

# CONVERSIONI E UNITÀ DI MISURA

## esercizio 1

Convertire una frazione molare di  $10^{-3}$  di CCl<sub>4</sub> in acqua in ppm (massivo) e in concentrazione (mol/L)

Dati:  $m_{CCl_4} = 154 \text{ g/mol}$

$$m_{H_2O} = 18 \text{ g/mol}$$

$$\rho_{sol} = 1030 \text{ g/L}$$

$$ppm = \frac{10^{-3} \cdot 154}{10^{-3} \cdot 154 + (1 - 10^{-3}) \cdot 18} \cdot 10^6 = 8490 \text{ ppm}$$

$$\tilde{c} = \frac{moli}{V_{tot}} \rightarrow X_i \frac{moli_i}{mole_{tot}} \cdot \rho \frac{\text{massa tot}}{\text{vol tot}} \cdot \frac{1}{MM} \frac{\text{massa tot}}{\text{moli tot}}$$

$$\hookrightarrow \tilde{c} = \frac{x_i \cdot \rho}{MM} \rightsquigarrow MM [\text{g/mol}] = 10^{-3} \cdot 154 + (1 - 10^{-3}) \cdot 18 = 18,136 \text{ g/mol}$$

$$\tilde{c} = \frac{10^{-3} \cdot 1030}{18,136} = 0,0568 \text{ mole/L}$$

## esercizio 2

Convertire 5 ppm (volume) di CH<sub>4</sub> in mg/Nm<sup>3</sup>

$$m_{CH_4} = 16 \text{ g/mol}$$

$$5 \text{ ppm} = \frac{5 \text{ mole CH}_4}{10^6 \text{ mole tot}} \rightarrow 5 \text{ mole CH}_4 \cdot 16 \frac{\text{g CH}_4}{\text{mole CH}_4} = 80 \text{ g CH}_4 = 80000 \text{ mg CH}_4$$

$$10^6 \text{ mole} \rightarrow V = \frac{10^6 \cdot 0,082 \cdot 273}{1} = 22,4 \cdot 10^6 \text{ NL} = 22,4 \cdot 10^3 \text{ Nm}^3$$

$$\tilde{c} = \frac{80000 \text{ mg CH}_4}{22,4 \cdot 10^3 \text{ Nm}^3} = 3,57 \text{ mg/Nm}^3$$

## esercizio 3

Calcolare la portata massica e volumetrica di NH<sub>3</sub> disiolto in un liquido che ha una portata di 20 m<sup>3</sup>/s e una concentrazione molaia di 3 mol/L.

$$\tilde{F}_i = 20 \cdot 10^3 \frac{\text{dm}^3 \text{ soluzione}}{\text{s}} \cdot 3 \cdot 10^{-3} \frac{\text{mole NH}_3}{\text{dm}^3 \text{ soluzione}} = 60 \text{ mole} \frac{\text{NH}_3}{\text{s}}$$

$$F_i = 60 \frac{\text{mole NH}_3}{\text{s}} \cdot 17 \frac{\text{g NH}_3}{\text{mole NH}_3} = 1020 \text{ g NH}_3/\text{s} = 1,02 \text{ kg NH}_3/\text{s}$$

#### esercizio 4

Sia dato un flusso di gas con  $X_i = 10^{-4}$  di SO<sub>3</sub> (massica).

Se  $F_i$  max è pari a 50 g/s, quanto vale  $Q_v$  max in Nm<sup>3</sup>/s?

Dati :  $F_i = 50 \frac{g}{s}$  gas  $\Rightarrow Q_v [Nm^3] ?$   
 $X_i = 10^{-4} \frac{g SO_3}{g gas}$

$$F_i \cdot X_i = 50 \cdot 10^{-4} = 5 \cdot 10^{-3} g SO_3 / s$$

$$MM_{SO_3} = 80 g/mole$$

$$\tilde{F}_i = \frac{5 \cdot 10^{-3}}{80} = 6,25 \cdot 10^{-5} mole SO_3 / s$$

$$1 : 22,4 = 6,25 \cdot 10^{-5} : x \longrightarrow x = 1,4 \cdot 10^{-3} NL = 1,4 \cdot 10^{-6} \frac{Nm^3 SO_3}{s}$$

#### esercizio 5

Una cartiera produce 30t/h di carta usando 20 kg/h di sbiancante, di cui il 2% è perso nelle acque di scarico.

Considerando che lo standard di qualità ambientale per lo sbiancante è di 5mg/L nelle acque di scarico e che il limite di emissione è di 3 g/h, calcolare:

- 1) Il flusso di sbiancante che deve essere rimosso durante il processo di trattamento delle acque reflue e l'efficienza della rimozione
- 2) La portata massima di acqua di scarico (a 5mg/L) che può essere scartata (e quindi la portata di acqua trattata)
- 3) Il fattore di emissione con e senza l'impianto di trattamento

#### punto 1 :

$$\frac{2}{100} \cdot 20 = 0,4 \text{ kg/h} = 400 \text{ g/h} \text{ di sbiancante}$$

↪ 397 g/h sono rimossi

$$\eta = \frac{397}{400} = 99,25 \%$$

#### punto 2

$$Q_v \left[ \frac{L}{h} \right] = \frac{3 \text{ g/h}}{5 \cdot 10^{-3} \text{ g/L}} = 600 \text{ L/h}$$

#### punto 3

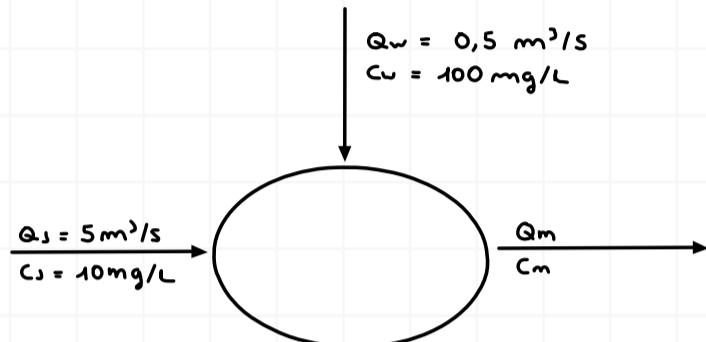
$$Fe = \frac{3 \text{ g/h}}{30 \cdot 10^3 \text{ kg/h}} = 10^{-4} \text{ g sb/kg carta con impianto}$$

$$Fe = \frac{400}{30 \cdot 10^3} = 0,0133 \text{ g sb/kg carta senza impianto}$$

# BILANCI DI MATERIA ED ENERGIA

## esercizio 1: Lago inquinato

Consideriamo di avere un lago di volume  $V=10.000.000 \text{ m}^3$  entro cui arriva un fiume che ha una corrente mediamente inquinata e una portata di  $5 \text{ m}^3/\text{s}$ . Esso porta con sé un inquinante ad una concentrazione di  $10 \text{ mg/L}$ . Oltre a questo fiume nel lago fluisce anche lo scarico di una fogna con una portata di  $0.5 \text{ m}^3/\text{s}$  e una concentrazione di inquinante di  $100 \text{ mg/L}$ . All'interno del lago è presente una popolazione di batteri che permette di abbattere questi inquinanti producendo  $\text{CO}_2$  e  $\text{H}_2\text{O}$ . La velocità di consumo di questo inquinante è proporzionale ad una costante cinetica  $K$  pari a  $0.2 \text{ 1/giorni}$ . Assumendo che ci sia un emissario a questo lago e supponendo che quelli appena elencati siano gli unici ingressi e le uniche uscite del lago, calcolare la concentrazione di stazionario di inquinante in uscita supponendo che il lago sia perfettamente miscelato.



$$\text{Bilancio generale di massa: } Q_s \cdot \rho + Q_w \cdot \rho = Q_m \cdot \rho \rightarrow Q_m = 5,5 \text{ m}^3/\text{s}$$

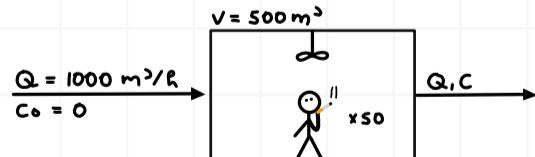
$$\text{Bilancio sulla specie: } 0 = Q_s \cdot C_s + Q_w \cdot C_w - Q_m \cdot C_m - K C_m V$$

$$C_m = \frac{Q_s \cdot C_s + Q_w \cdot C_w}{Q_m + KV}$$

$$C_m = \frac{5 \cdot 10^3 \frac{\text{L}}{\text{s}} \cdot 10 \frac{\text{mg}}{\text{L}} + 0,5 \cdot 10^3 \frac{\text{L}}{\text{s}} \cdot 100 \frac{\text{mg}}{\text{L}}}{5,5 \cdot 10^3 \frac{\text{L}}{\text{s}} + 10 \cdot 10^5 \text{L} \cdot 0,2 \frac{1}{86400 \text{s}}} = 3,45 \text{ mg/L}$$

## esercizio 2: Bar con fumatori

Supponiamo di avere un bar co un volume di  $500 \text{ m}^3$  e in cui si può fumare. Dentro questo bar ci sono 50 persone, ognuna delle quali fuma 2 sigarette all'ora e ogni singola sigaretta emette  $1,4 \text{ mg}$  di formaldeide  $\text{HCHO}$ . La formaldeide si degrada in anidride carbonica tramite una reazione che ha una costante cinetica  $K = 0,4 \text{ 1/ora}$ . L'aria fresca entra nel bar con una portata di  $1000 \text{ m}^3/\text{ora}$  e l'aria viziata esce con la stessa portata. Assumendo un miscela mento completo, stimare la concentrazione di formaldeide nell'aria a  $25^\circ\text{C}$  e  $1 \text{ atm}$ . Confrontare il valore trovato con quella che è una condizione limited  $0.05 \text{ ppm}$  al di sopra della quale non è possibile stare dentro al bar senza manifestare un'irritazione agli occhi.



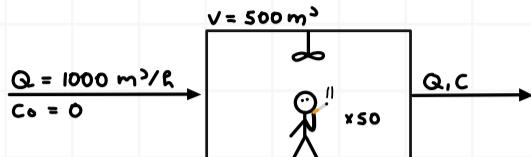
$$\text{Calcolo la quantità di HCHO emessa in un'ora: } 1,4 \text{ mg} \cdot 2 \cdot 50 = 140 \text{ mg/h di HCHO}$$

$$\text{Bilancio: } 0 = Q \cancel{C_0} + S - KCV - Q \cdot C \rightarrow C = \frac{S}{KV + Q} = \frac{140 \text{ mg/h}}{500 \text{ m}^3 \cdot 0,4 \frac{1}{\text{h}} + 1000 \text{ m}^3} = 0,117 \text{ mg/m}^3 = 3,9 \cdot 10^{-6} \frac{\text{mole}}{\text{m}^3}$$

$$1 \text{ m}^3 \text{ aria} \rightarrow n = \frac{101325 \text{ Pa} \cdot 1 \text{ m}^3}{8,31 \frac{\text{J}}{\text{molK}} \cdot 298 \text{ K}} = 40,9 \text{ mole} \rightarrow \frac{3,9 \cdot 10^{-6}}{40,9} \cdot 10^6 = 0,055 \text{ ppm} > 0,05 \text{ ppm}$$

### esercizio 3: Bar con fumatori non stazionario

Nelle stesse condizioni dell'esercizio 2, supponiamo che il bar apra alle 17:00 e che allo stesso orario l'area interna del bar sia pulita ( $C_0 = 0$ ). Monitorare la concentrazione di formaldeide all'interno del bar considerando una cinetica del primo ordine. Determinare la concentrazione dopo un'ora dall'apertura del bar.



$$C(t) = C_\infty + [C_0 - C_\infty] \cdot \exp\left(-\frac{t}{\tau}\right)$$

$$\cdot C_\infty = \frac{S}{Q + KV} = \frac{140 \text{ mg}/\text{h}}{500 \text{ m}^3 \cdot 0,4 \text{ h} + 1000 \text{ m}^3 \text{ h}} = 0,117 \text{ mg/m}^3 = 3,9 \cdot 10^{-6} \frac{\text{mol}}{\text{m}^3}$$

$$\cdot \frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{10}} = K + \frac{Q}{V} = 0,4 \frac{1}{\text{h}} + \frac{1000 \text{ m}^3/\text{h}}{500 \text{ m}^3} = 2,4 \text{ 1/h}$$

$$C(1 \text{ h}) = 0,117 \text{ mg/m}^3 + [0 - 0,117 \text{ mg/m}^3] \cdot \exp\left(-1 \text{ h} \cdot 2,4 \frac{1}{\text{h}}\right) = 0,106 \text{ mg/m}^3$$

### esercizio 4: Lago inquinato non stazionario

Nelle stesse condizioni dell'esercizio 1, supponiamo adesso di non avere più il contributo della corrente proveniente dalla fogna. Come cambia la concentrazione di inquinante dopo una settimana dalla rimozione di quest'ultima?



$$\begin{cases} V \cdot \frac{dC}{dt} = Q_s \cdot C_s - QC - KCV \\ C(0) = C_m = 3,5 \text{ mg/L} \quad (\text{dall'es. precedente}) \end{cases} \rightarrow Q_s = Q$$

$$\cdot C_\infty = \frac{Q_s \cdot C_s}{Q_s + KV} = \frac{5000 \cdot 10}{5000 + 0,2 \cdot 10 \cdot 10^6} = 1,78 \frac{\text{mg}}{\text{L}}$$

$$\cdot \frac{1}{\tau} = K + \frac{Q_s}{V} = 0,2 + \frac{5}{10 \cdot 10^6} = 0,2 \text{ 1/giorni}$$

$$C(7 \text{ gg}) = 1,78 + [3,5 - 1,78] \cdot \exp(-7 \cdot 0,2) = 2,2 \text{ mg/L}$$

### esercizio 5: inquinamento termico di un fiume

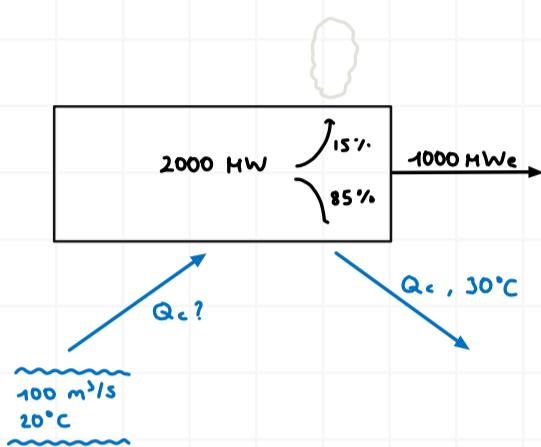
Supponiamo di avere una centrale a carbone che produce 1000MW elettrici con un'efficienza del 33%, quindi brucia l'equivalente di 3000MW termici di carbone di cui 2000MW devono essere dissipati.

Il 15% di questo calore se ne va nei fumi, mentre l'altro 85% deve essere smaltito nelle acque di raffreddamento.

In prossimità della centrale è presente un fiume con una portata di  $100 \text{ m}^3/\text{s}$  e una temperatura di  $20^\circ\text{C}$

Noi possiamo prendere una certa portata del fiume e utilizzarla come sistema di raffreddamento per asportare il calore della centrale (quindi l'85% dei 2000 MW termici). E' chiaro che a questo punto la temperatura dell'acqua in uscita dalla centrale sarà maggiore di quella in ingresso, e per legge non deve superare i  $30^\circ\text{C}$ .

Quanta acqua devo prelevare dal fiume? Posso farlo, cioè il fiume è abbastanza grande?



$$\dot{Q}_{\text{out}} = \frac{85}{100} \cdot 2000 = 1700 \text{ MW} = 1700 \cdot 10^6 \text{ J/s}$$

$$\dot{Q} = \dot{m}_c \cdot c_p \cdot (T_{\text{out}} - T_{\text{in}})$$

$$1700 \cdot 10^6 = \dot{m}_c \text{ kg/s} \cdot 4184 \frac{\text{J}}{\text{kg}\cdot\text{C}} \cdot 10^\circ\text{C} \rightarrow \dot{m}_c = 40631 \text{ kg/s}$$

$$Q_c = \frac{40631 \text{ kg/s}}{1000 \text{ kg/m}^3} = 40,63 \frac{\text{m}^3}{\text{s}} < 100 \text{ m}^3/\text{s}$$

↪ il fiume è abbastanza grande

ATTENZIONE: La temperatura del fiume per legge non deve cambiare di più di  $3^\circ\text{C}$ .

Supponendo di prendere tutta l'acqua proveniente dal fiume, siamo in regola?

$$\dot{Q} = \dot{m} \cdot c_p \cdot (T_{\text{out}} - T_{\text{in}})$$

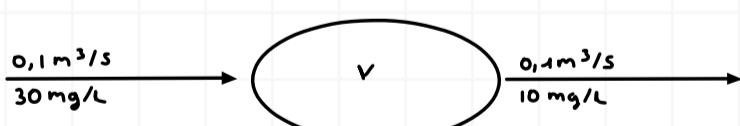
$$\Delta T = \frac{1700 \cdot 10^6}{100 \cdot 10^3 \cdot 4184} = 4,06^\circ\text{C}$$

### caso generico (impostazione)

$$\dot{m}_{\text{in}} \cdot c_p \cdot (T_{\text{in}} - T_{\text{ref}}) + \dot{Q} = \dot{m}_{\text{out}} \cdot c_p \cdot (T_{\text{out}} - T_{\text{ref}})$$

### esercizio 1.13 (Masters)

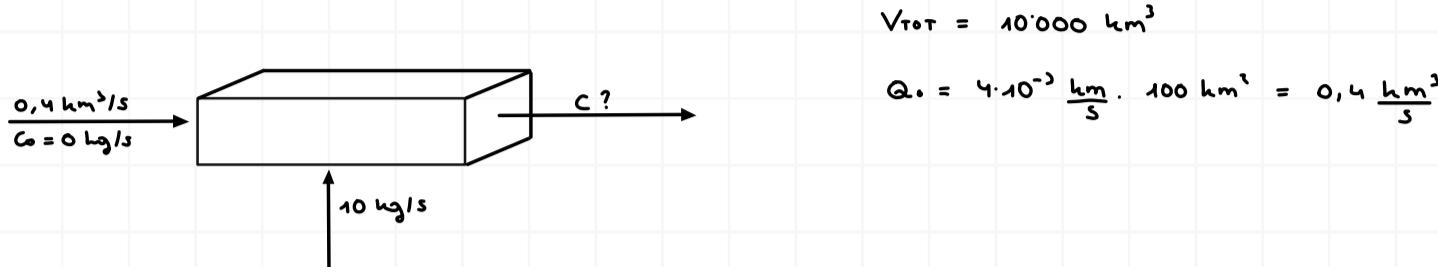
A lagoon is to be designed to accomodate an input flow of  $0.10 \text{ m}^3/\text{s}$  of nonconservative pollutant with concentration  $30.0 \text{ mg/L}$  and reaction rate  $0.20/\text{day}$ . The effluent from the lagoon must have pollutant concentration of less than  $10.0 \text{ mg/L}$ . Assuming complete mixing, how large must the lagoon be?



$$0 = 0,1 \cdot 10^3 \frac{\text{L}}{\text{s}} \cdot 30 \frac{\text{mg}}{\text{L}} - 0,1 \cdot 10^3 \frac{\text{L}}{\text{s}} \cdot 10 \frac{\text{mg}}{\text{L}} - \frac{0,2}{86400\text{s}} \cdot 10 \frac{\text{mg}}{\text{L}} \cdot V \rightarrow V = 86400000 \text{ L}$$

### esercizio 1.17

Consider the air over a city to be a box 100 km on a side that reaches up to an altitude of 1.0 km. Clean air is blowing into the box along one of its sides with a speed of 4 m/s. Suppose an air pollutant with reaction rate  $k = 0.20/\text{hr}$  is emitted into the box at a total rate of 10.0 kg/s. Find the steady-state concentration if the air is assumed to be completely mixed.



$$C_\infty = \frac{S}{Q + KV} = \frac{10 \text{ kg/s}}{0,4 \frac{\text{kg}}{\text{s}} + 10000 \text{ km}^3 \cdot \frac{0,2}{3600 \text{ s}}} = 10,5 \text{ kg/km}^3$$

### esercizi 1.19 e 1.20

A lagoon with volume 1,200 m<sup>3</sup> has been receiving a steady flow of a conservative waste at a rate of 100 m<sup>3</sup>/day for a long enough time to assume that steady-state conditions apply. The waste entering the lagoon has a concentration of 10 mg/L. Assuming completely mixed conditions,

- (a) What would be the concentration of pollutant in the effluent leaving the lagoon?
- (b) If the input waste concentration suddenly increased to 100 mg/L, what would the concentration in the effluent be 7 days later?

Repeat Problem 19 for a nonconservative pollutant with reaction rate  $k = 0.20/\text{d}$ .

- a) Se il bilancio è conservativo e stazionario, allora input = output

Dal bilancio materiale vediamo che la portata in ingresso è uguale a quella in uscita, quindi anche le concentrazioni

$$C = 10 \text{ mg/L}$$

$$\text{b)} \quad C(t) = C_\infty + [C_0 - C_\infty] \cdot \exp \left[ -t \cdot \left( K + \frac{Q}{V} \right) \right] \quad \longrightarrow \quad C_\infty = 100 \text{ mg/L}$$

$$\downarrow \quad K = 0$$

$$\begin{aligned} C(7) &= 100 + [10 - 100] \cdot \exp \left( -5 \cdot \frac{100}{1200} \right) \\ &= 40,67 \text{ mg/L} \end{aligned}$$

Adesso ripeto ma con  $K = 0,2/\text{day}$

$$\text{a)} \quad 0 = 100 \cdot 10^3 \frac{\text{L}}{\text{d}} \cdot 10 \frac{\text{mg}}{\text{L}} - 0,2 \frac{1}{\text{d}} \cdot C \frac{\text{mg}}{\text{L}} \cdot 1200 \cdot 10^3 \text{L} - 100 \cdot 10^3 \frac{\text{L}}{\text{a}} \cdot C$$

$$C = 2,94 \text{ mg/L}$$

$$\text{b)} \quad C(t) = C_\infty + [C_0 - C_\infty] \cdot \exp \left[ -t \cdot \left( K + \frac{Q}{V} \right) \right]$$

$$C_\infty = \frac{100 \cdot 10^3 \cdot 100}{0,2 \cdot 1200 \cdot 10^3 + 100 \cdot 10^3} = 29,41 \text{ mg/L}$$

$$C(7) = 29,41 + [2,94 - 29,41] \cdot \exp \left[ -7 \cdot \left( 0,2 + \frac{100}{1200} \right) \right] = 25,77 \text{ mg/L}$$

### esercizio bar

Supponiamo di avere un bar co un volume di  $500 \text{ m}^3$  e in cui si può fumare. Dentro questo bar ci sono 50 persone, ognuna delle quali fuma 2 sigarette all'ora e ogni singola sigaretta emette 1,4 mg di formaldeide HCHO. La formaldeide si degrada in anidride carbonica tramite una reazione che ha una costante cinetica  $K = 0,4 \text{ 1/ora}$ . L'aria fresca entra nel bar con una portata di  $1000 \text{ m}^3/\text{ora}$  e l'aria viziata esce con la stessa portata. Assumendo un miscela mento completo, stimare la concentrazione di formaldeide nell'aria a  $25^\circ\text{C}$  e 1 atm. Confrontare il valore trovato con quella che è una condizione limited 0.05 ppm al di sopra della quale non è possibile stare dentro al bar senza manifestare un'irritazione agli occhi.

Calcoliamo la portata di formaldeide all'ora

$$1,4 \frac{\text{mg}}{\text{sig}} \cdot 2 \frac{\text{sig}}{\text{h}} \cdot 50 = 140 \text{ mg/h}$$

$$\text{Stazionario: } O = 140 \frac{\text{mg}}{\text{h}} - C \cdot 1000 \frac{\text{m}^3}{\text{h}} - 0,4 \cdot C \cdot 500 \text{ m}^3$$

$$C = \frac{140 \text{ mg/h}}{1000 \text{ m}^3/\text{h} + 0,4 \cdot 500 \text{ m}^3/\text{h}} = 0,117 \frac{\text{mg}}{\text{m}^3} = 3,9 \cdot 10^{-6} \text{ mol/m}^3$$

$$\text{Quante mol di aria in } 1 \text{ m}^3? \quad n_{\text{aria}} = \frac{101325 \cdot 1 \text{ m}^3}{8,31 \cdot 298} = 40,92 \text{ mol/m}^3$$

$$\text{ppm} = \frac{3,9 \cdot 10^{-6}}{40,92} \cdot 10^6 = 0,095 \text{ ppm} > 0,05 \text{ ppm}$$

### esercizio 1.15

Your stomach is a chemical reactor. When you devour a fast food 99¢ special hamburger in about a minute, it acts like an instantaneous input of 325 g of food entering the stomach. In response, the stomach starts producing gastric liquids (acids), which are continuously excreted into the stomach at a rate of 12.0 mL/min as the hamburger is digested. The fluid also leaves the stomach to the small intestine at a flow rate of 12.0 mL/min, so the volume of liquid in the stomach stays constant at 1.15 L. The hamburger digestion rate constant is  $1.33 \text{ hr}^{-1}$ .

- (a) What kind of ideal reactor would you model your stomach as?
- (b) What fraction of the hamburger's mass will remain undigested in your stomach one hour after you eat the hamburger?

$$C(t) = C_\infty + [C_0 - C_\infty] \cdot \exp\left[-t\left(K + \frac{Q}{V}\right)\right]$$

- $C_\infty = 0 \text{ g/L}$
- $C_0 = \frac{325}{1,15} = 282,61 \text{ g/L}$
- $Q = 12 \text{ mL/min} = 720 \text{ mL/h}$
- $V = 1,15 \text{ L} = 1150 \text{ mL}$

$$C(1 \text{ h}) = 282,61 \cdot \exp\left[-\left(1,33 + \frac{720}{1150}\right)\right] = 39,96 \text{ g/L}$$

$$39,96 = \frac{X}{100} \cdot 282,61 \rightarrow X = 14,14 \% \text{ non digerito}$$

**esercizio 1.16**

Ozone is sometimes used as a disinfectant for drinking water. Ozone is highly reactive. It will react with many benign species found in water, as well as the pathogens it is intended to kill. It is found that ozone reaction in water nearly follows first-order kinetics in ozone concentration, such that its concentration decreases by 50 percent in 12 minutes ( $t_{1/2} = 12$  min). A water supplier wants to inject ozone into a pipe bringing water to the water treatment plant to predisinfect the influent. The 3.0-ft. diameter pipe is 3,400 feet long with a steady flow rate of 10,000 gal/min. What concentration of ozone (in mg/L) should be injected at the head of the pipe so that there will be an ozone concentration of 1.0 mg/L at the pipe's exit into the plant? The pipe may be realistically assumed to act as an ideal PFR.

$$\text{Volume del tubo: } 3400 \cdot \left(\frac{3}{2}\right)^2 \pi = 24033,18 \text{ ft}^3$$

$$10\,000 \text{ gal/min} = 1336,81 \text{ ft}^3/\text{min}$$

$$t = \frac{24033,18}{1336,81} = 17,98 \text{ min}$$

$$\frac{x_0}{2} = x_0 \cdot e^{-kt_{1/2}} \rightarrow k = 0,0578 \text{ 1/min}$$

$$\rightarrow 1 = x_0 \cdot e^{-0,0578 \cdot 17,98} \rightarrow x_0 = 2,83 \text{ mg/L}$$

# BOD, COD, TOC

## esercizio 1

Ho un campione di 300mL composto da 10mL di acqua inquinata e il resto è acqua di diluizione contenente microrganismi  
Quando inizialmente misuro l'ossigeno dissolto, leggo 9 mg/L

Affinchè si possa avere una misura apprezzabile, voglio che l'ossigeno dissolto finale sia almeno 2mg/L in meno a quello iniziale.  
Inoltre, non voglio che sia inferiore a 1mg/L per problematiche di sensibilità dello strumento di misura

Quale range di BOD (max e min) posso misurare in queste condizioni?

$$P \text{ (Fattore di diluizione)} = \frac{10}{300} = 0,033$$

### CASO 1

$$DO_i = 9 \text{ mg/L} \longrightarrow BOD = \frac{9-7}{0,033} = 60,6 \text{ mg O}_2/\text{L}$$

$$DO_f = 7 \text{ mg/L}$$

### CASO 2

$$DO_i = 9 \text{ mg/L} \longrightarrow BOD = \frac{9-1}{0,033} = 242,42 \text{ mg O}_2/\text{L}$$

$$DO_f = 1 \text{ mg/L}$$

## esercizio 2

In una bottiglia di 300 mL ho tutta acqua di diluizione con un BODd di 1 mg/L

In un'altra bottiglia di 300 ml ho 15 mL di acqua inquinata e il resto acqua di diluizione: il BOD della miscela è di 7,2 mg/L  
Quanto è il BODw dell'acqua inquinata?

$$V_m \cdot BOD_m = V_u \cdot BOD_u + V_d \cdot BOD_d \longrightarrow P = \frac{15}{300} = \frac{1}{20}$$

$$\begin{aligned} BOD_w &= \frac{V_m \cdot BOD_m - V_d \cdot BOD_d}{V_w} \\ &= \frac{V_m \cdot BOD_m}{V_w} - \frac{V_d \cdot BOD_d}{V_w} = \frac{BOD_m - BOD_d \cdot (1-P)}{P} = \frac{7,2 - 1 \cdot (1 - 1/20)}{1/20} = 125 \text{ mg/L} \end{aligned}$$

## esercizio 3

Supponiamo di avere un campione con un fattore di diluizione di 0.03

All'interno del campione inizialmente ho 9mg/L di ossigeno dissolto, e dopo 5 giorni ho 3mg/L

I microrganismi presenti hanno una costante cinetica pari a 0.22 1/giorni

Calcolare:

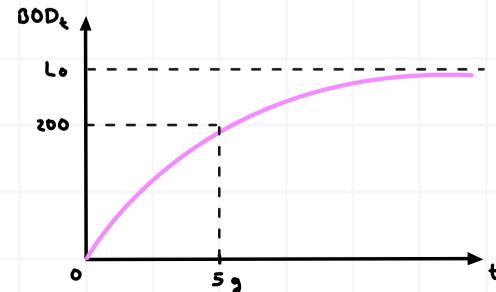
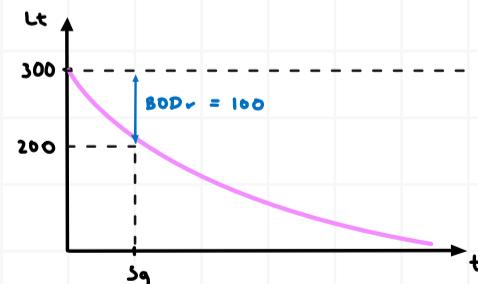
- BOD5
- BOD ultimo
- Carico organico ancora presente da ossidare dopo 5 giorni

$$BOD_s = \frac{9-3}{0,03} = 200 \text{ mg O}_2/\text{L}$$

$$L_t = L_0 \cdot e^{-kt} \longrightarrow L_0 - BOD_s = L_0 \cdot e^{-kt}$$

$$L_0 = \frac{BOD_s}{1 - e^{-kt}} = \frac{200}{1 - e^{-0,22 \cdot 5}} \approx 300 \text{ mg O}_2/\text{L}$$

$$L_t = 300 - 200 = 100 \text{ mg/L}$$



Supponiamo adesso che la temperatura sia di 25°C

Conoscendo il BOD ultimo (300 mg/L), quanto vale in queste condizioni il BOD<sub>5</sub>? (Chiaramente non vale più il fatto che all'inizio ho 9mg/L di ossigeno dissolto e alla fine 3mg/L)

$$\text{Aggiorno la } K : K(25^\circ\text{C}) = 0,22 \cdot 1,047^5 = 0,277 \text{ 1/giorno}$$

$$L_s = 300 \cdot e^{-0,277 \cdot 5} = 75 \text{ mg/L}$$

$$\text{BOD}_5 = 300 - 75 = 225 \text{ mg/L}$$

### esercizio 5.4 (Masters)

A BOD test is to be run on a sample of wastewater that has a five-day BOD of 230 mg/L. If the initial DO of a mix of distilled water and wastewater is 8.0 mg/L, and the test requires a decrease in DO of at least 2.0 mg/L with at least 2.0 mg/L of DO remaining at the end of the five days, what range of dilution factors ( $P$ ) would produce acceptable results? In 300-mL bottles, what range of wastewater volumes could be used?

$$\text{Dati: } \text{BOD}_5 = 230 \text{ mg/L}, \text{ DO}_i = 8 \text{ mg/L}, \text{ DO}_{f,\max} = 6 \text{ mg/L}, \text{ DO}_{f,\min} = 2 \text{ mg/L}$$

$$\left. \begin{array}{l} \text{CASO 1: } P_{\min} = \frac{8-6}{230} = 8,7 \cdot 10^{-3} \\ \text{CASO 2: } P_{\max} = \frac{8-2}{230} = 0,026 \end{array} \right\} 0,0087 < P < 0,026$$

$$\text{Se } V_{\text{TOT}} = 300 \text{ mL} \longrightarrow V_w, \min = 0,0087 \cdot 300 = 2,61 \text{ L}$$

$$V_w, \max = 0,026 \cdot 300 = 7,8 \text{ L}$$

### esercizio 5.8

A BOD test is run using 100 mL of treated wastewater mixed with 200 mL of pure water. The initial DO of the mix is 9.0 mg/L. After 5 days, the DO is 4.0 mg/L. After a long period of time, the DO is 2.0 mg/L, and it no longer seems to be dropping. Assume nitrification has been inhibited so the only BOD being measured is carbonaceous.

- (a) What is the five-day BOD of the wastewater?
- (b) Assuming no nitrification effects, estimate the ultimate carbonaceous BOD.
- (c) What would be the remaining BOD after five days have elapsed?
- (d) Estimate the reaction rate constant  $k$  (day<sup>-1</sup>).



$$a) P = \frac{100}{300} = \frac{1}{3}$$

$$\text{BOD}_5 = \frac{9-4}{1/3} = 15 \text{ mg O}_2/\text{L}$$

$$b) \text{BOD}_{\text{ultimo}} = \frac{9-2}{1/3} = 21 \text{ mg O}_2/\text{L}$$

$$c) L_s = 21 - 15 = 6 \text{ mg O}_2/\text{L}$$

$$d) L_t = L_0 \cdot e^{-k \cdot t} \longrightarrow 6 = 21 \cdot e^{-5k} \longrightarrow \frac{2}{7} = e^{-5k} = \ln(2/7) = -5k \longrightarrow k = 0,25 \text{ 1/giorni}$$

### esercizio 5.9

Suppose you are to measure the BOD removal rate for a primary wastewater treatment plant. You take two samples of raw sewage on its way into the plant and two samples of the effluent leaving the plant. Standard five-day BOD tests are run on the four samples, with no seeding, producing the following data:

Sample	Source	Dilution	DO <sub>i</sub> (mg/L)	DO <sub>f</sub> (mg/L)
1	Raw	1:30	9.2	2.2
2	Raw	1:15	9.2	?
3	Treated	1:20	9.0	2.0
4	Treated	?	9.0	>0

- (a) Find BOD<sub>5</sub> for the raw and treated sewage, and the percent removal of BOD in the treatment plant.
- (b) Find the DO that would be expected in Sample 2 at the end of the test.
- (c) What would be the maximum volume of treated sewage for Sample 4 that could be put into the 300-mL BOD bottle and still have the DO after five days remain above 2 mg/L?

$$a. \text{BOD}_5(R) = \frac{9.2 - 2.2}{1/30} = 210 \text{ mg O}_2/\text{L}$$

$$\text{BOD}_5(T) = \frac{9.2 - 2}{1/20} = 140 \text{ mg O}_2/\text{L}$$

$$\% \text{ rimozione} = \frac{210 - 140}{210} = 33\%$$

$$b. 210 = \frac{9.2 - \text{DO}_f}{1/15} \rightarrow \text{DO}_f = -4.8 \text{ mg O}_2/\text{L}$$

↳ impossibile  $\Rightarrow \text{DO}_f = 0 \text{ mg/L}$

$$c. \text{DO} > 2 \text{ mg/L} \rightarrow P < 1/20 \rightarrow V_w < \frac{300}{20} = 15 \text{ mL}$$

↳ volume massimo

### esercizio 5.11

A mixture consisting of 30 mL of waste and 270 mL of seeded dilution water has an initial DO of 8.55 mg/L; after five days, it has a final DO of 2.40 mg/L. Another bottle containing just the seeded dilution water has an initial DO of 8.75 mg/L and a final DO of 8.53 mg/L. Find the five-day BOD of the waste.

$$P_m = \frac{30}{300} = 0,1$$

$$\text{DO}_{i,m} = 8,55 \text{ mg/L} \rightarrow \text{BOD}_m = 8,55 - 2,4 = 6,15 \text{ mg/L}$$

$$\text{DO}_{f,m} = 2,4 \text{ mg/L}$$

$$\text{BOD}_d = 8,75 - 8,53 = 0,22 \text{ mg/L}$$

$$\text{BOD}_w = \frac{\text{BOD}_m - \text{BOD}_d \cdot (1-P)}{P} = 59,52 \text{ mg/L}$$

### esercizio 5.12

Some wastewater has a  $\text{BOD}_5$  of 150 mg/L at 20°C. The reaction rate  $k$  at that temperature has been determined to be 0.23/day.

- Find the ultimate carbonaceous BOD.
- Find the reaction rate coefficient at 15°C.
- Find  $\text{BOD}_5$  at 15°C.

a)  $L_s = L_o - \text{BOD}_s$

$$L_o - 150 = L_o \cdot e^{-0,23 \cdot 5} \rightarrow L_o = 219,5 \text{ mg/L}$$

b)  $K_{15} = 0,23 \cdot 1,047^{15-20} = 0,183 \text{ 1/day}$

c)  $L_s = 219,5 \cdot e^{-0,183 \cdot 5} = 87,9 \text{ mg/L}$

$$\text{BOD}_s = 219,5 - 87,9 = 131,6 \text{ mg/L}$$

### esercizio 5.8

A BOD test is run using 100 mL of treated wastewater mixed with 200 mL of pure water. The initial DO of the mix is 9.0 mg/L. After 5 days, the DO is 4.0 mg/L. After a long period of time, the DO is 2.0 mg/L, and it no longer seems to be dropping. Assume nitrification has been inhibited so the only BOD being measured is carbonaceous.

- What is the five-day BOD of the wastewater?
- Assuming no nitrification effects, estimate the ultimate carbonaceous BOD.
- What would be the remaining BOD after five days have elapsed?
- Estimate the reaction rate constant  $k$  ( $\text{day}^{-1}$ ).

a)  $P = \frac{100 \text{ mL W}}{300 \text{ mL M}} = \frac{1}{3} \frac{\text{mL W}}{\text{mL M}}$

$$\text{BOD}_s = \frac{9 - 4}{\frac{1}{3}} = 15 \text{ mg/L}$$

b)  $\text{BOD}_{\text{ultimo}} = \frac{9 - 2}{\frac{1}{3}} = 21 \text{ mg/L}$

c)  $\text{BOD}_n = 21 - 15 = 6 \text{ mg/L}$

d)  $6 = 21 \cdot e^{-k \cdot 5} \rightarrow k = 0,25 \text{ /giorni}$

### esercizio 5.11

A mixture consisting of 30 mL of waste and 270 mL of seeded dilution water has an initial DO of 8.55 mg/L; after five days, it has a final DO of 2.40 mg/L. Another bottle containing just the seeded dilution water has an initial DO of 8.75 mg/L and a final DO of 8.53 mg/L. Find the five-day BOD of the waste.

$$P = \frac{30}{300} = 0,1 \rightarrow \text{BOD}_n = 8,55 - 2,4 = 6,15 \text{ mg/L}$$

$$\text{BOD}_d = 8,75 - 8,53 = 0,22 \text{ mg/L}$$

$$\text{BOD}_n \cdot V_n = \text{BOD}_w \cdot V_w + \text{BOD}_d \cdot V_d$$

$$\text{BOD}_w = \frac{\text{BOD}_n \cdot V_n - \text{BOD}_d \cdot V_d}{V_w} = \frac{\text{BOD}_n - \text{BOD}_d \cdot (1-P)}{P} = 59,52 \text{ mg/L}$$





# TRASPORTO DEGLI INQUINANTI

## esercizio 1

Un fiume ha una portata di  $8.7 \text{ m}^3/\text{s}$ , e in questo fiume è presente del carico organico (BOD ultimo di  $6 \text{ mg/L}$ ). In questo fiume viene scaricata una fogna con una portata di  $1.1 \text{ m}^3/\text{s}$  e una concentrazione di carico organico di  $50 \text{ mg/L}$ . La costante di de-ossigenazione è  $0.2 \text{ 1/giorno}$

- Trovare la quantità di carico organico subito dopo la miscelazione
- Supponendo che il fiume scorra con una velocità  $u$  di  $0.3 \text{ m/s}$ , quanto è il carico organico residuo a  $30\text{km}$  dalla fogna?
- Di quanto si è ridotto l'ossigeno dissolto nel fiume?

$$\begin{array}{l} Q_w = 1,1 \text{ m}^3/\text{s} \\ L_{w,w} = 50 \text{ mg/L} \\ \\ \hline Q_r = 8,7 \text{ m}^3/\text{s} & Q_r, L_r \\ L_{r,r} = 6 \text{ mg/L} & \end{array}$$

$$2. \quad L_r = \frac{Q_w \cdot L_w + Q_r \cdot L_r}{Q_u + Q_r} = 10,5 \text{ mg/L}$$

$$b. \quad L(t) = L_0 \cdot e^{-k_d \cdot t}$$

$$\cdot \quad t = \frac{30000 \text{ m}}{0,3 \text{ m/s}} = 100000 \text{ s} = 1,16 \text{ day}$$

$$L(1,16) = 10,9 \cdot e^{-0,2 \cdot 1,16} = 8,7 \text{ mg/L}$$

$$c. \quad 10,9 - 8,7 = 2,2 \text{ mg/L}$$

## esercizio 2

Appena dopo il punto di mescolamento del fiume con la fogna, abbiamo un BOD di  $10.9 \text{ mg/L}$  è  $7.6 \text{ mg/L}$  di ossigeno dissolto. Il fiume e l'acqua di scarico hanno una temperatura di  $20^\circ\text{C}$ , una costante di de-ossigenazione di  $0.2 \text{ 1/giorno}$ , una velocità media di  $0.3 \text{ m/s}$ , una profondità media di  $3\text{m}$ .

- Calcolare la distanza a cui otteniamo il massimo valore di deficit di ossigeno
- Calcolare il minimo valore di ossigeno dissolto

$$a. \quad \text{Calcolare } k_r = \frac{3,9 \cdot \sqrt{0,3}}{\sqrt{3^3}} = 0,41 \text{ 1/day} \longrightarrow k_r > k_d, \text{ quindi avremo un minimo}$$

$$\text{Calcolo D}_0 \text{ (deficit iniziale)} = 9,1 - 7,6 = 1,5 \text{ mg/L}$$

$$t_c = \frac{1}{0,41 - 0,2} \cdot \ln \left( \frac{0,41}{0,2} \left( 1 - \frac{1,5 \cdot (0,41 - 0,2)}{0,2 \cdot 10,9} \right) \right) = 2,67 \text{ giorni}$$

$$S = 0,3 \frac{\text{m}}{\text{s}} \cdot 86400 \frac{\text{s}}{\text{day}} \cdot 2,67 \text{ day} = 69206 \text{ m}$$

$$b. \quad D(2,67) = \frac{0,2 \cdot 10,9}{0,41 - 0,2} \cdot (e^{-0,2 \cdot 2,67} - e^{-0,41 \cdot 2,67}) + 1,5 \cdot e^{-0,41 \cdot 2,67} = 3,1 \text{ mg/L}$$

$$D_{\text{min}} = 9,1 - 3,1 = 6 \text{ mg/L}$$

### esercizio 5.21

A wastewater treatment plant discharges 1.0 m<sup>3</sup>/s of effluent having an ultimate BOD of 40.0 mg/L into a stream flowing at 10.0 m<sup>3</sup>/s. Just upstream from the discharge point, the stream has an ultimate BOD of 3.0 mg/L. The deoxygenation constant  $k_d$  is estimated at 0.22/day.

- (a) Assuming complete and instantaneous mixing, find the ultimate BOD of the mixture of waste and river just downstream from the outfall.
- (b) Assuming a constant cross-sectional area for the stream equal to 55 m<sup>2</sup>, what ultimate BOD would you expect to find at a point 10,000 m downstream?

$$a. L_o = \frac{L_u \cdot Q_u + L_s \cdot Q_s}{Q_u + Q_s} = \frac{1 \text{ m}^3/\text{s} \cdot 40 \text{ mg/L} \cdot 10^3 \text{ L/m}^3 + 10 \text{ m}^3/\text{s} \cdot 3 \text{ mg/L} \cdot 10^3 \text{ L/m}^3}{1 \text{ m}^3/\text{s} + 10 \text{ m}^3/\text{s}} = 6363,6 \text{ mg/m}^3 = 6,36 \text{ mg/L}$$

$$b. 10000 \text{ m} \rightarrow t = \frac{10000 \text{ m}}{\frac{11 \text{ m}^3/\text{s}}{55 \text{ m}^2}} = 50000 \text{ s} = 0,58 \text{ day}$$

$$L(0,58) = 6,36 \cdot e^{-0,22 \cdot 0,58} = 5,6 \text{ mg/L}$$

### esercizio 5.22

The wastewater in Problem 21 has DO equal to 4.0 mg/L when it is discharged. The river has its own DO, just upstream from the outfall, equal to 8.0 mg/L. Find the initial oxygen deficit of the mixture just downstream from the discharge point. The temperatures of sewage and river are both 15°C.

$$DO_o = \frac{4 \cdot 1000 + 8 \cdot 10000}{11000} = 7,64 \text{ mg/L}$$

$$DO_s(15^\circ\text{C}) = 10,08 \text{ mg/L}$$

$$D_o = 10,08 - 7,64 = 2,44 \text{ mg/L}$$

### esercizio 5.23

A single source of BOD causes an oxygen sag curve with a minimum downstream DO equal to 6.0 mg/L. If the BOD of the waste is doubled (without increasing the waste flow rate), what would be the new minimum downstream DO? In both cases, assume that the initial oxygen deficit just below the source is zero, and the saturated value of DO is 10.0 mg/L. (Note that when the initial deficit is zero, the deficit at any point is proportional to the initial BOD.)

$$D(t) = \frac{k_d L_o}{K - k_d} \cdot (e^{-k_d t} - e^{-Kt}) + D_o e^{-Kt} \quad = 0$$

Se duplico  $L_o \rightarrow$  duplico il deficit

$$\text{Prima: } D = 10 - 6 = 4 \text{ mg/L}$$

$$\text{Dopo: } 2 \cdot 4 = 10 - DO_{min} \rightarrow DO_{min} = 2 \text{ mg/L}$$

### esercizio 5.24

The oxygen sag caused by a cannery reaches a minimum DO equal to 3.0 mg/L. Upstream from the cannery, the river DO is saturated at 10.0 mg/L, and it has no BOD of its own. Just downstream from the discharge point, the DO is still essentially saturated (i.e., consider the initial oxygen deficit to be zero so the downstream deficit is proportional to initial BOD). By what percentage should the BOD of the cannery waste be reduced to assure a healthy stream with at least 5.0 mg/L DO everywhere?

In queste condizioni:  $DO = 3 \text{ mg/L} \rightarrow D_{\max} = 10 - 3 = 7 \text{ mg/L}$

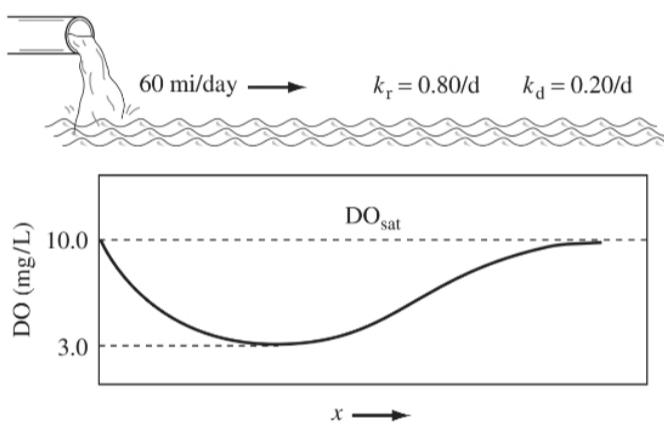
Dopo la riduzione:  $DO = 5 \text{ mg/L} \rightarrow D_{\max} = 10 - 5 = 5 \text{ mg/L}$

Il deficit è proporzionale al BOD ultimo se  $D_0 = 0$  (come in questo caso)

$$\rightarrow \text{riduzione \%} = \frac{7-5}{7} \cdot 100 = 28,6 \%$$

### esercizio 5.27

Suppose the only source of BOD in a river is untreated wastes that are being discharged from a food processing plant. The resulting oxygen sag curve has a minimum value of DO, somewhere downstream, equal to 3.0 mg/L (see Figure P27). Just below the discharge point, the DO of the stream is equal to the saturation value of 10.0 mg/L.



**FIGURE P27**

- By what percent should the BOD of the wastes be reduced to assure a healthy stream with at least 5.0 mg/L of DO everywhere? Would a primary treatment plant be sufficient to achieve this reduction?
- If the stream flows 60 miles per day, has a reaeration coefficient  $k_r$  equal to 0.80/day, and has a deoxygenation coefficient  $k_d$  of 0.20/day, how far downstream (miles) would the lowest DO occur?
- What ultimate BOD ( $L_0$  mg/L) of the mixture of river and wastes just downstream from the discharge point would cause the minimum DO to be 5.0 mg/L?
- Sketch the oxygen sag curve before and after treatment recommended in (a), labeling critical points ( $DO_{\min}$  location and value).

Dati:

$$DO_{\min} = 3 \text{ mg/L}$$

$$D_0 = 0$$

$$DO_s = 10 \text{ mg/L}$$

$$\mu = 60 \text{ mi/day}$$

$$k_r = 0,8 / \text{day}$$

$$k_d = 0,2 / \text{day}$$

a.  $D_{\max} = 10 - 3 = 7 \text{ mg/L}$

↪ noi vorremo  $D_{\max} = 10 - 5 = 5 \text{ mg/L}$

Poiché il deficit iniziale è nullo, so che il BOD ultimo del fiume è proporzionale al deficit di ossigeno, pertanto la riduzione percentuale del deficit corrisponde a quella del BOD

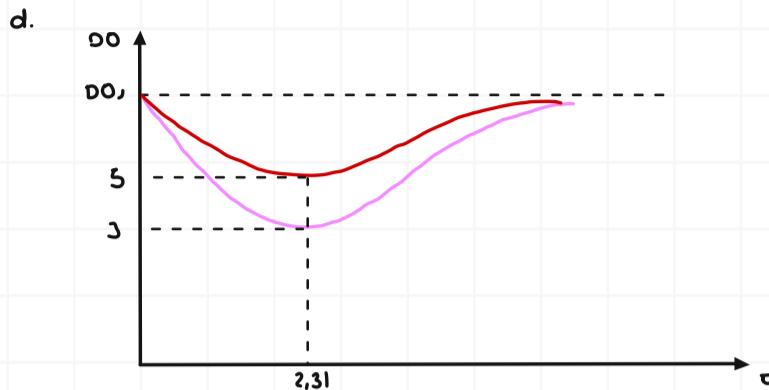
$$\% \text{ rid} = \frac{7-5}{7} = 29 \%$$

$$\text{b. } t_c = \frac{1}{k_r - k_d} \cdot \ln \left[ \frac{k_r}{k_d} \left( 1 - \frac{D_0 (k_r - k_d)}{k_d L_0} \right) \right] = \frac{1}{0.8 - 0.2} \cdot \ln \left( \frac{0.8}{0.2} \right) = 2,31 \text{ day}$$

$$x = 60 \text{ mi/day} \cdot 2,31 \text{ day} = 138,6 \text{ miles}$$

$$c) D(t) = \frac{k_d L_o}{K_r - k_d} \cdot (e^{-k_d t} - e^{-k_r t}) + D_o \cdot e^{-k_r t}$$

$$L_o = \frac{D(t) \cdot (K_r - k_d)}{k_d \cdot (e^{-k_d t} - e^{-k_r t})} = \frac{5 \cdot (0,8 - 0,2)}{0,2(e^{-0,2 \cdot 1,31} - e^{-0,1 \cdot 1,31})} = 31,75 \text{ mg/L}$$



#### esercizio 5.10

A city of 200,000 people deposits 37 cubic feet per second (cfs) of sewage having a BOD of 28.0 mg/L and 1.8 mg/L of DO into a river that has a flow rate of 250 cfs and a flow speed of 1.2 ft/s. Just upstream of the release point, the river has a BOD of 3.6 mg/L and a DO of 7.6 mg/L. The saturation value of DO is 8.5 mg/L. The deoxygenation coefficient  $k_d$  is 0.61/day, and the reaeration coefficient  $k_r$  is 0.76/day. Assume complete and instantaneous mixing of the sewage and river.

- (a) Find the initial oxygen deficit and ultimate BOD just downstream of the outfall.
- (b) Find the time and distance to reach the minimum DO.
- (c) Find the minimum DO.
- (d) Find the DO that could be expected 10 miles downstream.

$$\text{Dati: } Q_w = 37 \text{ ft}^3/\text{s}, L_w = 28 \text{ mg/L}, DO_w = 1,8 \text{ mg/L}$$

$$Q_r = 250 \text{ ft}^3/\text{s}, u = 1,2 \text{ ft/s}, L_r = 3,6 \text{ mg/L}, DO_r = 7,6 \text{ mg/L}$$

$$DO_s = 8,5 \text{ mg/L}, k_d = 0,61/\text{day}, k_r = 0,76/\text{day}$$

$$a) D_o = DO_s - DO_o$$

$$DO_o = \frac{DO_r \cdot Q_r + DO_w \cdot Q_w}{Q_r + Q_w} = 6,85 \text{ mg/L} \quad \rightarrow \quad D_o = 8,5 - 6,85 = 1,65 \text{ mg/L}$$

$$L_o = \frac{L_r Q_r + L_w Q_w}{Q_r + Q_w} = 6,74 \text{ mg/L}$$

$$b) t_c = \frac{1}{K_r - k_d} \cdot \ln \left[ \frac{K_r}{k_d} \left( 1 - \frac{D_o (K_r - k_d)}{K_d L_o} \right) \right] = 1,05 \text{ day}$$

$$x_c = 1,05 \cdot 1,2 \cdot 3600 \cdot 24 = 109033 \text{ ft}$$

$$c) D(t) = \frac{k_d L_o}{K_r - k_d} \cdot (e^{-k_d t} - e^{-k_r t}) + D_o \cdot e^{-k_r t}$$

$$D(1,05) = 2,85 \text{ mg/L}$$

$$DO_{min} = DO_s - D = 5,65 \text{ mg/L}$$

$$d) 10 \text{ miles} \rightarrow 52800 \text{ ft} \rightarrow t = \frac{52800}{1,2} = 44000 \text{ s} = 0,51 \text{ day}$$

$$D(0,51) = 2,6 \text{ mg/L} \rightarrow DO = DO_s - D = 5,9 \text{ mg/L}$$

**esercizio 5.28**

The ultimate BOD of a river just below a sewage outfall is 50.0 mg/L, and the DO is at the saturation value of 10.0 mg/L. The deoxygenation rate coefficient  $k_d$  is 0.30/day, and the reaeration rate coefficient  $k_r$  is 0.90/day. The river is flowing at the speed of 48.0 miles per day. The only source of BOD on this river is this single outfall.

- Find the critical distance downstream at which DO is a minimum.
- Find the minimum DO.
- If a wastewater treatment plant is to be built, what fraction of the BOD would have to be removed from the sewage to assure a minimum of 5.0 mg/L everywhere downstream?

$$a) t_c = \frac{1}{k_r - k_d} \cdot \ln \left( \frac{k_r}{k_d} \left[ 1 - \frac{D_0 (k_r - k_d)}{k_d L_0} \right] \right) = 1,83 \text{ day} \quad \rightarrow \quad S = 1,83 \cdot 48 = 88 \text{ miles}$$

$$b) D(1,83) = 9,62 \text{ mg/L}$$

$$DO_{min} = 10 - 9,62 = 0,38 \text{ mg/L}$$

$$c) D_{outf} = 5 \text{ mg/L} \quad \rightarrow \quad \frac{D_{outf}}{D_{row}} = \frac{5}{9,62} = 0,52 = 52\% \\ D_{row} = 9,62 \text{ mg/L} \quad \hookrightarrow \text{devo rimuovere il } 48\% \text{ del BOD}$$

**esercizio 5.33**

Just downstream of the outfall from a pollution point-source, the DO of a river is 6 mg/L, and the mix of river and wastes has a BOD of 20 mg/L. The saturation value of DO is 9 mg/L. The deoxygenation constant is  $k_d = 0.20/\text{day}$ .

- Estimate the reaeration coefficient using the O'Connor and Dobbins relationship (24), assuming that the river speed is 0.25 m/s, and the average stream depth is 3 m.
- Find the critical time downstream at which minimum DO occurs.
- Find the minimum DO downstream.
- If the outfall is the only source of BOD, what percent removal of BOD would be needed to assure a minimum DO of 5 mg/L?

$$a) k_{r,10} = \frac{3,9 \cdot \sqrt{0,25}}{\sqrt{3^3}} = 0,375 / \text{day}$$

$$b) t_c = \frac{1}{0,375 - 0,2} \cdot \ln \left[ \frac{0,375}{0,2} \left( 1 - \frac{3 \cdot (0,375 - 0,2)}{0,2 \cdot 20} \right) \right] = 2,79 \text{ day}$$

$$c) D(2,79) = \dots = 6,1 \text{ mg/L} \quad \rightarrow \quad DO = 9 - 6,1 = 2,9 \text{ mg/L}$$

$$d) \frac{D_{outf}}{D_{row}} = \frac{9 - 5}{6,1} = 0,66 \quad \rightarrow \quad \% \text{ BOD removal} = 33\%$$



# TRATTAMENTI DELLE ACQUE

## esercizio 1

Un equalizzatore ha una portata che varia rispetto al tempo con la relazione:  
 $k=20 \text{ m}^3/\text{h}$ ,  $a=15\text{m}^3/\text{h}$ ,  $t_f = 24\text{h}$

Stimare le dimensioni dell'equalizzatore affinché non si secchi e non strabordi

$$F_{in}(t) = k + a \cos \left( \frac{2\pi}{t_f} \left( t - \frac{t_f}{2} \right) \right)$$

Soluzione:

$$\begin{aligned} \text{Calcolo } F_{out} \rightarrow F_{out} &= \frac{\int_0^{14} k + a \cos \left[ \frac{2\pi}{t_f} \left( t - \frac{t_f}{2} \right) \right] dt}{24} = \frac{k t + a \sin \left[ \frac{2\pi}{t_f} \left( t - \frac{t_f}{2} \right) \right] \cdot \frac{t_f}{2\pi}}{24} \Big|_0^{24} \\ &= \frac{20 \cdot 24 + 15 \cdot \sin(2\pi - \pi) \cdot \frac{24}{2\pi}}{24} = 20 \text{ m}^3/\text{h} \end{aligned}$$

Ho un punto stazionario quando  $F_{in} = 20 \text{ m}^3/\text{h}$

Trovo il tempo  $t$  a cui avviene ciò:

$$20 = 20 + 15 \cos \left( \frac{\pi}{12} t - \pi \right) \rightarrow \frac{\pi}{12} t - \pi = \pm \frac{\pi}{2} \rightarrow t = 18 \text{ h} \\ t = 6 \text{ h}$$

$$\begin{aligned} V_{out} &= F_{out} \cdot t = 20 \cdot 6 = 120 \text{ m}^3 \\ V_{in} &= \int_0^t F_{in}(t) dt = k t + a \sin \left[ \frac{2\pi}{t_f} \left( t - \frac{t_f}{2} \right) \right] \cdot \frac{t_f}{2\pi} = 62,7 \text{ m}^3 \end{aligned} \quad \left. \begin{array}{l} 6 \text{ h} \\ \Delta V_1 = 57,3 \text{ m}^3 \end{array} \right\}$$

$$\begin{aligned} V_{out} &= 20 \cdot 18 = 360 \text{ m}^3 \\ V_{in} &= 62,7 \text{ m}^3 \end{aligned} \quad \left. \begin{array}{l} 18 \text{ h} \\ \Delta V_2 = 57,3 \end{array} \right\}$$

$$\Delta V = \Delta V_1 + \Delta V_2 = 114,6 \text{ m}^3$$

## esercizio 2

Dimensionare un bacino di decantazione per delle particelle sferiche con densità 2 kg/L e diametro di 0.5mm in una corrente di 2 m<sup>3</sup>/s. Assumere regime di Stokes

condizione di funzionamento:  $\tau_p > \tau_s$

$$\begin{aligned} \tau_s &= \frac{H}{v_s} \rightarrow \frac{L}{v_x} > \frac{H}{v_s} \rightarrow \frac{L}{H} > \frac{v_x}{v_s} \\ \tau_p &= \frac{L}{v_x} = \frac{V}{Q} \end{aligned}$$

$$v_s = \frac{(2-1) \cdot 10^3 \cdot 9,81 \cdot (0,5 \cdot 10^{-3})^2}{8,9 \cdot 10^{-4}} = 2,76 \text{ m/s}$$

$$\frac{L}{H} > \frac{2}{2,76} = 0,725$$

Le dimensioni del bacino devono essere tali che il lato sia almeno 0,725 volte più grande della profondità

### esercizio esempio

A drinking water treatment plant uses a circular sedimentation basin to treat 3.0 MGD of river water. (MGD stands for million gallons per day and is a common U.S. measure of flow rate used for water and wastewater treatment. 1.0 MGD = 0.0438 m<sup>3</sup>/s). After storms occur upstream, the river often carries 0.010 mm silt particles with an average density of 2.2 g/cm<sup>3</sup>, and the silt must be removed before the water can be used. The plant's clarifier is 3.5 m deep and 21 m in diameter. The water is 15°C.

- What is the hydraulic detention time of the clarifier?
- Will the clarifier remove all of the silt particles from the river water?

Dati:

$$Q = 0,1314 \text{ m}^3/\text{s}$$

$$d = 0,01 \text{ mm}$$

$$\rho_s = 2,2 \text{ g/cm}^3$$

$$H = 3,5 \text{ m}$$

$$D = 21 \text{ m}$$

$$T = 15^\circ\text{C}$$

$$a. \tau_{det} = \frac{\pi(D/2)^2 \cdot H}{Q} = 9225,74 \text{ s} = 2,56 \text{ h}$$

$$b. \tau_s = \frac{H}{v_s} \quad \text{con } v_s = \frac{(\rho_s - \rho) gd^2}{18 \mu} \quad (\text{Stokes})$$

$$\frac{(2200 - 1000) \text{ kg/m}^3 \cdot 9,81 \text{ m/s}^2 \cdot (0,01 \cdot 10^{-3})^2 \text{ m}^2}{18 \cdot 0,00114 \text{ kg/m.s}} = 5,74 \cdot 10^{-5} \text{ m/s}$$

$$\tau_s = \frac{3,5}{5,74 \cdot 10^{-5}} = 60975,6 \text{ s} = 16,94 \text{ h}$$

NO, non è sufficiente

### esercizio 6.5

You are trying to remove 10.0 μm diameter particles in a water treatment plant. The water is at 20°C, and the particle density is 1.2 g/mL. The plant treats 0.100 m<sup>3</sup>/s of water. It is proposed to use a 3.5 m deep, rectangular sedimentation tank with a length to width ratio of 5:1. What is the minimum required width of the basin?

Dati: H = 3,5 m, L = 5W, Q = 0,1 m<sup>3</sup>/s, ρ = 1,2 g/mL, d = 10 μm, T = 20°C

$$\rightarrow \tau_{det} > \tau_s \rightarrow \frac{V}{Q} > \frac{H}{v_s} \rightarrow \frac{W \cdot 5W \cdot H}{Q} > \frac{H}{v_s} \rightarrow W^2 > \frac{Q}{5v_s} \rightarrow W > \sqrt{\frac{Q}{5v_s}}$$

$$v_s = \frac{(1200 - 1000) \cdot 9,81 \cdot (10 \cdot 10^{-6})^2}{18 \cdot 0,001} = 1,09 \cdot 10^{-5} \text{ m/s}$$

$$W > 42,8 \text{ m}$$

## TRATTAMENTO BIOLOGICO

### esercizio 6.30

A perfectly mixed aeration pond with no recycle (return line) serves as the biological reactor for a small community. The pond receives  $30 \text{ m}^3/\text{d}$  of influent with a  $\text{BOD}_5$  of  $350 \text{ mg/L}$  that must be reduced to  $20 \text{ mg/L}$  before discharge. It has been found that the kinetic constants for the system are  $K_s = 100 \text{ mg/L BOD}_5$ ,  $k_d = 0.10 \text{ d}^{-1}$ ,  $\mu_m = 1.6 \text{ d}^{-1}$ , and  $Y$  is  $0.60 \text{ mg VSS/mg BOD}_5$ .

(a) What must the hydraulic detention time be in the aeration pond?

(b) What mass of microbes will be produced in the pond each day?

$$\text{Dati: } Q = 30 \text{ m}^3/\text{day} \quad K_s = 100 \text{ mg/L BOD}_5$$

$$S_0 = 350 \text{ mg/L} \quad k_d = 0.1 \text{ d}^{-1}$$

$$S = 20 \text{ mg/L} \quad \mu_{max} = 1.6 \text{ d}^{-1}$$

$$Y = 0.6 \text{ mg VSS/mg BOD}_5$$

Soluzione:

$$\text{Bilancio biomassa: } V \cdot \frac{\partial X}{\partial t} = -QX + r_x'V \rightarrow QX = \left[ \mu_{max} \cdot \frac{S}{S+K_s} X - k_d X \right] \cdot V \rightarrow$$

$$\frac{V}{Q} = \tau_{idr} = \frac{1}{\mu_{max} \cdot \frac{S}{S+K_s} - k_d} = 6 \text{ day}$$

$$\text{Bilancio substrato: } V \cdot \frac{\partial S}{\partial t} = QS_0 - QS - r_s V \rightarrow Q(S_0 - S) = V \cdot \frac{\mu_{max}}{Y} \cdot \frac{S}{K_s + S} \cdot X \rightarrow X = \frac{(S_0 - S)(K_s + S)}{\tau_{idr} \cdot S \cdot \mu_{max}} Y =$$

$$= \frac{(350 - 20) \text{ mg/L} \cdot (100 + 20) \text{ mg/L} \cdot 0.6}{6 \text{ day} \cdot 20 \text{ mg/L} \cdot 1.6 \text{ /day}} = 123.75 \text{ mg/L}$$

### esercizio 6.32

A wastewater treatment uses an activated sludge process for secondary treatment of  $0.300 \text{ m}^3/\text{s}$  of primary effluent. The mixed liquor has a concentration of  $2,100 \text{ mg VSS/L}$ , and the RAS concentration is  $10,000 \text{ mg VSS/L}$ . The substrate concentration in the primary effluent is  $220 \text{ mg BOD}_5/\text{L}$ . The  $F/M$  ratio for the activated sludge tank is  $0.52 \text{ mg BOD}_5 \cdot \text{mg VSS}^{-1} \cdot \text{d}^{-1}$ , and the cell residence time is  $9.0 \text{ d}$ .

- (a) What is the volume of the activated sludge tank?
- (b) What is the WAS flow rate?
- (c) What is the flow rate of secondary treated effluent?
- (d) What is the hydraulic residence time for the activated sludge tank?

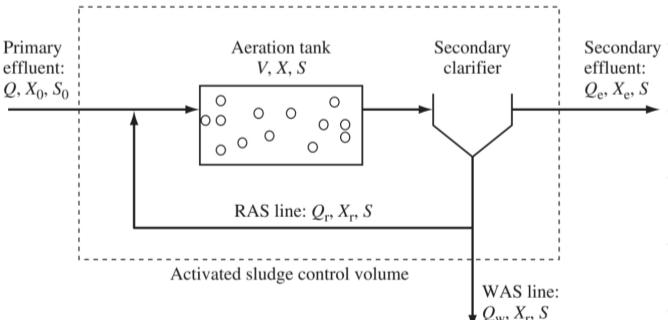
$$\text{Dati: } Q = 0.3 \text{ m}^3/\text{s}$$

$$S_0 = 220 \text{ mg BOD}_5/\text{L}$$

$$X_r = 10000 \text{ mg VSS/L}$$

$$X = 2100 \text{ mg VSS/L} \quad L_s = 0.52 \text{ mg BOD}/\text{mg VSS} \cdot \text{day}$$

$$\theta_c = 9 \text{ day}$$



$$\text{a. } V = \frac{Q \cdot S_0}{L_s \cdot X} = \frac{0.3 \cdot 10^3 \text{ L/s} \cdot 86400 \text{ s/day} \cdot 220 \text{ mg BOD/L}}{0.52 \frac{\text{mg BOD}}{\text{mg VSS} \cdot \text{day}} \cdot 10000 \frac{\text{mg VSS}}{\text{L}}} = 1.1 \cdot 10^6 \text{ L} \approx 1096.6 \text{ m}^3$$

$$\text{b. } \theta_c = \frac{VX}{Q_w X_w} \rightarrow Q_w = \frac{V \cdot X}{\theta_c \cdot X_w} = \frac{1096.6 \cdot 2100}{9 \cdot 10000} = 25.59 \text{ m}^3/\text{day}$$

$$\text{c. } Q_e = Q - Q_w = 0.3 \cdot 86400 - 25.59 = 25894 \text{ m}^3/\text{day}$$

$$\text{d. } \theta = \frac{1096.6}{0.3 \cdot 86400} = 0.04 \text{ day}$$



# DISPERSONE IN ATMOSFERA

## esercizio 7.29

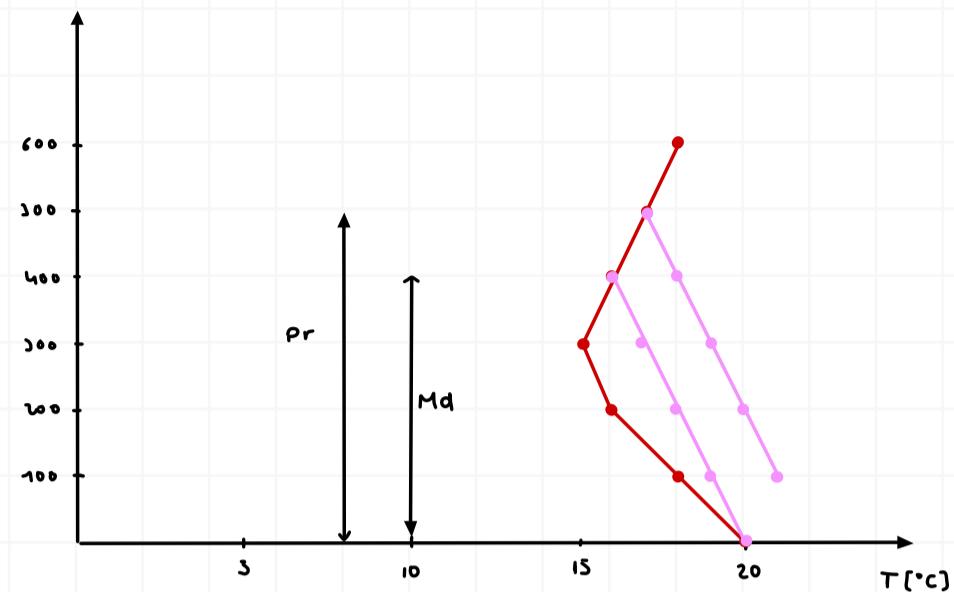
Suppose the following atmospheric altitude versus temperature data have been collected.

Altitude (m)	Temp (°C)
0	20
100	18
200	16
300	15
400	16
500	17
600	18

- (a) What would be the mixing depth?
- (b) How high would you expect a plume to rise if it is emitted at 21°C from a 100-m stack if it rises at the dry adiabatic lapse rate? Would you expect the plume to be looping, coning, fanning, or fumigating?

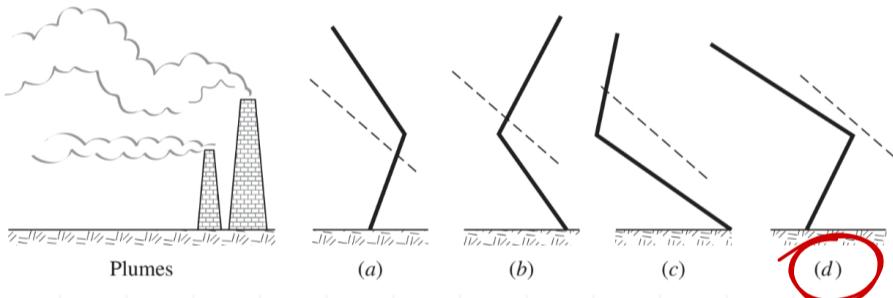
$$\text{Mixing depth} = 400 \text{ m}$$

$$\text{Plume rise} = 500 \text{ m}$$



## esercizio 7.31

A tall stack and a nearby short stack have plumes as shown in Figure P31. Which atmospheric temperature profile would be most likely to cause that pair of plumes? The dotted lines represent the dry adiabatic lapse rate.



## esercizio 7.32

A point source Gaussian plume model for a power plant uses 50 m as the effective stack height (see Figure P32). The night is overcast. (Note: This is not the same as "cloudy" in this model; check the footnotes in Table 8.) Your concern is with ground-level pollution at two locations, A and B, which are 1.2 and 1.4 km directly downwind from the stack.

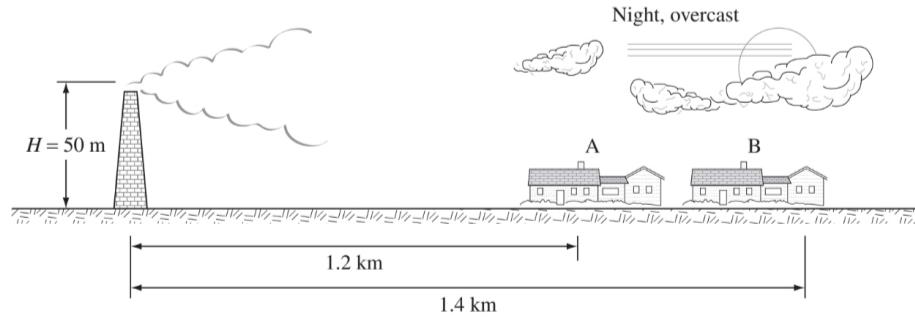


FIGURE P32

- (a) At what distance will the maximum concentration of pollution occur? Which location (A or B) would have the higher level of pollution?
- (b) Suppose the sky clears up, and the windspeed stays less than 5 m/s. Will the location downwind at which the maximum concentration occurs move? If so, will it move closer to the stack or further away from the stack? ( $H$  remains 50 m.)
- (c) Under the new conditions in part b, which house would experience the most pollution?

a) Wind < 5 m/s e cloud cover

→ Class D

Voglio  $x_{\max}$  ground-level

$$x_{\max} = \left( \frac{H^2 d}{(b+d)c^2} \right)^{\frac{1}{2+d}}$$

Prendo i parametri per  $x > 1 \text{ km}$

$$x_{\max} = \left( \frac{50^2 \cdot 0,516}{(0,516 + 0,894) \cdot 44,5^2} \right)^{\frac{1}{2+0,516}} = 0,45 \text{ km}$$

Prendo i parametri per  $x < 1 \text{ km}$

$$x_{\max} = \left( \frac{50^2 \cdot 0,725}{33,2^2 \cdot (0,725 + 0,894)} \right)^{\frac{1}{2+0,725}} = 1 \text{ km}$$

Scelgo  $x_{\max}$  maggiore → 1 km

La casa più inquinata sarà la A

b) Passiamo ad una classe B/C, perché di giorno l'aumento dei raggi solari comporta instabilità

le punto di max concentrazione si sposta verso la ciminiera

c) Sempre A

### esercizio 7.33

Suppose a bonfire emits CO at the rate of 20 g/s on a clear night when the wind is blowing at 2 m/s. If the effective stack height at the fire is 6 m, (a) what would you expect the ground-level CO concentration to be at 400 m downwind? (b) Estimate the maximum ground-level concentration.

Class F

$$\sigma_y = 34 \cdot 0,4^{0,894} = 14,99 \text{ m}$$

$$\sigma_z = 14,34 \cdot 0,4^{0,894} - 0,35 = 6,93 \text{ m}$$

$$c(x, 0) = \frac{20}{\pi \cdot 2 \cdot 14,99 \cdot 6,93} \cdot \exp\left(-\frac{6^2}{2 \cdot 6,93}\right) = 0,021 \text{ g/m}^3 = 21 \text{ mg/m}^3$$

$$x_{max} = \left( \frac{6^2 \cdot 0,74}{(0,74 + 0,894) \cdot 14,35^2} \right)^{\frac{1}{2+0,74}} = 0,18 \text{ km} = 180 \text{ m}$$

$$\sigma_y = 34 \cdot 0,18^{0,894} = 7,35$$

$$\sigma_z = 14,34 \cdot 0,18^{0,894} - 0,35 = 3,6847$$

$$c(180, 0) = \frac{20}{\pi \cdot 2 \cdot 3,6847} \cdot \exp\left(-\frac{6^2}{2 \cdot 3,6847}\right) = 31 \text{ mg/s}$$

### esercizio 7.36

A stack emitting 80 g/s of NO has an effective stack height of 100 m. The windspeed is 4 m/s at 10 m, and it is a clear summer day with the sun nearly overhead. Estimate the ground-level NO concentration

- (a) directly downwind at a distance of 2 km.
- (b) at the point downwind where NO is a maximum.
- (c) at a point located 2 km downwind and 0.1 km off the downwind axis.

Class: B

Aggiorno  $u_r$  del vento

$$u_{100} = 4 \cdot \left(\frac{100}{10}\right)^{0,15} = 5,65 \text{ m/s}$$

$$a) \sigma_y = 156 \cdot 2^{0,854} = 289,9 \text{ m}$$

$$\sigma_z = 108,2 \cdot 2^{0,898} + 2 = 233,61 \text{ m}$$

$$c(2, 0) = \frac{80}{\pi \cdot 5,65 \cdot 289,9 \cdot 233,61} \cdot \exp\left(-\frac{100^2}{2 \cdot 233,61^2}\right) = 6,1 \cdot 10^{-5} \text{ g/m}^3 = 61 \mu\text{g/m}^3$$

b)

$$x_{max} = \left( \frac{100^2 \cdot 1,149}{(1,149 + 0,894) \cdot 106,6^2} \right)^{\frac{1}{2+1,149}} = 0,736 \text{ km} \quad \rightarrow \quad \sigma_y = 161,945 \text{ m}$$

$$\sigma_z = 78,255 \text{ m}$$

$$c(x_{max}, 0) = \frac{80}{5,65 \pi \cdot 78,255 \cdot 161,945} \cdot \exp\left(-\frac{100^2}{2 \cdot 78,255^2}\right) = 157,2 \mu\text{g/m}^3$$

$$c) \quad c(2,0,1) = \frac{80}{\pi \cdot 3,65 \cdot 289,9 \cdot 233,61} \cdot \exp\left(-\frac{100^2}{2 \cdot 233,61^2}\right) \cdot \left(-\frac{100^2}{2 \cdot 289,9^2}\right) = 57,2 \mu\text{g}/\text{m}^3$$

### esercizio 7.38

The world's tallest stack is on a copper smelter in Sudbury, Ontario. It stands 380 m high and has an inner diameter at the top of 15.2 m. If 130°C gases exit the stack at 20 m/s while the ambient temperature is 10°C and the winds at stack height are 8 m/s, use the Briggs model to estimate the effective stack height. Assume a slightly unstable atmosphere, class C.

$$\Delta h = \frac{1,6 \cdot F^{1/3} \cdot X_i^{2/3}}{U}$$

$$\text{Fattore di galleggiamento: } F = 9,81 \cdot \left(\frac{15,2}{2}\right)^2 \cdot 20 \cdot \left(1 - \frac{10+273}{130+273}\right) = 3374,4 \text{ m}^4/\text{s}^3$$

$$X_i = 120 \cdot 3374,4^{0,4} = 3093,6$$

$$\Delta h = \frac{1,6 \cdot 3374,4^{1/3} \cdot 3093,6^{1/3}}{8} = 637 \text{ m}$$

$$H = 637 + 380 = 1017 \text{ m}$$

### esercizio 7.39

Repeat Problem 38 for a stable, isothermal atmosphere (no temperature change with altitude).

$$\Delta h = 2,6 \cdot \left(\frac{F}{U S}\right)^{1/3}$$

$$F = 3374,4 \text{ m}^4/\text{s}^3 \quad \longrightarrow \quad \Delta h = 277,59 \text{ m}$$

$$S = \frac{9,81}{283} \cdot 0,01 = 3,466 \cdot 10^{-4} \quad H = 277,59 + 380 = 657,59 \text{ m}$$

### esercizio 7.40

A power plant has a 100-m stack with an inside radius of 1 m. The exhaust gases leave the stack with an exit velocity of 10 m/s at a temperature of 120°C. Ambient temperature is 6°C, winds at the effective stack height are estimated to be 5 m/s, surface windspeed is 3 m/s, and it is a cloudy summer day. Estimate the effective height of this stack.

#### classe C

$$F = 9,81 \cdot 1^2 \cdot 10 \cdot \left(1 - \frac{6+273}{120+273}\right) = 28,46 \text{ m}^4/\text{s}^3$$

$$X_i = 50 \cdot 28,46^{5/8} = 405,38$$

$$\Delta h = \frac{1,6 \cdot 28,46^{1/3} \cdot 405,38^{1/3}}{5} = 53,5 \text{ m} \quad \longrightarrow \quad \Delta H = 53,5 + 100 = 153,5 \text{ m}$$

### esercizio 7.41

A 200-MW power plant has a 100-m stack with radius 2.5 m, flue gas exit velocity 13.5 m/s, and gas exit temperature 145°C. Ambient temperature is 15°C, windspeed at the stack is 5 m/s, and the atmosphere is stable, class E, with a lapse rate of 5°C/km. If it emits 300 g/s of SO<sub>2</sub>, estimate the concentration at ground level at a distance of 16 km directly downwind.

Calcolo il plume rise

$$\Delta h = 2,6 \left( \frac{F}{us} \right)^{1/3}$$

$$\cdot F = 9,81 \cdot 2,5 \cdot 13,5 \cdot \left( 1 - \frac{15+273}{145+273} \right) = 257,42 \text{ m}^4/\text{s}^3$$

$$\cdot S = \frac{9,81}{288} \cdot \left( 5 \cdot 10^{-3} + 0,01 \right) = 5,11 \cdot 10^{-4}$$

$$\Delta h = 2,6 \cdot \left( \frac{257,42}{5 \cdot 5,11 \cdot 10^{-4}} \right)^{1/3} = 121 \text{ m} \quad \longrightarrow \quad H = 121 + 100 = 221 \text{ m}$$

$$\sigma_y = 50,5 \cdot 16^{0,354} = 602,23 \text{ m}$$

$$\sigma_z = 35,4 \cdot 16^{0,305} - 34 = 95,05 \text{ m}$$

$$C(16,0) = \frac{300}{5\pi \cdot 95,05 \cdot 602,23} \cdot \exp \left( -\frac{221^2}{2 \cdot 95,05^2} \right) = 22,3 \mu\text{g}/\text{m}^3$$

### esercizio 7.44

A paper plant is being proposed for a location 1 km upwind from a town. It will emit 40 g/s of hydrogen sulfide, which has an odor threshold of about 0.1 mg/m<sup>3</sup>. Winds at the stack may vary from 4 to 10 m/s blowing toward the town. What minimum stack height should be used to assure concentrations are not more than 0.1 times the odor threshold at the near edge of town on a class B day? To be conservative, the stack will be designed assuming no plume rise. If the town extends beyond the 1-km distance, will any buildings experience higher concentrations than a residence at the boundary under these conditions?

$$\sigma_y = 156 \cdot 1^{0,354} = 156 \text{ m}$$

$$\sigma_z = 106,6 \cdot 1^{1,113} + 3,3 = 110 \text{ m}$$

$$C(x,0) = \frac{40}{\pi u \cdot 110 \cdot 156} \cdot \exp \left( -\frac{H^2}{2 \cdot 110^2} \right) = 0,1 \cdot 0,1 \cdot 10^{-3}$$

$$\rightarrow 0,1 \cdot 10^{-3} = \frac{3,42 \cdot 10^{-3}}{u} \cdot e^{-\frac{H^2}{24200}} \quad \longrightarrow \quad + \frac{H^2}{24200} = \ln \left( \frac{3,42}{0,1 \cdot u} \right) \quad \longrightarrow \quad H = \sqrt{24200 \cdot \ln \left( \frac{3,42}{0,1 \cdot u} \right)}$$

$$u = 4 \text{ m/s} \longrightarrow 266 \text{ m}$$

$$u = 10 \text{ m/s} \longrightarrow 220 \text{ m}$$



