Load the DimIrrepsLie package:

```
(* Get[
  "/Users/robert/Documents/MyMathematicaPrograms/DimIrrepsProj/DimIrrepsLie.wl"
  ]; *)

(* Get["…/src/DimIrrepsLie.wl"] on GitHub *)
```

In[ ]:=

Basic Examples

The Lie algebra Ar is the default:

In[4]:=
```
DimensionIrrepLie[{a, b}](* A2 = su(3) *)
```

Out[4]= $\frac{1}{2}(1+a)(1+b)(2+a+b)$

Various types of Lie algebras:

In[5]:=
```
DimensionIrrepLie ["D5", {1, 0, 0, 0, 0}] (* Vectorial representation of so(10) *)
```

Out[5]= 10

In[6]:=
```
DimensionIrrepLie ["D5", {0, 0, 0, 0, 1}]
(* Half-spinor representation of so(10) *)
```

Out[6]= 16

In[7]:=
```
DimensionIrrepLie ["D5", {0, 0, 0, 1, 0}]
(* Half-spinor representation of so(10) *)
```

Out[7]= 16

In[8]:=
```
DimensionIrrepLie ["E6", {2, 5, 13, 4, 11, 18}]
```

Out[8]= 199 815 954 869 278 740 594 812 606 136 365 625

In[9]:=
```
DimensionIrrepLie ["G2", {a, b}]
```

Out[9]= $\frac{9}{40}(1+a)\left(\frac{1}{3}+\frac{b}{3}\right)\left(\frac{4}{3}+a+\frac{b}{3}\right)\left(\frac{5}{3}+a+\frac{2b}{3}\right)(2+a+b)(3+2a+b)$

Quantum dimension (numeric):

In[10]:=
```
DimensionIrrepLie["A3", {2, 5, 3}, q]
```

Out[10]=

$$\frac{\text{qnum}[4,q]\times\text{qnum}[6,q]\times\text{qnum}[9,q]\times\text{qnum}[10,q]\times\text{qnum}[13,q]}{\text{qnum}[1,q]^3\,\text{qnum}[2,q]^2}$$

In[11]:=
```
% /. {qnum[n_, q] :> Subscript[Row[{"(", n, ")"}], q]}
```

Out[11]=

$$\frac{(4)_q\,(6)_q\,(9)_q\,(10)_q\,(13)_q}{(1)_q^3\,(2)_q^2}$$

Quantum dimension (symbolic):

In[12]:=
```
DimensionIrrepLie["A3", {a, b, c}, q]
```
Out[12]=

$$(\text{qnum}[1+a, q] \times \text{qnum}[1+b, q] \times \text{qnum}[2+a+b, q] \times \text{qnum}[1+c, q] \times \text{qnum}[2+b+c, q] \times \text{qnum}[3+a+b+c, q]) \Big/ \left(\text{qnum}[1, q]^3\, \text{qnum}[2, q]^2\, \text{qnum}[3, q]\right)$$

In[13]:=
```
% /. {qnum[n_, q] :> Subscript[Row[{"(", n, ")"}], q]}
```
Out[13]=

$$\frac{(1+a)_q\,(1+b)_q\,(2+a+b)_q\,(1+c)_q\,(2+b+c)_q\,(3+a+b+c)_q}{(1)_q^3\,(2)_q^2\,(3)_q}$$

In[14]:=
```
%% /. {qnum[n_, q] :> (q^n - q^(-n)) / (q - q^(-1))} // Simplify
```
Out[14]=

$$\left(q^{-3a-4b-3c}\left(-1+q^{2+2a}\right)\left(-1+q^{2(2+a+b)}\right)\left(-1+q^{2+2b}\right)\left(-1+q^{2(2+b+c)}\right)\left(-1+q^{2(3+a+b+c)}\right)\left(-1+q^{2+2c}\right)\right) \Big/ \left(\left(-1+q^2\right)^6\left(1+q^2\right)^2\left(1+q^2+q^4\right)\right)$$

Recovering the classical limit q -> 1:

In[15]:=
```
DimensionIrrepLie["A3", {2, 5, 3}, q];
```

In[16]:=
```
% /. { qnum[n_, q] :> (q^n - q^(-n)) / (q - q^(-1)) } // Simplify
```
Out[16]=

$$\frac{\left(-\frac{1}{q^6}+q^6\right)\left(-1+q^8\right)\left(-\frac{1}{q^9}+q^9\right)\left(-1+q^{20}\right)\left(-1+q^{26}\right)}{q^{23}\left(-\frac{1}{q}+q\right)^3\left(-1+q^4\right)^2}$$

In[17]:=
```
Limit[%, q → 1]
```
Out[17]=

7020

In[18]:=
```
DimensionIrrepLie["A3", {2, 5, 3}]
```
Out[18]=

7020

Same Lie algebra case as before, but symbolic:

In[19]:=
```
DimensionIrrepLie["A3", {a, b, c}, q];
```

In[20]:=
```
% /. { qnum[n_, q] :> (q^n - q^(-n)) / (q - q^(-1)) } // Simplify
```
Out[20]=

$$\left(q^{-3a-4b-3c}\left(-1+q^{2+2a}\right)\left(-1+q^{2(2+a+b)}\right)\left(-1+q^{2+2b}\right)\left(-1+q^{2(2+b+c)}\right)\left(-1+q^{2(3+a+b+c)}\right)\left(-1+q^{2+2c}\right)\right) \Big/ \left(\left(-1+q^2\right)^6\left(1+q^2\right)^2\left(1+q^2+q^4\right)\right)$$

In[21]:=
```
Limit[%, q → 1]
```
Out[21]=

$$\frac{1}{12}(1+a)(1+b)(2+a+b)(1+c)(2+b+c)(3+a+b+c)$$

In[22]:=
```
DimensionIrrepLie["A3", {a, b, c}]
```
Out[22]=

$$\frac{1}{12}(1+a)(1+b)(2+a+b)(1+c)(2+b+c)(3+a+b+c)$$

In[ ]:=

Applications

In[ ]:=

Classical dimensions (symbolic)

The Lie algebra is Ar, r = 2, 3, 4, …:

In[23]:=
```
DimensionIrrepLie[Table[Subscript[x, j], {j, 1, 2} ]]
```
Out[23]=

$$\frac{1}{2}(1+x_1)(1+x_2)(2+x_1+x_2)$$

In[24]:=
```
DimensionIrrepLie[Table[Subscript[x, j], {j, 1, 3} ]]
```
Out[24]=

$$\frac{1}{12}(1+x_1)(1+x_2)(2+x_1+x_2)(1+x_3)(2+x_2+x_3)(3+x_1+x_2+x_3)$$

In[25]:=
```
DimensionIrrepLie[Table[Subscript[x, j], {j, 1, 4} ]]
```
Out[25]=

$$\frac{1}{288}(1+x_1)(1+x_2)(2+x_1+x_2)(1+x_3)(2+x_2+x_3)(3+x_1+x_2+x_3)(1+x_4)(2+x_3+x_4)(3+x_2+x_3+x_4)(4+x_1+x_2+x_3+x_4)$$

With the above syntax the following equality is used:

In[26]:=

$$\text{Table}\left[\text{With}\left[\{r = \text{rval}\}, \text{DimensionIrrepLie}[\text{Table}[\text{Subscript}[x, j], \{j, 1, r\}]] == \frac{\prod_{p=1}^{r}\left(\prod_{s=1}^{1-p+r}\left(p + \sum_{j=s}^{-1+p+s} x_j\right)\right)}{\text{BarnesG}[2+r]}\right], \{\text{rval}, 1, 7\}\right]$$

Out[26]=

{True, True, True, True, True, True, True}

In[27]:=
```
DimensionIrrepLie[{a, b, c}] // Timing
```
Out[27]=

$$\left\{0.000153, \frac{1}{12}(1+a)(1+b)(2+a+b)(1+c)(2+b+c)(3+a+b+c)\right\}$$

In[28]:=
```
DimensionIrrepLie["A3", {a, b, c}] // Timing (* first call *)
```
Out[28]=

$$\left\{0.000129, \frac{1}{12}(1+a)(1+b)(2+a+b)(1+c)(2+b+c)(3+a+b+c)\right\}$$

In[29]:=
```
DimensionIrrepLie["A3", {a, b, c}] // Timing (* second call *)
```
Out[29]=

$$\left\{0.000116, \frac{1}{12}(1+a)(1+b)(2+a+b)(1+c)(2+b+c)(3+a+b+c)\right\}$$

Other examples of Lie algebras:

In[30]:=
```
DimensionIrrepLie["G2", {a, b}]
```
Out[30]=

$$\frac{9}{40}(1+a)\left(\frac{1}{3}+\frac{b}{3}\right)\left(\frac{4}{3}+a+\frac{b}{3}\right)\left(\frac{5}{3}+a+\frac{2b}{3}\right)(2+a+b)(3+2a+b)$$

In[31]:=
```
DimensionIrrepLie["B3", {a, b, c}]
```

Out[31]=

$$\frac{1}{90} (1+a) (1+b) (2+a+b) \left(\frac{1}{2}+\frac{c}{2}\right) \left(\frac{3}{2}+b+\frac{c}{2}\right) \left(\frac{5}{2}+a+b+\frac{c}{2}\right) (2+b+c) (3+a+b+c) (4+a+2 b+c)$$

In[32]:=
```
DimensionIrrepLie["C3", {a, b, c}]
```

Out[32]=

$$\frac{1}{90} \left(\frac{1}{2}+\frac{a}{2}\right) \left(1+\frac{a}{2}+\frac{b}{2}\right) (1+b) (1+c) \left(\frac{3}{2}+\frac{b}{2}+c\right) (2+b+c) (3+a+b+c) (4+a+b+2 c) (5+a+2 b+2 c)$$

In[33]:=
```
DimensionIrrepLie["D4", {a, b, c, d}]
```

Out[33]=

$$\frac{1}{4320} (1+a) (1+b) (2+a+b) (1+c) (2+b+c) (3+a+b+c) (1+d) (2+b+d) (3+a+b+d) (3+b+c+d) (4+a+b+c+d) (5+a+2 b+c+d)$$

In[34]:=
```
DimensionIrrepLie["A6", Table[Subscript[x, j], {j, 1, 6} ]]
```

Out[34]=

$$\frac{1}{24\,883\,200} (1+x_1) (1+x_2) (2+x_1+x_2) (1+x_3) (2+x_2+x_3) (3+x_1+x_2+x_3) (1+x_4) (2+x_3+x_4) (3+x_2+x_3+x_4) (4+x_1+x_2+x_3+x_4) (1+x_5) (2+x_4+x_5) (3+x_3+x_4+x_5) (4+x_2+x_3+x_4+x_5) (5+x_1+x_2+x_3+x_4+x_5) (1+x_6) (2+x_5+x_6) (3+x_4+x_5+x_6) (4+x_3+x_4+x_5+x_6) (5+x_2+x_3+x_4+x_5+x_6) (6+x_1+x_2+x_3+x_4+x_5+x_6)$$

In[35]:=
```
DimensionIrrepLie["E6", Table[Subscript[x, j], {j, 1, 6} ]]
```

Out[35]=

$$\frac{1}{23\,361\,421\,521\,715\,200\,000} (1+x_1) (1+x_2) (2+x_1+x_2) (1+x_3) (2+x_2+x_3) (3+x_1+x_2+x_3) (1+x_4) (2+x_3+x_4) (3+x_2+x_3+x_4) (4+x_1+x_2+x_3+x_4) (1+x_5) (2+x_4+x_5) (3+x_3+x_4+x_5) (4+x_2+x_3+x_4+x_5) (5+x_1+x_2+x_3+x_4+x_5) (1+x_6) (2+x_3+x_6) (3+x_2+x_3+x_6) (4+x_1+x_2+x_3+x_6) (3+x_3+x_4+x_6) (4+x_2+x_3+x_4+x_6) (5+x_1+x_2+x_3+x_4+x_6) (5+x_2+2 x_3+x_4+x_6) (6+x_1+x_2+2 x_3+x_4+x_6) (7+x_1+2 x_2+2 x_3+x_4+x_6) (4+x_3+x_4+x_5+x_6) (5+x_2+x_3+x_4+x_5+x_6) (6+x_1+x_2+x_3+x_4+x_5+x_6) (6+x_2+2 x_3+x_4+x_5+x_6) (7+x_1+x_2+2 x_3+x_4+x_5+x_6) (8+x_1+2 x_2+2 x_3+x_4+x_5+x_6) (7+x_2+2 x_3+2 x_4+x_5+x_6) (8+x_1+x_2+2 x_3+2 x_4+x_5+x_6) (9+x_1+2 x_2+2 x_3+2 x_4+x_5+x_6) (10+x_1+2 x_2+3 x_3+2 x_4+x_5+x_6) (11+x_1+2 x_2+3 x_3+2 x_4+x_5+2 x_6)$$

In[36]:=
```
DimensionIrrepLie["F4", Table[Subscript[x, j], {j, 1, 4} ]]
```
Out[36]=

$$\frac{1}{5\,893\,965\,000}\,(1+x_1)\,(1+x_2)\,(2+x_1+x_2)\left(\frac{1}{2}+\frac{x_3}{2}\right)\left(\frac{3}{2}+x_2+\frac{x_3}{2}\right)$$
$$\left(\frac{5}{2}+x_1+x_2+\frac{x_3}{2}\right)(2+x_2+x_3)\,(3+x_1+x_2+x_3)\,(4+x_1+2\,x_2+x_3)\left(\frac{1}{2}+\frac{x_4}{2}\right)$$
$$\left(1+\frac{x_3}{2}+\frac{x_4}{2}\right)\left(2+x_2+\frac{x_3}{2}+\frac{x_4}{2}\right)\left(3+x_1+x_2+\frac{x_3}{2}+\frac{x_4}{2}\right)\left(\frac{5}{2}+x_2+x_3+\frac{x_4}{2}\right)$$
$$\left(\frac{7}{2}+x_1+x_2+x_3+\frac{x_4}{2}\right)\left(\frac{9}{2}+x_1+2\,x_2+x_3+\frac{x_4}{2}\right)\left(5+x_1+2\,x_2+\frac{3\,x_3}{2}+\frac{x_4}{2}\right)$$
$$(3+x_2+x_3+x_4)\,(4+x_1+x_2+x_3+x_4)\,(5+x_1+2\,x_2+x_3+x_4)\left(\frac{11}{2}+x_1+2\,x_2+\frac{3\,x_3}{2}+x_4\right)$$
$$(6+x_1+2\,x_2+2\,x_3+x_4)\,(7+x_1+3\,x_2+2\,x_3+x_4)\,(8+2\,x_1+3\,x_2+2\,x_3+x_4)$$

In[ ]:=
Fundamental representations of exceptional simple Lie algebras

In[37]:=
```
Table[DimensionIrrepLie ["E6", UnitVector[6, p]], {p, 1, 6}]
```
Out[37]=

{27, 351, 2925, 351, 27, 78}

In[38]:=
```
Table[DimensionIrrepLie ["E7", UnitVector[7, p]], {p, 1, 7}]
```
Out[38]=

{56, 1539, 27 664, 365 750, 8645, 133, 912}

In[39]:=
```
Table[DimensionIrrepLie ["E8", UnitVector[8, p]], {p, 1, 8}]
```
Out[39]=

{248, 30 380, 2 450 240, 146 325 270, 6 899 079 264, 6 696 000, 3875, 147 250}

In[40]:=
```
Table[DimensionIrrepLie ["G2", UnitVector[2, p]], {p, 1, 2}]
```
Out[40]=

{14, 7}

In[41]:=
```
Table[DimensionIrrepLie ["F4", UnitVector[4, p]], {p, 1, 4}]
```
Out[41]=

{52, 1274, 273, 26}

In[ ]:=
Quantum dimensions (numeric input)

Classical dimension:

In[42]:=
```
res = DimensionIrrepLie ["A3", {3, 4, 1}]
```
Out[42]=

2310

Quantum dimension:

In[43]:=
```
qres = DimensionIrrepLie ["A3", {3, 4, 1}, q]
```
Out[43]=

$$\frac{\text{qnum}[4,\,q]\times\text{qnum}[5,\,q]\times\text{qnum}[7,\,q]\times\text{qnum}[9,\,q]\times\text{qnum}[11,\,q]}{\text{qnum}[1,\,q]^3\,\text{qnum}[2,\,q]\times\text{qnum}[3,\,q]}$$

A nicer output (Warning: One should use RuleDelayed (:>) in the following input, not Rule (->)):

In[44]:=
```
qres /. {qnum[n_, q] :> Subscript[Row[{"(", n, ")"}], q]}
```
Out[44]=

$$\frac{(4)_q\,(5)_q\,(7)_q\,(9)_q\,(11)_q}{(1)_q^3\,(2)_q\,(3)_q}$$

The quantum dimension in terms of the formal variable q:

In[45]:=
```
qresval = qres /. {qnum[n_, q] :> (q^n - q^(-n)) / (q - q^(-1))}
```
Out[45]=

$$\frac{\left(-\frac{1}{q^4}+q^4\right)\left(-\frac{1}{q^5}+q^5\right)\left(-\frac{1}{q^7}+q^7\right)\left(-\frac{1}{q^9}+q^9\right)\left(-\frac{1}{q^{11}}+q^{11}\right)}{\left(-\frac{1}{q}+q\right)^3\left(-\frac{1}{q^2}+q^2\right)\left(-\frac{1}{q^3}+q^3\right)}$$

In[46]:=
```
Limit[%, q → 1](* Checking the classical limit *)
```
Out[46]=

2310

The quantum dimension when q is a root of unity:

In[47]:=
```
qresvaltrig = FullSimplify[ExpToTrig[Expand[Together[qresval] /.{q → Exp[ⅈ π/κ]}]]]
```
Out[47]=

$$2\,\mathrm{Cos}\left[\frac{2\pi}{\kappa}\right]\left(1+2\,\mathrm{Cos}\left[\frac{2\pi}{\kappa}\right]+2\,\mathrm{Cos}\left[\frac{4\pi}{\kappa}\right]\right)\left(1+2\,\mathrm{Cos}\left[\frac{6\pi}{\kappa}\right]\right)\left(1+2\,\mathrm{Cos}\left[\frac{2\pi}{\kappa}\right]+2\,\mathrm{Cos}\left[\frac{4\pi}{\kappa}\right]+2\,\mathrm{Cos}\left[\frac{6\pi}{\kappa}\right]\right)\left(-1+2\,\mathrm{Cos}\left[\frac{8\pi}{\kappa}\right]+2\,\mathrm{Cos}\left[\frac{2\pi}{\kappa}\right]\left(1+2\,\mathrm{Cos}\left[\frac{2\pi}{\kappa}\right]+2\,\mathrm{Cos}\left[\frac{8\pi}{\kappa}\right]\right)\right)$$

In[48]:=
```
% /. κ → 12 (* a particular choice for
 κ. Warning: κ should be large enough for this irrep to exist *)
```
Out[48]=

$$\sqrt{3}\,\left(2+\sqrt{3}\right)^2$$

In[49]:=
```
Chop[N[%]]
```
Out[49]=

24.1244

The quantum dimension in terms of QPochhammer symbol:

In[50]:=
```
qres /. {qnum[n_, q] → -(q^(1-n) QPochhammer[q^2, q^2, n])/((-1 + q^2) QPochhammer[q^2, q^2, -1 + n])} // Simplify
```
Out[50]=

$$\left(\mathrm{QPochhammer}[q^2, q^2, 5]\,\mathrm{QPochhammer}[q^2, q^2, 7]\,\mathrm{QPochhammer}[q^2, q^2, 9]\,\mathrm{QPochhammer}[q^2, q^2, 11]\right) \Big/ \left(q^{28}\left(-1+q^2\right)^2 \mathrm{QPochhammer}[q^2, q^2, 3]^2\,\mathrm{QPochhammer}[q^2, q^2, 6]\,\mathrm{QPochhammer}[q^2, q^2, 8]\,\mathrm{QPochhammer}[q^2, q^2, 10]\right)$$

In[51]:=
```
% // FunctionExpand
```
Out[51]=

$$\frac{\left(1-q^8\right)\left(1-q^{10}\right)\left(1-q^{14}\right)\left(1-q^{18}\right)\left(1-q^{22}\right)}{q^{28}\left(1-q^2\right)\left(-1+q^2\right)^2\left(1-q^4\right)\left(1-q^6\right)}$$

In[52]:=
```
Chop[N[% /.{q → Exp[iπ/12]}], 10^-8]
```
Out[52]=
```
24.1244
```

In the framework of quantum Lie algebras (actually quantum enveloping algebras of Lie algebras) at roots of unity, the parameter κ (sometimes called the altitude) is
$\kappa = g + k$ where g is the dual Coxeter number of the chosen Lie algebra, and where k is a non-negative integer called the level.
The level k should be large enough for the chosen irreducible representation to exist (i.e. to be integrable).
For Ar = su(r+1), g=r+1, and the list of integrable irrep that exist at level k is given by the following command:

In[53]:=
```
integrableirrepsA[r_, k_] :=
 Flatten[Apply[Table, Module[{a, li1 = Append[Table[ToExpression[
         StringJoin[ToString[a], ToString[j]]], {j, r, 1, -1}], 0], li},
    li = Prepend[li1, k]; Sequence[Prepend[Table[{li⟦j⟧, 0, li⟦j - 1⟧},
       {j, 2, r + 1}], Drop[li1 - RotateLeft[li1], -1]]]]], r - 1]
```

For A3, g = 4, and for the irrep with highest weight {3, 4, 1} in the basis of fundamental weights, the minimal value of the level is k = 8. Indeed:

In[54]:=
```
Table[MemberQ[integrableirrepsA[3, k], {3, 4, 1}], {k, 1, 10}]
```
Out[54]=
```
{False, False, False, False, False, False, False, True, True, True}
```

In terms of q-dimensions, this can be checked as follows:
The level k (the altitude κ) should be large enough for the chosen irrep to exist.
For the irrep {3,4,1}, k= 8:

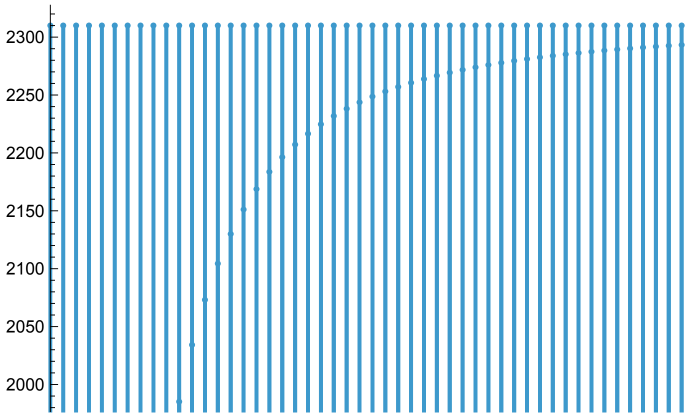
In[55]:=
```
Table[With[{g = 4}, Chop@N[qresvaltrig /. {κ → 4 + k} ]], {k, 7, 50}]
(* When k = 7, the q-dimension vanishes *)
```
Out[55]=
```
{0, 24.1244, 69.1698, 132.754, 210.538, 297.879, 390.66, 485.58, 580.165, 672.664,
 761.901, 847.143, 927.982, 1004.24, 1075.91, 1143.08, 1205.92, 1264.63,
 1319.45, 1370.62, 1418.38, 1462.97, 1504.61, 1543.51, 1579.9, 1613.94, 1645.81,
 1675.69, 1703.71, 1730.01, 1754.73, 1777.98, 1799.87, 1820.48, 1839.93,
 1858.28, 1875.62, 1892.01, 1907.53, 1922.22, 1936.15, 1949.36, 1961.9, 1973.81}
```

For very large k one recovers the classical limit:

In[56]:=
```
With[{g = 4}, Limit[qresvaltrig /. {κ → 4 + k}, k → Infinity ]]
```
Out[56]=
```
2310
```

In[57]:=
```
DiscretePlot[
 With[{g = 4}, {res, Chop@N[qresvaltrig /. {κ → 4 + k} ]}], {k, 1, 250, 5}]
```

Out[57]=



In[58]:=
```
Clear[res, qres, qresval, qresvaltrig, integrableirrepsA]
```

In[∘]:=

Quantum dimensions (symbolic input)

Usual (classical) dimension:

In[59]:=
```
res = DimensionIrrepLie ["A3", {a, b, c}]
```

Out[59]=

$$\frac{1}{12}(1+a)(1+b)(2+a+b)(1+c)(2+b+c)(3+a+b+c)$$

Quantum dimension:

In[60]:=
```
qres = DimensionIrrepLie ["A3", {a, b, c}, q]
```

Out[60]=

(qnum[1 + a, q] × qnum[1 + b, q] × qnum[2 + a + b, q] × qnum[1 + c, q] ×
 qnum[2 + b + c, q] × qnum[3 + a + b + c, q]) / (qnum[1, q]^3 qnum[2, q]^2 qnum[3, q])

Nicer output:

In[61]:=
```
qres /. {qnum[n_, q] :> Subscript[Row[{"(", n, ")"}], q]}
```

Out[61]=

$$\frac{(1+a)_q\,(1+b)_q\,(2+a+b)_q\,(1+c)_q\,(2+b+c)_q\,(3+a+b+c)_q}{(1)_q^3\,(2)_q^2\,(3)_q}$$

Explicit result in terms of q:

In[62]:=
```
qresval = qres /. {qnum[n_, q] :> (q^n - q^(-n)) / (q - q^(-1))}
```

Out[62]=

$$\left(\left(-q^{-1-a}+q^{1+a}\right)\left(-q^{-1-b}+q^{1+b}\right)\left(-q^{-2-a-b}+q^{2+a+b}\right)\left(-q^{-1-c}+q^{1+c}\right)\left(-q^{-2-b-c}+q^{2+b+c}\right)\left(-q^{-3-a-b-c}+q^{3+a+b+c}\right)\right)\Big/\left(\left(-\frac{1}{q}+q\right)^3\left(-\frac{1}{q^2}+q^2\right)^2\left(-\frac{1}{q^3}+q^3\right)\right)$$

Assume that q is a complex root of unity.
The following command is slow when the input (irrep) is not numeric:

In[63]:=
```
qresvaltrig = FullSimplify[ExpToTrig[Expand[Together[qresval] /.{q → Exp[ⅈ π/κ]}]],
    Assumptions → κ > 0] // PowerExpand(* Value for q, a root of unity *)
```

Out[63]=

$$\frac{1}{4+8\,\mathrm{Cos}\left[\frac{2\pi}{\kappa}\right]}\mathrm{Csc}\left[\frac{\pi}{\kappa}\right]^6\mathrm{Sec}\left[\frac{\pi}{\kappa}\right]^2\mathrm{Sin}\left[\frac{\pi}{\kappa}+\frac{a\,\pi}{\kappa}\right]\mathrm{Sin}\left[\frac{\pi}{\kappa}+\frac{b\,\pi}{\kappa}\right]\mathrm{Sin}\left[\frac{2\pi}{\kappa}+\frac{(a+b)\,\pi}{\kappa}\right]\mathrm{Sin}\left[\frac{\pi}{\kappa}+\frac{c\,\pi}{\kappa}\right]\mathrm{Sin}\left[\frac{2\pi}{\kappa}+\frac{(b+c)\,\pi}{\kappa}\right]\mathrm{Sin}\left[\frac{3\pi}{\kappa}+\frac{(a+b+c)\,\pi}{\kappa}\right]$$

For the following choice of a,b,c, the integer κ should be large enough (setting $\kappa = 4 + k$, the level k should be equal or larger than 8):

In[64]:=
```
(qresvaltrig /. {a → 3, b → 4, c → 1}) /. κ → 11
```

Out[64]=

0

In[65]:=
```
(qresvaltrig /. {a → 3, b → 4, c → 1}) /. κ → 12
```

Out[65]=

$$\frac{\sqrt{3}\left(-1+\sqrt{3}\right)^3\left(1+\sqrt{3}\right)^8}{8\left(4+4\sqrt{3}\right)}$$

In[66]:=
```
% // N
```

Out[66]=

24.1244

In[67]:=
```
Clear[res, qres, qresval, qresvaltrig]
```

In[∘]:=

Quantum dimensions : another example (G2)

In[68]:=
```
qres = DimensionIrrepLie ["G2", {2, 3}, q]
```

Out[68]=

$$\frac{\mathrm{qnum}\left[\frac{13}{3},q\right]\times\mathrm{qnum}\left[\frac{17}{3},q\right]\times\mathrm{qnum}[7,q]\times\mathrm{qnum}[10,q]}{\mathrm{qnum}\left[\frac{1}{3},q\right]\times\mathrm{qnum}[1,q]\times\mathrm{qnum}\left[\frac{5}{3},q\right]\times\mathrm{qnum}[2,q]}$$

A nicer output (Warning: One has to use RuleDelayed (:>) in the following input, not Rule (->)):

In[69]:=
```
qres /. {qnum[n_, q] :> Subscript[Row[{"(", n, ")"}], q]}
```

Out[69]=

$$\frac{\left(\frac{13}{3}\right)_q\left(\frac{17}{3}\right)_q(7)_q(10)_q}{\left(\frac{1}{3}\right)_q(1)_q\left(\frac{5}{3}\right)_q(2)_q}$$

The quantum dimension in terms of the formal variable q:

In[70]:=
```
qresval = qres /. {qnum[n_, q] :> (q^n - q^(-n)) / (q - q^(-1))}
```

Out[70]=

$$\frac{\left(-\frac{1}{q^{13/3}}+q^{13/3}\right)\left(-\frac{1}{q^{17/3}}+q^{17/3}\right)\left(-\frac{1}{q^7}+q^7\right)\left(-\frac{1}{q^{10}}+q^{10}\right)}{\left(-\frac{1}{q^{1/3}}+q^{1/3}\right)\left(-\frac{1}{q}+q\right)\left(-\frac{1}{q^{5/3}}+q^{5/3}\right)\left(-\frac{1}{q^2}+q^2\right)}$$

In[71]:=
```
Limit[%, q → 1](* Checking the classical limit *)
```
Out[71]=

1547

The quantum dimension when q is a root of unity:

In[72]:=
```
qresvaltrig = FullSimplify[ExpToTrig[Expand[Together[qresval] /.{q → Exp[ⅈ π/κ]}]]]
```
Out[72]=

$$\frac{1}{\left(e^{\frac{i\pi}{\kappa}}\right)^{2/3}}\left(1+2\operatorname{Cos}\left[\frac{2\pi}{\kappa}\right]+2\operatorname{Cos}\left[\frac{6\pi}{\kappa}\right]+2\operatorname{Cos}\left[\frac{10\pi}{\kappa}\right]\right)$$
$$\left(22\left(e^{\frac{i\pi}{\kappa}}\right)^{1/3}\operatorname{Cos}\left[\frac{\pi}{\kappa}\right]+8\operatorname{Cos}\left[\frac{\pi}{\kappa}\right]^2\operatorname{Cos}\left[\frac{2\pi}{\kappa}\right]^2\left(-1+2\operatorname{Cos}\left[\frac{2\pi}{\kappa}\right]\right)\left(1+2\operatorname{Cos}\left[\frac{2\pi}{\kappa}\right]\right)^2+\right.$$
$$8\,i\operatorname{Cos}\left[\frac{\pi}{\kappa}\right]^3\left(1-2\operatorname{Cos}\left[\frac{2\pi}{\kappa}\right]\right)^2\left(\operatorname{Sin}\left[\frac{\pi}{\kappa}\right]+\operatorname{Sin}\left[\frac{5\pi}{\kappa}\right]\right)+$$
$$\left(e^{\frac{i\pi}{\kappa}}\right)^{1/3}\left(11\left(e^{\frac{i\pi}{\kappa}}\right)^{1/3}+19\operatorname{Cos}\left[\frac{3\pi}{\kappa}\right]+15\operatorname{Cos}\left[\frac{5\pi}{\kappa}\right]+9\operatorname{Cos}\left[\frac{7\pi}{\kappa}\right]+5\operatorname{Cos}\left[\frac{9\pi}{\kappa}\right]+\right.$$
$$2\operatorname{Cos}\left[\frac{11\pi}{\kappa}\right]+2\left(e^{\frac{i\pi}{\kappa}}\right)^{1/3}\left(11\operatorname{Cos}\left[\frac{2\pi}{\kappa}\right]+9\operatorname{Cos}\left[\frac{4\pi}{\kappa}\right]+6\operatorname{Cos}\left[\frac{6\pi}{\kappa}\right]+4\operatorname{Cos}\left[\frac{8\pi}{\kappa}\right]+\right.$$
$$\left.\left.\left.2\operatorname{Cos}\left[\frac{10\pi}{\kappa}\right]+\operatorname{Cos}\left[\frac{12\pi}{\kappa}\right]\right)+i\left(\operatorname{Sin}\left[\frac{3\pi}{\kappa}\right]+\operatorname{Sin}\left[\frac{5\pi}{\kappa}\right]+\operatorname{Sin}\left[\frac{7\pi}{\kappa}\right]+\operatorname{Sin}\left[\frac{9\pi}{\kappa}\right]\right)\right)\right)$$

In[73]:=
```
% /. κ → 12 (* a particular choice for
 κ. Warning: κ should be large enough for this irrep to exist *)
```
Out[73]=

$$\frac{1}{\left(e^{\frac{i\pi}{12}}\right)^{2/3}}$$
$$\left(\frac{3}{4}\left(-1+\sqrt{3}\right)\left(1+\sqrt{3}\right)^4+\frac{i\left(1-\sqrt{3}\right)^2\left(1+\sqrt{3}\right)^3\left(\frac{-1+\sqrt{3}}{2\sqrt{2}}+\frac{1+\sqrt{3}}{2\sqrt{2}}\right)}{2\sqrt{2}}+\frac{11\left(1+\sqrt{3}\right)\left(e^{\frac{i\pi}{12}}\right)^{1/3}}{\sqrt{2}}+\right.$$
$$\left(e^{\frac{i\pi}{12}}\right)^{1/3}\left(7\sqrt{2}+\frac{3\left(-1+\sqrt{3}\right)}{\sqrt{2}}-\frac{1+\sqrt{3}}{\sqrt{2}}+i\left(\sqrt{2}+\frac{1+\sqrt{3}}{\sqrt{2}}\right)+11\left(e^{\frac{i\pi}{12}}\right)^{1/3}+\right.$$
$$\left.\left.2\left(\frac{3}{2}+\frac{9\sqrt{3}}{2}\right)\left(e^{\frac{i\pi}{12}}\right)^{1/3}\right)\right)$$

In[74]:=
```
Chop[N[%]]
```
Out[74]=

91.4794

The quantum dimension in terms of QPochhammer symbol:

In[75]:= qres /. $\left\{\text{qnum}[n\_, q] \to -\frac{q^{1-n}\,\text{QPochhammer}[q^2, q^2, n]}{(-1+q^2)\,\text{QPochhammer}[q^2, q^2, -1+n]}\right\}$ // Simplify

Out[75]=

$$\left(\text{QPochhammer}\left[q^2, q^2, -\frac{2}{3}\right] \text{QPochhammer}\left[q^2, q^2, \frac{2}{3}\right] \text{QPochhammer}\left[q^2, q^2, \frac{13}{3}\right] \text{QPochhammer}\left[q^2, q^2, \frac{17}{3}\right] \text{QPochhammer}[q^2, q^2, 7]\, \text{QPochhammer}[q^2, q^2, 10]\right) \Big/ \left(q^{22}\, \text{QPochhammer}\left[q^2, q^2, \frac{1}{3}\right] \text{QPochhammer}\left[q^2, q^2, \frac{5}{3}\right] \text{QPochhammer}[q^2, q^2, 2]\, \text{QPochhammer}\left[q^2, q^2, \frac{10}{3}\right] \text{QPochhammer}\left[q^2, q^2, \frac{14}{3}\right] \text{QPochhammer}[q^2, q^2, 6]\, \text{QPochhammer}[q^2, q^2, 9]\right)$$

In[76]:= % // FunctionExpand

Out[76]=

$$\frac{(1-q^{14})\,(1-q^{20})\,\left(1-(q^2)^{13/3}\right)\,\left(1-(q^2)^{17/3}\right)}{q^{22}\,(1-q^2)\,(1-q^4)\,\left(1-(q^2)^{1/3}\right)\,\left(1-(q^2)^{5/3}\right)}$$

In[77]:= Chop$\left[\text{N}\left[\% \,/.\left\{q \to \text{Exp}\left[\frac{i\,\pi}{12}\right]\right\}\right], 10\text{^}{-8}\right]$

Out[77]=

91.4794

In[78]:= Clear[res, qres, qresval, qresvaltrig]