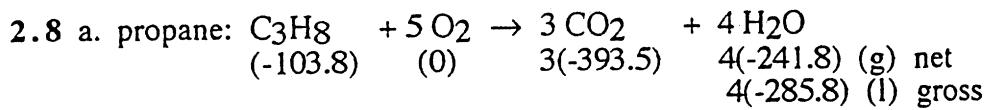
$$\begin{aligned} \text{CO}_2 \text{ emissions} &= 2.58 \times 10^{14} \text{ mol oil} \times \frac{2 \text{ mol CO}_2}{\text{mol oil}} \times \frac{44 \text{ g CO}_2}{\text{mol CO}_2} \times \frac{1 \text{ kg}}{10^3 \text{ g}} \\ &= 2.27 \times 10^{13} \text{ kg CO}_2 \end{aligned}$$

b. From example 2.3, if all U.S. energy came from methane:

$$\frac{4.1 \times 10^{12} \text{ kg CO}_2/\text{yr from methane}}{8.2 \times 10^{16} \text{ kJ/yr of energy from methane}} \times 3 \times 10^{17} \text{ kJ/yr} = 1.5 \times 10^{13} \text{ kg CO}_2$$

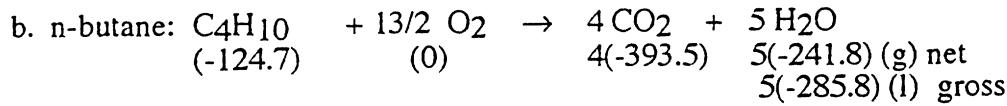
comparing that with the CO₂ released if all petroleum, yields

$$\frac{2.27 \times 10^{13} \text{ kg CO}_2/\text{yr if burn oil}}{1.5 \times 10^{13} \text{ kg CO}_2/\text{yr if burn CH}_4} = 1.51 \quad \text{that is, 51\% more CO}_2 \text{ if burn oil}$$



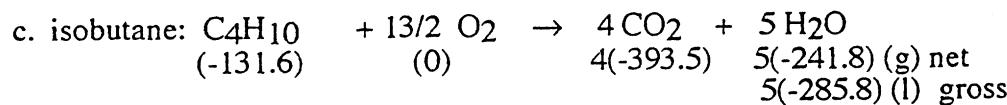
net: $\Delta H^\circ = 4x(-241.8) + 3(-393.5) - (-103.8) = -2043.9 \text{ kJ/mol}$

gross: $\Delta H^\circ = 4x(-285.8) + 3(-393.5) - (-103.8) = -2219.9 \text{ kJ/mol}$



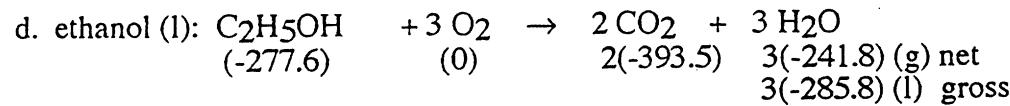
net: $\Delta H^\circ = 5x(-241.8) + 4(-393.5) - (-124.7) = -2658.3 \text{ kJ/mol}$

gross: $\Delta H^\circ = 5x(-285.8) + 4(-393.5) - (-124.7) = -2878.3 \text{ kJ/mol}$



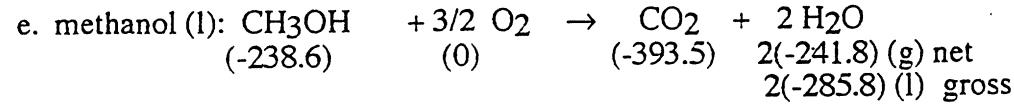
net: $\Delta H^\circ = 5x(-241.8) + 4(-393.5) - (-131.6) = -2651.4 \text{ kJ/mol}$

gross: $\Delta H^\circ = 5x(-285.8) + 4(-393.5) - (-131.6) = -2871.4 \text{ kJ/mol}$



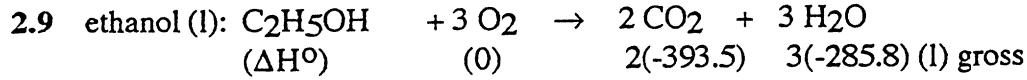
net: $\Delta H^\circ = 3x(-241.8) + 2(-393.5) - (-277.6) = -1234.8 \text{ kJ/mol}$

gross: $\Delta H^\circ = 3x(-285.8) + 2(-393.5) - (-277.6) = -1366.8 \text{ kJ/mol}$



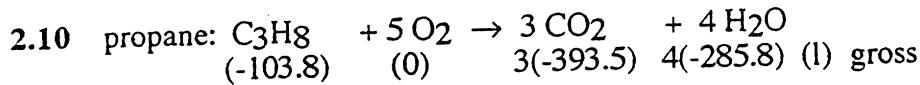
net: $\Delta H^\circ = 2x(-241.8) + (-393.5) - (-238.6) = -638.5 \text{ kJ/mol}$

gross: $\Delta H^\circ = 2x(-285.8) + (-393.5) - (-238.6) = -726.5 \text{ kJ/mol}$



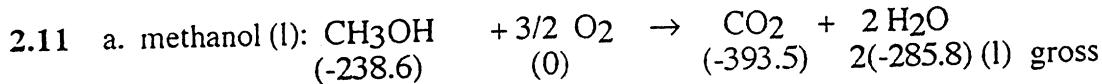
$3(-285.8) + 2(-393.5) - \Delta H^\circ = -1370 \text{ kJ/mol}$

$\Delta H^\circ = -274.4 \text{ kJ/mol}$



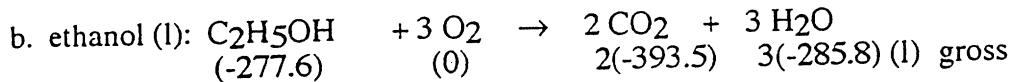
gross: $\Delta H^\circ = 4x(-285.8) + 3(-393.5) - (-103.8) = -2219.9 \text{ kJ/mol}$

$$2219.9 \frac{\text{kJ}}{\text{mol}} \times \frac{\text{mol}}{44 \text{ g}} \times 1000 \text{ g} = 50,452 \text{ kJ}$$



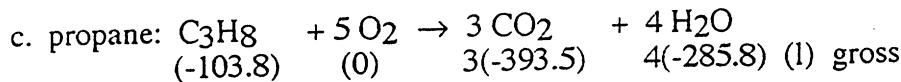
gross: $\Delta H^\circ = 2x(-285.8) + (-393.5) - (-238.6) = -726.5 \text{ kJ/mol}$

$$HHV = 726.5 \frac{\text{kJ}}{\text{mol}} \times \frac{\text{mol}}{32 \text{ g}} \times \frac{1000 \text{ g}}{2.2 \text{ lb}} \times 6.7 \frac{\text{lb}}{\text{gal}} \times \frac{\text{Btu}}{1.055 \text{ kJ}} = 65,537 \text{ Btu/gal}$$



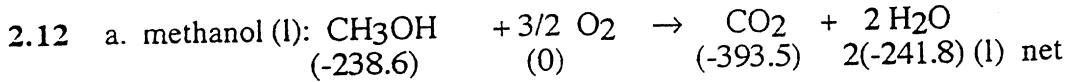
gross: $\Delta H^\circ = 3x(-285.8) + 2(-393.5) - (-277.6) = -1366.8 \text{ kJ/mol}$

$$HHV = 1366.8 \frac{\text{kJ}}{\text{mol}} \times \frac{\text{mol}}{46 \text{ g}} \times \frac{1000 \text{ g}}{2.2 \text{ lb}} \times 6.6 \frac{\text{lb}}{\text{gal}} \times \frac{\text{Btu}}{1.055 \text{ kJ}} = 84,492 \text{ Btu/gal}$$



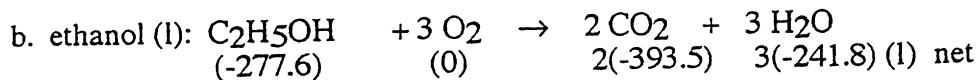
gross: $\Delta H^\circ = 4x(-285.8) + 3(-393.5) - (-103.8) = -2219.9 \text{ kJ/mol}$

$$HHV = 2219.9 \frac{\text{kJ}}{\text{mol}} \times \frac{\text{mol}}{44 \text{ g}} \times \frac{1000 \text{ g}}{2.2 \text{ lb}} \times 4.1 \frac{\text{lb}}{\text{gal}} \times \frac{\text{Btu}}{1.055 \text{ kJ}} = 89,123 \text{ Btu/gal}$$



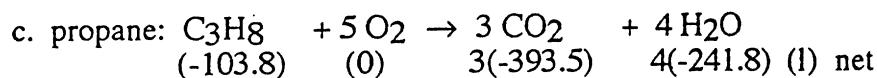
net: $\Delta H^\circ = 2x(-241.8) + (-393.5) - (-238.6) = -638.5 \text{ kJ/mol}$

$$LHV = 638.5 \frac{\text{kJ}}{\text{mol}} \times \frac{\text{mol}}{32 \text{ g}} \times \frac{1000 \text{ g}}{2.2 \text{ lb}} \times 6.7 \frac{\text{lb}}{\text{gal}} \times \frac{\text{Btu}}{1.055 \text{ kJ}} = 57,598 \text{ Btu/gal}$$



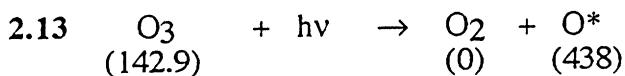
net: $\Delta H^\circ = 3x(-241.8) + 2(-393.5) - (-277.6) = -1234.8 \text{ kJ/mol}$

$$LHV = 1234.8 \frac{\text{kJ}}{\text{mol}} \times \frac{\text{mol}}{46 \text{ g}} \times \frac{1000 \text{ g}}{2.2 \text{ lb}} \times 6.6 \frac{\text{lb}}{\text{gal}} \times \frac{\text{Btu}}{1.055 \text{ kJ}} = 76,332 \text{ Btu/gal}$$



$$\text{net: } \Delta H^\circ = 4(-241.8) + 3(-393.5) - (-103.8) = -2043.9 \text{ kJ/mol}$$

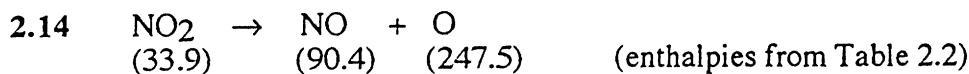
$$\text{LHV} = 2043.9 \frac{\text{kJ}}{\text{mol}} \times \frac{\text{mol}}{44 \text{ g}} \times \frac{1000 \text{ g}}{2.2 \text{ lb}} \times 4.1 \frac{\text{lb}}{\text{gal}} \times \frac{\text{Btu}}{1.055 \text{ kJ}} = 82,057 \text{ Btu/gal}$$



$$\Delta H^\circ = 438 - 142.9 = 295.1 \text{ kJ/mol} \quad (\text{endothermic})$$

$$\lambda \leq \frac{1.19 \times 10^{-4} \text{ kJ} \cdot \text{m/mol}}{\Delta H^\circ} \quad (2.12)$$

$$\leq \frac{1.19 \times 10^{-4} \text{ kJ} \cdot \text{m/mol}}{295.1 \text{ kJ/mol}} = 0.403 \times 10^{-6} \text{ m} = 0.403 \mu\text{m}$$



$$\Delta H^\circ = 90.4 + 247.5 - 33.9 = 304 \text{ kJ/mol} \quad (\text{endothermic})$$

$$\lambda \leq \frac{1.19 \times 10^{-4} \text{ kJ} \cdot \text{m/mol}}{\Delta H^\circ} \quad (2.12)$$

$$\leq \frac{1.19 \times 10^{-4} \text{ kJ} \cdot \text{m/mol}}{304 \text{ kJ/mol}} = 0.39 \times 10^{-6} \text{ m} = 0.39 \mu\text{m}$$



$$25 \frac{\text{mg}}{\text{L}} \times \frac{1 \text{ g}}{1000 \text{ mg}} \times \frac{\text{mol}}{36.45 \text{ g HCl}} = 6.8587 \times 10^{-4} \text{ mol/L} = [H^+]$$

$$pH = -\log [H^+] = -\log [6.8587 \times 10^{-4}] = 3.16$$

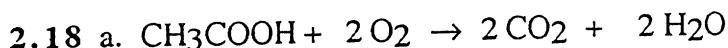
$$2.16 \quad [\text{OH}^-] = 3 \times 10^{-4} \frac{\text{mg}}{\text{L}} \times \frac{\text{g}}{1000 \text{ mg}} \times \frac{\text{mol}}{17 \text{ g}} = 1.7 \times 10^{-8} \text{ mol/L}$$

$$[\text{H}^+] = \frac{10^{-14}}{1.7 \times 10^{-8}} = 5.66 \times 10^{-7} \text{ mol/L}$$

$$\text{pH} = -\log [\text{H}^+] = -\log [5.66 \times 10^{-7}] = 6.25$$

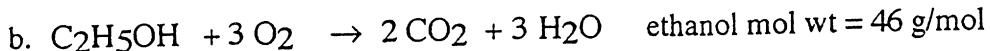
$$2.17 \quad \text{pH} = -\log [\text{H}^+] = 8.5 \quad \therefore [\text{H}^+] = 3.16 \times 10^{-9} \text{ mol/L}$$

$$[\text{OH}^-] = \frac{10^{-14}}{[\text{H}^+]} = \frac{10^{-14}}{3.16 \times 10^{-9}} = 3.16 \text{ mol/L}$$

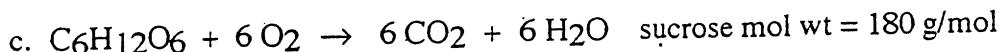


$$\text{Acetic acid mol wt} = 2 \times 12 + 4 \times 1 + 2 \times 16 = 60 \text{ g/mol}$$

$$\text{ThOD} = 200 \frac{\text{mg AA}}{\text{L}} \times \frac{\text{g}}{1000 \text{ mg}} \times \frac{\text{mol AA}}{60 \text{ g}} \times \frac{2 \text{ mol O}_2}{\text{mol AA}} \times \frac{32 \text{ g O}_2}{\text{mol O}_2} \times \frac{1000 \text{ mg}}{\text{g}} = 213 \text{ mg/L}$$



$$\text{ThOD} = 30 \frac{\text{mg eth}}{\text{L}} \times \frac{\text{g}}{1000 \text{ mg}} \times \frac{\text{mol}}{46 \text{ g eth}} \times \frac{3 \text{ mol O}_2}{\text{mol eth}} \times \frac{32 \text{ g O}_2}{\text{mol O}_2} \times \frac{1000 \text{ mg}}{\text{g}} = 62.6 \text{ mg/L}$$



$$\text{ThOD} = 50 \frac{\text{mg suc}}{\text{L}} \times \frac{\text{g}}{1000 \text{ mg}} \times \frac{\text{mol suc}}{180 \text{ g suc}} \times \frac{6 \text{ mol O}_2}{\text{mol suc}} \times \frac{32 \text{ g O}_2}{\text{mol O}_2} \times \frac{1000 \text{ mg}}{\text{g}} = 53.3 \text{ mg/L}$$



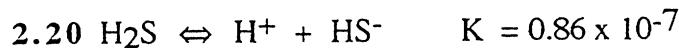
$$\frac{[\text{H}^+][\text{OCl}^-]}{[\text{HOCl}]} = 2.9 \times 10^{-8}$$

$$f = \frac{[\text{HOCl}]}{[\text{HOCl}] + [\text{OCl}^-]} = \frac{1}{1 + \frac{[\text{OCl}^-]}{[\text{HOCl}]}} = \frac{1}{1 + \frac{2.9 \times 10^{-8}}{[\text{H}^+]}} = \frac{1}{1 + \frac{2.9 \times 10^{-8}}{10^{-\text{pH}}}}$$

$$\text{pH} = 6: \quad f = \frac{1}{1 + \frac{2.9 \times 10^{-8}}{10^{-6}}} = 0.97$$

$$\text{pH} = 8: \quad f = \frac{1}{1 + \frac{2.9 \times 10^{-8}}{10^{-8}}} = 0.256$$

$$\text{pH} = 10: \quad f = \frac{1}{1 + \frac{2.9 \times 10^{-8}}{10^{-10}}} = 0.0034$$

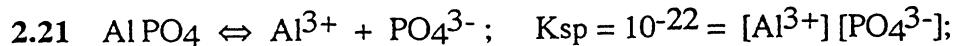


$$\frac{[\text{H}^+][\text{HS}^-]}{[\text{H}_2\text{S}]} = 0.86 \times 10^{-7}$$

$$f = \frac{[\text{H}_2\text{S}]}{[\text{H}_2\text{S}] + [\text{HS}^-]} = \frac{1}{1 + \frac{[\text{HS}^-]}{[\text{H}_2\text{S}]}} = \frac{1}{1 + \frac{0.86 \times 10^{-7}}{[\text{H}^+]}} = \frac{1}{1 + \frac{0.86 \times 10^{-7}}{10^{-\text{pH}}}}$$

$$\text{pH} = 6: \quad f = \frac{1}{1 + \frac{0.86 \times 10^{-7}}{10^{-6}}} = 0.92$$

$$\text{pH} = 8: \quad f = \frac{1}{1 + \frac{0.86 \times 10^{-7}}{10^{-8}}} = 0.104$$



$$[\text{Al}^{3+}] = [\text{PO}_4^{3-}] \quad \therefore [\text{PO}_4^{3-}] = 10^{-11} \text{ mol/L}$$

$$\text{mol wt PO}_4^{3-} = 31 + 4 \times 16 = 95 \text{ g/mol}$$

$$\text{PO}_4^{3-} \text{ concentration} = 10^{-11} \text{ mol/L} \times 95 \text{ g/mol} \times 1000 \text{ mg/g} = 9.5 \times 10^{-7} \text{ mg/L}$$

2.22 O₂ is 21% of air, so at *sea level* (1 atm), P O₂ = 0.21 x 1 atm = 0.21 atm

At 15°C, K_H = 0.0015236 for O₂ (Table 2.4)

$$[O_2] = K_H P_g = 0.0015236 \text{ mol/L-atm} \times 0.21 \text{ atm} = 0.00031996 \text{ mol/L}$$

$$\text{dissolved oxygen} = 3.1996 \times 10^{-4} \text{ mol/L} \times 32 \text{ g/mol} \times 1000 \text{ mg/g} = 10.2 \text{ mg/L}$$

$$\text{At } 2 \text{ km elevation, } P = 1 - 1.15 \times 10^{-4} H = 1 - 1.15 \times 10^{-4} \times 2000 = 0.77 \text{ atm}$$

$$[O_2] = K_H P_g = 0.0015236 \text{ mol/L-atm} \times 0.21 \times 0.77 \text{ atm} = 0.0002464 \text{ mol/L}$$

$$\text{dissolved oxygen} = 2.464 \times 10^{-4} \text{ mol/L} \times 32 \text{ g/mol} \times 1000 \text{ mg/g} = 7.9 \text{ mg/L}$$

2.23 P CO₂ = 2 atm and K_H = 0.033363 mol/L-atm at 25°C (Table 2.4)

$$[CO_2] = K_H P_g = 0.033363 \text{ mol/L-atm} \times 2 \text{ atm} = 0.066676 \text{ mol/L}$$

$$(\text{or, in g/L} = 0.066676 \text{ mol/L} \times 44 \text{ g/mol} = 2.9 \text{ g/L})$$



$$\frac{[H^+][HCO_3^-]}{[CO_2]} = K_1 = 4.47 \times 10^{-7} \text{ mol/L}$$

$$\text{and } [H^+] \approx [HCO_3^-] + [OH^-] \quad (2.43)$$

$$\text{and } [H^+] [OH^-] = K_w = 10^{-14} \quad (2.19)$$

$$\text{so } [H^+] = [HCO_3^-] + \frac{10^{-14}}{[H^+]} = \frac{K_1 [CO_2]}{[H^+]} + \frac{10^{-14}}{[H^+]}$$

$$[H^+]^2 = K_1 [CO_2] + 10^{-14} = 4.47 \times 10^{-7} \times 0.066676 + 10^{-14}$$

$$[H^+] = 0.000173 \text{ mol/L}$$

$$pH = -\log(0.000173) = 3.76$$

2.25 @ 275 ppm: $[CO_2] = K_H P_g = 0.033363 \text{ mol/L-atm} \times 275 \times 10^{-6} \text{ atm}$
 $= 9.17 \times 10^{-6} \text{ mol/L}$

using (2.46): $[H^+]^2 = K_1 [CO_2(aq)] + 10^{-14}$

$$[H^+]^2 = 4.47 \times 10^{-7} \times 9.17 \times 10^{-6} + 10^{-14} = 4.11 \times 10^{-12}$$
 $[H^+] = 2.0 \times 10^{-6}$

$pH = -\log [H^+] = -\log (2.0 \times 10^{-6}) = 5.69$

@ 600 ppm: $[CO_2] = 0.033363 \text{ mol/L-atm} \times 600 \times 10^{-6} \text{ atm} = 2.0 \times 10^{-5} \text{ mol/L}$

using (2.46): $[H^+]^2 = 4.47 \times 10^{-7} \times 2.0 \times 10^{-5} + 10^{-14} = 8.95 \times 10^{-12}$
 $[H^+] = 2.99 \times 10^{-6}$

$pH = -\log [H^+] = -\log (2.99 \times 10^{-6}) = 5.52$

2.26 Begin by writing the full set of equations that must be satisfied:

$$\frac{[H^+][HCO_3^-]}{[CO_2]} = K_1 = 4.47 \times 10^{-7} \text{ mol/L} \quad (2.37)$$

$$\frac{[H^+][CO_3^{2-}]}{[HCO_3^-]} = K_2 = 4.68 \times 10^{-11} \text{ mol/L} \quad (2.38)$$

$$[Ca^{2+}][CO_3^{2-}] = K_{sp} = 4.57 \times 10^{-9} \text{ mol/L} \quad (2.39)$$

$$[H^+] + 2[Ca^{2+}] = [HCO_3^-] + 2[CO_3^{2-}] + [OH^-] \approx [HCO_3^-] + [OH^-] \text{ charge balance}$$

$$[CO_2(aq)] = K_H P_g = 0.033363 \text{ mol/L-atm} \times 360 \times 10^{-6} \text{ atm} = 1.2 \times 10^{-5} \text{ mol/L}$$

Solve for the concentration of Ca^{2+} using (2.37), (2.38), and (2.39):

$$[Ca^{2+}] = \frac{K_{sp}}{[CO_3^{2-}]} = \frac{K_{sp}[H^+]}{K_2[HCO_3^-]} = \frac{K_{sp}[H^+]^2}{K_1 K_2 [CO_2]}$$

from the charge balance:

$$[H^+] = [HCO_3^-] + [OH^-] - 2[Ca^{2+}]$$

$$[\text{H}^+] = \frac{K_1[\text{CO}_2]}{[\text{H}^+]} + \frac{10^{-14}}{[\text{H}^+]} - \frac{2 K_{\text{sp}} [\text{H}^+]^2}{K_1 K_2 [\text{CO}_2]}$$

$$\text{so } [\text{H}^+]^2 = K_1[\text{CO}_2] + 10^{-14} - \frac{2 K_{\text{sp}} [\text{H}^+]^3}{K_1 K_2 [\text{CO}_2]}$$

$$[\text{H}^+]^2 = 4.47 \times 10^{-7} \times 1.2 \times 10^{-5} + 10^{-14} - \frac{2 \times 4.57 \times 10^{-9} [\text{H}^+]^3}{4.47 \times 10^{-7} \times 4.68 \times 10^{-11} \times 1.2 \times 10^{-5}}$$

$$[\text{H}^+]^2 = 5.37 \times 10^{-12} - 3.64 \times 10^{-13} [\text{H}^+]^3$$

$$\text{or } [\text{H}^+]^3 + 2.75 \times 10^{-14} [\text{H}^+]^2 = 1.47 \times 10^{-25}$$

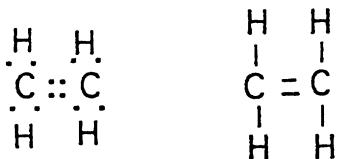
Imagine pH > 7 for example: $[\text{H}^+] < 10^{-7}$ then $2.75 \times 10^{-14} [\text{H}^+]^2 < 2.75 \times 10^{-28}$

which is negligible compared to 1.47×10^{-25} so we will ignore the $[\text{H}^+]$ term...

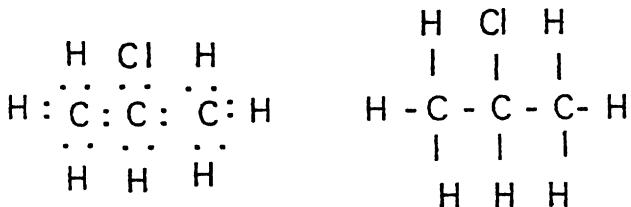
$$\text{then } [\text{H}^+]^3 \approx 1.47 \times 10^{-25} \text{ so } [\text{H}^+] = 5.27 \times 10^{-9}$$

$$\text{pH} = -\log(5.27 \times 10^{-9}) = 8.3$$

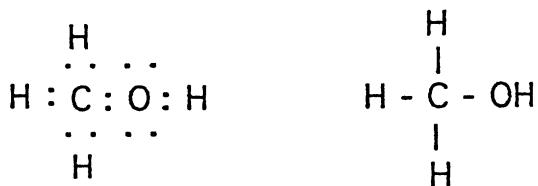
2.27 a. C₂H₄

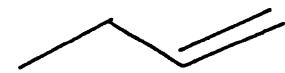
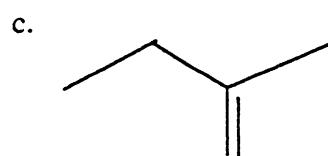
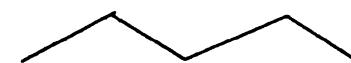


b. CH₃CHClCH₃

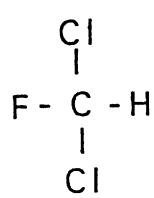
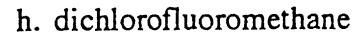
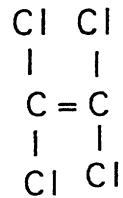
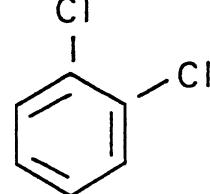
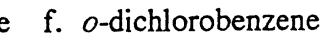
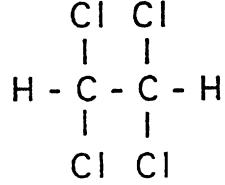
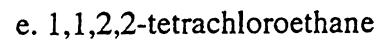
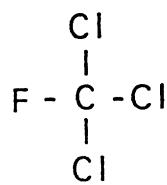
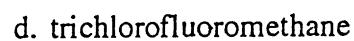
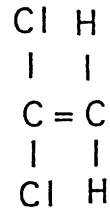
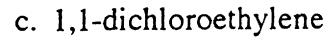
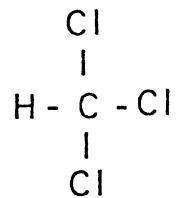
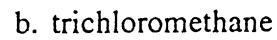
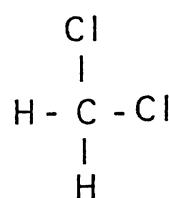
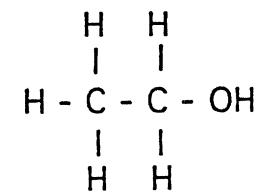
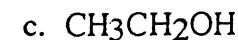
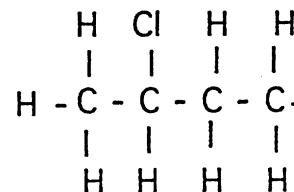
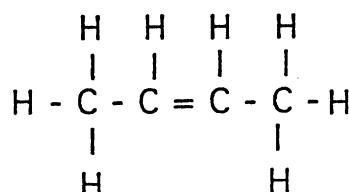
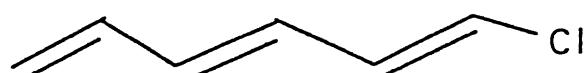


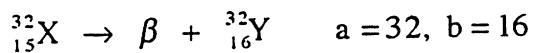
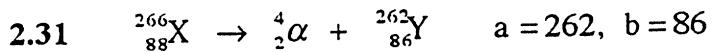
c. CH₃OH





d.





2.32 $64g \rightarrow 32g \rightarrow 16g \rightarrow 8g \rightarrow 4g \rightarrow 2g \rightarrow 1g$
elapsed days: 60 120 180 240 300 360 days

SOLUTIONS FOR CHAPTER 3

3.1 1 billion in 1850 growing to 4 billion in 1975:

a. by doubling time: 1 billion \rightarrow 2 billion \rightarrow 4 billion

means 2 doublings in 1975 - 1850 = 125 years.

$$T_d = \frac{125 \text{ yrs}}{2 \text{ doublings}} = 62.5 \text{ yrs/doubling}$$

$$r(\%) \approx \frac{70}{T_d} = \frac{70}{62.5} = 1.1\%/\text{yr}$$

b. by formula:

$$r = \frac{1}{t} \ln\left(\frac{N}{N_0}\right) = \frac{1}{125} \ln\left(\frac{4}{1}\right) = 0.011 = 1.1\%/\text{yr}$$

3.2 Tuition from \$1500 to \$20,000 in 1995 - 1962 = 33 yrs:

$$\text{a. } r = \frac{1}{t} \ln\left(\frac{N}{N_0}\right) = \frac{1}{33} \ln\left(\frac{20,000}{1500}\right) = 0.0785 = 7.85\%/\text{yr}$$

$$\text{b. In 25 yrs: } N = N_0 e^{rt} = 20,000 e^{0.0785 \times 25} = \$142,317/\text{yr}$$

3.3 5 million people 10,000 yrs ago leading to 1 billion in 1850:

$$r = \frac{1}{t} \ln\left(\frac{N}{N_0}\right) = \frac{1}{10,000} \ln\left(\frac{10^9}{5 \times 10^6}\right) = 0.00053 = 0.053\% \text{ (today's rate 30 x faster)}$$

increasing at that rate from 1850 to 2000, the population would be

$$N = N_0 e^{rt} = 1 \times 10^9 e^{0.00053 \times (2000 - 1850)} = 1.08 \text{ billion}$$

3.4

$$\text{Land} = (\text{land}/\text{food}) \times (\text{food}/\text{kcal}) \times (\text{kcal}/\text{person}) \times (\text{population})$$

@ -1% @0.5% @0.1% @1.5%

$$r = -1.0 + 0.5 + 0.1 + 1.5 = 1.1\%/\text{yr} \text{ growth in land required}$$

$$\text{a. doubling time for land needed} = T_d = \frac{\ln 2}{r} = \frac{0.693}{0.011} = 63 \text{ yrs}$$

$$\text{b. } \left(\frac{\text{land}}{\text{food}}\right)_{t=63 \text{ yrs}} = \left(\frac{\text{land}}{\text{food}}\right)_{t=0} e^{-0.01 \times 63} = 0.53 \times \left(\frac{\text{land}}{\text{food}}\right)_{t=0} \text{ that is, 47\% less land}$$

3.5 Carbon emissions = (Population) x (Energy/person) x (Carbon/energy)

$$@1.5\% \quad @1.5\% \quad @1\%$$

- a. to double the current rate of carbon emissions, at $r = 1.5 + 1.5 + 1 = 4\%/\text{yr}$

$$t = \frac{1}{r} \ln\left(\frac{N}{N_0}\right) = \frac{1}{0.04} \ln(2) = 17.33 \text{ yrs}$$

- b. in 17.33 yrs, the per capita energy demand will have increased by

$$\frac{(\text{Energy / person})_{t=17.33\text{yrs}}}{(\text{Energy / person})_{t=0\text{yrs}}} = e^{rt} = e^{0.015 \times 17.33} = 1.297 \quad (\text{up by } \approx 30\%)$$

- c. by then, total energy demand would have increased by

$$\text{Energy} = (\text{Energy/person}) \times (\text{population})$$

$$@1.5\% \quad @1.5\% \quad (\text{that is, total growth } @ 3\%/\text{yr})$$

$$\frac{(\text{Energy})_{t=17.33\text{yrs}}}{(\text{Energy})_{t=0\text{yrs}}} = e^{rt} = e^{0.03 \times 17.33} = 1.68 \quad (\text{up by } 68\%)$$

3.6 Starting with 5×10^9 tonnes C/yr @ 4%/yr and total atmospheric C = 700×10^9 tonnes

- a. Using (3.16), the time to emit 700×10^9 tonnes would be

$$T = \frac{1}{r} \ln\left(\frac{rQ}{P_0} + 1\right) = \frac{1}{0.04} \ln\left(\frac{0.04 \times 700 \times 10^9}{5 \times 10^9} + 1\right) = 47 \text{ yrs}$$

- b. To double atmospheric C given that half of the emissions remain in the atmosphere would require total emissions of $700 \times 2 = 1400$ billion tonnes:

$$T = \frac{1}{r} \ln\left(\frac{rQ}{P_0} + 1\right) = \frac{1}{0.04} \ln\left(\frac{0.04 \times 1400 \times 10^9}{5 \times 10^9} + 1\right) = 63 \text{ yrs}$$

3.7 Carbon(kg/yr) = Population x $\frac{\text{Energy(kJ/yr)}}{\text{person}} \times \frac{\text{Carbon(kgC)}}{\text{Energy(kJ)}}$
 $@0.6\% \quad @0.5\% \quad @-0.3\% \quad r_{\text{total}} = 0.8\% = 0.008$

- a. First, find the carbon emission rate in 1990:

$$C_0 = 250 \times 10^6 \text{ people} \times 320 \times 10^6 \text{ kJ/person} \times 15 \times 10^{-6} \text{ kgC/kJ} = 1.2 \times 10^{12} \text{ kgC}$$

if this grows at 0.8% for 2020 - 1990 = 30 yrs, emissions will be

$$C = C_0 e^{rt} = 1.2 \times 10^{12} \text{ kgC/yr} e^{0.008 \times 30} = 1.525 \times 10^{12} \text{ kgC/yr} = 1.525 \text{ GtC/yr}$$

b. During those 30 years, the total emitted will be (3.15):

$$Q = \frac{P_0}{r} (e^{rt} - 1) = \frac{1.2 \text{ GtC/yr}}{0.008/\text{yr}} (e^{0.008 \times 30} - 1) = 40.7 \text{ GtC}$$

c. Total energy demand in 2020 will be:

$$\begin{array}{lll} \text{Energy} = (\text{Energy/person}) \times (\text{population}) \\ @r & @0.5\% & @0.6\% \\ \text{total growth rate } r = 1.1\% = 0.011 \end{array}$$

$$\text{Energy (1990)} = 320 \times 10^6 \text{ (kJ/yr/person)} \times 250 \times 10^6 \text{ people} = 8 \times 10^{16} \text{ kJ/yr}$$

$$\text{Energy (2020)} = 8 \times 10^{16} \text{ kJ/yr} e^{0.011 \times 30} = 11.1 \times 10^{16} \text{ kJ/yr}$$

d. Per capita emissions of carbon in 2020 will be:

$$C/\text{person}(1990) = (\text{Energy kJ/yr/person}) \times (\text{Carbon/energy kgC/kJ})$$

$$= 320 \times 10^6 \text{ kJ/yr/person} \times 15 \times 10^{-6} \text{ kgC/kJ} = 4800 \text{ kgC/yr/person}$$

$$\text{growing at } r = 0.5\% - 0.3\% = 0.2\%/\text{yr} = 0.002/\text{yr}$$

$$C(2020) = C(1990) e^{rt} = 4800 \text{ kgC/yr/person} e^{0.002/\text{yr} \times 30\text{yr}} = 5.1 \times 10^3 \text{ kgC/yr}$$

3.8 Current usage 2 million tons Cr/yr; reserves 800 million tons of chromium; $r=2.6\%/\text{yr}$

$$T = \frac{1}{r} \ln \left(\frac{rQ}{P_0} + 1 \right) = \frac{1}{0.026} \ln \left(\frac{0.026 \times 800 \times 10^6}{2 \times 10^6} + 1 \right) = 93.6 \text{ yrs}$$

If resources are 5x reserves, the time to use them up would be

$$T = \frac{1}{0.026} \ln \left(\frac{0.026 \times 5 \times 800 \times 10^6}{2 \times 10^6} + 1 \right) = 152.7 \text{ yrs}$$

Note multiplying reserves $\times 5$ only increases the lifetime by a factor of 1.6.

3.9 Gaussian peaking at 6x current rate of 2 million tons/yr; resource of 4 billion tons:

$$\sigma = \frac{Q_\infty}{P_m \sqrt{2\pi}} = \frac{4000 \times 10^6 \text{ tons}}{6 \times 2 \times 10^6 \text{ tons/yr} \sqrt{2\pi}} = 133 \text{ yrs} \quad (\text{from 3.18})$$

To reach the maximum production rate, use (3.20):

$$t_m = \sigma \sqrt{2 \ln \frac{P_m}{P_0}} = 133 \sqrt{2 \ln 6} = 251.7 \approx 252 \text{ yrs}$$

To consume about 80% of the resource corresponds to $\pm 1.3\sigma$:

$$t_{80\%} = 2 \times 1.3 \sigma = 2 \times 1.3 \times 133 = 346 \text{ yrs}$$

- 3.10** At current rates P_0 it would take 100 yrs to add Q tons of CFC to the already existing Q tons. That is,

$$100 P_0 = Q \quad \text{or} \quad \frac{Q}{P_0} = 100$$

Then using (3.16), the time required to add those Q tons and double CFCs is

$$T = \frac{1}{r} \ln \left(\frac{rQ}{P_0} + 1 \right) = \frac{1}{0.02} \ln(0.02 \times 100 + 1) = 54.9 \text{ yrs}$$

- 3.11** Bismuth half life is 4.85 days so using (3.8) the corresponding reaction rate K is

$$T_{1/2} = \frac{\ln 2}{K} \quad \text{so,} \quad K = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.85 \text{d}} = 0.143 / \text{day}$$

After 7 days the initial 10 g is reduced to

$$N = N_0 e^{-Kt} = 10 \text{g} e^{-0.143 \times 7} = 3.68 \text{g}$$

- 3.12** Reaction rate $K = 0.2/\text{day}$, so from (3.8) the half-life is

$$T_{1/2} = \frac{\ln 2}{K} = \frac{\ln 2}{0.2/\text{d}} = 3.466 \text{ days}$$

The fraction remaining after 5 days is

$$\frac{N}{N_0} = e^{-Kt} = e^{-0.2/\text{d} \times 5\text{d}} = 0.368 \text{ that is, about 37\% of the sewage remains}$$

- 3.13** Using the logistic curve (3.26) starting with $N_0=6.3$ billion in 2000, growing at $R_0=0.015/\text{yr}$ to a maximum of $K=10.3$ billion, first find growth rate r

$$r = \frac{R_0}{\left(1 - \frac{N_0}{K}\right)} = \frac{0.015}{\left(1 - \frac{6.3}{10.3}\right)} = 0.0386$$

To reach 9 billion, we need first to find t^* the time when size is $K/2=5.15$ billion:

$$t^* = \frac{1}{r} \ln \left(\frac{K}{N_0} - 1 \right) = \frac{1}{0.0386} \ln \left(\frac{10.3}{6.3} - 1 \right) = -11.7 \text{ yrs before 2000 (that is, 1988)}$$

We can use (3.27) to find when will it reach 9 billion:

$$t = t^* - \frac{1}{r} \ln\left(\frac{K}{N} - 1\right) = -11.7 - \frac{1}{0.0386} \ln\left(\frac{10.3}{9} - 1\right) = 38.4 \text{ yrs (that is 2038)}$$

which is quite similar to Figure 3.20.

3.14 Similar to Problem 3.13 but starting with 3.65 billion in 1970 and 2.0% growth:

$$r = \frac{R_0}{\left(1 - \frac{N_0}{K}\right)} = \frac{0.02}{\left(1 - \frac{3.65}{10.3}\right)} = 0.03098$$

to find when $N = 10.3/2 = 5.15$ billion:

$$t^* = \frac{1}{r} \ln\left(\frac{K}{N_0} - 1\right) = \frac{1}{0.03098} \ln\left(\frac{10.3}{3.65} - 1\right) = 19.4 \text{ yrs } (19.4 + 1970 = 1989)$$

Projected out to 1995 (25 yrs later) using Eq. 3.22:

$$N = \frac{K}{1 + e^{-r(t - t^*)}} = \frac{10.3}{1 + e^{-0.0309825 - 19.4}} = 5.6 \text{ billion (actual was 5.7)}$$

3.15 When $N_0=100$ the doubling time eqn lets us find the growth rate R_0 :

$$R_0 = \frac{\ln 2}{T_d} = \frac{\ln 2}{1} = 0.693/\text{yr}$$

With no growth constraints use (3.26),

$$r = \frac{R_0}{\left(1 - \frac{N_0}{K}\right)} = \frac{0.693}{\left(1 - \frac{100}{4000}\right)} = 0.711/\text{yr}$$

a. Max sustainable yield when population is half the carrying capacity

$$N = 2000/2 = 1000 \text{ fish}$$

using (3.29) the maximum yield is:

$$\text{max yield} = \frac{rK}{4} = \frac{0.711 \times 2000}{4} = 355 \text{ fish/yr}$$

b. If the pond is kept at 1500 fish (instead of the optimum 1000), yield (3.21) is

$$\text{yield} = rN\left(1 - \frac{N}{K}\right) = 0.711 \times 1500 \left(1 - \frac{1500}{2000}\right) = 267 \text{ fish/yr}$$

3.16 Begin by finding the early growth rate r from (3.26)

$$r = \frac{R_0}{\left(1 - \frac{N_0}{K}\right)} = \frac{0.693}{\left(1 - \frac{100}{4000}\right)} = 0.71/\text{yr}$$

Yield is given by (3.21), now with non-optimal $N=3000$ fish:

$$\text{yield} = rN \left(1 - \frac{N}{K}\right) = 0.71 \times 3000 \left(1 - \frac{3000}{4000}\right) = 533 \text{ fish/yr}$$

This is less than the maximum sustainable yield of 710 fish found in Example 3.9.

3.17 At present, yield = $dN/dt = 2000/\text{yr}$; $K = 10,000$ fish; $N = 4000$; and we want maximize yield. From (3.21)

$$r = \frac{dN/dt}{N \left(1 - \frac{N}{K}\right)} = \frac{2000}{4000 \left(1 - \frac{4000}{10,000}\right)} = 0.8333$$

To maximize sustainable yield, the population should be allowed to grow to $K/2 = 5000$ fish, at which point the yield would be:

$$\text{max yield} = \frac{rK}{4} = \frac{0.8333 \times 10,000}{4} = 2083 \text{ fish/yr}$$

3.18 India: $N=762$ million; $b=34/1000$; $d=13/1000$; infant mort.=118 per 1000 live births:

$$\text{a. births} = 762 \times 10^6 \times \frac{34}{1000} = 25.9 \text{ million / yr}$$

$$\text{infant deaths} = 25.9 \times 10^6 \text{ births} \times \frac{118 \text{ deaths}}{1000 \text{ births}} = 3.06 \text{ million/yr}$$

$$\text{total deaths} = 762 \times 10^6 \times \frac{13}{1000} = 9.91 \text{ million deaths/yr}$$

$$\text{fraction of deaths that are infants} = \frac{3.06}{9.91} = 0.309 \approx 31\%$$

$$\text{b. infant deaths } @ \frac{10}{1000} = 25.9 \times 10^6 \text{ births} \times \frac{10 \text{ deaths}}{1000 \text{ births}} = 0.26 \text{ million/yr}$$

$$\text{"avoidable deaths"} = 3.06 - 0.26 M = 2.8 \text{ million/yr}$$

$$\text{c. annual increase} = N(b - d) = 762 \times 10^6 \left(\frac{34}{1000} - \frac{13}{1000} \right) = 16 \text{ million/yr}$$

3.19 India: N=931 million; b=29/1000; d=13/1000; infant mort.=74 per 1000 live births:

$$a. \text{births} = 931 \times 10^6 \times \frac{29}{1000} = 27.0 \text{ million / yr}$$

$$\text{infant deaths} = 27.0 \times 10^6 \text{ births} \times \frac{74 \text{ deaths}}{1000 \text{ births}} = 2.0 \text{ million/yr}$$

$$\text{total deaths} = 931 \times 10^6 \times \frac{9}{1000} = 8.38 \text{ million deaths/yr}$$

$$\text{fraction of deaths that are infants} = \frac{2.0}{8.38} = 0.239 \approx 24\%$$

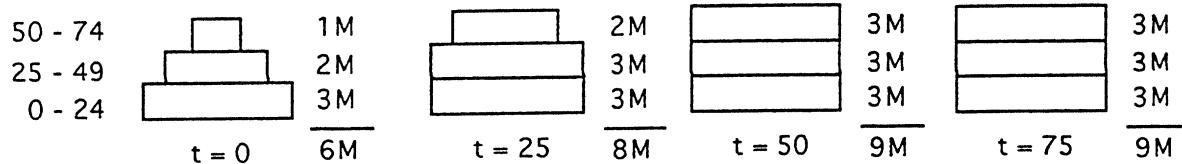
$$b. \text{infant deaths } @ \frac{10}{1000} = 27.0 \times 10^6 \text{ births} \times \frac{10 \text{ deaths}}{1000 \text{ births}} = 0.27 \text{ million/yr}$$

$$\text{"avoidable deaths"} = 2.0 - 0.27 \text{ M} = 1.73 \text{ million/yr}$$

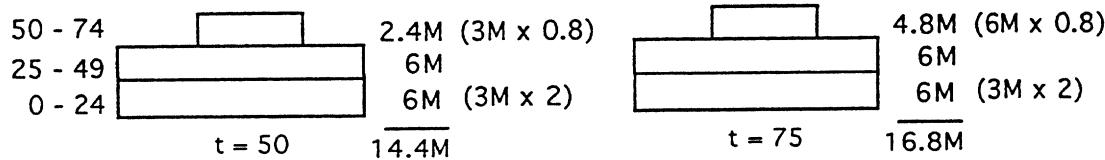
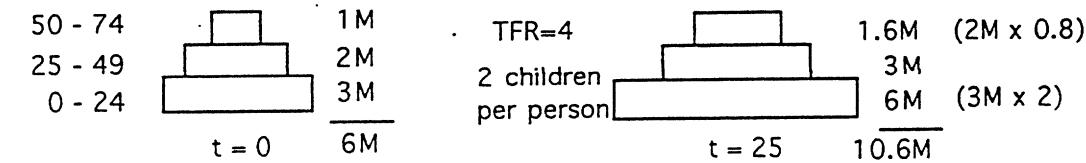
$$c. \text{annual increase} = N(b - d) = 931 \times 10^6 \left(\frac{29}{1000} - \frac{9}{1000} \right) = 18.6 \text{ million/yr}$$

Notice even though birth rates are down, population is growing faster than it did in 1985 (Problem 3.18).

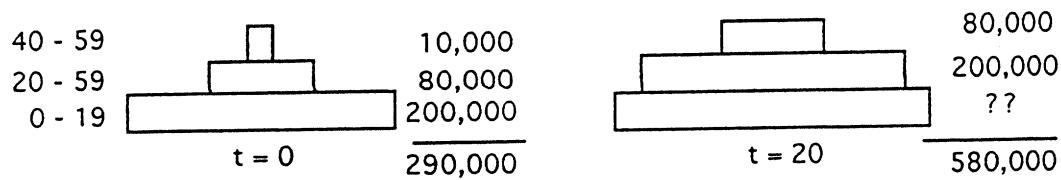
3.20 Replacement level fertility starting at t=0:



3.21 Same as Prob. 3.20 at beginning, but TFR=4 for 25yrs, and 20% death at 50:



3.22 Since $r = 3.5\%$, the doubling time is $T_d \approx 70/r = 70/3.5 = 20$ yrs. So in 20 years the population will have doubled from 290,000 to 580,000. The bottom of the pyramid must be the amount that will make the total equal to 580,000:



So the bottom of the pyramid must have $580,000 - (80,000 + 200,000) = 300,000$ people. For 200,000 people to have had 300,000 births means on average each person in the 0-19 category had 1.5 children, or each woman had 3.0 children. That is, TFR = 3.0.

3.23 Sample calculations for 1990:

$$\begin{aligned} \text{ages 0 - 9 in 1990} &= P_0(1990) = b_{10}P_{10}(1980) + b_{20}P_{20}(1980) \\ &= 0.25 \times 224 \text{ M} + 0.25 \times 182 \text{ M} = 101.5 \text{ M} \end{aligned}$$

$$\text{ages 10 - 19: } P_{10}(1990) = P_0(1980) \frac{L_{10}}{L_0} = 235 \text{ M} \times 0.957 = 224.9$$

similarly,

$$\begin{aligned} P_{20}(1990) &= 224 \times 0.987 = 221.1 \text{ M} \\ P_{30}(1990) &= 182 \times 0.980 = 178.4 \text{ M} \\ P_{40}(1990) &= 124 \times 0.964 = 119.5 \text{ M} \\ P_{50}(1990) &= 95 \times 0.924 = 87.8 \text{ M} \\ P_{60}(1990) &= 69 \times 0.826 = 57.0 \text{ M} \\ P_{70}(1990) &= 42 \times 0.633 = 26.6 \text{ M} \\ P_{80}(1990) &= 24 \times 0.316 = \underline{7.6 \text{ M}} \end{aligned}$$

$$\text{TOTAL} = 1024.3 \text{ M}$$

3.24 Using a spreadsheet makes it straightforward:

Problem 3.24: China Age Structure with 1-child family:								
Age Interval	Fraction Surviving	Births per Person	Population 1980	Population 1990	Population 2000	Population 2010	Population 2020	Population 2030
0 - 9	0.957	0	235	101.50	111.50	79.78	50.64	45.42
10 - 19	0.987	0.25	224	224.90	97.14	106.70	76.35	48.47
20 - 29	0.98	0.25	182	221.09	221.97	95.87	105.31	75.35
30 - 39	0.964	0	124	178.36	216.67	217.53	93.96	103.21
40 - 49	0.924	0	95	119.54	171.94	208.87	209.70	90.57
50 - 59	0.826	0	69	87.78	110.45	158.87	192.99	193.76
60 - 69	0.633	0	42	56.99	72.51	91.23	131.23	159.41
70 - 79	0.316	0	24	26.59	36.08	45.90	57.75	83.07
80 -	0	0	6	7.58	8.40	11.40	14.50	18.25
TOTAL			1001	1024.32	1046.64	1016.15	932.43	817.51

3.25 Now delay births by one 10-yr interval:

Problem 3.25: China Age Structure with 1-child family, delayed births:								
Age Interval	Fraction Surviving	Births per Person	Population 1980	Population 1990	Population 2000	Population 2010	Population 2020	Population 2030
0 - 9	0.957	0	235	76.50	99.86	109.66	72.45	41.28
10 - 19	0.987	0	224	224.90	73.21	95.57	104.94	69.33
20 - 29	0.98	0.25	182	221.09	221.97	72.26	94.33	103.58
30 - 39	0.964	0.25	124	178.36	216.67	217.53	70.81	92.44
40 - 49	0.924	0	95	119.54	171.94	208.87	209.70	68.26
50 - 59	0.826	0	69	87.78	110.45	158.87	192.99	193.76
60 - 69	0.633	0	42	56.99	72.51	91.23	131.23	159.41
70 - 79	0.316	0	24	26.59	36.08	45.90	57.75	83.07
80 -	0	0	6	7.58	8.40	11.40	14.50	18.25
TOTAL			1001	999.32	1011.09	1011.29	948.71	829.39

The peak in Prob. 3.24 was 1047 million; with delayed births the peak drops to 1011 million.

3.26 Using the more realistic 2-child per family birth rates gives:

Problem 3.26: China Age Structure with 2-child family:								
Age Interval	Fraction Surviving	Births per Person	Population 1980	Population 1990	Population 2000	Population 2010	Population 2020	Population 2030
0 - 9	0.957	0	235	182.00	221.09	221.97	171.91	208.83
10 - 19	0.987	0	224	224.90	174.17	211.58	212.43	164.52
20 - 29	0.98	1	182	221.09	221.97	171.91	208.83	209.67
30 - 39	0.964	0	124	178.36	216.67	217.53	168.47	204.65
40 - 49	0.924	0	95	119.54	171.94	208.87	209.70	162.41
50 - 59	0.826	0	69	87.78	110.45	158.87	192.99	193.76
60 - 69	0.633	0	42	56.99	72.51	91.23	131.23	159.41
70 - 79	0.316	0	24	26.59	36.08	45.90	57.75	83.07
80 -	0	0	6	7.58	8.40	11.40	14.50	18.25
TOTAL			1001	1104.82	1233.27	1339.26	1367.81	1404.57

↑ ↑

SOLUTIONS FOR CHAPTER 4

4.1 From the slope of the figure, potency = $0.001/0.1(\text{mg/kg-d}) = 0.01 (\text{mg/kg-d})^{-1}$

$$\text{CDI} = \frac{10^{-3} \text{ mg/m}^3 \times 20 \text{ m}^3/\text{d}}{70 \text{ kg}} = 0.000285 \text{ mg/kg-d}$$

$$\text{Risk} = \text{CDI} \times \text{Potency} = 0.000285 \text{ mg/kg-d} \times 0.01 (\text{mg/kg-d})^{-1} = 2.9 \times 10^{-6}$$

4.2 0.2 ppb of PCB:

a. $\text{CDI} = \frac{0.2 \times 10^{-3} \text{ mg/L} \times 2 \text{ L/d} \times 350 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} = 2.35 \times 10^{-6} \text{ mg/kg-d}$

b. $\text{Risk} = \text{CDI} \times \text{Potency} = 2.35 \times 10^{-6} \text{ mg/kg-d} \times 7.7 (\text{mg/kg-d})^{-1} = 18.1 \times 10^{-6}$

c. $\text{Extra cancers/yr} = \frac{10^6 \text{ people} \times 18.1 \times 10^{-6} \text{ cancer/person-life}}{70 \text{ yr/life}} = 0.26 \text{ cancer/yr}$

d. $\text{Expected cancer rate} = \frac{193 \text{ deaths/yr}}{100,000 \text{ people}} \times 10^6 \text{ people} = 1930 \text{ deaths/yr}$
adding 0.26 cancers per year would not be detectable.

4.3 Rat data:

		D	\bar{D}
		E	\bar{E}
E	D	a 30	b 470
	\bar{D}	c 10	d 290

a. Relative risk = $\frac{a/(a+b)}{c/(c+d)} = \frac{30/500}{10/300} = 1.8$

b. Attributable risk = $\frac{a}{a+b} - \frac{c}{c+d} = \frac{30}{500} - \frac{10}{300} = 0.0267$

c. Odds ratio = $\frac{ad}{bc} = \frac{30 \times 290}{470 \times 10} = 1.85$

All three indicators are consistent with a relationship between exposure and risk.

4.4 Filling in the matrix gives:

		D	\bar{D}
		E	\bar{E}
E	D	a 5	b 95
	\bar{D}	c 2	d 98

$$a. \text{ Relative risk} = \frac{a/(a+b)}{c/(c+d)} = \frac{5/100}{2/100} = 2.5$$

$$b. \text{ Attributable risk} = \frac{a}{a+b} - \frac{c}{c+d} = \frac{5}{100} - \frac{2}{100} = 0.03$$

$$c. \text{ Odds ratio} = \frac{ad}{bc} = \frac{5 \times 98}{95 \times 2} = 2.58$$

4.5 70 kg individual, 2L/day, 0.1 mg/L of 1,1-dichloroethylene, 20 years:

$$a. \text{ Hazard quotient} = \frac{\text{average dose during exposure}}{\text{RfD}} = \frac{2\text{L/d} \times 0.1\text{mg/L} / 70\text{kg}}{0.009 \text{ mg/kg-d}} = 0.32$$

$$b. \text{ CDI} = \frac{2\text{L/d} \times 0.1 \text{ mg/L} \times 20 \text{ yrs} \times 365 \text{ d/yr}}{70 \text{ kg} \times 70 \text{ yrs} \times 365 \text{ d/yr}} = 0.000816 \text{ mg/kg-d}$$

$$\text{Risk} = \text{CDI} \times \text{Potency} = 0.000816 \text{ mg/kg-d} \times 0.58 (\text{mg/kg-d})^{-1} = 4.7 \times 10^{-4}$$

c. Drinking the water for 30 years instead of 20 doesn't change the Hazard Quotient since that is based only on the period of time when the individual is exposed. The cancer risk does change, however:

$$\text{CDI} = \frac{2\text{L/d} \times 0.1 \text{ mg/L} \times 30 \text{ yrs} \times 365 \text{ d/yr}}{70 \text{ kg} \times 70 \text{ yrs} \times 365 \text{ d/yr}} = 0.00122 \text{ mg/kg-d}$$

$$\text{Risk} = 0.00122 \text{ mg/kg-d} \times 0.58 (\text{mg/kg-d})^{-1} = 7.1 \times 10^{-4}$$

4.6 DWEL is the concentration of pollutant that leads to a 10^{-6} risk,

$$\text{CDI} = \frac{\text{Risk}}{\text{Potency}} = \frac{10^{-6}}{7.5 \times 10^{-3}} = 1.33 \times 10^{-4} \text{ mg/kg-d}, \text{ and using the definition of CDI:}$$

$$\text{CDI} = \frac{C(\text{mg/L}) \times 2 \text{ L/d}}{70 \text{ kg}} = 1.33 \times 10^{-4} \text{ mg/kg-d}, \text{ solving for C}$$

$$C(\text{mg/L}) = \text{DWEL} = 70 \times 1.33 \times 10^{-4} / 2 = 4.67 \times 10^{-3} \text{ mg/L} \approx 5 \mu\text{g/L}$$

$$4.7 \text{ Risk} = \text{CDI} \times \text{Potency} = 10^{-6} = \frac{2\text{L/d} \times 0.01 \text{ mg/L}}{70 \text{ kg}} \times \text{Potency}$$

$$\text{so, Potency} = \frac{70 \text{ kg} \times 10^{-6}}{0.02 \text{ mg/d}} = 3.5 \times 10^{-3} (\text{mg/kg-d})^{-1}$$

4.8 dioxin standard = 3×10^{-8} mg/L, using EPA suggested exposure factors (Table 4.10):

$$CDI = \frac{2L/d \times 3 \times 10^{-8} \text{ mg/L} \times 350 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} = 3.5 \times 10^{-10} \text{ mg/kg-d}$$

$$\text{Risk} = CDI \times \text{Potency} = 3.5 \times 10^{-10} \text{ mg/kg-d} \times 1.56 \times 10^5 \text{ (mg/kg-d)}^{-1} = 5.5 \times 10^{-5}$$

4.9 Tetrachloroethylene standard = 0.005 mg/L

$$\text{Risk} = CDI \times \text{Potency}$$

$$= \frac{2L/d \times 0.005 \text{ mg/L} \times 350 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times 5.1 \times 10^2 \text{ (mg/kg-d)}^{-1} = 3.0 \times 10^{-6}$$

4.10 Radiation potency = 1 cancer death per 8000 person-rems:

a.

$$\text{Cancer} = \frac{1 \text{ cancer death}}{8000 \text{ person-rems}} \times 260 \times 10^6 \text{ people} \times 0.130 \text{ rem/yr} = 4.2 \times 10^3 \text{ deaths/yr}$$

b.

$$\text{Risk} = 4 \text{ mrem/flight} \times N \text{ flights} \times \frac{1 \text{ cancer}}{8000 \text{ rems} \times 10^3 \text{ mrem/rem}} = 10^{-6}$$

$$N = \frac{10^{-6} \times 8 \times 10^6}{4} = 2 \text{ cross country flights}$$

(agrees with Table 4.3)

4.11

$$\text{Denver risk} = 0.12 \text{ rem/yr} \times 70 \text{ yr} \times \frac{1 \text{ cancer death}}{8000 \text{ rems}} = 1.05 \times 10^{-3}$$

a.

$$\text{Sea level risk} = 0.04 \text{ rem/yr} \times 70 \text{ yr} \times \frac{1 \text{ cancer death}}{8000 \text{ rems}} = 0.35 \times 10^{-3}$$

b. Denver deaths due to cosmic radiation exposure:

$$\frac{0.12 \text{ rem/person-yr} \times 0.5 \times 10^6 \text{ people} \times 1 \text{ death}}{8000 \text{ people-rems}} = 7.5 \approx 8 \text{ deaths/yr}$$

$$\text{Expected} = 0.5 \times 10^6 \text{ people} \times \frac{193 \text{ deaths/yr}}{100,000 \text{ people}} = 965 \text{ annual cancer deaths}$$

c. Incremental risk = $10^{-6} = (0.12 \text{ rem/yr} - 0.04 \text{ rem/yr}) \times N \text{ yr} \times \frac{1 \text{ death}}{8000 \text{ rem}}$

$$N = \frac{8000 \times 10^{-6}}{0.08} = 0.1 \text{ yr} \quad (\text{Table 4.3 suggests 2 months})$$

d. $260 \times 10^6 \text{ people} \times 0.040 \text{ rem/yr} \times \frac{1 \text{ death}}{8000 \text{ person - rems}} = 1300 \text{ deaths/yr}$

4.12 Radon exposure of 1.5 piC/L equivalent to 400 mrem/yr (0.4 rem/yr):

a. $0.4 \text{ rem/yr} \times \frac{1 \text{ cancer}}{8000 \text{ pers - rem}} \times 260 \times 10^6 \text{ people} = 13,000 \text{ cancer deaths/yr}$

b. Risk = $\frac{0.4 \text{ rem}}{\text{yr}} \times \frac{1 \text{ cancer}}{8000 \text{ person - rem}} \times \frac{70 \text{ yr}}{\text{lifetime}} = 3.5 \times 10^{-3} \text{ cancer / lifetime}$

4.13 For 75 million people exposed to 0.4 rem of radiation:

a. extra cancer deaths = $0.4 \text{ rem} \times \frac{1 \text{ cancer}}{8000 \text{ person - rems}} \times 75 \times 10^6 \text{ people} = 3750$

b. normal cancer deaths = $0.22 \times 75 \times 10^6 \text{ people} = 16.5 \times 10^6$

4.14 Living within 5 miles of a reactor for 50 yrs--what mrem/yr exposure gives a 10^{-6} risk:

$$10^{-6} = \frac{X \text{ rem}}{\text{yr}} \times 50 \text{ yrs} \times \frac{1 \text{ cancer death}}{8000 \text{ rem}}$$

$$X = \frac{8000 \times 10^{-6}}{50} = 0.16 \times 10^{-3} \text{ rem / yr} = 0.16 \text{ mrem/yr}$$

4.15 Concentrations yielding acceptable risks:

a. benzene, oral, 10^{-5} risk, potency = $2.9 \times 10^{-2} (\text{mg/kg-d})^{-1}$

$$\text{Risk} = \text{CDI} \times \text{Potency}$$

$$10^{-5} = \frac{2L/d \times C \text{ mg/L} \times 350 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{2.9 \times 10^{-2}}{\text{mg/kg - d}}$$

$$C = \frac{10^{-5} \times 70 \times 365 \times 70}{2 \times 350 \times 30 \times 2.9 \times 10^{-2}} = 0.03 \text{ mg/L}$$

b. trichloroethylene in air, risk 10^{-6} , inhalation potency 1.3×10^{-2} :

$$10^{-6} = \frac{20 \text{ m}^3/\text{d} \times C \text{ mg/m}^3 \times 350 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{1.3 \times 10^{-2}}{\text{mg/kg - d}}$$

$$C = \frac{10^{-6} \times 70 \times 365 \times 70}{20 \times 350 \times 30 \times 1.3 \times 10^{-2}} = 6.6 \times 10^{-4} \text{ mg/m}^3$$

c. benzene in air, risk 10^{-5} , potency 2.9×10^{-2} :

$$10^{-6} = \frac{20 \text{ m}^3 / \text{d} \times C \text{ mg/m}^3 \times 350 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{2.9 \times 10^{-2}}{\text{mg/kg - d}}$$

$$C = \frac{10^{-6} \times 70 \times 365 \times 70}{20 \times 350 \times 30 \times 2.9 \times 10^{-2}} = 2.9 \times 10^{-4} \text{ mg/m}^3$$

d. vinyl chloride in water, risk 10^{-4} , potency 2.3:

$$10^{-4} = \frac{2L/d \times C \text{ mg/L} \times 350 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{2.3}{\text{mg/kg - d}}$$

$$C = \frac{10^{-4} \times 70 \times 365 \times 70}{2 \times 350 \times 30 \times 2.3} = 3.7 \times 10^{-3} \text{ mg/L}$$

4.16 Trichloroethylene in an industrial facility; risk 10^{-4} :

Risk = CDI x Potency

$$10^{-4} = \frac{20 \text{ m}^3 / \text{d} \times C \text{ mg/m}^3 \times 250 \text{ d/yr} \times 25 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{1.3 \times 10^{-2}}{\text{mg/kg - d}}$$

$$C = \frac{10^{-4} \times 70 \times 365 \times 70}{20 \times 250 \times 25 \times 1.3 \times 10^{-2}} = 0.11 \text{ mg/m}^3$$

To convert to ppm, we need the molecular weight of $\text{C}_2\text{Cl}_3\text{H}$, which is $2 \times 12 + 3 \times 35.5 + 1 = 131.5 \text{ g/mol}$. From (1.8):

$$\text{ppm} = \frac{24.465 \times C (\text{mg/m}^3)}{\text{mol wt}} = \frac{24.465 \times 0.11}{131.5} = 0.02 \text{ ppm}$$

4.17 Benzene in fish = C (mg/L) in river x BCF (L/kg); From Table 4.12, BCF = 5.2 L/kg

Risk = CDI x Potency

$$10^{-6} = \frac{C \text{ mg/L} \times 5.2 \text{ L/kg} \times 0.054 \text{ kg/d} \times 350 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{2.9 \times 10^{-2}}{\text{mg/kg - d}}$$

$$C = \frac{10^{-6} \times 70 \times 365 \times 70}{5.2 \times 0.054 \times 350 \times 30 \times 2.9 \times 10^{-2}} = 0.021 \text{ mg/L}$$

4.18 DDT in fish = C (mg/L) x BCF (L/kg); from Table 4.12, BCF = 54,000 L/kg

Risk = CDI x Potency

$$\text{Risk} = \frac{0.020 \text{ mg/L} \times 54,000 \text{ L/kg} \times 0.002 \text{ kg/d}}{70 \text{ kg}} \times \frac{0.34}{\text{mg/kg - d}} = 0.01$$

4.19 Hazard index = Sum of the hazard quotients:

2 mg/L of 1,1,1-trichloroethane; RfD = 0.035 mg/kg-d

$$ADD = \frac{1 \text{ L/d} \times 2 \text{ mg/L}}{50 \text{ kg}} = 0.04 \text{ mg/kg - d}$$

$$HQ = \frac{ADD}{RfD} = \frac{0.04 \text{ mg/kg - d}}{0.035 \text{ mg / kg - d}} = 1.14$$

0.04 mg/L of tetrachloroethylene; RfD = 0.010 mg/kg-d

$$HQ = \frac{ADD}{RfD} = \frac{1 \text{ L/d} \times 0.04 \text{ mg/L}}{0.010 \text{ mg/kg - d}} = \frac{0.0008}{0.010} = 0.08$$

0.1 mg/L of 1,1-dichloroethylene; RfD = 0.009 mg/kg-d

$$HQ = \frac{ADD}{RfD} = \frac{1 \text{ L/d} \times 0.1 \text{ mg/L}}{0.009 \text{ mg/kg - d}} = \frac{0.002}{0.009} = 0.22$$

Hazard Index = 1.14 + 0.08 + 0.22 = 1.44 , cause for concern.

4.20 1.0 g/day of heptachlor leaking into a 30,000 m³ pond; K=0.35/day:

a. Input rate = Output rate + Decay rate = KCV (1.18)

$$1.0 \text{ g/day} = 0.35/\text{day} \times C(\text{mg/L}) \times 10^{-3} \text{ g/mg} \times 30,000 \text{ m}^3 \times 10^3 \text{ L/m}^3$$

$$C = \frac{1.0}{30,000 \times 0.35} = 0.95 \times 10^{-4} \text{ mg/L}$$

b. 70-kg person, drinking 2 L/day for 5 years:

$$\text{Risk} = CDI \times \text{potency}$$

$$= \frac{2 \text{ L/d} \times 0.95 \times 10^{-4} \text{ mg/L} \times 5 \text{ yrs}}{70 \text{ kg} \times 70 \text{ yrs}} \times \frac{3.4}{\text{mg/kg - d}} = 0.66 \times 10^{-6} \approx 0.7 \times 10^{-6}$$

4.21 0.03 mg of BaP per cigarette, 20 cigarettes/day for 40 years, potency 6.11 (mg/kg-d)⁻¹

$$CDI = \frac{20 \text{ cig/d} \times 0.03 \text{ mg/cig} \times 40 \text{ yrs}}{70 \text{ kg} \times 70 \text{ yrs}} = 0.0049 \text{ mg/kg - d}$$

$$\text{Risk} = CDI \times \text{potency} = 0.0049 \text{ mg/kg - d} \times 6.11 (\text{mg/kg - d})^{-1} = 0.03$$

4.22 Sidestream smoke, $6 \times 10^{-4} \text{ mg/m}^3$ while breathing $0.83 \text{ m}^3/\text{hr}$, 10^{-6} risk, BaP potency $6.11 (\text{mg/kg-d})^{-1}$

$$\text{CDI} = \frac{6 \times 10^{-4} \text{ mg/cig} - \text{m}^3 \times 0.83 \text{ m}^3 \times N \text{ cigs}}{70 \text{ kg} \times 70 \text{ yrs} \times 365 \text{ d/yr}} = 2.8 \times 10^{-10} N \text{ mg/kg - d}$$

$$\text{Risk} = \text{CDI} \times \text{Potency} = 10^{-6} = 2.8 \times 10^{-10} N (\text{mg/kg - d}) \times \frac{6.11}{\text{mg/kg - d}}$$

$$N = \frac{10^{-6}}{2.8 \times 10^{-10} \times 6.11} = 584 \text{ cigarettes}$$

At eight cigarettes per day smoked in this poor fellow's presence:

$$\frac{584 \text{ cigs}}{8 \text{ cigs/d}} = 73 \text{ days}$$

4.23 Infiltration of $120 \text{ m}^3/\text{hr}$, $0.1 \text{ mg BaP/cigarette}$, 1 cigarette per hour:

- a. Find the steady-state concentration

Input rate = Output rate

$$1 \text{ cig/hr} \times 0.1 \text{ mg/cig} = 120 \text{ m}^3/\text{hr} \times C \text{ mg/m}^3$$

$$C = 0.1/120 = 0.00083 \text{ mg/m}^3$$

- b. living for a year with a smoker:

$$\text{CDI} = \frac{0.00083 \frac{\text{mg}}{\text{m}^3} \times 20 \frac{\text{m}^3}{\text{day}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times 8 \frac{\text{hr}}{\text{day}} \times 365 \text{ day}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} = 1.1 \times 10^{-6} \text{ mg/kg - d}$$

$$\text{Risk} = \text{CDI} \times \text{Potency} = 1.1 \times 10^{-6} \text{ mg/kg-d} \times 6.11 (\text{mg/kg-d})^{-1} = 7 \times 10^{-6}$$

4.24 260 million people, $2\text{L}/\text{day}$, $360\text{d}/\text{yr}$, 30yr , find risk and incremental cancers for,

- a. trichloroethylene at 0.005 mg/L :

Risk = CDI x Potency

$$\text{Risk} = \frac{2\text{L/d} \times 0.005 \text{ mg/L} \times 360 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{1.1 \times 10^{-2}}{\text{mg/kg - d}} = 6.6 \times 10^{-7}$$

$$\Delta \text{ cancer} = \frac{260 \times 10^6 \text{ people} \times 6.6 \times 10^{-7} \text{ cancer / person - life}}{70 \text{ yr/lifetime}} = 2.5 \text{ cancer/yr}$$

- b. benzene at 0.005 mg/L :

$$\text{Risk} = \frac{2\text{L/d} \times 0.005 \text{ mg/L} \times 360 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{2.9 \times 10^{-2}}{\text{mg/kg - d}} = 1.75 \times 10^{-6}$$

$$\Delta \text{cancer} = \frac{260 \times 10^6 \text{ people} \times 1.75 \times 10^{-6} \text{ cancer / person - life}}{70 \text{ yr/lifetime}} = 6.5 \text{ cancer/yr}$$

c. arsenic at 0.05 mg/L:

$$\text{Risk} = \frac{2\text{L/d} \times 0.05 \text{ mg/L} \times 360 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{1.75}{\text{mg/kg - d}} = 1.05 \times 10^{-3}$$

$$\Delta \text{cancer} = \frac{260 \times 10^6 \text{ people} \times 1.05 \times 10^{-3} \text{ cancer / person - life}}{70 \text{ yr/lifetime}} = 3925 \text{ cancer/yr}$$

d. carbon tetrachloride at 0.005 mg/L:

$$\text{Risk} = \frac{2\text{L/d} \times 0.005 \text{ mg/L} \times 360 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{0.13}{\text{mg/kg - d}} = 7.9 \times 10^{-6}$$

$$\Delta \text{cancer} = \frac{260 \times 10^6 \text{ people} \times 7.9 \times 10^{-6} \text{ cancer / person - life}}{70 \text{ yr/lifetime}} = 29 \text{ cancer/yr}$$

e. vinyl chloride at 0.002 mg/L:

$$\text{Risk} = \frac{2\text{L/d} \times 0.002 \text{ mg/L} \times 360 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{2.3}{\text{mg/kg - d}} = 5.6 \times 10^{-5}$$

$$\Delta \text{cancer} = \frac{260 \times 10^6 \text{ people} \times 5.6 \times 10^{-5} \text{ cancer / person - life}}{70 \text{ yr/lifetime}} = 206 \text{ cancer/yr}$$

f. PCBs at 0.0005 mg/L:

$$\text{Risk} = \frac{2\text{L/d} \times 0.0005 \text{ mg/L} \times 360 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{7.7}{\text{mg/kg - d}} = 4.6 \times 10^{-5}$$

$$\Delta \text{cancer} = \frac{260 \times 10^6 \text{ people} \times 4.6 \times 10^{-5} \text{ cancer / person - life}}{70 \text{ yr/lifetime}} = 172 \text{ cancer/yr}$$

4.25 Formaldehyde at $50 \mu\text{g/m}^3$ with potency 1.3×10^{-5} cancer per $\mu\text{g/m}^3$

$$\text{Risk} = 50 \mu\text{g/m}^3 \times 1.3 \times 10^{-5} \text{ cancer per } \mu\text{g/m}^3 = 6.5 \times 10^{-4}$$

4.26 Groundwater with 10 ppb TCE vs. surface water with 50 ppb chloroform:

$$\text{TCE risk} = \frac{2\text{L/d} \times 0.010 \text{ mg/L} \times 360 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{1.1 \times 10^{-2}}{\text{mg/kg - d}} = 1.3 \times 10^{-6}$$

$$\text{Chloroform risk} = \frac{2L/d \times 0.050 \text{ mg/L} \times 360 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} \times \frac{6.1 \times 10^{-3}}{\text{mg/kg - d}} = 3.7 \times 10^{-6}$$

Stick with the groundwater.

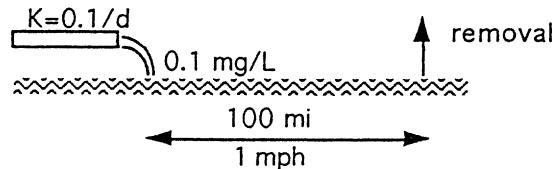
4.27 70 kg man, 0.1 mg/m³ tetrachloroethylene, 1 m³/hr, 8hr/d, 5 d/wk, 50 wk/yr, 30 yrs, 0.9 absorption, potency 2x10⁻³ (mg/kg-d)⁻¹, risk:

$$\text{CDI} = \frac{0.1 \text{ mg/m}^3 \times 1 \text{ m}^3/\text{hr} \times 8 \text{ hr/d} \times 5 \text{ d/wk} \times 50 \text{ wk/yr} \times 30 \text{ yr} \times 0.90}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} = 3.0 \times 10^{-3} \text{ mg/kg-d}$$

$$\text{Risk} = \text{CDI} \times \text{potency} = 3.0 \times 10^{-3} \text{ mg/kg-d} \times \frac{2 \times 10^{-3}}{\text{mg/kg-d}} = 6 \times 10^{-6} \text{ for the 70kg man}$$

$$\text{Risk for the 50 - kg woman} = 6 \times 10^{-6} \times \frac{70 \text{ kg}}{50 \text{ kg}} = 8.4 \times 10^{-6}$$

4.28 Potency 0.3 (mg/kg-d)⁻¹



$$\text{time downstream} = \frac{100 \text{ mi}}{1 \text{ mi/hr} \times 24 \text{ hr/day}} = 4.1667 \text{ days}$$

$$C_{100\text{mi}} = C_0 e^{-Kt} = 0.1 \text{ mg/L} \times e^{-0.1/d \times 4.1667d} = 0.066 \text{ mg/L}$$

$$\text{CDI} = \frac{0.066 \text{ mg/L} \times 2L/d \times 360 \text{ d/yr} \times 30 \text{ yr}}{70 \text{ kg} \times 365 \text{ d/yr} \times 70 \text{ yr}} = 0.000796$$

$$\text{Risk} = 0.000796 \text{ mg/kg-d} \times \frac{0.30}{\text{mg/kg-d}} = 2.4 \times 10^{-4}$$

4.29 One-hit model:

$$P(d) = 1 - \exp[-(q_0 + q_1 d)] = 1 - \exp[-(0.01209 + 0.001852 x d)]$$

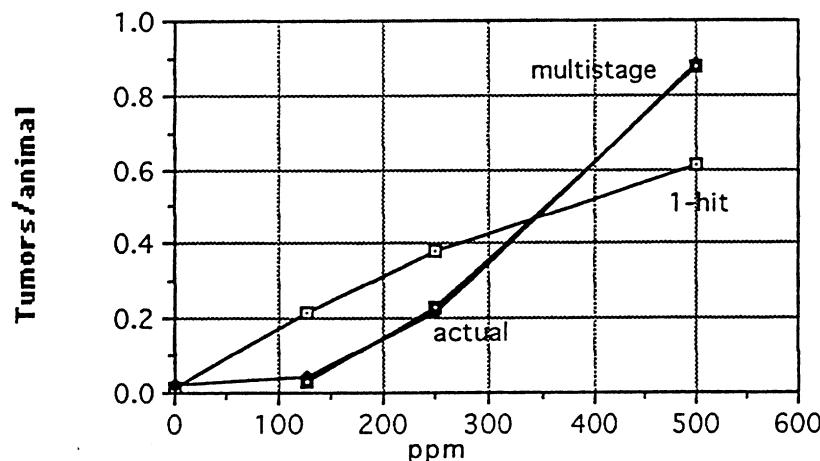
Multistage model:

$$\begin{aligned} P(d) &= 1 - \exp[-(q_0 + q_1 d + q_2 d^2 + q_3 d^3 + q_4 d^4)] \\ &= 1 - \exp[-(0.02077 + 1.101 \times 10^{-8} d^3 + 1.276 \times 10^{-11} d^4)] \end{aligned}$$

Dose (ppm)	actual tumors	one-hit	multistage
---------------	------------------	---------	------------

125	0.03	0.216	0.044
250	0.23	0.378	0.215
500	0.88	0.608	0.888
1	?	0.0138	0.0206

Problem 4.29 Model Predictions and Actual tumors



4.30 10 million people, 10^{-4} risk,

$$\text{cancer rate} = \frac{10 \times 10^6 \text{ people} \times 10^{-4} \text{ cancer / person - life}}{70 \text{ yr / life}} = 14.3 \text{ cancer/yr}$$

@ 10^{-5} risk, there would be 1.4 cancers, saving $14.3 - 1.4 = 12.9$ cancer/yr

$$\text{cost per cancer} = \frac{\$1/\text{yr - person} \times 10^7 \text{ people}}{12.9 \text{ cancers avoided}} = \$0.77 \text{ million / cancer avoided}$$

SOLUTIONS FOR CHAPTER 5

5.1 In a standard BOD test:

- a. stoppered to prevent reaeration
- b. black to prevent photosynthesis
- c. to keep final DO above zero
- d. to assure adequate microbial population
- e. would take too long

5.2 $BOD_5 = 200 \text{ mg/L}$; 90% efficient removal; initial DO = 9.2 mg/L

$$\text{a. } BOD_s = (1 - 0.9) \times 200 \text{ mg/L} = 20 \text{ mg/L} = \frac{DO_i - DO_f}{P}$$

$$P = \frac{(9.2 - 2.0) \text{ mg/L}}{20 \text{ mg/L}} = 0.36 \text{ (36\% of the 300 mL is wastewater)}$$

$$\text{Volume of treated wastewater} = 0.36 \times 300 \text{ mL} = 108 \text{ mL}$$

b. 50% treated wastewater ($P=0.50$):

$$DO_f = DO_i - P \times BOD_s = 9.2 - 0.50 \times 20 = -0.8 \text{ mg/L}$$

but DO cannot be negative, so $DO_f = 0$.

$$\text{5.3 } BOD_s = \frac{9.0 - 1.0}{1/5} = 40 \text{ mg/L}$$

5.4 To have at least a 2.0 mg/L drop in DO during the test:

$$P_{\min} = \frac{DO_i - DO_f}{BOD_s} = \frac{2.0 \text{ mg/L}}{230 \text{ mg/L}} = 0.0087$$

which corresponds to a volume of wastewater = $0.0087 \times 300 \text{ mL} = 2.6 \text{ mL (min)}$

To have final DO at least 2.0 mg/L means:

$$P_{\max} = \frac{DO_i - DO_f}{BOD_s} = \frac{8.0 - 2.0 \text{ mg/L}}{230 \text{ mg/L}} = 0.026$$

which means max waste volume = $0.026 \times 300 \text{ mL} = 7.8 \text{ mL}$

5.5 The BOD for treated and untreated waste is

$$\text{Untreated } BOD_s = \frac{DO_i - DO_f}{P} = \frac{6.0 - 2.0 \text{ mg/L}}{5/300 \text{ mg/L}} = 240 \text{ mg/L}$$

$$\text{Treated BOD}_5 = \frac{\text{DO}_i - \text{DO}_f}{P} = \frac{9.0 - 4.0 \text{ mg/L}}{15/300 \text{ mg/L}} = 100 \text{ mg/L}$$

$$\% \text{ BOD removal} = \frac{(240 - 100)}{240} \times 100 = 58.3\% \text{ not nearly the 85\% removal expected}$$

5.6 From the graph:

- a. Ultimate BOD, $L_0 = 40 \text{ mg/L}$
- b. $\text{BOD}_5 = 40 - 15 = 25 \text{ mg/L}$
- c. Remaining BOD after 5 days = $L_5 = 15 \text{ mg/L}$

5.7 $\text{BOD}_5 = 200 \text{ mg/L}$ and $L_0 = 300 \text{ mg/L}$

$$\text{BOD}_5 = L_0(1 - e^{-kt}) = 200 \text{ mg/L} = 300 \text{ mg/L}(1 - e^{-ks})$$

$$k = -\frac{1}{5} \ln\left(1 - \frac{200}{300}\right) = 0.22 \text{ / day}$$

$$\text{BOD}_5 = L_0(1 - e^{-kt}) = 200 \text{ mg/L} = 300 \text{ mg/L}(1 - e^{-ks})$$

$$K = -\frac{1}{5} \ln\left(1 - \frac{200}{300}\right) = 0.095 \text{ / day}$$

5.8 Given $P = 1/3$, $\text{DO}_i = 9.0 \text{ mg/L}$, $\text{DO}_5 = 4.0 \text{ mg/L}$, $\text{DO}_{\infty} = 2.0 \text{ mg/L}$:

$$\text{a. } \text{BOD}_5 = \frac{\text{DO}_i - \text{DO}_5}{P} = \frac{9.0 - 4.0}{1/3} = 15 \text{ mg/L}$$

$$\text{b. } \text{BOD}_{\infty} = L_0 = \frac{\text{DO}_i - \text{DO}_{\infty}}{P} = \frac{9.0 - 2.0}{1/3} = 21 \text{ mg/L}$$

$$\text{c. } L_5 = 21 - 15 = 6 \text{ mg/L}$$

$$\text{d. } k = -\frac{1}{t} \ln\left(\frac{L_t}{L_0}\right) = -\frac{1}{5} \ln\left(\frac{L_5}{L_0}\right) = -\frac{1}{5} \ln\left(\frac{6}{21}\right) = 0.25 \text{ / day}$$

5.9 a. Performance of the treatment plant:

$$\text{treated BOD}_5 = \frac{9.0 - 2.0}{1/20} = 140 \text{ mg/L}$$

$$\text{treated BOD}_5 = \frac{9.0 - 2.0}{1/20} = 140 \text{ mg/L}$$

$$\% \text{ removal} = \frac{210 - 140}{210} = 0.333 = 33.3\%$$

b. Sample 2 at end of test:

$$DO_f = DO_i - P \times BOD_s = 9.2 - \frac{1}{15} \times 210 = -4.8 \text{ mg/L}$$

but DO can't be negative, so $DO_f = 0 \text{ mg/L}$

$$\text{c. } DO_f > 2.0, \text{ so: } P \leq \frac{DO_i - DO_f}{BOD_s} = \frac{9.0 - 2.0}{140} = 0.05$$

$$\text{max Vol treated wastewater} = 0.05 \times 300 \text{ mL} = 15 \text{ mL}$$

5.10 Seeded waste: 1:30, $DO_i = 9.2$, $DO_f = 2.0 \text{ mg/L}$

Blank: $B_i = 9.2$, $B_f = 8.0 \text{ mg/L}$, Using (5.8) gives,

$$BOD_w = \frac{(DO_i - DO_f) - (B_i - B_f)(1 - P)}{P} = \frac{9.2 - 2.0 - (9.2 - 8.0)(1 - \frac{1}{30})}{1/30} = 181 \text{ mg/L}$$

5.11 Using (5.8) gives,

$$\begin{aligned} BOD_w &= \frac{(DO_i - DO_f) - (B_i - B_f)(1 - P)}{P} \\ &= \frac{8.55 - 2.40 - (8.75 - 8.53)(1 - \frac{1}{10})}{1/10} = 59.5 \text{ mg/L} \end{aligned}$$

5.12 $BOD_5 = 150 \text{ mg/L}$ at 20°C , $k = 0.23/\text{day}$:

a. To find ultimate BOD, rearrange (5.12):

$$L_0 = \frac{BOD_s}{(1 - e^{-ks})} = \frac{150 \text{ mg/L}}{(1 - e^{-0.23/d \times 5d})} = 219.5 \text{ mg/L}$$

b. To find k at 15°C , use (5.15):

$$k_T = k_{20} \theta^{(T-20)}; \quad k_{15} = 0.23 \times (1.047)^{(15-20)} = 0.183 / \text{day}$$

c. At 15°C,

$$\text{BOD}_5 = L_0(1 - e^{-k_5}) = 219.5(1 - e^{-0.183 \times 5}) = 131.5 \text{ mg/L}$$

5.13 BOD₅ at 20°C = 210 mg/L and L₀ = 350 mg/L, find BOD₅ at 25°C:

$$L_t = L_0 e^{-kt}$$

$$(350 - 210) = 350 e^{-k_5}$$

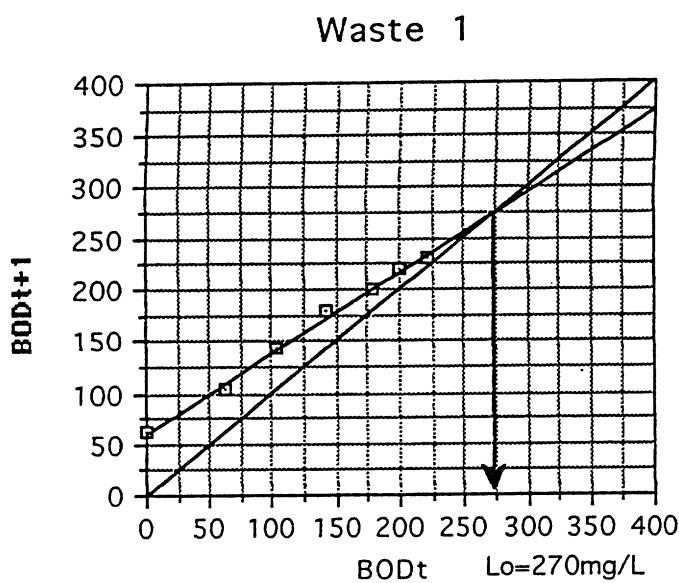
$$k_{20^\circ} = -\frac{1}{5} \ln\left(\frac{140}{350}\right) = 0.1832 / \text{day}$$

To find k at 25°C, use (5.15),

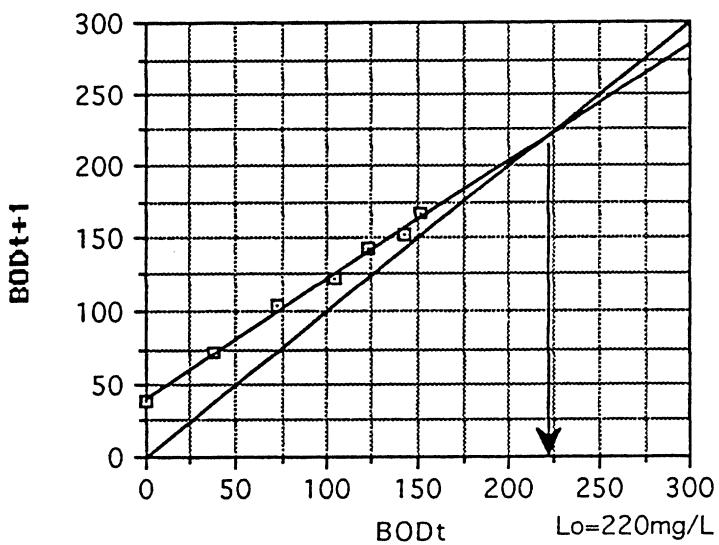
$$k_T = k_{20^\circ} \theta^{(T-20)}; \quad k_{25} = 0.1832 \times (1.047)^{(25-20)} = 0.231 / \text{day}$$

$$\text{Then, } \text{BOD}_5(25^\circ\text{C}) = L_0(1 - e^{-k_5}) = 350(1 - e^{-0.231 \times 5}) = 239.5 \text{ mg/L}$$

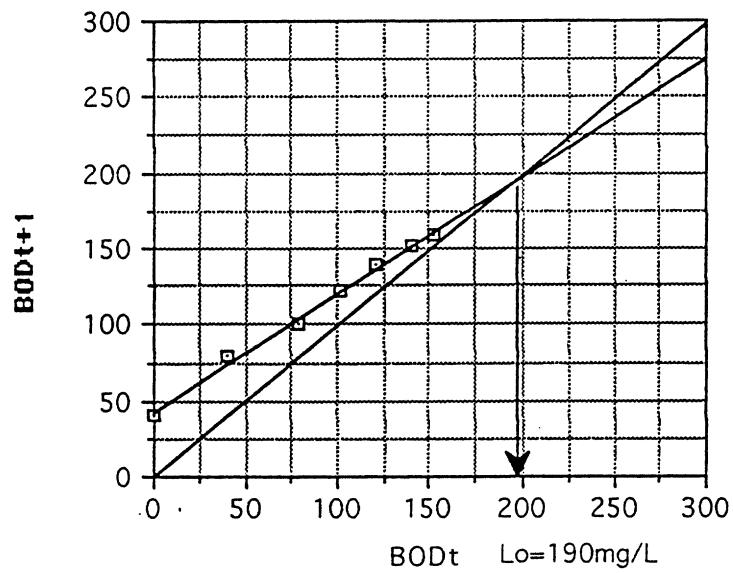
5.14 Plots of BOD_{t+1} vs BOD_t along with the line representing BOD_{t+1}=BOD_t are shown. Finding L₀ has considerable room for error.



Waste 2



Waste 3



5.15 Show $BOD_{t+1} = a BOD_t + b$, that is, that it is linear:

$$BOD_{t+1} = L_0(1 - e^{-kt+1}) = L_0(1 - e^{-kt}e^{-k})$$

now add and subtract e^{-k}

$$BOD_{t+1} = L_0(1 - e^{-kt}e^{-k} + e^{-k} - e^{-k}) = L_0[e^{-k}(1 - e^{-kt}) + 1 - e^{-k}]$$

$$BOD_{t+1} = e^{-k}L_0(1 - e^{-kt}) + L_0(1 - e^{-k}) = e^{-k}BOD_t + L_0(1 - e^{-k})$$

$$BOD_{t+1} = a BOD_t + b \quad \text{where } a = e^{-k}, \text{ and } b = L_0(1 - e^{-k})$$

5.16 BOD₅=180mg/L, k=0.22/day, TKN=30mg/L:

$$a. \text{ CBOD} = L_0 = \frac{\text{BOD}_t}{(1 - e^{-kt})} = \frac{180}{(1 - e^{-0.22 \times 5})} = 270 \text{ mg/L}$$

$$b. \text{ NBOD} = 4.57 \times \text{TKN} = 4.57 \times 30 \text{ mg/L} = 137 \text{ mg/L}$$

$$c. \text{ BOD}_{\text{remaining}} = (270 + 137) - 180 = 227 \text{ mg/L}$$

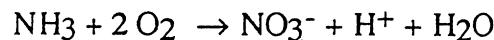
5.17 C₆H₁₅O₆N + 6 O₂ → 6 CO₂ + 6 H₂O + NH₃

$$\text{mol wt algae} = 6 \times 12 + 15 + 6 \times 16 + 14 = 197 \text{ g/mol}$$

a. 6 moles of O₂ (6x2x16=192 g) oxidizes 1 mole (197 g) of algae

$$\text{carbonaceous ThOD} = \frac{10 \text{ mg algae}}{L} \times \frac{192 \text{ g O}_2}{197 \text{ g algae}} = 9.75 \text{ mg/L}$$

b. Nitrogenous portion:



1 mole NH₃ (17g) needs 2 moles O₂ (64g) so,

$$\text{NBOD} = \frac{10 \text{ mg algae}}{L} \times \frac{17 \text{ mg NH}_3}{197 \text{ mg algae}} \times \frac{64 \text{ mg O}_2}{17 \text{ mg NH}_3} = 3.25 \text{ mg/L}$$

$$\text{Total theoretical oxygen demand} = 9.75 + 3.25 = 13.0 \text{ mg/L}$$

5.18 2 CH₂(NH₂)COOH + 3 O₂ → 4 CO₂ + 2 H₂O + 2 NH₃

$$\text{mol wt glycine} = 2 \times 12 + 5 \times 1 + 1 \times 14 + 2 \times 16 = 75 \text{ g/mol}$$

$$a. \text{ CBOD} = \frac{3 \text{ mol O}_2}{2 \text{ mol glycine}} \times \frac{32 \text{ g O}_2 / \text{ mol}}{75 \text{ g glycine/mol}} \times \frac{200 \text{ mg glycine}}{L} = 128 \text{ mg/L}$$

b. Nitrogenous portion: NH₃ + 2 O₂ → NO₃⁻ + H⁺ + H₂O

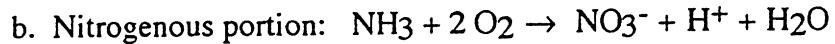
$$\text{NBOD} = \frac{2 \text{ mol O}_2}{1 \text{ mol NH}_3} \times \frac{2 \text{ mol NH}_3}{1 \text{ mol glycine}} \times \frac{32 \text{ g / mol O}_2}{75 \text{ g/mol glycine}} \times \frac{200 \text{ mg glycine}}{L} = 171 \text{ mg/L}$$

$$c. \text{ Total theoretical oxygen demand} = 128 + 171 = 299 \text{ mg/L}$$

5.19 C₈H₁₂O₃N₂ + 8 O₂ → 8 CO₂ + 3 H₂O + 2 NH₃

$$\text{mol wt casein} = 8 \times 12 + 12 \times 1 + 3 \times 16 + 2 \times 14 = 184 \text{ g/mol}$$

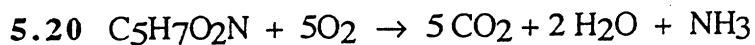
a. CBOD = $\frac{8\text{ mol O}_2}{\text{mol casein}} \times \frac{32\text{ g O}_2 / \text{mol}}{184\text{ g casein/mol}} \times \frac{200\text{ mg casein}}{\text{L}} = 278\text{ mg/L}$



$$\text{NBOD} = \frac{2\text{ mol O}_2}{1\text{ mol NH}_3} \times \frac{2\text{ mol NH}_3}{1\text{ mol casein}} \times \frac{32\text{ g/mol O}_2}{184\text{ g/mol casein}} \times \frac{200\text{ mg casein}}{\text{L}} = 139\text{ mg/L}$$

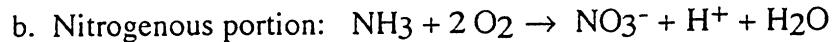
c. Total BOD = $278 + 139 = 417\text{ mg/L}$

d. $\text{BOD}_5 = \text{CBOD}(1 - e^{-kt}) = 278(1 - e^{-0.15 \times 5}) = 198\text{ mg/L}$



mol wt cells = $5 \times 12 + 7 \times 1 + 2 \times 16 + 14 = 113\text{ g/mol}$

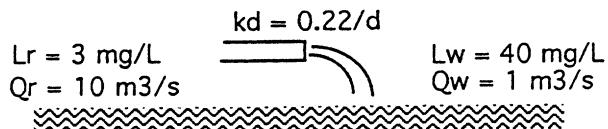
a. CBOD = $\frac{5\text{ mol O}_2}{\text{mol cells}} \times \frac{32\text{ g O}_2 / \text{mol}}{113\text{ g cells/mol}} \times 1\text{ g cells} = 1.416\text{ g O}_2$



$$\text{NBOD} = \frac{2\text{ mol O}_2}{1\text{ mol NH}_3} \times \frac{1\text{ mol NH}_3}{1\text{ mol cells}} \times \frac{32\text{ g/mol O}_2}{113\text{ g/mol cells}} \times 1\text{ g cells} = 0.566\text{ g O}_2$$

c. Total BOD = $1.416 + 0.566 = 1.98\text{ g O}_2 \text{ per g cells}$

5.21



a. just downstream:

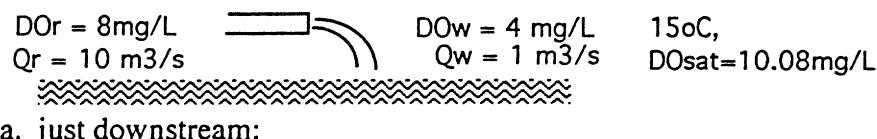
$$L_0 = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r} = \frac{1\text{ m}^3/\text{s} \times 40\text{ mg/L} + 10\text{ m}^3/\text{s} \times 3\text{ mg/L}}{10 + 1\text{ m}^3/\text{s}} = 6.4\text{ mg/L}$$

b. at 10,000 m downstream:

$$t = \frac{\text{distance}}{\text{speed}} = \frac{10,000\text{ m}}{1\text{ m}^3/\text{s} / 55\text{ m}^2} \times \frac{\text{hr}}{3600\text{ s}} \times \frac{\text{day}}{24\text{ hr}} = 0.578\text{ days}$$

$$L_t = L_0 e^{-kt} = 6.4 e^{-0.22/d \times 0.578d} = 5.6\text{ mg/L}$$

5.22



a. just downstream:

$$DO = \frac{4.0 \text{ mg/L} \times 1 \text{ m}^3/\text{s} + 8 \text{ mg/L} \times 10 \text{ m}^3/\text{s}}{1 + 10 \text{ m}^3/\text{s}} = 7.64 \text{ mg/L}$$

From Table 5.11, DO_{sat} = 10.08 mg/L

$$\text{Initial deficit } D_O = DO_{sat} - DO = 10.08 - 7.64 = 2.44 \text{ mg/L}$$

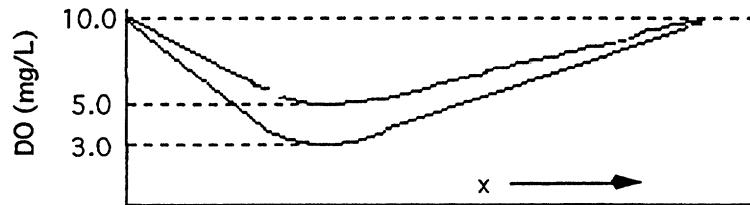
5.23 Notice when $D_0 = 0$ then $D = \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t})$ that is, the deficit is proportional to the initial BOD. Doubling L_0 will therefore double the deficit at every point.

Before: maximum deficit $D_{max} = DO_{sat} - DO_{min} = 10.0 - 6.0 = 4 \text{ mg/L}$

After: maximum deficit $D_{max} = 2 \times 4 \text{ mg/L} = 8 \text{ mg/L}$

$$\text{now } DO_{min} = DO_{sat} - D_{max} = 10.0 - 8.0 = 2.0 \text{ mg/L}$$

5.24



As in Prob. 5.23, deficit is proportional to BOD:

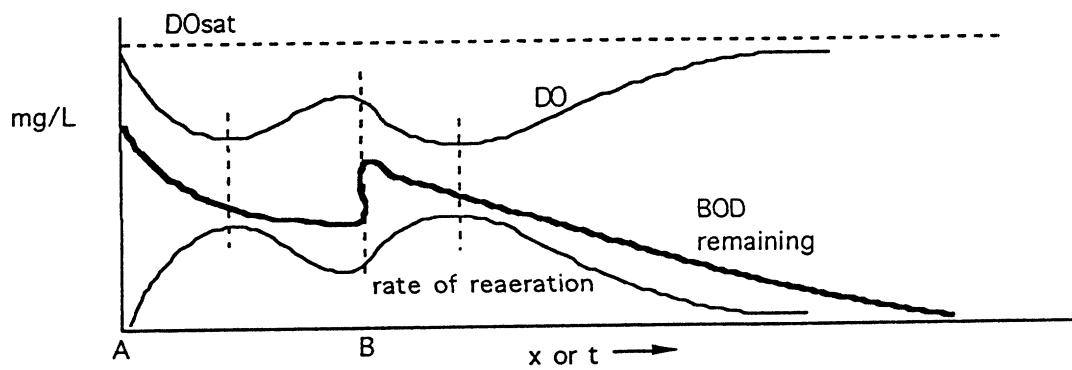
$$\text{original } D_{max} = 10.0 - 3.0 = 7.0 \text{ mg/L}$$

$$\text{desired } D_{max} = 10.0 - 5.0 = 5.0 \text{ mg/L}$$

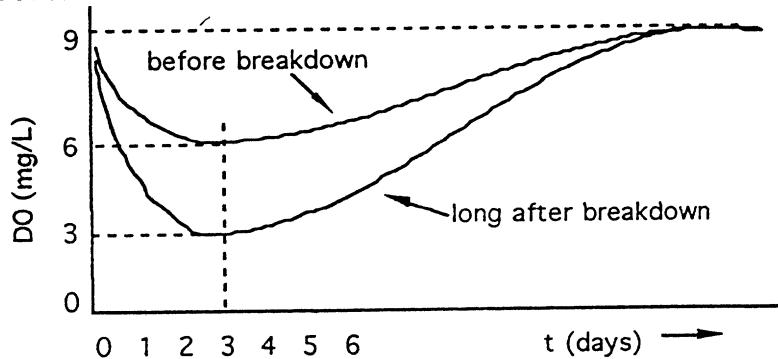
$$\frac{\text{desired } D_{max}}{\text{original } D_{max}} = \frac{5.0 \text{ mg/L}}{7.0 \text{ mg/L}} = 0.71$$

Therefore, need to remove $1.0 - 0.71 = 0.29 = 29\%$ of initial BOD.

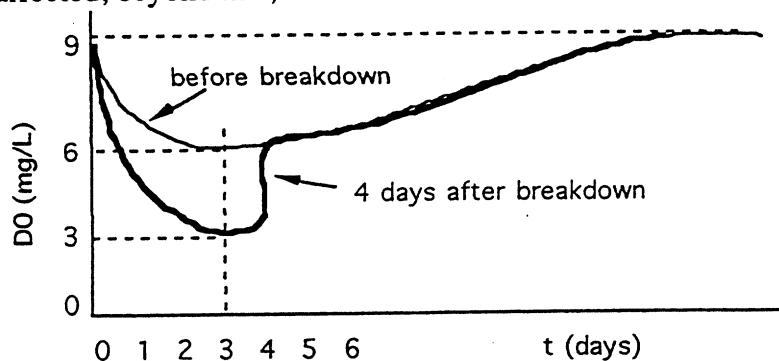
5.25



- 5.26 a. Long after treatment plant breaks down, deficit is doubled since the BOD is doubled.



- b. Only 4 days after the breakdown, the first 4-days beyond the outfall have been affected, beyond that, the DO is same as before the breakdown:



- 5.27 a. The minimum DO of 3.0 mg/L means the maximum deficit (before fixing it) is $D_{max} = 10 - 3 = 7 \text{ mg/L}$. For healthy conditions, we want DO_{min} to be 5 mg/L so that means we want $D_{max(\text{new})} = 10 - 5 = 5 \text{ mg/L}$.

$$\frac{D_{max,want}}{D_{max,have}} = \frac{5.0 \text{ mg/L}}{10.0 - 3.0 \text{ mg/L}} = 0.71,$$

so, we need to remove 29% of the BOD. Since a primary treatment plant removes about 35% of the BOD (Chapter 6), it should do the job.

- b. Using (5.31) we can find the critical time and distance downstream:

$$t_c = \frac{1}{k_r - k_d} \ln\left(\frac{k_r}{k_d}\right) = \frac{1}{0.80 - 0.20/d} \ln\left(\frac{0.8}{0.2}\right) = 2.31 \text{ days}$$

$$\text{distance} = 60 \text{ mi/day} \times 2.31 \text{ days} = 138.6 \text{ miles}$$

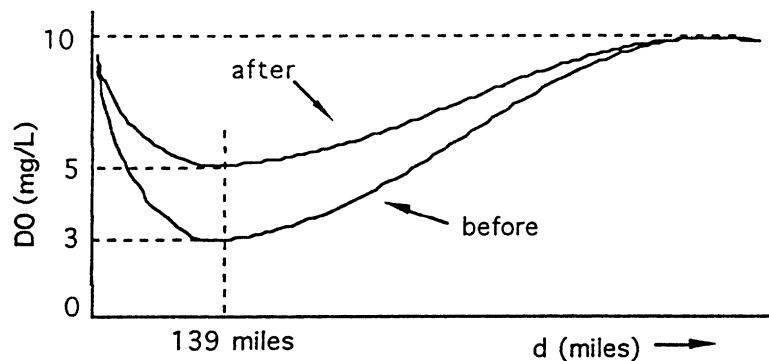
- c. What ultimate BOD to assure 5 mg/L (Eqn. 5.27):

$$D_{max} = \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t})$$

$$5 \text{ mg/L} = \frac{0.2 L_0}{0.8 - 0.2} (e^{-0.2 \times 2.31} - e^{-0.8 \times 2.31}) = 0.157 L_0$$

$$L_0 = \frac{5}{0.157} = 31.7 \text{ mg/L}$$

d.



5.28 a. $t_c = \frac{1}{k_r - k_d} \ln\left(\frac{k_r}{k_d}\right) = \frac{1}{0.90 - 0.30/d} \ln\left(\frac{0.9}{0.3}\right) = 1.83 \text{ days}$

$$\text{assuming } x = v t: \text{ distance} = 48.0 \text{ miles/day} \times 1.83 \text{ days} = 87.9 \text{ miles}$$

b. $D_{max} = \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t})$

$$= \frac{0.3/d \times 50.0 \text{ mg/L}}{(0.9 - 0.3)/d} (e^{-0.3 \times 1.83} - e^{-0.9 \times 1.83}) = 9.6 \text{ mg/L}$$

$$DO_{min} = DO_{sat} - D_{max} = 10.0 - 9.6 = 0.4 \text{ mg/L}$$

- c. when $D_O = 0$, D is proportional to L_O :

$$\frac{D_{\max, \text{want}}}{D_{\max, \text{have}}} = \frac{10.0 - 5.0}{9.6} = 0.52$$

therefore, need to remove $1.0 - 0.52 = 48\%$ of the BOD.

- 5.29** Now the river has an initial deficit that is not 0, so we can't use the simple proportionality between L_0 and D_{\max} .

a.

$$t_c = \frac{1}{k_r - k_d} \ln \left(\frac{k_r}{k_d} \left[1 - \frac{D_0(k_r - k_d)}{k_d L_0} \right] \right) = \frac{1}{0.90 - 0.30/d} \ln \left(\frac{0.9}{0.3} \left[1 - \frac{2(0.9 - 0.3)}{0.3 \times 50} \right] \right) = 1.69 \text{ days}$$

$$\text{critical distance} = 48 \text{ mi/day} \times 1.69 \text{ days} = 81.2 \text{ miles}$$

$$\begin{aligned} b. \quad D_{\max} &= \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_0 e^{-k_r t} \\ &= \frac{0.3/d \times 50.0 \text{ mg/L}}{(0.9 - 0.3)/d} (e^{-0.3 \times 1.69} - e^{-0.9 \times 1.69}) + 2.0 e^{-0.9 \times 1.69} = 10.03 \text{ mg/L} \end{aligned}$$

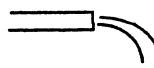
$$DO_{\min} = 10.0 - 10.03 < 0 \text{ anaerobic, } \therefore DO_{\min} = 0$$

5.30

$$DO_r = 7.6 \text{ mg/L}$$

$$L_r = 3.6 \text{ mg/L}$$

$$Q_r = 250 \text{ cfs}$$



$$DO_w = 1.8 \text{ mg/L}$$

$$L_w = 28 \text{ mg/L}$$

$$Q_w = 37 \text{ cfs}$$

$$u = 1.2 \text{ ft/s} \quad DO_{\text{sat}} = 8.5 \text{ mg/L} \quad kr = 0.76/d \quad kd = 0.61/d$$

- a. initial conditions:

$$DO = \frac{37 \text{ cfs} \times 1.8 \text{ mg/L} + 250 \text{ cfs} \times 7.6 \text{ mg/L}}{37 + 250 \text{ cfs}} = 6.85 \text{ mg/L}$$

$$\text{Initial deficit} = D_0 = 8.5 \text{ mg/L} - 6.85 \text{ mg/L} = 1.65 \text{ mg/L}$$

$$\text{Initial BOD} = L_0 = \frac{37 \text{ cfs} \times 28 \text{ mg/L} + 250 \text{ cfs} \times 3.6 \text{ mg/L}}{37 + 250 \text{ cfs}} = 6.75 \text{ mg/L}$$

- b. critical point:

$$t_c = \frac{1}{k_r - k_d} \ln \left(\frac{k_r}{k_d} \left[1 - \frac{D_0(k_r - k_d)}{k_d L_0} \right] \right)$$

$$= \frac{1}{0.76 - 0.61/d} \ln \left(\frac{0.76}{0.61} \left[1 - \frac{1.65(0.76 - 0.61)}{0.61 \times 6.75} \right] \right) = 1.05 \text{ days}$$

$$\text{critical distance } = x_c = 1.2 \frac{\text{ft}}{\text{s}} \times 3600 \frac{\text{s}}{\text{hr}} \times 24 \frac{\text{hr}}{\text{d}} \times 5280 \frac{\text{ft}}{\text{mi}} \times 1.05 \text{ day} = 20.7 \text{ miles}$$

c. minimum DO:

$$\begin{aligned} D_{\max} &= \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_0 e^{-k_r t} \\ &= \frac{0.61/d \times 6.75 \text{ mg/L}}{(0.76 - 0.61)/d} (e^{-0.61 \times 1.05} - e^{-0.76 \times 1.05}) + 1.65 e^{-0.76 \times 1.05} = 2.85 \text{ mg/L} \end{aligned}$$

$$DO_{\min} = DO_{\text{sat}} - D_{\max} = 8.5 - 2.85 = 5.6 \text{ mg/L}$$

d. 10 miles downstream:

$$t = \frac{10 \text{ mi} \times 5280 \text{ ft/mi}}{1.2 \text{ ft/s} \times 3600 \text{ s/hr} \times 24 \text{ hr/d}} = 0.51 \text{ days}$$

$$\begin{aligned} D &= \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_0 e^{-k_r t} \\ &= \frac{0.61 \times 6.75}{0.76 - 0.61} (e^{-0.61 \times 0.51} - e^{-0.76 \times 0.51}) + 1.65 e^{-0.76 \times 0.51} = 2.6 \text{ mg/L} \end{aligned}$$

$$DO = 8.5 - 2.6 = 5.9 \text{ mg/L}$$

5.31

$$DO_r = 6.0 \text{ mg/L}$$

$$L_r = 7.0 \text{ mg/L}$$

$$Q_r = 0.90 \text{ m}^3/\text{s}$$

$$DO_w = 1.0 \text{ mg/L}$$

$$L_w = 6.40 \text{ mg/L}$$

$$Q_w = 0.30 \text{ m}^3/\text{s}$$

$$u = 0.65 \text{ m/s} \quad DO_{\text{sat}} = 8.0 \text{ mg/L} \quad kr = 0.37/\text{d} \quad kd = 0.20/\text{d}$$

Initial conditions:

$$DO = \frac{0.3 \text{ m}^3/\text{s} \times 1.0 \text{ mg/L} + 0.9 \text{ m}^3/\text{s} \times 6 \text{ mg/L}}{0.3 + 0.9 \text{ m}^3/\text{s}} = 4.75 \text{ mg/L}$$

$$\text{Initial deficit } D_O = 8.0 - 4.75 = 3.25 \text{ mg/L}$$

$$\text{Initial BOD} = L_0 = \frac{0.3 \text{ m}^3/\text{s} \times 6.4 \text{ mg/L} + 0.9 \text{ m}^3/\text{s} \times 7.0 \text{ mg/L}}{0.3 + 0.9 \text{ m}^3/\text{s}} = 6.85 \text{ mg/L}$$

Critical time:

$$t_c = \frac{1}{k_r - k_d} \ln \left(\frac{k_r}{k_d} \left[1 - \frac{D_0(k_r - k_d)}{k_d L_0} \right] \right)$$

$$= \frac{1}{0.37 - 0.20/d} \ln \left(\frac{0.37}{0.20} \left[1 - \frac{3.25(0.37 - 0.20)}{0.20 \times 6.85} \right] \right) = 0.582 \text{ days}$$

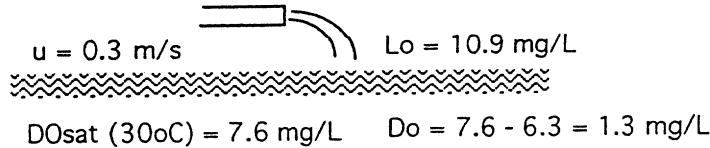
Maximum deficit:

$$D_{\max} = \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_0 e^{-k_r t}$$

$$= \frac{0.20/d \times 6.85 \text{ mg/L}}{(0.37 - 0.20)/d} (e^{-0.20 \times 0.582} - e^{-0.37 \times 0.582}) + 3.25 e^{-0.37 \times 0.582} = 3.3 \text{ mg/L}$$

$$DO_{\min} = 8.0 - 3.3 = 4.7 \text{ mg/L}$$

5.32



Need to adjust for 30°C temperature (instead of 20°C):

from (5.15),

$$k_d(30^\circ\text{C}) = k_d(20^\circ\text{C})(1.047)^{(30-20)} = 0.20/d \times (1.047)^{10} = 0.317/\text{day}$$

from page 201,

$$k_r(30^\circ\text{C}) = k_r(20^\circ\text{C})(1.024)^{(30-20)} = 0.41/d \times (1.024)^{10} = 0.520/\text{day}$$

critical time, $t_c = \frac{1}{k_r - k_d} \ln \left(\frac{k_r}{k_d} \left[1 - \frac{D_0(k_r - k_d)}{k_d L_0} \right] \right)$

$$= \frac{1}{0.520 - 0.317/d} \ln \left(\frac{0.520}{0.317} \left[1 - \frac{1.3(0.520 - 0.317)}{0.317 \times 10.9} \right] \right) = 2.05 \text{ days}$$

critical distance $= x_c = 0.3 \frac{\text{m}}{\text{s}} \times 3600 \frac{\text{s}}{\text{hr}} \times 24 \frac{\text{hr}}{\text{d}} \times 2.05 \text{ day} = 53,050 \text{ m}$
(for comparison, at 20°C , $x_c = 69,300 \text{ m}$)

maximum deficit,

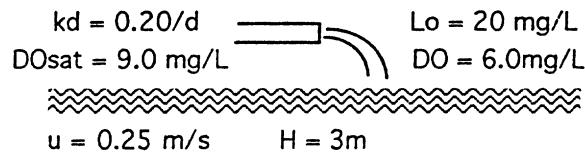
$$D_{\max} = \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_0 e^{-k_r t}$$

$$= \frac{0.317 \times 10.9}{0.520 - 0.317} (e^{-0.317 \times 2.05} - e^{-0.520 \times 2.05}) + 1.3 e^{-0.520 \times 2.05} = 3.5 \text{ mg/L}$$

$$DO_{min} = DO_{sat}(30^{\circ}C) - D_{max} = 7.6 - 3.5 = 4.1 \text{ mg/L}$$

(for comparison, at 20°C it was 6.0 mg/L)

5.33



a. Finding k_r using (5.24):

$$k_r = \frac{3.9 u^{1/2}}{H^{3/2}} = \frac{3.9 \times 0.25^{0.5}}{3^{1.5}} = 0.375 / d$$

$$\begin{aligned} \text{b. critical time, } t_c &= \frac{1}{k_r - k_d} \ln \left(\frac{k_r}{k_d} \left[1 - \frac{D_0(k_r - k_d)}{k_d L_0} \right] \right) \\ &= \frac{1}{0.375 - 0.20/d} \ln \left(\frac{0.375}{0.2} \left[1 - \frac{3.0(0.375 - 0.20)}{0.20 \times 20} \right] \right) = 2.79 \text{ days} \end{aligned}$$

c. minimum DO,

$$\begin{aligned} D_{max} &= \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_0 e^{-k_r t} \\ &= \frac{0.20 \times 20}{0.375 - 0.20} (e^{-0.20 \times 2.79} - e^{-0.375 \times 2.79}) + 3.0 e^{-0.375 \times 2.79} = 6.1 \text{ mg/L} \end{aligned}$$

$$DO_{min} = 9.0 - 6.1 = 2.9 \text{ mg/L}$$

d. What percent removal needed to get $DO_{min} = 5.0 \text{ mg/L}$

$$\frac{D_{max,want}}{D_{max,have}} = \frac{9.0 - 5.0}{6.1} = 0.66, \text{ so must remove } 1.00 - 0.66 = 0.34 = 34\%$$

5.34 Algae represented by $C_{106}H_{263}O_{110}N_{16}P$,

$$\text{mol wt} = 106 \times 12 + 263 \times 1 + 110 \times 16 + 16 \times 14 + 31 = 3550 \text{ g/mol}$$

$$C = \frac{106 \times 12}{3550} = 0.3583 = 358.3 \text{ mg}$$

$$H = \frac{263}{3550} = 0.0741 = 74.1 \text{ mg}$$

$$O = \frac{16 \times 16}{3550} = 0.4958 = 495.8 \text{ mg}$$

a. per 1 g of algae:

$$N = \frac{16 \times 14}{3550} = 0.0631 = 63.1 \text{ mg}$$

$$P = \frac{31}{3550} = 0.0087 = 8.7 \text{ mg}$$

b. with 0.10 mg N and 0.04 mg of P available,

$$\text{N allows: } \frac{0.10 \text{ mgN}}{\text{L}} \times \frac{1000 \text{ mg algae}}{63.1 \text{ mgN}} = 1.6 \text{ mg/L algae}$$

$$\text{P allows: } \frac{0.04 \text{ mgP}}{\text{L}} \times \frac{1000 \text{ mg algae}}{8.7 \text{ mgP}} = 4.6 \text{ mg/L algae}$$

So, N is limiting nutrient.

c. mass of algae can be 1.6 mg algae/L

d. cutting N by 50%

a. cutting P by 50%, would allow P to make 2.3 mg/L of algae, but N is still limiting.
So, the mass of algae is still 1.6 mg/L.

e

5.35 Given 60mg N and 10mg P per 1000 mg algae. At 0.12 mgN/L and 0.03 mgP/L available:

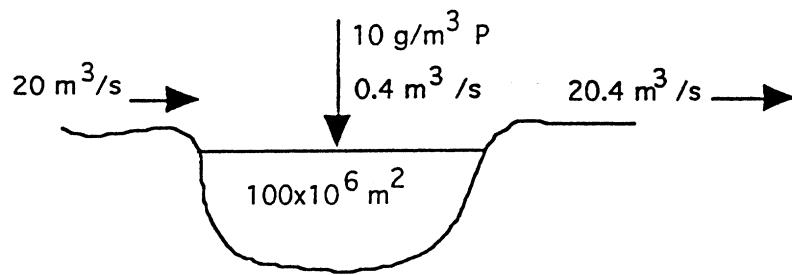
$$\frac{0.12 \text{ mgN}}{\text{L}} \times \frac{1000 \text{ mg algae}}{60 \text{ mgN}} = 2.0 \text{ mg algae/L}$$

$$\frac{0.03 \text{ mgP}}{\text{L}} \times \frac{1000 \text{ mg algae}}{10.0 \text{ mgP}} = 3.0 \text{ mg algae/L}$$

Nitrogen is the limiting nutrient.

- a. To control algal production to 1.0 mg/L means N needs to be reduced to 0.06 mgN/L, a 50% reduction.
- b. To control algal production by reducing P requires 0.01 mgP/L, a 66.6% reduction.

5.36



from (5.35), the phosphorus input allows a concentration of phosphorus of:

$$C = \frac{QC_{in} + S}{Q + v_s A} = \frac{0.4 \text{ m}^3/\text{s} \times 10 \text{ gP/m}^3}{20.4 \text{ m}^3/\text{s} + \left(\frac{10 \text{ m/yr} \times 100 \times 10^6 \text{ m}^2}{3600 \text{ s/hr} \times 24 \text{ hr/d} \times 365 \text{ d/yr}} \right)} = 0.077 \text{ g/m}^3 = 0.077 \text{ mg/L}$$

To keep phosphorus at less than 0.010 mg/L,

$$\frac{\text{Source want}}{\text{Source have}} = \frac{C \text{ want}}{C \text{ have}} = \frac{0.010 \text{ mg/L}}{0.077 \text{ mg/L}} = 0.13$$

Need to reduce the phosphorus input by $1 - 0.13 = 0.87 = 87\%$

5.37 a. porosity = $\frac{\text{void volume}}{\text{total volume}} = \frac{(50 - 35) \text{ cm}^3}{50 \text{ cm}^3} = 0.30$

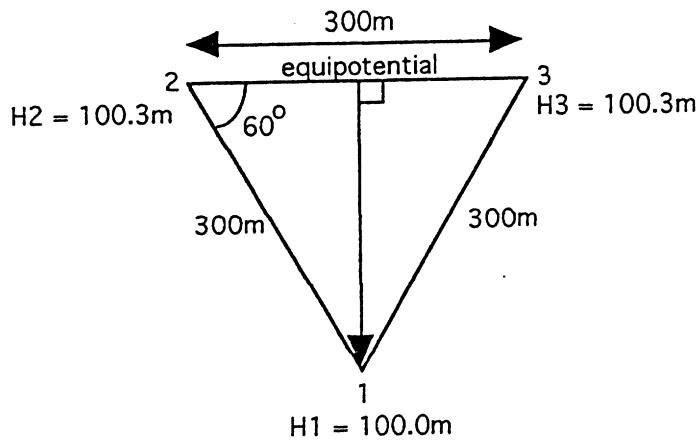
b. solids density = $\frac{100 \text{ g}}{35 \text{ cm}^3} = 2.86 \text{ g/cm}^3$

5.38 Gravel aquifer, Table 5.12, porosity = 25%, specific yield = 22%:

$$\text{yield} = \text{specific yield} \times \text{volume} = 0.22 \times 10,000 \text{ m}^2 \times 1.0 \text{ m} = 2200 \text{ m}^3$$

$$\text{fraction removed} = \frac{2200 \text{ m}^3}{\text{porosity} \times \text{volume}} = \frac{2200 \text{ m}^3}{0.25 \times 10,000 \text{ m}^2 \times 1 \text{ m}} = 0.88 = 88\%$$

5.39



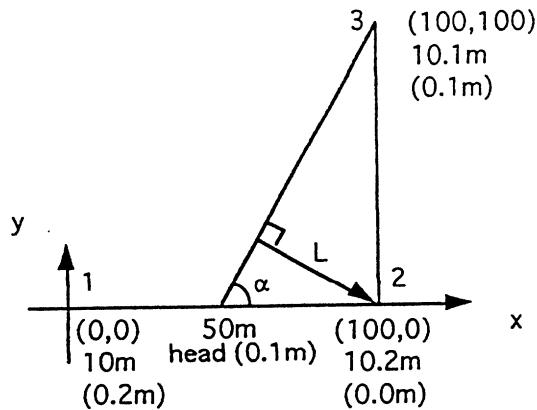
$$\text{gradient} = \frac{\Delta h}{L} = \frac{100.3 - 100.0}{300 \sin 60^\circ} = 0.00115$$

5.40 Lowest head = #2, use as datum

head #1 = 0.2m above datum

head #2 = 0 m above datum

head #3 = 0.1m above datum



Halfway between #1 and #2 head is 0.1m. Draw equipotential between #3 and that point. From geometry,

$$\alpha = \tan^{-1}\left(\frac{100}{50}\right) = 63.43^\circ$$

$$\text{gradient} = \frac{\Delta h}{L} = \frac{0.1}{50 \sin 63.43^\circ} = 0.00224$$

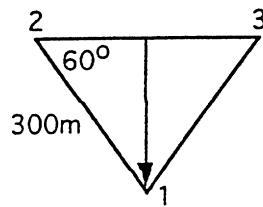
5.41 From Prob. 5.39,

grad = 0.00115, hydraulic conductivity K = 1000m/d, porosity = 0.23

a. Darcy velocity = $v = K \frac{dh}{dL} = 1000 \text{ m/d} \times 0.00115 = 1.15 \text{ m/day}$

b. avg. linear velocity $v' = \frac{\text{Darcy velocity}}{\text{porosity}} = \frac{1.15 \text{m/day}}{0.23} = 5.0 \text{m/day}$

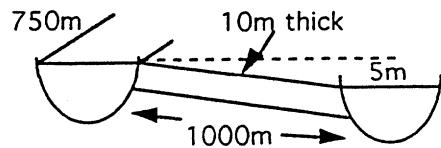
c.



without accounting for retardation, $t = \frac{\text{distance, } d}{\text{avg. velocity } v'} = \frac{300 \sin 60^\circ}{5.0 \text{m/day}} = 52 \text{days}$

with retardation factor of 2: $t = R \times 52d = 2 \times 52 \text{ days} = 104 \text{ days}$

5.42



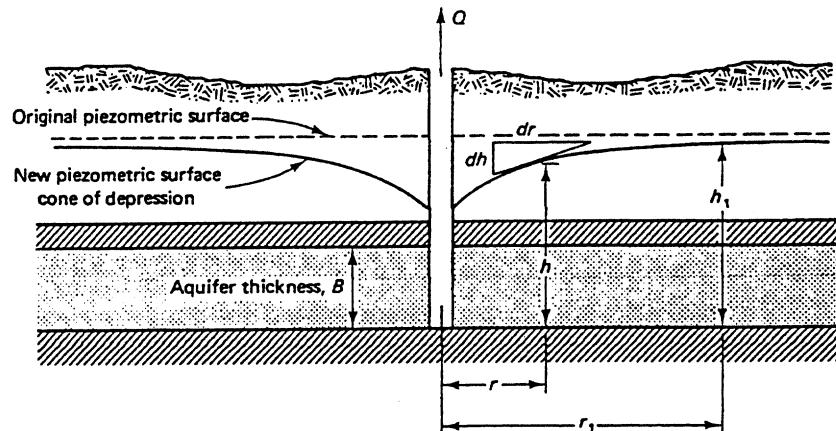
$$Q = KA \frac{dh}{dL} = 7.0 \frac{\text{m}}{\text{day}} \times (10 \text{m} \times 750 \text{m}) \times \frac{5 \text{m}}{1000 \text{m}} = 262.5 \text{m}^3/\text{day}$$

5.43 $v' = 1.0 \text{m/day}$, grad = 0.0005, porosity $\eta = 0.20$

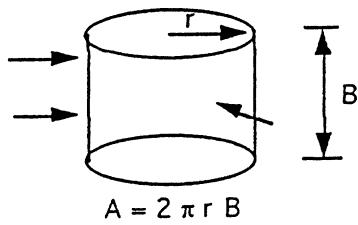
$$\text{Darcy velocity } v = K \times \text{grad}$$

$$K = \frac{v}{\text{grad}} = \frac{v' \eta}{\text{grad}} = \frac{1.0 \text{m/day} \times 0.20}{0.0005} = 400 \text{m/day}$$

5.44



$$Q = KA \frac{dh}{dL} = K2\pi rB \frac{dh}{dr}$$

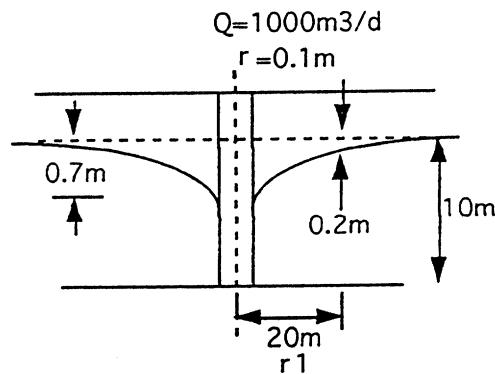


$$\int_{r_1}^{r_2} Q \frac{dr}{r} = K2\pi B \int_{h_1}^{h_2} dh$$

$$Q \ln\left(\frac{r_1}{r}\right) = 2\pi KB(h_1 - h)$$

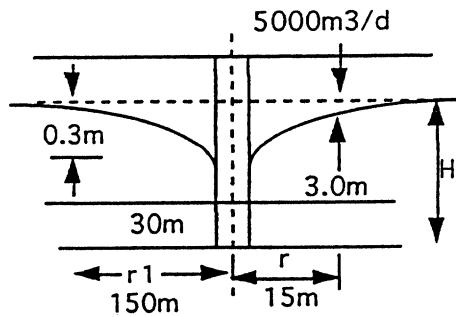
$$Q = \frac{2\pi KB(h_1 - h)}{\ln(r_1/r)}$$

5.45



$$K = \frac{Q \ln(r_1/r)}{\pi(h_1^2 - h^2)} = \frac{1000\text{m}^3/\text{d} \ln(20/0.1)}{\pi(9.8^2 - 9.3^2)\text{m}^2} = 176.6\text{m/d}$$

5.46



$$Q = \frac{2\pi KB(h_1 - h)}{\ln\left(\frac{r_1}{r}\right)} \quad (5.57)$$

$$K = \frac{Q \ln\left(\frac{r_1}{r}\right)}{2\pi B(h_1 - h)} = \frac{5000 \text{ m}^3/\text{d} \ln\left(\frac{150}{15}\right)}{2\pi 30[(H-0.3)-(H-3.0)] \text{ m}^2} = 22.6 \text{ m/d}$$

5.47

$$Q = 2\pi rBK\left(\frac{dh}{dr}\right)$$

$$v' = -\frac{K}{\eta} \frac{dh}{dr} = \frac{dr}{dt}, \text{ that is, } \frac{dh}{dr} = -\frac{\eta}{K} \left(\frac{dr}{dt}\right)$$

$$Q = -2\pi rBK\left(\frac{\eta}{K} \frac{dr}{dt}\right) = -2\pi rB\eta\left(\frac{dr}{dt}\right)$$

$$\int_0^t Q dt = -2\pi B\eta \int_R^{r_w} r dr = -2\pi B\eta \left(\frac{r^2}{2}\right)_R^{r_w} = \pi \eta B(R^2 - r_w^2)$$

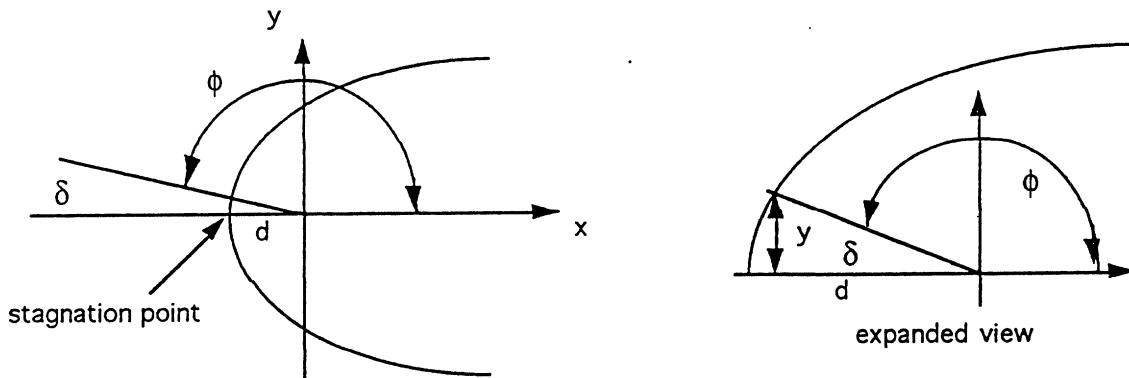
$$Qt = \pi \eta B(R^2 - r_w^2)$$

$$t = \frac{\pi \eta B(R^2 - r_w^2)}{Q} \quad \text{Q.E.D.}$$

5.48 From Prob. 5.46: $B=30\text{m}$, $Q=5000\text{m}^3/\text{d}$, $R=15\text{m}$, $r_w=0.2\text{m}$, $\eta=0.30$.

$$t = \frac{\pi 0.30 \times 30 \text{ m} (15^2 - 0.2^2) \text{ m}^2}{5000 \text{ m}^3 / \text{d}} = 1.27 \text{ days}$$

5.49 Show $d = Q/(2Bv\pi)$



$$y = \frac{Q}{2Bv} \left(1 - \frac{\phi}{\pi}\right) \quad (5.60)$$

from expanded view,

$$y = \frac{Q}{2Bv} \left[1 - \left(\frac{\pi - \delta}{\pi} \right) \right] = \frac{Q\delta}{2Bv\pi}$$

$$\left(\frac{y}{d} \right) = \tan \delta \approx \delta \text{ for small values of } \delta$$

Therefore, for small δ , $y \approx \frac{Q}{2Bv\pi} \left(\frac{y}{d} \right)$

$$\text{so, } d = \frac{Q}{2Bv\pi}, \text{ that is, } x = -\frac{Q}{2Bv\pi} \quad \text{Q.E.D.}$$

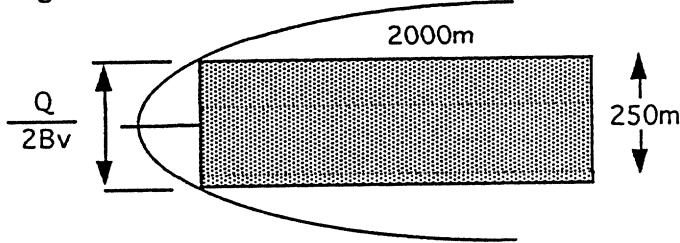
5.50 0.1m³ TCE plume, B=10m, 2000m x 250m, grad=0.001, K=0.001m/s, h=0.4:

- a. Can it dissolve? Using the specific gravity given in Table 5.15, the concentration C of TCE in the plume (if it could all dissolve) would be

$$C = \frac{0.1\text{m}^3 \times 1.47 \text{ kg/L} \times 10^3 \text{ L/m}^3 \times 10^6 \text{ mg/kg}}{2000\text{m} \times 250\text{m} \times 10\text{m} \times 0.40 \times 10^3 \text{ L/m}^3} = \frac{1.47 \times 10^8 \text{ mg}}{2 \times 10^9 \text{ L}} = 0.073 \text{ mg/L}$$

Since the solubility (Table 5.15) is 1100 mg/L, YES it could all dissolve.

- b. Try a single well:



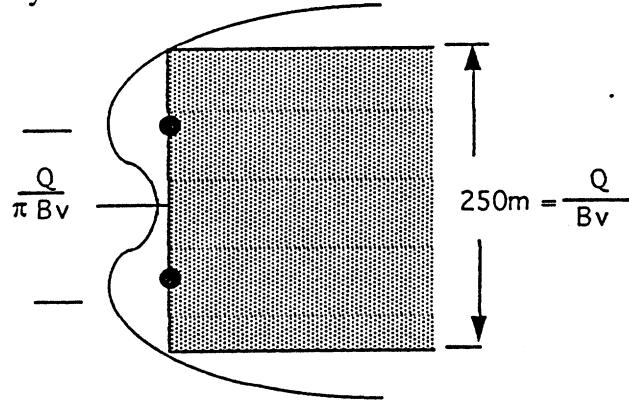
$$\frac{Q}{2Bv} = 250\text{m}$$

$$v = K \frac{dh}{dL} = 0.001\text{m/s} \times 0.001 = 1 \times 10^{-6}\text{m/s}$$

$$Q = 250 \times 2Bv = 250\text{m} \times 2 \times 10\text{m} \times 1 \times 10^{-6}\text{m/s} = 0.005\text{m}^3/\text{s}$$

this exceeds the maximum pumping rate, which is given as 0.003 m³/s.

Try 2 wells:



$$\frac{Q}{Bv} = 250 \text{ m}$$

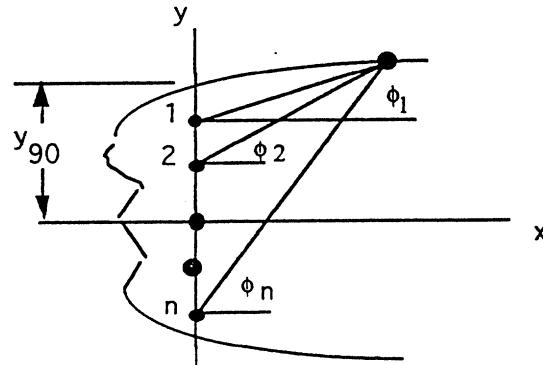
$$Q = 250 \text{ m} \times 10 \text{ m} \times 1 \times 10^{-6} \text{ m/s} = 0.0025 \text{ m}^3/\text{s} < 0.003 \text{ m}^3/\text{s}$$

so, 2 wells will work.

c. spacing for minimum pumping rate:

$$\text{spacing} = \frac{Q}{\pi Bv} = \frac{0.0025 \text{ m}^3/\text{s}}{\pi \times 10 \text{ m} \times 10^{-6} \text{ m/s}} = 79.6 \approx 80 \text{ m apart}$$

5.51 Show width of capture zone for n wells is $nQ/(2Bv)$.



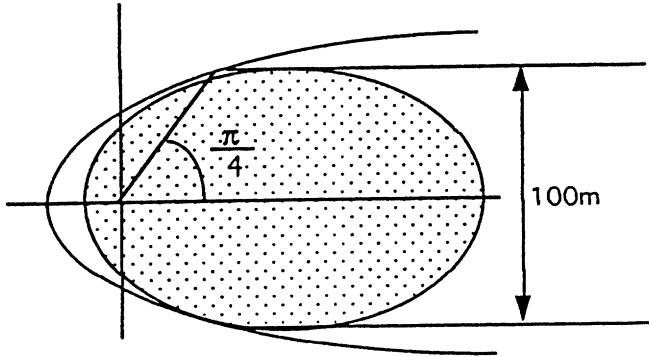
$$\text{Eqn. 5.61: } y = \frac{Q}{2Bv} \left(n - \frac{1}{\pi} \sum_{i=1}^n \phi_i \right)$$

As shown in the figure, when $x = 0$, call the y value of the capture zone curve y_{90} . For $x = 0$, and $y = y_{90}$, and n wells, all n of the ϕ_i are $\pi/2$, so

$$y_{90} = \frac{Q}{2Bv} \left[n - \frac{1}{\pi} \left(\frac{\pi}{2} + \frac{\pi}{2} + \dots + \frac{\pi}{2} \right) \right] = \frac{Q}{2Bv} \left[n - \frac{1}{\pi} \left(\frac{n\pi}{2} \right) \right] = \frac{Q}{2Bv} \left(\frac{n}{2} \right) = \frac{Qn}{4Bv}$$

The width of the capture zone along the y -axis is just $2x y_{90} = Qn/(2Bv)$ Q.E.D.

5.52 $B=20\text{m}$, $h = 0.30$, $K = 10^{-4}\text{m/s}$, grad = 0.0015



starting with (5.60),

$$y = \frac{Q}{2Bv} \left[1 - \frac{\phi}{\pi} \right] = \frac{Q}{2Bv} \left(1 - \frac{\pi/4}{\pi} \right) = \frac{3Q}{8Bv}$$

$$Q = \frac{8Bvy}{3} = \frac{8}{3} \times 20\text{m} \times 1 \times 10^{-4}\text{m/s} \times 0.0015 \times \left(\frac{100}{2}\right)\text{m} = 0.0004\text{m}^3/\text{s}$$

5.53 1 m^3 of a sand-gravel aquifer, 20L of tetrachloroethylene, 20% of solubility,

- a. From Table 5.12 the porosity is estimated at 0.20, from Table 5.15 the solubility is given as 150 mg/L. Since the amount dissolved is only 20% of its solubility:

$$\text{dissolved perc} = 0.20 \times 150 \text{ mg/L} \times 1 \text{ m}^3 \times 10^3 \text{ L/m}^3 \times 0.20 = 6000 \text{ mg} = 6\text{g}$$

- b. To find remaining tetrachloroethylene, use the sp. gr. from Table 5.15 of 1.63,

$$\text{total mass} = 20 \text{ L} \times 1.63 \text{ kg/L} \times 10^3 \text{ g/kg} = 32,600\text{g}$$

$$\text{undissolved mass} = 32,600 - 6 = 32,594 \text{ g} \text{ (almost all not dissolved)}$$

- c. gradient = 0.001, porosity = 0.20,

$$\text{average linear velocity} = \frac{K \left(\frac{dh}{dl} \right)}{\eta} = \frac{410 \text{ m/d} \times 0.001}{0.20} = 2.05 \approx 2.1 \text{ m/day}$$

- d. using a 1 m^2 cross section for our cubic meter of aquifer gives, and the 6 g/m^3 contaminant concentration, gives,

$$\text{aquifer flow rate} = 2.05 \text{ m/d} \times 1 \text{ m}^2 = 2.05 \text{ m}^3/\text{d}$$

$$\text{contaminant removal rate} = 2.05 \text{ m}^3/\text{d} \times 6 \text{ g/m}^3 = 12.3 \text{ g/day}$$

to remove the total 32,600 g would take:

$$t = \frac{32,600\text{g}}{12.3\text{g/day} \times 365\text{day/yr}} = 7.3 \text{ years}$$

SOLUTIONS FOR CHAPTER 6

6.1 Hardness in meq/L:

$$\text{Ca}^{2+} = \frac{150 \text{mg/L}}{20.05 \text{mg/meq}} = 7.5 \text{meq/L}$$

$$\text{Mg}^{2+} = \frac{60 \text{mg/L}}{12.15 \text{mg/meq}} = 4.9 \text{meq/L}$$

$$\text{Total hardness} = 7.5 + 4.9 = 12.4 \text{ meq/L}$$

Hardness as CaCO_3 :

$$\text{Hardness} = 12.4 \frac{\text{meq}}{\text{L}} \times 50 \frac{\text{mg as CaCO}_3}{\text{meq}} = 622 \text{mg/L as CaCO}_3$$

Table 6.5 would classify this as *very hard water*.

6.2 $\text{pH} = 9$,

$$[\text{H}^+] = 10^{-\text{pH}} = 1 \times 10^{-9} \text{ mol/L}$$

$$\begin{aligned} (\text{H}^+) &= \frac{1 \times 10^{-9} \text{ mol}}{\text{L}} \times \frac{1 \text{ g}}{\text{mol}} \times \frac{10^3 \text{ mg}}{\text{g}} \times \frac{\text{meq}}{1 \text{ mg}} = 1 \times 10^{-6} \text{ meq/L} \\ &= 10^{-6} \frac{\text{meq}}{\text{L}} \times 50 \frac{\text{mg CaCO}_3}{\text{meq}} = 5 \times 10^{-5} \text{ mg/L as CaCO}_3 \end{aligned}$$

$$[\text{OH}^-] = \frac{10^{-14}}{[\text{H}^+]} = \frac{10^{-14}}{10^{-9}} = 10^{-5} \text{ mol/L}$$

$$(\text{OH}^-) = \frac{10^{-5} \text{ mol}}{\text{L}} \times \frac{17 \text{ g}}{\text{mol}} \times \frac{10^3 \text{ mg}}{\text{g}} \times \frac{\text{meq}}{17 \text{ mg}} = 10^{-2} \text{ meq/L}$$

$$= 10^{-2} \frac{\text{meq}}{\text{L}} \times 50 \frac{\text{mg CaCO}_3}{\text{meq}} = 0.5 \text{ mg/L as CaCO}_3$$

6.3 First, ignoring the contribution of H^+ and OH^- , with $\text{pH}=10.5$, 39.0mg/L of CO_3^{2-} and 24.5 mg/L of HCO_3^- ,

$$\text{a. } \text{CO}_3^{2-} = \frac{(12 + 3 \times 16)}{2} = 30.0 \frac{\text{mg}}{\text{meq}}$$

$$(\text{CO}_3^{2-}) = \frac{39.0 \text{mg/L}}{30 \text{mg/meq}} \times \frac{50 \text{mg of CaCO}_3}{\text{meq}} = 65 \text{ mg/L as CaCO}_3$$

$$\text{HCO}_3^- = \frac{(1+12+3 \times 16)}{1} = 61.0 \frac{\text{mg}}{\text{meq}}$$

$$(\text{HCO}_3^-) = \frac{24.5 \text{ mg/L}}{61.0 \text{ mg/meq}} \times \frac{50 \text{ mg of CaCO}_3}{\text{meq}} = 20.1 \text{ mg/L as CaCO}_3$$

Ignoring H⁺ and OH⁻, alkalinity = (HCO₃⁻) + (CO₃²⁻)

$$= 20.1 + 65.0 = 85.1 \text{ mg/L as CaCO}_3$$

b. Including H⁺ and OH⁻,

$$\text{pH} = 10.5, [\text{H}^+] = 10^{-10.5} = 3.16 \times 10^{-11} \text{ mol/L}$$

$$(\text{H}^+) = \frac{3.16 \times 10^{-11} \text{ mol}}{\text{L}} \times \frac{1 \text{ g}}{\text{mol}} \times \frac{10^3 \text{ mg}}{\text{g}} \times \frac{1 \text{ meq}}{\text{mg}} \times \frac{50 \text{ mg CaCO}_3}{\text{meq}}$$

$$= 1.5 \times 10^{-6} \text{ mg/L as CaCO}_3$$

$$\text{pH} = 10.5, [\text{OH}^-] = \frac{10^{-14}}{10^{-10.5}} = 3.16 \times 10^{-4} \text{ mol/L}$$

$$(\text{OH}^-) = \frac{3.16 \times 10^{-4} \text{ mol}}{\text{L}} \times \frac{17 \text{ g}}{\text{mol}} \times \frac{10^3 \text{ mg}}{\text{g}} \times \frac{\text{meq}}{17 \text{ mg}} \times \frac{50 \text{ mg CaCO}_3}{\text{meq}}$$

$$= 15.8 \text{ mg/L as CaCO}_3$$

$$\text{alkalinity} = (\text{HCO}_3^-) + (\text{CO}_3^{2-}) + (\text{OH}^-) - (\text{H}^+)$$

$$= 20.1 + 65.0 + 15.8 - 1.5 \times 10^{-6} = 100.9 \text{ mg/L as CaCO}_3$$

6.4 a. alkalinity = (HCO₃⁻) + (CO₃²⁻) + (OH⁻) - (H⁺),

but pH near neutral so (H⁺), (OH⁻), and (CO₃²⁻) are negligible,

$$(\text{HCO}_3^-) = \frac{165 \text{ mg/L}}{61.0 \text{ mg/meq}} \times \frac{50 \text{ mg of CaCO}_3}{\text{meq}} = 135.2 \text{ mg/L as CaCO}_3$$

$$\text{alkalinity} = 135.2 \text{ mg/L as CaCO}_3$$

b. hardness = (Ca²⁺) + (Mg²⁺)

$$= \left(\frac{90 \text{ mg}}{\text{L}} \times \frac{\text{meq}}{20 \text{ mg}} + \frac{30 \text{ mg}}{\text{L}} \times \frac{\text{meq}}{12.2 \text{ mg}} \right) \times \frac{50 \text{ mg of CaCO}_3}{\text{meq}} = 348 \text{ mg/L as CaCO}_3$$

c. total dissolved solids (TDS):

$$\text{TDS} = 90 + 30 + 72 + 6 + 120 + 225 + 165 = 708 \text{ mg/L}$$

6.5 Questionable Cl⁻

Ions	mg/L	mg/meq	meq/L
Ca ²⁺	90	20.0	4.5
Mg ²⁺	30	12.2	2.46
Na ⁺	72	23.0	3.13
K ⁺	6	39.1	0.15
		Total =	10.24
Cl ⁻	(Cl ⁻)	35.5	(Cl ⁻)/35.5
SO ₄ ²⁻	225	48.0	4.69
HCO ₃ ⁻	165	61.0	2.70
		Total =	7.39 + (Cl ⁻)/35.5

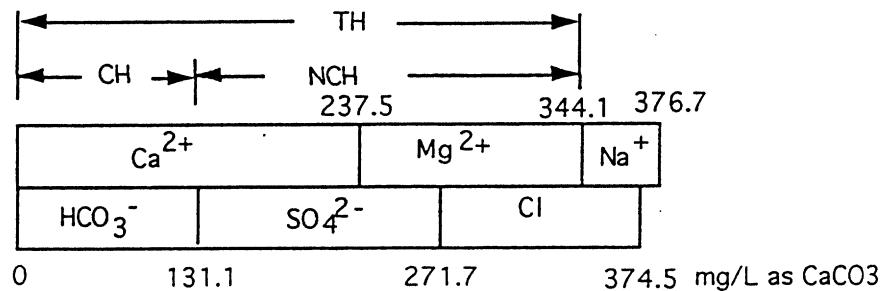
to balance, 10.24 = 7.39 + (Cl⁻)/35.5

Cl⁻ = 101 mg/L so there probably was an error.

6.6 Water analysis:

Ion	mg/L	mg/meq	meq/L	mg/L as CaCO ₃
Ca ²⁺	95	20.0	4.75	237.5
Mg ²⁺	26	12.2	2.13	106.6
Na ⁺	15	23.0	0.65	32.6
HCO ₃ ⁻	160	61.0	2.63	131.1
SO ₄ ²⁻	135	48.0	2.81	140.6
Cl ⁻	73	35.5	2.06	102.8

- total hardness (TH) = (Ca²⁺) + (Mg²⁺) = 237.5 + 106.6 = 344.1 mg/L as CaCO₃
- carbonate hardness (CH) = (HCO₃⁻) = 131.1 mg/L as CaCO₃
- noncarbonate hardness (NCH) = 344.1 - 131.1 = 213 mg/L as CaCO₃
- alkalinity = (HCO₃⁻) = 131.1 mg/L as CaCO₃ (since pH near neutral)
- TDS = 95 + 26 + 15 + 160 + 135 + 73 = 504 mg/L



6.7 Water analysis:

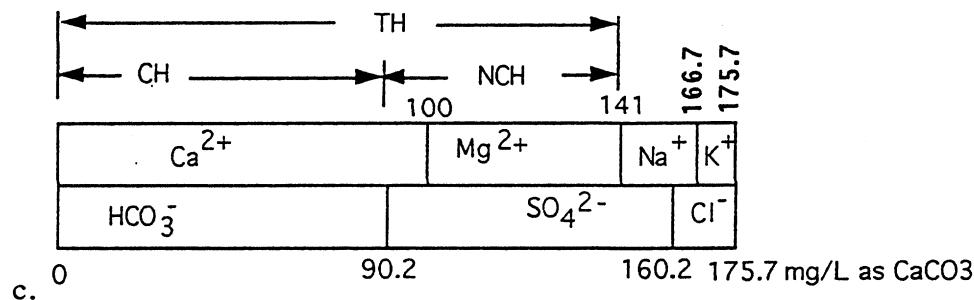
<u>Ion</u>	<u>mg/L</u>	<u>mg/meq</u>	<u>meq/L</u>	<u>mg/L as CaCO₃</u>
Ca ²⁺	40.0	20.0	2.0	100.0
Mg ²⁺	10.0	12.2	0.82	41.0
Na ⁺	X?	23.0	Y?	Z?
K ⁺	7.0	39.1	0.18	9.0
HCO ₃ ⁻	110.0	61.0	1.8	90.2
SO ₄ ²⁻	67.2	48.0	1.4	70.0
Cl ⁻	11.0	35.5	0.31	15.5

a. Equivalents balance to find Na concentration:

$$100 + 41 + Z + 9 = 90.2 + 70 + 15.5 \quad Z = 25.7 \text{ mg/L as CaCO}_3$$

$$\text{or, } \frac{25.7 \text{ mg/L as CaCO}_3}{50 \text{ mg/meq}} \times 23.0 \text{ mg/meq} = 11.8 \text{ mg/L}$$

b. TH = (Ca²⁺) + (Mg²⁺) = 100.0 + 41.0 = 141 mg/L as CaCO₃

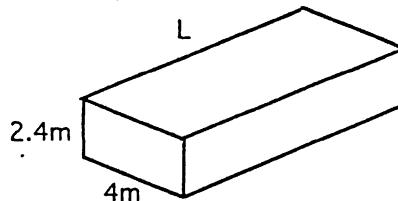


6.8 Simple salt balance:

$$5 \times 10^6 \text{ L/d} \times 1500 \text{ mg/L} = 3 \times 10^6 \text{ L/d} \times 75 \text{ mg/L} + 2 \times 10^6 \text{ L/d} \times C \text{ mg/L}$$

$$C = \frac{5 \times 1500 - 3 \times 75}{2} = 3638 \text{ mg/L}$$

6.9 Primary clarifier, overflow rate $32\text{m}^3/\text{m}^2\text{-day}$.

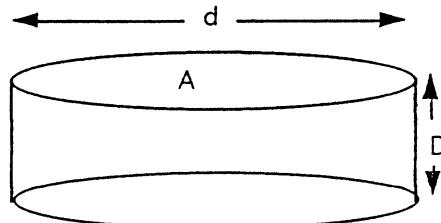


$$\text{Overflow rate} = \frac{\text{flow rate}}{\text{area}} = \frac{2000\text{m}^3 / \text{day}}{4\text{m} \times L \text{ m}} = 32\text{m}^3 / \text{m}^2 - \text{day}$$

$$L = \frac{2000}{4 \times 32} = 15.6\text{m}$$

$$\text{Detention time} = \frac{\text{volume}}{\text{flow rate}} = \frac{2.4\text{m} \times 4\text{m} \times 15.6\text{m} \times 24\text{hr/day}}{2000\text{m}^3 / \text{day}} = 1.8 \text{ hr}$$

6.10 2 mgd, 800g/d-ft^2 , DT>2 hr, depth>11 ft, circular,



$$\text{Area} = \frac{\text{flow rate}}{\text{overflow rate}} = \frac{2 \times 10^6 \text{ gal / d}}{800 \text{ gal / d - ft}^2} = 2500 \text{ ft}^2 = \frac{\pi}{4} d^2$$

$$d = \sqrt{\frac{4 \times 2500 \text{ ft}^2}{\pi}} = 56.4 \text{ ft}$$

Detention time constraint:

$$DT = \frac{\text{volume}}{\text{flow rate}} = \frac{2500 \text{ ft}^2 \times D \text{ ft}}{2 \times 10^6 \text{ gal / d} \times \frac{\text{ft}^3}{7.48 \text{ gal}}} \times \frac{24 \text{ hr}}{\text{day}} = 0.2244D(\text{hr}) \geq 2 \text{ hr}$$

$$D \geq \frac{2}{0.2244} = 8.9 \text{ ft}$$

Minimum depth constraint says depth must be at least 11 ft (more than the detention time constraint). Therefore depth D = 11 ft.

SOLUTIONS FOR CHAPTER 7

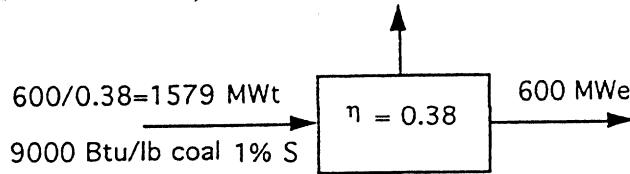
7.1 From (1.8), $\text{mg/m}^3 = \frac{\text{ppm} \times \text{mol wt}}{24.465}$ (at 1 atm and 25°C)

$$\text{a. CO}_2 \text{ mg/m}^3 = \frac{5000 \text{ ppm} \times (12 + 2 \times 16)}{24.465} = 8992 \text{ mg/m}^3 \approx 9000 \text{ mg/m}^3$$

$$\text{b. HCHO ppm} = \frac{24.465 \times 3.6 \text{ mg/m}^3}{(2 \times 1 + 12 + 16)} = 2.94 \text{ ppm}$$

$$\text{c. NO mg/m}^3 = \frac{25 \text{ ppm} \times (14 + 16)}{24.465} = 30.7 \text{ mg/m}^3$$

7.2 70% efficient scrubber, find S emission rate:



$$\text{Input} = \frac{600,000 \text{ kWe}}{0.38} \times \frac{3412 \text{ Btu}}{\text{kWhr}} \times \frac{1 \text{ lb coal}}{9000 \text{ Btu}} \times \frac{0.01 \text{ lb S}}{1 \text{ lb coal}} = 5986 \text{ lb S/hr}$$

70% efficient, says release $0.3 \times 5986 \text{ lb S/hr} = 1796 \text{ lb S/hr} \approx 1800 \text{ lb S/hr}$

7.3 If all S converted to SO₂ and now using a 90% efficient scrubber:

$$\text{SO}_2 = 0.1 \times \frac{5986 \text{ lb S}}{\text{hr}} \times \frac{(32 + 2 \times 16) \text{ lb SO}_2}{32 \text{ lb S}} = 1197 \text{ lb SO}_2 / \text{hr} \approx 1200 \text{ lb SO}_2 / \text{hr}$$

7.4 70% scrubber, 0.6 lb SO₂/10⁶ Btu in, find % S allowable:

$$\text{a. } \frac{X \text{ lbs S}}{\text{lbs coal}} \times \frac{0.3 \text{ lbs S out}}{1 \text{ lb S in}} \times \frac{2 \text{ lbs SO}_2}{1 \text{ lb S}} \times \frac{1 \text{ lb coal}}{15,000 \text{ Btu}} = \frac{0.6 \text{ lb SO}_2}{10^6 \text{ Btu}}$$

$$X = \frac{15,000 \times 0.6}{0.3 \times 2 \times 10^6} = 0.015 = 1.5\% \text{ S fuel}$$

$$\text{b. } \frac{X \text{ lbs S}}{\text{lbs coal}} \times \frac{0.3 \text{ lbs S out}}{1 \text{ lb S in}} \times \frac{2 \text{ lbs SO}_2}{1 \text{ lb S}} \times \frac{1 \text{ lb coal}}{9,000 \text{ Btu}} = \frac{0.6 \text{ lb SO}_2}{10^6 \text{ Btu}}$$

$$X = \frac{9,000 \times 0.6}{0.3 \times 2 \times 10^6} = 0.009 = 0.9\% \text{ S fuel}$$

7.9



for $R'CHO$ to be $HCHO$, R' must be H so that

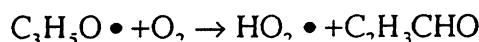
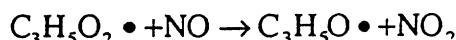
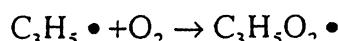
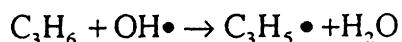


for the reaction to balance, $R = CH_3$

which says RH in (7.16) must be CH_4 (methane)

7.10 $RH = \text{propene} = CH_2=CH-CH_3 = C_3H_6$ so, $R = C_3H_5$

so the sequence of reactions (7.16) to (7.19) are:



The end product is acrolein, CH_2CHCHO .

7.11 U. S. Power plants:

$$\text{heat input} = 685 \times 10^6 \text{ tons} \times 2000 \frac{\text{lb}}{\text{ton}} \times 10,000 \frac{\text{Btu}}{\text{lb}} = 1.37 \times 10^{16} \text{ Btu}$$

$$\text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{1400 \times 10^9 \text{ kWh} \times 3412 \text{ Btu/kWh}}{1.37 \times 10^{16} \text{ Btu}} = 0.349 \approx 35\%$$

At NSPS of 0.03 lb particulates per 10^6 Btu input, emissions would have been:

$$\text{emissions} = \frac{0.03 \text{ lb}}{10^6 \text{ Btu heat input}} \times 1.37 \times 10^{16} \text{ Btu} \times \frac{1000 \text{ g}}{2.2 \text{ lb}} = 1.87 \times 10^{11} \text{ g}$$

$$\text{For comparison, } \frac{\text{emissions at NSPS}}{\text{actual emissions}} = \frac{1.87 \times 10^{11} \text{ g}}{0.39 \times 10^{12} \text{ g}} = 0.48 = 48\%$$

7.12 Derivation for the dry adiabatic lapse rate:

$$dQ = dU + dW \quad \text{where } dU = C_v dt \text{ and } dW = PdV$$

$$dQ = C_v dt + PdV \quad (1)$$

ideal gas law says $PV = nRT$

$$\text{so, } d(PV) = PdV + VdP = nRT$$

$$\text{or, } PdV = nRT - VdP$$

plugged into (1) gives:

$$dQ = C_v dT + nRdT - VdP$$

$$\frac{dQ}{dT} = C_v + nR - V \frac{dP}{dT} \quad (2)$$

at constant pressure :

$$\frac{dQ}{dT} = C_v + nR = C_p$$

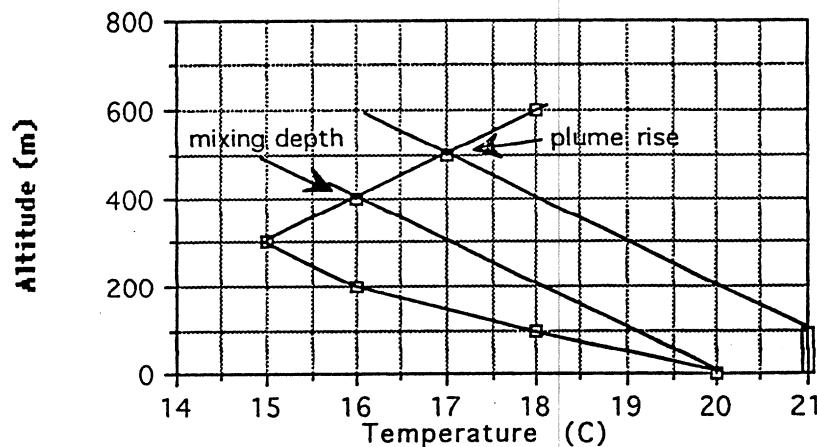
putting that into (2) gives,

$$\frac{dQ}{dT} = C_p - V \frac{dP}{dT}$$

or,

$$dQ = C_p dT - VdP \quad \text{which is (7.35)}$$

7.13 Plotting the data, extending from groundlevel to crossing with ambient profile at the adiabatic lapse rate, and extending from the stack height gives:



a) mixing depth (projecting from 20°C at 0m at slope -1°/100m) = 400 m

b) plume rise (projecting from 21°C at 100m) = 500m

7.14 Projection from the ground at 22°C crosses ambient at 500m.

Need the windspeed at 250 m (halfway up) using (7.43) and Table 7.7 for Class C,

$$\frac{u_H}{u_a} = \left(\frac{H}{z_a} \right)^p \quad \text{so, } \frac{u_{250}}{4 \text{ m/s}} = \left(\frac{250 \text{ m}}{10 \text{ m}} \right)^{0.20} = 1.90$$

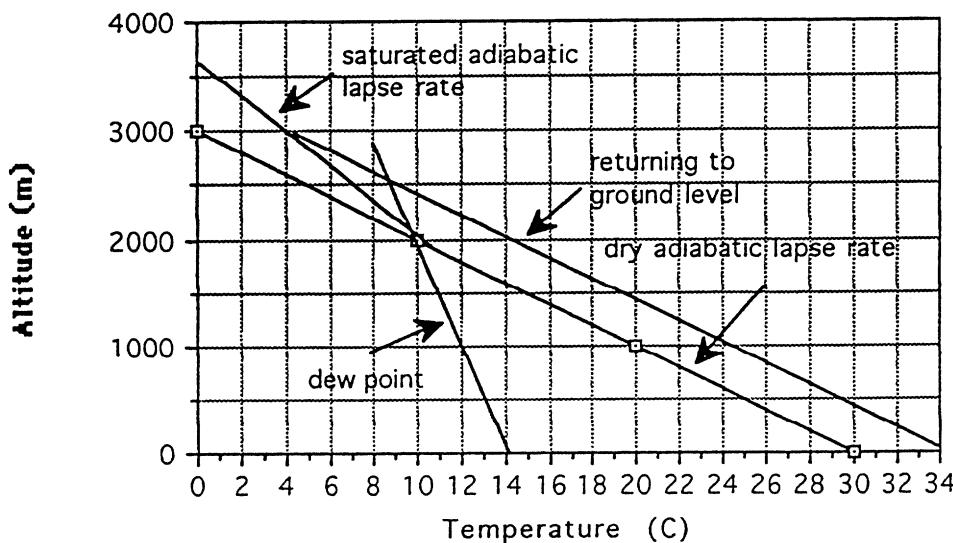
$$u_{250} = 1.90 \times 4 = 7.6 \text{ m/s}$$

$$\text{Ventilation coeff} = 500 \text{ m} \times 7.6 \text{ m/s} = 3.8 \times 10^3 \text{ m}^2/\text{s}$$

- 7.15 Below the knee, the plume is fanning which suggests a stable atmosphere, which could be profile (a), (b) or (d).

Above the knee, the plume is looping, which suggests superadiabatic, which is d.

- 7.16 Projecting the dry adiabatic lapse rate, and the dew point lapse rate, gives:



- crossover point is where clouds begin to form (saturated air) = 2000m.
- at 3 km, the temperature would be $10^\circ - (6^\circ/\text{km} \times 1\text{km}) = 4^\circ\text{C}$.
- falling from 3km at 4°C and increasing 10°C per km, reaches $4+3\times 10=34^\circ\text{C}$

- 7.17 A $20\mu\text{m}$ article blown to 8000 m, settling velocity,

$$v = \frac{d^2 \rho g}{18\eta} = \frac{(20 \times 10^{-6} \text{ m})^2 \times 1.5 \times 10^6 \text{ g/m}^3 \times 9.80 \text{ m/s}^2}{18 \times 0.0172 \text{ g/m-s}} = 0.019 \text{ m/s}$$

$$\text{Time to reach the ground} = \frac{8000 \text{ m}}{0.019 \text{ m/s} \times 3600 \text{ s/hr} \times 24 \text{ hr/d}} = 4.87 \text{ days}$$

$$\text{horizontal distance} = 4.87 \text{ days} \times 10 \text{ m/s} \times 3600 \text{ s/hr} \times 24 \text{ hr/d} \times 10^{-3} \text{ km/m} = 4200 \text{ km}$$

- 7.18 Residence time for $10 \mu\text{m}$ particle, unit density, at 1000m:

$$\text{settling velocity } v = \frac{d^2 \rho g}{18\eta} = \frac{(10 \times 10^{-6} \text{ m})^2 \times 10^6 \text{ g/m}^3 \times 9.80 \text{ m/s}^2}{18 \times 0.0172 \text{ g/m-s}} = 0.00317 \text{ m/s}$$

$$\text{residence time} = \tau = \frac{h}{v} = \frac{1000\text{m}}{0.00317\text{m/s} \times 3600\text{s/hr}} = 87.6\text{hrs}$$

7.19 Settling velocity and Reynolds numbers:

$$\text{a. } 1 \mu\text{m: } v = \frac{d^2 \rho g}{18\eta} = \frac{(1 \times 10^{-6}\text{m})^2 \times 10^6\text{g/m}^3 \times 9.80\text{m/s}^2}{18 \times 0.0172\text{g/m-s}} = 3.2 \times 10^{-5}\text{ m/s}$$

$$Re = \frac{\rho_{\text{air}} dv}{\eta} = \frac{1.29 \times 10^3 \text{g/m}^3 \times 1 \times 10^{-6}\text{m} \times 3.17 \times 10^{-5}\text{m/s}}{0.0172 \text{m/s}} = 2.4 \times 10^{-6}$$

$$\text{b. } 10 \mu\text{m: } v = \frac{d^2 \rho g}{18\eta} = \frac{(10 \times 10^{-6}\text{m})^2 \times 10^6\text{g/m}^3 \times 9.80\text{m/s}^2}{18 \times 0.0172\text{g/m-s}} = 3.2 \times 10^{-3}\text{ m/s}$$

$$Re = \frac{\rho_{\text{air}} dv}{\eta} = \frac{1.29 \times 10^3 \text{g/m}^3 \times 10 \times 10^{-6}\text{m} \times 3.17 \times 10^{-3}\text{m/s}}{0.0172 \text{m/s}} = 2.4 \times 10^{-3}$$

$$\text{c. } 20 \mu\text{m: } v = \frac{d^2 \rho g}{18\eta} = \frac{(20 \times 10^{-6}\text{m})^2 \times 10^6\text{g/m}^3 \times 9.80\text{m/s}^2}{18 \times 0.0172\text{g/m-s}} = 0.0127\text{ m/s}$$

$$Re = \frac{\rho_{\text{air}} dv}{\eta} = \frac{1.29 \times 10^3 \text{g/m}^3 \times 20 \times 10^{-6}\text{m} \times 0.0127\text{m/s}}{0.0172 \text{m/s}} = 0.02$$

7.20 H=50m, overcast so Class D, A at 1.2km, B at 1.4km.

- a. Fig 7.50, Class D, H=50m, max concentration at 1km. Since concentration is decreasing past 1 km, the higher level of pollution will be at site "A".
- b. Clear sky, wind < 5m/s: Class is now A,B or C. At 50m, Class A,B, or C, Fig 7.50 shows us that the maximum point moves closer to the stack.
- c. It will still be house at site "A."

7.21 Bonfire, 20g/s CO, wind 2 m/s, H=6m, distance = 400m:

Table 7.8, clear night, stability classification = F

$$C(x,0) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(-\frac{H^2}{2\sigma_z^2}\right) \quad (7.46)$$

- a. At 400m, $\sigma_y = 15\text{m}$, $\sigma_z = 7\text{m}$

$$C = \frac{20 \times 10^6 \mu\text{g/s}}{\pi 2\text{m/s} \times 15\text{m} \times 7\text{m}} \exp\left(-\frac{6^2}{2 \times 7^2}\right) = 21 \times 10^3 \mu\text{g/m}^3 = 21\text{mg/m}^3$$

- b. At the maximum point, Fig. 7.50

$$\left(\frac{Cu_H}{Q} \right)_{\max} \approx 3.8 \times 10^{-3} / m^2$$

$$C_{\max} = \frac{Q}{u_H} \left(\frac{Cu_H}{Q} \right)_{\max} = \frac{20 \times 10^3 \text{ mg/s}}{2 \text{ m/s}} \times \frac{3.8 \times 10^{-3}}{\text{m}^2} = 38 \approx 40 \text{ mg/m}^3$$

7.22 H=100m, Q=1.2g/s, u_H=4m/s, u_a=3⁺m/s, C<365μg/m³.

The more unstable the atmosphere, the higher the peak downwind concentration (see Fig. 7.49). From Table 7.8, with wind > 3m/s, B is the most unstable so it leads to the worst concentration:

$$\text{From Fig. 7.50, } X_{\max} = 0.7 \text{ km; } \left(\frac{Cu_H}{Q} \right)_{\max} \approx 1.5 \times 10^{-5} / m^2$$

$$C_{\max} = \frac{Q}{u_H} \left(\frac{Cu_H}{Q} \right)_{\max} = 365 \times 10^{-6} \text{ g/s} = \frac{Q}{4 \text{ m/s}} \times \frac{1.5 \times 10^{-5}}{\text{m}^2}$$

$$Q \approx \frac{4 \times 365 \times 10^{-6}}{1.5 \times 10^{-5}} = 97 \text{ g/s}$$

$$\text{Maximum power plant size} = 97 \text{ g/s} \times \frac{\text{MW}}{1.2 \text{ g/s}} = 80 \text{ MW}$$

7.23 Atmospheric conditions, stack height, and groundlevel concentration restrictions same as Prob. 7.22 so that:

$$\text{Emissions } Q \approx 97 \text{ g/s}$$

$$97 \text{ g/s} = \frac{0.6 \text{ lb SO}_2}{10^6 \text{ Btu in}} \times \frac{1 \text{ Btu in}}{0.35 \text{ Btu out}} \times \frac{3412 \text{ Btu out}}{\text{kWh}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{10^3 \text{ g}}{2.2 \text{ lb}} \times P_{\text{kW}}$$

$$P_{\text{kW}} = \frac{97 \times 10^6 \times 0.35 \times 3600 \times 2.2}{0.6 \times 3412 \times 1000} = 131,000 \text{ KW} = 130 \text{ MW}$$

7.24 H=100m, u_a=4 m/s, Q=80g/s, clear summer day so Class B:

First, find the windspeed at the effective stack height using (7.43) and Table 7.7:

$$u_H = u_a \left(\frac{H}{z_a} \right)^p = 4 \text{ m/s} \left(\frac{100}{10} \right)^{0.15} = 5.65 \text{ m/s}$$

a. at 2 km: σ_y = 290m, σ_z = 234m

$$C(x, 0) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(-\frac{H^2}{2\sigma_z^2}\right)$$

$$C = \frac{80 \times 10^6 \mu g/s}{\pi 5.65 m/s \times 290 m \times 234 m} \exp\left(-\frac{100^2}{2 \times 234^2}\right) = 61 \mu g/m^3$$

b. At the maximum point, 0.7 km (Fig. 7.50),

$$\left(\frac{Cu_H}{Q}\right)_{max} \approx 1.5 \times 10^{-5} / m^2$$

$$C_{max} = \frac{Q}{u_H} \left(\frac{Cu_H}{Q}\right)_{max} = \frac{80 \times 10^6 \mu g/s}{5.65 m/s} \left(\frac{1.5 \times 10^{-5}}{m^2}\right) = 212 \mu g/m^3$$

c. At $x=2km$, $y=0.1km$:

$$\begin{aligned} C(x, y) &= \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(-\frac{H^2}{2\sigma_z^2}\right) \exp\left(\frac{-y^2}{2\sigma_y^2}\right) \\ &= 61 \mu g/m^3 \times \exp\left(\frac{-100^2}{2 \times 290^2}\right) = 57 \mu g/m^3 \end{aligned}$$

7.25 For class C, notice from (7.44) and (7.45) with Table 7.9

$$\frac{\sigma_y}{\sigma_z} = \frac{ax^{0.894}}{cx^d + f} = \frac{104x^{0.894}}{61x^{0.911}} = 1.7x^{-0.017} \approx \text{fairly constant} \approx k (\text{about } 1.7)$$

So, assume $\sigma_y = k \sigma_z$, then

$$a. C(x, 0) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(-\frac{H^2}{2\sigma_z^2}\right) = \frac{Q}{\pi u k \sigma_z^2} \exp\left(-\frac{H^2}{2\sigma_z^2}\right)$$

to find the maximum concentration, differentiate and set equal to zero:

$$\frac{dC}{d\sigma_z} = \frac{Q}{\pi u k} \left\{ \frac{1}{\sigma_z^2} \left(-\frac{H^2}{2} \right) \left(\frac{-2}{\sigma_z^3} \right) e^{-\frac{H^2}{2\sigma_z^2}} + e^{-\frac{H^2}{2\sigma_z^2}} \left(\frac{-2}{\sigma_z^3} \right) \right\} = 0$$

multiply through by σ_z^5 and cancel lots of terms to get,

$$\left(\frac{2H^2}{2} \right) - 2\sigma_z^2 = 0 \quad \text{or} \quad \sigma_z = \frac{H}{\sqrt{2}} = 0.707H$$

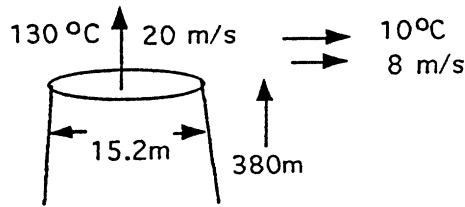
b. substituting the newly found value for σ_z ,

$$C_{\max} = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(-\frac{H^2}{2(H^2/2)}\right) = \frac{Q}{\pi u \sigma_y \sigma_z} e^{-1} = \frac{0.117Q}{u \sigma_y \sigma_z}$$

c. using $\sigma_y = k \sigma_z$

$$C_{\max} = \frac{0.117Q}{uk \sigma_z^2} = \frac{0.117Q}{uk(0.707H)^2} = \frac{f(Q, u)}{H^2} \text{ (varies as inverse of } H^2)$$

7.26 Sudbury stack:



Using (7.49) for buoyancy flux parameter

$$F = gr^2 v_s \left(1 - \frac{T_a}{T_s} \right) = 9.8 \frac{m}{s^2} \times \left(\frac{15.2m}{2} \right)^2 \times 20 \frac{m}{s} \times \left(1 - \frac{10 + 273}{130 + 273} \right) = 3370 m^4/s^3$$

and distance downwind to final plume rise x_f given on page 420 (with $F > 55$),

$$x_f = 120F^{0.4} = 120 \times (3370)^{0.4} = 3092$$

for stability classification C, use (7.51) for plume rise,

$$\Delta h = \frac{1.6 F^{1/3} x_f^{2/3}}{u_h} = \frac{1.6 \times (3370)^{1/3} (3092)^{2/3}}{8} = 635 \text{ m plume rise}$$

$$H = \text{effective stack height} = h + \Delta h = 380 + 635 = 1015 \text{ m}$$

7.27 Repeat P7.26 with a stable, isothermal atmosphere:

$F = 3370 \text{ m}^4/\text{s}^3$ from Prob. 7.26, but for isothermal atmosphere, need the stability parameter, S

$$S = \frac{g}{T_a} \left(\frac{\Delta T_a}{\Delta z} + 0.01 K/m \right) = \frac{9.8 m/s^2}{(10 + 273) K} (0 + 0.01 K/m) = 3.46 \times 10^{-4} / s^2$$

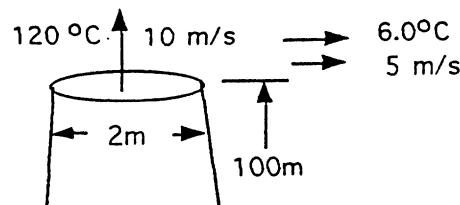
which is used in (7.48) for plume rise under these conditions,

$$\Delta h = 2.6 \left(\frac{F}{u_h S} \right)^{1/3} = 2.6 \left(\frac{3370 \text{ m}^4/\text{s}^3}{8 \text{ m/s} \times 3.46 \times 10^{-4} / \text{s}^2} \right)^{1/3} = 278 \text{ m}$$

$$H = \text{effective stack height} = h + \Delta h = 380 + 278 = 657 \text{ m}$$

(notice the atmospheric stability lowered effective stack height vs Prob. 7.26)

7.28 Cloudy summer day, stability classification C (Table 7.8),



Using (7.49) for buoyancy flux parameter

$$F = gr^2 v_s \left(1 - \frac{T_a}{T_s} \right) = 9.8 \frac{\text{m}}{\text{s}^2} \times \left(\frac{2\text{m}}{2} \right)^2 \times 10 \frac{\text{m}}{\text{s}} \times \left(1 - \frac{6+273}{120+273} \right) = 28.4 \text{ m}^4 / \text{s}^3$$

and distance downwind to final plume rise x_f given on page 420 (with $F < 55$),

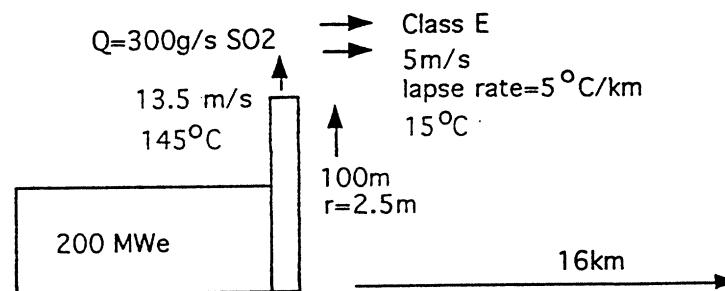
$$x_f = 50F^{5/8} = 50 \times (28.4)^{5/8} = 406$$

for stability classification C, use (7.51) for plume rise,

$$\Delta h = \frac{1.6 F^{1/3} x_f^{2/3}}{u_h} = \frac{1.6 \times (28.4)^{1/3} (406)^{2/3}}{5} = 54 \text{ m plume rise}$$

$$H = \text{effective stack height} = h + \Delta h = 100 + 54 = 154 \text{ m}$$

7.29 Power plant, find groundlevel pollution 16 km away. Need first find H.



First, find buoyancy flux parameter (7.49),

$$F = gr^2 v_s \left(1 - \frac{T_a}{T_s} \right) = 9.8 \frac{\text{m}}{\text{s}^2} \times (2.5\text{m})^2 \times 13.5 \frac{\text{m}}{\text{s}} \times \left(1 - \frac{15+273}{145+273} \right) = 257 \text{ m}^4 / \text{s}^3$$

plume rise for stable (Class E) atmosphere needs S from (7.50),

$$S = \frac{g}{T_a} \left(\frac{\Delta T_a}{\Delta z} + 0.01 K/m \right) = \frac{9.8 m/s^2}{(15 + 273) K} \left(\frac{5^\circ}{1000 m} + 0.01 K/m \right) = 5.1 \times 10^{-4} / s^2$$

plume rise is given by (7.48),

$$\Delta h = 2.6 \left(\frac{F}{u_h S} \right)^{1/3} = 2.6 \left(\frac{257 m^4/s^3}{5 m/s \times 5.1 \times 10^{-4} / s^2} \right)^{1/3} = 121 m$$

so the effective height is $H = 100 m + 121 m = 221 m$

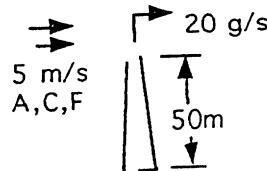
Concentration downwind at 16km: (Table 7.10) $\sigma_y = 602 m$, $\sigma_z = 95 m$

$$C(x,0) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp \left(-\frac{H^2}{2\sigma_z^2} \right)$$

$$C = \frac{300 \times 10^6 \mu g/s}{\pi 5 m/s \times 602 m \times 95 m} \exp \left(-\frac{221^2}{2 \times 95^2} \right) = 22 \mu g/m^3$$

7.30

$$C_{max} = \frac{Q}{u_H} \left(\frac{Cu_H}{Q} \right) = \frac{20 \times 10^6 \mu g/s}{5 m/s} \left(\frac{Cu_H}{Q} \right) = 4 \times 10^6 \left(\frac{Cu_H}{Q} \right) \mu g/s$$

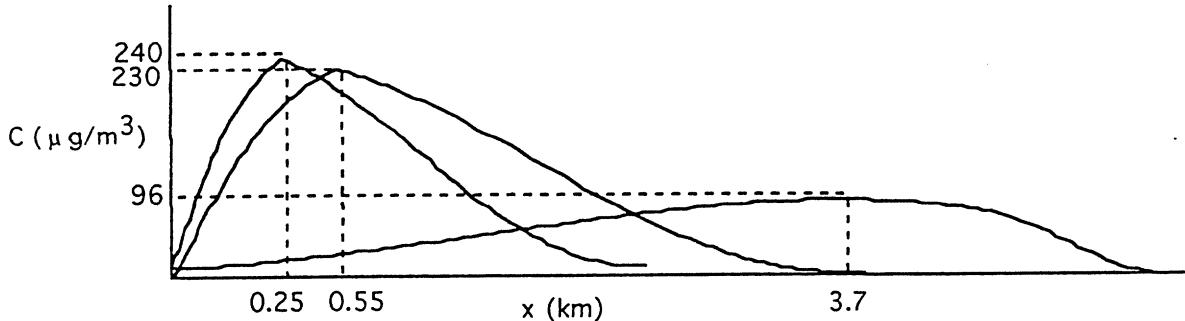


Using Fig. 7.50 gives,

$$\text{"A"} \quad \left(\frac{Cu_H}{Q} \right) \approx 6 \times 10^{-5} \text{ at } 0.25 \text{ km}, C_{max} = 4 \times 10^6 \times 6 \times 10^{-5} = 240 \mu g/m^3$$

$$\text{"C"} \quad \left(\frac{Cu_H}{Q} \right) \approx 5.8 \times 10^{-5} \text{ at } 0.55 \text{ km}, C_{max} = 4 \times 10^6 \times 5.8 \times 10^{-5} = 230 \mu g/m^3$$

$$\text{"F"} \quad \left(\frac{Cu_H}{Q} \right) \approx 2.4 \times 10^{-5} \text{ at } 3.7 \text{ km}, C_{max} = 4 \times 10^6 \times 2.4 \times 10^{-5} = 96 \mu g/m^3$$



7.31 $H = 50\text{m}, 100\text{m}, 200\text{m}$; Class C, 20 g/s:

$$C_{\max} = \frac{Q}{u_H} \left(\frac{Cu_H}{Q} \right) = \frac{20 \times 10^6 \mu\text{g/s}}{5 \text{m/s}} \left(\frac{Cu_H}{Q} \right) = 4 \times 10^6 \left(\frac{Cu_H}{Q} \right) \mu\text{g/s}$$

Using Fig. 7.50,

$$@50\text{m}: C_{\max} \approx 4 \times 10^6 \times 5.7 \times 10^{-5} = 228 \mu\text{g/m}^3$$

$$@100\text{m}: C_{\max} \approx 4 \times 10^6 \times 1.5 \times 10^{-5} = 60 \mu\text{g/m}^3$$

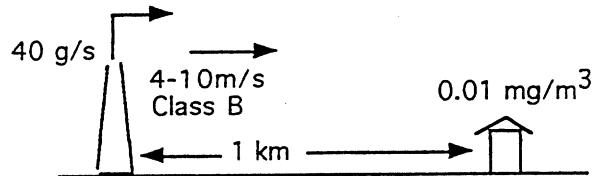
$$@200\text{m}: C_{\max} \approx 4 \times 10^6 \times 3.4 \times 10^{-6} = 14 \mu\text{g/m}^3$$

Do they drop as $(1/H)^2$, that is expectation: $\frac{C(2H)}{C(H)} = \frac{1}{4}$

$$\text{test them: } \frac{C(100\text{m})}{C(50\text{m})} = \frac{6}{22.8} = 0.26 \quad \frac{C(200\text{m})}{C(100\text{m})} = \frac{1.4}{6} = 0.23 \text{ not bad!}$$

$$\text{expect } \frac{C(200\text{m})}{C(50\text{m})} = \frac{1}{16} = 0.0625 \quad \frac{C(200\text{m})}{C(50\text{m})} = \frac{1.4}{22.8} = 0.061 \text{ again, not bad.}$$

7.32 Paper mill emitting H_2S , 1km away want $0.1 \times$ odor threshold:



Class B, at 1 km, (Table 7.10) $\sigma_y = 156\text{m}$, $\sigma_z = 110\text{m}$

$$C(x,0) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(-\frac{H^2}{2\sigma_z^2}\right)$$

$$0.01 \times 10^{-3} \text{ g/m}^3 = \frac{40 \text{ g/s}}{\pi u \text{ m/s} \times 156 \text{ m} \times 110 \text{ m}} \exp\left(-\frac{H^2}{2 \times 10^2}\right)$$

$$\text{rearranging: } e^{\frac{H^2}{24,200}} = \frac{40}{\pi u \times 156 \times 110 \times 0.01 \times 10^{-3}} = \frac{74.2}{u}$$

$$\text{or, } H = \left[24,200 \ln\left(\frac{74.2}{u}\right) \right]^{0.5}$$

so, at each end of the wind speed range we can find the height needed:

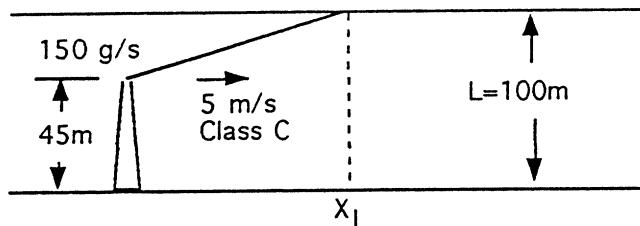
$$H_{u=4} = \left[24,200 \ln\left(\frac{74.2}{4}\right) \right]^{0.5} = 265 \text{ m} \quad \text{says to be conservative use } H=265 \text{ m}$$

$$H_{u=10} = \left[24,200 \ln\left(\frac{74.2}{10}\right) \right]^{0.5} = 220 \text{ m}$$

If the town extends beyond 1 km, from Fig 7.50 at H=265, Class B, $X_{\max} \approx 1.8 \text{ km}$

Therefore, with the peak occurring beyond the 1 km house, the concentration will rise for buildings located > 1km away. YES

7.33 Stack under an inversion:



$$\text{at } x = X_L \quad \sigma_z = 0.47(L-H) = 0.47(100 - 45) = 26 \text{ m}$$

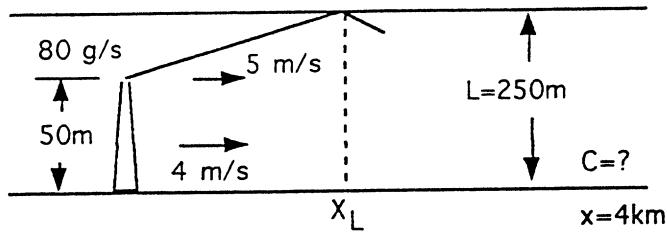
for class C, $\sigma_z = 26 \text{ m}$ at $x = 0.4 \text{ km}$ (Table 7.10), therefore $X_L = 0.4 \text{ km}$, and also from Table 7.10, $\sigma_y = 46 \text{ m}$.

$$\begin{aligned} \text{at } x = X_L: C(X_L, 0) &= \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(-\frac{H^2}{2\sigma_z^2}\right) \\ &= \frac{150 \times 10^3 \text{ mg/s}}{\pi 5 \text{ m/s} \times 46 \text{ m} \times 26 \text{ m}} \exp\left(-\frac{45^2}{2 \times 26^2}\right) = 1.8 \text{ mg/m}^3 \end{aligned}$$

$$\text{at } x = 2X_L: \sigma_y = 85 \text{ m} \text{ (Table 7.10),}$$

$$C(2X_L, 0) = \frac{Q}{\sqrt{2\pi} u \sigma_y L} = \frac{150 \times 10^3 \text{ mg/s}}{\sqrt{2\pi} \times 5 \text{ m/s} \times 85 \text{ m} \times 100 \text{ m}} = 1.4 \text{ mg/m}^3$$

7.34 Stack under an inversion layer:



We need the stability classification: clear summer day, 4m/s, Table 7.8 says Class B.

$$\text{at } x = X_L \quad \sigma_z = 0.47(L-H) = 0.47(250 - 50) = 94 \text{ m}$$

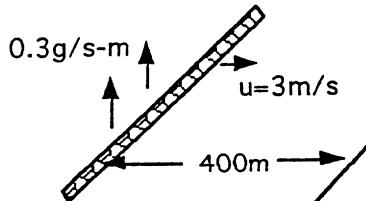
- a. From Table 7.10, at $\sigma_z = 94\text{m}$ Class B, $X_L \approx 0.9\text{km}$. Since our point of interest is at 4 km, we are well past the point at which reflections first occur so we can use (7.52). We need σ_y at 4km, which is given in Table 7.10 as 539m:

$$C(4\text{km}, 0) = \frac{Q}{\sqrt{2\pi} u \sigma_y L} = \frac{80 \times 10^6 \mu\text{g/s}}{\sqrt{2\pi} \times 5\text{ m/s} \times 539\text{m} \times 250\text{m}} = 47 \mu\text{g/m}^3$$

- b. Without the inversion layer, at 4km $\sigma_z = 498\text{m}$, $\sigma_y = 539\text{m}$ so,

$$C(4\text{km}, 0) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(\frac{-H^2}{2\sigma_z^2}\right) = \frac{80 \times 10^6 \mu\text{g/s}}{\pi \times 5\text{ m/s} \times 539\text{m} \times 498\text{m}} \exp\left(\frac{-50^2}{2 \times 498^2}\right) = 19 \mu\text{g/m}^3$$

7.35 Agricultural burn

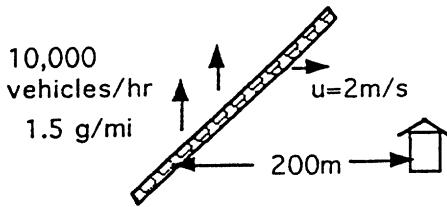


Clear fall afternoon, winds 3 m/s, so stability class "C" (Table 7.8),

and $\sigma_z = 26\text{m}$ (Table 7.10). Using (7.54),

$$C(0.4\text{km}) = \frac{2q}{\sqrt{2\pi} u \sigma_z} = \frac{2 \times 300 \text{ mg/m-s}}{\sqrt{2\pi} \times 3 \text{ m/s} \times 26 \text{ m}} = 3.0 \text{ mg/m}^3$$

7.36 A freeway modelled as a line source:



Clear summer, 2 m/s, Table 7.8 suggests Class A or A-B.

At 0.2 km, $\sigma_z = 29\text{m}$ for Class A; $\sigma_z = 20\text{m}$ for Class B. What should we use? Since it is somewhere between Class A and Class B, but closer to A, let's use $\sigma_z \approx 26\text{m}$:

To find the linear emission rate:

$$q = 10,000 \frac{\text{vehicles}}{\text{hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{1.5 \text{ g}}{\text{mi} - \text{vehicle}} \times \frac{\text{mi}}{5280 \text{ ft}} \times \frac{\text{ft}}{0.3048 \text{ m}} \\ = 2.58 \times 10^{-3} \text{ g/m-s} = 2.58 \text{ mg/m-s}$$

Then, using (7.54),

$$C(0.2 \text{ km}) = \frac{2q}{\sqrt{2\pi} u \sigma_z} = \frac{2 \times 2.58 \text{ mg/m-s}}{\sqrt{2\pi} \times 2 \text{ m/s} \times 26 \text{ m}} = 0.04 \text{ mg/m}^3$$

7.37 Box model, 250,000 vehicles between 4 and 6pm, driving 40km ea, emitting 4g/km CO.

a. $q_s = 250,000 \text{ veh.} \times \frac{40 \text{ km}}{\text{veh}} \times \frac{4 \text{ g CO}}{\text{km}} \times \frac{1}{2 \text{ hrs}} \times \frac{\text{hr}}{3600 \text{ s}} \times \frac{1}{15 \times 80 \times 10^6 \text{ m}^2} = 4.6 \times 10^{-6} \text{ g CO/m}^2 \text{s}$

b. Using (7.58) with $t = 2 \text{ hrs} \times 3600 \text{ s/hr} = 7200 \text{ s}$,

$$C(t) = \frac{q_s L}{u H} (1 - e^{-ut/L}) \\ = \frac{4.6 \times 10^{-6} \text{ g/m}^2 \text{s} \times 15,000 \text{ m}}{0.5 \text{ m/s} \times 15 \text{ m}} (1 - e^{-0.5 \text{ m/s} \cdot 7200 \text{ s} / 15000 \text{ m}}) = 0.002 \text{ g/m}^3 = 2 \text{ mg/m}^3$$

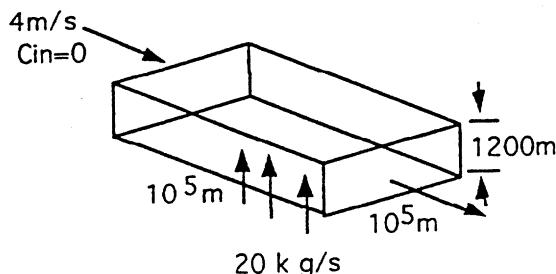
c. With no wind, go back to (7.55) and solve the differential equation:

$$LWH \frac{dC}{dt} = q_s LW$$

$$dC = \frac{q_s LW}{LWH} dt \quad \text{so, } C = \frac{q_s}{H} t$$

$$C = \frac{q_s}{H} t = \frac{4.6 \times 10^{-6} \text{ g CO/m}^2 \cdot \text{s}}{15 \text{ m}} \times 2 \text{ hrs} \times \frac{3600 \text{ s}}{\text{hr}} = 0.0022 \text{ g CO/m}^3 = 2.2 \text{ mg/m}^3$$

7.38 Box model, 10^5 m on a side, $H=1200$ m, $u=4$ m/s, $\text{SO}_2=20$ kg/s, steady state:



$$\text{input rate} = \text{output rate}$$

$$20 \frac{\text{kg}}{\text{s}} \times \frac{10^9 \mu\text{g}}{\text{kg}} = 4 \frac{\text{m}}{\text{s}} \times 10^5 \text{m} \times 1200 \text{m} \times C \left(\frac{\mu\text{g}}{\text{m}^3} \right)$$

$$C = \frac{20 \times 10^9}{4 \times 10^5 \times 1200} = 41.7 \mu\text{g/m}^3$$

7.39 Assume steady-state conditions were achieved by 5pm Friday so that $C(0)=41.7 \mu\text{g/m}^3$.

With $q_s = 0$, and $C_{in} = 0$, (7.57 gives us $C(t) = C(0)e^{-ut/L}$.

a. At midnight, $t=7\text{hrs} \times 3600\text{s/hr} = 2.52 \times 10^4 \text{ s}$

$$C(t) = C(0)e^{-ut/L} = 41.7 \mu\text{g/m}^3 \cdot e^{-4\text{m/s} \cdot 2.52 \times 10^4 \text{ s} / 10^5 \text{ m}} = 15.2 \mu\text{g/m}^3$$

b. Starting up again at 8am on Monday, by 5pm (9hrs later):

first check to see concentration left from Friday at 5pm (63 hrs earlier):

$$C(t) = C(0)e^{-ut/L} = 41.7 \mu\text{g/m}^3 \cdot e^{-4\text{m/s} \cdot 63\text{hrs} \times 3600\text{s/hr} / 10^5 \text{ m}} = .005 \mu\text{g/m}^3 \approx 0$$

so we can ignore that and let $C(0) = 0$ at 8am Monday. First find the emissions per unit area,

$$q_s = \frac{\text{emission rate}}{\text{area}} = \frac{20 \text{kg/s} \times 10^9 \mu\text{g/kg}}{10^5 \text{m} \times 10^5 \text{m}} = 2.0 \mu\text{g/m}^2 \cdot \text{s}$$

Then use (7.57) with $C_{in} = 0$:

$$\begin{aligned} C(t) &= \frac{q_s L}{u H} (1 - e^{-ut/L}) \\ &= \frac{2.0 \mu\text{g/m}^2 \cdot \text{s} \times 10^5 \text{m}}{4 \text{m/s} \times 1200 \text{m}} (1 - e^{-(4 \text{m/s} \times 9 \text{hr} \times 3600 \text{s/hr})}) = 30.2 \mu\text{g/m}^3 \end{aligned}$$

- 7.40 Steady-state conditions from Prob 7.38, wind drops to 2 m/s, 2hrs later:

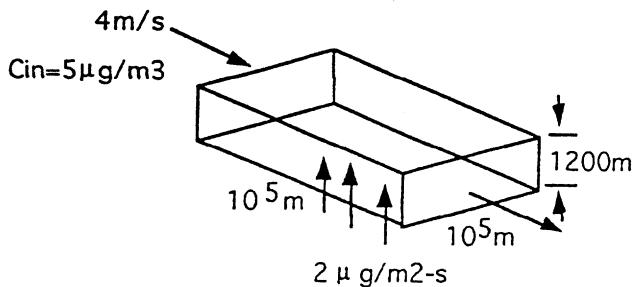
From Prob. 7.38, emission rate $q_s = 2.0 \mu\text{g}/\text{m}^2\cdot\text{s}$, and $C = 41.7 \mu\text{g}/\text{m}^3$. Using (7.57),

$$C(t) = \frac{q_s L}{uH} \left(1 - e^{-ut/L}\right) + C(0)e^{-ut/L}$$

$$C(2\text{hr}) = \frac{2.0 \mu\text{g}/\text{m}^2 \cdot \text{s} \times 10^5 \text{m}}{2\text{m/s} \times 1200\text{m}} \left(1 - e^{-(2\text{m/s} \times 2\text{hr} \times 3600\text{s/hr} \cdot 5\text{hr})}\right) + 41.7e^{-(2\text{m/s} \times 2\text{hr} \times 3600\text{s/hr} \cdot 5\text{hr})}$$

$$= 47.3 \mu\text{g}/\text{m}^3$$

- 7.41 Modified Prob. 7.38, now incoming air has $5 \mu\text{g}/\text{m}^3$ and there are $10 \mu\text{g}/\text{m}^3$ already there at 8am. Find the concentration at noon:



$$C(t) = \left(\frac{q_s L}{uH} + C_{in} \right) \left(1 - e^{-ut/L}\right) + C(0)e^{-ut/L}$$

$$C(4\text{hr}) = \left(\frac{2.0 \mu\text{g}/\text{m}^2 \cdot \text{s} \times 10^5 \text{m}}{4\text{m/s} \times 1200\text{m}} + 5 \mu\text{g}/\text{m}^3 \right) \left(1 - e^{-(4\text{m/s} \times 4\text{hr} \times 3600\text{s/hr} \cdot 5\text{hr})}\right)$$

$$+ 10e^{-(4\text{m/s} \times 4\text{hr} \times 3600\text{s/hr} \cdot 5\text{hr})}$$

$$C(4\text{hr} = \text{noon}) = 26.1 \mu\text{g}/\text{m}^3.$$

- 7.42 Now using conditions of Prob 7.38, but for a nonconservative pollutant with $K=0.23/\text{hr}$:

Rate into box = Rate out of box + Rate of decay

$$S = u W H C + K C V$$

$$20 \frac{\text{kg}}{\text{s}} \times \frac{10^9 \mu\text{g}}{\text{kg}} = 4 \frac{\text{m}}{\text{s}} \times 10^5 \text{m} \times 1200\text{m} \times C \left(\frac{\mu\text{g}}{\text{m}^3} \right)$$

$$+ \left(\frac{0.23}{\text{hr}} \times C \frac{\mu\text{g}}{\text{m}^3} \times \frac{1\text{hr}}{3600\text{s}} \times 10^5 \text{m} \times 10^5 \text{m} \times 1200\text{m} \right)$$

$$20 \times 10^9 = 4.8 \times 10^8 C + 7.6 \times 10^8 C$$

$$C = 16 \mu\text{g}/\text{m}^3$$

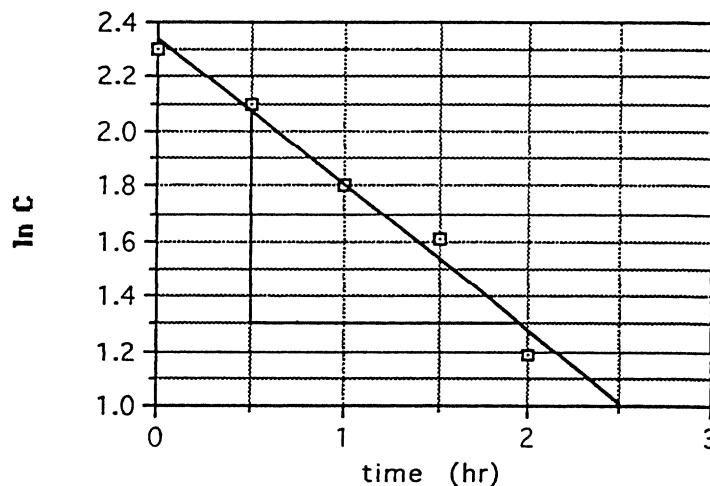
- 7.43** Starting with (7.61) and using the special conditions of this tracer-gas study; that is, a conservative tracer ($K=0$), no tracer in the air leaking into the room ($C_a=0$), and the tracer source turned off at $t=0$ ($S=0$) gives the exponential decay of tracer as:

$$C(t) = C_0 e^{-It}$$

$\ln[C(t)] = \ln(C_0) - It$ which is of the form

$y=mx + b$, where $y = \ln C$, $m=I$, and $b=\ln C_0$

time (hr)	C (ppm)	$\ln C$
0	10.0	2.303
0.5	8.0	2.079
1.0	6.0	1.792
1.5	5.0	1.609
2.0	3.3	1.194



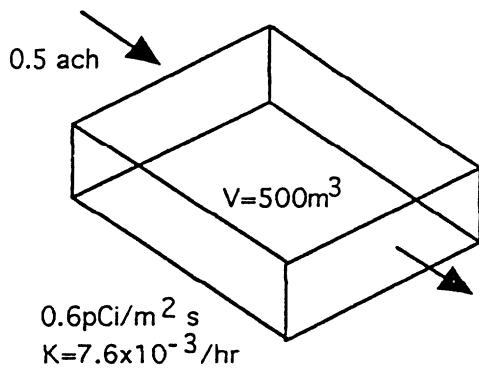
From the graph, the slope is about: $slope \approx \frac{(2.1-1.3)}{2.0-0.5} = 0.53$

Thus, the infiltration rate is about 0.53 air changes per hour.

- 7.44** Infiltration 0.5ach, 500m^3 volume, 200 m^2 floor space, radon $0.6\text{pCi/m}^2\text{s}$:

Using (7.60) with $K=7.6 \times 10^{-3}/\text{hr}$ (Table 7.15),

$$S = \frac{(S/V)}{I + K} = \frac{\left(0.6\text{pCi/m}^2\text{s} \times 200\text{m}^2\right) / 500\text{m}^3}{(0.5/\text{hr} + 7.6 \times 10^{-3}/\text{hr}) \times \frac{1\text{hr}}{3600\text{s}}} = 1700\text{pCi/m}^3 = 1.7\text{pCi/L}$$



7.45 Just have half as much ground-floor area to let radon in, so:

$$S = \frac{(S/V)}{I + K} = \frac{\left(0.6 \text{ pCi/m}^2 \text{ s} \times 100 \text{ m}^2\right) / 500 \text{ m}^3}{(0.5 / \text{hr} + 7.6 \times 10^{-3} / \text{hr}) \times \frac{1 \text{ hr}}{3600 \text{ s}}} = 850 \text{ pCi/m}^3 = 0.85 \text{ pCi/L}$$

7.46 Using exposure factors from Table 4.10 and potency from Prob. 4.10 and 4.12,

$$\text{Exposure} = \frac{1.7 \text{ pCi/L} \times 360 \frac{\text{day}}{\text{yr}} \times 30 \text{ yr}}{365 \text{ day/yr} \times 70 \text{ yr}} = 0.72 \text{ pCi/L} \text{ average over 70 yrs}$$

$$\text{potency} = \frac{400 \text{ mrem/yr}}{1.5 \text{ pCi/L}}$$

$$\text{Risk} = 0.72 \text{ pCi/L} \times \frac{400 \text{ mrem/yr}}{1.5 \text{ pCi/L}} \times \frac{1 \text{ cancer death}}{8000 \text{ rem}} \times \frac{\text{rem}}{10^3 \text{ mrem}} \times 70 \text{ yr} = 0.0017 \approx 0.2\%$$

7.47 300m³ house, 0.2ach, oven+2burners 6pm to 7pm, find CO 7pm and 10pm. For these circumstances, (7.62) is appropriate:

$$C(t) = \frac{S}{IV} (1 - e^{-kt})$$

the source strength, S is

$$6 - 7\text{pm} : \text{oven} + 2 \text{ burners}, 1900 \text{ mg/hr} + 2 \times 1840 \text{ mg/hr} = 5580 \text{ mg/hr}$$

solving for C after 1 hr:

$$C(1\text{hr}, 7\text{pm}) = \frac{5580 \text{ mg/hr}}{0.2 \frac{\text{airchange}}{\text{hr}} \times 300 \frac{\text{m}^3}{\text{ac}}} (1 - e^{-0.2/\text{hr} \times 1\text{hr}}) = 16.8 \text{ mg/m}^3$$

Now turn off the burners and watch CO coast down until 10pm, 3hrs later:

$$C(3\text{hrs}, 10\text{pm}) = C(0, 7\text{pm})e^{-lt} = 16.8e^{-0.2/\text{hr} \times 3\text{hrs}} = 9.3 \text{mg/m}^3$$

7.48 $I=0.39 \text{ach}$, $V=27\text{m}^3$, after 1-hr $\text{NO} = 4.7 \text{ppm}$; find source strength, S :

first convert NO in ppm to mg/m^3 using (1.8) and assuming $T=25^\circ\text{C}$,

$$\text{mg/m}^3 = \frac{\text{ppm} \times \text{mol wt}}{24.465} = \frac{4.7 \times (14+16)}{24.465} = 5.76 \text{mg/m}^3$$

a. Source strength, rearrange (7.62)

$$S = \frac{IVC}{(1-e^{-lt})} = \frac{0.39 \frac{\text{ac}}{\text{hr}} \times 27 \frac{\text{m}^3}{\text{ac}} \times 5.76 \frac{\text{mg}}{\text{m}^3}}{(1-e^{-0.39/\text{hr} \times 1\text{hr}})} = 188 \text{mgNO/hr}$$

b. 1-hr after turning off the heater,

$$C = C_0 e^{-lt} = 4.7 \text{ppm} \times e^{-0.39/\text{hr} \times 1\text{hr}} = 3.2 \text{ppmNO}$$

c. in a house with 0.2 ach, 300m^3 ,

$$C(\infty) = \frac{S}{IV} = \frac{188 \text{mg/hr}}{0.2 \frac{\text{ac}}{\text{hr}} \times 300 \frac{\text{m}^3}{\text{ac}}} = 3.1 \text{mg/m}^3 \times \frac{24.465}{30} = 2.6 \text{ppm}$$

7.49 100MW coal plant, 33.3% efficient, $CF=0.70$,

a. electricity generated per year,

$$\text{Energy} = 100,000 \text{kW} \times 24 \text{hr/day} \times 365 \text{day/yr} \times 0.70 = 613 \times 10^6 \text{kWh/yr}$$

$$\text{b. heat input} = 613 \times 10^6 \text{kWh/yr} \times \frac{3 \text{kWh}_t \text{in}}{1 \text{kWh}_e \text{out}} \times \frac{3412 \text{Btu}}{\text{kWh}} = 6.28 \times 10^{12} \text{Btu/yr}$$

c. Shut it down and sell the allowances,

$$\text{SO}_2 \text{ saved by shutting down} = 6.28 \times 10^{12} \frac{\text{Btu}}{\text{yr}} \times \frac{0.6 \text{ lb SO}_2}{10^6 \text{ Btu}} \times \frac{\text{ton}}{2000 \text{ lb}} = 1883 \text{ tons/yr}$$

$$1883 \frac{\text{tons}}{\text{yr}} \times \frac{1 \text{ allowance}}{\text{ton}} \times \frac{\$1500}{\text{allowance}} = \$2.85 \text{ million/yr}$$

SOLUTIONS FOR CHAPTER 8

8.1 From (8.1),

$$\delta^{18}\text{O}(\text{\%}) = \left[\frac{\left(\frac{^{18}\text{O}}{^{16}\text{O}} \right)_{\text{sample}} - \left(\frac{^{18}\text{O}}{^{16}\text{O}} \right)_{\text{standard}}}{\left(\frac{^{18}\text{O}}{^{16}\text{O}} \right)_{\text{standard}}} \right] \times 10^3 = \left[\frac{0.00199 - 0.00200}{0.00200} \right] \times 10^3 = -5$$

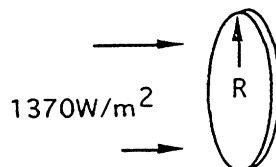
The negative sign means the sample corresponds to a warmer climate.

8.2 From the equation given for the ice core,

$$T(^{\circ}\text{C}) = 1.5 \delta^{18}\text{O}(\text{\%}) + 20.4 = 1.5 \times (-35) + 20.4 = -32.1^{\circ}\text{C}$$

notice, by the way, that since this sample is for glacial ice, not ocean water or sediment, the negative sign on $\delta^{18}\text{O}(\text{\%})$ means colder temperatures.

8.3 The flat earth!



$$E_{\text{absorbed}} = E_{\text{radiated}}$$

$$S\pi R^2 = \sigma T^4 A = \sigma T^4 (2\pi R^2)$$

$$T = \left(\frac{S}{2\sigma} \right)^{\frac{1}{4}} = \left(\frac{1370 \text{ W/m}^2}{2 \times 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4} \right)^{\frac{1}{4}} = 331.5 \text{ K} - 273.1 = 58.4^{\circ}\text{C}$$

8.4 The basic relationship is $S = \frac{k}{d^2}$, using data for Earth from Table 8.2 lets us find k:

$$k = S d^2 = 1370 \text{ W/m}^2 \times (150 \times 10^6 \text{ km} \times 10^3 \text{ m/km})^2 = 3.083 \times 10^{25} \text{ W}$$

a. Mercury: $S = \frac{k}{d^2} = \frac{3.083 \times 10^{25} \text{ W}}{(58 \times 10^6 \text{ km} \times 10^3 \text{ m/km})^2} = 9163 \text{ W/m}^2$

b. The effective temperature (8.7) of Mercury would be:

$$T_e = \left[\frac{S(1-\alpha)}{4\sigma} \right]^{\frac{1}{4}} = \left[\frac{9163 \text{ W/m}^2 (1-0.06)}{4 \times 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4} \right]^{\frac{1}{4}} = 441 \text{ K} \quad (168^{\circ}\text{C})$$

c. Peak wavelength:

$$\lambda_{\max} = \frac{2898}{T(K)} = \frac{2898}{441} = 6.6 \mu\text{m}$$

8.5 Solar flux variation of $\pm 3.3\%$, gives a range of S

$$S_{\max} = 1370 (1+0.033) = 1415.2 \text{ W/m}^2$$

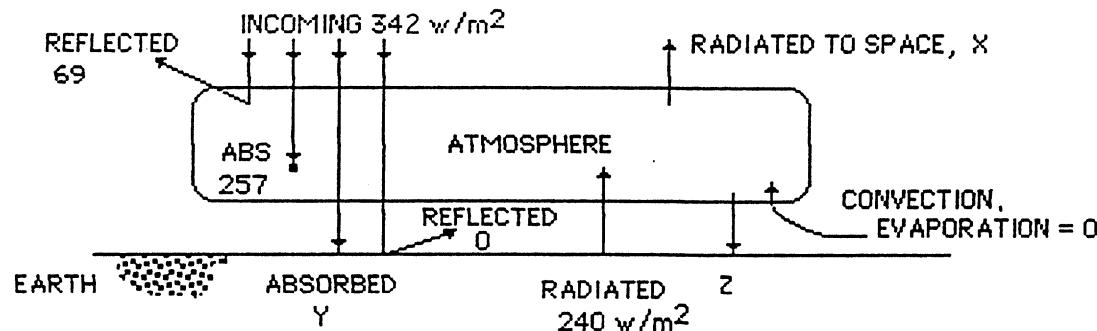
$$S_{\min} = 1370 (1 - 0.033) = 1324.8 \text{ W/m}^2$$

$$T_e, \max = \left[\frac{S(1-\alpha)}{4\sigma} \right]^{\frac{1}{4}} = \left[\frac{1415.2 \text{ W/m}^2 (1 - 0.31)}{4 \times 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4} \right]^{\frac{1}{4}} = 256.2 \text{ K} \quad (-17^\circ \text{C})$$

$$T_e, \min = \left[\frac{S(1-\alpha)}{4\sigma} \right]^{\frac{1}{4}} = \left[\frac{1324.8 \text{ W/m}^2 (1 - 0.31)}{4 \times 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4} \right]^{\frac{1}{4}} = 252 \text{ K} \quad (-21^\circ \text{C})$$

a difference of about 4°C .

8.6 A nuclear war:



a. Surface temperature,

$$\sigma T_s^4 = 240 \text{ W/m}^2$$

$$T_s = \left[\frac{240 \text{ W/m}^2}{5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4} \right]^{\frac{1}{4}} = 255 \text{ K} \quad (-18^\circ \text{C})$$

b. X, atmosphere to space: balance incoming from space = outgoing to space

$$342 = 69 + X \quad X = 273 \text{ W/m}^2$$

c. Y, absorbed by earth: incoming solar has to go somewhere,

$$342 = 69 + 257 + Y$$

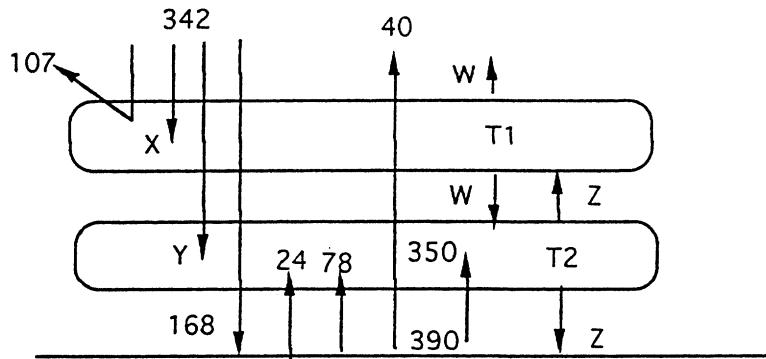
$$Y = 16 \text{ W/m}^2$$

d. Z, radiation from atmosphere to surface: balance earth's surface radiation,

$$Y + Z = 240 = 16 + Z$$

$$Z = 224 \text{ W/m}^2$$

8.7 A 2-layer atmosphere:



a. at the surface: $168 + Z = 24 + 78 + 390$ $Z = 324 \text{ W/m}^2$

b. extraterrestrial: $342 = 107 + 40 + W$ $W = 195 \text{ W/m}^2$

c. lower atmosphere:

$$Y + 24 + 78 + 350 + 195 = 2 \times 324 \quad Y = 1 \text{ W/m}^2$$

d. incoming: $342 = 107 + X + 1 + 168$ $X = 66 \text{ W/m}^2$

e. temperatures T_1 and T_2 , from

$$\sigma T_1^4 = W = 195 \quad T_1 = \left(\frac{195 \text{ W/m}^2}{5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4} \right)^{1/4} = 242 \text{ K } (-31^\circ \text{C})$$

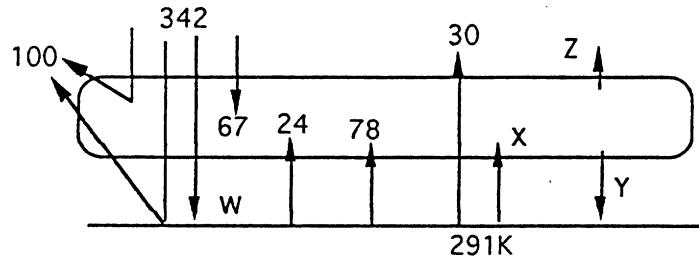
$$\sigma T_2^4 = Z = 324 \quad T_2 = \left(\frac{324 \text{ W/m}^2}{5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4} \right)^{1/4} = 275 \text{ K } (2^\circ \text{C})$$

8.8 Hydrologic cycle:

$$\text{evaporation} = \frac{78 \text{ W/m}^2 \times 5.1 \times 10^{14} \text{ m}^2 \times \frac{1 \text{ J/s}}{\text{W}} \times 3600 \frac{\text{s}}{\text{hr}} \times 24 \frac{\text{hr}}{\text{d}} \times 365 \frac{\text{d}}{\text{yr}}}{2465 \text{ kJ/kg} \times 10^3 \text{ kg/m}^3 \times 10^3 \text{ J/kJ}} = 5.1 \times 10^{14} \text{ m}^3$$

averaged over the globe, with area $5.1 \times 10^{14} \text{ m}^2$ is very close to 1 m of annual precipitation.

8.9 Greenhouse enhanced earth:



a. incoming energy: $342 = 100 + 67 + W$ $W = 175 \text{ W/m}^2$

b. find Z from radiation to space:

$$342 = 100 + 30 + Z \quad Z = 212 \text{ W/m}^2$$

c. to find X , need the energy radiated by a 291 K surface:

$$\text{surface radiated} = \sigma T^4 = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \times (291\text{K})^4 = 406.6 \text{ W/m}^2$$

$$\text{so that, } 406.6 = X + 30 \quad X = 376.6 \text{ W/m}^2$$

d. can find Y several ways; at the surface, or in the atmosphere,

$$W + Y = 406.6 + 24 + 78 = 175 + Y \quad Y = 333.6 \text{ W/m}^2$$

$$\text{or, } 67 + 24 + 78 + X = Y + Z \\ 67 + 24 + 78 + 376.6 = Y + 212 \quad (Y = 333.6)$$

8.10 Solar constant $S = 1370$, increase by 0.1%, $\Delta S = 0.001 \times 1370 = 1.37 \text{ W/m}^2$

The change in forcing is per unit area of earth,

$$\Delta F = \frac{\text{change in radiation hitting earth}}{\text{surface area of earth}} = \frac{1.37 \text{ W/m}^2 \times \pi R^2}{4\pi R^2} = \frac{1.37}{4} = 0.343 \text{ W/m}^2$$

Climate sensitivity of 0.55,

$$\Delta T = \lambda \Delta F = 0.55 \times 0.343 = 0.19^\circ\text{C}$$

8.11 Changing the albedo from 0.31 to 0.30 changes radiative forcing by

$$(0.31 - 0.30) \times 342 \text{ W/m}^2 = 3.42 \text{ W/m}^2$$

$$\text{Total } \Delta F = 3.42 + 4.0 = 7.42 \text{ W/m}^2$$

$$\text{and } \Delta T = \lambda \Delta F = 0.55 \times 7.42 = 4.1^\circ\text{C}$$

- 8.12 a. C_3HF_7 is an HFC (no Cl), $3 + 1 + 7 - 90 = 227$, HFC-227
 b. $C_2H_3FCl_2$ is an HCFC, $2 + 3 + 1 - 90 = 141$, HCFC-141
 c. $C_2F_4Cl_2$ is a CFC, $2 + 0 + 4 - 90 = 114$, CFC-114
 d. CF_3Br is a Halon, H-1301
- 8.13 a. HCFC-225, $225 + 90 = 315$ (3C, 1H, 5F), 8 sites - (1+5) = 2 Cl, $\therefore C_3HF_5Cl_2$
 b. HFC-32, $32 + 90 = 122$ (1C, 2H, 2F), 4 sites, 0 Cl, $\therefore CH_2F_2$
 c. H-1301, (1C, 3F, 0Cl, 1Br) $\therefore CF_3Br$
 d. CFC-114, $114 + 90 = 204$ (2C, 0H, 4F), 6 sites - 4 = 2 Cl, $\therefore C_2F_4Cl_2$

8.14 Radiative forcing for N_2O ,

$$\Delta F = k_2(\sqrt{C} - \sqrt{C_0})$$

$$k_2 = \frac{\Delta F}{(\sqrt{C} - \sqrt{C_0})} = \frac{0.14}{(\sqrt{311} - \sqrt{275})} = 0.133$$

If it reaches 417 ppb, added forcing would be:

$$\Delta F = k_2(\sqrt{C} - \sqrt{C_0}) = 0.133(\sqrt{417} - \sqrt{311}) = 0.37 W/m^2$$

8.15 Combined radiative forcings (ignoring the complication mentioned in Prob. 8.19):

$$a. \quad \Delta F_{CO_2} = 6.3 \ln \frac{[(CO_2)]}{[(CO_2)_0]} = 6.3 \ln \left(\frac{356}{278} \right) = 1.558 W/m^2$$

$$\Delta F_{CH_4} = 0.031 \left(\sqrt{CH_4} - \sqrt{(CH_4)_0} \right) = 0.031 \left(\sqrt{1714} - \sqrt{700} \right) = 0.463 W/m^2$$

$$\Delta F_{N_2O} = 0.133 \left(\sqrt{N_2O} - \sqrt{(N_2O)_0} \right) = 0.133 \left(\sqrt{311} - \sqrt{275} \right) = 0.140 W/m^2$$

$$\Delta F_{CFC-11} = 0.22 [(CFC-11) - (CFC-11)_0] = 0.22 (0.268 - 0) = 0.059 W/m^2$$

$$\Delta F_{CFC-12} = 0.28 [(CFC-12) - (CFC-12)_0] = 0.28 (0.503 - 0) = 0.141 W/m^2$$

$$\text{Combined forcing} = 1.558 + 0.463 + 0.140 + 0.059 + 0.141 = 2.36 W/m^2$$

b. From 1992 to 2100:

$$\Delta F_{CO_2} = 6.3 \ln \left[\frac{[CO_2]}{[CO_2]_0} \right] = 6.3 \ln \left(\frac{710}{356} \right) = 4.35 \text{ W/m}^2$$

$$\Delta F_{CH_4} = 0.031 \left(\sqrt{CH_4} - \sqrt{(CH_4)_0} \right) = 0.031 \left(\sqrt{3616} - \sqrt{1714} \right) = 0.581 \text{ W/m}^2$$

$$\Delta F_{N_2O} = 0.133 \left(\sqrt{N_2O} - \sqrt{(N_2O)_0} \right) = 0.133 \left(\sqrt{417} - \sqrt{311} \right) = 0.370 \text{ W/m}^2$$

$$\Delta F_{CFC-11} = 0.22 \left[(CFC-11) - (CFC-11)_0 \right] = 0.22(0.040 - 0.268) = -0.050 \text{ W/m}^2$$

$$\Delta F_{CFC-12} = 0.28 \left[(CFC-12) - (CFC-12)_0 \right] = 0.28(0.207 - 0.503) = -0.083 \text{ W/m}^2$$

$$\text{Combined forcing} = 4.35 + 0.581 + 0.370 - 0.050 - 0.083 = 5.17 \text{ W/m}^2$$

c. $\Delta F = 6.3 \ln \left(\frac{710}{278} \right) + 0.031 \left(\sqrt{3616} - \sqrt{700} \right) + 0.133 \left(\sqrt{417} - \sqrt{275} \right)$
 $+ 0.22 \times 0.040 + 0.28 \times 0.207 = 7.53 \text{ W/m}^2$

$$(\text{alternatively: } \Delta F = 2.36 + 5.17 = 7.53 \text{ W/m}^2)$$

8.16 From Prob. 8.15,

$$\Delta F = 6.3 \ln \left(\frac{710}{278} \right) + 0.031 \left(\sqrt{3616} - \sqrt{700} \right) + 0.133 \left(\sqrt{417} - \sqrt{275} \right)$$
 $+ 0.22 \times 0.040 + 0.28 \times 0.207 = 7.53 \text{ W/m}^2$

$$\Delta T_s = \lambda \Delta F = 0.57 \text{ } ^\circ\text{C/(W/m}^2\text{)} \times 7.53 \text{ W/m}^2 = 4.3 \text{ } ^\circ\text{C}$$

8.17 From Prob. 8.15

$$\Delta F = 6.3 \ln \left(\frac{710}{278} \right) + 0.031 \left(\sqrt{3616} - \sqrt{700} \right) + 0.133 \left(\sqrt{417} - \sqrt{275} \right)$$
 $+ 0.22 \times 0.040 + 0.28 \times 0.207 = 7.53 \text{ W/m}^2$

and from (8.32),

$$(CO_2)_{\text{equiv}} = 278 e^{\Delta F_{6.3}} = 278 e^{7.53} = 919 \text{ ppm}$$

8.18 From (8.34)

a. $\lambda = \frac{\Delta T_{2x}}{4.37} = \frac{2.0^\circ C}{4.37} = 0.46^\circ C/(W/m^2)$

b. Forcing by CO₂ alone (8.30): $\Delta F = 6.3 \ln\left(\frac{500}{278}\right) = 3.70 W/m^2$

c. Total forcing: $\Delta F = 6.3 \ln\left(\frac{600}{278}\right) = 4.85 W/m^2$

non-CO₂ forcing $\Delta F = 4.85 - 3.70 = 1.15 W/m^2$

d. Equilibrium temperature change,

$$\Delta T = \frac{\Delta T_{2x}}{\ln 2} \cdot \ln\left[\frac{CO_2}{(CO_2)_0}\right] = \frac{2.0^\circ C}{\ln 2} \cdot \ln\left(\frac{600}{278}\right) = 2.22^\circ C$$

e. Warming by CO₂: $\frac{\Delta F_{CO_2}}{\Delta F_{total}} = \frac{3.70}{4.85} = 0.76 = 76\%$

8.19 Overlapping absorption bands correction factor:

$$M=3616 \quad M_0=1714 \quad N=417 \quad N_0=311$$

$$f(M_0, N_0) = 0.47 \ln \left[1 + 2.01 \times 10^{-5} (M_0 N_0)^{0.75} + 5.31 \times 10^{-15} M_0 (M_0 N_0)^{1.52} \right]$$

$$f(M_0, N_0) = 0.47 \ln \left[1 + 2.01 \times 10^{-5} (1714 \times 311)^{0.75} + 5.31 \times 10^{-15} \times 1714 (1714 \times 311)^{1.52} \right] = 0.159$$

$$f(M, N_0) = 0.47 \ln \left[1 + 2.01 \times 10^{-5} (3616 \times 311)^{0.75} + 5.31 \times 10^{-15} \times 3616 (3616 \times 311)^{1.52} \right] = 0.256$$

$$\begin{aligned} \Delta F_{CH_4} &= 0.036 \left(\sqrt{CH_4} - \sqrt{(CH_4)_0} \right) - f(M, N_0) + f(M_0, N_0) \\ &= 0.036 \left(\sqrt{3616} - \sqrt{1714} \right) - 0.256 + 0.159 = 0.577 \approx 0.58 \end{aligned}$$

(this result is virtually the same as obtained in Example 8.6)

8.20 From (8.38), $GWP = \frac{\Delta F_g}{\Delta F_{CO_2}} \cdot \frac{\tau (1 - e^{-\gamma_t})}{\int R_{CO_2}(t) dt}$

a. $GWP_{20} = 1630 \cdot \frac{42(1 - e^{-2\gamma_{42}})}{13.2} = 1965$

$$b. GWP_{100} = 1630 \cdot \frac{42(1 - e^{-10\%_{42}})}{43.1} = 1440$$

$$c. GWP_{500} = 1630 \cdot \frac{42(1 - e^{-50\%_{42}})}{138} = 495$$

8.21	<u>years</u>	<u>gas</u>	<u>rate(10^9kg)</u>	<u>GWP</u>	<u>GWPxrate</u>	<u>fraction</u>
	20	CO ₂	44,700	1	44,700	0.65
	20	CH ₄	320	56	17,920	0.26
	20	N ₂ O	22	280	6,160	0.09
	100	CO ₂	44700	1	44,700	0.77
	100	CH ₄	320	21	6,720	0.11
	100	N ₂ O	22	310	6,820	0.12
	500	CO ₂	44700	1	44,700	0.88
	500	CH ₄	320	6.5	2,080	0.04
	500	N ₂ O	22	170	3,740	0.07

8.22 The actual $\Delta T_{\text{realized}}$ is estimated to be about 0.6°C , which is 75% of the equilibrium ΔT

$$\Delta T_{\text{realized}} = 0.6^{\circ}\text{C} = 0.75 \Delta T_{\text{equilibrium}}$$

$$\text{so, } \Delta T_{\text{equilibrium}} = 0.6 / 0.75 = 0.8^{\circ}\text{C}$$

$$\text{but, } \Delta T_{\text{equilibrium}} = \lambda \Delta F_{\text{actual}} = 0.57 \times \Delta F_{\text{actual}} = 0.8$$

$$\text{that is, } \Delta F_{\text{actual}} = \frac{0.8}{0.57} = 1.40 \text{ W/m}^2$$

The direct forcing is 2.45 W/m^2 , so aerosols etc are $2.45 - 1.40 = 1.05 \text{ W/m}^2$

8.23 Energy sources and carbon intensity:

Coal	25%	@ 24.2 gC/MJ
Oil	45%	@ 19.7 gC/MJ
Gas	20%	@ 13.8 gC/MJ
Other	10%	@ 0

$$a. \text{ avg C intensity} = 0.25 \times 24.2 + 0.45 \times 19.7 + 0.20 \times 13.8 + 0.10 \times 0 = 17.68 \text{ gC/MJ}$$

b. Coal replaced by non-carbon emitting sources:

$$\text{avg C intensity} = 0.25 \times 0 + 0.45 \times 19.7 + 0.20 \times 13.8 + 0.10 \times 0 = 11.63 \text{ gC/MJ}$$

c. modeled as an exponential growth function,

$$C = C_0 e^{rt}$$

$$r = \frac{1}{t} \ln\left(\frac{C}{C_0}\right) = \frac{1}{100} \ln\left(\frac{11.63}{17.68}\right) = -0.0042 = -0.42\%/\text{yr}$$

8.24 Out of oil and gas, demand = $2 \times 330 \text{ EJ/yr}$, 28% coal, 60% syn gas/oil@44gC/MJ,

a. carbon emission rate:

$$\text{avg carbon intensity} = 0.28 \times 25.8 + 0.60 \times 44 + 0.12 \times 0 = 33.6 \text{ gC/MJ}$$

$$\text{Emissions} = \frac{2 \times 330 \times 10^{18} \text{ J}}{\text{yr}} \times \frac{\text{MJ}}{10^6 \text{ J}} \times \frac{33.6 \text{ gC}}{\text{MJ}} \times \frac{\text{GtC}}{10^{15} \text{ gC}} = 22.2 \text{ GtC/yr}$$

b. growth from 6.0 GtC/yr to 22.2 GtC/yr in 100 yrs,

$$r = \frac{1}{100} \ln\left(\frac{22.2}{6.0}\right) = 0.013 = 1.3\%/\text{yr}$$

c. amount remaining with 50% airborne fraction, use (3.15):

$$\text{total emitted} = Q = \frac{P_0}{r} (e^{rt} - 1) = \frac{6.0 \text{ GtC/yr}}{0.01308} (e^{0.01308 \text{ yr} \times 100 \text{ yr}} - 1) = 1239 \text{ GtC}$$

$$\text{amount remaining in atmosphere} = 0.50 \times 1239 = 619 \text{ GtC}$$

d. amount in atmosphere in 100 yrs = $750 + 619 = 1369 \text{ GtC}$

$$(\text{CO}_2) = \frac{1369 \text{ GtC}}{2.12 \text{ GtC/ ppmCO}_2} = 646 \text{ ppm}$$

e. equilibrium temperature increase, with $\Delta T_{2x} = 3^\circ\text{C}$,

$$\Delta T = \frac{\Delta T_{2x}}{\ln 2} \cdot \ln\left[\frac{(\text{CO}_2)}{(\text{CO}_2)_0}\right] = \frac{3.0}{\ln 2} \cdot \ln\left(\frac{645}{356}\right) = 2.57^\circ\text{C}$$

8.25 Repeat of Prob. 8.24, but now conservation scenario:

a. carbon emission rate:

$$\text{avg carbon intensity} = 0.20 \times 25.8 + 0.30 \times 15.3 + 0.10 \times 20 = 11.75 \text{ gC/MJ}$$

$$\text{Emissions} = \frac{330 \times 10^{18} \text{ J}}{\text{yr}} \times \frac{\text{MJ}}{10^6 \text{ J}} \times \frac{11.75 \text{ gC}}{\text{MJ}} \times \frac{\text{GtC}}{10^{15} \text{ gC}} = 3.88 \text{ GtC/yr}$$

b. growth from 6.0 GtC/yr to 3.88 GtC/yr in 100 yrs,

$$r = \frac{1}{100} \ln\left(\frac{3.88}{6.0}\right) = -0.0044 = -0.44\%/\text{yr}$$

c. amount remaining with 50% airborne fraction, use (3.15):

$$\text{total emitted} = Q = \frac{P_0}{r} (e^{rt} - 1) = \frac{6.0 \text{GtC/yr}}{-0.0044} (e^{-0.0044 \cdot 100 \text{yr}} - 1) = 483 \text{GtC}$$

$$\text{amount remaining in atmosphere} = 0.50 \times 483 = 242 \text{ GtC}$$

d. amount in atmosphere in 100 yrs = $750 + 242 = 992 \text{ GtC}$

$$(CO_2) = \frac{992 \text{GtC}}{2.12 \text{GtC/ppmCO}_2} = 468 \text{ppm}$$

e. equilibrium temperature increase, with $\Delta T_{2x} = 3^\circ\text{C}$,

$$\Delta T = \frac{\Delta T_{2x}}{\ln 2} \cdot \ln\left[\frac{CO_2}{(CO_2)_0}\right] = \frac{3.0}{\ln 2} \cdot \ln\left(\frac{468}{356}\right) = 1.18^\circ\text{C}$$

8.26 Do it by scenario:

$$(A) \quad r = 1.0 + 0.3 - 2.0 - 0.7 = -1.4\%/\text{yr}$$

$$Q = \frac{P_0}{r} (e^{rt} - 1) = \frac{6.0 \text{GtC/yr}}{-0.014} (e^{-0.014 \cdot 70 \text{yr}} - 1) = 268 \text{GtC}$$

$$(CO_2) = \frac{750 \text{GtC} + Q \times AF}{2.12 \text{GtC/ppmCO}_2} = \frac{750 + 268 \times 0.4 \text{GtC}}{2.12 \text{GtC/ppmCO}_2} = 404 \text{ppm}$$

$$\Delta T = \frac{\Delta T_{2x}}{\ln 2} \cdot \ln\left[\frac{CO_2}{(CO_2)_0}\right] = \frac{3}{\ln 2} \cdot \ln\left(\frac{404}{356}\right) = 0.55^\circ\text{C}$$

$$\begin{aligned} t_d &= \frac{1}{r} \ln\left(\frac{Q}{P_0} r + 1\right) = \frac{1}{r} \ln\left[\frac{(750/AF)r}{P_0} + 1\right] \\ &= \frac{1}{-0.014} \ln\left[\frac{(750/0.4)(-0.014)}{6.0} + 1\right] = \text{never!!} \end{aligned}$$

$$(B) \quad r = 1.5 + 1.5 - 0.2 + 0.4 = 3.2\%/\text{yr}$$

$$Q = \frac{P_0}{r} (e^{rt} - 1) = \frac{6.0 \text{ GtC/yr}}{0.032} (e^{0.032t \text{ yr} \times 70 \text{ yr}} - 1) = 1574 \text{ GtC}$$

$$(\text{CO}_2) = \frac{750\text{GtC} + Q \times \text{AF}}{2.12\text{GtC / ppmCO}_2} = \frac{750 + 1574 \times 0.5\text{GtC}}{2.12\text{GtC / ppmCO}_2} = 725\text{ppm}$$

$$\Delta T = \frac{\Delta T_{2x}}{\ln 2} \cdot \ln \left[\frac{CO_2}{(CO_2)_0} \right] = \frac{2}{\ln 2} \cdot \ln \left(\frac{725}{356} \right) = 2.05^\circ C$$

$$t_d = \frac{1}{r} \ln \left[\frac{(750\% / AF)}{P_0} + 1 \right] = \frac{1}{0.032} \ln \left[\frac{(750\% / 0.5) 0.032}{6.0} + 1 \right] = 69 \text{ yrs}$$

$$(C) \quad r = 1.4 + 1.0 - 1.0 - 0.2 = 1.2\%/\text{yr}$$

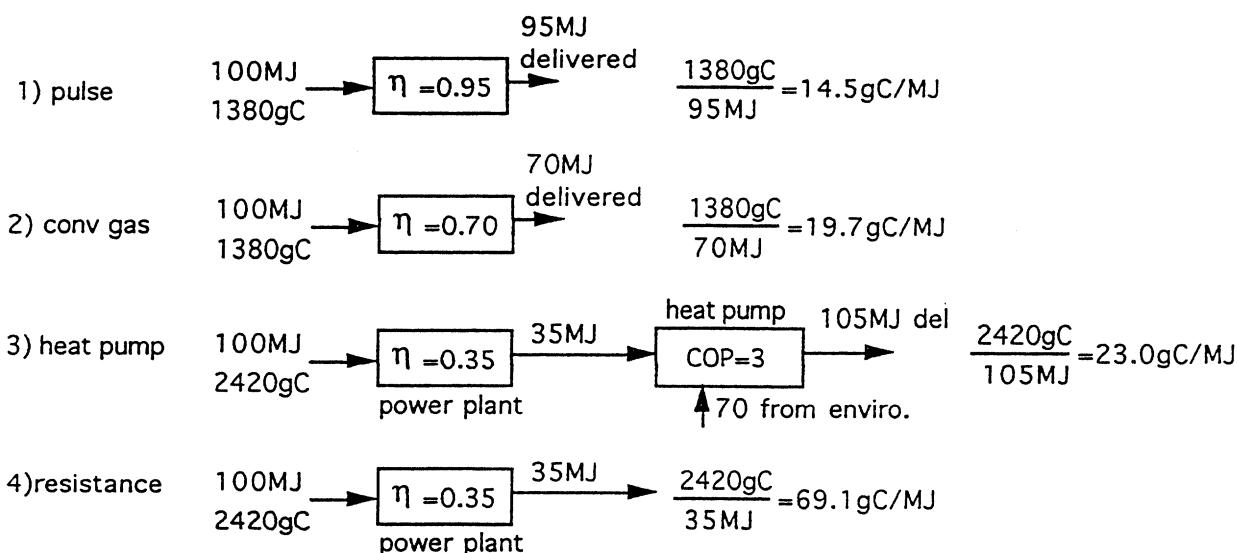
$$Q = \frac{P_0}{r} (e^{rt} - 1) = \frac{6.0 \text{GtC/yr}}{0.012} (e^{0.012 \cdot 70 \text{yr}} - 1) = 658 \text{GtC}$$

$$(\text{CO}_2) = \frac{750\text{GtC} + Q \times \text{AF}}{2.12\text{GtC/ ppmCO}_2} = \frac{750 + 658 \times 0.5\text{GtC}}{2.12\text{GtC/ ppmCO}_2} = 509\text{ppm}$$

$$\Delta T = \frac{\Delta T_{2x}}{\ln 2} \cdot \ln \left[\frac{CO_2}{(CO_2)_0} \right] = \frac{3}{\ln 2} \cdot \ln \left(\frac{509}{356} \right) = 1.55^\circ C$$

$$t_d = \frac{1}{r} \ln \left[\frac{(750\% / AF)_r}{P_0} + 1 \right] = \frac{1}{0.012} \ln \left[\frac{(750\% / 0.5) 0.012}{6.0} + 1 \right] = 116 \text{ yrs}$$

8.27 Using LHV carbon intensities from Table 8.9, the four options are:

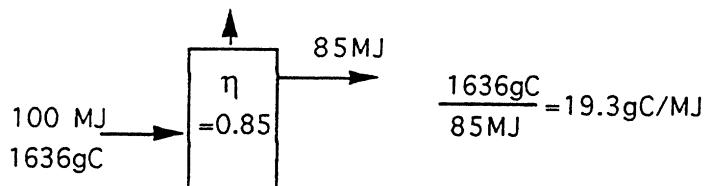


Notice the tremendous range: 14.5 to 69.1 gC/MJ, almost 5:1 !

8.28 Propane-fired water heater:

$$a. \ C_3H_8 = \frac{3 \times 12 \text{ gC/mol}}{2200 \text{ kJ/mol}} \times \frac{10^3 \text{ kJ}}{\text{MJ}} = 16.36 \text{ gC/MJ}$$

b. delivering heat at 85% efficiency to hot water,



c. savings vs 34.0 gC/MJ with an electric water heater:

$$\frac{\text{propane}}{\text{electric}} = \frac{19.3}{34.0} = 0.57 \quad \text{so there is a 43\% savings}$$

8.29 Using (3.18) to find σ , and (3.20) to find t_m , then plotting (3.17) gives,

$$a. \ Q_\infty = 200,000 \text{ EJ} \times \frac{25.8 \text{ gC}}{\text{MJ}} \times \frac{\text{GtC}}{10^{15} \text{ gC}} \times \frac{10^{12} \text{ MJ}}{\text{EJ}} = 5160 \text{ GtC}$$

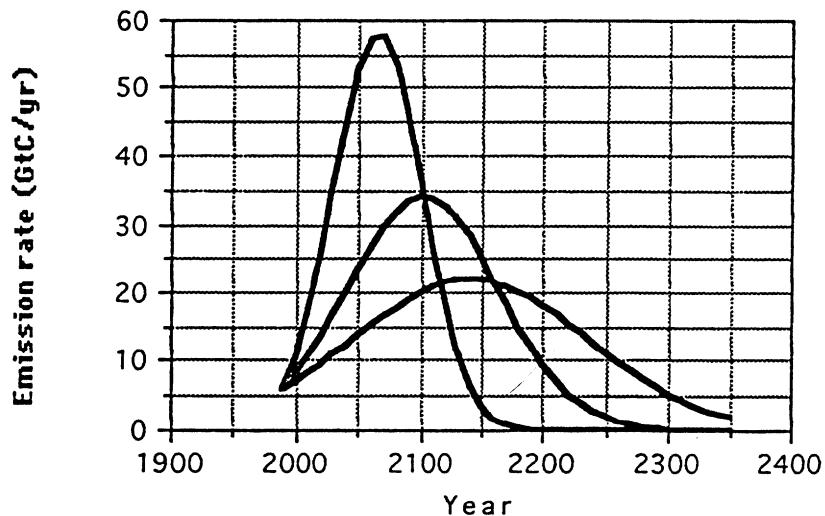
$$\sigma = \frac{Q_\infty}{P_m \sqrt{2\pi}} = \frac{5160 \text{ GtC}}{22 \text{ GtC/yr} \sqrt{2\pi}} = 93.57 \text{ yr}$$

$$t_m = \sigma \sqrt{2 \ln \left(\frac{P_m}{P_0} \right)} = 93.57 \text{ yr} \sqrt{2 \ln \left(\frac{22 \text{ GtC/yr}}{6.0 \text{ GtC/yr}} \right)} = 150.8 \text{ yr}$$

then put these into $P = P_m \exp \left[\frac{-1}{2} \left(\frac{t - t_m}{\sigma} \right)^2 \right]$

Qinf	200000	25.8	5160
Pm	22	34	58
Pm/Po	3.67	5.67	9.67
sigma	93.57	60.55	35.49
tm	150.84	112.77	75.60
delta time	10		
year	P		
1990	6.00	6.00	6.00
2000	7.09	8.05	10.51

Prob. 8.29 Solution



$$8.30 \quad Q_{\infty} = 200,000 \text{EJ} \times \frac{25.8 \text{gC}}{\text{MJ}} \times \frac{\text{GtC}}{10^{15} \text{gC}} \times \frac{10^{12} \text{MJ}}{\text{EJ}} = 5160 \text{GtC}$$

$$\Delta \text{CO}_2 = 5160 \text{GtC} \times 0.73 \times \frac{1 \text{ ppmCO}_2}{2.12 \text{ GtC}} = 1776 \text{ ppm}$$

$$\text{CO}_2 = 1776 + 280 = 2056 \text{ ppm}$$

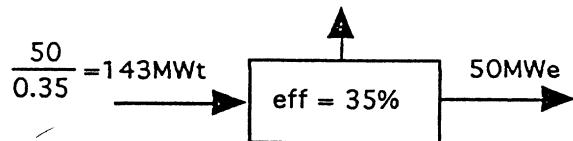
$$\Delta T = \frac{3}{\ln 2} \cdot \ln\left(\frac{2056}{280}\right) = 8.6^\circ\text{C}$$

$$8.31 \quad \text{a. } \text{C}_2\text{H}_6 : \frac{2 \times 12 \text{gC/mol}}{1542 \text{ kJ/mol}} \times \frac{10^3 \text{ kJ}}{\text{MJ}} = 15.56 \text{gC/MJ}$$

$$\text{b. } \text{C}_3\text{H}_8 : \frac{3 \times 12 \text{gC/mol}}{2220 \text{ kJ/mol}} \times \frac{10^3 \text{ kJ}}{\text{MJ}} = 16.22 \text{gC/MJ}$$

$$\text{c. } \text{C}_4\text{H}_{10} : \frac{4 \times 12 \text{gC/mol}}{2878 \text{ kJ/mol}} \times \frac{10^3 \text{ kJ}}{\text{MJ}} = 16.68 \text{gC/MJ}$$

8.32 A carbon tax of \$20/tonne of C (as CO₂):



- a. assuming a 100% capacity factor (plant operates all of the time),

$$C_{\text{emissions}} = \frac{50\text{MW}}{0.35} \times \frac{1\text{MJ/s}}{\text{MW}} \times \frac{3600\text{s}}{\text{hr}} \times \frac{8760\text{hr}}{\text{yr}} \times \frac{24\text{gC}}{\text{MJ}} \times \frac{\text{tonneC}}{10^6\text{gC}} = 1.08 \times 10^5 \text{tonneC / yr}$$

$$\text{tax} = 1.08 \times 10^5 \text{tonneC / yr} \times \frac{\$20}{\text{tonneC}} = \$2.16 \text{million / yr}$$

- b. carbon sequestering,

$$\text{Area} = \frac{1.08 \times 10^5 \text{tonneC / yr}}{5000\text{kgC / yr} \cdot \text{acre}} \times \frac{10^3 \text{kg}}{\text{tonne}} = 21,600 \text{acres}$$

- c. biomass instead of tax,

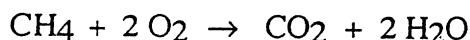
$$\text{Forestry cost} = \frac{\$2.16 \times 10^6 / \text{yr}}{21,600 \text{acres}} = \$100 / \text{yr per acre}$$

8.33 Leaky landfill, 10 tonnes CH₄ per year:

- a. 20-yr GWP for methane = 56,

$$10 \text{ tonnes CH}_4 / \text{yr} \times 56 = 560 \text{ tonnes CO}_2 (\text{equivalent})$$

- b. burning the methane,



$$\begin{aligned} \text{CO}_2 \text{ emitted} &= \frac{1 \text{ mol CO}_2}{\text{mol CH}_4} \times \frac{(12 + 2 \times 16) \text{ tonneCO}_2 / \text{mol}}{(12 + 4 \times 1) \text{ tonneCH}_4 / \text{mol}} \times \frac{10 \text{ tonneCH}_4}{\text{yr}} \\ &= 27.5 \text{ tonne CO}_2 / \text{yr} \end{aligned}$$

- c. equivalent CO₂ savings = 560 - 27.5 = 532.5 tonne CO₂

$$\text{as C: } 532.5 \text{ tonneCO}_2 / \text{yr} \times \frac{12 \text{ tonneC}}{44 \text{ tonne CO}_2} = 145.2 \text{ tonneC / yr saved}$$

- d. carbon tax saved = 145.2 tonneC/yr x \$20/tonneC = \$2900 / yr

- e. same thing, 532.5 tonne CO₂ saved x \$5.45/tonneCO₂ = \$2900/yr

8.34 Gasoline C₇H₁₅, 6.15 lbs/gal, fully combusted,

a. gasoline = $\frac{6.15 \text{ lb gas}}{\text{gal}} \times \frac{(7 \times 12 = 84) \text{ lbs C}}{(7 \times 12 + 15 \times 1 = 99) \text{ lb gas}} = 5.22 \text{ lbsC/gal}$

$$C = \frac{40,000 \text{ miles}}{12 \text{ miles/gal}} \times \frac{5.22 \text{ lbs C}}{\text{gal}} = 17,394 \text{ lbsC that will be released}$$

b. 4000 lb car, 10,000 mi/yr,

$$C = \frac{17,394 \text{ lbsC}}{40,000 \text{ mi}} \times \frac{10,000 \text{ mi}}{\text{yr}} = 4348 \text{ lbs C/yr}$$

$$\frac{\text{carbon}}{\text{vehicle wt.}} = \frac{4348 \text{ lbs C/yr}}{4000 \text{ lb}} = 1.09$$

the car emits slightly more carbon per year than it weighs!

c. carbon tax = $\frac{5.22 \text{ lbs C}}{\text{gal}} \times \frac{\$15}{2000 \text{ lbs C}} = \$0.039/\text{gal} = 3.9\text{¢/gal}$

d. new car @40mpg, for 40,000 miles:

$$\text{C reduction} = 17,394 \text{ lbsC} - \frac{40,000 \text{ mi}}{40 \text{ mi/gal}} \times \frac{5.22 \text{ lbsC}}{\text{gal}} = 12,174 \text{ lbs C saved}$$

e. trading in the clunker for the 40 mpg vehicle would save

$$\text{tax savings} = 12,174 \text{ lbsC} \times \frac{\$15}{2000 \text{ lbsC}} = \$91 \text{ per car}$$

that is, those C offsets would save the utility \$91, which they could spend to get the clunker off the road.

8.35 Electrics vs gasoline powered cars:

a. gas car emissions = $\frac{5.22 \text{ lbs C/gal}}{40 \text{ miles/gal}} \times \frac{1000 \text{ g}}{2.2 \text{ lbs}} = 59.3 \text{ gC/mi}$

b. with the very efficient natural-gas-fired power plant,

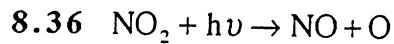
$$\text{electric emissions} = \frac{8000 \text{ kJ}}{\text{kWh}} \times \frac{13.8 \text{ gC}}{\text{MJ}} \times \frac{\text{MJ}}{10^3 \text{ kJ}} \times \frac{\text{kWh}}{5 \text{ mi}} = 22.1 \text{ gC/mi}$$

c. with the typical old coal plant,

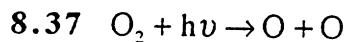
$$\text{coal plant heat rate} = \frac{1\text{kW heat in}}{0.30 \text{ kW electric out}} \times \frac{1\text{kJ/s}}{\text{kW heat in}} \times \frac{3600\text{s}}{\text{hr}} = 12,000\text{kJ/kWhe}$$

$$\text{electric emissions} = \frac{12,000\text{kJ}}{\text{kWh}} \times \frac{24\text{gC}}{\text{MJ}} \times \frac{\text{MJ}}{10^3\text{kJ}} \times \frac{\text{kWh}}{5\text{mi}} = 57.6\text{gC/mi}$$

So, carbon can be saved with electric cars when efficient natural-gas power plants are assumed, but for the typical old coal plant, very little if any savings accrue.



$$\lambda_{\max} = \frac{1.19 \times 10^{-4} \text{ kJ} \cdot \text{m/mol}}{\Delta H^0 \text{ kJ/mol}} = \frac{1.19 \times 10^{-4}}{306} = 388 \times 10^{-9} \text{ m} = 388 \text{ nm}$$



$$\lambda_{\max} = \frac{1.19 \times 10^{-4} \text{ kJ} \cdot \text{m/mol}}{\Delta H^0 \text{ kJ/mol}} = \frac{1.19 \times 10^{-4}}{495} = 240 \times 10^{-9} \text{ m} = 240 \text{ nm}$$

SOLUTIONS FOR CHAPTER 9

9.1 A 30 yd³ packer truck, 750 lb/yd³, 100' between stops, 5mph, 1 min to load 200 lb.

$$\text{time} = 100 \frac{\text{ft}}{\text{stop}} \times \frac{\text{mi}}{5280 \text{ft}} \times \frac{\text{hr}}{5 \text{mi}} \times \frac{60 \text{min}}{\text{hr}} + 1 \text{min} = 1.227 \text{min/ stop}$$

a.

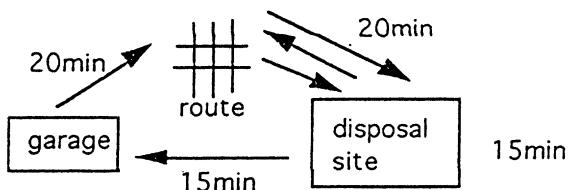
$$\text{loading} = 200 \frac{\text{lb}}{\text{stop}} \times \frac{\text{stop}}{1.227 \text{min}} = 163 \text{ lb/stop}$$

$$\text{time to fill} = 30 \text{yd}^3 \times 750 \frac{\text{lb}}{\text{yd}^3} \times \frac{\text{min}}{163 \text{ lbs}} \times \frac{\text{hr}}{60 \text{ min}} = 2.3 \text{ hrs}$$

$$\text{b. } \frac{\text{customers}}{\text{truckload}} = \frac{30 \text{yd}^3 \times 750 \text{ lb/yd}^3 \times 4 \text{ homes/stop}}{200 \text{ lb/stop}} = 450 \text{ homes / truckload}$$

$$\text{customers} = \frac{450 \text{ customers}}{\text{truckload}} \times \frac{2 \text{ loads}}{\text{day}} \times \frac{5 \text{ days}}{\text{week}} = 4500 \text{ customers}$$

9.2 Route timing.



$$\text{a. time not on route} = 20\text{min} + 3 \times 20\text{min} + 2 \times 15\text{min} + 15\text{min} + 40\text{min} = 165\text{min/day}$$

to fill a truck takes:

$$25 \text{yd}^3 \text{ truck} \times \frac{4 \text{ yd}^3 \text{ curb}}{\text{yd}^3 \text{ in truck}} \times \frac{\text{customer}}{0.2 \text{ yd}^3 \text{ curb}} \times \frac{\text{stop}}{4 \text{ customer}} \times \frac{1.5 \text{ min}}{\text{stop}} = 187.5 \frac{\text{min}}{\text{load}}$$

two loads per day takes:

$$\left[2 \text{ loads} \times 187.5 \frac{\text{min}}{\text{load}} + 165 \text{ min (travel, breaks)} \right] \times \frac{\text{hr}}{60 \text{ min}} = 9.0 \text{ hrs / day}$$

$$\text{b. Customers} = 25 \text{yd}^3 \text{ truck} \times \frac{4 \text{ yd}^3 \text{ curb}}{\text{yd}^3 \text{ in truck}} \times \frac{\text{customer}}{0.2 \text{ yd}^3 \text{ curb}} = 500 \text{ customers / load}$$

$$\# \text{ customers} = \frac{500 \text{ customers}}{\text{truckload}} \times \frac{2 \text{ loads}}{\text{day}} \times \frac{5 \text{ days}}{\text{week}} = 5000 \text{ customers}$$

$$\text{c. labor} = \left(\frac{\$40}{\text{hr}} \times \frac{8 \text{ hrs}}{\text{day}} + \frac{\$60}{\text{hr}} \times \frac{1 \text{ hr}}{\text{day}} \right) \times \frac{5 \text{ day}}{\text{wk}} \times \frac{52 \text{ wks}}{\text{yr}} = \$98,800 / \text{yr}$$

$$\text{trucks} = \frac{\$10,000}{\text{yr}} + \frac{\$3500 / \text{yr}}{\text{yd}^3} \times 25 \text{yd}^3 = \$97,500 / \text{yr}$$

$$\text{Customer cost} = \frac{(\$98,800 + \$97,500) / \text{yr}}{5000 \text{ customers}} = \$39.26 / \text{yr}$$

- 9.3 To avoid overtime pay, working 8 hrs per day and needing 165min to make the runs back and forth to the disposal site, breaks, etc (see Prob. 9.2),

$$\text{collection time} = 8 \text{hr} \times 60 \text{min/hr} - 165 \text{min} = 315 \text{ min/day}$$

$$\text{customers} = \frac{315 \text{min}}{\text{day}} \times \frac{\text{stop}}{1.5 \text{min}} \times \frac{4 \text{customers}}{\text{stop}} \times \frac{5 \text{day}}{\text{week}} = 4200 \text{customers}$$

notice the annual cost of service per customer is now

$$\$40/\text{hr} \times 8 \text{hr}/\text{dx} 5 \text{d/wk} \times 52 \text{wk/yr} + \$97,500 / 4200 = \$43/\text{yr}$$

- 9.4 So, with 8-hr days a smaller truck can be used. As in 9.3,

$$\text{collection time} = 8 \text{hr} \times 60 \text{min/hr} - 165 \text{min} = 315 \text{ min/day}$$

with 2 truckloads per day,

$$\text{customers} = \frac{315 \text{min}}{\text{day}} \times \frac{\text{stop}}{1.5 \text{min}} \times \frac{4 \text{customers}}{\text{stop}} = 840 \text{customers / day}$$

or 420 customers per truckload. At 2 loads per day and 5 days per week, that would give 4200 customers once a week service. Truck size needed is therefore,

$$\text{truck size} = \frac{420 \text{customers}}{\text{truckload}} \times \frac{0.2 \text{yd}^3 \text{at curb}}{\text{customer}} \times \frac{\text{yd}^3 \text{in truck}}{4 \text{yd}^3 \text{ at curb}} = 21 \text{yd}^3$$

$$\text{costing, trucks} = \frac{\$10,000}{\text{yr}} + \frac{\$3500 / \text{yd}^3}{\text{yr}} \times 21 \text{yd}^3 = \$83,500 / \text{yr}$$

$$\text{labor} = \left(\frac{\$40}{\text{hr}} \times \frac{8 \text{hrs}}{\text{day}} \right) \times \frac{5 \text{day}}{\text{wk}} \times \frac{52 \text{wks}}{\text{yr}} = \$83,200 / \text{yr}$$

$$\text{resulting in: Customer cost} = \frac{(\$83,200 + \$83,500) / \text{yr}}{4200 \text{ customers}} = \$39.69 / \text{yr}$$

(compared with \$39.26 per customer in Prob. 9.2)

9.5 Comparing two truck sizes,

a. customers for each truck:

(A) $27m^3$ truck:

$$27m^3 \text{ truck} \times \frac{4 m^3 \text{ curb}}{1 m^3 \text{ in truck}} \times \frac{\text{customer}}{0.25m^3 \text{ curb}} = 432 \text{ customers / load}$$

$$\# \text{ customers} = \frac{432 \text{ customers}}{\text{truckload}} \times \frac{2 \text{ loads}}{\text{day}} \times \frac{5 \text{ days}}{\text{week}} = 4320 \text{ customers } (27m^3)$$

(B) $15m^3$ truck:

$$15m^3 \text{ truck} \times \frac{4 m^3 \text{ curb}}{1 m^3 \text{ in truck}} \times \frac{\text{customer}}{0.25m^3 \text{ curb}} = 240 \text{ customers / load}$$

$$\# \text{ customers} = \frac{240 \text{ customers}}{\text{truckload}} \times \frac{3 \text{ loads}}{\text{day}} \times \frac{5 \text{ days}}{\text{week}} = 3600 \text{ customers } (15m^3)$$

b. hours per day for the crew:

(A) $27m^3$ truck:

$$432 \frac{\text{customers}}{\text{load}} \times \frac{0.4 \text{ min}}{\text{customer}} \times \frac{2 \text{ loads}}{\text{day}} = 346 \text{ min}$$

$$(346 \text{ min} + 160 \text{ min}) \times \frac{\text{hr}}{60 \text{ min}} = 8.43 \text{ hrs / day}$$

(B) $15m^3$ truck:

$$240 \frac{\text{customers}}{\text{load}} \times \frac{0.4 \text{ min}}{\text{customer}} \times \frac{3 \text{ loads}}{\text{day}} = 288 \text{ min}$$

$$(288 \text{ min} + 215 \text{ min}) \times \frac{\text{hr}}{60 \text{ min}} = 8.38 \text{ hrs / day}$$

c. cost per customer:

(A) $27m^3$ truck:

$$\frac{\$40}{\text{hr}} \times \frac{8.43 \text{ hr}}{\text{day}} \times \frac{5 \text{ day}}{\text{wk}} \times \frac{52 \text{ wk}}{\text{yr}} + \$120,000/\text{yr} = \$207,672/\text{yr } (27m^3)$$

$$\text{cost per customer} = \frac{\$207,672/\text{yr}}{4320 \text{ customers}} = \$48.07/\text{yr } (27m^3)$$

(B) $15m^3$ truck:

$$\frac{\$40}{hr} \times \frac{8.38\text{hr}}{\text{day}} \times \frac{5\text{day}}{\text{wk}} \times \frac{52\text{wk}}{\text{yr}} + \$70,000/\text{yr} = \$157,152/\text{yr}$$

$$\text{cost per customer} = \frac{\$157,152/\text{yr}}{3600\text{customers}} = \$43.65/\text{yr} \quad (15\text{m}^3)$$

The 15m³ truck results in a lower cost of customer service.

9.6 A \$150,000 truck, 2gal/mi, \$1.50/gal, 10,000mi/yr, \$20k/yr maintenance:

a. amortized at 12%, 8-yrs,

$$CRF(8\text{yr}, 12\%) = \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] = \left[\frac{0.12(1+0.12)^8}{(1+0.12)^8 - 1} \right] = 0.201/\text{yr}$$

$$\text{amortization} = \$150,000 \times 0.201/\text{yr} = \$30,195/\text{yr}$$

$$\text{fuel} = \frac{10,000\text{mi}}{\text{yr}} \times \frac{2\text{gal}}{\text{mi}} \times \frac{\$1.50}{\text{gal}} = \$30,000/\text{yr}$$

$$\text{total truck cost} = \$30,195 + 30,000 + 20,000 \text{ (maint)} = \$80,195/\text{yr}$$

b. labor = $\frac{\$25}{\text{hr} \cdot \text{person}} \times \frac{2\text{people}}{\text{truck}} \times \frac{40\text{hr}}{\text{wk}} \times \frac{52\text{wk}}{\text{yr}} = \$104,000/\text{yr}$

c. cost = $\frac{\$80,195 + \$104,000/\text{yr}}{10\text{tonnes/day} \times 260\text{day/yr}} = \$70.84/\text{tonne}$

9.7 Reworking Examples 9.1 - 9.3, using:

a. 1-run per day,

8hr - (gar to route) -(route to dump) - (time at dump) - (dump to gar) - (breaks)

$$\text{time left to collect} = 8 - 0.4 - 0.4 - 0.2 - 0.25 - 1 = 5.75 \text{ hrs}$$

which allows N stops per day (1 load per day),

$$N = \frac{5.75\text{hr/d} \times 3600\text{s/hr}}{60\text{s/stop} \times 1 \text{load/day}} = 345 \text{stops/ load}$$

truck volume needed is,

$$V = \frac{0.25\text{m}^3/\text{stop} \times 345 \text{ stops/load}}{3.5 \text{ m}^3 \text{ curb}/\text{m}^3 \text{in truck}} = 24.64\text{m}^3$$

with economics,

labor = \$62,400/yr as before,

truck = \$10,000 + 4000x24.64 = \$108,570/yr

serving 345 stops/load x 1 load/day x 5 days/wk = 1725 customers

$$\frac{\$108,570 + 62,400 / \text{yr}}{1725 \text{customers}} = \frac{\$170,970}{1725 \text{cust}} = \$99.11 / \text{yr per customer}$$

$$\text{refuse} = 0.25 \frac{\text{m}^3}{\text{house} \cdot \text{wk}} \times 52 \frac{\text{wk}}{\text{yr}} \times 120 \frac{\text{kg}}{\text{m}^3} \times 1725 \text{homes} \times \frac{\text{tonne}}{1000 \text{kg}} = 2691 \text{ tonne / yr}$$

$$\text{costing} = \frac{\$170,970 / \text{yr}}{2691 \text{tonne/yr}} = \$63.53 / \text{tonne} \quad (\text{all agrees with Table 9.9})$$

b. 3-runs per day:

8hr - (gar to route) -(route to dumpx5) - (time at dumpx3) - (dump to gar) - (breaks)

$$\text{time left to collect} = 8 - 0.4 - 0.4 \times 5 - 0.2 \times 3 - 0.25 - 1 = 3.75 \text{ hrs}$$

which allows N stops per day (1 load per day),

$$N = \frac{3.75 \text{hr/d} \times 3600 \text{s/hr}}{60 \text{s/stop} \times 3 \text{load/day}} = 75 \text{stops/load}$$

truck volume needed is,

$$V = \frac{0.25 \text{m}^3 / \text{stop} \times 75 \text{ stops/load}}{3.5 \text{ m}^3 \text{ curb/m}^3 \text{in truck}} = 5.36 \text{m}^3$$

with economics,

labor = \$62,400/yr as before,

truck = \$10,000 + 4000x5.36 = \$31,428/yr

serving 75 stops/load x 3 load/day x 5 days/wk = 1125 customers

$$\frac{\$31,428 + 62,400 / \text{yr}}{1125 \text{customers}} = \frac{\$93,828}{1125 \text{cust}} = \$83.40 / \text{yr per customer}$$

$$\text{refuse} = 0.25 \frac{\text{m}^3}{\text{house} \cdot \text{wk}} \times 52 \frac{\text{wk}}{\text{yr}} \times 120 \frac{\text{kg}}{\text{m}^3} \times 1125 \text{homes} \times \frac{\text{tonne}}{1000 \text{kg}} = 1755 \text{ tonne / yr}$$

$$\text{costing} = \frac{\$93,828 / \text{yr}}{1755 \text{tonne/yr}} = \$53.46 / \text{tonne} \quad (\text{all agrees with Table 9.9})$$

- 9.8** 200 tonnes/d, 5d/wk, \$3million, \$100,000/yr, trucks \$120,000, 20 tonne/trip, \$80k/yr, 4 trip/day, 5 d/wk, 10%, 10yr amortization,

$$\text{Station costs: } \text{CRF}(10\text{yr}, 10\%) = \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] = \left[\frac{0.10(1+0.10)^{10}}{(1+0.10)^{10} - 1} \right] = 0.16275/\text{yr}$$

$$\$3\text{million} \times 0.16275/\text{yr} + \$100,000/\text{yr} = \$588,236/\text{yr}$$

$$\text{to handle: } 200 \frac{\text{tonnes}}{\text{day}} \times 5 \frac{\text{day}}{\text{wk}} \times 52 \frac{\text{wk}}{\text{yr}} = 52,000 \text{tonnes/yr}$$

$$\text{which is } \text{cost} = \frac{\$588,236/\text{yr}}{52,000 \text{tonnes/yr}} = \$11.31/\text{tonne}$$

Truck costs:

$$\text{depreciation} = \$120,000 \times \text{CRF}(10\text{yr}, 10\%) = \$120,000 \times 0.16275/\text{yr} = \$19,530/\text{yr}$$

$$\text{driver + maintenance + depreciation} = \$80,000 + 19,530 = \$99,530/\text{yr}$$

$$\text{to haul} \quad 20 \frac{\text{tonnes}}{\text{trip}} \times 4 \frac{\text{trip}}{\text{day}} \times 5 \frac{\text{day}}{\text{wk}} \times 52 \frac{\text{wk}}{\text{yr}} = 20,800 \text{tonnes/truck - yr}$$

$$\text{which is } \frac{\$99,530/\text{yr}}{20,800 \text{tonne/yr}} = \$4.79/\text{tonne}$$

$$\text{for a total of } \$4.79 + \$11.31 = \$16.10 \text{ per tonne}$$

- 9.9** a. Cost of direct haul to the disposal site,

$$\$40 + 30 t_1 = 40 + 30 \times 1.5 = \$85/\text{tonne}$$

- b. transfer station 0.3 hr from collection route,

$$\$40 + 30 t_1 = 40 + 30 \times 0.3 = \$49/\text{tonne} \text{ to get to the transfer station}$$

for the transfer station,

$$\$10 + 10 t_2 = 10 + 10 (1.5 \text{hr} - 0.3 \text{hr}) = \$22/\text{tonne}$$

$$\text{total cost} = \$49 + 22 = \$71/\text{tonne}$$

- c. minimum distance from the transfer station to the disposal site,

direct haul \$ = \$ to transfer station + \$ for transfer station

$$\$40 + \$30/\text{hr} 1.5\text{hr} = (\$40 + 30 t_1) + \$10 + 10(1.5 - t_1)$$

$$85 = 40 + 10 + 15 + 30 t_1 - 10 t_1 = 65 + 20 t_1$$

$$t_1 = 1 \text{ hr} \quad t_2 = 1.5 - 1 = 0.5 \text{ hr}$$

that is, the transfer station must be at least 0.5 hrs away from the disposal site.

- 9.10** At 50% recycling rate, a 16g can has 8g of new aluminum and 8 g of recycled aluminum. Using data from Table 9.12,

$$\text{new aluminum} = 8\text{g} \times 235,000 \frac{\text{J}}{\text{g}} \times \frac{\text{kJ}}{10^3 \text{J}} = 1880 \text{kJ}$$

$$\text{recycled aluminum} = 8\text{g} \times 5150 \frac{\text{J}}{\text{g}} \times \frac{\text{kJ}}{10^3 \text{J}} = 41.2 \text{kJ}$$

$$\text{total energy required} = 1880 + 41.2 = 1921 \text{ kJ/can for the aluminum}$$

From Table 9.13,

$$\text{energy/can} = 1921 \text{ kJ} + 421 + 4 + 955 + 9\text{g recycled} \times \frac{3\text{kJ}}{10\text{g}} = 3303 \text{ kJ}$$

$$\text{per liter, energy} = \frac{3303 \text{kJ}}{0.355 \text{L}} = 9306 \text{ kJ / can}$$

- 9.11** Heavier cans from yesteryear, 0.0205 kg/can, and at 25% recycling rate, says

$$\text{new aluminum per can is } 0.75 \times 20.5\text{g/can} = 15.375 \text{ g,}$$

$$\text{and the recycled amount is } 20.5 - 15.375 = 5.125 \text{ g.}$$

$$\text{new aluminum} = 15.375\text{g} \times 235,000 \frac{\text{J}}{\text{g}} \times \frac{\text{kJ}}{10^3 \text{J}} = 3613 \text{kJ}$$

$$\text{recycled aluminum} = 5.125\text{g} \times 5150 \frac{\text{J}}{\text{g}} \times \frac{\text{kJ}}{10^3 \text{J}} = 26.4 \text{kJ}$$

$$\text{total energy for aluminum} = 3613 + 26 = 3639 \text{ kJ/can}$$

From Example 9.5, primary energy for aluminum today is about 1443 kJ/can

$$\frac{1443 \text{kJ / can}}{3639 \text{kJ / can}} = 0.396 \quad \text{so the savings is a bit over 60% !}$$

9.12 U.S. using 1.6×10^6 tonnes/yr Al, 63% recycling rate,

- a. Primary energy for aluminum,

$$\text{new aluminum} = 0.37 \times 1.6 \times 10^6 \text{ tonne} \times 1000 \frac{\text{kg}}{\text{tonne}} \times 235,000 \frac{\text{kJ}}{\text{kg}} = 139.1 \times 10^{12} \text{ kJ}$$

$$\text{recycled aluminum} = 0.63 \times 1.6 \times 10^6 \text{ tonne} \times 1000 \frac{\text{kg}}{\text{tonne}} \times 5150 \frac{\text{kJ}}{\text{kg}} = 5.2 \times 10^{12} \text{ kJ}$$

$$\text{Total primary energy for aluminum} = (139.1 + 5.2) \times 10^{12} = 144.3 \times 10^{12} \text{ kJ}$$

- b. With no recycling,

$$\text{all new aluminum} = 1.6 \times 10^6 \text{ tonne} \times 1000 \frac{\text{kg}}{\text{tonne}} \times 235,000 \frac{\text{kJ}}{\text{kg}} = 376 \times 10^{12} \text{ kJ}$$

- c. CO₂ emissions (Table 9.12)

- 1) With recycling:

$$\text{new aluminum} = 0.37 \times 1.6 \times 10^6 \text{ tonneAl} \times 13.1 \frac{\text{tonneCO}_2}{\text{tonneAl}} = 7.75 \times 10^6 \text{ tonneCO}_2$$

$$\text{recycled Al} = 0.63 \times 1.6 \times 10^6 \text{ tonneAl} \times 0.48 \frac{\text{tonneCO}_2}{\text{tonneAl}} = 0.48 \times 10^6 \text{ tonneCO}_2$$

$$\text{Total} = (7.75 + 0.48) \times 10^6 = 8.23 \times 10^6 \text{ tonne CO}_2/\text{yr}$$

- 2) Without recycling:

$$\text{new aluminum} = 1.6 \times 10^6 \text{ tonneAl} \times 13.1 \frac{\text{tonneCO}_2}{\text{tonneAl}} = 20.96 \times 10^6 \text{ tonneCO}_2/\text{yr}$$

$$\text{CO}_2 \text{ reduction with recycling} = (20.96 - 8.23) \times 10^6 \text{ tonne} = 12.7 \times 10^6 \text{ tonne CO}_2/\text{yr}$$

9.13 3 million tonnes/yr of Al, 35% recovery rate,

- a. CO₂ emissions:

$$\text{new Al} = 0.65 \times 3.0 \times 10^6 \text{ tonneAl} \times 13.1 \frac{\text{tonneCO}_2}{\text{tonneAl}} = 25.54 \times 10^6 \text{ tonneCO}_2/\text{yr}$$

$$\text{recycled AL} = 0.35 \times 3.0 \times 10^6 \text{ tonneAl} \times 0.48 \frac{\text{tonneCO}_2}{\text{tonneAl}} = 0.50 \times 10^6 \text{ tonneCO}_2/\text{yr}$$

$$\text{Total} = (25.54 + 0.50) \times 10^6 = 26 \times 10^6 \text{ tonne CO}_2/\text{yr}$$

b. Primary energy for the aluminum:

$$\text{new Al} = 0.65 \times 3.0 \times 10^6 \text{tonneAl} \times 235,000 \frac{\text{kJ}}{\text{kg}} \times 1000 \frac{\text{kg}}{\text{tonne}} = 458 \times 10^{12} \text{kJ/yr}$$

$$\text{recycled Al} = 0.35 \times 3.0 \times 10^6 \text{tonneAl} \times 5150 \frac{\text{kJ}}{\text{kg}} \times 1000 \frac{\text{kg}}{\text{tonne}} = 5.4 \times 10^{12} \text{kJ/yr}$$

$$\text{Total energy} = (458 + 5.4) \times 10^{12} \text{ kJ/yr} = 463 \times 10^{12} \text{ kJ/yr}$$

9.14 Newsprint, 5.97% moisture, HHV=18540kJ/kg, 6.1% H. Starting with 1 kg of as received waste,

$$\text{energy vaporizing moisture} = 0.0597 \text{kgH}_2\text{O} \times 2440 \frac{\text{kJ}}{\text{kg}} = 145.6 \text{kJ}$$

$$1 \text{kg as received: dry weight} = 1 - 0.0597 = 0.9403 \text{ kg}$$

$$\text{hydrogen in the dry waste} = 0.061 \times 0.9403 = 0.0574 \text{ kg}$$

$$\text{energy as H becomes H}_2\text{O} = 0.0574 \text{kgH} \times \frac{9 \text{kgH}_2\text{O}}{\text{kgH}} \times 2440 \frac{\text{kJ}}{\text{kg}} = 1259.6 \text{kJ}$$

$$\text{total energy lost in water vapor} = 145.6 + 1259.6 = 1405 \text{ kJ/kg}$$

$$\text{LHV} = \text{HHV} - 1405 = 18,540 - 1,405 = 17,135 \text{ kJ/kg}$$

9.15 Corrugated boxes, 5.2% moisture, HHV=16,380kJ/kg, 5.7%H in dried material,

$$\text{Using (9.8)} \quad Q_L = 2440(W + 9H)$$

$$W = 0.052 \text{ kgH}_2\text{O/kg waste}, \quad H = (1 - 0.052) \times 0.057 = 0.054 \text{ kgH/kgwaste}$$

$$Q_L = 2440 \frac{\text{kJ}}{\text{kgH}_2\text{O}} (0.052 + 9 \times 0.054) \frac{\text{kgH}_2\text{O}}{\text{kg}} = 1313 \frac{\text{kJ}}{\text{kg}}$$

$$\text{LHV} = \text{HHV} - Q_L = 16,380 - 1313 = 15,067 \text{ kJ/kg}$$

9.16 2L PET bottle, 54g, 14% H, HHV = 43,500kJ/kg,

$$Q_L = 2440 \frac{\text{kJ}}{\text{kgH}_2\text{O}} \times 9 \frac{\text{kgH}_2\text{O}}{\text{kgH}} \times 0.14 \frac{\text{kgH}}{\text{kgPET}} \times \frac{54 \text{gPET}}{10^3 \text{g/kg}} = 166 \text{ kJ/bottle}$$

$$\text{LHV} = \text{HHV} - Q_L = \left(43,500 \frac{\text{kJ}}{\text{kgPET}} \times \frac{54 \text{gPET / bottle}}{10^3 \text{g/kg}} - 166 \frac{\text{kJ}}{\text{bottle}} \right) = 2183 \text{ kJ/bottle}$$

9.17 Energy estimates based on HHV = 339(C) + 1440(H) - 139(O) + 105(S)

a. corrugated boxes: based on dry weight,

$$\text{HHV(dry)} = 339 \times 43.73 + 1440 \times 5.70 - 139 \times 44.93 + 105 \times 0.21 = 16,809 \text{ kJ/kg}$$

"as received" there are $(1 - 0.052) = 0.948 \text{ kg dry material / kg of "as received"}$

$$\text{HHV as received} = 0.948 \text{ kg(dry)} \times 16,809 \text{ kJ/kg(dry)} = 15,935 \text{ kJ/kg}$$

b. junk mail:

$$\text{HHV(dry)} = 339 \times 37.87 + 1440 \times 5.41 - 139 \times 42.74 + 105 \times 0.09 = 14,697 \text{ kJ/kg}$$

$$\text{HHV(as received)} = (1 - 0.0456) \times 14,697 = 14,027 \text{ kJ/kg}$$

c. mixed garbage:

$$\text{HHV(dry)} = 339 \times 44.99 + 1440 \times 6.43 - 139 \times 28.76 + 105 \times 0.52 = 20,568 \text{ kJ/kg}$$

$$\text{HHV(as received)} = (1 - 0.72) \times 20,568 = 5759 \text{ kJ/kg}$$

d. lawn grass:

$$\text{HHV(dry)} = 339 \times 46.18 + 1440 \times 5.96 - 139 \times 36.43 + 105 \times 0.42 = 19,218 \text{ kJ/kg}$$

$$\text{HHV(as received)} = (1 - 0.7524) \times 19,218 = 4758 \text{ kJ/kg}$$

e. demolition softwood:

$$\text{HHV(dry)} = 339 \times 51.0 + 1440 \times 6.2 - 139 \times 41.8 + 105 \times 0.1 = 20,417 \text{ kJ/kg}$$

$$\text{HHV(as received)} = (1 - 0.077) \times 20,417 = 18,845 \text{ kJ/kg}$$

f. tires:

$$\text{HHV(dry)} = 339 \times 79.1 + 1440 \times 6.8 - 139 \times 5.9 + 105 \times 1.5 = 35,944 \text{ kJ/kg}$$

$$\text{HHV(as received)} = (1 - 0.0102) \times 35,944 = 35,578 \text{ kJ/kg}$$

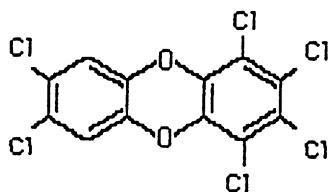
g. polystyrene:

$$\text{HHV(dry)} = 339 \times 87.10 + 1440 \times 8.45 - 139 \times 3.96 + 105 \times 0.02 = 41,147 \text{ kJ/kg}$$

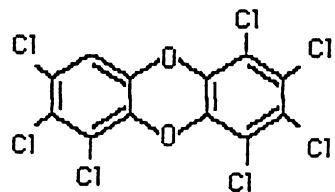
$$\text{HHV(as received)} = (1 - 0.002) \times 41,147 = 41,064 \text{ kJ/kg}$$

9.18 Draw the chemical structures:

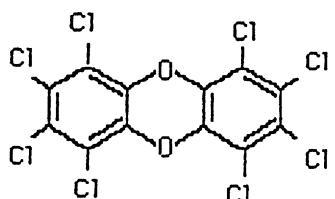
a. 1,2,3,4,7,8-hexachlorodibenzo-p-dioxin



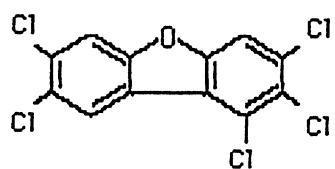
b. 1,2,3,4,6,7,8-heptachlorodibenzo-p-dioxin



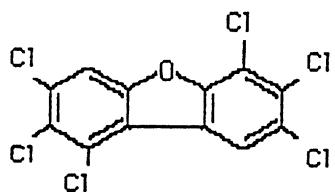
c. octachlorodibenzo-p-dioxin



d. 2,3,4,7,8-pentachlorodibenzofuran



e. 1,2,3,6,7,8-hexachlorodibenzofuran



9.19 U.S. 129 million tons, 800 lb/yd³, cell 10ft, 1 lift/yr, 80% is MSW, 1000 people:

$$V_{MSW} = 129 \times 10^6 \frac{\text{ton}}{\text{yr}} \times \frac{2000 \text{ lb}}{\text{ton}} \times \frac{\text{yd}^3}{800 \text{ lb}} \times \frac{27 \text{ ft}^3}{\text{yd}^3} = 8.71 \times 10^9 \text{ ft}^3 / \text{yr}$$

@80% per cell,

$$V_{landfill} = \frac{8.71 \times 10^9 \text{ ft}^3 / \text{yr}}{0.80} = 10.9 \times 10^9 \text{ ft}^3 / \text{yr}$$

$$A_{lift} = \frac{10.9 \times 10^9 \text{ ft}^3 / \text{yr}}{10 \text{ ft/lift}} \times \frac{\text{acre}}{43,560 \text{ ft}^2} = 24,987 \approx 25,000 \text{ acres/yr}$$

population of the U.S. is around 260 million, so the area per 1000 people is

$$A_{lift \text{ per } 1000} = \frac{24,987 \text{ acres/yr}}{260,000 \text{ thousand people}} = 0.096 \approx 0.1 \text{ acre/1000 people - yr}$$

9.20 50,000 people, 40,000 tons/yr, 22% recovery, 1000 lb/yd³, 10ft lift, 80%MSW:

a.

$$V_{landfill} = 40,000 \frac{\text{ton}}{\text{yr}} \times (1 - 0.22) \times \frac{2000 \text{ lb}}{\text{ton}} \times \frac{\text{yd}^3}{1000 \text{ lb}} \times \frac{27 \text{ ft}^3}{\text{yd}^3} \times \frac{\text{ft}^3 \text{ landfill}}{0.80 \text{ ft}^3 \text{ MSW}} = 2.11 \times 10^6 \text{ ft}^3 / \text{yr}$$

$$A_{lift} = \frac{2.11 \times 10^6 \text{ ft}^3 / \text{yr}}{10 \text{ ft/lift}} = 2.11 \times 10^5 \text{ ft}^2 / \text{yr} \quad \times \frac{\text{acre}}{43,560 \text{ ft}^2} = 4.83 \text{ acre/yr}$$

b. to complete the landfill will take:

$$\text{time remaining} = \frac{40 \frac{\text{acres}}{\text{lift}} \times 2 \text{ lifts}}{4.83 \text{ acre/yr}} = 16.5 \text{ yrs}$$

9.21 By increasing its recovery rate to 40 percent,

$$V_{landfill} = 40,000 \frac{\text{ton}}{\text{yr}} \times (1 - 0.40) \times \frac{2000 \text{ lb}}{\text{ton}} \times \frac{\text{yd}^3}{1000 \text{ lb}} \times \frac{27 \text{ ft}^3}{\text{yd}^3} \times \frac{\text{ft}^3 \text{ landfill}}{0.80 \text{ ft}^3 \text{ MSW}} = 1.62 \times 10^6 \text{ ft}^3 / \text{yr}$$

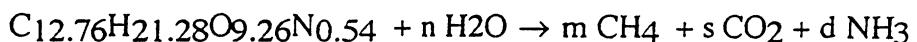
$$A_{lift} = \frac{1.62 \times 10^6 \text{ ft}^3 / \text{yr}}{10 \text{ ft/lift}} = 1.62 \times 10^5 \text{ ft}^2 / \text{yr} \quad \times \frac{\text{acre}}{43,560 \text{ ft}^2} = 3.72 \text{ acre/yr}$$

$$\text{time remaining} = \frac{40 \frac{\text{acres}}{\text{lift}} \times 2 \text{ lifts}}{3.72 \text{ acre/yr}} = 21.5 \text{ yrs} \quad (\text{added 5 more years vs P9.20})$$

9.22 Lawn trimmings, 620g moisture, 330 g decomposable organics represented by C_{12.76}H_{21.28}O_{9.26}N_{0.54}

$$1 \text{ mol of trimmings} = 12 \times 12.76 + 1 \times 21.28 + 16 \times 9.26 + 14 \times 0.54 = 330.2 \text{ g/mol}$$

that is, 1 kg of as received trimmings has 330g of decomposable organics (1 mole) using (9.9) gives



$$\text{where } m = (4 \times 12.76 + 21.28 - 2 \times 9.26 - 3 \times 0.54)/8 = 6.5225$$

so, 6.5225 moles of methane are produced per mole of dry trimmings, which is the same as 6.5225 moles of methane produced per kg of "as received" trimmings.

a. $V_{\text{CH}_4} = \frac{0.0224 \text{ m}^3 \text{ CH}_4}{\text{mol CH}_4} \times \frac{6.5225 \text{ mol CH}_4}{\text{kg "as received"}} = 0.146 \text{ m}^3 \text{ CH}_4 / \text{kg}$

b. $\text{CH}_4 \text{ energy} = \frac{6.5225 \text{ mol CH}_4}{\text{kg "as received"}} \times \frac{890 \text{ kJ}}{\text{mol}} = 5805 \text{ kJ / kg}$

9.23 1 kg of food wastes has 720g of water and 280 g of dry C_aH_bO_cN_d

a.	C	45%	$0.45 \times 280 = 126 \text{ g}$
	H	6.4%	$0.064 \times 280 = 17.92 \text{ g}$
	O	28.8%	$0.288 \times 280 = 80.64 \text{ g}$
	N	3.3%	$0.033 \times 280 = 9.24 \text{ g}$
Total = 233.8g/mol			

$$\text{C: } 12 \text{ g/mol} \times a \text{ mol} = 126 \text{ g, so } a = 126/12 = 10.5 \text{ mol}$$

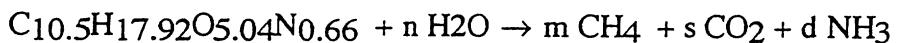
$$\text{H: } 1 \text{ g/mol} \times b \text{ mol} = 17.92 \text{ g, so } b = 17.92/1 = 17.92 \text{ mol}$$

$$\text{O: } 16 \text{ g/mol} \times c \text{ mol} = 80.64 \text{ g, so } c = 80.64/16 = 5.04 \text{ mol}$$

$$\text{N: } 14 \text{ g/mol} \times d \text{ mol} = 9.24 \text{ g, so } d = 9.24/14 = 0.66 \text{ mol}$$

The chemical formula for dry food wastes: C_{10.5}H_{17.92}O_{5.04}N_{0.66}

b. chemical reaction:

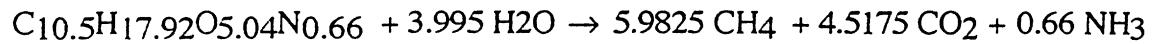


$$\text{where } n = (4 \times 10.5 - 17.92 - 2 \times 5.04 + 3 \times 0.66)/4 = 3.995$$

$$m = (4 \times 10.5 + 17.92 - 2 \times 5.04 - 3 \times 0.66)/8 = 5.9825$$

$$s = (4 \times 10.5 - 17.92 + 2 \times 5.04 + 3 \times 0.66)/8 = 4.5175$$

$$d = 0.66$$



c. $\text{CH}_4 = \frac{5.9825 \text{ moles CH}_4}{(5.9825 + 4.5175 + 0.66) \text{ moles gas}} = 0.536 = 53.6\%$

d. $V_{\text{CH}_4} = \frac{0.0224 \text{ m}^3 \text{ CH}_4}{\text{mol CH}_4} \times \frac{5.9825 \text{ mol CH}_4}{\text{kg "as received"}} = 0.134 \text{ m}^3 \text{ CH}_4 / \text{kg}$

e. $\text{CH}_4 \text{ energy} = \frac{5.9825 \text{ mol CH}_4}{\text{kg "as received"}} \times \frac{890 \text{ kJ}}{\text{mol}} = 5324 \text{ kJ/kg "as received"}$