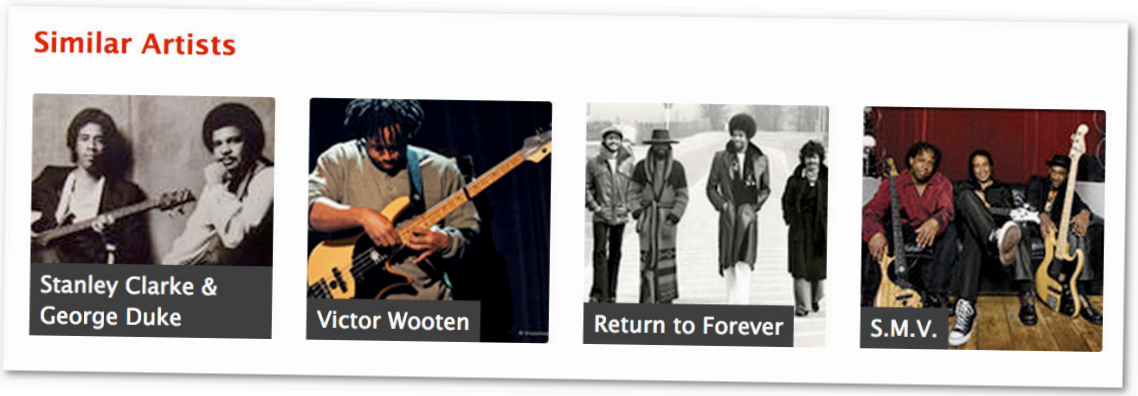
Chapter 2: Collaborative Filtering

I like what you like

We are going to start our exploration of data mining by looking at recommendation systems. Recommendation systems are everywhere—from Amazon:

to last.fm recommending music or concerts:

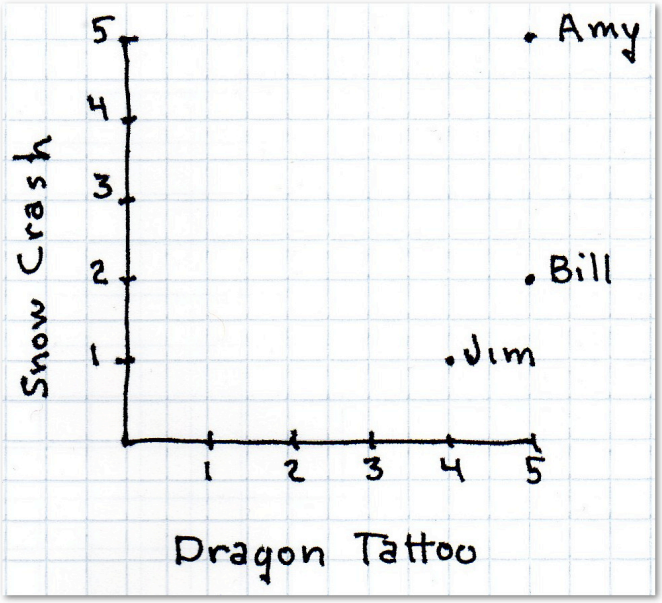


In the Amazon example, above, Amazon combines two bits of information to make a recommendation. The first is that I viewed *The Lotus Sutra* translated by Gene Reeves; the second, that customers who viewed that translation of the *Lotus Sutra* also viewed several other translations.

The recommendation method we are looking at in this chapter is called collaborative filtering. It's called collaborative because it makes recommendations based on other people—in effect, people collaborate to come up with recommendations. It works like this. Suppose the task is to recommend a book to you. I search among other users of the site to find one that is similar to you in the books she enjoys. Once I find that similar person I can see what she likes and recommend those books to you—perhaps Paolo Bacigalupi's *The Windup Girl*.

How do I find someone who is similar?

So the first step is to find someone who is similar. Here's the simple 2D (dimensional) explanation. Suppose users rate books on a 5 star system—zero stars means the book is terrible, 5 stars means the book is great. Because I said we are looking at the simple 2D case, we restrict our ratings to two books: Neal Stephenson's *Snow Crash* and the Steig Larsson's *The Girl with the Dragon Tattoo*.



First, here's a table showing 3 users who rated these books

	Snow Crash	Girl with the Dragon Tattoo
Amy	5☆	5☆
Bill	2☆	5☆
Jim	1☆	4☆

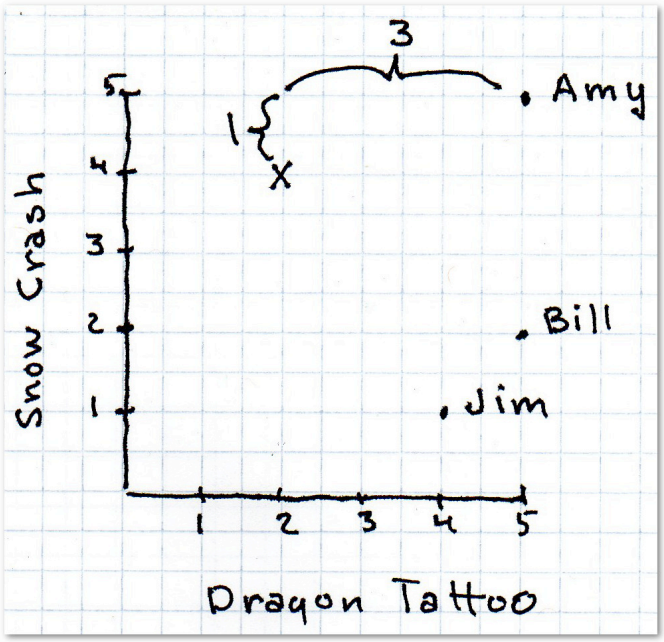
I would like to recommend a book to the mysterious Ms. X who rated *Snow Crash* 4 stars and *The Girl with the Dragon Tattoo* 2 stars. The first task is to find the person who is most similar, or closest, to Ms. X. I do this by computing distance.

Manhattan Distance

The easiest distance measure to compute is what is called Manhattan Distance or cab driver distance. In the 2D case, each person is represented by an (x, y) point. I will add a subscript to the x and y to refer to different people. So (x_1, y_1) might be Amy and (x_2, y_2) might be the elusive Ms. X. Manhattan Distance is then calculated by

$$| x_1 - x_2 | + | y_1 - y_2 |$$

(so the absolute value of the difference between the x values plus the absolute value of the difference between the y values). So the Manhattan Distance for Amy and Ms. X is 4:



Computing the distance between Ms. X and all three people gives us:

	Distance from Ms. X
Amy	4
Bill	5
Jim	5

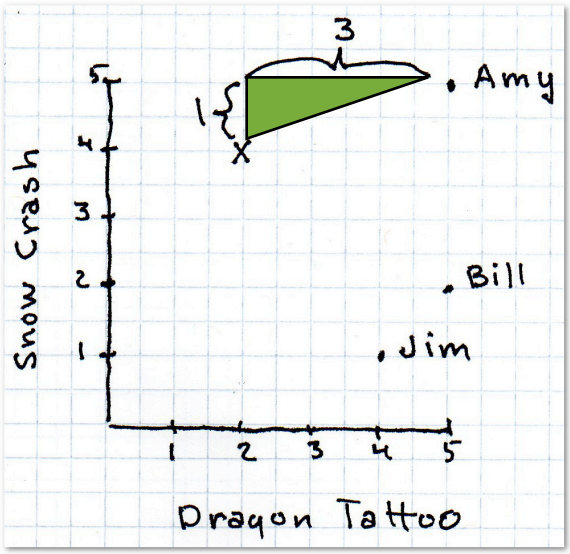
Amy is the closest match. We can look in her history and see, for example, that she gave five stars to Paolo Bacigalupi's *The Windup Girl* and we would recommend that book to Ms. X.

Euclidean Distance

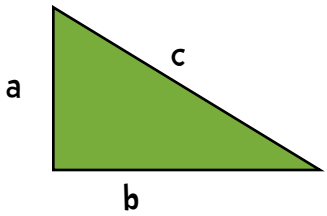
One benefit of Manhattan Distance is that it is fast to compute. If we are Facebook and are trying to find who among one million users is most similar to little Danny from Kalamazoo, fast is good.

Pythagorean Theorem

You may recall the Pythagorean Theorem from your distant educational past. Here, instead of finding the Manhattan Distance between Amy and Ms. X (which was 4) we are going to figure out the straight line, as-the-crow-flies, distance



The Pythagorean Theorem tells us how to compute that distance.



$$c = \sqrt{a^2 + b^2}$$

This straight-line, as-the-crow-flies distance we are calling Euclidean Distance. The formula is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Recall that x_1 is how well person 1 liked *Dragon Tattoo* and x_2 is how well person 2 liked it; y_1 is how well person 1 liked *Snow Crash* and y_2 is how well person 2 liked it.

Amy rated both *Snow Crash* and *Dragon Tattoo* a 5; The elusive Ms. X rated *Dragon Tattoo* a 2 and *Snow Crash* a 4. So the Euclidean distance between

$$\sqrt{(5-2)^2 + (5-4)^2} = \sqrt{3^2 + 1^2} = \sqrt{10} = 3.16$$

Computing the rest of the distances we get

	Distance from Ms. X
Amy	3.16
Bill	3.61
Jim	3.61

N-dimensional thinking

Let's branch out slightly from just looking at rating two books (and hence 2D) to looking at something slightly more complex. Suppose we work for an online streaming music service and we want to make the experience more compelling by recommending bands. Let's say users can rate bands on a star system 1-5 stars and they can give half star ratings (for example, you can give a band 2.5 stars). The following chart shows 8 users and their ratings of eight bands.

	Angelica	Bill	Chan	Dan	Hailey	Jordyn	Sam	Veronica
Blues Traveler	3.5	2	5	3	-	-	5	3
Broken Bells	2	3.5	1	4	4	4.5	2	-
Deadmau5	-	4	1	4.5	1	4	-	-
Norah Jones	4.5	-	3	-	4	5	3	5
Phoenix	5	2	5	3	-	5	5	4
Slightly Stoopid	1.5	3.5	1	4.5	-	4.5	4	2.5
The Strokes	2.5	-	-	4	4	4	5	3
Vampire Weekend	2	3	-	2	1	4	-	-

The hyphens in the table indicate that a user didn't rate that particular band. For now we are going to compute the distance based on the number of bands they both reviewed. So, for example, when computing the distance between Angelica and Bill, we will use the ratings for *Blues Traveler*, *Broken Bells*, *Phoenix*, *Slightly Stoopid*, and *Vampire Weekend*. So the Manhattan Distance would be:

	Angelica	Bill	Difference
Blues Traveler	3.5	2	1.5
Broken Bells	2	3.5	1.5
Deadmau5	-	4	
Norah Jones	4.5	-	
Phoenix	5	2	3
Slightly Stoopid	1.5	3.5	2
The Strokes	2.5	-	-
Vampire Weekend	2	3	1
Manhattan Distance:			9

The Manhattan Distance row, the last row of the table, is simply the sum of the differences: (1.5 + 1.5 + 3 + 2 + 1).

Computing the Euclidean Distance is similar. We only use the bands they both reviewed:

	Angelica	Bill	Difference	Difference^2
Blues Traveler	3.5	2	1.5	2.25
Broken Bells	2	3.5	1.5	2.25
Deadmau5	-	4		
Norah Jones	4.5	-		
Phoenix	5	2	3	9
Slightly Stoopid	1.5	3.5	2	4
The Strokes	2.5	-	-	
Vampire Weekend	2	3	1	1
Sum of squares				18.5
Euclidean Distance				4.3

To parse that out a bit more:

$$Euclidean = \sqrt{(3.5-2)^2 + (2-3.5)^2 + (5-2)^2 + (1.5-3.5)^2 + (2-3)^2}$$

$$= \sqrt{1.5^2 + (-1.5)^2 + 3^2 + (-2)^2 + (-1)^2}$$

$$= \sqrt{2.25 + 2.25 + 9 + 4 + 1}$$

$$= \sqrt{18.5} = 4.3$$

Got it?

Try an example on your own...

	Angelica	Bill	Chan	Dan	Hailey	Jordyn	Sam	Veronica
Blues Traveler	3.5	2	5	3	-	-	5	3
Broken Bells	2	3.5	1	4	4	4.5	2	-
Deadmau5	-	4	1	4.5	1	4	-	-
Norah Jones	4.5	-	3	-	4	5	3	5
Phoenix	5	2	5	3	-	5	5	4
Slightly Stoopid	1.5	3.5	1	4.5	-	4.5	4	2.5
The Strokes	2.5	-	-	4	4	4	5	3
Vampire Weekend	2	3	-	2	1	4	-	-

sharpen your pencil

Compute the Euclidean Distance between Hailey and Veronica.

Compute the Euclidean Distance between Hailey and Jordyn

sharpen your pencil - solution

Compute the Euclidean Distance between Hailey and Veronica.

$$= \sqrt{(4-5)^2 + (4-3)^2} = \sqrt{1+1} = \sqrt{2} = 1.414$$

Compute the Euclidean Distance between Hailey and Jordyn

$$= \sqrt{(4-4.5)^2 + (1-4)^2 + (4-5)^2 + (4-4)^2 + (1-4)^2}$$

$$= \sqrt{(-0.5)^2 + (-3)^2 + (-1)^2 + (0)^2 + (-3)^2}$$

$$= \sqrt{.25 + 9 + 1 + 0 + 9} = \sqrt{19.25} = 4.387$$

A Flaw

It looks like we discovered a flaw with using these distance measures. When we computed the distance between Hailey and Veronica, we noticed they only rated two bands in common (Norah Jones and The Strokes), whereas when we computed the distance between Hailey and Jordyn, we noticed they rated five bands in common. This seems to skew our distance measurement, since the Hailey-Veronica distance is in 2 dimensions while the Hailey-Jordyn

distance is in 5 dimensions. Manhattan Distance and Euclidean Distance work best when there are no missing values. Dealing with missing values is an active area of scholarly research. Later in the book we will talk about how to deal with this problem. For now just be aware of the flaw as we continue our first exploration into building a recommendation system.

A generalization

We can generalize Manhattan Distance and Euclidean Distance to what is called the Minkowski Distance Metric:

$$d(x,y) = (\sum_{k=1}^{n} | x_k - y_k |^r)^{\frac{1}{r}}$$

When

- r = 1: The formula is Manhattan Distance.
- r = 2: The formula is Euclidean Distance
- r = ∞: Supremum Distance

Arghhhh Math!

When you see formulas like this in a book you have several options. One option is to see the formula--brain neurons fire that say *math formula*--and then you quickly skip over it to the next English bit. I have to admit that I was once a skipper. The other option is to see the formula, pause, and dissect it.

Many times you'll find the formula quite understandable. Let's dissect it now. When $r = 1$ the formula reduces to Manhattan Distance:

$$d(x,y) = \sum_{k=1}^{n} | x_k - y_k |$$

So for the music example we have been using throughout the chapter, x and y represent two people and $d(x, y)$ represents the distance between them. n is the number of bands they both rated (both x and y rated that band). We've done that calculation a few pages back:

	Angelica	Bill	Difference
Blues Traveler	3.5	2	1.5
Broken Bells	2	3.5	1.5
Deadmau5	-	4	
Norah Jones	4.5	-	
Phoenix	5	2	3
Slightly Stoopid	1.5	3.5	2
The Strokes	2.5	-	-
Vampire Weekend	2	3	1
Manhattan Distance:			9

That difference column represents the absolute value of the difference and we sum those up to get 9.

When r = 2, we get the Euclidean distance:

$$d(x,y) = \sqrt{\sum_{k=1}^{n} (x_k - y_k)^2}$$

Here's the scoop!

The greater the r, the more a large difference in one dimension will influence the total difference.

Representing the data in Python (finally some coding)

There are several ways of representing the data in the table above using Python. I am going to use Python's dictionary (also called an associative array or hash table):

Remember,

All the code for the book is available at www.guidetodatamining.com.

```
users = {"Angelica": {"Blues Traveler": 3.5, "Broken Bells": 2.0,
                      "Norah Jones": 4.5, "Phoenix": 5.0,
                        "Slightly Stoopid": 1.5,
                        "The Strokes": 2.5, "Vampire Weekend": 2.0},

         "Bill":     {"Blues Traveler": 2.0, "Broken Bells": 3.5,
                      "Deadmau5": 4.0, "Phoenix": 2.0,
                      "Slightly Stoopid": 3.5, "Vampire Weekend": 3.0},

         "Chan":     {"Blues Traveler": 5.0, "Broken Bells": 1.0,
                      "Deadmau5": 1.0, "Norah Jones": 3.0,
                      "Phoenix": 5, "Slightly Stoopid": 1.0},

         "Dan":      {"Blues Traveler": 3.0, "Broken Bells": 4.0,
                      "Deadmau5": 4.5, "Phoenix": 3.0,
                      "Slightly Stoopid": 4.5, "The Strokes": 4.0,
                      "Vampire Weekend": 2.0},

         "Hailey":   {"Broken Bells": 4.0, "Deadmau5": 1.0,
                      "Norah Jones": 4.0, "The Strokes": 4.0,
                      "Vampire Weekend": 1.0},

         "Jordyn":   {"Broken Bells": 4.5, "Deadmau5": 4.0, "Norah Jones": 5.0,
                      "Phoenix": 5.0, "Slightly Stoopid": 4.5,
                      "The Strokes": 4.0, "Vampire Weekend": 4.0},

         "Sam":      {"Blues Traveler": 5.0, "Broken Bells": 2.0,
                      "Norah Jones": 3.0, "Phoenix": 5.0,
                      "Slightly Stoopid": 4.0,  "The Strokes": 5.0},

         "Veronica": {"Blues Traveler": 3.0, "Norah Jones": 5.0,
                      "Phoenix": 4.0,  "Slightly Stoopid": 2.5,
                      "The Strokes": 3.0}}
```

We can get the ratings of a particular user as follows:

```
>>> users["Veronica"]
{"Blues Traveler": 3.0, "Norah Jones": 5.0, "Phoenix": 4.0,
"Slightly Stoopid": 2.5, "The Strokes": 3.0}

>>>
```

The code to compute Manhattan distance

I'd like to write a function that computes the Manhattan distance as follows:

```
def manhattan(rating1, rating2):
   """Computes the Manhattan distance. Both rating1 and rating2 are
   dictionaries of the form
   {'The Strokes': 3.0, 'Slightly Stoopid': 2.5 ..."""

   distance = 0
   for key in rating1:
      if key in rating2:
         distance += abs(rating1[key] - rating2[key])
   return distance
```

To test the function:

```
>>> manhattan(users['Hailey'], users['Veronica'])

2.0
>>> manhattan(users['Hailey'], users['Jordyn'])
7.5
>>>
```

Now a function to find the closest person (actually this returns a sorted list with the closest person first):

```
def computeNearestNeighbor(username, users):
    """creates a sorted list of users based on their distance to
    username"""
    distances = []
    for user in users:
        if user != username:
            distance = manhattan(users[user], users[username])
            distances.append((distance, user))
    # sort based on distance -- closest first
    distances.sort()
    return distances
```

And just a quick test of that function:

```
>>> computeNearestNeighbor("Hailey", users)
[(2.0, ''Veronica'), (4.0, 'Chan'),(4.0, 'Sam'), (4.5, 'Dan'), (5.0,
'Angelica'), (5.5, 'Bill'), (7.5, 'Jordyn')]
```

Finally, we are going to put this all together to make recommendations. Let's say I want to make recommendations for Hailey. I find her nearest neighbor—Veronica in this case. I will then find bands that Veronica has rated but Hailey has not. Also, I will assume that Hailey would have rated the bands the same as (or at least very similar to) Veronica. For example, Hailey has not rated the great band Phoenix. Veronica has rated Phoenix a '4' so we will assume Hailey is likely to enjoy the band as well. Here is my function to make recommendations.

```
def recommend(username, users):
    """Give list of recommendations"""
    # first find nearest neighbor
    nearest = computeNearestNeighbor(username, users)[0][1]
    recommendations = []
    # now find bands neighbor rated that user didn't
    neighborRatings = users[nearest]
    userRatings = users[username]
    for artist in neighborRatings:
        if not artist in userRatings:
            recommendations.append((artist, neighborRatings[artist]))
    # using the fn sorted for variety - sort is more efficient
    return sorted(recommendations,
                  key=lambda artistTuple: artistTuple[1],
                  reverse = True)
```

And now to make recommendations for Hailey:

```
>>> recommend('Hailey', users)
[('Phoenix', 4.0), ('Blues Traveler', 3.0), ('Slightly Stoopid', 2.5)]
```

That fits with our expectations. As we saw above, Hailey's nearest neighbor was Veronica and Veronica gave Phoenix a '4'. Let's try a few more:

```
>>> recommend('Chan', users)
```

```
[('The Strokes', 4.0), ('Vampire Weekend', 1.0)]

>>> recommend('Sam', users)
[('Deadmau5', 1.0)]
```

We think Chan will like The Strokes and also predict that Sam will not like Deadmau5.

```
>>> recommend('Angelica', users)
[]
```

Hmm. For Angelica we got back an empty set meaning we have no recommendations for her. Let us see what went wrong:

```
>>> computeNearestNeighbor('Angelica', users)
[(3.5, 'Veronica'), (4.5, 'Chan'), (5.0, 'Hailey'), (8.0, 'Sam'), (9.0,
'Bill'), (9.0, 'Dan'), (9.5, 'Jordyn')]
```

Angelica's nearest neighbor is Veronica. When we look at their ratings:

	Angelica	Bill	Chan	Dan	Hailey	Jordyn	Sam	Veronica
Blues Traveler	3.5	2	5	3	-	-	5	3
Broken Bells	2	3.5	1	4	4	4.5	2	-
Deadmau5	-	4	1	4.5	1	4	-	-
Norah Jones	4.5	-	3	-	4	5	3	5
Phoenix	5	2	5	3	-	5	5	4
Slightly Stoopid	1.5	3.5	1	4.5	-	4.5	4	2.5
The Strokes	2.5	-	-	4	4	4	5	3
Vampire Weekend	2	3	-	2	1	4	-	-

We see that Angelica rated every band that Veronica did. We have no new ratings, so no recommendations.

Shortly, we will see how to improve the system to avoid these cases.

exercise

1) Implement the Minkowski Distance function.

2) Alter the computeNearestNeighbor function to use Minkowski Distance.

exercise - solution

1) Implement the Minkowski Distance function.

```
def minkowski(rating1, rating2, r):
    """Computes the Minkowski distance.
    Both rating1 and rating2 are dictionaries of the form
    {'The Strokes': 3.0, 'Slightly Stoopid': 2.5}"""
    distance = 0
    commonRatings = False
    for key in rating1:
        if key in rating2:
            distance +=
                 pow(abs(rating1[key] - rating2[key]), r)
            commonRatings = True
    if commonRatings:
        return pow(distance,  1/r)
    else:
        return 0 #Indicates no ratings in common
```

2) Alter the computeNearestNeighbor function to use Minkowski Distance.

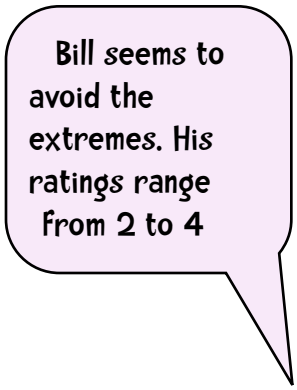
just need to alter the distance = line to

```
distance = minkowski(users[user], users[username], 2)
```

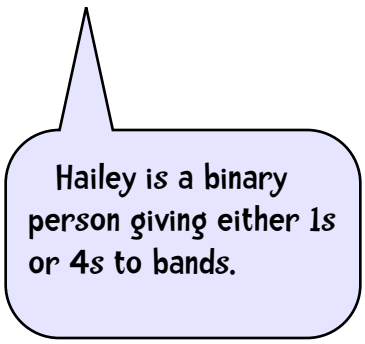
(the 2 as the r argument implements Euclidean)

Blame the users

Let's take a look at the user ratings in a bit more detail. We see that users have very different behaviors when it comes to rating bands



	Angelica	Bill	Chan	Dan	Hailey	Jordyn	Sam	Veronica
Blues Traveler	3.5	2	5	3	-	-	5	3
Broken Bells	2	3.5	1	4	4	4.5	2	-
Deadmau5	-	4	1	4.5	1	4	-	-
Norah Jones	4.5	-	3	-	4	5	3	5
Phoenix	5	2	5	3	-	5	5	4
Slightly Stoopid	1.5	3.5	1	4.5	-	4.5	4	2.5
The Strokes	2.5	-	-	4	4	4	5	3
Vampire Weekend	2	3	-	2	1	4	-	-



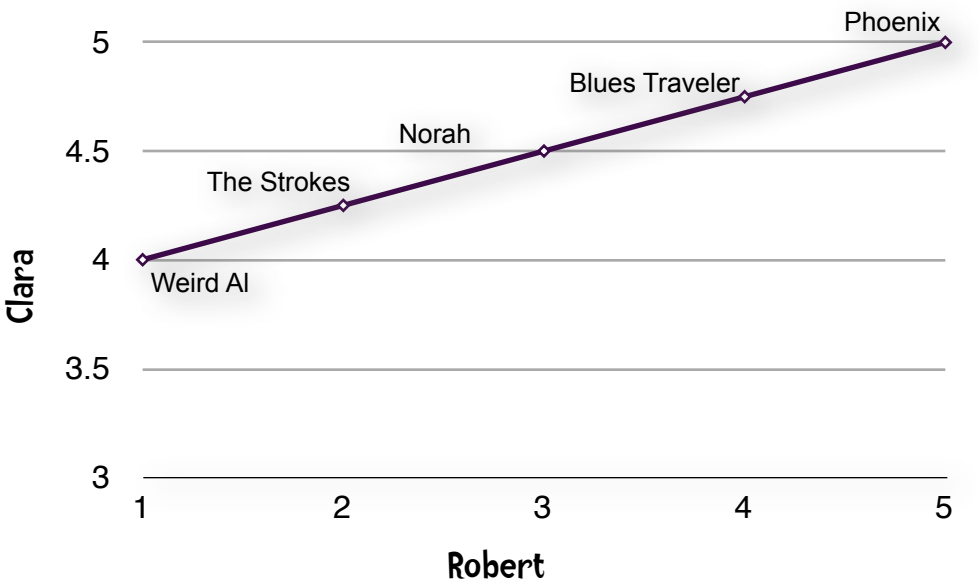
So how do we compare, for example, Hailey to Jordan? Does Hailey's '4' mean the same as Jordyn's '4' or Jordyn's '5'? I would guess it is more like Jordyn's '5'. This variability can create problems with a recommendation system.

Pearson Correlation Coefficient

One way to fix this problem is to use the Pearson Correlation Coefficient. First, the general idea. Consider the following data (not from the data set above):

	Blues Traveler	Norah Jones	Phoenix	The Strokes	Weird Al
Clara	4.75	4.5	5	4.25	4
Robert	4	3	5	2	1

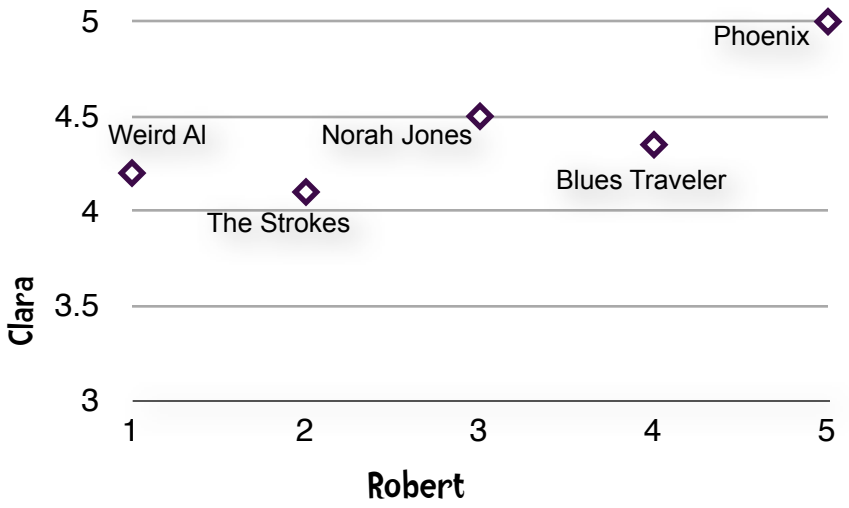
This is an example of what is called 'grade inflation' in the data mining community. Clara's lowest rating is 4—all her rating are between 4 and 5. If we are to graph this chart it would look like



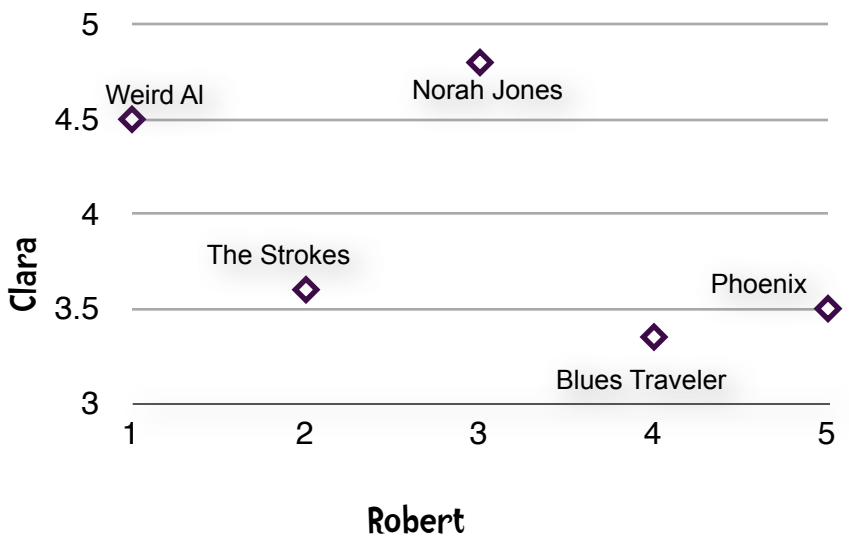
Straight line = Perfect Agreement!!!

The fact that this is a straight line indicates a perfect agreement between Clara and Robert. Both rated Phoenix as the best band, Blues Traveler next, Norah Jones after that, and so on. As Clara and Robert agree less, the less the data points reside on a straight line:

Pretty Good Agreement:



Not So Good Agreement:



So chart-wise, perfect agreement is indicated by a straight line. The Pearson Correlation Coefficient is a measure of correlation between two variables (in this specific case the correlation between Angelica and Bill). It ranges between -1 and 1 inclusive. 1 indicates perfect agreement. -1 indicates perfect disagreement. To give you a general feel for this, the chart above with the straight line has a Pearson of 1, the chart above that I labelled 'pretty good agreement' has a Pearson of 0.91, and the 'not so good agreement' chart has a Pearson of 0.81 So we can use this to find the individual who is most similar to the person we are interested in.

The formula for the Pearson Correlation Coefficient is

$$r = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}}$$

Arghhhh Math Again!

Here's a personal confession. I have a Bachelor of Fine Arts degree in music. While I have taken courses in ballet, modern dance, and costume design, I did not have a single math course as an undergrad. Before that, I attended an all boys trade high school where I took courses in plumbing and automobile repair, but no courses in math other than the basics. Either due to this background or some innate wiring in my brain, when I read a book that has formulas like the one above, I tend to skip over the formulas and continue with the text below them. If you are like me I would urge you to fight

that urge and actually look at the formula. Many formulas that on a quick glimpse look complex are actually understandable by mere mortals.

Other than perhaps looking complex, the problem with the formula above is that the algorithm to implement it would require multiple passes through the data. Fortunately for us algorithmic people, there is an alternative formula, which is an approximation of Pearson:

$$r = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}}{\sqrt{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}} \sqrt{\sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n}}}$$

(Remember what I said two paragraphs above about not skipping over formulas) This formula, in addition to looking initially horribly complex is, more importantly, numerically unstable meaning that what might be a small error is amplified by this reformulation. The big plus is that we can implement it using a single-pass algorithm, which we will get to shortly. First, let's dissect this formula and work through the example we saw a few pages back:

	Blues Traveler	Norah Jones	Phoenix	The Strokes	Weird Al
Clara	4.75	4.5	5	4.25	4
Robert	4	3	5	2	1

To start with, let us compute

$$\sum_{i=1}^{n} x_i y_i$$

Which is in the first expression in the numerator. Here the x and y represent Clara and Robert.

	Blues Traveler	Norah Jones	Phoenix	The Strokes	Weird Al
Clara	4.75	4.5	5	4.25	4
Robert	4	3	5	2	1

For each band we are going to multiple Clara's and Robert's rating together and sum the results:

$$(4.75 \times 4) + (4.5 \times 3) + (5 \times 5) + (4.25 \times 2) + (4 \times 1)$$

$$= 19 + 13.5 + 25 + 8.5 + 4 = 70$$

Sweet! Now let's compute the rest of the numerator:

$$\frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}$$

So the

$$\sum_{i=1}^{n} x_i$$

is the sum of Clara's ratings, which is 22.5. The sum of Robert's is 15 and they rated 5 bands:

$$\frac{22.5 \times 15}{5} = 67.5$$

So the numerator in the formula on page 26 is 70 - 67.5 = 2.5

Now let's dissect the denominator.

$$\sqrt{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}$$

First,

	Blues Traveler	Norah Jones	Phoenix	The Strokes	Weird Al
Clara	4.75	4.5	5	4.25	4
Robert	4	3	5	2	1

$$\sum_{i=1}^{n} x_i^2 = (4.75)^2 + (4.5)^2 + (5)^2 + (4.25)^2 + (4)^2 = 101.875$$

We've already computed the sum of Clara's ratings, which is 22.5. Square that and we get 506.25. We divide that by the number of co-rated bands (5) and we get 101.25.

Putting that together:

$$\sqrt{101.875 - 101.25} = \sqrt{.625} = .79057$$

Next we do the same computation for Robert:

$$\sqrt{\sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n}} = \sqrt{55 - 45} = 3.162277$$

Putting this altogether we get:

$$r = \frac{2.5}{.79057(3.162277)} = \frac{2.5}{2.5} = 1.00$$

So 1 means there was perfect agreement between Clara and Robert!

Take a break before moving on!!

exercise

Before going to the next page, implement the algorithm in Python. You should get the following results.

```
>>> pearson(users['Angelica'], users['Bill'])
-0.90405349906826993
>>> pearson(users['Angelica'], users['Hailey'])
0.42008402520840293
>>> pearson(users['Angelica'], users['Jordyn'])
0.76397486054754316
>>>
```

For this implementation you will need 2 Python functions sqrt (square root) and power operator ** which raises its left argument to the power of its right argument:

```
>>> from math import sqrt
>>> sqrt(9)
3.0
>>> 3**2
9
```

exercise - solution

Here is my implementation of Pearson

```
def pearson(rating1, rating2):
    sum_xy = 0
    sum_x = 0
    sum_y = 0
    sum_x2 = 0
    sum_y2 = 0
    n = 0
    for key in rating1:
        if key in rating2:
            n += 1
            x = rating1[key]
            y = rating2[key]
            sum_xy += x * y
            sum_x += x
            sum_y += y
            sum_x2 += x**2
            sum_y2 += y**2
    # if no ratings in common return 0
    if n == 0:
        return 0
    # now compute denominator
    denominator = sqrt(sum_x2 - (sum_x**2) / n) *
                  sqrt(sum_y2 - (sum_y**2) / n)
    if denominator == 0:
        return 0
    else:
        return (sum_xy - (sum_x * sum_y) / n) / denominator
```

One last formula – Cosine Similarity

I would like to present one last formula, which is very popular in text mining but also used in collaborative filtering—cosine similarity. To see when we might use this formula, let's say I change my example slightly. We will keep track of the number of times a person played a particular song track and use that information to base our recommendations on.

	number of plays		
	The Decemberists The King is Dead	Radiohead The King of Limbs	Katy Perry E.T.
Ann	10	5	32
Ben	15	25	1
Sally	12	6	27

Just by eye-balling the above chart (and by using any of the distance formulas mentioned above) we can see that Sally is more similar in listening habits to Ann than Ben is.

So what is the problem?

I have around four thousand tracks in iTunes. Here is a snapshot of the top few ordered by number of plays:

✓	Name	Time	Artist	Album	Genre	Plays ▾
✓	Moonlight Sonata	7:38	Marcus Miller	Silver Rain	Jazz+Funk	25
✓	Blast!	5:43	Marcus Miller	Marcus	Jazz	20
✓	Art Isn't Real (City of Sin)	2:48	Deer Tick	War Elephant	Alt-Country	19
✓	Between the Lines	4:35	Sara Bareilles	Little Voice	Folk	19
✓	Stay Around A Little Longer (Feat. B.B. King)	5:00	BUDDY GUY	Living Proof	Blues	18
✓	My Companjera	3:22	Gogol Bordello	Trans-Continental...	Alternative...	18
✓	Rebellious Love	3:57	Gogol Bordello	Trans-Continental...	Alternative...	18
✓	Immigraniada (We Comin' Rougher)	3:46	Gogol Bordello	Trans-Continental...	Alternative...	18
✓	Love Song	4:19	Sara Bareilles	Little Voice	Folk	18

So my top track is Moonlight Sonata by Marcus Miller with 25 plays. Chances are that you have played that track zero times. In fact, chances are good that you have not played any of my top tracks. In addition, there are over 15 million tracks in iTunes and I have only four thousand. So the data for a single person is sparse since it has relatively few non-zero attributes (plays of a track). When we compare two people by using the number of plays of the 15 million tracks, mostly they will have shared zeros in common. However, we do not want to use these shared zeros when we are computing similarity.

A similar case can be made when we are comparing text documents using words. Suppose we liked a certain book, say T*om Corbett Space Cadet: The Space Pioneers* by Carey Rockwell and we want to find a similar book. One possible way is to use word frequency. The attributes will be individual words and the values of those attributes will be the frequency of those words in the book. So 6.13% of the words in *The Space Pioneers* are occurrences of the word *the*, 0.89% are the word *Tom*, 0.25% of the words are *space*. I can compute the similarity of this book to others by using these word frequencies. However, the same problem related to sparseness of data occurs here. There are 6,629 unique words in *The Space Pioneers* and there are a bit over one million unique words in English. So if our attributes are English words, there will be relatively few non-zero attributes for *The Space Pioneers* or any other book. Again, any measure of similarity should not depend on the shared-zero values.

Cosine similarity ignores 0-0 matches. It is defined as

$$\cos(x,y) = \frac{x \cdot y}{||x|| \times ||y||}$$

where · indicates the dot product and ||x|| indicates the length of the vector x. The length of a vector is

$$\sqrt{\sum_{i=1}^{n} x_i^2}$$

Let's give this a try with the perfect agreement example used above:

	Blues Traveler	Norah Jones	Phoenix	The Strokes	Weird Al
Clara	4.75	4.5	5	4.25	4
Robert	4	3	5	2	1

The two vectors are:

$$x = (4.75, 4.5, 5, 4.25, 4)$$
$$y = (4, 3, 5, 2, 1)$$

then

$$||x|| = \sqrt{4.75^2 + 4.5^2 + 5^2 + 4.25^2 + 4^2} = \sqrt{101.875} = 10.09$$

$$||y|| = \sqrt{4^2 + 3^2 + 5^2 + 2^2 + 1^2} = \sqrt{55} = 7.416$$

The dot product is

$$x \cdot y = (4.75 \times 4) + (4.5 \times 3) + (5 \times 5) + (4.25 \times 2) + (4 \times 1) = 70$$

And the cosine similarity is

$$\cos(x, y) = \frac{70}{10.093 \times 7.416} = \frac{70}{74.85} = 0.935$$

The cosine similarity rating ranges from 1 indicated perfect similarity to -1 indicate perfect negative similarity. So 0.935 represents very good agreement.

sharpen your pencil

Compute the Cosine Similarity between Angelica and Veronica (from our dataset). (Consider dashes equal to zero)

	Blues Traveler	Broken Bells	Deadmau 5	Norah Jones	Phoenix	Slightly Stoopid	The Strokes	Vampire Weekend
Angelica	3.5	2	-	4.5	5	1.5	2.5	2
Veronica	3	-	-	5	4	2.5	3	-

sharpen your pencil - solution

Compute the Cosine Similarity between Angelica and Veronica (from our dataset).

	Blues Traveler	Broken Bells	Deadmau 5	Norah Jones	Phoenix	Slightly Stoopid	The Strokes	Vampire Weekend
Angelica	3.5	2	-	4.5	5	1.5	2.5	2
Veronica	3	-	-	5	4	2.5	3	-

$x = (3.5, 2, 0, 4.5, 5, 1.5, 2.5, 2)$

$y = (3, 0, 0, 5, 4, 2.5, 3, 0)$

$$||x|| = \sqrt{3.5^2 + 2^2 + 0^2 + 4.5^2 + 5^2 + 1.5^2 + 2.5^2 + 2^2} = \sqrt{74} = 8.602$$

$$||y|| = \sqrt{3^2 + 0^2 + 0^2 + 5^2 + 4^2 + 2.5^2 + 3^2 + 0^2} = \sqrt{65.25} = 8.078$$

The dot product is

$$x \cdot y =$$
$$(3.5 \times 3) + (2 \times 0) + (0 \times 0) + (4.5 \times 5) + (5 \times 4) + (1.5 \times 2.5) + (2.5 \times 3) + (2 \times 0) = 64.25$$

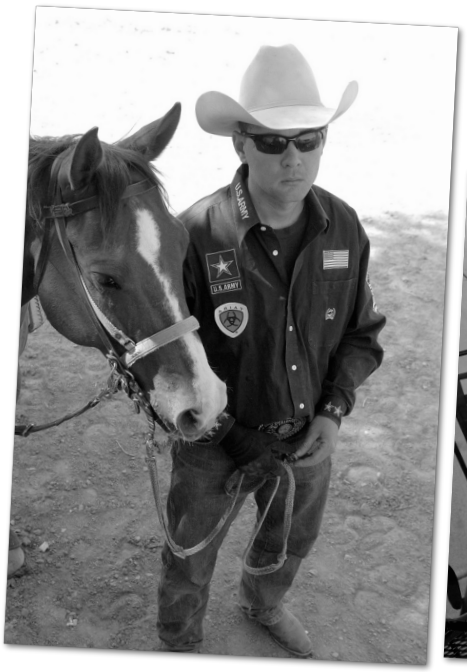
Cosine Similarity is

$$\cos(x, y) = \frac{64.25}{8.602 \times 8.078} = \frac{64.25}{69.487} = 0.9246$$

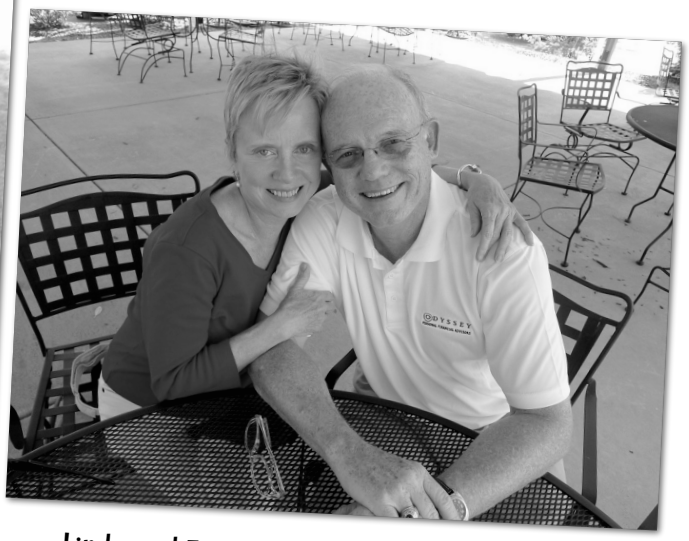
Which similarity measure to use?

We will be exploring this question throughout the book. For now, here are a few helpful hints:

So, if the data is dense (nearly all attributes have non-zero values) then Manhattan and Euclidean are reasonable to use. What happens if the data is not dense? Consider an expanded music rating system and three people, all of which have rated 100 songs on our site:



Jake: hardcore fan of Country



Linda and Eric: love, love, love 60s rock!

Linda and Eric enjoy the same kind of music. In fact, among their ratings, they have 20 songs in common and the difference in their ratings of those 20 songs (on a scale of 1 to 5) averages only 0.5!! The Manhattan Distance between them would be 20 x .5 = 10. The Euclidean Distance would be:

$$d = \sqrt{(.5)^2 \times 20} = \sqrt{.25 \times 20} = \sqrt{5} = 2.236$$

Linda and Jake have rated only one song in common: Chris Cagle's *What a Beautiful Day*. Linda thought it was okay and rated it a 3, Jake thought it was awesome and gave it a 5. So the Manhattan Distance between Jake and Linda is 2 and the Euclidean Distance is

$$d = \sqrt{(3-5)^2} = \sqrt{4} = 2$$

So both the Manhattan and Euclidean Distances show that Jake is a closer ~~match~~ to Linda than Eric is. So in this case both distance measures produce poor results.

Hey, I have an idea that might fix this problem.

Right now, people rate tunes on a scale of 1 to 5. How about for the tunes people don't rate I will assume the rating is 0. That way we solve the problem of sparse data as every object has a value for every attribute!

Good idea, but that doesn't work either. To see why we need to bring in a few more characters into our little drama: Cooper and Kelsey. Jake, Cooper and Kelsey have amazingly similar musical tastes. Jake has rated 25 songs on our site.

Cooper

Kelsey

Cooper has rated 26 songs, and 25 of them are the same songs Jake rated. They love the same kind of music and the average distance in their ratings is only 0.25!!

Kelsey loves both music and our site and has rated 150 songs. 25 of those songs are the same as the ones Cooper and Jake rated. Like Cooper, the average distance in her ratings and Jake's is only 0.25!!

Our gut feeling is that Cooper and Kelsey are equally close matches to Jake.

Now consider our modified Manhattan and Euclidean distance formulas where we assign a 0 for every song the person didn't rate.

With this scheme, Cooper is a much closer match to Jake than Kelsey is.

Why?

To answer why, let us look at a the following simplified example (again, a 0 means that person did not rate that song):

Song:	1	2	3	4	5	6	7	8	9	10
Jake	0	0	0	4.5	5	4.5	0	0	0	0
Cooper	0	0	4	5	5	5	0	0	0	0
Kelsey	5	4	4	5	5	5	5	5	4	4

Again, looking at the songs they mutually rated (songs 4, 5, and 6), Cooper and Kelsey seem like equally close matches for Jake. However, Manhattan Distance using those zero values tells a different story:

$$d_{Cooper,Jake} = (4-0)+(5-4.5)+(5-5)+5-4.5) = 4+0.5+0+0.5 = 5$$

$$d_{Kelsey,Jake} = (5-0)+(4-0)+(4-0)+(5-4.5)+(5-5)+(5-4.5)+(5-0)$$

$$+(5-0)+(4-0)+(4-0)$$

$$= 5+4+4+0.5+0+.5+5+5+4+4 = 32$$

The problem is that these zero values tend to dominate any measure of distance. So the solution of adding zeros is no better than the original distance formulas. One workaround people have used is to compute—in some sense—an 'average' distance where one computes the distance by using songs they rated in common divided that by the number of songs they rated in common.

Again, Manhattan and Euclidean work spectacularly well on dense data, but if the data is sparse it may be better to use Cosine Similarity.

Weirdnesses

Suppose we are trying to make recommendations for Amy who loves Phoenix, Passion Pit and Vampire Weekend. Our closest match is Bob who also loves Phoenix, Passion Pit, and Vampire Weekend. His father happens to play accordion for the Walter Ostanek Band, this year's Grammy winner in the polka category. Because of familial obligations, Bob gives 5 stars to the Walter Ostanek Band. Based on our current recommendation system, we think Amy will absolutely love the band. But common sense tells us she probably won't.

Or think of Professor Billy Bob Olivera who loves to read data mining books and science fiction. His closest match happens to be me, who also likes data mining books and science fiction. However, I like standard poodles and have rated The Secret Lives of Standard Poodles highly. Our current recommendation system would likely recommend that book to the professor.

The problem is that we are relying on a single “most similar” person. Any quirk that person has is passed on as a recommendation. One way of evening out those quirks is to base our recommendations on more than one person who is similar to our user. For this we can use the k-nearest neighbor approach.

K-nearest neighbor

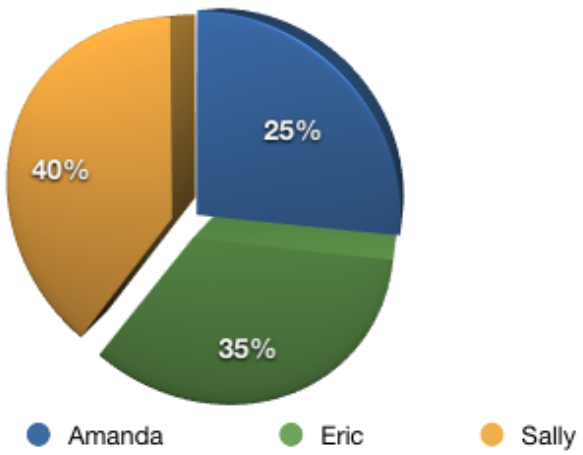
In the k-nearest neighbor approach to collaborative filtering we use k most similar people to determine recommendations. The best value for k is application specific—you will need to do some experimentation. Here's an example to give you the basic idea.

Suppose I would like to make recommendations for Ann and am using k-nearest neighbor with k=3. The three nearest neighbors and their Pearson scores are shown in the following table:

Person	Pearson
Sally	0.8
Eric	0.7
Amanda	0.5

0.8 + 0.7 + 0.5 = 2.0

Each of these three people are going to influence the recommendations. The question is how can I determine how much influence each person should have. If there is a Pie of Influence™, how big a slice should I give each person? If I add up the Pearson scores I get 2. Sally's share is 0.8/2 or 40%. Eric's share is 35% (0.7 / 2) and Amanda's share is 25%.



Suppose Amanda, Eric, and Sally, rated the band, The Grey Wardens as follows

Person	Grey Wardens Rating
Amanda	4.5
Eric	5
Sally	3.5

Person	Grey Wardens Rating	Influence
Amanda	4.5	25.00%
Eric	5	35.00%
Sally	3.5	40.00%

Projected rating = (4.5 x 0.25) + (5 x 0.35) + (3.5 x 0.4)

= 4.275

sharpen your pencil

Suppose I use the same data as above but use a k-nearest neighbor approach with k=2. What is my projected rating for Grey Wardens?

Person	Pearson
Sally	0.8
Eric	0.7
Amanda	0.5

Person	Grey Wardens Rating
Amanda	4.5
Eric	5
Sally	3.5

solution

Person	Pearson
Sally	0.8
Eric	0.7
Amanda	0.5

Person	Grey Wardens Rating
Amanda	4.5
Eric	5
Sally	3.5

Projected rating = Sally's portion + Eric's portion

= (3.5 x (0.8 / 1.5)) + (5 x (0.7 / 1.5))

= (3.5 x .5333) + (5 x 0.4667)

= 1.867 + 2.333

= 4.2

A Python Recommendation Class

I combined some of what we covered in this chapter in a Python Class. Even though it is slightly long I have included the code here (don't forget you can download the code at http://www.guidetodatamining.com).

```
import codecs
from math import sqrt

users = {"Angelica": {"Blues Traveler": 3.5, "Broken Bells": 2.0,
                      "Norah Jones": 4.5, "Phoenix": 5.0,
                      "Slightly Stoopid": 1.5,
                      "The Strokes": 2.5, "Vampire Weekend": 2.0},

         "Bill":{"Blues Traveler": 2.0, "Broken Bells": 3.5,
                 "Deadmau5": 4.0, "Phoenix": 2.0,
                 "Slightly Stoopid": 3.5, "Vampire Weekend": 3.0},

         "Chan": {"Blues Traveler": 5.0, "Broken Bells": 1.0,
                  "Deadmau5": 1.0, "Norah Jones": 3.0, "Phoenix": 5,
                  "Slightly Stoopid": 1.0},

         "Dan": {"Blues Traveler": 3.0, "Broken Bells": 4.0,
                 "Deadmau5": 4.5, "Phoenix": 3.0,
                 "Slightly Stoopid": 4.5, "The Strokes": 4.0,
                 "Vampire Weekend": 2.0},

         "Hailey": {"Broken Bells": 4.0, "Deadmau5": 1.0,
                    "Norah Jones": 4.0, "The Strokes": 4.0,
                    "Vampire Weekend": 1.0},

         "Jordyn":  {"Broken Bells": 4.5, "Deadmau5": 4.0,
                     "Norah Jones": 5.0, "Phoenix": 5.0,
                     "Slightly Stoopid": 4.5, "The Strokes": 4.0,
                     "Vampire Weekend": 4.0},
```

```
         "Sam": {"Blues Traveler": 5.0, "Broken Bells": 2.0,
                 "Norah Jones": 3.0, "Phoenix": 5.0,
                 "Slightly Stoopid": 4.0, "The Strokes": 5.0},

         "Veronica": {"Blues Traveler": 3.0, "Norah Jones": 5.0,
                      "Phoenix": 4.0, "Slightly Stoopid": 2.5,
                      "The Strokes": 3.0}
        }



class recommender:

    def __init__(self, data, k=1, metric='pearson', n=5):
        """ initialize recommender
        currently, if data is dictionary the recommender is initialized
        to it.
        For all other data types of data, no initialization occurs
        k is the k value for k nearest neighbor
        metric is which distance formula to use
        n is the maximum number of recommendations to make"""
        self.k = k
        self.n = n
        self.username2id = {}
        self.userid2name = {}
        self.productid2name = {}
        # for some reason I want to save the name of the metric
        self.metric = metric
        if self.metric == 'pearson':
            self.fn = self.pearson
        #
        # if data is dictionary set recommender data to it
        #
        if type(data).__name__ == 'dict':
            self.data = data
```

```
def convertProductID2name(self, id):
    """Given product id number return product name"""
    if id in self.productid2name:
        return self.productid2name[id]
    else:
        return id


def userRatings(self, id, n):
    """Return n top ratings for user with id"""
    print ("Ratings for " + self.userid2name[id])
    ratings = self.data[id]
    print(len(ratings))
    ratings = list(ratings.items())
    ratings = [(self.convertProductID2name(k), v)
               for (k, v) in ratings]
    # finally sort and return
    ratings.sort(key=lambda artistTuple: artistTuple[1],
                 reverse = True)
    ratings = ratings[:n]
    for rating in ratings:
        print("%s\t%i" % (rating[0], rating[1]))



def loadBookDB(self, path=''):
    """loads the BX book dataset. Path is where the BX files are
    located"""
    self.data = {}
    i = 0
    #
    # First load book ratings into self.data
    #
    f = codecs.open(path + "BX-Book-Ratings.csv", 'r', 'utf8')
    for line in f:
        i += 1
```

```
        # separate line into fields
        fields = line.split(';')
        user = fields[0].strip('"')
        book = fields[1].strip('"')
        rating = int(fields[2].strip().strip('"'))
        if user in self.data:
            currentRatings = self.data[user]
        else:
            currentRatings = {}
        currentRatings[book] = rating
        self.data[user] = currentRatings
    f.close()
    #
    # Now load books into self.productid2name
    # Books contains isbn, title, and author among other fields
    #
    f = codecs.open(path + "BX-Books.csv", 'r', 'utf8')
    for line in f:
        i += 1
        # separate line into fields
        fields = line.split(';')
        isbn = fields[0].strip('"')
        title = fields[1].strip('"')
        author = fields[2].strip().strip('"')
        title = title + ' by ' + author
        self.productid2name[isbn] = title
    f.close()
    #
    #  Now load user info into both self.userid2name and
    #  self.username2id
    #
    f = codecs.open(path + "BX-Users.csv", 'r', 'utf8')
    for line in f:
        i += 1
        # separate line into fields
        fields = line.split(';')
        userid = fields[0].strip('"')
```

```
            location = fields[1].strip('"')
            if len(fields) > 3:
                age = fields[2].strip().strip('"')
            else:
                age = 'NULL'
            if age != 'NULL':
                value = location + '  (age: ' + age + ')'
            else:
                value = location
            self.userid2name[userid] = value
            self.username2id[location] = userid
        f.close()
        print(i)


    def pearson(self, rating1, rating2):
        sum_xy = 0
        sum_x = 0
        sum_y = 0
        sum_x2 = 0
        sum_y2 = 0
        n = 0
        for key in rating1:
            if key in rating2:
                n += 1
                x = rating1[key]
                y = rating2[key]
                sum_xy += x * y
                sum_x += x
                sum_y += y
                sum_x2 += pow(x, 2)
                sum_y2 += pow(y, 2)
        if n == 0:
            return 0
        # now compute denominator
        denominator = (sqrt(sum_x2 - pow(sum_x, 2) / n)
                       * sqrt(sum_y2 - pow(sum_y, 2) / n))
```

```
        if denominator == 0:
            return 0
        else:
            return (sum_xy - (sum_x * sum_y) / n) / denominator


    def computeNearestNeighbor(self, username):
        """creates a sorted list of users based on their distance to
        username"""
        distances = []
        for instance in self.data:
            if instance != username:
                distance = self.fn(self.data[username],
                                   self.data[instance])
                distances.append((instance, distance))
        # sort based on distance -- closest first
        distances.sort(key=lambda artistTuple: artistTuple[1],
                       reverse=True)
        return distances

    def recommend(self, user):
       """Give list of recommendations"""
       recommendations = {}
       # first get list of users  ordered by nearness
       nearest = self.computeNearestNeighbor(user)
       #
       # now get the ratings for the user
       #
       userRatings = self.data[user]
       #
       # determine the total distance
       totalDistance = 0.0
       for i in range(self.k):
          totalDistance += nearest[i][1]
       # now iterate through the k nearest neighbors
       # accumulating their ratings
       for i in range(self.k):
```

```
            # compute slice of pie
            weight = nearest[i][1] / totalDistance
            # get the name of the person
            name = nearest[i][0]
            # get the ratings for this person
            neighborRatings = self.data[name]
            # get the name of the person
            # now find bands neighbor rated that user didn't
            for artist in neighborRatings:
                if not artist in userRatings:
                    if artist not in recommendations:
                        recommendations[artist] = (neighborRatings[artist]
                                                   * weight)
                    else:
                        recommendations[artist] = (recommendations[artist]
                                                   + neighborRatings[artist]
                                                   * weight)
        # now make list from dictionary
        recommendations = list(recommendations.items())
        recommendations = [(self.convertProductID2name(k), v)
                           for (k, v) in recommendations]
        # finally sort and return
        recommendations.sort(key=lambda artistTuple: artistTuple[1],
                             reverse = True)
        # Return the first n items
        return recommendations[:self.n]
```

Example of this program executing

First, I will construct an instance of the recommender class with the data we previously used:

```
>>> r = recommender(users)
```

Some simple examples using these band ratings:

```
>>> r.recommend('Jordyn')
[('Blues Traveler', 5.0)]
>>> r.recommend('Hailey')
[('Phoenix', 5.0), ('Slightly Stoopid', 4.5)]
```

A New Dataset

Ok, it is time to look at a more realistic dataset. Cai-Nicolas Zeigler collected over one million ratings of books from the Book Crossing website. This ratings are of 278,858 users rating 271,379 books. This anonymized data is available at http://www.informatik.uni-freiburg.de/~cziegler/BX/ both as an SQL dump and a text file of comma-separated-values (CSV). I had some problems loading this data into Python due to apparent character encoding problems. My fixed version of the CSV files are available on this book's website.

The CSV files represent three tables:

- BX-Users, which, as the name suggests, contains information about the users. There is an integer user-id field, as well as the location (i.e., Albuquerque, NM) and age. The names have been removed to anonymize the data.
- BX-Books. Books are identified by the ISBN, book title, author, year of publication, and publisher.
- BX-Book-Ratings, which includes a user-id, book ISBN, and a rating from 0-10.

The function loadBookDB in the recommender class loads the data from these files.

Now I am going to load the book dataset. The argument to the loadBookDB function is the path to the BX book files.

```
>>> r.loadBookDB('/Users/raz/Downloads/BX-Dump/')
1700018
```

> Note:
> This is a large dataset and may take a bit of time to load on your computer. On my Hackintosh (2.8 GHz i7 860 with 8GB RAM) it takes 24 seconds to load the dataset and 30 seconds to run a query.

Now I can get recommendations for user 17118, a person from Toronto:

```
>>> r.recommend('171118')
[("The Godmother's Web by Elizabeth Ann Scarborough", 10.0), ("The Irrational
Season (The Crosswicks Journal, Book 3) by Madeleine L'Engle", 10.0), ("The
Godmother's Apprentice by Elizabeth Ann Scarborough", 10.0), ("A Swiftly
Tilting Planet by Madeleine L'Engle", 10.0), ('The Girl Who Loved Tom Gordon by
Stephen King', 9.0), ('The Godmother by Elizabeth Ann Scarborough', 8.0)]
```

```
>>> r.userRatings('171118', 5)
Ratings for toronto, ontario, canada
2421
The Careful Writer by Theodore M. Bernstein   10
Wonderful Life: The Burgess Shale and the Nature of History by Stephen Jay
Gould  10
Pride and Prejudice (World's Classics) by Jane Austen      10
The Wandering Fire (The Fionavar Tapestry, Book 2) by Guy Gavriel Kay   10
Flowering trees and shrubs: The botanical paintings of Esther Heins by Judith
Leet   10
```

Projects

You won't really learn this material unless you play around with the code. Here are some suggestions of what you might try.

1. Implement Manhattan distance and Euclidean distance and compare the results of these three methods.

2. The book website has a file containing movie ratings for 25 movies. Create a function that loads the data into your classifier. The recommend method described above should recommend movies for a specific person.