Is liquidity risk still priced in the cross-section of US stocks?

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Abstract

The subject of the thesis is the examination of the liquidity risk premium in the U.S. capital market over the period 1920 to 2020. Liquidity risk is analyzed using the Ľuboš Pástor and Robert F. Stambaugh liquidity gamma index, with numerous model modifications and their empirical implications. The statistical significance of the calculated liquidity risk premium is affected by the data basis, the data frequency and filtering techniques with regard to the monthly share price. In addition, the time point and the frequency of portfolio rebalancing also show statistical effects that represent robustness tests for the risk premium. The statistical significance of the calculated liquidity risk premium varies across different sub-periods, but can generally be shown to be robust. Analyses of different data frequencies and version controls of the used database may offer further future insight for replication studies and empirical proves.

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List of abbreviations

AMEX	American Stock Exchange
CAPM	Capital Asset Pricing Model
cf.	compare
CRSP	Center for Research in Security Prices
Diss.	Dissertation
et al.	et alii, and others
FF	Fama and French
fig.	figure
i. i. d.	identical and independent distributed
Jg.	Jahrgang
NASDAQ	National Association of Securities Dealer Automated Quotations
NYSE	New York Stock Exchange
OLS	Ordinary Least Squares
PS	Pástor and Stambaugh
s.	see
tab.	table
t. m.	that means
USD	US-Dollar

Introduction

„ […] expected market illiquidity positively affects ex ante stock excess return, suggesting that expected stock excess return partly represents an illiquidity premium."[1]

One of the cornerstones of efficient capital markets is the liquidity of trading, which affects the currently available information, i.e. the immediate price formation, through various dimensions.[2] Traded securities and other financial products are dependent on certain risks, which is ultimately reflected in the traded price. Especially in times of crisis that affect the overall market and/or large market segments, liquidity emerges as a potential risk factor.[3]

According to findings on previously widely accepted risk factor models, such as the Capital Asset Pricing Model (CAPM) according to Sharpe[4] and Lintner[5], the Fama and French (FF) three-factor model [6] and the Carhart four-factor model[7], the question arises whether a form of liquidity risk should be included in factor models to explain securities prices.[8] Taking into account the theory of utility and an assumed risk aversion of investors[9], it seems logical that if the value of a security held is overly dependent on a particular variable, investors will demand higher compensation.[10] This scenario arises above all when taking out loans (leverage) where the investor must comply with certain payment terms with deposited collateral when purchasing securities. If the value of the positions held falls below the collateral limits, the positions held must be liquidated in the market. These liquidations are therefore more likely to occur if the overall market liquidity also suffers collapses.[11] This scenario befell the hedge fund Long-Term Capital Management in 1998.[12] Unforeseen effects of shocks at the overall market level, taking liquidity into account, led to the end of the hedge fund, which triggered academic interest in a closer analysis of the exact interrelationships.

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1 Cf. Amihud, Y. (2002), p. 31.
2 Cf. Fama, E. F. (2021), p. 387f, Jacobs, H./Hillert, A. (2015), p. 695, Amihud, Y./Mendelson, H. (1991), p. 1424, Chordia, T./Sarkar, A./Subrahmanyam, A. (2004), p. 85.
3 Cf. Chordia, T./Sarkar, A./Subrahmanyam, A. (2004), p. 85.
4 Cf. Sharpe, W. F. (1964), p. 442.
5 Cf. Lintner, J. (1975), p. 13.
6 Cf. Fama, E. F./French, K. R. (2021), p. 51.
7 Cf. Carhart, M. M. (1997), p. 80.
8 Cf. Acharya, V. V./Pedersen, L. H. (2005), p. 394, Vayanos, D./Wang, J. (2013), p. 1308, Chen, Y./Eaton, G. W./Paye, B. S. (2018), p. 49.
9 Cf. Pratt, J. W. (1978), p. 122.
10 Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 643.
11 Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 643.
12 Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 643f.

The previous academic discussion of the liquidity factor at the overall market level has received increasing attention since it was first initiated by Amihud and Mendelson (1986).[13] A shift away from initial liquidity ratios, which refer to individual securities variables, and towards examination at higher levels, such as dependencies on a constructed overall market liquidity, can be observed in this context.[14] Research into whether liquidity is a priced-in factor has since spanned different financial products, different levels, a variety of market types, and geographically categorised markets.[15] Despite the intensive research of the last 35 years or so[16] there is still a discussion about the exact methodological procedure of testing whether liquidity can be modelled equivalently, such as the market risk premium in the CAPM as sensitivity in the form of a beta factor (β).[17] In particular, the combinations of key figures, frequencies of data collection and modifications to the data basis are[18] debatable, as measurement methods, especially in relation to aggregate total market liquidity, prove to be highly sensitive to those changes.[19]

The focus of this paper is to analyse whether liquidity risk itself, and under what conditions, can be empirically proven to be a systematic risk factor in US capital market returns. It is shown whether a conceptualised liquidity risk factor should be considered in future multifactor models to explain stock returns. The relationship between stock price and measurement concept to aggregate total market liquidity is analysed in more detail.

First, in chapter two, the theoretical foundations of the liquidity concept and selected valuation dimensions are presented in order to be able to evaluate the analysed PS model. Chapter three deals with the liquidity gamma index according to PS, which forms the basis for answering the research question. In chapter four, model modifications of the model presented in chapter three are shown. Against this background, empirical results based on historical statistics are presented in chapter five, which

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[13] Cf. Marks, J. M./Shang, C. (2019), p. 147, Keene, M. A./Peterson, D. R. (2007), p. 91, Pástor, Ľ./Stambaugh, R. F. (2019), p. 16, Goyenko, R. Y./Holden, C. W./Trzcinka, C. A. (2009), p. 153.
[14] Cf. Amihud, Y./Mendelson, H. (2015), p. 150, Piqueira, N. S. (2005), p. 2, Vayanos, D./Wang, J. (2013), p. 1340.
[15] Cf. Amihud, Y./Mendelson, H. (2015), p. 150, Pástor, Ľ./Stambaugh, R. F. (2019), p. 1.
[16] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 1, Spiegel, M. I./Wang, X. (2005), p. 1.
[17] Cf. Liu, Y. (2018), p. 7.
[18] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 3.
[19] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 1, Pástor, Ľ./Stambaugh, R. F. (2003), p. 658f.

serve to answer the research question. Chapter six concludes with conclusions, implications and an outlook for future liquidity risk research.

Theoretical foundations of liquidity measurement

Within this chapter, conceptual definitions of liquidity are first differentiated in order to transition to the formulation of the risk factor.

1.1 Delimitation of the concept of liquidity

In the following, the concept of liquidity is distinguished from closely related concepts and specified in more detail in order to concretise the aspect of liquidity analysed here.

1.2 Definition of liquidity

The concept of liquidity depends on the application in various ways and is therefore first defined more precisely in order to be able to refer to it in the further course of the work.
A distinction is made between three main types of liquidity, as shown in fig. 1: macroeconomic, monetary liquidity, market liquidity of individual securities and refinancing liquidity.[20]

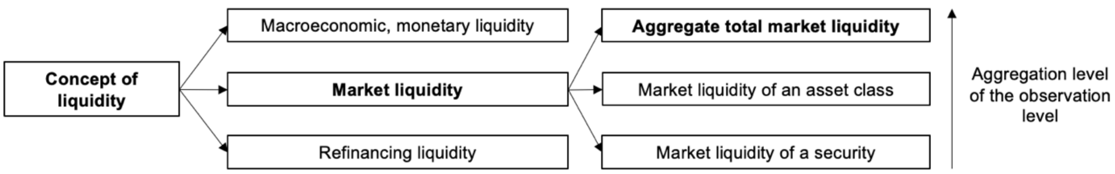

Figure 1: Delimitation and overview of different liquidity concepts. [21]

The first characteristic describes the extent to which the economy as a whole and/or on a global level influences other macroeconomic parameters.[22] These include, for example, inflation, long-term interest rates, risk premiums and cross-border financial flows.[23] Market liquidity is understood as the ability to trade large volumes quickly, at

[20] Cf. Rösch, C. (2012), p. 14, Liu, Y. (2018), p. 2.
[21] Own presentation.
[22] Cf. Rösch, C. (2012), p. 14.
[23] Cf. Baks, K./Kramer, M. C. F. (1999), p. 3, Clark, P. B./Polak, J. J. (2004), p. 52, Rüffer, R./Stracca, L. (2006), p. 9-14.

low cost and without price movements on the market.[24] Funding liquidity is understood as the availability of investors to maintain the positions and solvency of other investors through funding.[25]
In addition, a distinction is made between asset liquidity of individual securities, the liquidity of financial institutions and the liquidity of the financial market as a whole and/or individual asset classes.[26] The liquidity of the market as a whole is therefore dependent on the liquidity of all the individual securities traded.[27]

It should be emphasised above all that " [...] there is no unambiguous, theoretically correct or universally accepted definition of liquidity".[28] Depending on the market environment, there are shifts in the relevance of characterising properties of liquidity. All characterising parameters are time-variable.[29] In times of high financial market stability, transaction costs appear to be highly relevant, whereas in times of crisis rapid equilibrium price discovery through the mapping of new information appears to be the main feature.[30]
Market liquidity, as one of the main types of liquidity mentioned above (s. fig. 1), which is the subject of this analysis, is characterised by three main features:

- Price stability ("without price movements")
- Immediate order execution ("fast")
- Order size ("large volumes")

In order to enable a better assessability of these characteristics, they are formulated as components of transaction costs.[31] Market liquidity can be understood as the trading cost of a securities transaction in the capital market, relative to its fair value.[32] Total transaction costs comprise four components: The price effect PI(q), The search

[24] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 644, Sarr, A./Lybek, T. (2002), p. 4, Chordia, T./Sarkar, A./Subrahmanyam, A. (2004), p. 85, Longstaff, F. A. (1995), p. 1767 f.
[25] Cf. Liu, Y. (2018), p. 2, Rösch, C. (2012), p. 14 f, Malz, A. M. (2018), p. 36.
[26] Cf. Sarr, A./Lybek, T. (2002), p. 7, Sadka, R. (2014),
[27] Cf. Sarr, A./Lybek, T. (2002), p. 7.
[28] Cf. Baker, H. K. (1996), p. 1, Le, H./Gregoriou, A. (2020), p. 1170.
[29] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 1.
[30] Cf. Sarr, A./Lybek, T. (2002), p. 5.
[31] Cf. Hibbert, J., et al. (2009), p. 3.
[32] Cf. Amihud, Y./Mendelson, H. (2006), p. 20, Pástor, Ľ./Stambaugh, R. F./Taylor, L. A. (2020), p. 1.

costs S(q), The delay costs D(q) and the direct trade costs TC(q).[33] In sum, the components represent the market liquidity of a security as follows:

$$ML(q) = PI(q) + S(q) + D(q) + TC(q) \tag{1}$$

and mean in detail:

The price effect PI(q) measures how strongly a transaction entered into the market influences the current market price of the security in question. Accordingly, price effect costs can be understood as the difference between the realised transaction price and the fair value. Price effect includes the premium or discount the trader pays to execute his transaction at the current market conditions.[34] The price effect component can be divided into a short-term and a long-term effect.[35] In the short term, the transaction creates an imbalance between supply and demand, to which the price adjusts. This effect results from the lack of market liquidity and usually has no significant impact on the fundamental value of the security. Accordingly, a rapid recovery to the level of the fair value takes place again after the order has been settled (engl.: reversal).[36] The long-term effect, on the other hand, results from the traders' informational interpretation of the transactions. Transactions may have been triggered by traders who possess superior information, which represents a potential risk for the securities traders, against which they hedge in part with a constant price reflection, which is ultimately reflected in the market as part of the price effect.[37]

The search costs S(q) comprise all costs associated with finding a suitable trading partner for the transaction. Depending on the asset class, this share can vary greatly and, for example in the case of more exotic goods such as art or classic cars, reach a considerable size in the total transaction costs. However, even with very large block transactions of shares, search costs usually reach a larger dimension, so that private agreements between both trading parties are more worthwhile than accepting the risk of price effects when placing on the market.[38]

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[33] Cf. Amihud, Y./Mendelson, H. (2006), p. 20, Stange, S./Kaserer, C. (2011), p. 271.
[34] Cf. Rösch, C. (2012), p. 19.
[35] Cf. Amihud, Y./Mendelson, H./Pedersen, L. H. (2006), p. 43.
[36] Cf. Vayanos, D./Wang, J. (2013), p. 1292.
[37] Cf. Amihud, Y. (2002), p. 33, Kyle, A. S. (1985), p. 1316, Sadka, R. (2006), p. 312.
[38] Cf. Rösch, C. (2012), p. 19 f.

The delay cost D(q) describes the potential cost to the market of changes that occur during delays in the transaction. A distinction is made between intended and unintended delays, whereby unintended delays can consist of the trader splitting large block transactions in order to avoid too large price effects. Intended delays, on the other hand, occur when transactions cannot be settled in full, and during this period of suspense, trading is subject to the risk of changing prices.[39]
Direct trading costs TC(q) refer to all costs incurred during the transaction, such as trading venue fees, which are the only costs that can be controlled by the trading venue itself, and exogenous costs, such as trader fees and taxes.[40]

Further specifying, the focus of this work, in addition to market liquidity, is on the component of the price effect of transaction costs in order to analyse the sensitivity of the share price in response to changes in overall market liquidity. A special role in the choice of this aspect is played by the perspective adopted, since from the point of view of small investors who, for example, can trade according to the difference between the bid and ask price (bid-ask spread) as an indicator and the price effect is negligible due to small transactions, whereas institutional investors have to take the price effect into account due to significantly larger transactions or block trading.[41]
However, the difference between the bid and ask price cannot fully capture the facet of the price effect, as the market depth is not taken into account (s. fig. 2).[42]

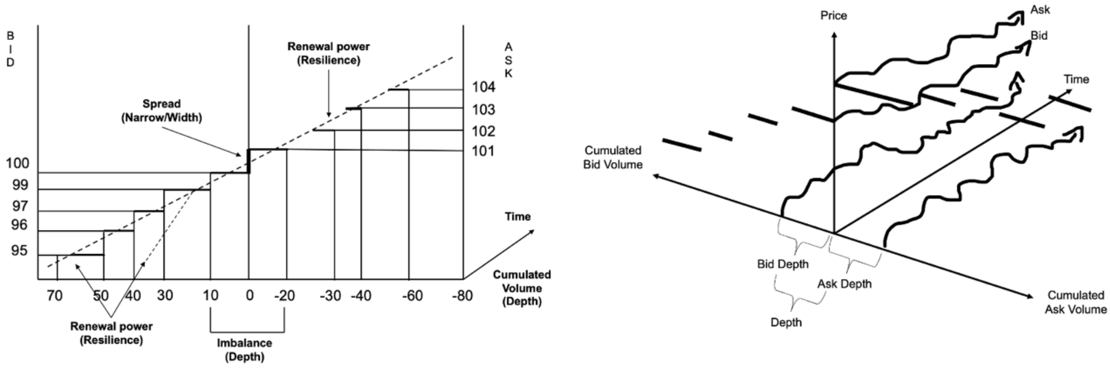

Figure 2: Exemplary presentation of the liquidity dimensions depth, breadth and renewal power on the basis of a) bid and ask price developments and b) in temporal variation.[43]

[39] Cf. Rösch, C. (2012), p. 21.
[40] Cf. Rösch, C. (2012), p. 22.
[41] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 9.
[42] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 9, Malz, A. M. (2018), p. 45.
[43] Own presentation based on Ranaldo, A. (2001), p. 312, Wyss, R. v. (2004), p. 8.

This capture is also not possible through purely volume-based characteristics, such as trading volume[44]: „For example, high volume levels may indicate that a particular security is very liquid. But volume does not tell us how costly it is to actually trade the security."[45], which underlines the importance of the mostly combined concepts of price effect ratios as suitable liquidity ratios. In the following, market liquidity is operationalised on the various market liquidity dimensions in order to ensure a better quantitative assessment of the measurement concepts.

1.3 Multidimensionality of liquidity

The exact number of different dimensions of market liquidity, like the concept of liquidity itself, is subject to ongoing debate.[46] However, the three basic dimensions of depth, breadth and resiliency emerge as a minimum consensus.[47] Time is added as an additional dimension.[48]

The depth of a market is determined by the presence of both limited buy and sell orders that can be executed at a price close to the market price. If there is an imbalance in the market in terms of buy and sell orders, the depth ensures that this is redressed by the orders in the market without causing significant price jumps.[49] The lower a market is, the more stable securities prices behave.[50] The depth dimension here refers to the definition aspect of price stability.

A market has breadth if, on the one hand, orders are available in large numbers, there are many suppliers and demanders, and, on the other hand, if they are available in a corresponding volume.[51] The broader a market is, the more likely it is to cover a more diverse range of orders, leading to greater price stabilisation potential. Accordingly, larger volumes can potentially be traded better.[52]

The renewal power of a market is understood as the process when short-term price changes, caused by temporary order imbalances, are immediately offset by new orders flowing into the market.[53] In the case of a rise in the price of a share due to a

[44] Cf. Li, J./Mooradian, R./Zhang, W. D. (2007), p. 29.
[45] Cf. Spiegel, M. I./Wang, X. (2005), p. 2.
[46] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 646, Saali, T. (2014), p. 14.
[47] Cf. Bernstein, P. L. (1987), p. 55, Kyle, A. S. (1985), p. 1316 f.
[48] Cf. Lippman, S. A./McCall, J. J. (1986), p. 44.
[49] Cf. Bernstein, P. L. (1987), p. 55.
[50] Cf. Oesterhelweg, O./Schiereck, D. (1993), p. 3.
[51] Cf. Bernstein, P. L. (1987), p. 55.
[52] Cf. Oesterhelweg, O./Schiereck, D. (1993), p. 5.
[53] Cf. Garbade, K. (1982), p. 428.

short-term surplus of buy orders, a market that is capable of renewal enables the immediate rebalancing to the original level of the price through the arrival of sell orders. The renewal power is necessary to ensure trading on financial markets at all times and thus represents the basis for the dimensions depth and breadth.[54]

The time dimension describes the speed with which an order of a given size can be executed in the market at applicable costs. The time dimension represents the definition aspect of immediate execution (s. also fig. 2).[55]

Exemplary presentation of various market liquidity dimensions based on the bid price				
Market:	1	2	3	4
Bid	Thin and flat	Thin and deep	Wide and flat	Wide and deep
50 $	100	100	500	500
49 $	200	200	500	500
48 $	0	300	0	700
47 $	0	300	0	900
46 $	0	300	0	1500

Table 1: Exemplary presentation of different market liquidity dimensions of a capital market from the perspective of the bid price.[56]

As an example, all dimensions, except for the speed of execution, are shown in tab. 1.

All four dimensions together fully represent market liquidity. Measurement concepts that are to measure market liquidity must be able to depict it sufficiently. Structural components of the market set-up, such as market organisation forms and market microstructures, which have a significant influence on the interaction of the four dimensions, will not be discussed further due to the focus of the paper.

1.4 Delineation of the concept of risk: liquidity - characteristics, risk factor and premium

After the theoretical foundation of the first component of the research question, market liquidity, was presented in chapter 2, the following section is devoted to its influence on share prices: the price of liquidity as a liquidity premium.[57] As before for the

[54] Cf. Oesterhelweg, O./Schiereck, D. (1993), p. 5.
[55] Cf. Lippman, S. A./McCall, J. J. (1986), p. 5.
[56] Table taken from Garbade, K. (1982), p. 421.
[57] Cf. Hibbert, J., et al. (2009), p. 3.

liquidity concept, a more precise delimitation is also made for the risk concept, because liquidity has complex links to risk factors such as company size, company value, company quality and the liquidity status of the market.[58] The link between risk and premium is as follows: “Stock excess return, traditionally called ‘risk premium’, has been considered a compensation for risk“[59], according to which the liquidity premium represents a price premium for a security relative to a hypothetical, perfectly liquid security with the same characteristics.[60]

On the capital market, a distinction is made between systematic and unsystematic, i.e. idiosyncratic, risk according to CAPM considerations.[61] (s. Appendix A1).
The systematic part is accounted for by the sensitivity of equity returns to the returns of the broad market, which is modelled in the beta factor as a linear covariance relationship: $\beta_i = \frac{Cov(R_i,R_m)}{Var[R_m]}$.[62] The expected return on shares is based on this systematic risk, as the relationship of the equations shows[63]:

$$E[R_i] = \gamma_0 + \gamma_1 \cdot \beta_i \tag{2}$$

$$\beta_i = \frac{Cov(R_i,R_m)}{Var(R_m)}{}^{64} \tag{3}$$

where$E[R_i]$ is the expected stock return, γ_0 is the return on an assumed risk-free investment and γ_1 is the excess market return $\gamma_1 = (R_m - \gamma_0)$.
Companies face a fundamental risk factor whose changes over time influence the cross-section of equity returns through the covariance relationship.[65] Systematic risk cannot be offset by holding a diversified portfolio.[66] The idiosyncratic part, on the other hand, is the part of the expected return that is uncorrelated with the market return.[67]
The liquidity risk of a security is based on the variation of its price over time in response to unexpected changes in aggregate total market liquidity[68] and, according

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[58] Cf. Liu, Y. (2018), p. 1 f.
[59] Cf. Amihud, Y. (2002), p. 32.
[60] Cf. Hibbert, J., et al. (2009), p. 4.
[61] Cf. Sharpe, W. F. (1964), p. 439.
[62] Cf. Keim, D. B. (2016), p. 1, Acharya, V. V./Pedersen, L. H. (2019), p. 113.
[63] Cf. Acharya, V. V./Pedersen, L. H. (2019), p. 113, Goyal, A./Santa-Clara, P. (2003), p. 976.
[64] Cf. Jagannathan, R./Wang, Z. (1996), p. 6.
[65] Cf. Harvey, C. R./Liu, Y./Zhu, H. (2016), p. 9.
[66] Cf. Amihud, Y./Mendelson, H. (1986), p. 44.
[67] Cf. Bartram, S. M./Brown, G./Stulz, R. M. (2016), p. 1.
[68] Cf. Sadka, R. (2014), p. 20, Pástor, Ľ./Stambaugh, R. F. (2003), p. 642, Sadka, R. (2006), p. 311, Lou, X./Sadka, R. (2011), p. 51.

to Acharya and Pedersen (2005), can be divided into three individual risk components: (i) the covariance of individual securities liquidity with overall market liquidity (commonality)[69], (ii) the covariance of the security return with the total market liquidity and (iii) the covariance of the security liquidity with the total market return (s. also fig. 6).[70] Previous research on liquidity primarily refers to the first two components: Liquidity as a firm characteristic (Level of Liquidity) and Liquidity as a systematic risk factor (Liquidity beta), both of which relate to the systematic risk component.[71]

Both approaches reflect different aspects of the liquidity profile of a share, which is why the characteristics are also referred to as an "average effect"[72] and the risk factor can be referred to as a "volatility or correlation effect".[73]
The central approaches to liquidity are shown in fig. 3.

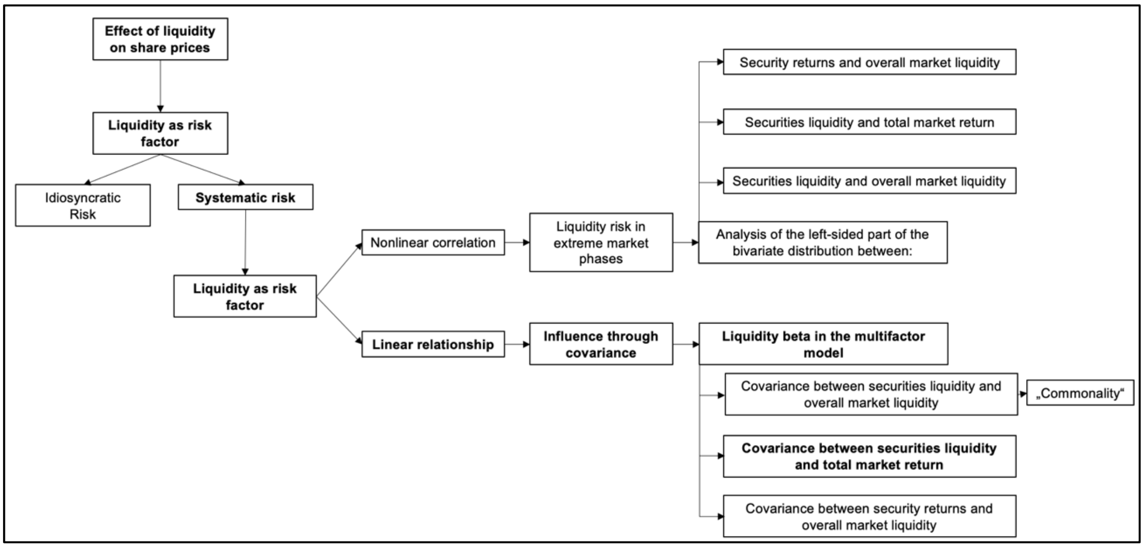

Figure 3: Classification and overview of liquidity as a risk factor.[74]

In this paper, we refer to the linear relationship between aggregate market liquidity and stock returns in the context of systematic risk, which corresponds to the middle aspect of the "liquidity betas in the multifactor model" branch in fig. 3. Since there has already been an intensive examination of the first component, which is also referred to as "commonality", there is a need for a more detailed analysis“[75] (s. fig. 3),

[69] Cf. Rösch, C. (2012), p. 9, Piqueira, N. S. (2005), p. 4.
[70] Cf. Acharya, V. V./Pedersen, L. H. (2005), p. 375.
[71] Cf. Le, H./Gregoriou, A. (2020), p. 1171, Lou, X./Sadka, R. (2011), p. 51, Sadka, R. (2006), p. 311, Papavassiliou, V. (2013), p. 1, Acharya, V. V./Pedersen, L. H. (2005), p. 376, Al-Haji, A. (2020), p. 2.
[72] Cf. Acharya, V. V./Pedersen, L. H. (2005), p. 376.
[73] Cf. Lou, X./Sadka, R. (2011), p. 51.
[74] Own presentation.
[75] Cf. Karolyi, G. A./Lee, K.-H./van Dijk, M. A. (2012), p. 83, Acharya, V. V./Pedersen, L. H. (2005), p. 376.

this paper will limit itself to the second component in order to analyse the link between stock prices and overall market liquidity.

The liquidity gamma index according to Pástor and Stambaugh

In order to check whether a form of liquidity is taken into account as a risk factor in share prices, an overall market ratio is first determined from individual liquidity measures (s. fig. 1), which can be used to check how share prices behave when this overall market ratio varies, in order to be able to conclude on a corresponding compensation for risk.

The integration of factors describing liquidity is primarily limited to the firm level and less to the aggregation of these individual measures to the next higher level, the market level.[76] Similarly, liquidity is rarely included as a component in multifactor models, which results from the highly controversial disagreement about the accuracy of the measurement.[77] The following statement, in relation to trading volume, reveals the difficulties: "The more complex part of measuring liquidity risk is how to measure the impact of trading volume on price.“[78]

One of the most important approaches in this direction is presented by Pástor and Stambaugh (PS) 2003, in which they try to answer the question posed by Chordia, Roll and Subrahmanyam (2001): "Do unanticipated liquidity variations represent a risk factor priced in the cross section of asset returns?.“[79].[80] Their approach was to test whether there is a relationship between expected stock returns and the sensitivity of these to changes in the aggregate market liquidity you constructed.[81] The aim of their model is to provide empirical evidence as to whether liquidity as a systematic risk factor is priced into expected returns and thus remunerated with a premium for investors, provided that the price of a share, for example, resonates more strongly with changes in aggregate market liquidity.[82] The connection between company level and market level is relevant as a risk factor for investors, especially during times of

[76] Cf. Gibson, R./Mougeot, N. (2004), p. 159.
[77] Cf. Jensen, G. R./Moorman, T. (2010), p. 338, Guillaume, F. (2015), p. 214, Goyenko, R. Y./Holden, C. W./Trzcinka, C. A. (2009), p. 153, Amihud, Y./Mendelson, H. (1986), p. 43, Johnson, T. C. (2008), p. 404.
[78] Cf. Muranaga, J./Ohsawa, M. (1997), p. 195.
[79] Chordia, T./Roll, R./Subrahmanyam, A. (2001),
[80] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 1.
[81] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 642.
[82] Cf. Liu, Y. (2018), p. 18.

crisis, because it is precisely then that the risk materialises.[83] The point is that the return of a share depends not only on liquidity, understood as a characteristic, but also on its sensitivity, in relation to the constructed aggregate market liquidity.[84] Thus, the PS concept is to be understood as an example of an environment-dependent metric because the liquidity status of the market is also taken into account.

1.5 Data basis

The data basis of PS's analyses is based on data from the US capital market of the Center for Research in Security Prices (CRSP), in which all stocks with the codes 10 and 11 are included in daily frequency in the period from August 1962 to December 1999.[85]

The calculation of total market liquidity is based on the period August 1962 to December 1999 and data from the NYSE and AMEX, as is the time series of the change in total market liquidity. The sensitivity of stock returns to this constructed market liquidity is calculated per listed stock using the codes 10 and 11 of the NYSE, AMEX and NASDAQ stock exchanges in the period January 1968 to December 1999.[86] The time lag of the sensitivity calculation results from the precision of the measurement of the historical liquidity betas, which is discussed in more detail in chapters 3.2 and 3.3.

1.6 Methodical approach

The methodology of measuring the relationship between overall market liquidity and stock returns is done in two steps. First, the liquidity of the individual companies is determined numerically with the help of suitable measures.[87] Subsequently, the aggregation of all individual values is carried out by, e.g. value or equal weighting, in order to determine the liquidity of the entire market under consideration.[88]

PS pursue their own approach as a liquidity measure at the company level, because existing measures primarily serve to assess the liquidity of individual companies in a

[83] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 6.
[84] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 1, Li, H./Novy-Marx, R./Velikov, M. (2019), p. 1.
[85] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 646.
[86] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 663.
[87] Cf. Eckbo, B. E./Norli, Ø. (2002), p. 4.
[88] Cf. Chen, J. (2005), p. 4.

short time horizon.[89] Its aim is to illustrate the price effect as a component of transaction costs, which in this case focuses on temporary share price changes triggered by transaction entries.[90] A more illiquid share experiences a larger return revearsal, resulting in a negative liquidity gamma index value ($\hat{\gamma}_i$).[91] This represents the short-term, transitory component of the price effect, which is largely characterised by the extent of market liquidity.[92] Basically, this means that when market liquidity is low, each transaction moves the price of the security in question more, but only in a temporary time frame.[93] Consequently, the liquidity of a stock reflects the extent to which the USD volume traded on a given day appears to predict that the stock will outperform (or underperform) the broad market, and underperform (or outperform) on the following day.[94]

Liquidity measurement at company level

The liquidity ratio for share i in the month t is the Ordinary Least Squares (OLS) estimator of the regression coefficient $\gamma_{i,t}$ of the regression equation:[95]

$$r^e_{i,d+1,t} = \theta_{i,t} + \phi_{i,t} r_{i,d,t} + \gamma_{i,t} \cdot sign\left(r^e_{i,d,t}\right) \cdot v_{i,d,t} + \epsilon_{i,d+1,t}, d = 1, \dots, D, \qquad (4)$$

With the parameters $r_{i,d,t}$ as the return of the stock i at the day d in the month t; the excess return $r^e_{i,d,t} = r_{i,d,t} - r_{m,d,t}$ oft he realized return $r_{i,d,t}$ over the value-weighted CRSP Market return $r_{m,d,t}$ at the day d in the month t; $v_{i,d,t}$ as the trading volume of the stock i at the day d in the month t, $sign\left(r^e_{i,d,t}\right)$ takes on the following values:

$$sign\left(r^e_{i,d,t}\right) = \begin{cases} 1, for\ values\ r_{i,d,t} > 0 \\ -1, for \text{ values } r_{i,d,t} < 0 \\ 0, for \text{ values } r_{i,d,t} = 0 \end{cases}.$$ [96]

Even if not explicitly mentioned by PS, days with no trading volume are excluded, so that the following applies to the entire data basis: $v_{i,d,t} > 0$.[97]

Further restrictions arise as follows: At least 15 observations per stock in one month are required for the regression. ($D > 15$), a stock is removed from the first and the

[89] Cf. Gibson, R./Mougeot, N. (2004), p. 159.
[90] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 646.
[91] Cf. Piqueira, N. S. (2005), p. 5, Amihud, Y./Mendelson, H./Pedersen, L. H. (2006), p. 40 f.
[92] Cf. Vayanos, D./Wang, J. (2013), p. 1292.
[93] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 1.
[94] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 3.
[95] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 647.
[96] Cf. Samuel Xin, L./John, K. C. W. (2012), p. 3278.
[97] Cf. Pontiff, J./Singla, R. (2019), p. 260, Pástor, Ľ./Stambaugh, R. F. (2019), p. 6, Li, H./Novy-Marx, R./Velikov, M. (2019), p. 6.

last started month of the CRSP data set in which it occurs there. The individual daily records do not have to be consistently related, but two consecutive daily records are sufficient. Shares are excluded from the calculations if their share price is either below 5 US dollars (USD) or above 1000 USD at the end of the previous month. Volume is measured in millions of USD.[98]

Liquidity measurement at overall market level

From the share-specific liquidity measurement figures, in total N each month t, the total market level is aggregated to by taking the simple average: [99]

$$\hat{\gamma}_i = \frac{1}{N}\sum_{i=1}^{N}\hat{\gamma}_{i,t} \tag{5}$$

whereby an overall market ratio, the liquidity gamma index $\hat{\gamma}_i$ is generated on a monthly basis for the period of August 1962 to December 1999.[100]

Due to the difference in value of the trade size of 1 million USD, compared to the beginning of the database in the 1960s and the end in the 1990s, the resulting time series is scaled to the relative market value of the respective period, i.e. it is inflation-adjusted:[101]

$$\frac{m_t}{m_1} \cdot \hat{\gamma}_i \tag{6}$$

with the parameter m_t, as the market value of all in the month t in the average included stocks i at the end of the month $t-1$ in USD and m_1, as the corresponding market value in the first month of the data basis, i.e. here August 1962.[102] The course of this aggregated total market liquidity is shown in fig. 4 in the original period of the PS study from August 1962 to December 1999. Except for individual swings of the time series to the positive or negative, it fluctuates around the value 0 over time.

[98] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 647.
[99] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 650.
[100] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 650.
[101] Cf. Samuel Xin, L./John, K. C. W. (2012), p. 3278.
[102] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 651.

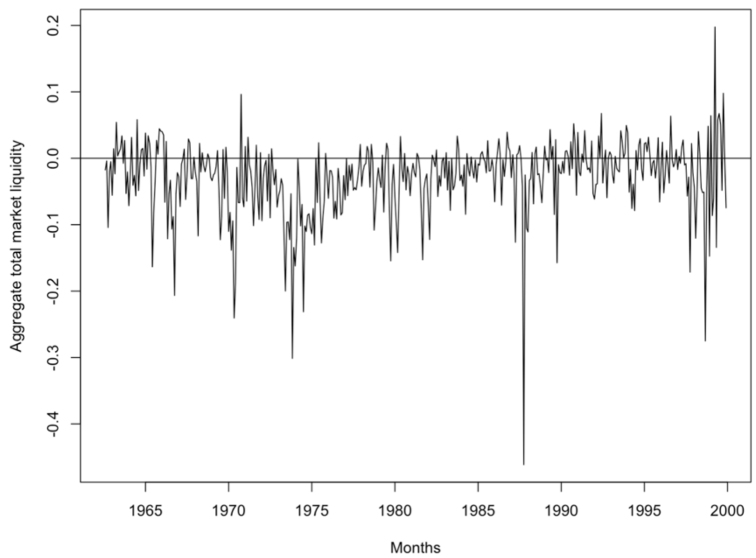

Figure 4: Aggregate total market liquidity in the period August 1962 to December 1999 by PS (2003). [103]

This is also shown by descriptive measures such as the average of -0.029, a minimum of 0.198 and a minimum of -0.461 and a minimum of -0.461 of the time series in the period shown (s. Appendix A2).

In order to calculate the changes (innovations) in the total market liquidity time series from fig. 5, we proceed as follows:

$$\triangle\,\hat{\gamma}_t = \left(\frac{m_t}{m_1}\right)\frac{1}{N_t}\sum_{i=1}^{N_t}(\hat{\gamma}_{i,t} - \hat{\gamma}_{i,t-1}) \text{ [104]} \tag{7}$$

The computed delat $\triangle\,\hat{\gamma}_t$ is set to the previous value $\triangle\,\hat{\gamma}_{t-1}$ and to the previous value of the scaled time series $\left(\frac{m_t}{m_1}\right)\hat{\gamma}_{t-1}$ regressed:

$$\triangle\,\hat{\gamma}_t = a + b\,\triangle\,\hat{\gamma}_{t-1} + c\left(\frac{m_t}{m_1}\right)\hat{\gamma}_{t-1} \text{ [105]} \tag{8}$$

The regression calculated in this way allows the unforeseen change to be[106], depends on the most recent change on the one hand and on the most recent deviation from the long-term average on the other, which is reflected in the parameter a concentrated in equation 8.[107] An important property for empirical investigations is that the results of this regression are serially uncorrelated. This means that they can be

[103] Own presentation based on Pástor, Ľ./Stambaugh, R. F. (2003), p. 651.
[104] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 652.
[105] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 652.
[106] Cf.Li, H./Novy-Marx, R./Velikov, M. (2019), p. 6.
[107] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 652.

regarded as realisations of a random variable due to their fulfilment of the identical and independent distribution (i.i.d.).[108] These residuals, scaled by 100 $\hat{u}_t$, form the time series of changes in total market liquidity (Innovations in Liquidity) L_t :

$$L_t = \frac{1}{100}\hat{u}_t \qquad (9)$$

The resulting time series is used as an explanatory variable in regressions for stock returns and is shown in fig. 5.

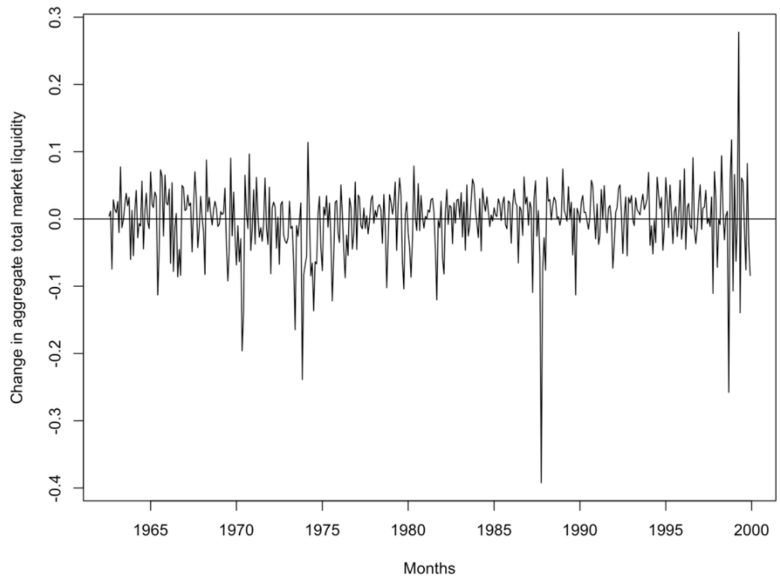

Figure 5: Change in total market liquidity in the period from August 1962 to December 1999 by PS (2003).[109]

If there is no link between stock returns and this variable L_t, so there seems to be no liquidity premium.[110]

Detailed descriptive statistics of both time series are included in Appendix A2, which also contains data for the period after the original study.

Interpretation of the Liquidity Gamma Index

The value of the regression coefficient $\gamma_{i,t}$ in equation 4 can be interpreted as the cost of liquidity, t. m. as a return reversal, which accrues when USD 1 million of share i will be traded. In terms of the overall market ratio, this is what it would cost if each share in the market traded for \$1 million in transaction volume.[111]

[108] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 652, Samuel Xin, L./John, K. C. W. (2012), p. 3278.
[109] Own presentation based on Pástor, Ľ./Stambaugh, R. F. (2003), p. 651.
[110] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 652 f.
[111] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 650 f.

1.7 Portfolio formation and alpha formation

Since all relevant variables for calculating market liquidity were presented in chapters 3.1 and 3.2, the econometric methodology in empirical capital market studies to determine systematic risk factors in the cross-section of stock returns is presented below.

Equity return sensitivity, in relation to changes in overall market liquidity L_t , is comparable to a regression coefficient, t. m. as the liquidity beta β_i^L , in a multiple regression model with additional variables identified as explanatory components of stock returns. [112] (s. equation 10): „Measuring the effects of illiquidity on stock prices and expected returns requires separating them from the effects of risk."[113] For this purpose, at the end of each year, starting in 1967 , 10 portfolios (decile portfolios) of PS, based on the values of β_i^L, are formed.[114] The recalculated returns of the following 12 months of these portfolio constellations are recorded per portfolio as a time series up to the year 1999. The resulting excess returns of these portfolio constellations are regressed on recognised variables explaining the stock returns.[115] If the intercepts β_i^0, also referred to as alpha (α), are different from zero, the liquidity beta β_i^L explains risk components of equity returns that cannot be explained by previously known risk factors.[116] For portfolio sorting purposes, the liquidity beta β_i^L is calculated according to the following multiple regression equation for each share of the data base:

$$r_{i,t} = \beta_i^0 + \beta_i^L L_t + \beta_i^M MKT_t + \beta_i^S SMB_t + \beta_i^H HMK_t + \epsilon_{i,t} \quad (10)$$ [117]

With the parameters $r_{i,t}$ as the excess return oft he stock i, MKT (Market) as the excess return above the market index, SMB (Small minus Big) and HML (High minus Low) as the excess return between sorted portfolios based on market capitalisation and the book to market value ratio. This specification of the liquidity beta allows for a capture of the synchronicity with total market liquidity, captured in β_i^L, which differs from that with other variables.[118] The values of the liquidity beta β_i^L, on which the

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[112] Cf. Pástor, L'./Stambaugh, R. F. (2003), p. 662.
[113] Cf. Vayanos, D./Wang, J. (2013), p. 1347.
[114] Cf. Pástor, L'./Stambaugh, R. F. (2003), p. 673.
[115] Cf. Pástor, L'./Stambaugh, R. F. (2003), p. 663.
[116] Cf. Pástor, L'./Stambaugh, R. F. (2003), p. 662 f, Keene, M. A./Peterson, D. R. (2007), p. 92.
[117] Cf. Pástor, L'./Stambaugh, R. F. (2003), p. 663.
[118] Cf. Pástor, L'./Stambaugh, R. F. (2003), p. 663.

sorting of the portfolios is carried out,are computed in two different ways.[119] The first method forecasts the liquidity betas according to equation 10, taking into account six other factors, in addition to the historical liquidity beta.[120] Due to the scope of the methodological and econometric approach of this method[121], this paper will not discuss it any further. Because the historical liquidity beta in the first method is the most important and robust parameter, its calculation is referred to here as the second method.[122] In addition, PS 2019 noted: "Years later, we noticed that the beta pattern flipped in the expanded sample: historical betas now do a better job than predicted betas in creating dispersion in the post-ranking betas." [123]. This is reflected in the results of the extended calculation according to equation 10 by December 2018 in a significantly more significant beta value of 8.04 with a t-statistic of t = 3.16 for the historical liquidity betas.[124] In comparison, the predicted beta value, 3.24 with a t-statistic of t = 0.91 for the same extended period is.[125]

	August 1962 -		
	December 1998	December 1983	December 1968
Intercept	-1,79 (-6,75)	-4,39 (-12,94)	-2,75 (-2,95)
Historical liquidity beta	2,30 (9,97)	3,75 (10,87)	9,18 (9,99)
Average liquidity	-0,87 (-4,12)	-0,02 (-0,08)	-0,48 (-0,61)
Average volume	1,54 (3,29)	-3,37 (-5,03)	0,07 (0,05)
Cumulative return	-0,04 (-0,14)	1,00 (2,86)	0,93 (0,86)
Return volatility	-0,24 (-1,60)	-1,13 (-3,39)	-2,61 (-2,25)
Price	0,59 (1,85)	7,51 (15,00)	4,32 (3,38)
Shares outstanding	-1,43 (-3,37)	0,67 (1,26)	-0,69 (-0,54)

Table 2: Regression parameters and T-statistics of projected liquidity betas in the period August 1962 to December 1998 PS (2003).[126]

Hence PS concluded: "To maintain the quality of our traded liquidity factor, we decided about ten years ago to focus on historical betas, and we have been doing that ever since."[127]

[119] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 663.
[120] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 664 ff.
[121] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 13.
[122] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 665 f.
[123] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 12 f.
[124] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 12
[125] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 13.
[126] Table taken from Pástor, Ľ./Stambaugh, R. F. (2003), p. 664.
[127] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 13.

In the excerpt from Table 2, the seven parameters used to forecast the liquidity betas of the first method are shown with their T-statistic values for three time periods. The historical liquidity beta has the most robust T-statistics over all three periods and, in absolute terms, the greatest influence on the predicted beta values according to the second method.[128]

Restricting the calculation to historical liquidity betas also allows for better comparability with replication studies that primarily use the same calculation method.[129]

Portfolio sorting based on historical liquidity betas

The second method of calculation of β_i^L is based on the use of only historical OLS values in the equation 10 to originally ensure that the results of the first method are not based exclusively on the other variables used, which can be found in tab. 2.[130]

Because the historical beta is the only explanatory liquidity variable, the minimum data availability for the historical calculation is set at five years for a more precise calculation.[131] The time series of the change in total market liquidity L_t is calculated again according to equation 9, at the end of each year in the period January 1968 to December 1999 for all shares for which at least 5 years of historical monthly returns are available. The stocks are then divided into 10 value-weighted decile portfolios per month based on their historically determined liquidity betas according to equation 10.

The Liquidity Gamma Index - Model Modifications

After the PS concept for capturing and evaluating aggregate total market liquidity has been presented, model modifications from replication studies with regard to the construction of aggregate market liquidity are presented below. The aim of this analysis is the differentiated evaluation of the empirical findings derived from the PS model in chapter 5. In particular, the construction sensitivity of the PS approach to model specifications is addressed[132], because: „Ideally empirical results should be insensitive to

[128] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 666.

[129] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 3 f, Pontiff, J./Singla, R. (2019), p. 259 f, Chen, J. (2005), p. 8, Eckbo, B. E./Norli, Ø. (2002), p. 6 f, Goyenko, R. Y./Holden, C. W./Trzcinka, C. A. (2009), p. 160, Samuel Xin, L./John, K. C. W. (2012), p. 3278.

[130] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 663.

[131] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 674.

[132] Cf. Vayanos, D./Wang, J. (2013), p. 1351.

the specific choices made regarding the details of the empirical design. That is, results should be robust to alternative specifications."[133]
In the following, alternative specifications are first presented, which are empirically examined in the following chapter 5. When the liquidity gamma index according to PS is mentioned below, the time series according to equation 9 is meant.

1.8 Data basis

The data basis is the starting point of any empirical capital market research. With regard to the results in the empirical part in chapter 5, there are significant differences in the choice of modifications to the data basis in the approaches presented, and these are shown here. The PS concept is primarily based on CRSP data.[134] The ongoing adjustment of the CRSP database for the US capital market, in which the historical data is corrected for error identifications, should be emphasised.[135] Consequently, if the same data are used again years later, differences may arise that affect the final empirical results.[136]

1.8.1 Zero-Volume days

With respect to the liquidity gamma index according to PS, the implicit exclusion of days when no volume of a stock was traded is a key modification of the data set, which is used by both Li et al. (2019)[137], as well as Pontiff and Singla (2019)[138] that will be discussed. The importance of this adjustment is illustrated by Liu's (2006) measure of the number of zero-volume days as a liquidity indicator, and also by Kang and Zhang's (2014) adjustment of the Amihud ratio *ILLIQ* to *AdjILLIQ*.[139] Not taking this modification into account leads to a reduction in the correlation of replicated time series, which, compared to the time series of PS for the period from August 1962 to December 1999, is reduced from 98.9% when taking it into account to 39.2% when not taking it into account.[140] In the extended period from August 1962 to December 2017, this results in a correlation reduction from 93.2% when taking into account to

133 Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 14.
134 Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 646.
135 Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 3.
136 Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 3.
137 Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 6.
138 Cf. Pontiff, J./Singla, R. (2019), p. 260.
139 Cf. Liu, W. (2006), p. 632, Kang, W./Zhang, H. (2014), p. 49.
140 Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 8.

27% when not taking into account.[141] If zero volume days are removed, this affects comparatively few data points, despite the large correlation effect: 4.6 million out of a total of about 52 million daily observations, which is about 9% of the total observations.[142]

This reduction therefore results from the included small, illiquid shares. This shows that the PS Liquidity Gamma Index anticipates a crucial assumption regarding these stocks. Smaller companies may be underrepresented in the database, but in reality they have an impact on market dynamics. The excluded companies are on average one-thirtieth the size of those that do not fall under the filter criterion and account for about 0.44% of the NYSE market capitalisation.[143]

1.8.2 Price restrictions

Shares are excluded for months in which their share price is less than USD 5 or more than USD 1000 at the end of the previous month-end.[144] This is not an unusual filter criterion[145], however, this adjustment also creates a difference between reality and its modelled representation. Taking into account the lower price limit of prices below USD 5 results in an exclusion of 24.5% of the daily CRSP data that would have been included in the liquidity gamma calculation.[146] These primarily smaller companies react particularly sensitively to changes in aggregate market liquidity. Their company-specific liquidity ratio has a greater effect on the overall ratio due to the simple average calculation compared to a value weighting. Including these stocks can therefore lead to a more realistic representation of the overall market. The importance of handling companies with small to very small market capitalisation is illustrated by Fama and French (2008): "From a general economic perspective, it is important to know whether anomalous patterns in returns are marketwide or limited to illiquid stocks that represent a small portion of market wealth.“[147] However, the retention of filtering techniques previously recognised from the literature ensure a certain comparability between studies and anomalies, as well as the mapping of essential market segments.[148]

[141] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 6, Li, H./Novy-Marx, R./Velikov, M. (2019), p. 7.
[142] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 8.
[143] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 8.
[144] Cf. Pontiff, J./Singla, R. (2019), p. 260.
[145] Cf. Pontiff, J./Singla, R. (2019), p. 268.
[146] Cf. Pontiff, J./Singla, R. (2019), p. 268.
[147] Cf. Fama, E. F./French, K. R. (2008), p. 1655.
[148] Cf. Jacobs, H. (2015), p. 3.

1.8.3 Value-weighting

PS's liquidity gamma index, which tracks aggregate total market liquidity, is calculated as a simple average over all stock-specific liquidity metrics, which contradicts the widely accepted practice of a value-weighted average.[149] In addition to PS (2003), the approach of equal weighting in the context of liquidity measurement is followed, for example, by Amihud (2002) and Acharya and Pedersen (2005).[150] This potentially overweights small stocks, gives them too much influence on the overall result and distorts observations (noise).[151] The results are far more extreme than with other weighting methods, such as the classification of an equal number of NYSE shares in portfolios: "NYSE breaks", popular in Academia" or the classification according to equal market capitalisation: "cap breaks", popular in industry"[152], The second variant is rather uncommon in analyses of the cross-section of stock returns.[153] The second variant would also result in a large imbalance in the number of shares per portfolio, up to the extreme case that a decile portfolio contains very few, extremely large shares.[154] This in turn affects the diversification of idiosyncratic risk within this portfolio, which can cause a distortion of results.[155] Contrast this with the view that value weighting tends to under-represent small stocks and value-weighted portfolio returns are dominated by a few large stocks.[156]

A comparison of value weighting and equal weighting provides information on whether observations are primarily driven by small, extreme data points.[157]

1.9 Restriction of regression parameters

Another adaptation of the PS methodology is the restriction of the axis intercept to the value 0, which is often used in regressions and can also be a possible modification in equation 4.[158] When applying this modification, it is important to consider the

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[149] Cf. Pontiff, J./Singla, R. (2019), p. 268.
[150] Cf. Amihud, Y. (2002), p. 37, Acharya, V. V./Pedersen, L. H. (2005), p. 388.
[151] Cf. Ruenzi, S./Ungeheuer, M./Weigert, F. (2013), p. 21.
[152] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 14.
[153] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 7.
[154] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 7.
[155] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 7.
[156] Cf. Hou, K./Xue, C./Zhang, L. (2017), p. 9, Fama, E. F./French, K. R. (2015), p. 19.
[157] Cf. Ruenzi, S./Ungeheuer, M./Weigert, F. (2013), p. 21, Jacobs, H./Müller, S. (2020), p. 11.
[158] Cf. Pontiff, J./Singla, R. (2019), p. 268.

reasonableness of the assumed relationships. The original intention of this assumption is that if the explanatory variables of the regression equation 4 previous return and the signed volume are zero, then stock-specific return forecasts for the next day correspond to the market return.[159] This assumption is quite appropriate in a general context as a prediction, as Pontiff and Singla (2019) note: "This restriction is in line with previous research.", but in this particular case, assuming that both variables are zero, to be discussed.[160] This results in the trade-off of either maintaining a degree of freedom or accepting a possible distortion.[161]

1.10 Portfolio formation

Further modifications to which the liquidity gamma index of PS has proven to be highly sensitive can be located in the portfolio formation. These concern, on the one hand, the frequency of the reallocation of the portfolio weights and, on the other hand, the specific months of execution.[162]

1.10.1 Frequency of portfolio rebalancing

PS perform their portfolio adjustment on an annual basis at the end of December, whereas the construction of their liquidity gamma index takes place monthly.[163] A significantly more frequent portfolio reallocation is countered by the potentially higher transaction costs incurred as a result, which reduce a possible risk factor premium.[164]

1.10.2 Timing of the portfolio rebalancing

Following on from the portfolio turnover frequency, the corresponding month, in the case of PS at the end of December, is also a significant factor influencing the measurement results. Both the first and the last month of the year seem plausible. Pontiff and Singla (2019) use January as the reallocation time and a monthly frequency as shown in chapter 5.2.

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159 Cf. Pontiff, J./Singla, R. (2019), p. 268 f.
160 Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 8 f.
161 Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 8 f.
162 Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 14.
163 Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 14 f.
164 Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 7 f.

1.11 Time variability of the liquidity risk premium

A key factor influencing the liquidity risk premium is its time variability. Investors are uncertain what transaction costs they can expect in the future when they have to sell shares. As exemplified in chapter 1, liquidations are more likely to occur during periods of market stress. Because the premium mainly occurs during these disruptive market phases (s. figs. 6 and 7), it is therefore particularly dependent on these asymmetric liquidity shocks.:[165] "Further, if liquidity matters and liquidity varies, then must not liquidity risk matter? In other words, if the level of liquidity affects the price, then liquidity shocks lead to price shocks, so shouldn't investors care about this risk?"[166] Consequently, the effect of liquidity on expected stock returns also varies, which can partly explain different high measurement results of the premium by using different time periods with different strong market shocks.[167] Furthermore, it should be noted that during normal market phases the time series in figs. 6 and 7 fluctuate around 0, which makes it difficult to use them as explanatory variables in regressions. Therefore, in comparative empirical studies, care should be taken to include market shocks for more precise measurement.[168] High time variability particularly affects individual components of the PS concept, such as market depth and trading volume, which each exhibit higher volatility than returns.[169]

The US Capital Market and the Liquidity Risk Premium - Empirical Results and Market Liquidity Shocks

The sensitivity of stock returns, in relation to aggregate total market liquidity, can be analysed especially during crisis phases that affect the total market and/or larger market segments.[170] As shown by a variety of authors, overall market liquidity can decrease drastically or even disappear in such phases.[171] Liquidity then materialises as a systematic risk factor.[172] The applications of the models to new data are perceived as construction sensitivities, against which the measurement concept should

[165] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 13.
[166] Cf. Acharya, V. V./Pedersen, L. H. (2019), p. 116.
[167] Cf. Amihud, Y. (2018), p. 215.
[168] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 13 f.
[169] Cf. Chordia, T./Roll, R./Subrahmanyam, A. (2001), p. 526.
[170] Cf. Ruenzi, S./Ungeheuer, M./Weigert, F. (2013), p. 1.
[171] Cf. Chordia, T./Sarkar, A./Subrahmanyam, A. (2004), p. 86, Chen, J. (2005), p. 2.
[172] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 13.

deliver robust results when correctly mapping the liquidity risk relationship. Therefore, the empirical consideration differentiates between replication within the original study period of PS and replication in periods outside this period. Optimally, an application to the unknown data that are not from the design's database should yield similar, if not the same, statistically significant results to support the initial observations. This prevents a "data snooping effect", where the same anomaly results are found over and over again in data sets where similar results have been found before.[173]

Besides the analysis of the significance of regression coefficients[174], the extent to which liquidity effects are captured can also be checked by visually examining time series that are intended to depict overall market liquidity: [175] „you know it when you see it“[176]. The significance of regression coefficients is assessed in this paper primarily on the basis of the values of the t-statistics, where the empirical t-value of the data used must be greater than or equal to the value of the normal distribution, provided that a one-sided t-test with a positive t-value is used. If the resulting empirical t-value is less than 0, it is compared with the corresponding negative critical limits, since the normal distribution is symmetrical. If the empirical t-value is greater than or equal to 1.645, it is significantly different from 0 at the 5% level. If the empirical t-value is greater than or equal to 1.960, it is significantly different from 0 at the 2.5% level.[177] For a large sample size, the t-distribution approaches the normal distribution, which is why the quantile values of the normal distribution are used as critical values for comparison.[178]

1.12 The liquidity risk premium in the period 1962-1999

Whether there is a statistically significant correlation between stock returns on the US capital market and the aggregate total market liquidity ratio constructed by PS is first examined in the period of the original PS study. The liquidity risk premiums of 7.5% p.a. calculated by PS against the four-factor model and of around 9% against the three-factor model (s. tab. 4 in PS Study[179]) are based on the forecast liquidity betas, which are not discussed in detail in this paper. Equivalent values for historical

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[173] Cf. Keim, D. B. (2016), p. 9 f, Malkiel, B. G./Xu, Y. (2002), p. 14, Jacobs, H./Müller, S. (2020), p. 2.
[174] Cf. Acharya, V. V./Pedersen, L. H. (2019), p. 116.
[175] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 8.
[176] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 1.
[177] Cf. Acharya, V. V./Pedersen, L. H. (2019), p. 116.
[178] Cf. Stock, J. H./Watson, M. W. (2012), p. 171.
[179] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 669.

liquidity betas are 4.87% p.a. against the four-factor model and 4.15% against the three-factor model (s. Appendix A3).

The empirical results after the portfolio sorting, based on the historical liquidity betas as presented in chapter 3.3, are analysed below.

Portfolio sorting based on historical liquidity betas

Table 3 shows the historical liquidity betas, the average market capitalisation and the company liquidity, measured as the average value of all $\hat{\gamma}_{i,t}$ values, with their respective regression coefficients in three time periods per portfolio presented.[181] The same data for the three FF factors and the momentum factor (MOM) can be taken from the table in Appendix A4.

	Decile portfolios										
	A. Liquidity betas										
Periods	1	2	3	4	5	6	7	8	9	10	10-1
Jan. 1968 – Dec. 1999	-6,02 (-2,57)	-0,65 (-0,37)	-0,62 (-0,48)	-0,54 (-0,41)	1,12 (0,96)	-1,58 (-1,24)	1,37 (1,00)	2,00 (1,49)	3,04 (1,99)	-0,04 (-0,02)	5,99 (1,88)
Jan. 1968 – Dec. 1983	-7,59 (-1,84)	-1,17 (-0,44)	3,87 (1,86)	-1,54 (-0,68)	-0,48 (-0,25)	1,65 (0,71)	-1,18 (-0,55)	0,02 (0,01)	1,26 (0,54)	0,41 (0,14)	7,99 (1,60)
Jan. 1984 – Dec. 1999	-4,17 (-1,52)	-1,49 (-0,63)	-4,10 (-2,46)	-0,30 (-0,18)	2,55 (1,72)	-2,75 (-2,00)	2,80 (1,56)	3,79 (2,08)	4,38 (2,07)	1,18 (0,39)	5,35 (1,26)
	B. Additional data, Januar 1968 – December 1999										
Market capitalisation	7,11	7,69	10,44	17,65	16,67	22,18	16,26	11,64	9,89	6,97	
Liquidity	-0,52	-0,19	-0,06	-0,04	-0,02	-0,05	-0,05	-0,05	-0,05	-0,12	

Table 5: Liquidity betas, market capitalisation and liquidity of the decile portfolios based on sorting by historical liquidity betas according to PS (2003).[180]

tab. 3 illustrates that over the entire period under study, from January 1968 to December 1999, the liquidity beta difference of the first and tenth decile portfolios is significant at the 5% level. The difference is 5.99 with a value of the T-statistic of t = 1.88.[182] Liquidity beta differences in sub-periods are significant at low levels with t = 1.60 (January 1968 to December 1983) and with t = 1.26 (January 1984 to December 1999). Looking at the average market capitalisation, it is noticeable that no clear

[180] Table taken from Pástor, Ľ./Stambaugh, R. F. (2003), p. 675.
[181] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 674.
[182] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 674.

trend can be observed per decile portfolio.[183] The decile portfolio with the lowest liquidity beta does not contain the largest companies on average. The largest companies on average are found in the midfield in portfolios 6, 4, 5 and 7.

For comparison, the exact replication of this table by Li et al. (2019) can be used, which is presented in tab. 4, but with expanded values (s. Appendix A5). The signs

	Decile portfolios										
	A. Liquidity betas										
Zeiträume	1	2	3	4	5	6	7	8	9	10	10-1
Jan. 1968 – Dec. 1999	-7,16 (-2,93)	-0,29 (-0,17)	-1,29 (-0,85)	-0,69 (-0,50)	3,31 (2,57)	-0,01 (-0,01)	1,09 (0,75)	0,63 (0,39)	3,06 (1,74)	-0,15 (-0,07)	7,01 (2,12)
Jan. 1968 – Dec. 1983	-10,11 (-2,31)	-0,32 (-0,12)	1,22 (0,48)	-1,94 (-0,82)	2,21 (0,98)	3,85 (1,55)	-3,34 (-1,45)	0,98 (0,36)	2,04 (0,74)	-0,99 (-0,32)	9,12 (1,78)
Jan. 1984 – Dec. 1999	-4,63 (-1,55)	-0,80 (-0,34)	-3,32 (-1,82)	-0,73 (-0,44)	4,97 (3,18)	-1,86 (-1,17)	3,22 (1,65)	1,22 (0,59)	5,32 (2,26)	0,77 (0,23)	5,39 (1,19)
	B. Additional data, Januar 1968 – Dezember 1999										
Market capitalisation	8,35	8,92	17,09	21,51	16,12	24,32	17,05	14,50	9,55	8,23	
Liquidity	-0,09	-0,09	-0,09	-0,01	-0,02	-0,04	-0,05	-0,05	-0,13	-0,08	

Table 6: Liquidity betas, market capitalisation and liquidity of decile portfolios based on sorting by historical liquidity betas according to Li et al (2019).[184]

of the approximated values of tab. 4 largely match those of tab. 3 of PS, although significant differences in values result, despite the use of the same methodology and data as of PS. A similar picture emerges from the comparison of the market capitalisation values, in which the general trend of PS can be confirmed, but value differences result in higher values.

Alphas of the portfolios sorted by historical liquidity betas

To the extent that market liquidity constructed by PS affects stock returns, there should be systematic return differences in portfolio returns sorted by this historical liquidity beta.[185] The excess returns of the 10 value-weighted portfolios are regressed on various recognised factor models, such as the CAPM, the FF three-factor model and the three-factor model supplemented by the momentum factor (four-factor model). If the intercept of these regressions is different from 0 with statistical significance, this represents the alpha value of the portfolio (α).[186] Alpha refers to the excess return compared to a benchmark[187], which therefore cannot be fully explained by these factor models and is therefore due to the liquidity risk factor. These alpha

[183] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 674.
[184] Table taken from Li, H./Novy-Marx, R./Velikov, M. (2019), p. 45.
[185] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 668.
[186] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 668.
[187] Cf. Holmes, C. (2009), p. 432.

values in percent per year can be found in Appendix A3. Looking at the entire period from January 1968 to December 1999, significant alpha values are shown as the difference between portfolios 10 and 1 of 4.66% in the CAPM, 4.15% in the case of the FF three-factor model and a value of 4.87% in the case of the four-factor model per year, with corresponding t-statistics of t = 2.36, t = 2.08 and t = 2.38.[188]
Corresponding alpha values for the periods January 1968 - December 1983 and January 1984 - December 1999 are insignificant.
Table 5 shows the comparison of the alpha values calculated by PS for different factor models and the values replicated by Li et al. (2019) in the period January 1968 to December 1999..

	Liquidity factor according to website data				Replicated liquidity factor			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Pastor-Stambaugh period, January 1968 - December 1999								
α	0,28	0,26	0,37	0,20	0,30	0,28	0,37	0,20
	(1,73)	(1,60)	(2,14)	(1,16)	(1,90)	(1,73)	(2,22)	(1,19)
MKT		-0,03	-0,03	-0,02		-0,01	-0,01	0,00
		(-0,63)	(-0,65)	(-0,45)		(-0,23)	(-0,24)	(0,03)
SMB		-0,14	-0,17	-0,11		-0,13	-0,16	-0,09
		(-2,50)	(-2,86)	(-1,77)		(-2,37)	(-2,70)	(-1,54)
HML		0,14	0,10	0,20		0,11	0,09	0,17
		(2,10)	(1,59)	(2,14)		(1,80)	(1,33)	(1,83)
MOM			-0,10				-0,09	
			(-2,13)				(-1,94)	
Profitability				0,26				0,29
				(2,35)				(2,69)
Investment				-0,01				0,02
				(-0,09)				(0,16)

Table 7: Comparison of replicated average monthly alpha values across different factor models in the period January 1968 to December 1999 from Li et al. (2019).[189]

The alpha values calculated by PS are the average monthly excess returns of the trading strategy (buy stocks with high liquidity beta: portfolio 10, sell stocks with low liquidity beta: portfolio 1) of the value-weighted decile portfolio return differential (s. authors' website data). These return time series are regressed on the factor models: FF three- and five-factor model (1993, 2015) and the Carhart four-factor model (1997) as explanatory variables. Comparison shows a high degree of agreement between the original values and the replicated values. During the analysed period, alpha values are only significant in relation to the four-factor model, with values of the T-statistics of t = 2.14, calculated by PS and t = 2.22, calculated by Li et al. (2019).[190]

[188] Cf. Pástor, Ľ./Stambaugh, R. F. (2003), p. 674.
[189] Table taken from Li, H./Novy-Marx, R./Velikov, M. (2019), p. 13.
[190] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 12.

	r^e	α_{FF3}	α_{FF4}	α_{FF5}
Panel A: Posted liquidity factor				
PS Sample	0,28 (1,73)	0,26 (1,60)	0,37 (2,14)	0,20 (1,16)
Panel B: Posted liquidity factor (annual December rebalancing)				
PS Sample	0,30 (1,90)	0,28 (1,73)	0,37 (2,22)	0,20 (1,19)
Panel C: Liquidity factor constructed using annual Januar rebalancing				
PS Sample	0,06 (0,34)	0,05 (0,30)	0,23 (1,33)	0,10 (0,55)
Panel D: Liquidity factor constructed using (natural) monthly rebalancing				
PS Sample	0,18 (1,02)	0,20 (1,10)	0,42 (2,30)	0,19 (1,03)

Table 6: Monthly alpha values across different factor models with different timing of portfolio rebalancing in the period of the PS study.[191]

Based on the data provided by PS, Pontiff and Singla (2019) succeed in replicating the alpha value exactly, with an annual liquidity risk premium (alpha) of 3.91% across the FF three-factor model and a t-statistic of t = 2.00 (s. Table A6). Deviations compared to the value of PS s. Appendix A3 can be attributed to changes in the CRSP data set.

In their extended study (s. chapter 5.2) the values deviate with an annual liquidity risk premium of 3.82% instead of 3.91% and a T-value of t = 1.93, because the base time series of total market liquidity already begins in 1932 and not in 1962 as in PS.[192] Table 6 shows alpha values via the FF three- and five-factor model as well as via the Carhart four-factor model with annual portfolio reallocations in December and January and on a monthly basis for the comparison period of PS.

If other times are chosen for rebalancing, as in Panel C in January and Panel D monthly, the alpha values become largely insignificant.

Measured alpha via the FF three-factor model in the period January 1968 to December 1999 using decile portfolios sorted based on historical liquidity betas			
Original study by PS (2003) (s. appendix A3)	Study by Pontiff and Singla (2019) based on website data from PS. (s. appendix A6)	Study by Pontiff and Singla (2019) (s. appendix A6)	Study by Li et al. (2019) based on website data from PS. (s. appendix A7)
4,15 (2,08)	3,91 (2,00)	3,82 (1,93)	3,37 (1,73)

Table 7: Comparison of the average annual liquidity risk premium of four studies in the period January 1986 to December 1999.[193]

[191] Table taken from Li, H./Novy-Marx, R./Velikov, M. (2019), p. 13.
[192] Cf. Pontiff, J./Singla, R. (2019), p. 264.
[193] Own presentation.

Tab. 7 shows an overview of the calculated liquidity risk premium of the studies analysed in this paper for the period January 1968 to December 1999.

Although the methodology used is the same as for PS, there are deviations in the values, which, however, allow for comparability due to the rather low level. The calculated premiums at the 5% level are statistically significant in all cases and at the 2.5% level in two of the four studies.

1.13 The liquidity risk premium in the period 1920-1962 and 2000-2020

"There are two ways to deal with the bias introduced by multiple testing: out-of-sample validation and using a statistical framework that allows for multiple testing. When feasible, out-of-sample testing is the cleanest way to rule out spurious factors."[194], therefore, the robustness of the findings from the previous subchapter 5.1 is verified using unknown data. Based on this, the replication of the PS Liquidity Gamma Index is analysed for the period from 1920 to 1962, i.e. before the actual study period of the original PS study, and in the period from January 2000 to December 2015, i.e. after the original study period. In order to ensure a valid analysis, it is necessary to record the data for as long as possible, because only a historical retrospective can be carried out and not a real time analysis.[195] However, it should also be noted here that: „ […] a successful outcome of in-sample replication is not guaranteed even if the methodology used by the replication study is identical to that used in the original study"[196], which can already be seen in tab. 4.

[194] Cf. Harvey, C. R./Liu, Y./Zhu, H. (2016), p. 11.
[195] Cf. Harvey, C. R./Liu, Y./Zhu, H. (2016), p. 11.
[196] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 3.

Replication of total market liquidity and its change over time

The application of equations 4-10 to new data is done continuously by the authors PS themselves (s. websites of both Robert F. Stambaugh and Ľuboš Pástor).[197] These time series of the total market liquidity and the change in it, continued until December 2020, are illustrated in fig. 6.

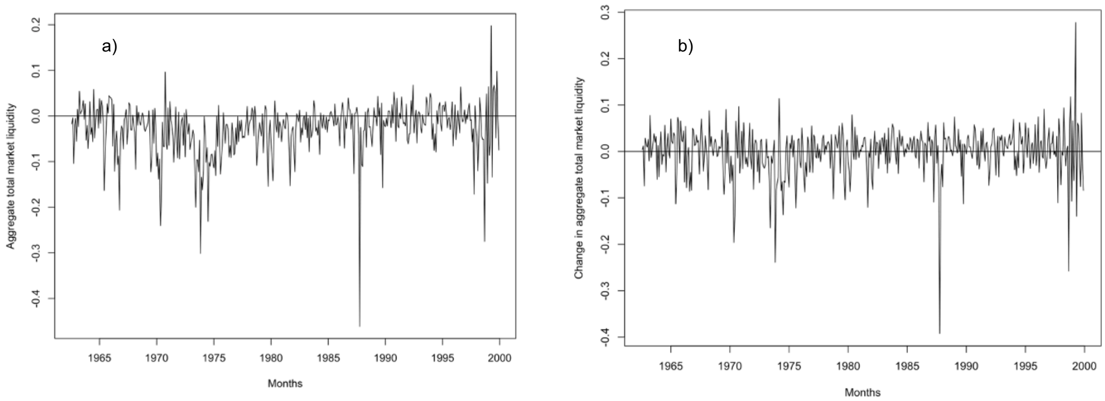

Figure 6: Time series of a) total market liquidity and b) change in total market liquidity in the period August 1962 to December 2020 by PS (2003).[198]

The period of the original study by PS from 2003, (August 1962 to December 1999) is delimited by dashed vertical lines. Both figures can be found separately in Appendices A8 and A9 for better comparison.

On the basis of purely visual examinations, time series of models depicting overall market liquidity must reflect significant market events such as crises and shocks.[199] In the period from 1962 to 2020, the main such events are the financial crisis of 2007 to 2009, the terrorist attack on 11 September 2001 and effects of the ongoing Covid-19 pandemic in March 2020.

These major events can be clearly seen in fig. 6[200], which is a first confirming indication that liquidity is adequately represented and therefore that sensitivities of the measurement concept in the present specification provide valid results. Replications are provided by the studies of Li et al. (2019) and Pontiff and Singla (2019), which

[197] Further data on the portfolios' returns can be accessed on Lubos Pástor's website.
[198] Own presentation based on Pástor, Ľ./Stambaugh, R. F. (2003), p. 651.
[199] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 5.
[200] Vgl. Ľuboš Pástor: <URL: https://faculty.chicagobooth.edu/lubos-pastor/data> (Zugriff: 15.09.2021)

are analysed below in comparison with the original time series of PS. The total market liquidity in the period from 1920 to 2015 , calculated by Pontiff and Singla (2019), is shown in fig. 7 a). CRSP data before 1962 are not available at the time of the PS study, which is why these data are included ex-post of the PS study.[201]

In direct comparison, the replicated time series shows lower volatility both before 1962 and after 1999, but to a more pronounced extent before 1962.[202] Market shocks, such as the 2007/2008 financial crisis, are clearly reflected over several months. In the period from 1999 onwards, October 2002 forms the most negative point of the extended time series, corresponding to the extended time series of PS itself, with a minimum value of -0.308 at this point (s. Appendix A2). This date, however, cannot be explained by plausible events concerning market liquidity.[203] A visual analysis of the replicated time series of Li et al. (2019) for the period January 1962 to December 2015 is shown in fig. 7 b), in which on the one hand the quality of the replication of the PS methodology succeeds with a correlation of 99% in the period of the original study and a correlation of 93% in the years thereafter.

The financial crisis is again clearly visible in the curve. This speaks for a high degree

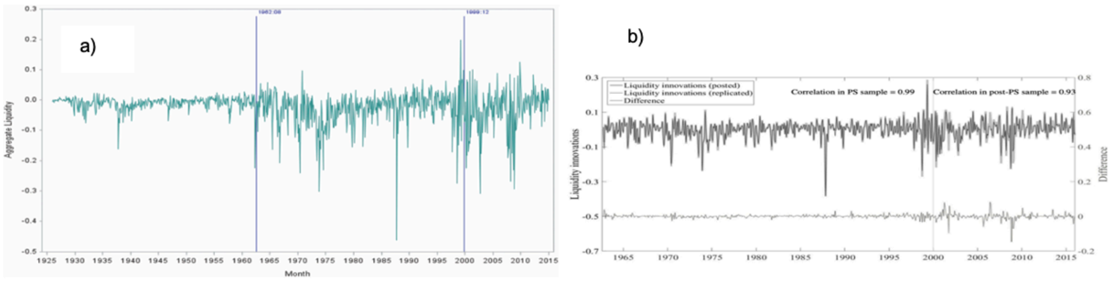

Figure 7: a) replication of total market liquidity and b) the change in this by Pontiff and Singla (2019) and Li et al. (2019).[204]

of replication of the calculation steps presented in chapter 3. Both figures are included separately in Appendices A10 and A11 for better comparison.

Replication of the alpha values of the decile portfolios

In addition to the visual analysis of the underlying time series of overall market liquidity and its changes, the replication of the alpha values of the decile portfolios is also examined in more detail.

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[201] Cf. Pontiff, J./Singla, R. (2019), p. 262.
[202] Cf. Pontiff, J./Singla, R. (2019), p. 263.
[203] Cf. Pontiff, J./Singla, R. (2019), p. 263.
[204] Figures taken from Li, H./Novy-Marx, R./Velikov, M. (2019), p. 7, Pontiff, J./Singla, R. (2019), p. 263.

Calculations of the liquidity risk premium are also performed by Pontiff and Singla (2019) for further periods before and after the original PS period and are shown in tab. 8.

	(1)	(2)	(3)	(4)	(5)
	Period before the PS study	Period of the PS study	Period after the PS study	Entire CRSP dataset	Extended PS study
Start time	1932:1	1968:1	2000:1	1932:1	1968:1
End time	1967:12	1999:12	2017:12	2017:12	2017:1
	-2,72 (-1,36)	3,82 (1,93)	5,56 (1,84)	1,45 (1,13)	4,45 (2,67)

Table 8: Liquidity risk premiums and T-statistics according to Pontiff and Singla (2019) for different time periods.[205]

Strikingly, before the PS period, the alpha value is negative and insignificant with a value of -2.72%, with a value of the T-statistic of t = -1.36. After the calculations, there is an increasing trend of the liquidity risk premium from 3.82%, in the period January 1968 to December 1999, to 5.56%, in the period January 2000 to December 2017. Adjustments of the study periods provide significant measurement results of the premium for the period January 1968 to December 2017, whereas the entire CRSP database rejects the existence of a premium with a t-statistic of t = 1.13 in the Pontiff and Singla (2019) study. Accordingly, the question arises as to which time periods are given more importance. Depending on the view, greater relevance can be attributed to more recent data due to their better representation of the existing reality.

	Liquidity factor according to website data				Replicated liquidity factor			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel B: Post Pastor-Stambaugh period, January 2000 - December 2015								
α	0,69	0,69	0,68	0,53	0,70	0,72	0,66	0,53
	(2,38)	(2,38)	(2,31)	(1,76)	(2,35)	(2,38)	(2,21)	(1,71)
MKT		-0,01	0,01	0,07		0,01	0,09	0,10
		(-0,19)	(0,15)	(0,84)		(0,17)	(1,25)	(1,26)
SMB		0,15	0,14	0,32		0,06	0,02	0,26
		(1,69)	(1,51)	(2,93)		(0,69)	(0,18)	(2,35)
HML		-0,13	-0,12	-0,21		-0,12	-0,09	-0,20
		(-1,43)	(-1,33)	(-1,55)		(-1,24)	(-0,94)	(-1,44)
MOM			0,04				0,15	
			(0,76)				(2,62)	
Profitability								0,43
								(2,85)
Investment				-0,13				-0,18
				(-0,74)				(-0,99)

Table 9: Comparison of replicated average monthly alpha values across different factor models in the period January 2000 to December 2015 from Li et al. (2019).[206]

Table 9 compares the alpha values measured by PS across different factor models and those replicated by Li et al. (2019) for the period January 2000 to December

[205] Table taken from Pontiff, J./Singla, R. (2019), p. 264.
[206] Table taken from Li, H./Novy-Marx, R./Velikov, M. (2019), p. 13.

2015. The alpha values calculated by PS are obtained as the average monthly excess returns of the long-short strategy of the 10-1 value-weighted decile portfolio return differential (s. authors' website data). This return time series was regressed on the following factor models as explanatory variables: FF three- and five-factor model (1993, 2015) and the Carhart four-factor model (1997). Comparatively, there is a high degree of agreement between the values carried forward by PS and the replicated values, within the period January 2000 to December 2015.

Replication studies and modifications of the PS model

After the basic equation of the PS model has been replicated in different time frames, the modifications discussed in chapter 4 and their effects on the statistical results are analysed below.

Panel A: Correlation between four variants of the PS Index of changes in market liquidity in the period August 1962 to December 2017				
	Value-weighted Index	All price levels included	Zero volume days included	Base PS Index
Zero volume days included				0,27
All price levels included			0,11	0,17
Value-weighted Index		0,17	0,26	0,95
$\theta_{i,t}$ restricted to 0	0,92	0,15	0,24	0,97
Panel B: Correlations between the returns of the four variants of the PS Index of Changes in Market Liquidity in the period August 1962 to December 2017				
	Value-weighted Index	All price levels included	Zero volume days included	Base PS Index
Zero volume days included				0,26
All price levels included			0,02	0,10
Value-weighted Index		0,15	0,25	0,82
$\theta_{i,t}$ restricted to 0	0,77	0,18	0,25	0,87
Panel C: Average annual returns of the trading strategy based on the four variants of the traded liquidity factor in the period January 1968 to December 2017				
$\theta_{i,t}$ restricted to 0	Value-weighted Index	All price levels included	Zero volume days included	Base PS Index
3,13 (1,89)	1,02 (0,59)	0,78 (0,15)	0,82 (0,50)	4,45 (2,67)

Table 10: Correlations between four variations of the liquidity gamma index as well as average annual returns compared to the PS index.[207]

The time series correlation of the following four adjustments with the PS time series according to equation 9 are shown in Table 10 for the periods August 1962 to December 2017 and January 1968 to December 2017. Panel A contains the correlation of decile portfolio returns (buy portfolio 10, sell portfolio 1) of the four variants with the original time series. The average annual return premium of this trading strategy of these four variants is additionally included in Panel C.[208] The variants of value weighting and parameter restriction to 0 show high correlations of over 90% with the PS time series.[209] Further correlations between the time series of PS and modifications amount to values below 30%, which is also reflected in correlations among the modifications. Only the variants of the value weighting and the restricted parameter

[207] Table taken from Pontiff, J./Singla, R. (2019), p. 273.
[208] Cf. Pontiff, J./Singla, R. (2019), p. 269.
[209] Cf. Pontiff, J./Singla, R. (2019), p. 269 f.

have a correlation of 92%. Correlations of the decile portfolio returns of the four variants generally show lower values compared to Part A.[210] Thus, the correlations between the time series of PS and the value-weighted variant were reduced from 95% to 82% and with the restricted variant from 97% to 87%.

The average annual return premiums amount to consistently lower and statistically insignificant values, compared to the PS study.[211] The only significant value with 3.13 and a t-statistic of t = 1.89 is provided by the restricted variant of a total of four modifications.

Alpha values relating to the modification of the adjusted portfolio rebalancing dates are shown in the tables in Appendices A12 and A13. Table A12 shows alpha values as average monthly excess returns of the trading strategy over the FF three- and five-factor model, as well as over the Carhart four-factor model with annual portfolio rebalancing in December and January and on a monthly basis. Comparison periods are the original PS period, January 2000 to December 2015 and January 1968 to December 2015. As previously shown in Table 7, the replicated alpha values with annual reallocations in December, show high agreement with the original PS values. Annual reallocations in January, on the other hand, (s. Table A12 Panel C) show mostly lower and insignificant alpha values. Monthly portfolio shifts result in consistently more robust, though also lower, values than PS alpha values for all three time periods. Table A13 presents T-statistics of the average excess returns of the portfolios over the period January 1968 to December 2015, and of the three factor models used with the three portfolio modifications discussed in Section 4.1.3. In addition, Li et al. (2019) vary the number of portfolios from 2, 3, 5, 10 and 20.[212]

The methodical procedure of annual portfolio rebalancing at the end of December and the exclusion of price levels below USD 5 and above USD 1,000 are retained for the sake of comparability.[213] With the exception of the market capitalisation limits, the variant of 10 portfolios (decile portfolio approach) always generates the highest T-statistics in absolute terms. Within these values, the variant of PS with the market capitalisation limits generates the highest value of t = 3.04 across all T-statistics shown in tab. A13.[214] Like the variation of observation periods before, the portfolio

[210] Cf. Pontiff, J./Singla, R. (2019), p. 270.
[211] Cf. Pontiff, J./Singla, R. (2019), p. 270.
[212] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 17 f.
[213] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 17.
[214] Cf. Li, H./Novy-Marx, R./Velikov, M. (2019), p. 17.

modifications also provide differentiated results with even minor changes in the PS methodology.

Conclusion

Based on the PS concept for analysing whether liquidity risk exists in the US equity market and is remunerated with a premium for investors, it can be seen that it is not possible to guarantee a completely holistic recording of all relevant market and liquidity dimensions in one key figure. This fact is documented by the multitude of approaches to liquidity measurement.[215] Regardless of the different approaches, however, it is important to note that there are theoretically implied links between stock returns and liquidity costs and that this link is independent of the calculation method used.[216]

Accordingly, it follows that if liquidity affects stock prices, then both theory and practice should take this liquidity effect into account: “Investment decisions and portfolio composition should be based on the liquidity of capital assets as well as on their expected return and risk (however measured).”[217] The attempt to document this correlation is accompanied by numerous limitations. This manifests itself in the varying extents of the liquidity risk premium found, although a statistically significant liquidity risk premium can be assumed overall.[218] Data basis, availability and filtering represent one of the greatest challenges of empirical liquidity studies.[219] The primary question is the frequency of the data.[220] Because high-frequency records are only available globally for a few trading venues in long time series[221], So far, primarily low-frequency data has been chosen in order to be able to include longer recordings and data with potentially higher statistical significance across more markets globally.[222] This favours US-centred research due to a high degree of data coverage of American

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[215] Cf. Rösch, C. (2012), p. 28, Amihud, Y. (2018), p. 214 f, Le, H./Gregoriou, A. (2020), p. 1172.
[216] Cf. Amihud, Y. (2018), p. 214.
[217] Cf. Amihud, Y./Mendelson, H. (1991), p. 1424.
[218] Cf. Piqueira, N. S. (2005), p. 2.
[219] Cf. Rösch, C. (2012), p. 28.
[220] Cf. Vayanos, D./Wang, J. (2013), p. 1345.
[221] Cf. Le, H./Gregoriou, A. (2020), p. 1182, Chen, J. (2005), p. 4, Goyenko, R. Y./Holden, C. W./Trzcinka, C. A. (2009), p. 153 f.
[222] Cf. Le, H./Gregoriou, A. (2020), p. 1171, Ma, R./Anderson, H. D./Marshall, B. R. (2016), p. 122, Piqueira, N. S. (2005), p. 8.

markets.[223] Its results do not necessarily have to be representative on a global level. In view of the fact that strong fluctuations in liquidity, market depth and trading volume can already be detected within a week and within a day, the results of this study are not necessarily representative[224], The question arises as to whether these effects are sufficiently taken into account in higher aggregation, such as a monthly view.[225] Possible implications arise in that low-frequency ratios should only be used when high-frequency data are not available, as is often the case for empirical capital market studies.[226] Accordingly: "An alternative interpretation […] is that different frequencies capture different phenomena."[227]

When analysing the US capital market, reference should also be made to the ongoing adjustments to the CRSP data set, which has established itself as the standard data basis for empirical capital market research.[228] Therefore, variations in results in replication studies can also be attributed to these changes to a certain extent.[229]
The restriction of days without trading volume is also a limitation for a variety of liquidity measures. However, by excluding such days, valuable information on market liquidity can be lost.[230]
Another key factor concerning liquidity is its time-varying nature. Analyses of different time periods in chapter 5 suggest that over time the liquidity premium has increased, whereas it seems to be insignificant before the period of the original PS study, but is even more detectable after this period. In this context, it is important to consider within the selected time period whether it includes stress phases and how strongly these are reflected in a historical comparison with other crisis phases in which the liquidity risk materialises and can be better calculated. The importance of time variability is thereby clarified.[231]

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[223] Cf. Jacobs, H. (2016), p. 271, Jacobs, H./Müller, S. (2020), p. 1, Harvey, C. R./Liu, Y./Zhu, H. (2016), p. 5. Hochfrequenzdaten sind in den USA seit 1983 verfügbar Cf. Hasbrouck, J. (2009), p. 1445, Goyenko, R. Y./Holden, C. W./Trzcinka, C. A. (2009), p. 153, Pontiff, J./Singla, R. (2019), p. 258.
[224] Cf. Chordia, T./Roll, R./Subrahmanyam, A. (2001), p. 526.
[225] Based on the intraday view of the NYSE, a U-shape of bid-ask spreads can be observed Cf.Upson, J./Van Ness, R. A. (2017), p. 3, Brock, W. A./Kleidon, A. W. (1992), p. 470 f, Chan, K./Chung, Y. P./Johnson, H. (1995), p. 343, Chung, K. H./Van Ness, B. F./Van Ness, R. A. (1999), p. 255.
[226] Cf. Vayanos, D./Wang, J. (2013), p. 1345.
[227] Cf. Vayanos, D./Wang, J. (2013), p. 1345.
[228] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 3, Harvey, C. R./Liu, Y./Zhu, H. (2016), p. 36.
[229] Cf. Pástor, Ľ./Stambaugh, R. F. (2019), p. 3, Jacobs, H./Müller, S. (2020), p. 9.
[230] Cf. Le, H./Gregoriou, A. (2020), p. 1171.
[231] Cf. Nageswaran, S. (2012), p. 1, Vayanos, D./Wang, J. (2013), p. 1293.

Another temporal influence in this context is the holding period of investors[232], which will require more intensive analysis in the future.[233] As a possible theoretical implication, there are clientele effects that can be derived from this.[234]
Market-side drivers also need to be considered in the historical context, such as the design of the market microstructure[235], the increasing integration of international capital markets[236], Increasing trading volume and trading place competition[237], which in sum underlines the complex, time-variable character of the liquidity risk premium.
Whether these complex interactions can be captured by simple linear multifactor models or whether non-linear statistical methods capture these effects better remains to be tested in the future.[238] As a future approach to capital market research, in relation to liquidity, the combination of several measurement concepts into one large model can serve to be able to sufficiently depict several facets of liquidity.[239]

In conclusion, it can be stated that a highly complex[240], no single measurement concept can do justice to a multidimensional construct such as liquidity.[241] It is important to pay particular attention to which facet of liquidity is being measured[242] and whether there are not several different ones at the same time, which are calculated by different approaches and influence each other by colinearity, whereby not all dimensions have to be remunerated with a premium.[243]

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232 Cf. Novy-Marx, R. (2004), p. 1.
233 Cf. Nageswaran, S. (2012), p. 22.
234 Cf. Amihud, Y./Mendelson, H./Pedersen, L. H. (2006), p. 10, Hibbert, J., et al. (2009), p. 3.
235 Cf. Malz, A. M. (2018), p. 46, Vayanos, D./Wang, J. (2013), p. 1294.
236 Cf. Ma, R./Anderson, H. D./Marshall, B. R. (2016), p. 118.
237 Cf. Piqueira, N. S. (2005), p. 23.
238 Cf. Chulia, H./Koser, C./Uribe, J. M. (2021), p. 21, Ruenzi, S./Ungeheuer, M./Weigert, F. (2013), p. 22.
239 Cf. Amihud, Y. (2018), p. 215.
240 Cf. Malz, A. M. (2018), p. 36.
241 Cf. Amihud, Y. (2018), p. 214 f, Amihud, Y./Mendelson, H./Pedersen, L. H. (2006), p. 32, Vayanos, D./Wang, J. (2013), p. 1351, Spiegel, M. I./Wang, X. (2005), p. 30, Stambaugh, R. F./Yuan, Y. (2017), p. 1273.
242 Cf. Vayanos, D./Wang, J. (2013), p. 1346, Goyenko, R. Y./Holden, C. W./Trzcinka, C. A. (2009), p. 179., Cf. Ruenzi, S./Ungeheuer, M./Weigert, F. (2013), p. 16, Liu, Y. (2018), p. 27, Eckbo, B. E./Norli, Ø. (2002), p. 3., Pástor, Ľ./Stambaugh, R. F. (2019), S. 3.
243 Cf. Acharya, V. V./Pedersen, L. H. (2019), p. 115.

Appendix

Figure A1: Risk distribution after consideration of the CAPM**.**

Own illustration based on:

Heinze and Radinger (2011): Der Beta-Faktor in der Unternehmensbewertung, in: Controller Magazin, p. 49.

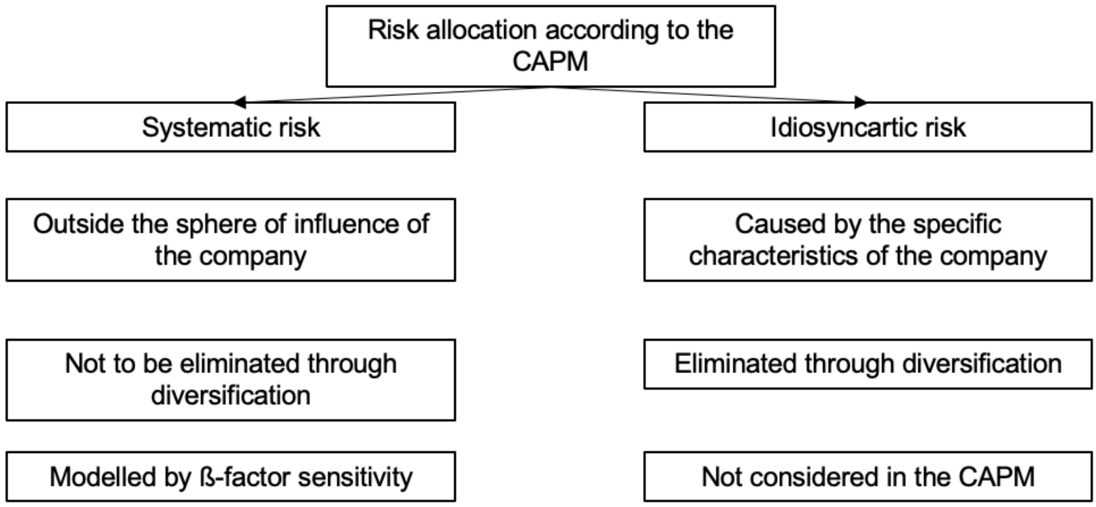

Table A2: Descriptive statistics on time series of total market liquidity according to equation 6 and changes in total market liquidity according to equation 9 in the period August 1962 to December 2020 according to PS (2003).

Own table based on data from:
Website of the author **Robert F. Stambaugh**: <URL: http://finance.wharton.up-enn.edu/~stambaug/> (accessed 11 September 2021) and **Ľuboš Pástor**: <URL: https://faculty.chicagobooth.edu/lubos-pastor/data> (accessed 11 September 2021).

	Year, month	Total market liquidity	Year, month	Changes in Total market liquidity
		A. August 1962 bis December 1999		
Minimum	1987, 10	-0,461393	1987, 10	-0,39228
1.Qunatil		-0,053426		-0,025344
Median		-0,020615		0,00687
Mean		-0,029474		-0,001634
2.Quantil		0,005437		0,026555
Maximum	1999, 4	0,197723	1999, 4	0,277805
		B. January 1968 - December 1999		
Minimum	1987, 10	-0,461393	1987, 10	-0,39228
1.Qunatil		-0,056342		-0,025452
Median		-0,021539		0,005156
Mean		-0,031132		-0,002816
2.Quantil		0,002692		0,026158
Maximum	1999, 4	0,197723	1999, 4	0,277805
		C. January 1968 - December 2020		
Minimum	1987, 10	-0,461393	1987, 10	-0,3922797
1.Qunatil		-0,054058		-0,0250543
Median		-0,019263		0,0059635
Mean		-0,028344		-0,0005397
2.Quantil		0,007431		0,0282581
Maximum	1999, 4	0,197723	1999, 4	0,2778051
		D. January 1968 - December 1983		
Minimum	1973, 11	-0,30129	1973, 11	-0,238851
1.Qunatil		-0,07443		-0,035251
Median		-0,03441		-0,002071
Mean		-0,04611		-0,009229
2.Quantil		-0,01037		0,024605
Maximum	1970, 10	0,09627	1974, 3	0,113961
		E. January 1984 - December 1999		
Minimum	1987, 10	-0,461393	1987, 10	-0,392280
1.Qunatil		-0,031122		-0,012773
Median		-0,009411		0,10217
Mean		-0,016152		0,003597
2.Quantil		0,010120		0,030039
Maximum	1999, 4	0,197723	1999, 4	0,277805
		F. Dezember 1999 - December 2020		
Minimum	2002, 10	-0,30824	2008, 9	-0,236076
1.Qunatil		-0,05346		-0,023013
Median		-0,01325		0,007829
Mean		-0,0243		0,002585
2.Quantil		0,01797		0,035223
Maximum	2009, 12	0,12457	2008, 11	0,14392
		G. Differenz der Zeiträume Januar 1984 - Dezember 1999 und Januar 1968 bis Dezember 1983		
Minimum		-0,160103		-0,153429
1.Qunatil		0,043308		0,022478
Median		0,024999		0,104241
Mean		0,029958		0,012826
2.Quantil		0,02049		0,005434
Maximum		0,101453		0,163844

Table A3: Alpha values and T-statistics of the decile portfolios via the CAPM, FF three-factor model and the Carhart four-factor model in the period January 1968 to December 1999 as well as various sub-periods according to PS (2003).

Taken from:

Pástor und Stambaugh (2003): Liquidity risk and expected stock returns, in: Journal of Political economy, 111. Jg., p. 676.

	Decile portfolios										
	A. January 1968 – December 1999										
	1	2	3	4	5	6	7	8	9	10	10-1
CAPM Alpha	-2,06 (-1,30)	-0,36 (-0,34)	0,63 (0,76)	0,49 (0,57)	0,07 (0,10)	0,49 (0,58)	1,42 (1,64)	1,36 (1,63)	-0,02 (-0,02)	2,60 (1,96)	4,66 (2,36)
Fama-French Alpha	-0,62 (-0,42)	-0,09 (-0,08)	0,46 (0,57)	0,57 (0,68)	-0,62 (-0,86)	-0,28 (-0,35)	0,90 (1,06)	0,84 (1,00)	0,03 (0,03)	3,53 (2,71)	4,15 (2,08)
Four factor Alpha	-1,20 (-0,79)	-0,04 (-0,04)	0,22 (0,26)	0,34 (0,40)	-0,29 (-0,40)	-0,25 (-0,31)	1,05 (1,20)	0,71 (0,82)	0,29 (0,29)	3,67 (2,74)	4,87 (2,38)
	B. January 1968 – December 1983										
CAPM Alpha	-1,10 (-0,46)	1,04 (0,70)	0,94 (0,79)	0,35 (0,27)	-0,28 (-0,26)	0,46 (0,34)	0,09 (0,08)	0,83 (0,72)	0,33 (0,25)	2,51 (1,51)	3,62 (1,32)
Fama-French Alpha	-1,24 (-0,53)	2,32 (1,56)	1,66 (1,41)	1,53 (1,21)	-1,05 (-0,98)	-0,49 (-0,38)	-0,06 (-0,05)	-0,07 (-0,06)	0,17 (0,13)	1,61 (1,01)	2,85 (1,01)
Four factor Alpha	-3,74 (-1,58)	1,50 (0,96)	0,87 (0,71)	0,86 (0,66)	-0,20 (-0,18)	0,21 (0,16)	0,59 (0,47)	-0,18 (-0,15)	0,59 (0,43)	1,64 (0,98)	5,38 (1,86)
	C. January 1984 – December 1999										
CAPM Alpha	-2,79 (-1,31)	-1,63 (-1,04)	0,21 (0,18)	0,40 (0,36)	0,37 (0,36)	0,23 (0,23)	3,12 (2,51)	1,70 (1,40)	-0,11 (-0,08)	2,70 (1,28)	5,49 (1,90)
Fama-French Alpha	0,03 (0,02)	-2,04 (-1,29)	-0,60 (-0,53)	-0,33 (-0,30)	-0,40 (-0,40)	-0,55 (-0,59)	2,21 (1,83)	1,50 (1,22)	-0,11 (-0,07)	4,41 (2,20)	4,38 (1,54)
Four factor Alpha	0,57 (0,30)	-1,50 (-0,94)	-0,50 (-0,44)	-0,28 (-0,25)	-0,39 (-0,38)	-0,87 (-0,93)	2,06 (1,68)	1,35 (1,08)	0,02 (0,01)	4,55 (2,23)	3,98 (1,38)

Table A4: Historical liquidity betas, average market capitalisation, firm liquidity and the four-factor model with their respective regression coefficients and T-statistics for three periods per decile portfolio according to PS (2003).

Taken from:

Pástor und Stambaugh (2003): Liquidity risk and expected stock returns, in: Journal of Political economy, 111. Jg., p. 675.

	Decile portfolios										
	A. Liquidity betas										
Periods	1	2	3	4	5	6	7	8	9	10	10-1
Jan. 1968 – Dec. 1999	-6,02 (-2,57)	-0,65 (-0,37)	-0,62 (-0,48)	-0,54 (-0,41)	1,12 (0,96)	-1,58 (-1,24)	1,37 (1,00)	2,00 (1,49)	3,04 (1,99)	-0,04 (-0,02)	5,99 (1,88)
Jan. 1968 – Dec. 1983	-7,59 (-1,84)	-1,17 (-0,44)	3,87 (1,86)	-1,54 (-0,68)	-0,48 (-0,25)	1,65 (0,71)	-1,18 (-0,55)	0,02 (0,01)	1,26 (0,54)	0,41 (0,14)	7,99 (1,60)
Jan. 1984 – Dec. 1999	-4,17 (-1,52)	-1,49 (-0,63)	-4,10 (-2,46)	-0,30 (-0,18)	2,55 (1,72)	-2,75 (-2,00)	2,80 (1,56)	3,79 (2,08)	4,38 (2,07)	1,18 (0,39)	5,35 (1,26)
	B. Additional data, Januar 1968 – December 1999										
Market capitalisation	7,11	7,69	10,44	17,65	16,67	22,18	16,26	11,64	9,89	6,97	
Liquidity	-0,52	-0,19	-0,06	-0,04	-0,02	-0,05	-0,05	-0,05	-0,05	-0,12	
MKT Beta	1,12 (37,25)	1,09 (48,37)	1,02 (61,23)	0,96 (56,23)	0,98 (65,92)	0,99 (59,99)	1,02 (58,01)	1,01 (58,52)	1,02 (51,53)	1,09 (40,84)	-0,03 (-0,74)
SMB Beta	0,37 (8,02)	-0,00 (-0,02)	-0,13 (-5,11)	-0,16 (-6,03)	-0,09 (-4,21)	-0,15 (-6,10)	-0,11 (-4,19)	-0,00 (-0,02)	0,04 (1,20)	0,16 (4,06)	-0,20 (-3,25)
HML Beta	-0,20 (-4,04)	-0,05 (-1,31)	0,02 (0,87)	-0,02 (-0,80)	0,10 (4,22)	0,12 (4,40)	0,07 (2,60)	0,09 (3,27)	-0,01 (-0,38)	-0,15 (-3,39)	0,05 (0,76)
MOM Beta	0,04 (1,64)	-0,00 (-0,18)	0,02 (1,25)	0,01 (1,13)	-0,02 (-1,91)	-0,00 (-0,17)	-0,01 (-0,76)	0,01 (0,65)	-0,02 (-1,11)	-0,01 (-0,46)	-0,05 (-1,51)

Table A5: Historical liquidity betas, average market capitalisation, firm liquidity and the four-factor model with their respective regression coefficients and T-statistics for three periods per decile portfolio according to Li et al. (2019).

Taken from:

Li et al. (2019): Liquidity risk and asset pricing, in: Critical Finance Review, 8. Jg., p. 45.

Decile portfolios											
A. Liquidity betas											
Zeiträume	1	2	3	4	5	6	7	8	9	10	10-1
Jan. 1968 – Dec. 1999	-7,16 (-2,93)	-0,29 (-0,17)	-1,29 (-0,85)	-0,69 (-0,50)	3,31 (2,57)	-0,01 (-0,01)	1,09 (0,75)	0,63 (0,39)	3,06 (1,74)	-0,15 (-0,07)	7,01 (2,12)
Jan. 1968 – Dec. 1983	-10,11 (-2,31)	-0,32 (-0,12)	1,22 (0,48)	-1,94 (-0,82)	2,21 (0,98)	3,85 (1,55)	-3,34 (-1,45)	0,98 (0,36)	2,04 (0,74)	-0,99 (-0,32)	9,12 (1,78)
Jan. 1984 – Dec. 1999	-4,63 (-1,55)	-0,80 (-0,34)	-3,32 (-1,82)	-0,73 (-0,44)	4,97 (3,18)	-1,86 (-1,17)	3,22 (1,65)	1,22 (0,59)	5,32 (2,26)	0,77 (0,23)	5,39 (1,19)
B. Additional data, January 1968 – Dezember 1999											
Market capitalisation	8,35	8,92	17,09	21,51	16,12	24,32	17,05	14,50	9,55	8,23	
Liquidity	-0,09	-0,09	-0,09	-0,01	-0,02	-0,04	-0,05	-0,05	-0,13	-0,08	
MKT Beta	1,12 (38,24)	1,07 (51,93)	1,01 (57,14)	0,96 (59,90)	0,96 (62,82)	0,99 (63,03)	1,00 (59,08)	0,99 (52,41)	1,02 (49,90)	1,11 (40,82)	-0,01 (-0,22)
SMB Beta	0,30 (6,87)	-0,00 (0,04)	-0,16 (-6,15)	-0,13 (-5,32)	-0,11 (-4,62)	-0,15 (-6,19)	-0,13 (-5,10)	0,04 (1,50)	0,10 (3,16)	0,15 (3,67)	-0,15 (-2,57)
HML Beta	-0,19 (-4,03)	-0,04 (-1,21)	-0,01 (-0,45)	0,01 (0,43)	0,14 (5,48)	0,13 (5,15)	0,03 (0,93)	0,13 (4,34)	0,01 (0,35)	-0,09 (-2,11)	0,10 (1,54)
MOM Beta	0,01 (0,29)	-0,01 (-0,81)	0,02 (1,66)	-0,00 (-0,22)	-0,00 (-0,39)	0,00 (0,30)	-0,03 (-2,42)	0,01 (0,62)	-0,01 (-0,81)	-0,02 (-0,84)	-0,02 (-0,79)

Table A6: Replication of the liquidity risk premium via the FF three-factor model in the period January 1968 to December 1999 according to Pontiff and Singla (2019).

Taken from:

Pontiff und Singla (2019): Liquidity Risk?, in: Critical Finance Review, 8. Jg., p. 262.

Annual Alpha of a long-short strategy based on extreme liquidity portfolios		
(1)	(2)	(3)
Replication value	Original value of table 8 according to Pástor and Stambaugh	Replication value according to website data
3,91 (2,00)	4,15 (2,08)	3,91 (2,00)

Tabelle A7: Alpha values and T-statistics of the decile portfolios via the CAPM, FF three-factor model and Carhart four-factor model for different time periods according to Li et al. (2019).

Taken from:

Li et al. (2019): Liquidity risk and asset pricing, in: Critical Finance Review, 8. Jg., p. 47.

	Decile portfolios										
	A. January 1968 – December 1999										
	1	2	3	4	5	6	7	8	9	10	10-1
CAPM Alpha	-1,62	-1,03	0,22	0,83	0,29	0,09	0,46	1,96	0,22	2,47	4,09
	(-1,06)	(-1,04)	(0,24)	(1,03)	(0,37)	(0,11)	(0,55)	(2,12)	(0,22)	(1,83)	(2,12)
Fama-French Alpha	-0,34	-0,82	0,29	0,71	-0,56	-0,72	0,20	1,22	0,16	3,03	3,37
	(-0,24)	(-0,80)	(0,34)	(0,91)	(-0,75)	(-0,94)	(0,24)	(1,32)	(0,16)	(2,25)	(1,73)
Four factor Alpha	-0,48	-0,64	-0,06	0,75	-0,50	-0,78	0,68	1,08	0,36	3,27	3,75
	(-0,32)	(-0,61)	(-0,06)	(0,93)	(-0,64)	(-0,98)	(0,80)	(1,14)	(0,35)	(2,37)	(1,87)
	B. January 1968 – December 1983										
CAPM Alpha	-1,40	0,13	0,86	0,48	-0,23	0,03	-0,09	1,90	0,22	2,73	4,13
	(-0,61)	(0,09)	(0,64)	(0,41)	(-0,20)	(0,03)	(-0,08)	(1,44)	(0,16)	(1,64)	(1,59)
Fama-French Alpha	-1,18	1,04	2,68	1,06	-1,19	-0,48	0,00	0,70	-0,70	1,72	2,90
	(-0,53)	(0,77)	(2,16)	(0,90)	(-1,04)	(-0,38)	(0,00)	(0,53)	(-0,51)	(1,09)	(1,09)
Four factor Alpha	-2,75	0,27	1,62	0,87	-0,76	-0,03	0,88	0,78	0,12	1,67	4,41
	(-1,20)	(0,19)	(1,28)	(0,71)	(-0,64)	(-0,02)	(0,73)	(0,57)	(0,09)	(1,01)	(1,59)
	C. January 1984 – December 1999										
CAPM Alpha	-1,05	-1,02	-0,16	0,70	0,46	-0,07	1,41	1,42	0,36	2,57	3,62
	(-0,48)	(-0,68)	(-0,12)	(0,62)	(0,40)	(-0,07)	(1,06)	(1,04)	(0,23)	(1,13)	(1,18)
Fama-French Alpha	0,78	-1,45	-1,08	0,07	-0,48	-0,88	0,70	1,25	0,92	3,55	2,77
	(0,39)	(-0,96)	(-0,88)	(0,06)	(-0,46)	(-0,93)	(0,55)	(0,91)	(0,58)	(1,59)	(0,92)
Four factor Alpha	1,34	-0,93	-1,02	0,20	-0,60	-1,13	0,88	1,04	0,79	3,81	2,47
	(0,66)	(-0,62)	(-0,82)	(0,18)	(-0,57)	(-1,18)	(0,68)	(0,75)	(0,50)	(1,69)	(0,81)

Figure A8: Time series of total market liquidity for the period August 1962 to December 2020 according to PS (2003).

Own illustration based on:

Pástor und Stambaugh (2003): Liquidity risk and expected stock returns, in: Journal of Political economy, 111. Jg., p. 651.

Based on data from:

Website of the author **Robert F. Stambaugh**: <URL: http://finance.wharton.upenn.edu/~stambaug/> (accessed 11 September 2021) and **Ľuboš Pástor**: <URL: https://faculty.chicagobooth.edu/lubos-pastor/data> (accessed 11 September 2021).

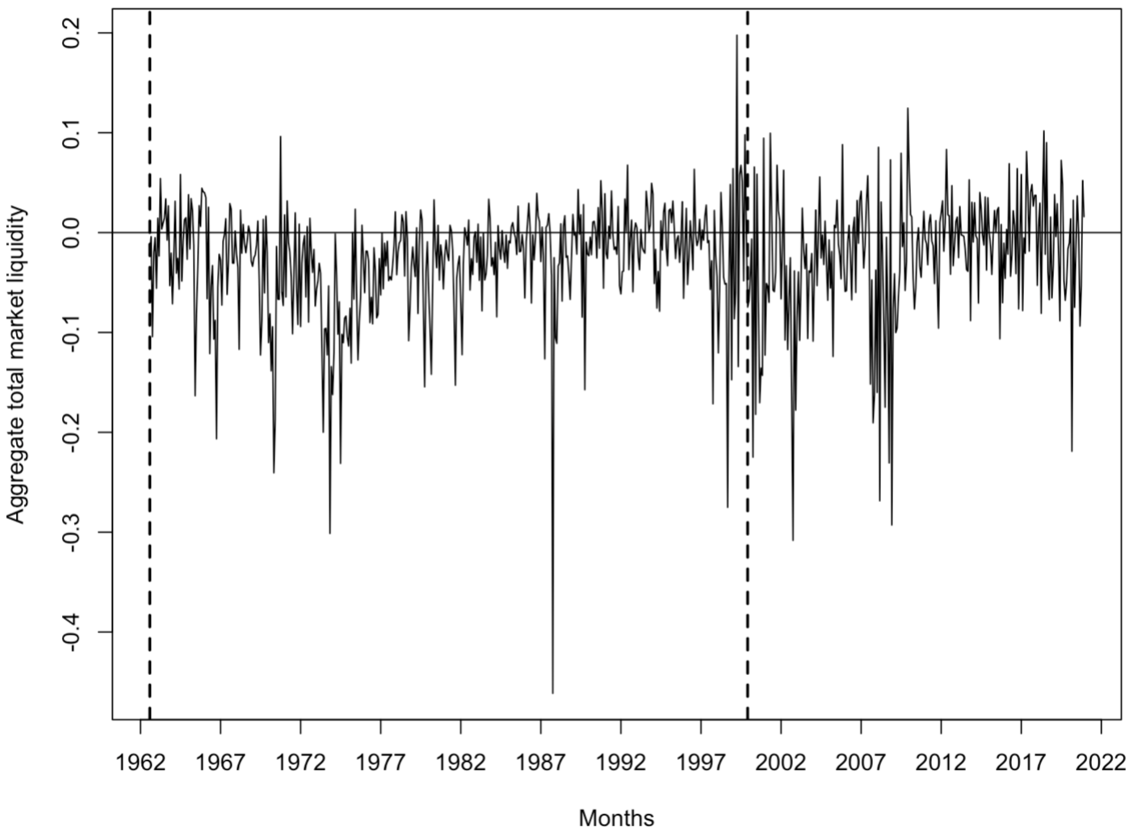

Figure A9: Time series of the change in total market liquidity for the period August 1962 to December 2020 according to PS (2003).

Own illustration based on:

Pástor und Stambaugh (2003): Liquidity risk and expected stock returns, in: Journal of Political economy, 111. Jg., p. 651.

Based on data from:

Website of the author **Robert F. Stambaugh**: <URL: http://finance.wharton.up-enn.edu/~stambaug/> (accessed 11 September 2021) and **Ľuboš Pástor**: <URL: https://faculty.chicagobooth.edu/lubos-pastor/data> (accessed 11 September 2021).

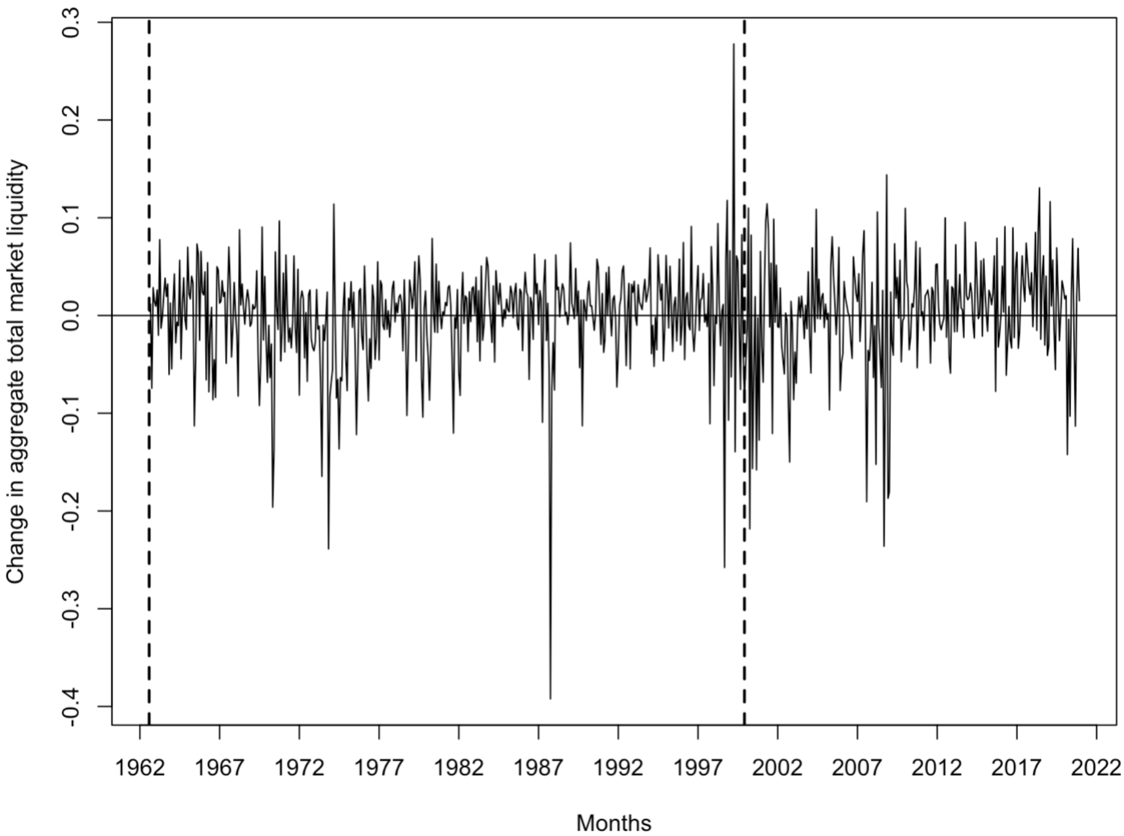

Figure A10: Total market liquidity for the period from 1920 to 2015 according to Pontiff and Singla (2019).

Taken from:

Pontiff und Singla (2019): Liquidity risk?, in: Critical Finance Review, 8. Jg., p. 263.

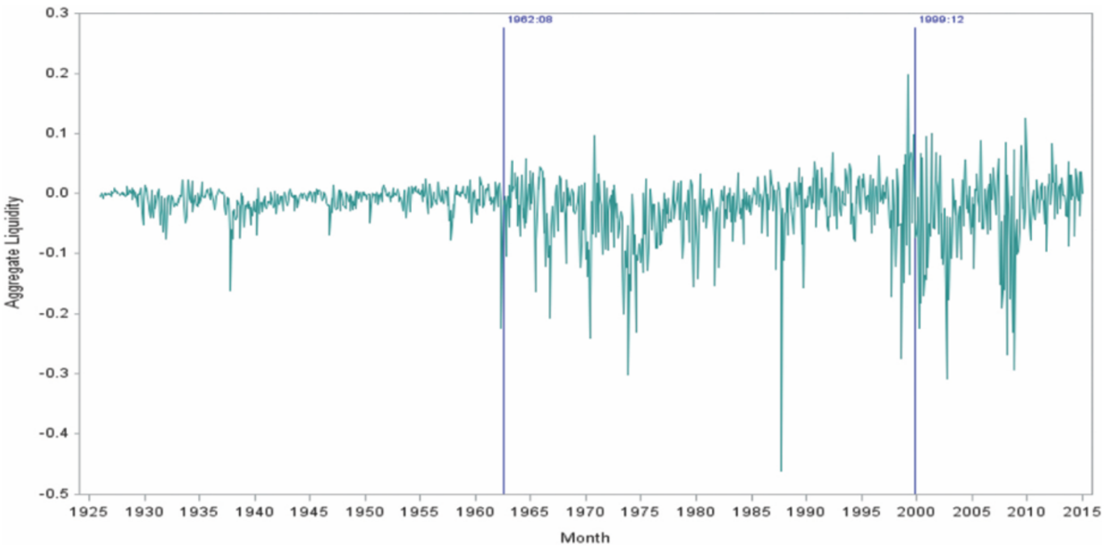

Figure A11: Changes in total market liquidity for the period January 1962 to December 2018 from Li et al. (2019).

Taken from:

Li et al. (2019): Liquidity risk and asset pricing, in: Critical Finance Review, 8. Jg., p. 7.

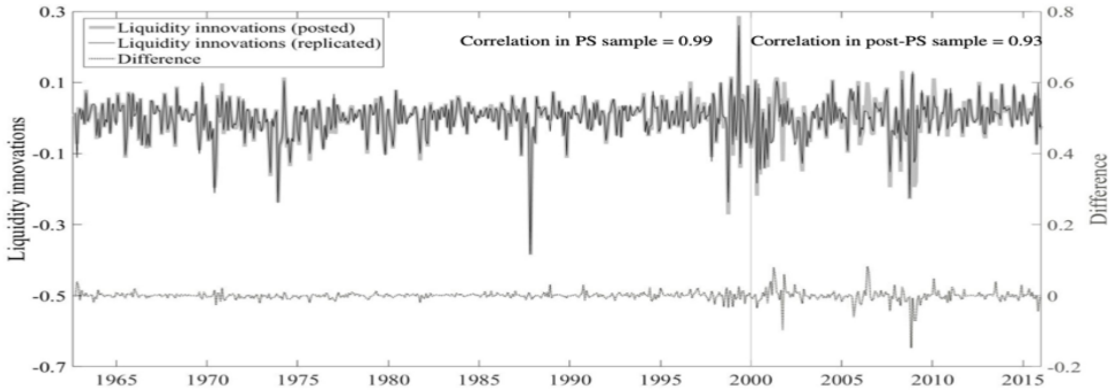

Table A12: Average monthly alpha values across the FF three-factor and five-factor models and the Carhart four-factor model with annual portfolio shifts in December and January and on a monthly basis for three comparison periods.

Taken from:

Li et al. (2019): Liquidity risk and asset pricing, in: Critical Finance Review, 8. Jg., p. 16.

	r^e	α_{FF3}	α_{FF4}	α_{FF5}
Panel A: Liquidity factor according to website data				
PS period	0,28	0,26	0,37	0,20
	(1,73)	(1,60)	(2,14)	(1,16)
Post-PS period	0,69	0,69	0,68	0,53
	(2,38)	(2,38)	(2,31)	(1,76)
Entire period	0,42	0,44	0,46	0,44
	(2,87)	(2,97)	(3,02)	(2,85)
Panel B: Liquidity factor according to website data (annual December rebalancing)				
PS period	0,30	0,28	0,37	0,20
	(1,90)	(1,73)	(2,22)	(1,19)
Post-PS period	0,70	0,72	0,66	0,53
	(2,35)	(2,38)	(2,21)	(1,71)
Entire period	0,44	0,45	0,42	0,43
	(2,99)	(3,04)	(2,78)	(2,82)
Panel C: Liquidity factor according to website data (annual January rebalancing)				
PS period	0,06	0,05	0,23	0,10
	(0,34)	(0,30)	(1,33)	(0,55)
Post-PS period	0,57	0,55	0,49	0,39
	(1,97)	(1,88)	(1,68)	(1,29)
Entire period	0,23	0,23	0,23	0,27
	(1,54)	(1,52)	(1,52)	(1,77)
Panel D: Liquidity factor according to website data (monthly rebalancing)				
PS period	0,18	0,20	0,42	0,19
	(1,02)	(1,10)	(2,30)	(1,03)
Post-PS period	0,75	0,76	0,73	0,49
	(2,55)	(2,56)	(2,45)	(1,61)
Entire period	0,37	0,38	0,45	0,36
	(2,40)	(2,42)	(2,84)	(2,24)

Table A13: Alpha values and T-statistics of the decile portfolios and the CAPM, the Carhart four-factor model, and the FF five-factor model for the period January 1968 to December 2015.

Taken from:

Li et al. (2019): Liquidity risk and asset pricing, in: Critical Finance Review, 8. Jg., p. 18.

$n^{Portfolio}$	Name breaks	NYSE breaks	Market capitalisation breaks
Panel A: T-statistics of average excess returns			
2	2,08	2,07	1,85
3	2,02	1,88	2,06
5	1,94	1,51	1,54
10	2,99	2,63	1,87
20	0,08	1,33	1,25
Panel B: T-statistics of alpha values vs. the Fama and French (1993) three-factor model			
2	1,77	1,76	1,54
3	1,73	1,55	1,82
5	1,94	1,48	1,53
10	3,04	2,68	1,81
20	0,08	1,40	1,27
Panel C: T-statistics of alpha values versus the Carhart (1997) four-factor model.			
2	1,95	1,99	1,82
3	2,14	1,99	2,26
5	2,19	1,74	1,77
10	2,78	2,64	1,78
20	0,34	1,57	1,22
Panel D: T-statistics of alpha values versus the Fama and French (2015) five-factor model			
2	1,93	1,88	1,68
3	2,19	2,13	2,09
5	2,33	2,03	1,85
10	2,82	2,55	1,63
20	-0,09	1,29	0,68

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Affidavit

"I declare in lieu of oath by my signature that I have written the above thesis independently and without outside help and that I have marked all passages that I have taken verbatim or approximately verbatim from publications as such, and that I have not used any literature or other aids other than those indicated. The thesis has not been submitted in this or a similar form to any other examination authority."

Hünxe, den 23. September 2021

Place, date Signature (Robert Hennings)