Original Article

Pricing liquidity risk and cost in the stock market: How different was the financial crisis?

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Xue Han

Xue Han is a Junior Economist of Research and Analytical Services (RAS), Towers Watson. She is the local segment leader of Risk and Financial Services (RFS), and also manages Mortality Service Line and Structural Reward System. Her research interest focuses on financial econometrics and mathematics, in terms of liquidity risk modeling, stochastic mortality analysis, dynamic reward pricing and so on.

Zheng Jian

Zheng Jian was an Economist and Head of Financial and Economic Modeling of Research and Analytical Services (RAS), Towers Watson. He is currently an Associate Economic Affairs Officer at the United Nations Economic Commission for Africa (UNECA).

Correspondence: Xue Han, Research and Analytical Services (RAS), Towers Watson, 10th Floor, Fiber Home Building, Youkeyuan, Luoyu Road, East Lake Development District, Wuhan 430074, Hubei, China
E-mail: Xue.Han@towerswatson.com

ABSTRACT Past literature has proven that liquidity plays a role in stock pricing, but our study shows that this role is remarkably bigger during an unstable period – the subprime crisis. The market became more sensitive to illiquidity costs and investors were paying a higher premium in crisis than in boom. This conclusion holds even when the overall illiquidity cost is divided into an expected part and an unexpected part. In contrast, while the market remained largely as sensitive to liquidity risk as before, liquidity risk itself, represented by the liquidity betas, changed remarkably in the crisis period compared to the prior boom period.

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INTRODUCTION

Investors usually seem to pay limited attention to illiquidity cost or liquidity risk when investing in the equity market, probably because 'liquidity', which is invisible, intangible and hard to measure, does not pose a big problem in this most active market. However, numerous research papers report that liquidity has been taken into account by investors in stock markets and there are significantly positive premiums on illiquidity in stock returns.[1]

During extreme market environments, such as the financial turmoil in late 2008, liquidity generally tends to be an even greater concern. The unanticipated liquidity problem turned out to spread so deeply into the financial markets and deteriorate for so long with such dreadful consequences that the number of record-breaking losses was refreshed many times in just a few months from September 2008. In fact, liquidity has always been an important pricing factor and a few questions naturally arise regarding its role in asset pricing during the crisis. Can asset allocation be adjusted timely and smoothly? Can portfolios be rebalanced at low cost? What will the price gap be from the figure when loss-control orders are given to the actual figure when the transaction is finally made?

With rising interest in liquidity issues and newly acquired data, it is an appropriate moment to reconsider the pricing of liquidity in the stock market. Past research on the topic largely focuses on the observations in long-term market equilibrium. But simple economic intuition tells us there is a sharp difference between a happy period when returns are rising and more and more players are attracted to the market; and a crisis period marked by panic and investors eager to cut their losses and get out. Liquidity should have been priced differently in the subprime crisis and the main objective of this study is to find out how.

REVIEW OF PREVIOUS LITERATURE

We first give a brief review of past literature in the area. Amihud and Mendelson (1986) find that illiquidity measured as transaction costs on the bid-ask spread is significantly priced by expected returns for NYSE-listed stocks. This makes sense as investors would require higher expected returns for stocks with higher bid-ask spreads to offset the higher trading cost. The findings of Eleswarapu (1997) support the significantly positive relationship between illiquidity and expected return for Nasdaq stocks. He argues furthermore, that the liquidity priced effect is much stronger in January than in other months. However, both Amihud and Mendelson (1986) and Eleswarapu (1997) define the transaction cost[2] caused by illiquidity as the cost of immediate execution rather than the actual trades between bid price and ask price, so their measurement of transaction cost, for example, the bid-ask spread, is a bit larger than the actual one.

In recent studies, more researchers take holding periods into account.[3] Vayanos (1998) develops a general dynamic equilibrium model with transaction costs in a continuous time-overlapping generation economy and also argues that stock prices may rise because of transaction costs. Moreover, Vayanos (1998) finds that a more liquid stock is less adversely affected by an increase in transaction costs. Chalmers and Kadlec (1998) obtain amortized effective spread by capturing both the actual transaction price between spread and the average length of stock-holding periods. They find stronger priced evidence for amortized illiquidity than unamortized illiquidity. But this amortized method relies on daily turnover rates to measure average holding periods, so that the transaction cost differences among various stocks are much more volatile due to the great volatility of daily turnover rates.

Amihud (2002) considers another aspect of illiquidity cost: selling cost, and measures it with a new instrument called ILLIQ, which is the absolute daily return divided by dollar volume and averaged over a month or a year, so that it can reflect the average price response to every selling dollar. His empirical results estimated with selling cost also indicate the positive return-illiquidity relationship across different stocks. What is more, the illiquidity priced effect is much stronger for small-firm stocks. From then on, illiquidity cost is always measured by joining transaction cost and selling cost together or analyzing the two aspects respectively, in similar or extensional forms, which is more accurate and close to the actual illiquidity cost in equity market.

Pastor and Stambaugh (2003, p. 683) suggest that illiquidity is priced not only in the average cost, but also in its own volatility and sensitivity to other factors, which can be defined as liquidity risk, that is, 'stocks that are more sensitive to aggregate liquidity have substantially higher expected returns'. This finding is extended by Acharya and Pedersen (2005), who explore a complete theoretical framework of Liquidity-Adjusted CAPM to complement the traditional capital asset-pricing model with constant trading frictions[4] by adding a new pricing factor, liquidity. They prove that liquidity risk is priced not only in return sensitivity to market liquidity, but also in liquidity sensitivity to market return and liquidity commonality to the market, based on monthly average data of NYSE and AMEX stocks over a 37-year long period from 1963 to 1999.

Following this string, we seek to investigate further whether the pricing of liquidity exhibits different patterns in medium and short terms especially between a booming/normal period and a crisis/slump period. We choose the same established framework, Liquidity-Adjusted CAPM, in this article and apply it to two different periods of US stock market: a booming period from 1 January 2004 to 31 December 2006 and a crisis period from 1 January 2007[5] to 31 December 2008. As we have much shorter data ranges, daily stock returns and liquidity indicators are used.

In the following section we provide a more detailed description of our methodology and models. Data profiles and manipulation are described in the section 'Data'. The section 'Empirical Analysis' presents our empirical analysis and findings, while the 'Conclusion' section concludes the article.

METHODOLOGY

The measure of illiquidity cost by matching ILLIQ with Effective Spread

Following Acharya and Pedersen (2005), we construct a normalized joint illiquidity cost measured by combining transaction cost and selling cost together, but drop the maximum restriction of 30 per cent to capture the complete fluctuations of illiquidity cost in our much shorter data range.

For the measurement of transaction cost, Effective Spread[6] instead of the ordinary bid-ask spread is adopted. Effective Spread is defined as the absolute difference between actual transaction prices and the 'fair' price estimated to be the average of bid price and ask price; thus it is closer to the real transaction cost borne by investors. The formula below shows how Effective Spread is calculated for any individual stock j on day t.

$$\text{Transaction cost: Effective Spread}_t^j = \frac{\left|\text{Transaction Price} - \frac{\text{Bid Price}+\text{Ask Price}}{2}\right|}{\text{Transaction Price}} \quad (1)$$

For the measurement of selling cost, ILLIQ (Amihud, 2002) is used but adjusted from the average of a month or a year to a day to capture the shorter-term fluctuations. That is:

$$\text{Selling Cost}: ILLIQ_t^j = \frac{\left|R_t^j\right|}{DV_t^j} \quad (2)$$

where R_t^j and DV_t^j are, respectively, the return and dollar volume (in millions) of individual stock j on day t. That is, selling cost is measured as the average price response to every 1 million trading volume. Therefore, ILLIQ is a measurement in terms of 'per cent per dollar', whereas the asset pricing model in our following methodology is specified in terms of 'dollar cost per dollar invested'. This problem will be solved in the following cost-combining process.

Effective Spread is used as a proxy of transaction cost, while ILLIQ is used to measure the selling cost, the individual stock's price movement in response to trading volume. Therefore, by combining the two costs together as Acharya and Pedersen (2005) did,[7] the normalized joint illiquidity cost of individual stock j on day t, denoted by C_t^j, is then estimated by combining Effective Spread and ILLIQ,

$$\text{Normalized joint illiquidity cost:} \quad C_t^j = aILLIQ_t^j P_t^M + b \tag{3}$$

where P_t^M is the ratio of the market capitalization of S&P 500 at day t and the one at the start day of our data range, so as to include inflation.

The coefficients a and b are chosen in such a way that the cross-sectional distribution of illiquidity cost C_t^j has the same mean and variance as Effective Spread$_t^j$. Assume the mean and variance of Effective Spread$_t^j$ is m_1 and σ_1^2, respectively, and the mean and variance of $ILLIQ_t^j P_t^M$ is m_2 and σ_2^2, respectively. We can match them by

$$\begin{cases} am_2 + b = m_1 \\ a^2\sigma_2^2 = \sigma_1^2 \end{cases} \Rightarrow \begin{cases} a = \sigma_1/\sigma_2 \\ b = m_1 - m_2\sigma_1/\sigma_2 \end{cases} \tag{4}$$

The measure of liquidity risk based on Liquidity-Adjusted CAPM

The Liquidity-Adjusted CAPM has become a standard framework to analyze how liquidity friction is priced in stock markets since it was first introduced by Acharya and Pedersen (2005). The model inherits the main structure of traditional CAPM but liquidity enters as an additional element except for market return, therefore it complements the traditional pricing model with constant trading frictions. The pricing formula is as below, with C_t^j representing illiquidity cost for asset j at time t.

$$E(R_t^j - R_f) = E(C_t^j) + \gamma\beta_{net}^j \tag{5}$$

Where

$$\gamma = E(R_t^M - C_t^M - R_f) \tag{6}$$

$$\beta_{net}^j = \frac{\text{cov}(R_t^j - C_t^j,\, R_t^M - C_t^M)}{\text{var}(R_t^M - C_t^M)} = \beta_1^j + \beta_2^j - \beta_3^j - \beta_4^j \tag{7}$$

Liquidity will affect the excess return of risky assets in two ways, directly increasing excess return demanded by the expected illiquidity cost EC_t^j and also adding to the risk factor β. Owing to the existence of illiquidity cost faced by both the risky asset and the market portfolio, β can be decomposed further into four components:

$$\beta_1^j = \frac{\text{cov}(R_t^j,\, R_t^M)}{\text{var}(R_t^M - C_t^M)}, \text{ which is return commonality between individual and the market} \tag{8}$$

$$\beta_2^j = \frac{\text{cov}(C_t^j,\, C_t^M)}{\text{var}(R_t^M - C_t^M)}, \text{ which is illiquidity commonality between individual and the market} \tag{9}$$

$$\beta_3^j = \frac{\text{cov}(R_t^j,\, C_t^M)}{\text{var}(R_t^M - C_t^M)}, \text{ which is individual return sensitivity to market illiquidity} \tag{10}$$

$$\beta_4^j = \frac{\text{cov}(C_t^j,\, R_t^M)}{\text{var}(R_t^M - C_t^M)}, \text{ which is individual illiquidity sensitivity to market return} \tag{11}$$

β_1^j is the standard market beta in traditional CAPM, which captures only the risk of return fluctuations, while β_2^j, β_3^j and β_4^j capture the additional risk caused by fluctuation of illiquidity cost. Together $\beta_2^j - \beta_3^j - \beta_4^j$ measures total liquidity risk. Hereafter, we call β_1^j market beta, and β_2^j, β_3^j, β_4^j liquidity risk betas.

For β_2^j, Liquidity-Adjusted CAPM shows how much return is required to compensate investors for holding a more illiquid stock that co-moves with the market when aggregate liquidity is scarcer, by multiplying the liquidity commonality risk with positive premium.

Pastor and Stambaugh (2003) find that stocks with higher return sensitivity to market liquidity will fetch higher prices, reflected in Liquidity-Adjusted CAPM as the negative premium of β_3^j. It can be interpreted as the willingness of investors to accept a lower average return on a stock whose expected return will be higher when the whole market is illiquid.

For β_4^j, this sensitivity measure also attracts a negative risk premium. Acharya and Pedersen (2005, p. 382) suggest that investors prefer to 'accept a lower expected return on a security that is liquid in a down market'.

DATA

Time-invariant individual stocks

We employ the historical daily data from Bloomberg from 1 January 2004 to 31 December 2008 for all the 500 individual stocks on the constituent list of the S&P 500 at 5 January 2009. Our measures of stock daily return R_t^j and illiquidity cost C_t^j are based on closing transaction price, closing bid price and closing ask price, which are all adjusted for stock splits and dividends. Risk-free interest rate R_f is transformed from 90-day Treasury bill rate r into daily return by $e^{r/360}-1$. Then excess return is defined as $R_t^j-R_f$. Data range is divided into two periods: booming period from 1 January 2004 to 31 December 2006; financial crisis from 1 January 2007[8] to 31 December 2008, so that we can draw comparisons of the two periods by their own empirical results to capture the unique nature of liquidity during the last financial crisis.

Time-variant portfolios

Following Acharya and Pedersen (2005), our empirical analysis is based on two panels of time-variant portfolios formed from S&P 500 stocks. To create the first panel, for each quarter we sort all the individual S&P 500 stocks into 25 equal-size[9] portfolios, $p \in \{1, 2, \ldots, 25, m\}$, based on an increasing ranking of the individual stocks by their average market capitalizations of the previous quarter. Put differently, the 20 stocks with smallest market cap form portfolio 1; the next 20 stocks with smallest market cap become portfolio 2 and so forth. As the ranking of the stocks can change from quarter to quarter, the components of each portfolio also change. We call the first panel of portfolios the time-variant market-cap portfolios. The second portfolio panel is constructed in the same way except that the ranking on individual stocks is made according to average normalized joint illiquidity cost of the previous quarter instead of market cap, and we call it the time-variant illiquidity portfolios. We use equal-weighted[10] R_t^p and C_t^p throughout this article. Market portfolio is defined as the overall portfolio with equal-weighted R_t^m and C_t^m calculated with all of the 500 individual stocks.

From the results of Augmented Dickey-Fuller Unit-Root Test, we find that all the series of R_t^p, C_t^p and excess return $(R_t^p-R_f)$, $p \in \{1, 2, \ldots, 25, m\}$ follow stationary process.

EMPIRICAL ANALYSIS

In this section we present the findings of our empirical analysis and try to answer the fundamental questions below regarding the role of liquidity in stock markets:

Question 1: Has the time-series trend of illiquidity cost changed during crisis period?

Question 2: Do stock returns react differently to illiquidity cost in boom and in crisis?

Question 3: What about the components of liquidity risk? Do they show different features during crisis?

Question 4: How much is paid on liquidity risk in the crisis period?

The analysis is divided into two steps. We start with time-series modeling on illiquidity cost and focus on the first two questions above. Then we move on to the pricing of liquidity risk and cost together using the more sophisticated Liquidity-Adjusted CAPM.

Time-series of illiquidity cost

Illiquidity persistency

The Autocorrelations (AC) and Partial Autocorrelations (PAC) tests show a link between daily market illiquidity and its lags in both the booming and the crisis period (please refer to Figures A1 to A4 in Appendix), but this autocorrelation is much stronger during the financial crisis. A similar strengthening of autocorrelation effect during the crisis period is also found with all illiquidity cost series of individual stocks. This universal observation implies a generally more persistent disagreement between buyers and sellers during the market slump, which results in lasting transaction frictions represented by illiquidity cost.

Differentiate expected and unexpected parts of illiquidity cost

Applying the Auto-Regression (AR) model to the illiquidity-cost time-series of individual stocks, we are able to deconstruct illiquidity further into an expected part and an unexpected liquidity shock. All the individual stocks from the S&P 500 index are analyzed using the regression equation below for both boom and crisis period, respectively:

$$C_t^p = \alpha_0 + \alpha_1 C_{t-1}^p + \alpha_2 C_{t-2}^p + \ldots + \alpha_k C_{t-k}^p + u_t^p \qquad (12)$$

In this model the expected part of illiquidity is $E_{t-1}(C_t^p) = \hat{C}_t^p = C_t^p - \hat{u}_t^p$, which is the estimated value of C_t^p, while the unexpected illiquidity shock is $\hat{u}_t^p$, which is also the residual of regression (12).

We choose lags = 5 for all the 500 individual stocks by AIC (Akaike's information criterion) and the series tail characteristics of AC and PAC. Run AR(5) as regression (12) on the market portfolio's illiquidity cost C_t^m to get its unexpected illiquidity $\hat{u}_t^m$ in two periods, respectively, and then combine the results together as shown in Figure A5 of the Appendix. It is easy to see that the unexpected liquidity shocks became considerably larger and more volatile during the past 4 months of 2008.

What happened in those 4 months? On 15 September 2008, the largest bankruptcy in US history was experienced by Lehman Brothers, then the fourth-biggest US investment bank. Lehman ended its 158-year existence with a debt of $613 billion, putting a spark to a crisis fuse waiting to be lit. Investor expectation uncertainty exploded and in the following months markets moved frequently and swiftly in reaction to the hot inflows of news and rumor. Our finding provides proof that in this chaos not only had previous illiquidity cost become more persistent, but there had also been more random shocks to market liquidity. Such high illiquidity-cost variations caused by the shocks add to our cause to investigate how liquidity risk, together with the average illiquidity cost, has been priced (affecting asset returns) during the crisis period.

Excess return as time-series of illiquidity cost

We construct a panel data model as regression (13) on 500 time-invariant individual stocks with random effects, which is a specification supported by the Hausman Test.

$$R_t^p - R_f = \alpha + \lambda_1 E_{t-1}(C_t^p) + \lambda_2 \hat{u}_t^p + \lambda_1' D \cdot E_{t-1}(C_t^p) + \lambda_2' D \cdot \hat{u}_t^p + v^p + \varepsilon_t^p \qquad (13)$$

where $D = \begin{cases} 0 & \textit{in boom,} \\ 1 & \textit{in crisis} \end{cases}$ is a dummy variable to differentiate between two periods.

Applying the dummy D to draw the period-specific effects, it's much easier to find the relations between excess stock returns and components of illiquidity cost in two periods, respectively. Here the parameters λ_1' and λ_2' measure the difference between two periods. That means λ_1 and $\lambda_1+\lambda_1'$ measure how the expected illiquidity is priced in boom and in crisis, respectively. While λ_2 and $\lambda_2+\lambda_2'$ work the same for the unexpected illiquidity in two periods.

The regression results are summarized in Table A1 of the Appendix, together with the Wald Test to tell whether $\lambda_1=\lambda_2$ and whether $\lambda_1+\lambda_1'=\lambda_2+\lambda_2'$.

From Table A1, all the estimated coefficients $\hat{\lambda}_1$, $\hat{\lambda}_2$ $\hat{\lambda}_1'$, $\hat{\lambda}_2'$ are positive and highly significant, which means that expected or unexpected illiquidity cost are both significantly priced by excess return in boom and in crisis.

Significantly positive $\hat{\lambda}_1'$ and $\hat{\lambda}_2'$ also imply that stock returns are more sensitively linked to both the expected and the unexpected part of illiquidity cost in the crisis period. The market requires a higher premium for liquidity sacrifice in such a negative environment. It is bad news for someone who is holding illiquid stocks but has to sell them during the low ebb to meet certain short-term liquidity needs. On the other side of the trade, for investors with fewer short-term liquidity constraints, the over-compensation on illiquidity can be a potential boon.

Another notable finding is that the Wald Test significantly supports $\hat{\lambda}_1<\hat{\lambda}_2$ and $\hat{\lambda}_1+\hat{\lambda}_1' < \hat{\lambda}_2+\hat{\lambda}_2'$, which means unexpected illiquidity costs always have a larger influence on excess return than expected illiquidity costs, in both boom and crisis. A natural explanation is that investors react more forcibly to the unknown liquidity shocks than the predictable illiquidity cost. Thus an additional premium is required for the cost embedded in the unexpected part of illiquidity.

So far we have studied how stock markets price illiquidity and had a cursory discussion of premiums on illiquidity innovation (unexpected illiquidity). In the next section, we investigate the pricing of liquidity risk in a more sophisticated fashion.

How liquidity risk is priced

Liquidity risk betas in boom and crisis

We use the two panels of time-variant portfolios constructed in the section 'Time-variant portfolios', in contrast to individual stocks in the previous discussion, for our detailed analysis of liquidity risk. We first calculate the ordinary market beta β_1^p and all the three liquidity risk betas β_2^p, β_3^p, β_4^p of each portfolio p in all the 25 time-variant illiquidity portfolios according to formula (8)–(11) using the entire daily time series of booming period and crisis period, respectively.[11] These betas and other relevant statistics are reported in Table A2 (see Appendix).

It can be seen in Table A2 that as the portfolios are placed in an increasing order of illiquidity, their market-caps (Mcap) tend to decrease. This observation is consistent with previous studies such as Amihud (2002) and Acharya and Pedersen (2005). In fact illiquidity and market-cap are inversely correlated so well that analysis of the illiquidity portfolios and analysis of the market-cap portfolios lead to the same conclusions. Therefore we only use the results on illiquidity portfolios to present our findings in the discussion. However, all conclusions are almost equally applicable to market-cap portfolios.

These increasingly illiquid portfolios have increasingly higher expected illiquidity cost E(C); Bid-Ask Spread Ratios (Spread) also increase, showing consistency between our illiquidity measure and more traditional ones.

All trends discussed above persist in both boom and crisis periods. However, when it comes to liquidity risk betas, the change between the two periods is clear. β_2^p, the commonality (co-movement) in liquidity of portfolio illiquidity cost and market illiquidity cost rises moderately in the crisis period and the rise is more significantly for portfolios consisting of the most liquid stocks. In other words, the fluctuation of illiquidity cost faced by highly liquid, large-cap stocks becomes more consistent with the fluctuation of market illiquidity cost. This, like the traditional beta or β_1^p in our model, implies more difficulty for investors to 'diversify' liquidity risk among the large-cap stocks.

β_3^p and β_4^p demonstrate even more obvious changes that they decreased uniformly from positive in booming period to negative in crisis period. These two betas describe the correlation between stock return and liquidity cost; a negative value implies that return and liquidity deteriorate at the same time in the crisis period. As a result, during this time when stock prices fall and someone wants to liquidate stocks to stop losses, a higher illiquidity cost will be experienced. In sharp contrast, positive β_3^p and β_4^p in the boom period imply that the return and illiquidity loss can to some extent hedge each other. One practical piece of advice for investors from this finding is that as liquidity tends to decline together with return, it should be taken into account more seriously when facing an unfavorable market environment.

Premium paid to liquidity risk

According to Liquidity-Adjusted CAPM, equation (5), regression below must hold with $\alpha = 0$ and $\delta = 1$.

$$E(R_t^p - R_f) = \alpha + \delta E(C_t^p) + \gamma \beta_{net}^p \quad (14)$$

However, in reality, average illiquidity cost does not merely depend on the expected illiquidity cost for every unit of transaction E(C). It also depends on how frequently transactions are made, and actually the holding period[12] is always much longer than just one trading day which $\delta = 1$ means. Therefore, δ is more reasonably calibrated as an approximation of turnover rate rather than being 1. Since E(C) have similar increasing trends as the liquidity risk betas,[13] especially during the crisis period, we choose to use a new regression equation (15)[14] instead to estimate the premium on betas more accurately.

$$E(R_t^p - R_f) - Turnover \cdot E(C_t^p) = \alpha + \gamma(\beta_1^p + \beta_2^p - \beta_3^p - \beta_4^p) \quad (15)$$

Adding a dummy variable $D = \begin{cases} 0 & in\ boom, \\ 1 & in\ crisis \end{cases}$ into regression (15) to differentiate between two periods, we get the following:

$$E(R_t^p - R_f) - Turnover \cdot E(C_t^p) = \alpha + \gamma_0 \beta_{net}^p + \gamma_1 D \cdot \beta_{net}^p \quad (16)$$

Then the risk premium γ of illiquidity risk is γ_0 in boom, and $\gamma_0 + \gamma_1$ in crisis.

We can see from Table A3 (see Appendix) that with either illiquidity portfolios or market-cap portfolios, the risk premium $\hat{\gamma}$ of liquidity risk, which is the coefficient on β_{net}^p, is always positive and significant at 1 per cent level. That means the liquidity risk has been significantly priced by excess return in boom or crisis, and its risk premium $\hat{\gamma}$ is as follows:

$$\text{Liquidity risk premium } \hat{\gamma} = \begin{cases} \hat{\gamma}_0 & \in [0.0036,\ 0.0049] \quad in \quad boom, \\ \hat{\gamma}_0 + \hat{\gamma}_1 & \in [0.0033,\ 0.0046] \quad in \quad crisis \end{cases}$$

Coefficient $\hat{\gamma}_1$ on $D \cdot \beta_{net}^p$ is significantly negative at about 5 per cent level, but its estimated value is extremely small at around -0.0003 (Table A3). It means that for every unit increase in liquidity risk, investors tend to accept a marginally lower return premium in crisis than in boom. The decrease is, however, marginal. As discussed in the section 'Liquidity risk betas in boom and crisis', investors actually do pay a much higher total premium on liquidity risk in crisis because all the three components of liquidity risk rise dramatically during this period.

For comparison,[15] we compute the annualized expected return premium on the liquidity betas for illiquidity portfolio 2 and illiquidity portfolio 22, respectively, a portfolio of most liquid stocks and a portfolio of least liquid stocks in S&P 500 with 252 trading days in a year. The results are shown in Table A4 in the Appendix.

As shown in Table A4, the S&P 500's most liquid stocks, usually also the largest,[16] on average earned an annualized liquidity risk premium from 0.30 per cent to 0.41 per cent. This annualized liquidity risk premium had been much higher for the least liquid stocks in the same index, usually with small market caps, at an average level from 1.46 per cent to 2.03 per cent. In the long-term equilibrium, however, the premium on liquidity risk is around 0.15 to 1.20 per cent.[17]

CONCLUSIONS

Liquidity has been proven by past literature to have played a role in stock pricing, while our study shows that this role is remarkably bigger in an unstable market condition – the subprime crisis.

This change in importance can be attributed to both illiquidity cost and liquidity risk but in different patterns. We do not observe any statistically significant change in the size of average illiquidity costs. Our study on the time-series link between excess stock return and illiquidity costs, however, reveals that stock markets became more sensitive to them in the crisis than the previous boom period. In other words, for the same level of illiquidity, the US stock market was paying higher illiquidity premiums in times of crisis. Even when the overall illiquidity cost is divided into an expected and an unexpected part this conclusion still holds. Moreover, investors are more concerned about the unknown liquidity shocks than the predictable illiquidity cost. Thus, an additional premium is required for unexpected illiquidity cost whether in times of boom or crisis.

In contrast, while the market remained largely as sensitive to liquidity risk, liquidity risk itself, represented by the liquidity betas, changed remarkably in the crisis period compared to the booming period before it. β_2^p increased significantly, indicating a much stronger similarity between the fluctuation in market illiquidity cost and the fluctuations in illiquidity cost of individual stocks. β_3^p and β_4^p, the commonality between liquidity cost and stock returns, traveled in the opposite direction, dropping from positive in boom to negative in crisis. All these changes led to higher liquidity risk as it became more likely that liquidity conditions would deteriorate simultaneously (due to β_2^p) and even worse together with stock returns (due to β_3^p and β_4^p).

On the basis of the Liquidity-Adjusted CAPM model, we measured the potential annualized premium assigned to the overall liquidity risk during the subprime crisis period. As shown in Table A4, the most liquid stocks, usually also the largest, from S&P 500 on average earned an annualized liquidity risk premium from 0.30 per cent to 0.41 per cent. This annualized liquidity risk premium had been much higher for the least liquid stocks, usually with small market caps, in S&P 500 at an average level from 1.46 per cent to 2.03 per cent. In the long-term equilibrium, however, premium on liquidity risk is at around 0.15 to 1.20 per cent.

It should be noted that these estimations generated from our modelling may not exactly reflect the true level of liquidity risk premium, especially in such a crisis period when the asset-pricing mechanism may deviate strongly from its long-term equilibrium. But we do think two general trends discovered in this study are instructive, that is, the stock returns are more sensitively linked to the illiquidity cost and the liquidity risk itself rises remarkably during a crisis period.

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NOTES

1. See the literature review part in following paragraphs.
2. See Glosten and Milgrom (1985), Vayanos (1998), Chalmers and Kadlec (1998), Korajczyk and Sadka (2008), for the description and comparison of different measurements of transaction cost.
3. For a thorough discussion on this issue please refer to Vayanos (2004), Spiegel and Wang (2005), Bekaert *et al* (2007), and Easley *et al* (2008).
4. For example, Amihud and Mendelson (1986), Vayanos (1998) and so on.
5. The market was still trending upward at this time and the peak was around mid-2007, but we have to keep our data range of crisis for at least two complete trading years for the estimation efficiency and comparability of our empirical results in two periods.
6. Widely used in liquidity related research such as Chalmers and Kadlec (1998), Acharya and Pedersen (2005) or Korajczyk and Sadka(2008).
7. We drop the maximum restriction of 30 per cent for normalized joint illiquidity cost in Acharya and Pedersen (2005) to capture the complete fluctuations of illiquidity cost in our much shorter data range.
8. The market was still trending upward at this time and the peak was around mid-2007, but we have to keep our data range of crisis for at least two complete trading years for the estimation efficiency and comparability of our empirical results in two periods.
9. It means 20 stocks for each portfolio.
10. For each portfolio p, we can calculate its R_t^p and C_t^p by $R_t^p = \sum_{j \in p} w_t^j R_t^j$ and $C_t^p = \sum_{j \in p} w_t^j C_t^j$, $w_t^j (j \in p)$, are either value-weighted or equal-weighted, and we prefer the latter one. Eleswarapu (1997), Chordia *et al* (2000), Amihud (2002), Acharya and Pedersen (2005) adopt equal-weighted return and illiquidity to compensate for the over-representation in their samples of highly liquid securities compared to the actual situations in the economy.
11. The results for all the time-variant market-cap portfolios and illiquidity portfolios are available from the authors upon request.
12. We can also find evidence in past literature regarding this, please refer to footnote 3.
13. Please refer to Table A2 in the Appendix: Beta 3 and Beta 4 contribute to liquidity risk with a negative sign before them, thus they also lead to increasing liquidity risk in Table A2.
14. Acharya and Pedersen (2005) also used a similar approach.
15. Portfolio 1 and portfolios 23–25 are inappropriate as examples due to the missing data and outliers.
16. It means the stocks with largest market caps.
17. Please refer to calculation results from Tables 1 and 4 in Acharya and Pedersen (2005), based on monthly average data of NYSE and AMEX stocks over a 37-year long period from 1963 to 1999.

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APPENDIX

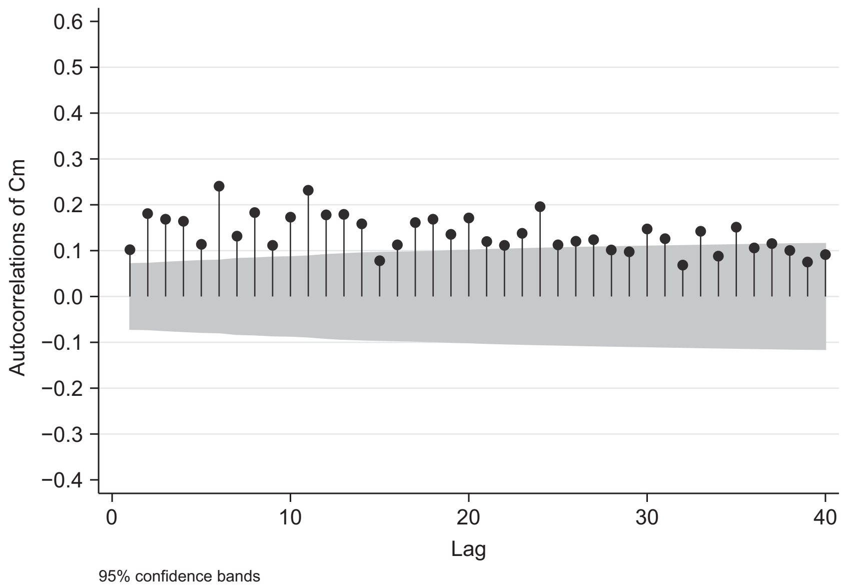

Figure A1: AC of the market illiquidity C_t^m in booming period
Source: Towers Watson calculations based on data from Bloomberg.

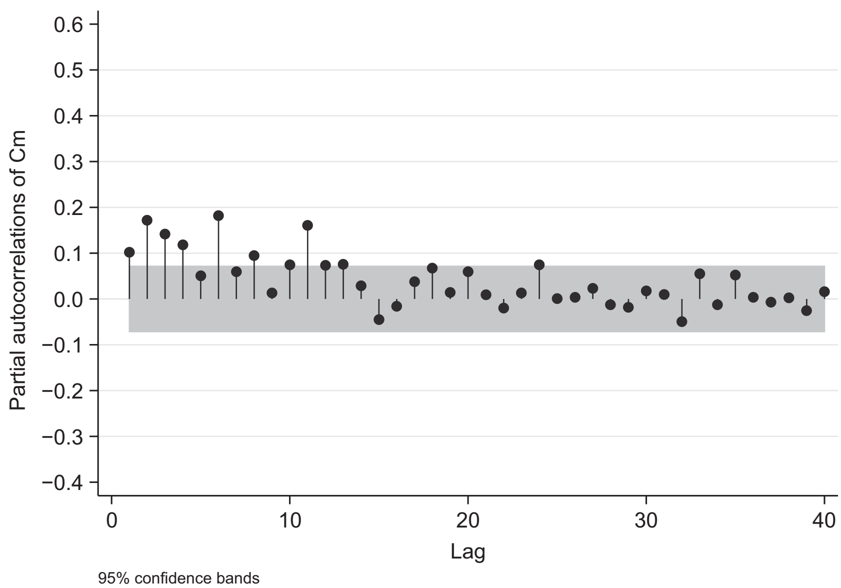

Figure A2: PAC of the market illiquidity cost C_t^m in booming period
Source: Towers Watson calculations based on data from Bloomberg.

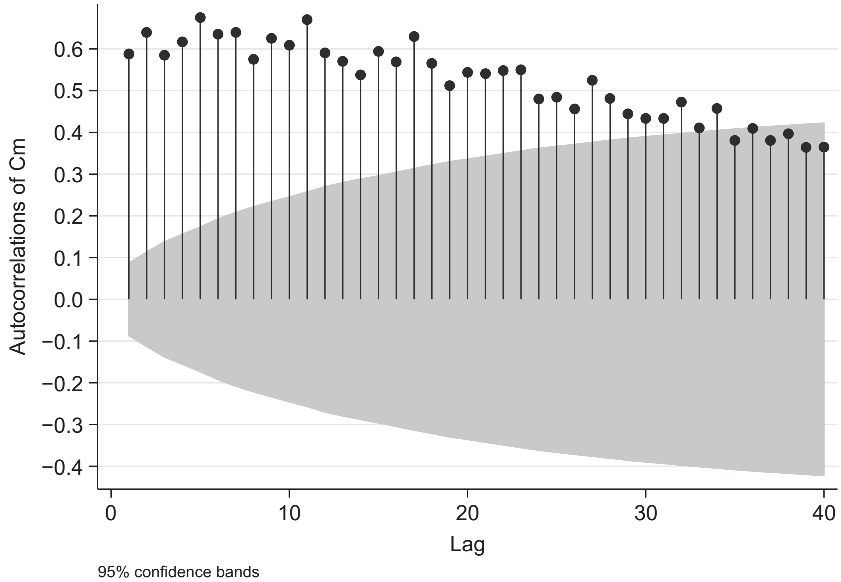

Figure A3: AC of the market illiquidity cost C_t^m in financial crisis
Source: Towers Watson calculations based on data from Bloomberg.

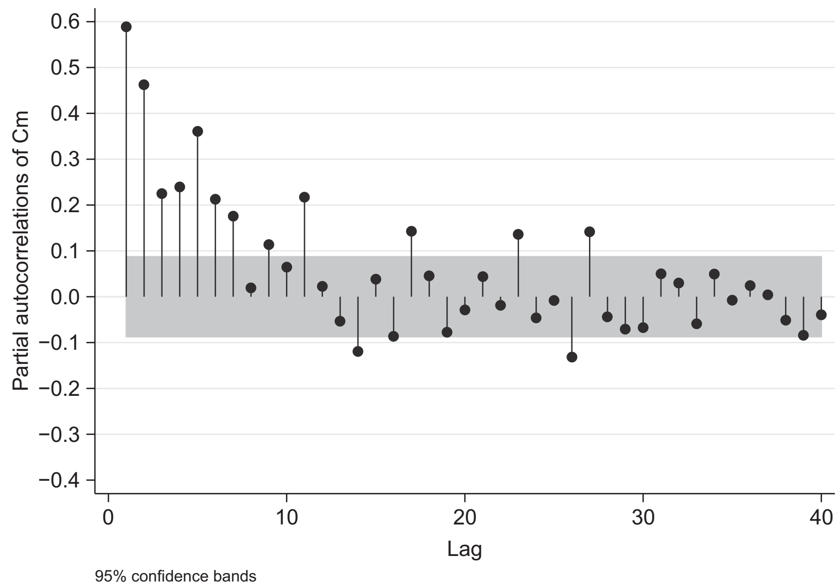

Figure A4: PAC of the market illiquidity cost C_t^m in financial crisis
Source: Towers Watson calculations based on data from Bloomberg.

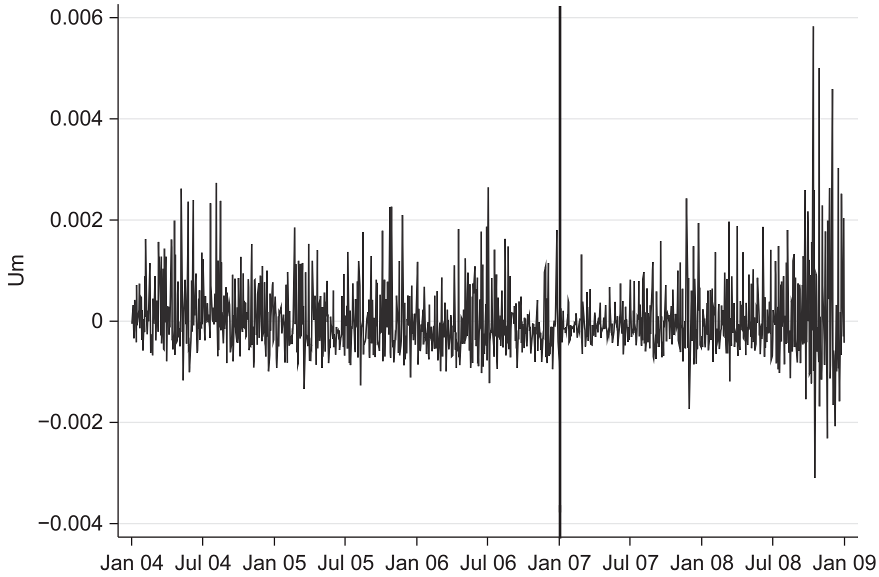

Figure A5: The market unexpected illiquidity $\hat{u}_t^m$ in two periods
Source: Towers Watson calculations based on data from Bloomberg.

Table A1: Panel data results

Constant	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}'_1$	$\hat{\lambda}'_2$	*Wald Test* (x^2) H_0: $\hat{\lambda}_1=\hat{\lambda}_2$ H_a: $\hat{\lambda}_1<\hat{\lambda}_2$	*Wald Test* (x^2) H_0: $\hat{\lambda}_1+\hat{\lambda}'_1=\hat{\lambda}_2+\hat{\lambda}'_2$ H_a: $\hat{\lambda}_1+\hat{\lambda}'_1<\hat{\lambda}_2+\hat{\lambda}'_2$
−0.008 (0.000)	0.016 (0.000)	0.080 (0.000)	0.101 (0.000)	0.207 (0.000)	44.880 (0.000)	80.230 (0.000)

This table reports the estimated coefficients from the panel data regressions (13) with GLS random-effects estimators for 500 time-invariant individual stocks, using daily data during two periods: the boom period from 1 January 2004 to 31 December 2006 and the crisis period from 1 January 2007 to 31 December 2008. The *P*-values are reported in the parentheses.
Source: Towers Watson calculations based on data from Bloomberg.

Table A2: Betas and relevant properties of 25 time-variant illiquidity portfolios[a]

Period	Illiquidity Portfolio	Mcap (USbillion$)	E(C) (*100)	Spread (%)	Beta1 (*100)	Beta2 (*100)	Beta3 (*100)	Beta4 (*100)	Betanet (*100)	R (%)
	2	51.324	0.02	0.023	88.374	0.063	0.588	0.091	87.758	0.03
	4	27.315	0.024	0.027	92.223	0.073	0.375	0.077	91.844	0.027
	6	26.315	0.027	0.029	89.374	0.072	0.397	0.075	88.974	0.053
	8	18.553	0.028	0.03	95.872	0.089	0.408	0.059	95.495	0.072
	10	17.01	0.031	0.032	94.493	0.101	0.723	0.114	93.756	0.056
Boom	12	15.398	0.034	0.035	94.905	0.112	0.55	0.149	94.318	0.069
	14	14.3	0.038	0.039	101.633	0.132	0.591	0.154	101.02	0.088
	16	10.176	0.044	0.043	109.07	0.157	0.598	0.099	108.53	0.082
	18	14.623	0.055	0.052	107.825	0.226	0.478	0.144	107.429	0.115
	20	10.435	0.085	0.075	124.609	0.364	0.688	0.07	124.215	0.148
	22	19.23	2.835	3.005	101.478	5.688	0.64	2.229	104.297	0.064
	2	51.666	0.045	0.059	87.001	0.144	−0.08	−0.131	87.356	−0.114
	4	38.394	0.047	0.054	84.218	0.112	−0.057	−0.009	84.395	−0.078
	6	33.732	0.051	0.055	83.961	0.111	−0.161	−0.079	84.312	−0.117
	8	25.827	0.055	0.058	94.665	0.124	−0.16	−0.028	94.977	−0.047
Crisis	10	17.199	0.063	0.064	102.267	0.155	−0.282	−0.135	102.839	−0.057
	12	18.614	0.062	0.061	94.296	0.142	−0.109	−0.076	94.623	−0.045
	14	18.2	0.062	0.066	97.443	0.15	−0.232	−0.106	97.931	−0.036
	16	17.825	0.078	0.074	108.904	0.188	−0.311	−0.125	109.527	−0.004
	18	12.568	0.083	0.079	100.288	0.199	−0.172	−0.135	100.794	−0.043
	20	15.131	0.126	0.107	122.931	0.376	−0.264	−0.307	123.878	0.025
	22	12.561	0.338	0.391	123.536	0.386	−0.552	−0.813	125.287	0.034

[a]The results for all the time-variant market-cap portfolios and illiquidity portfolios are available from the authors upon request.
This table reports betas and other relevant characteristics of even number of 25 illiquidity portfolios (the odd number portfolios demonstrate similar features) formed each quarter during the boom period from 1 January 2004 to 31 December 2006 and the crisis period from 1 January 2007 to 31 December 2008, respectively. Betanet, Beta1, Beta2, Beta3 and Beta4 are computed as equations (7) – (11) using the entire time series in boom or crisis, with E(C) as average illiquidity cost. Market Cap (Mcap), Stock Return (R) and Bid-Ask Spread Ratio (Spread) are calculated for each portfolio as daily time-series averages in boom or crisis. Portfolio 24 is not reported because its components are less than 20 stocks owing to missing data.
Source: Towers Watson calculations based on data from Bloomberg.

Table A3: Estimation results of Liquidity-Adjusted CAPM

Portfolio type	*Dependent variable*	*Constant*	β^p_{net}	$D \cdot \beta^p_{net}$	R^2
Illiquidity portfolios	$E(R^p_t - R_f) - Turnover \cdot E(C^p_t)$	−0.0129 (0.000)	0.0049 (0.000)	−0.0003 (0.007)	0.6777
Market-cap portfolios	$E(R^p_t - R_f) - Turnover \cdot E(C^p_t)$	−0.0116 (0.000)	0.0036 (0.005)	−0.0003 (0.003)	0.5307

This table reports the cross-sectional regression results of Liquidity-Adjusted CAPM with a dummy variable as equations (16) for 25 time-variant illiquidity portfolios and 25 time-variant market-cap portfolios using calculated betas as Table A2 reports. The *P*-values are in the parentheses.
Source: Towers Watson calculations based on data from Bloomberg.

Table A4: Annualized expected return premium of illiquidity portfolios in crisis

Annualized return premium	$\hat{\gamma}\beta_2$*252(%)	$-\hat{\gamma}\beta_3$*252(%)	$-\hat{\gamma}\beta_4$*252(%)	$\hat{\gamma}(\beta_2-\beta_3-\beta_4)$*252(%)
Portfolio 2	[0.12 0.17]	[0.07 0.09]	[0.11 0.15]	[0.30 0.41]
Portfolio 22	[0.32 0.45]	[0.46 0.64]	[0.68 0.94]	[1.46 2.03]

Source: Towers Watson calculations based on data from Bloomberg.

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