Measurement of liquidity risk in the context of market risk calculation

by

Jun Muranaga and Makoto Ohsawa*

Institute for Monetary and Economic Studies

Bank of Japan

Abstract

This paper aims at shedding light on liquidity risk, which has been left behind in the pursuit of more sophisticated market risk measurements both by market practitioners and by central banks. We first define liquidity risk and show that it can be divided into execution cost and opportunity cost. In the light of stylized facts regarding tick-by-tick dynamics of market liquidity and price/spread movements, which have been documented previously by finance literature, we propose several modified market risk measures reflecting intraday liquidity patterns and price movements. We then demonstrate, by applying those risk measures to the Japanese equity market, to what extent the quantified liquidity effects could effect conventional measurement of market risk represented by VaR.

* The views expressed in the paper are those of the authors and do not necessarily reflect those of the Bank of Japan, the Institute for Monetary and Economic Studies or the Euro-currency Standing Committee. Although the paper benefited from comments by central bank economists participating in the research project on the aggregation of market risk, the authors are solely responsible for any remaining errors.

I. Introduction

The quantification of market risk has been a major issue for financial institutions as well as central banks over the past few years. Considerable technical efforts have been made to measure market risk as accurately as possible.[1]

One of the areas which has attracted a great deal of attention is how one can appropriately make assumptions about future movements in market risk factors. One direction of such research is studying the characteristics of ***daily price changes*** more carefully. Mori, Ohsawa, and Shimizu (1996), for instance, proposed techniques for explicitly taking account of fat-tail distributions of market prices and correlation breakdown between various risk factors to see to what extent these characteristics of market prices affect value-at-risk (VaR) calculations. Another type of research [e.g. Fallon (1996) and Alexander (1996)] evaluates the effectiveness of various types of volatility models such as the moving average method, GARCH, and implied volatility, for improving the accuracy of the VaR model.

Another potential direction of research on future price movements used as an input for market risk calculation, which we follow in this paper, is to analyze ***intraday price changes*** more carefully, since the prices at which one can trade are not necessarily end-of-day trade prices or mid-prices – as assumed in the ordinary framework for market risk calculation, but, as a matter of fact, reflect bid or ask prices which fluctuate during the day. As quite a few pieces of academic literature after Black Monday documented, the key to understanding second-by-second price changes is the investigation of a relationship between trading activity and price changes. In other words this is the price impact of trading activity, which can be labeled "liquidation or liquidity risk".

Although it is generally recognized among market practitioners that liquidity risk is a very serious concern for firms, especially in a stressful market situation, quantitative techniques measuring liquidity risk both in normal and stressful situations appear to be very premature.[2] In marking to market positions, the current standard practice is to use mid-rates (or the latest trade prices, which tend to be determined somewhere within bid-ask spreads); a more realistic and conservative method, i.e. using bid and ask prices for long and short positions respectively, does not in practice prevail. Even if bid/ask prices are used, the decision regarding which bid/ask prices should be used is usually left to individual traders and is not centrally monitored and controlled by middle offices. With regard to the quantification of liquidity risk in the context of daily risk management, the effort to incorporate the liquidity effect into market risk and a stress test is still at an embryonic stage.

---

[1] Two recent additions to the literature by private and central bank communities, which provide a comprehensive review of the measurement of market risk, are Wilson (1996) and Session 4 of Part II in Board of Governors of the Federal Reserve System ed. (1996), respectively.

[2] The following description is based on informal interviews with several leading Japanese and U.S. trading houses conducted in the summer of 1996.

Recently, however, we observe some advancement in this direction. In order to take account implicitly of the abrupt leaps in observed market prices which often occur in less liquid markets (e.g. emerging markets), a few firms use worst-case scenarios in which worst historical price movements over relatively longer periods such as 5-10 days are assumed.[3] Some firms are considering the adjustment of holding periods of their positions in value-at-risk calculations based on the liquidities of individual products. Certain firms regularly check the difference between the necessary liquidation time for less liquid positions forecast by traders and the actual time needed to close the positions. The more complex part of measuring liquidity risk is how to measure the impact of trading volume on price. Currently, no firms appear to take this effect into account in daily risk management. However, the prevailing use of computer-driven asset management and the growing interest in evaluating the performance of pension funds more accurately tend to increase the need to quantify the price impact.

The objective of this paper is to propose several measures for quantifying liquidity risk, based on intraday price and trading data, which can be of some value for more accurate market risk quantification. First, Section 2 of this paper defines the scope of liquidity risk in our discussion, making clear distinction between execution and opportunity cost components of liquidity risk. Section 3 surveys stylized facts regarding intraday price and bid-ask spread movements and the price impact of trading activity, which are documented in finance literature. In light of the stylized facts Section 4 proposes several different modified market risk measures reflecting intraday liquidity patterns and price movements and demonstrates to what extent the quantified liquidity effects could affect measurement of market risk – represented by VaR – for the Japanese equity market. Section 5 discusses areas for future research.

II. Definition of liquidity risk

Liquidity risk in this paper is defined as the risk of being unable to liquidate a position in a timely manner at a reasonable price.[4] Theoretically, liquidity risk in this sense can be divided into the variability of ***execution cost*** (the cost of immediacy) and that of ***opportunity cost*** (the cost of waiting). As Chart 1 shows, the execution costs, comprising (1) "bid-ask spreads" representing transaction cost and (2) the "price impact of trading activity" (hereafter called "market impact"), both of which are sometimes not easily separable from each other, decrease with the time needed to

---

[3] The difficulty deriving historical volatility in illiquid markets lies in the fact that effective quotations of the bid/ask prices at which one can actually trade disappear and cannot be observed for quite a long period (which sometimes lasts for several days). If we calculate daily historical volatility by extrapolating effective quotes from the data obtained before the trade halt, it is highly likely that we will end up with an underestimation of underlying market volatility.

[4] One might imagine ***funding risk*** from the term "liquidity risk". As a matter of fact, funding risk could trigger a rise in market rates, in particular under the assumption of there being no lender of last resort, which could lead to losses in trading positions. However, the scope of our analysis in this paper does not include the measurement of funding risk.

complete an intended transaction. In contrast, opportunity cost, i.e. the cost of being forced to postpone trading, tends to increase with execution time. What traders need to do is to strike a balance between these two costs so as to minimize liquidity risk. Chart 1 shows that the sum of both costs is usually regarded as liquidity risk, which traders implicitly try to minimize in their daily trading activity.

As a number of pieces of market microstructure literature show [e.g. Glosten and Harris (1988)], the bid-ask spread, which is an important component of the execution costs faced by investors, is divided into (1) order-processing cost and (2) adverse selection cost. The adverse selection component exists because a market maker may trade with investors who possess superior information. This component is believed to represent the market maker's profits from uninformed traders who compensate him for the expected losses to informed traders [Glosten and Milgrom (1985)]. An empirical study by George, Kaul, and Nimalendran (1991) using daily and weekly data of both AMEX/NYSE and NASDAQ stocks found out that the predominant component of quoted spread is order-processing cost, while the adverse selection component comprises only 8-13% of quoted spreads.

With regard to the opportunity cost component of liquidity risk, the only theoretical effort worth noting here seems to be the work by Longstaff (1995). Since liquidity premium is basically determined by price changes during the period of restricted trade, the maximum value of the liquidity premium is derived in his model as a premium for a lookback option, the underlying asset price path of which is determined by the optimal price path calculated under conditions of perfect foresight of the future price path and market timing, and the strike price of which is determined by a Black-Scholes stochastic process. In practice, however, it is not at all easy to take explicit account of the variability of opportunity cost in market risk calculation. As mentioned in the Introduction to this paper, certain firms consider the arbitrary adjustment of holding periods of particular positions in order to reflect the opportunity cost component of liquidity risk, though there exist no objective and scientific ways to determine what is the appropriate holding period. Hence in the following analysis we focus mainly on the execution cost element of liquidity risk. This implies, however, that, as Chart 1 shows schematically, our measurement of liquidity risk tends to be understated, especially in a stressful situation in which shortage of liquidity prevents virtually all transactions for quite a long time period.

III. Stylized facts regarding liquidity effects documented by previous academic literature

This section reviews stylized facts regarding the relationship between trading activity (market liquidity) and bid/ask price changes as well as spread changes in different markets and time periods as well as theoretical explanations for them, based on previously published academic literature. Here we do not intend to conduct a comprehensive survey but rather focus on stylized facts which should be kept in mind in measuring liquidity risk in the context of market risk calculation in the ensuing chapter.

1. Volatility of intraday price movements

A first well-known fact documented by Amihud and Mendelson (1987) with regard to NYSE stocks' intraday price movement is that ***open-to-open return volatility is higher than close-to-close return volatility*** in the markets applying the call-clearing procedure at the opening session. There seem to be two streams of theoretical explanation about what causes this phenomenon. Stoll and Whaley (1990) argue that the wider bid-ask spreads caused by specialists using their monopoly position at the opening call makes prices more volatile, since transaction prices tend to bounce between bid and ask prices. In contrast Lee and Lin (1995) postulate that the specialist encourages trading by putting a smaller cost on immediacy in order to reveal more private information – accumulated during the overnight trading halt, which reduces the adverse-selection problem specialists face and makes subsequent trades in the continuous market more profitable. In any case this could have important implications for market risk management, since almost all the firms use closing price data in their firm-wide market risk calculation.

Another stylized fact documented also by Amihud and Mendelson (1987) is the ***greater deviation of opening returns from the normal distribution – with fatter tails – than closing returns***, which could also be worth noting in quantifying market risk.

2. Determinants of bid-ask spreads

A rather trivial fact about bid-ask spreads is that ***infrequently traded stocks are characterized by large bid-ask spreads***. There are several conjectured explanations for these large spreads.[5] The first explanation involves inventory and liquidity effects. A second reason is monopolistic market power exercised by a single market maker providing liquidity for inactive stocks. A third explanation is that the large spreads arise as the natural consequence of the greater risk of informed trading in illiquid stocks which market makers incur.

Another stylized fact about bid-ask spreads is ***the turn-of-the-year seasonal movement of bidask spreads***: A significant decline in the spreads of NYSE stocks from the end of December to the end of the following January, which is believed to cause excess January returns – since this tendency is pronounced especially for small or low-priced stocks, this seasonal anomaly is sometimes labeled "size-related anomaly". For instance, Clark, McConnell and Singh (1992) detects such a decline at the turn of the year by observing 1982-1987 NYSE stock price data. Ritter and Chopra (1989) attribute the January excess returns to portfolio rebalancing strategy – i.e. investment in riskier stocks – taken

---

5 The following summary is from Easley, Kiefer, O'Hara, and Paperman (1996).

by those who sold losers at year end for various reasons ranging from tax-loss selling, portfolio window dressing and parking-the-proceeds.

In the Japanese stock market, which has two sessions a day, bid-ask spreads exhibit a ***W-shape pattern***, where they peak at the opening, just before and after lunch time, and during the closing session. This is in parallel with the W-shape of transaction volume and volatility movement during a day. This contrasts with U-shape movement of U.S. stock market, where there is only one session in a day. What Bollerslev and Melvin (1994) found for foreign exchange markets is that ***the size of bid-ask spreads in the foreign exchange market (DM/$ rates) is positively related to the underlying exchange rate (conditional) volatility***, by using ordered probit regression to cope with discreteness in the spreads data.

In contrast, however, Bollerslev and Domowitz (1993) suggest that ***quotation activity of foreign exchange does not influence bid-ask spread changes – seemingly contradicting the fact derived from stock markets*** – while spreads have a positive effect on return volatility. Glassman (1987) concluded that trading volume in foreign exchange markets is rather negatively correlated with bid-ask spreads. In fact, spreads widen a significant amount prior to weekends and holidays. Locke and Sarkar (1996) also show that bid-ask spreads at several futures markets do not increase even on higher volatility days. These results might suggest that there is a certain difference in the readiness of liquidity supply between the stock and forex/futures market.

3. Market impact

The stylized facts which are most important for dynamic liquidity analysis involve the characteristics of ***market impact,*** defined as the slope (sensitivity) of the bid or ask price schedule (hereafter called λ) to trading volume or order flow. The following features of λ for U.S. stock markets are identified by the numerous additions to the empirical literature which emerged after Black Monday [see a survey by Hebner (1996)]:

a. $\lambda > 0$, which is supported almost unanimously.

b. λ is concave in transaction size (Effects of stealth trading and more intensive brokerage search for larger transaction). However, if the ratio of informed/uninformed trading increases with transaction size, λ should be convex in transaction size.

c. λ is lower for more active securities.

d. λ is asymmetrical between buyer – and seller – initiated transactions. However, which of the two is larger varies from study to study.

Watanabe (1996), who analyzes intraday data of Japanese Government Bond Futures, finds that a significant causality from volume to volatility exists, while many other studies in this area either focus on contemporaneous relationships or find no strong causality.

IV. Modified market risk measures reflecting intraday liquidity pattern and price movements

Keeping the aforementioned stylized facts with respect to intraday price movement and trading activity in mind, we now propose several liquidity risk measures which could be useful for achieving more accurate market risk quantification. The market to which we will apply the proposed risk measures is the Tokyo Stock Exchange (TSE), tick-by-tick trade data of which have kindly been provided by Nikko Securities for the purpose of this research.[6] Although we have to confine ourselves to focusing on the Japanese equity market because of the data availability constraint, the following measures are, in principle, applicable to other markets in other countries. Simulations based on a hypothetical portfolio of stocks listed in the TSE will demonstrate to what extent those quantified liquidity risks could affect measurement of market risk obtained in the form of VaR.

The first modified measure for market risk we propose takes into account intraday tick-by-tick price movement which reflects intraday liquidity patterns. As a first attempt in this direction we will check whether opening sessions of the TSE, where actual trading is at its most active and significant quantities of trades of less liquid equities take place, also exhibit higher volatility and kurtosis, as evidenced for the NYSE. Second, in order to relate intraday price fluctuations to the intraday trading pattern more precisely and quantify "execution timing risk" during the day, we introduce the notion of "trade volume-weighted average prices (VWAP)" and construct a market risk measure reflecting the daily volatility of VWAP and intraday histograms of actual trade prices which clearly display the risk of deviating from average trade execution performance during a single day. The second modification we will propose in obtaining a more accurate measure for market risk is the explicit incorporation of intraday variability of bid-ask spreads into the price risk calculation. Finally we propose a market liquidity measure based on λ,which represents the market impact of trading activity, and the historical distribution of which could potentially be used for augmenting market risk calculation.

1. Modified market risk measure reflecting intraday price movement

Since many less liquid stocks on the TSE are traded mostly in the opening clearing process, it is important to take a close look at the volatility and distribution of opening prices in order to measure market risk more accurately. Using the daily data, we will check whether a stylized fact of higher volatility and kurtosis of opening sessions than closing sessions is also applicable to the Japanese equity market. Since there are two (morning and afternoon) sessions a day in the TSE, we compute two opening and closing volatility indices, respectively. The result presented in Chart 2 shows, in contrast with our conjecture, that there is no significant difference in volatility and kurtosis on average between opening sessions and closing sessions in the TSE, although 237 and 230 stocks

---

[6] The data are on quoted bid and ask prices as well as trade prices and volumes of over 1200 stocks listed in the first section of the TSE and the data observation period ranges from 2nd October 1995 to 30th September 1996.

out of 500 samples exhibit higher open-to-open volatility and kurtosis, respectively (the average open-to-open volatility and kurtosis of these equities are higher by 5.9% and 29.4%). This result clearly contrasts with the stylized fact observed in the NYSE. This could be partly because the TSE's closing sessions follow the exactly same call-clearing procedure as in the morning sessions, while in the case of the NYSE the closing sessions are characterized by "market-on-close (MOC) order"[7] which tends to prevent prices from moving by wider margins. In the next part of this section, therefore, we will take a more careful look at the tick-by-tick intraday price movement of the TSE-listed equities in order to reflect liquidity effects on market risk measurement.

Transaction prices move widely during a day, as shown in Chart 3 which shows a histogram of trade prices of a particular stock in our sample. The execution timing risk during one day can be defined as the risk of deviating from the average transaction cost for the day and ending up by trading at unfavorable prices compared with other participants in the market. Although the execution timing cost could arise simply from an inappropriate trading strategy, liquidity constraint prevalent in the market could also force traders to execute transactions with bad timing, particularly in stressful situations. In this sense the execution timing risk can be interpreted as also including opportunity cost.

Judging from Chart 3, the distribution of daily trading prices which are accumulated over the observation time period and standardized by daily volume-weighted average price (VWAP) can be roughly assumed to be a normal distribution. Assuming that the daily movement of VWAP follows a lognormal stochastic process, we now conduct a two-step Monte Carlo simulation (100,000 times) based on two normal distributions of daily VWAP changes and daily trade prices.[8] The

---

7 A market-on-close order is guaranteed execution at the closing price according to prescribed pricing and order entry procedures. When there is an imbalance of MOC orders, the imbalance is executed against the prevailing bid or offer on the Exchange at the close of trading, thus setting the closing price. An excess of buy orders is executed against the offer and an excess of sell orders is executed against the bid. The remaining buy and sell MOC orders are then paired off at the price at which the imbalance was executed. When the aggregate size of the buy MOC orders equal the aggregate size of the sell MOC orders, the buy and sell orders are paired off at the price of the previous NYSE trade. The result of these pricing procedures is that all executed MOC orders receive the same closing price.

8 This simulation process can be mathematically expressed as follows:

$$P_{ex} = P^0_{VWAP} \exp\left(\sigma_{VWAP}\,\varepsilon_a \sqrt{t}\right) + \sigma_H\,\varepsilon_b$$

P_{ex} : expected execution price at the end of the holding period

P^0_{VWAP} : VWAP on the risk evaluation day (Sep 1996)

σ_{VWAP} : historical volatility of VWAP over the observation period

σ_H : SE of distribution of daily trade prices accumulated over the observation period (Oct. 1995-Sept. 1996) and standardized by each-day VWAP

t : holding period (one day in this simulation)

$\varepsilon_a , \varepsilon_b$: standard normal random numbers simulation result (Chart 4) based on a hypothetical portfolio comprising of five stocks chosen randomly from the sample (basic statistics of the portfolio are summarized in Chart 5) shows that VaR taking into consideration execution timing risk is significantly higher than both VWAP-based VaR and ordinary VaR using end-of-day mid-prices.

2. Modified market risk measure taking account of intraday variability of bid-ask spreads

As past market microstructure literature indicates, the bid-ask spreads of Japanese equities fluctuate widely even during a single day (Chart 6). By analogy with the simulation with respect to execution timing risk described above, we quantify the risk involved in the intraday variability of bid-ask spreads, by using the same portfolio introduced in the preceding section. We again assume a lognormal stochastic process for end-of-day mid-prices. Ask and bid prices are defined as mid-price plus and minus ($\frac{1}{2}$*bid-ask spreads) respectively, where the probability density function of bid-ask spreads is based on a historical (non-parametric) simulation using the data covering the one-year observation period.[9] The result of the simulation for bid prices is presented in Chart 7. First, we note that the expected value of the distribution is lower than that of mid-prices. Moreover, the modified market risk measure represented by 99 percentile VaR turns out to be higher than ordinary VaR, which underscores the importance of bid-ask spreads in market risk calculation.[10]

---

[9] The simulation process can be formally described as follows:

$$P_{bid} = P_m^0 \exp\left(\sigma_m \varepsilon \sqrt{t}\right) - \frac{1}{2} f(u)$$

$$P_{ask} = P_m^0 \exp\left(\sigma_m \varepsilon \sqrt{t}\right) + \frac{1}{2} f(u)$$

P_{bid}	: bid price at the end of the holding period
P_{ask}	: ask price at the end of the holding period
P_m^0	: end-of-day mid-price on the risk evaluation day
t	: holding period (one day in this simulation)
σ_m	: volatility of mid-prices
$f(\bullet)$	: probability density function of bid-ask spreads
ε	: standard normal random numbers
u	: uniform random numbers

[10] In conducting the historical simulation for bid-ask spreads we arbitrarily excluded equities, the spread distribution of which looks like Chart 8. The tail events in this case simply reflect the fact that either quoted bid or ask price disappear from the market. Although it is desirable for these illiquid market conditions to be investigated more carefully and incorporated into liquidity risk calculation, we do not deal with the issue in this paper. This might contribute to a spuriously small difference between ordinary and modified VaR.

3. Risk in λ for securities with different liquidity

As we saw in the preceding chapter, the market impact often defined by λ (sensitivity of bid or ask prices to trading volume) has attracted greater attention in market microstructure literature. While most of the previous literature focuses on characteristics of λ in a deterministic sense as described in Chapter 3, we try to proceed one step further and investigate the statistical distribution of λ in order to determine the implications of market impact for market risk calculation.

First we construct λ as a ratio of price impact to adjacent trading volume standardized by normal market size. Normal market size is approximated by a daily average of trading volumes per transaction, which is assumed to be constant during one day.[11] Price impact is measured by a change from before-trade quoted bid or ask price to after-trade quoted bid or ask price. If there is a difference between a change in ask price and that in bid price, we take the larger price change – if bid (ask) price change is larger, then we can interpret it as seller (buyer)-initiated market impact.[12] Charts 9 and 10 show that expected value of λ and 90 percentile point of λ distribution of are negatively correlated with the level of liquidity for securities. In obtaining λ we pool both seller-initiated and buyer-initiated transaction data, though there might be an asymmetry between two transactions, as documented by previous studies.

Chart 11 clearly demonstrates that market impact represented by λ cannot be neglected in market risk calculation. Hence, we conduct VaR simulation to show how λ affects ordinary market risk measurement. We again assume a lognormal stochastic process for mid-prices. In quantifying market impact, we additionally assume that our trader having the same hypothetical portfolio used for the previous simulations liquidates his position not all at once but only gradually. In this simulation he divides his position into a piece of transactions, each of which is assumed to be equal to the average trading amount per transaction of the evaluation date. This assumption means that a trader has to execute at least a few transactions before he liquidates his entire positions, each transaction of

[11] Normal market size might be better represented by time-varying orders in the market, which are not available in our data set.

[12] A conventional method of classifying trades compares the trading price to the quote prices in effect at the time of the trade. However, it should be pointed out that there are several shortcomings in this method. Another method relies solely on trading prices, avoiding data quality problems with quote prices. Lee and Ready (1991) provide a succinct review of this issue.

which causes downward price pressure. The simulation result is shown in Chart 12.[13] It is clear that the market risk measurement taking account of λ ("λ-augmented VaR") is much larger than the ordinary VaR. It is also notable that the distribution of the λ-augmented VaR is of fat-tail feature which gives rise to unproportionately large 99-percentile risk compared with 95-percentile risk. Another characteristic of this market risk measurement is that as the initial holding position increases in size, the respective risk amounts increase in an unproportionate fashion. For instance, if we double the initial position, the risk amount almost triples.

However, we should interpret this simulation result with caution. First, our λ is derived under the assumption of deterministic normal market size, though, as a matter of fact, normal market size seems to follow a definite stochastic process which should be incorporated into risk measurement. In stressful situations, in particular, outstanding orders (normal market size) could easily diminish, resulting in higher market impact. Secondly, although our λ is certainly a departure from a static risk calculation in the sense that it takes account of first-round price impacts of individual trades, it does not capture second-round price effects which would, for instance, arise from the herding behavior of other participants. If aggregated sensitivity data for the market as a whole is available, one possible way of capturing this kind of second-round effect is first to estimate future trading patterns from information on aggregated market sensitivities under certain scenarios about future price movements, the procedure of which is exemplified by Shimizu (1997), and then derive

---

13 The simulation process can be mathematically expressed as follows:

$$P^i_{bid} = P^{i-1}_{bid} \cdot \exp\left(-\frac{V^i g(u_2)}{NMS}\right) \quad \text{where} \quad P^0_{bid} = P^0_m \exp\left(\sigma_m \varepsilon\sqrt{t}\right) - \frac{1}{2}E[f(u_1)]$$

$$P^i_{ask} = P^{i-1}_{ask} \cdot \exp\left(\frac{V^i g(u_2)}{NMS}\right) \quad \text{where} \quad P^0_{ask} = P^0_m \exp\left(\sigma_m \varepsilon\sqrt{t}\right) + \frac{1}{2}E[f(u_1)]$$

P^i_{bid} : after-trade quoted bid price

P^i_{ask} : after-trade quoted ask price

P^0_m : end-of-day mid-price on the risk evaluation day

t : holding period (one day in this simulation)

σ_m : volatility of mid-prices

$f(\bullet)$: probability density function of bid-ask spreads

$g(\bullet)$: probability density function of λ

ε : standard normal random numbers

u_1, u_2 : uniform random numbers

V^i : trading volume per transaction

NMS : normal market size (a daily average of trading volumes per transaction assumed constant in this simulation).

the price impact by using λ. Although this issue is worth pursuing, it is beyond the scope of this paper.

V. Summary and areas for future research

This paper aims to shed light on liquidity risk, which has been left behind in the pursuit of more sophisticated market risk measurements both by market practitioners and by central banks. We first defined liquidity risk and showed that it can be divided into execution cost and opportunity cost. In the light of stylized facts regarding dynamics of liquidity and price/spread movements, which have been documented previously by finance literature, we proposed several modified market risk measures reflecting intraday liquidity patterns and price movements. We then demonstrated, by applying the measures to the Japanese equity market, to what extent the quantified liquidity effects affect conventional measurement of market risk represented by VaR.

Although our proposed market risk measures clearly reflect liquidity risk, we did not analyze the time-varying properties of market liquidity and the dynamic interaction between trading activity and price/spread movements. A possible next step would be to investigate the dynamics of market impact by applying time-series modeling. Furthermore, the market impact of trading on prices might not be one-time, as implicitly assumed in our measurement, but could trigger second-round effects, which might need to be analyzed in an experimental simulation as formulated by Shimizu (1997). Since the driving forces behind tick-by-tick price movements are not only trading activity but also the arrival of new information, identification of these two factors would be the key to understanding intraday price movements in a dynamic context.

References

Alexander, C. (1996): "Volatility and Correlation Forecasting", in C. Alexander, ed., *The Handbook of Risk Management and Analysis*, West Sussex: John Wiley.

Bollerslev, T. and I. Domowitz (1993): "Trading Patterns and Prices in the Interbank Foreign Exchange Market", *Journal of Finance*, 4, p.1421-1443.

Bollerslev, T. and M. Melvin (1994): Bid-ask Spreads and Volatility in the Foreign Exchange Market", *Journal of International Economics*, 36, p.355-372.

Board of Governors of the Federal Reserve System, ed. (1996): *Risk Measurement and Systemic Risk - Proceedings of a Joint Central Bank Research Conference, November 1995*, Washington, D.C.

Clark, R.A., J.J. McConnell and M. Singh (1992): "Seasonalities in NYSE Bid-Ask Spreads and Stock Returns in January", Journal of Finance, 5, p.1999-2014.

Easley, D., N.M. Kiefer, M. O'Hara and J.B. Paperman: "Liquidity, Information, and Infrequently Traded Stocks", Journal of Finance, 4, p.1405-1436.

Fallon, W. (1996): "Calculating Value-at-Risk", a paper presented at the Wharton Conference on *Risk Management in Banking* on 13th-15th October 1996.

George, T.J., G. Kaul and M. Nimalendran (1991): "Estimation of the Bid-Ask Spread and Its Components: A New Approach", *Review of Financial Studies*, 4-4, p.623-656

Glassman D. (1987): "Exchange Rate Risk and Transactions Costs: Evidence from Bid-Ask Spreads", *Journal of International Money and Finance*, 6, p.479-490.

Glosten, L.R. and L.E. Harris (1988): "Estimating the Components of the Bid-Ask Spread", *Journal of Financial Economics*, 21, p.123-142

Glosten, L.R. and P.R. Milgrom (1985): "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders", *Journal of Financial Economics*, 14, p.71-100.

Hebner, K. (1996): "Liquidity in Financial Markets: Theory and Evidence", *mimeo*.

Lee, J-H and J-C Lin (1995): "Volatility and Liquidity at NYSE Opening Calls: A Closer Look", *Journal of Financial Economics*, 4, p.479-493.

Lee, C. M. and M. J. Ready (1991): "Inferring Trade Direction from Intraday Data", *Journal of Finance*, 2, p.733-746.

Locke P.R. and A. Sarkar (1996): "Volatility and Liquidity in Futures Markets", *Federal Reserve Bank of New York Research Paper No. 9612*, May.

Longstaff, F.A. (1995): "How Much Can Marketability Affect Security Values?", *Journal of Finance*, 5, p.1767-1774.

Mori, A., M. Ohsawa and T. Shimizu (1996): "A Framework for More Effective Stress Testing", in FRB, ed., *Risk Measurement and Systemic Risk – Proceedings of a Joint Central Bank Research Conference, November 1995*, Washington D.C.

Shimizu, T. (1997): "Dynamic Macro Stress Exercise Including Feedback Effects", *mimeo*, Institute for Monetary and Economic Studies, Bank of Japan.

Stoll, H.R. and R.E. Whaley (1990): "Stock Market Structure and Volatility, *Review of Financial Studies*, 44, p.115-134.

Watanabe, T.: "Intraday Price Volatility and Trading Volume: A Case of the Japanese Government Bond Futures, in FRB, ed., *Risk Measurement and Systemic Risk*, Washington D.C. (forthcoming).

Wilson, T. (1996): "Calculating Risk Capital", in C. Alexander, ed., *The Handbook of Risk Management and Analysis*, West Sussex: John Wiley.

Chart 1

Components of liquidity risk

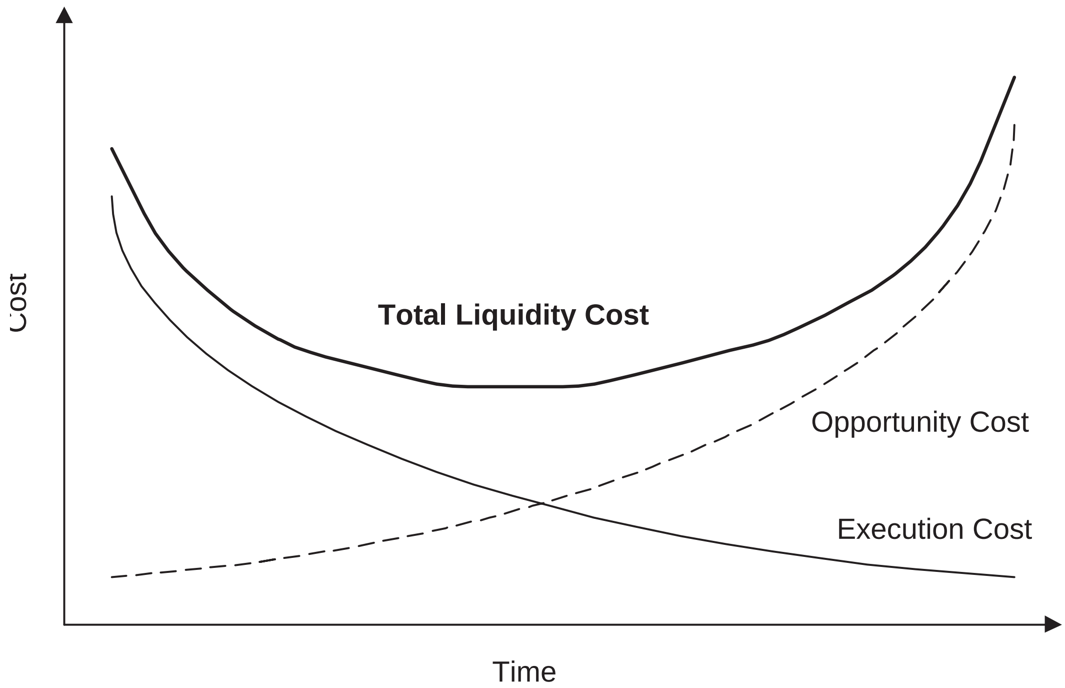
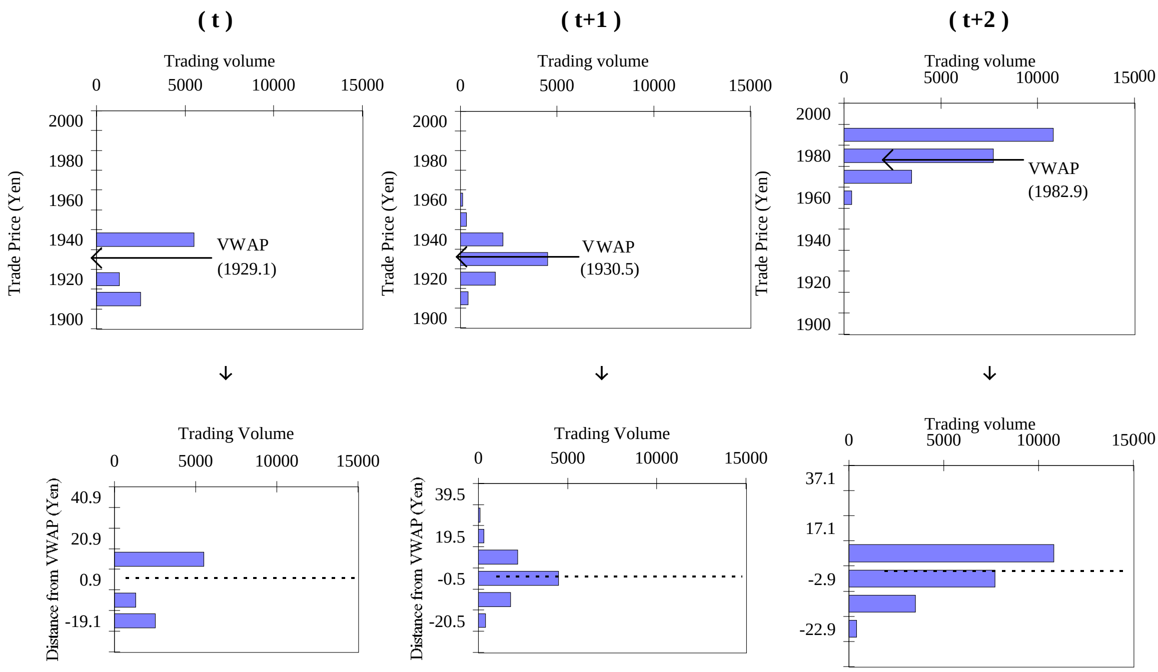


Chart 2

Volatility and kurtosis of daily return

	AM Open	AM Close	PM Open	PM Close
Volatility	0.02000	0.01930	0.01896	0.01999
Kurtosis	6.5503	6.7363	7.2008	6.5145

Chart 3
Histogram of Trade Prices



Accumulated Histogram

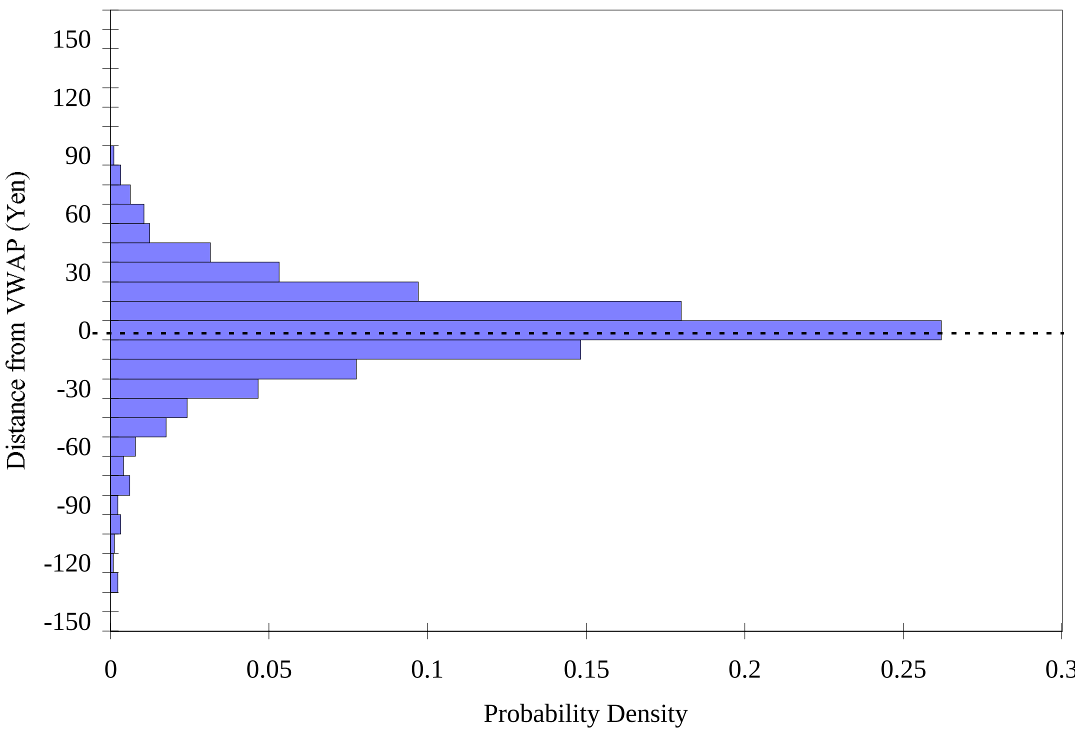
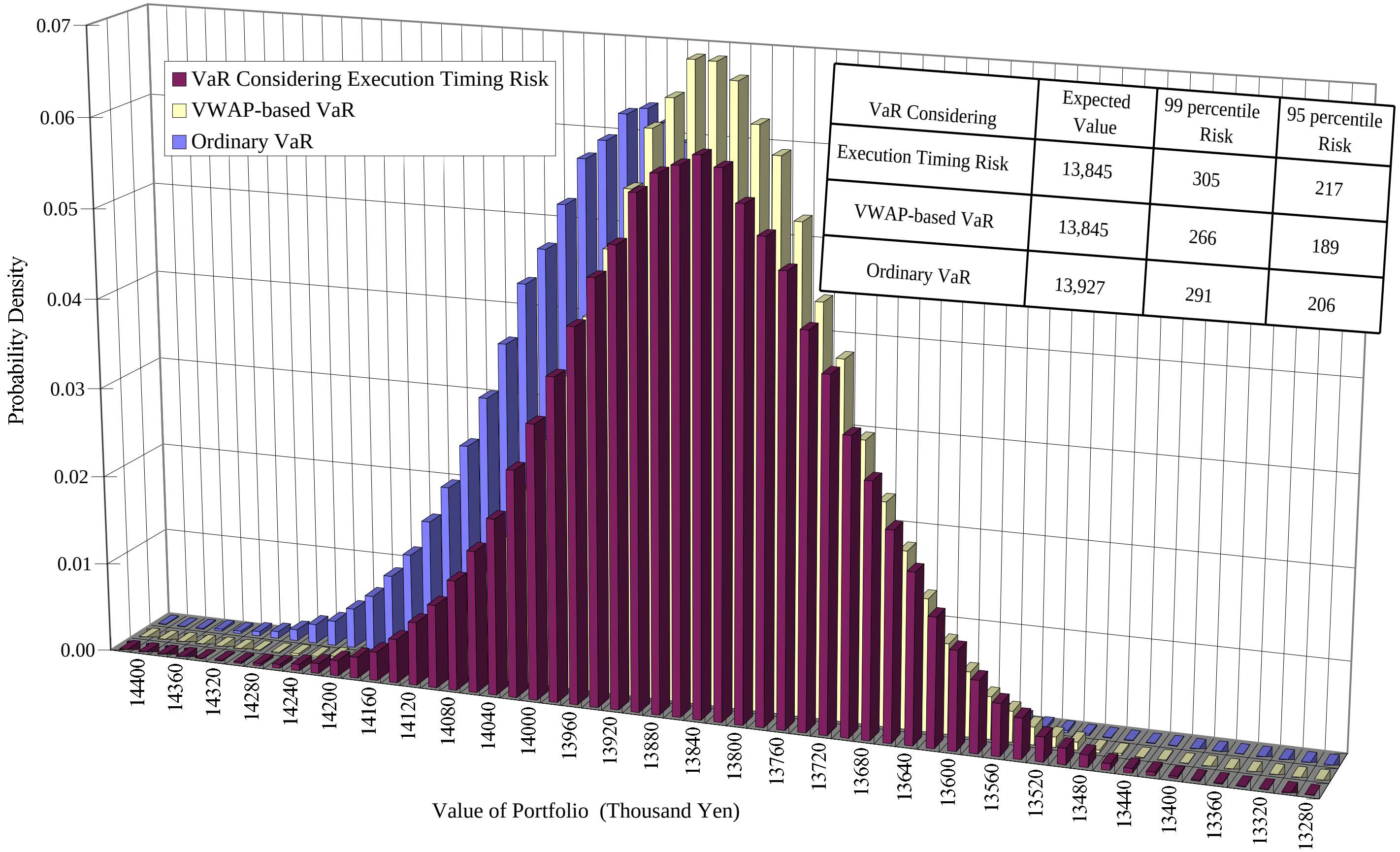


Chart 4 Market Risk Considering Execution Timing Risk



VaR Considering	Expected Value	99 percentile Risk	95 percentile Risk
Execution Timing Risk	13,845	305	217
VWAP-based VaR	13,845	266	189
Ordinary VaR	13,927	291	206

Chart 5

Basic statistics of the portfolio

	Stock A	Stock B	Stock C	Stock D	Stock E
Position (units of stocks)	+ 1,000	+ 1,000	+ 1,000	+ 1,000	+ 1,000
Mid-price on the day of risk evaluation (Yen)	2,275	5,730	1,580	2,355	1,985
HV of mid-price (%)	1.957	1.420	1.886	2.167	1.863
VWAP on the day of risk evaluation (Yen)	2,256	5,718	1,524	2,335	2,010
HV of VWAP (%)	1.749	1.314	1.567	1.867	1.683
Standard deviation of VWAP (Yen)	25.72	43.43	13.60	33.89	20.84
Average bid-ask spread (Yen)	22.62	45.90	17.33	25.60	21.76
Daily average of trading volumes per transaction	608	386	500	950	375

Chart 6

Histogram of bid-ask spread

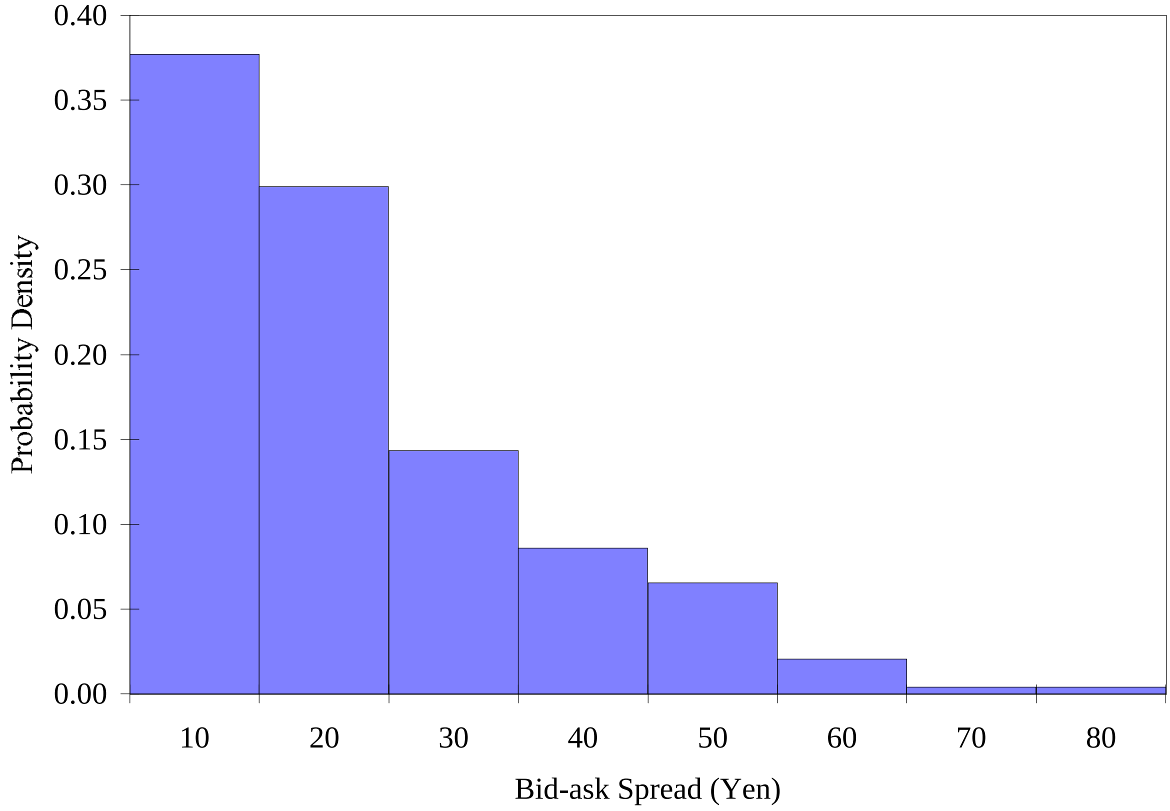
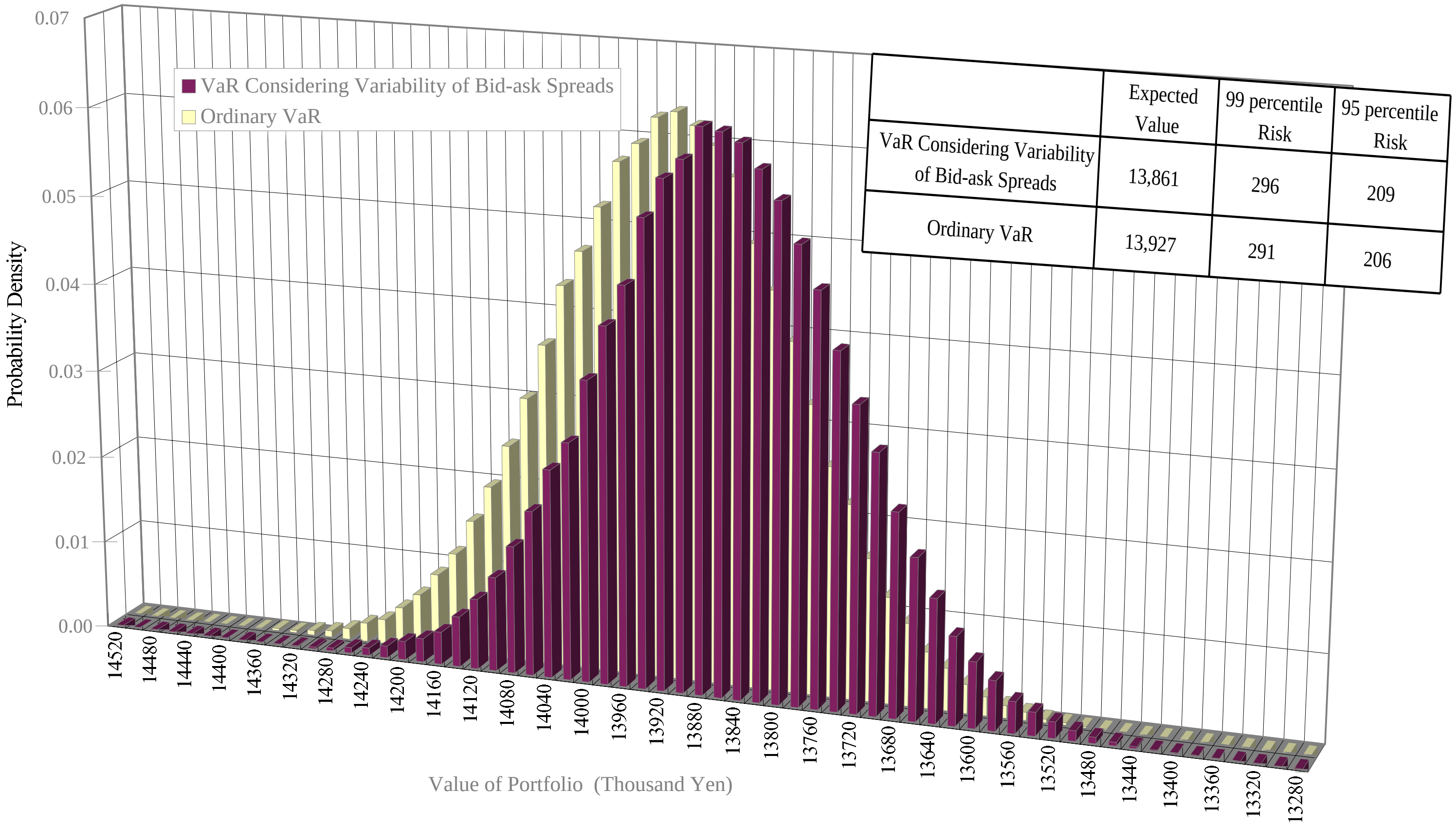


Chart 7 Market Risk Considering Intraday Variability of Bid-ask Spreads



	Expected Value	99 percentile Risk	95 percentile Risk
VaR Considering Variability of Bid-ask Spreads	13,861	296	209
Ordinary VaR	13,927	291	206

Chart 8

Tail events of bid-ask spreads

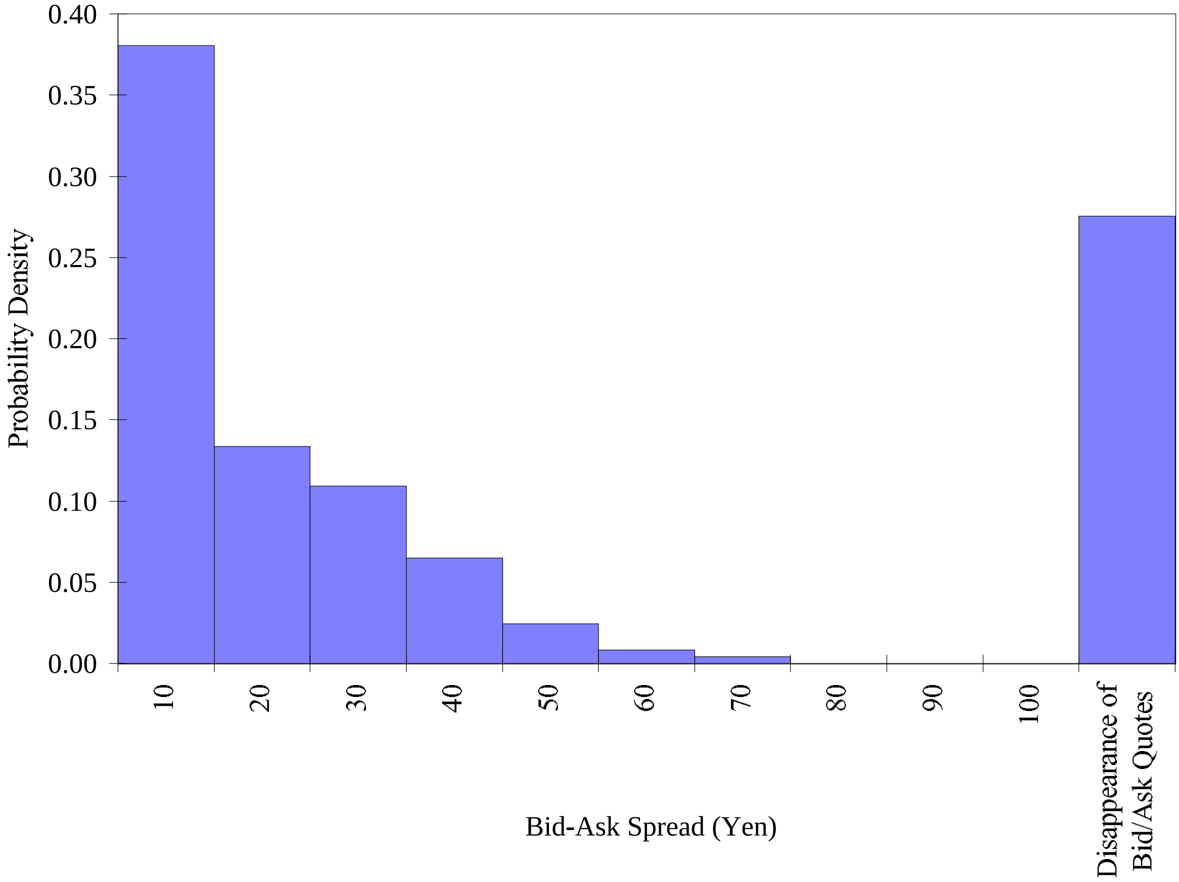


Chart 9

Relationship between λ and turnovers

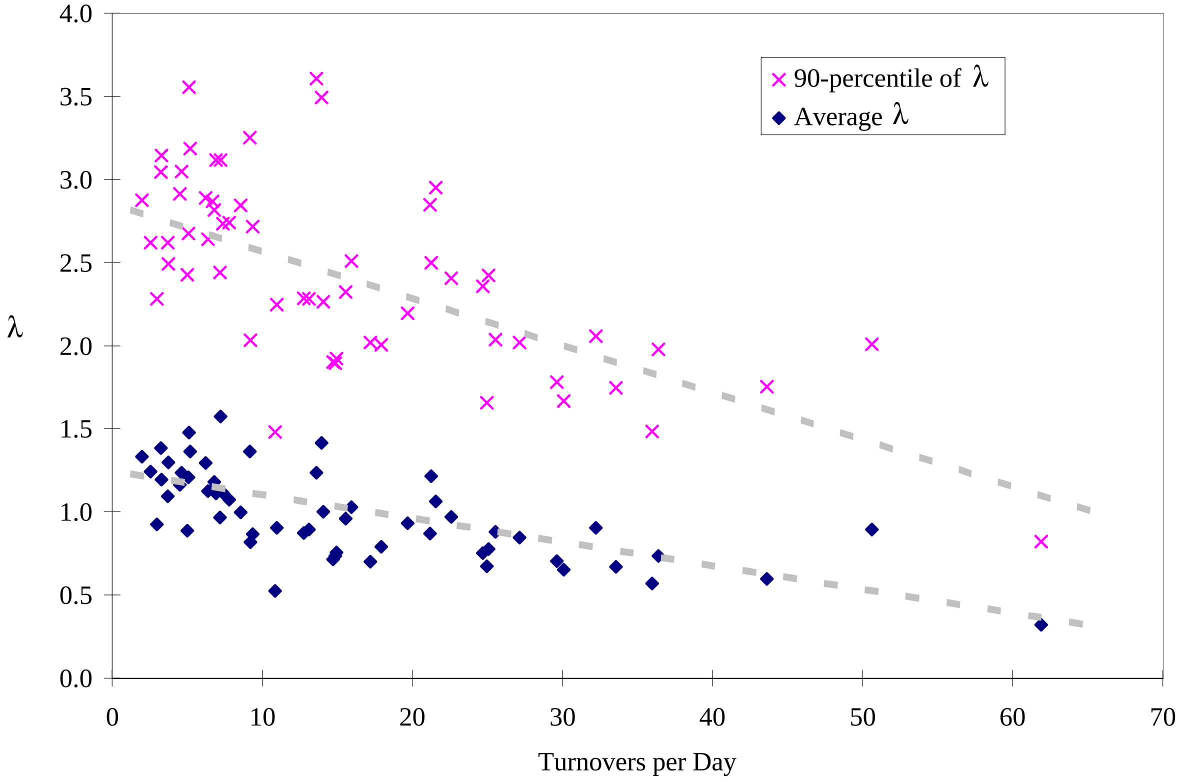


Chart 10

Relationship between λ and trading volume

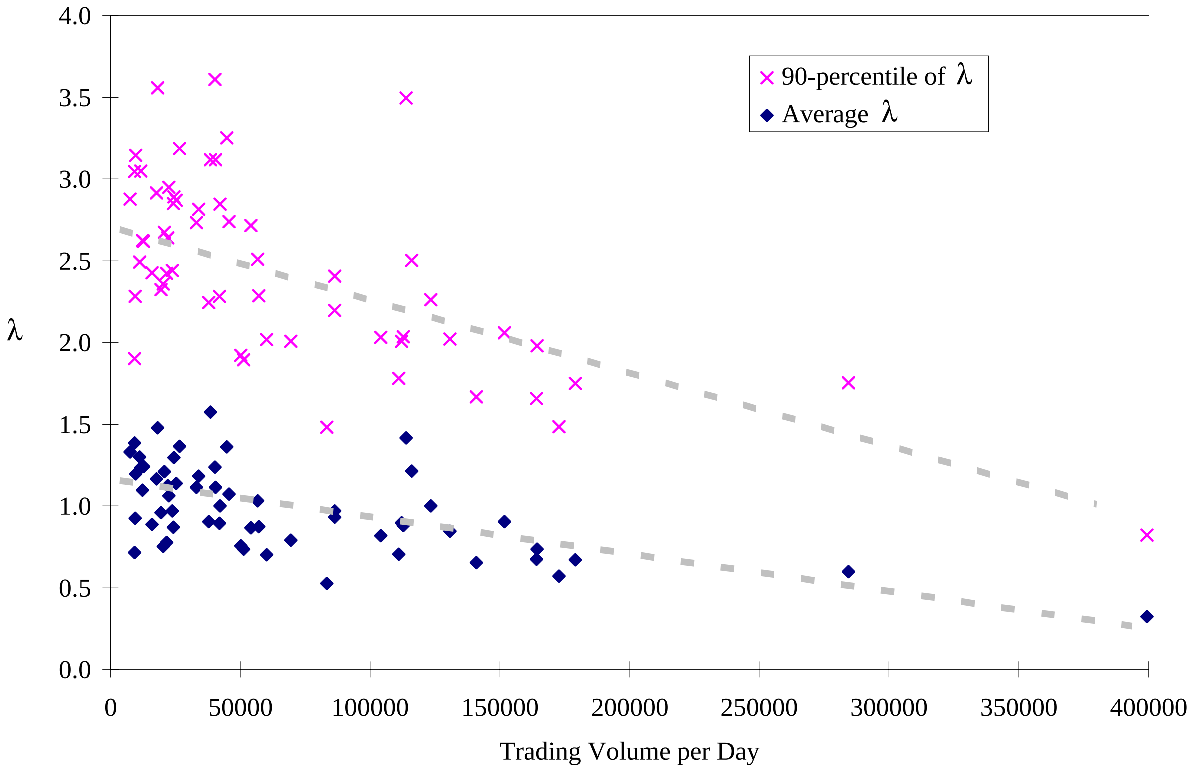


Chart 11

Histogram of λ

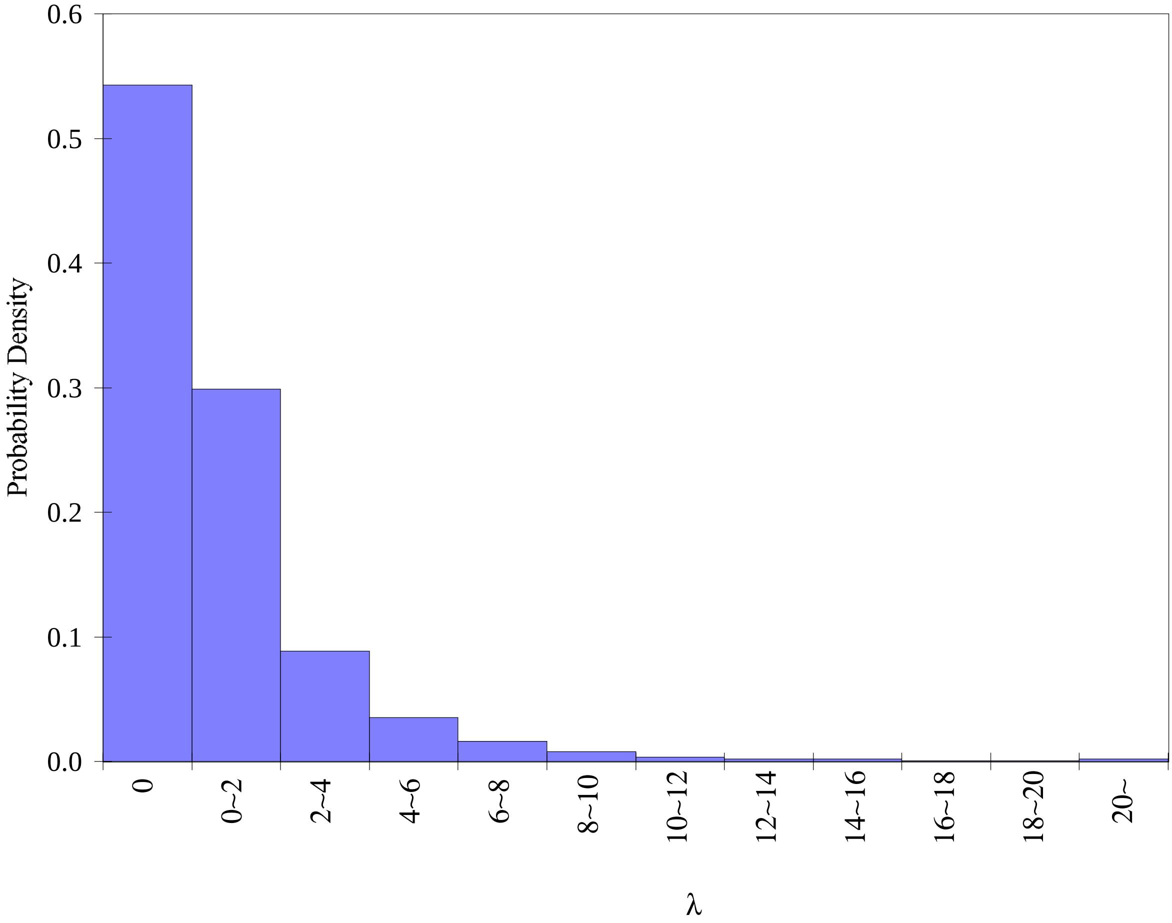
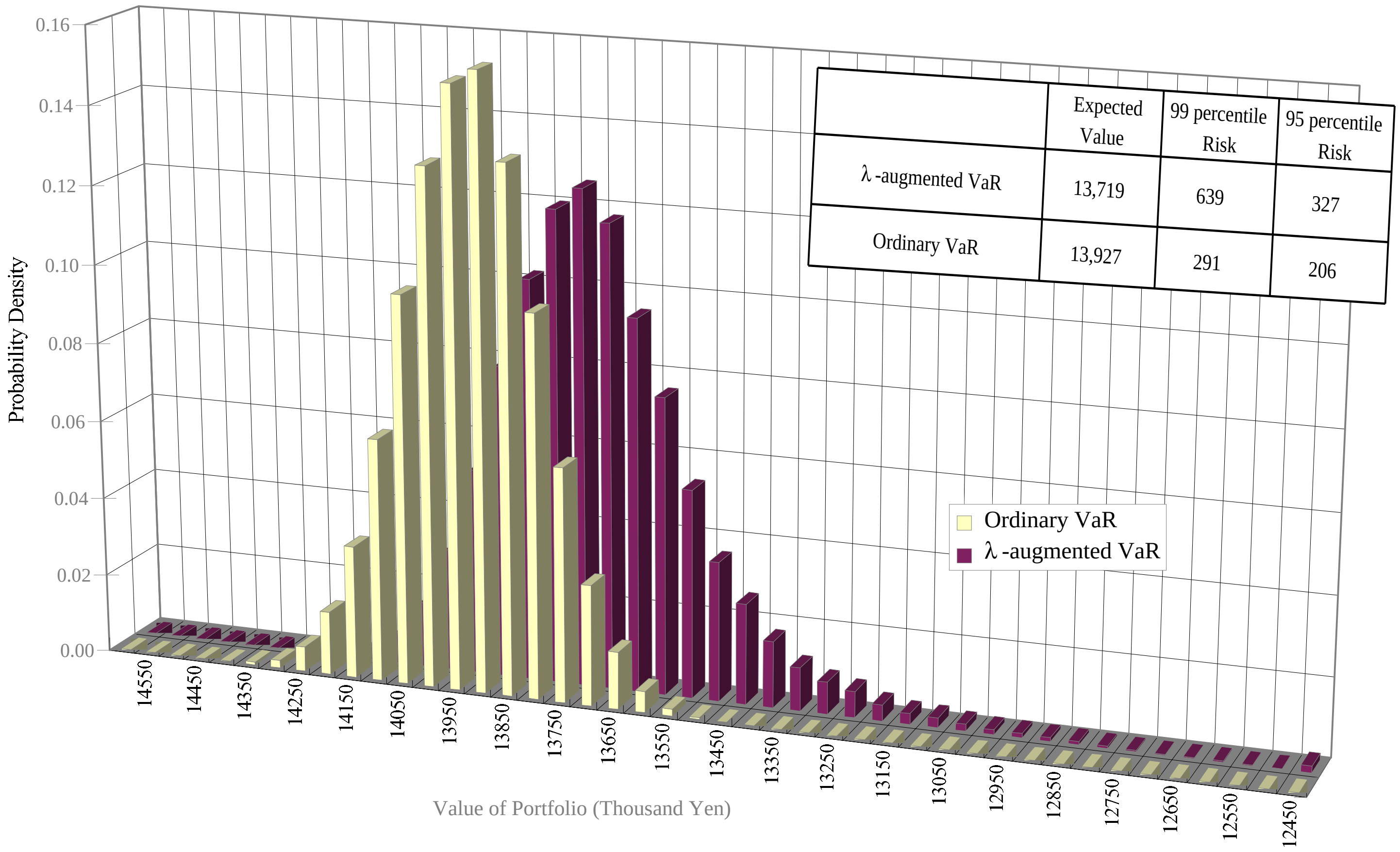


Chart 12 Market Risk Considering Variability of λ



	Expected Value	99 percentile Risk	95 percentile Risk
λ -augmented VaR	13,719	639	327
Ordinary VaR	13,927	291	206