Economic Modelling 29 (2012) 361–368

Contents lists available at SciVerse ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Liquidity-adjusted conditional capital asset pricing model

Jinan Wang [a,1], Langnan Chen [b,*]

[a] Finance and Economics College, Jimei University, Xiamen 361021, China
[b] Institute for Economics and Lingnan (University) College, Sun Yat-sen University, Guangzhou 510275, China

ARTICLE INFO

Article history:
Accepted 18 November 2011

JEL classification:
G0
G11
G12

Keywords:
CAPM
Liquidity
Liquidity risk
Conditional coskewness

ABSTRACT

This paper derives a liquidity-adjusted conditional two-moment capital asset pricing model (CAPM) and a liquidity-adjusted conditional three-moment CAPM respectively based on theory of stochastic discount factor. The liquidity-adjusted conditional two-moment CAPM shows that a security's conditional expected excess return consists of three parts: its conditional expected liquidity cost, the systemic risk premium and the liquidity risk premium. The liquidity-adjusted conditional three-moment CAPM shows that a security's conditional expected excess return depends on its conditional expected liquidity cost, the conditional covariance between its return and the market return, the conditional covariance between its liquidity cost and the market liquidity cost, and the conditional coskewness of its return and the market return.

1. Introduction

Although liquidity is extremely difficult to be defined accurately, it is no doubt that liquidity is very important to the security market. The U.S. stock market crash in 1987, the bankruptcy of LTCM in 1998, and the global financial crisis in recent years clearly indicate that the possibility that liquidity might disappear from a market and might not be available when it is needed is a big source of risk to investors.

The earliest study on liquidity can be traced back to Keynes' descriptive definition of liquidity from the perspective of monetary economics. The recent studies on liquidity have focused on asset pricing and market microstructure disciplines. The scholars in the field of market microstructure usually use bid–ask spread as a proxy for liquidity while the scholars in the field of asset pricing employ transaction costs and the tradability of assets to measure liquidity. This paper intends to investigate the liquidity from the perspective of asset pricing.

The capital asset pricing model (CAPM) (Sharpe (1964); Lintner (1965); Mossin (1966); Black (1972)) has a history of more than forty years. Although the CAPM has received early empirical support (see, for instance, Black et al. (1972); Fama and MacBeth (1973)), it has been challenged on the basis of incompleteness. The incompleteness has been examined by numbers of papers, among which, the most important ones are Fama and French (1992), Kraus and Litzenberger (1976), Harvey and Siddique (2000b), Amihud and Mendelson (1986, 1989), Merton (1987), Jacoby et al. (2000); Acharya and Pedersen (2005), and He and Kryzanowski (2006).

Fama and French (1992) develop a three-factor model which incorporates such variables as size, book-to-market value ratio and price to earnings ratio. Their empirical results show that the CAPM has no explanatory power while size and book-to-market value ratio play an important role in explaining the cross-section of average returns on NYSE, Amex, and NASDAQ stocks for the period 1963–1990. However, their study has been challenged on the basis of methodology, periodic effects, and the data used (see Kenz and Ready (1997) and Kothari et al. (1995)).

Kraus and Litzenberger (1976) extend the CAPM to incorporate the effect of skewness in the return distributions based on the assumption that investors have a preference for positive skewness in their portfolios. They derive an unconditional three-moment CAPM and find that a measure of coskewness can be used as a supplement to the covariance measure of risk to explain the cross-sectional variation of returns on individual NYSE stocks.

Other studies also find that the skewness plays an important role in portfolio pricing. Friend and Westerfield (1980) provide some but no conclusive evidence that the coskewness in addition to covariance is required to explain the returns on individual risky assets, suggesting that investors may be willing to pay a premium for positive skewness in their portfolios. Lee et al. (1996) investigate the importance of coskewness in asset pricing by employing the multivariate test procedure developed by Gibbons (1982) and find that the risk premium of a portfolio should be explained by a weighted average of covariance and coskewness though the coskewness is weaker in explaining risk premium of a portfolio than covariance. As far as portfolios across countries are

* Corresponding author. Tel.: +86 13570115839; fax: +86 2084115553.
E-mail addresses: anjiali@163.com (J. Wang), lnscln@mail.sysu.edu.cn (L. Chen).
[1] Tel.: +86 13959237668.

0264-9993/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.econmod.2011.11.007

concerned, Chunhachinda et al., 1997 find that the coskewness has an important impact on the determination of weighting factors for an optimal investment portfolio. Prakash et al. (2003) also support the view that the coskewness plays an important role in the formation of an optimal investment portfolio.

In addition, Lim (1989) uses GMM to test the three-moment CAPM of Kraus and Litzenberger (1976). The testing method adopted by Lim (1989) circumvents the impact of the errors-in-variable problem and the parameter estimates are statistically consistent. The results of Lim (1989) reveal that the coskewness is crucial in asset pricing. Nevertheless, the Kraus and Litzenberger (1976) three-moment model does not fully explain the expected returns of portfolios.

Harvey and Siddique (1999, 2000a, 2000b) have conducted a few studies on the role of coskewness in portfolio pricing. Harvey and Siddique (1999) propose a method to estimate the autoregressive conditional skewness. Harvey and Siddique (2000a) explore the relationship between the risk premium of market portfolio and time-varying conditional skewness. Harvey and Siddique (2000b) further investigate empirically the role of skewness of different definitions in portfolio pricing based on monthly data from NYSE, AMEX and NASDAQ and find that the conditional coskewness has an important impact on portfolio pricing under either the traditional CAPM or the Fama–French three factor model.

Yang et al. (2010) characterize the conditional coskewness between stock and bond excess returns in the context of a three-moment Intertemporal CAPM specification by utilizing a bivariate regime-switching model and find that both conditional U.S. stock coskewness and bond coskewness command significantly negative ex ante risk premiums from statistical and economic points of view.

Other scholars extend the CAPM from the perspective of liquidity. One of the earliest studies on the relationship between the stock returns and the liquidity is Amihud and Mendelson (1986). Using the bid–ask spread as a measure of liquidity, they empirically test the implications of their theoretical model and find a linear relationship between excess returns and beta, and confirm a concave relationship between the excess returns and the relative spread (bid–ask spread divided by price). Amihud and Mendelson (1989) find that, except for beta, the other three factors identified by Merton (1987) as those related to risk-adjusted returns are no longer significant when relative bid–ask spread is included as an explanatory variable.[2] They conclude that the expected asset returns are positively related to the beta and the relative bid–ask spread. Some other scholars also confirm the positive relationship between the bid–ask spread or trading volume and the returns (see, for instance, Eleswarapu (1997); Amihud et al. (1990); Jones (2001); Datar et al. (1998); and Brennan and Subrahmanyam (1996)). However, there are also a few studies that conclude the negative relationship between the return and the liquidity (Haugen and Baker (1996); Brennan et al. (1998); Datar et al. (1998); Brennan et al. (1998); Chordia et al. (2001); Baker and Stein (2004); and Marshall and Young (2003)).

Chordia et al. (2000) investigate the systematic liquidity risk and its risk premium based on the tick by tick data from NYSE. They also assess the different liquidity measures (quoted spread, and effective spread and depth) and find that the liquidity of an individual stock has certain commonality. Since then the empirical tests on systematic liquidity risk have been growing and a few papers have been published. Huberman and Halka (2001) find the existence of commonality of liquidity among stocks from American Stock Exchange based on the daily data. Emilios and Giouvris (2009) and Martinez et al. (2003) reach the same conclusion based on the British stock market and Spanish stock market respectively. However, Hasbrouk and Seppi (2001) reach the opposite conclusion by employing the high frequency data of 30 stocks including the Dow Jones Industrial Index: no commonality among the liquidity of the individual stocks.

Though there is no clear view regarding the existence of a common factor to individual liquidity risk, most researchers do accept that liquidity risk has commonality. Pastor and Stambaugh (2003) find that market-wide liquidity is an important state variable in asset pricing. By using a measure of liquidity estimated with stock's within-month daily returns and volume, they find that the expected stock returns are related cross-sectionally to the sensitivities of returns to the fluctuation in aggregate liquidity.

Acharya and Pedersen (2005) present a liquidity adjusted CAPM (a 4-β model), in which the liquidity risk is decomposed into 3 parts. The first liquidity risk is that the return increases with the covariance between a security's illiquidity and the market illiquidity. The second liquidity risk is due to covariation between a security's return and the market liquidity. The third liquidity risk is due to covariation between a security's illiquidity and the market return. They use the illiquidity ratio of Amihud (2002) and confirm the existence of a liquidity risk premium.

He and Kryzanowski (2006) construct a weighted β model based on the bid–ask spread. Unlike Acharya and Pedersen (2005), they use the amortized spread as a direct illiquidity measure that is theoretically grounded (Amihud and Mendelson, 1986) and empirically supported (Chalmers and Kadlec, 1998). Their empirical results suggest that the commonality of liquidity plays an important role in asset-pricing beyond that contained in the level of illiquidity.

Eckbo and Norli (2005) find that the bid–ask spread ratios of individual stocks have co-movement feature to some extent and confirm that the liquidity risk resulted from the co-movement is priced in the cross-section portfolios. Gibson and Mougeot (2004) find the evidence that liquidity risk is priced in the US stock markets based on monthly data. Avramov et al. (2002) observe the impact of market liquidity risk on the market portfolio efficiency in the context of a intertemporal capital asset pricing model (ICAPM) specification and find that the market portfolio efficiency can be improved by hedging the market liquidity risk and the improvement is especially significant when short-selling is restricted. This finding suggests that the market liquidity risk is priced from another perspective. Martinez et al. (2003) find that there is no evidence of liquidity risk premium in the Spanish stock market.

Brunnermeier and Pedersen (2009) propose a model that links an asset's market liquidity and traders' funding liquidity. According to their model, margins are destabilizing and market liquidity and funding liquidity are mutually reinforcing, leading to liquidity spirals. Their model provides the following findings: (i) market liquidity's suddenly drying up, (ii) commonality in liquidity, (iii) flight to quality, and (iv) market liquidity's co-movement.

Jiekun Huang (2010) examines the relationship between the expected market volatility and the demand for liquidity in open-end mutual funds. Using data on mutual fund holdings and trading, he finds that mutual fund managers tend to hold more liquid stocks and sell illiquid stocks during periods when the expected market volatility is high, i.e. the phenomenon of dynamic preference for liquidity.

However, no literature that considers both liquidity factor and conditional high moments at the same time has been found until 2010. Chen et al. (2011) propose a capital asset pricing model (CAPM) by incorporating liquidity and skewness factors at the same time. Using the data from Chinese stock market, their empirical results indicate that illiquidity cost, liquidity risk and as well as skewness have important impacts on asset pricing in the Chinese stock market. However, the model proposed in that paper is an unconditional version. This paper derives a liquidity-adjusted conditional two-moment CAPM and a liquidity-adjusted conditional three-moment CAPM respectively by utilizing a general stochastic discount factor pricing framework. Based on the proposed model, the risk premium for a risky asset depends on the four factors: (i) the conditional expected liquidity cost

[2] The four factors are beta, firm size, availability of information about the security (proxied by the fraction of investors who purchase it) and residual risk caused by the application of the traditional CAPM.

of the asset; (ii) the conditional covariance between the asset return and the market return; (iii) the conditional covariance between the asset liquidity cost and the market liquidity cost; and (iv) the conditional coskewness of the asset return and the market return.

The remainder of the paper is organized as follows. Section 2 presents the assumptions for the proposed models. Section 3 derives a liquidity-adjusted conditional two-moment CAPM. Section 4 derives a liquidity-adjusted conditional three-moment CAPM. Section 5 concludes. Proofs are in Appendix 1.

2. Assumptions

Consider a simple two period economy, in which a new generation of agents is born at period t and die at period $t+1$. Generation consists of N agents, n = 1, 2,..., N, who live for two periods: t and $t+1$. These agents are not only investors but also consumers. Agent n has wealth W_t at period t and W_{t+1} at period $t+1$.The consumption level of the agent is denoted by C. Assume that there are I risky assets (i = 1, 2,..., I) and one risk free asset. Denote ξ numbers of asset i a representative investor chooses to buy at price $P_{i,t}$ at period t. He will sell the asset at price $P_{i,t+1}$ at period $t+1$ with a round-trip liquidity cost of $l_{i,t+1}$.[3] The dividend of asset is $D_{i,t+1}$ to be paid at period $t+1$. Hence, the gross return of asset i is $R_{i,t+1}=\frac{D_{i,t+1}+P_{i,t+1}}{P_{i,t}}$, the net return of asset i is $R^*_{i,t+1}=\frac{P_{i,t+1}+D_{i,t+1}-l_{i,t+1}}{P_{i,t}}=R_{i,t+1}-L_{i,t+1}$, the net market return is $R_{M,t+1}{}^*=R_{M,t+1}-L_{M,t+1}$[4], and the relative liquidity cost is $L_{i,t+1}=\frac{l_{i,t+1}}{P_{i,t}}$. In this paper, we assume that the relative liquidity costs of risky asset are normally distributed and the returns are asymmetrically distributed. Furthermore, we assume that the relative liquidity costs and the returns of risky asset are not correlated, i.e. $\mathrm{cov}(R_i,L_i)=0$.[5]

The representative investor derives utility from consumption c_t at period t and consumption c_{t+1} at period $t+1$. A utility function defining current and future consumption is:

$$U(c_t,c_{t+1})=u(c_t)+\beta E_t[u(c_{t+1})]$$

where β is the subjective discount factor, which captures investors' impatience and their aversion to risk. We use a convenient utility form, $u(c_t)=\frac{1}{1-\gamma}c_t^{1-\gamma}$. The limit, as $\gamma\to1$, is $u(c)=\ln(c)$. Obviously, the utility function satisfies the following three conditions (Arrow, 1971):

A1) Marginal utility is positive;
A2) Marginal utility is decreasing;
A3) Absolute risk aversion is non-increasing.

Note that Condition A1 captures the fundamental desire for more consumption, rather than a desire for intermediate objectives such as mean and variance of portfolio returns. Here consumption c_{t+1} is random, or the investor does not know his wealth tomorrow and hence how much he will decide to consume. Condition A2 (i.e. $U''<0$) implies that investors exhibit aversion variance whereas Condition A3 (i.e. $U'''>0$) implies that the investor prefers positive skewness.[6]

Assume that an investor can freely buy or sell as many of risky assets as he wishes, he needs to choose the optimal numbers of risky assets to trade so as to maximize his utility. Then the representative investor's portfolio selection is:

$$\max_{\{\xi\}} u(c_t)+E_t\beta u(c_{t+1})$$
$$s.t.\begin{cases} c_t=W_t-P_{i,t}\xi \\ c_{t+1}=W_{t+1}+\left(P_{i,t+1}+D_{i,t+1}-l_{i,t+1}\right)\xi \end{cases}.$$

Substituting the constraints into the objective function and setting the derivative with respect to ξ equal to zero, we obtain the first-order condition for an optimal consumption and portfolio choice:

$$P_{i,t}u'(c_t)=E_t\left[\beta u'(c_{t+1})\left(P_{i,t+1}+D_{i,t+1}-l_{i,t+1}\right)\right]. \tag{1}$$

Eq. (1) can be rewritten as

$$E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\left(\frac{P_{i,t+1}+D_{i,t+1}-l_{i,t+1}}{P_{i,t}}\right)\right]=1. \tag{2}$$

Denote $\beta\frac{u'(c_{t+1})}{u'(c_t)}$, the marginal rate of substitution of the investor between period t and $t+1$, as m_{t+1} which can be viewed a stochastic discount factor, and substitute $R^*_{i,t+1}=R_{i,t+1}-L_{i,t+1}=\frac{P_{i,t+1}+D_{i,t+1}-l_{i,t+1}}{P_{i,t}}$ into Eq. (2), the first-order condition can simply be expressed as

$$E_t\left[R^*_{i,t+1}m_{t+1}\right]=1 \tag{3}$$

Eq. (3) can be rewritten as

$$Cov_t\left[m_{t+1},R^*_{i,t+1}\right]+E_t\left[R^*_{i,t+1}\right]E_t\left[m_{t+1}\right]=1 \tag{4}$$

$$\text{Or } E_t\left[R^*_{i,t+1}\right]=\frac{1}{E_t\left[m_{t+1}\right]}-\frac{Cov_t\left[m_{t+1},R^*_{i,t+1}\right]}{E_t\left[m_{t+1}\right]}. \tag{5}$$

In the following two parts of this paper, we derive a liquidity-adjusted conditional two-moment CAPM and a liquidity-adjusted conditional three-moment CAPM respectively by utilizing the first-order condition, i.e. Eqs. (3), (4), and (5).

3. Liquidity-adjusted conditional two-moment CAPM

In order to derive a liquidity-adjusted conditional two-moment CAPM, we assume that the stochastic discount factor is linear to the net market return[7]:

$$m_{t+1}=a_t+b_tR^*_{M,t+1}. \tag{6}$$

Assume that a conditional risk free asset exists and its return is represented by R_F, according to the definition of m_{t+1}, for risk free asset and market portfolio, we have

$$E_t\left[R_Fm_{t+1}\right]=R_FE_t\left[m_{t+1}\right]=1 \tag{7}$$

$$E_t\left[R^*_{M,t+1}m_{t+1}\right]=1. \tag{8}$$

[3] No transaction cost is assumed for the risk free asset in this paper.

[4] Here "net" means to consider the liquidity cost.

[5] This assumption is critical to this paper, the rational is as follows. Many liquidly shocks are due to innovations in trading system and trading technology, thus they are not related to the cash flows of a company. Amihud and Mendelson (1986), Amihud (2002), and Barber and Odean (2002) hold the same view. However, there are also studies such as Baker and Stein (2004) and Acharya and Pedersen (2005) who have the opposite view.

[6] $d\left[-U''/U'\right]/dW\le0\Rightarrow\left[-U'U'''+\left(U''\right)^2\right]/\left(U'\right)^2 0\Rightarrow U'''\ge\left(U''\right)^2/U'0.$

[7] The logarithmic utility form of representative agent we assumed in part 2 of this paper guarantees that the discount factor is linear in the value weighted portfolio of wealth. If we relate the discount factor to the marginal rate of substitution between period t and $t+1$ in a two-period economy, we can receive some more implications. Also, see Harvey and Siddique (2000a, 2000b) for more detailed discussions on the relation between the stochastic discount factor and the marginal rate of substitution.

From Eqs. (6), (7) and (8), we can derive a liquidity-adjusted conditional two-moment CAPM:[8]

$$E_t\left[R_{i,t+1}\right]-R_F=E_t\left[L_{i,t+1}\right]+\frac{Cov_t\left[R_{M,t+1},R_{i,t+1}\right]}{Var_t\left[R_{M,t+1}-L_{M,t+1}\right]}E_t\left[R_{M,t+1}-R_F-L_{M,t+1}\right]+\frac{Cov_t\left[L_{M,t+1},L_{i,t+1}\right]}{Var_t\left[R_{M,t+1}-L_{M,t+1}\right]}E_t\left[R_{M,t+1}-R_F-L_{M,t+1}\right]. \tag{9}$$

Eq. (9) can be expressed as:

$$E_t\left[r_{i,t+1}\right]=E_t\left[L_{i,t+1}\right]+\frac{Cov_t\left[r_{M,t+1},r_{i,t+1}\right]}{Var_t\left[r_{M,t+1}-L_{M,t+1}\right]}E_t\left[r_{M,t+1}-L_{M,t+1}\right]+\frac{Cov_t\left[L_{M,t+1},L_{i,t+1}\right]}{Var_t\left[r_{M,t+1}-L_{M,t+1}\right]}E_t\left[r_{M,t+1}-L_{M,t+1}\right] \tag{10}$$

where

$r_{i,\,t+1}(=R_{i,\,t+1}-R_F)$ represents return of individual stock in excess of the risk-free return;
$r_{M,\,t+1}(=R_{M,\,t+1}-R_F)$ represents market return in excess of the risk-free return.

Eq. (10) clearly demonstrates that the risk premium for a risky asset can be decomposed into three parts:

(1) the conditional expected liquidity cost of asset;
(2) the product of fundamental beta (or covariance between the asset return and the market return) times net risk premium;
(3) the product of liquidity beta (or covariance between the asset liquidity cost and the market liquidity cost) times net risk premium.

As in the standard CAPM, the required return on an asset increases linearly with the market beta, that is, the covariance between an asset's return and the market return. Our model differs from the standard CAPM in considering the level of liquidity cost and the commonality of liquidity. According to our model, an investor requires not only the premium for the level of liquidity cost, but also the premium for liquidity risk. In other words, besides the level of liquidity cost, the required return on an asset increases linearly with the liquidity beta, i.e. the covariance between the asset liquidity cost and the market liquidity cost. This is because investors want to be compensated for holding a security that becomes illiquid (i.e. high liquidity cost) when the market in general becomes illiquid.

In fact, many studies (to name a few, Chordia et al. (2000); Hasbrouk and Seppi (2001); and Huberman and Halka (2001).) find that most stocks' illiquidity is positively related to the market illiquidity. Therefore, an investor requires risk premium because of the commonality-in-liquidity effect. Acharya and Pedersen (2005) find that the commonality of liquidity, as captured by covariance between the asset liquidity and the market liquidity, is one of the three liquidity risks. They argue that the risk premium associated with commonality in liquidity is caused by the wealth effects of illiquidity.

[8] See Appendix for the derivation.

If we assume that the relative liquidity cost is related to the return like Barber and Odean (2002) and Acharya and Pedersen (2005), i.e., our liquidity-adjusted conditional two-moment CAPM can then be rewritten as

$$E_t\left[R_{i,t+1}\right]-R_F=E_t\left[L_{i,t+1}\right]+\left(\frac{Cov_t\left[R_{M,t+1},R_{i,t+1}\right]}{Var_t\left[R_{M,t+1}-L_{M,t+1}\right]}+\frac{Cov_t\left[L_{M,t+1},L_{i,t+1}\right]}{Var_t\left[R_{M,t+1}-L_{M,t+1}\right]}-\frac{Cov_t\left[R_{i,t+1},L_{M,t+1}\right]}{Var_t\left[R_{M,t+1}-L_{M,t+1}\right]}-\frac{Cov_t\left[L_{i,t+1},R_{M,t+1}\right]}{Var_t\left[R_{M,t+1}-L_{M,t+1}\right]}\right)E_t\left[R_{M,t+1}-L_{M,t+1}-R_F\right]. \tag{11}$$

While LCAPM of Acharya and Pedersen (2005) is

$$E_t\left[r_{t+1}^i\right]-r^f=E_t\left[c_{t+1}^i\right]+\left(\frac{Cov_t\left[r_{t+1}^i,r_{t+1}^M\right]}{Var_t\left[r_{t+1}^M-c_{t+1}^M\right]}+\frac{Cov_t\left[c_{t+1}^i,c_{t+1}^M\right]}{Var_t\left[r_{t+1}^M-c_{t+1}^M\right]}-\frac{Cov_t\left[r_{t+1}^i,c_{t+1}^M\right]}{Var_t\left[r_{t+1}^M-c_{t+1}^M\right]}-\frac{Cov_t\left[c_{t+1}^i,r_{t+1}^M\right]}{Var_t\left[r_{t+1}^M-c_{t+1}^M\right]}\right)E_t\left[r_{t+1}^M-c_{t+1}^M-r^f\right] \tag{12}$$

where

r^f	is the market risk-free interest rate;
r_{t+1}^i	is an asset's expected (gross) return;
c_{t+1}^i	is an asset's relative illiquidity cost;
r_{t+1}^M	is the market return;
c_{t+1}^M	is the relative market illiquidity.

After comparing Eq. (11) with Eq. (12), it is not difficult to find that our model, i.e. Eq. (11), which is derived under the assumption of $\mathrm{cov}(R_i,L_i)\neq 0$, is the same as LCAPM of Acharya and Pedersen (2005). If Acharya and Pedersen (2005) assume that the relative illiquidity cost is not related to the return, their model (i.e. Eq. (12)) can be rewritten as

$$E_t\left[r_{t+1}^i\right]=r^f+E_t\left[c_{t+1}^i\right]+\left(\frac{Cov_t\left[r_{t+1}^i,r_{t+1}^M\right]}{Var_t\left[r_{t+1}^M-c_{t+1}^M\right]}+\frac{Cov_t\left[c_{t+1}^i,c_{t+1}^M\right]}{Var_t\left[r_{t+1}^M-c_{t+1}^M\right]}\right)\times E_t\left[r_{t+1}^M-c_{t+1}^M-r^f\right]. \tag{13}$$

It is easy to find that Eq. (13) is the same as our model or Eq. (9).

In fact, our liquidity-adjusted conditional two-moment CAPM (i.e. Eq. (9)) is most closely related to a cross-sectional model developed by He and Kryzanowski (2006). In their paper, they also assume that the fundamental return process is uncorrelated with the spread ratio process and focus on the relative importance between the level of liquidity cost and the commonality of liquidity. The difference is that they use amortized spread which is developed by Chalmers and Kadlec in 1998 to measure liquidity. Their model is as follows:

$$E\left[\tilde{R}_j\right]=R_z+\beta_j^w\left(E\left[\tilde{R}_m\right]-R_z\right)+\left(E\left[\tilde{S}_j^a\right]-E\left[\tilde{S}_m^a\right]\right) \tag{14}$$

where

R_f	is the market observed risk-free rate;
$E\left[\tilde{S}_j^a\right]$	is the expected amortized spreads of asset j;
$E\left[\tilde{S}_m^a\right]$	is the expected amortized spreads of the market portfolio;

$R_z = R_f + E\left[\tilde{S}_m^a\right]$ is interpreted as the market equilibrium zero-beta rate;
$\beta_j^w = \rho^v \beta_j^v + \rho^s \beta_j^s$ is the weighted average of the fundamental and spread betas;
$\beta_j^v = \frac{\mathrm{cov}\left(\tilde{R}_j, \tilde{R}_m\right)}{\mathrm{var}\left(\tilde{R}_m\right)}$ is the fundamental beta;
$\beta_j^s = \frac{\mathrm{cov}\left(\tilde{S}_j^a, \tilde{S}_m^a\right)}{\mathrm{var}\left(\tilde{S}_m^a\right)}$ is the spread beta;
$\rho^v = \frac{\mathrm{var}\left[\tilde{R}_m\right]}{\mathrm{var}\left[\tilde{R}_m\right] + \mathrm{var}\left[\tilde{S}_m^a\right]}$ is the proportional weight of the fundamental beta;
$\rho^s = \frac{\mathrm{var}\left[\tilde{S}_m^a\right]}{\mathrm{var}\left[\tilde{R}_m\right] + \mathrm{var}\left[\tilde{S}_m^a\right]}$ is the proportional weight of the spread beta.

Their model can be expressed as:

$$E\left[\tilde{R}_j\right] = R_f + E\left[\tilde{S}_j^a\right] + \rho^v \frac{Cov\left[\tilde{R}_j, \tilde{R}_m\right]}{Var\left[\tilde{R}_m\right]}\left(E\left[\tilde{R}_m\right] - R_z\right) + \rho^s \frac{Cov\left[\tilde{S}_j^a, \tilde{S}_m^a\right]}{Var\left[\tilde{S}_m^a\right]}\left(E\left[\tilde{R}_m\right] - R_z\right). \tag{15}$$

Comparing Eq. (9) with Eq. (15), it is not difficult to find that both our model and their model have variable measuring of the level of liquidity cost and variable measuring of the liquidity risk. The difference is that they use amortized spread to measure the liquidity while we use relative liquidity cost.

In a word, our liquidity-adjusted conditional two-moment CAPM based on the theory of stochastic discount factor and other liquidity-adjusted models have much in common.

Many researches have concluded that the rate of return on the risky assets is asymmetrically distributed. Under the condition that all investors hold the market portfolios, investors would be willing to pay for a premium for assets that possess the positive coskewness with the market if that portfolio is characterized by positive skewness. If the market has negative skewness, investors will be averse to positive coskewness with the market. Therefore, we need to consider moments higher than two orders when we develop capital asset pricing model. However, the standard CAPM considers only mean and variance when pricing risky assets. As a result, pricing bias is inevitable in the standard CAPM. Our liquidity-adjusted conditional two-moment CAPM has the same shortfall as the standard CAPM. For this reason, it is natural to extend the liquidity-adjusted conditional two-moment CAPM by incorporating the effect of skewness on valuation. In the next part of the paper, we derive a liquidity-adjusted conditional three-moment CAPM by using the first-order condition equation again.

4. Liquidity-adjusted conditional three-moment CAPM

In order to derive a liquidity-adjusted conditional three-moment CAPM, we assume that the stochastic discount factor is quadratic in the net market return,[9] that is

$$m_{t+1} = a_t + b_t R_{M,t+1}^* + c_t R_{M,t+1}^{*2}. \tag{16}$$

According to the definition of m_{t+1}, for risk free asset, market portfolio and the asset that has return of $R_{M,t+1}^{*2}$, we have

$$\begin{cases} 1 = E_t\left[R_{M,t+1}^* m_{t+1}\right] \\ 1 = E_t\left[R_{M,t+1}^{*2} m_{t+1}\right] \\ 1 = E_t\left[m_{t+1}\right] R_F \end{cases}. \tag{17}$$

From Eqs. (5), (16), and (17), we have[10]

$$E_t\left[r_{i,t+1}\right] = E_t\left[L_{i,t+1}\right] + \lambda_{1,t}^* Cov_t\left[r_{i,t+1}, r_{M,t+1}\right] + \lambda_{2,t}^* Cov_t\left[L_{i,t+1}, L_{M,t+1}\right] + \lambda_{3,t}^* Cov_t\left[r_{i,t+1}, r_{M,t+1}^2\right] \tag{18}$$

where

$$r_{i,t+1}^* = r_{i,t+1} - L_{i,t+1};\ r_{M,t+1}^* = r_{M,t+1} - L_{M,t+1};$$

$$\lambda_{1,t}^* = \frac{Var_t\left[r_{M,t+1}^{*2}\right] E_t\left[r_{M,t+1}^*\right] - Skew_t\left[r_{M,t+1}^*\right] E_t\left[r_{M,t+1}^{*2}\right]}{Var_t\left[r_{M,t+1}^*\right] Var_t\left[r_{M,t+1}^{*2}\right] - \left(Skew_t\left[r_{M,t+1}^*\right]\right)^2} + 2E_t\left[L_{M,t+1}\right];$$

$$\lambda_{2,t}^* = \frac{Var_t\left[r_{M,t+1}^{*2}\right] E_t\left[r_{M,t+1}^*\right] - Skew_t\left[r_{M,t+1}^*\right] E_t\left[r_{M,t+1}^{*2}\right]}{Var_t\left[r_{M,t+1}^*\right] Var_t\left[r_{M,t+1}^{*2}\right] - \left(Skew_t\left[r_{M,t+1}^*\right]\right)^2} + 2E_t\left[r_{M,t+1}\right].$$

$$\lambda_{3,t}^* = \frac{Var_t\left[r_{M,t+1}^*\right] E_t\left[r_{M,t+1}^{*2}\right] - Skew_t\left[r_{M,t+1}^*\right] E_t\left[r_{M,t+1}^*\right]}{Var_t\left[r_{M,t+1}^*\right] Var_t\left[r_{M,t+1}^{*2}\right] - \left(Skew_t\left[r_{M,t+1}^*\right]\right)^2}.$$

Eq. (18) clearly demonstrates that the risk premium for a risky asset depends on the following four factors:

(1) The conditional expected liquidity cost of the asset;
(2) The conditional covariance between the asset return and the market return;
(3) The conditional covariance between the asset liquidity cost and the market liquidity cost;
(4) The conditional coskewness of the asset return and the market return.

As in the three moment CAPM of Harvey and Siddique (2000a, 2000b), the required return on an asset increases linearly with the market beta, that is, the covariance between an asset's return and the market return, and increases linearly with the market gamma, that is, the coskewness of the asset return and the market return.This model differs from the three moments CAPM of Harvey and Siddique (2000a, 2000b) in considering the level of liquidity cost and the commonality of liquidity. This model differs from LCAPM of Acharya and Pedersen (2005) in considering higher moments. According to our model, investors require not only the premium for the market beta and gamma, but also premium for the level of liquidity cost and the liquidity risk. That is, besides the level of liquidity cost, the required return on an asset increases linearly with the market beta, the market gamma and the liquidity beta, i.e. the covariance between the asset liquidity cost and the market liquidity cost.

Eq. (18) can be rewritten as:

$$E_t\left[r_{i,t+1}\right] = E_t\left[L_{i,t+1}\right] + 2E_t\left[L_{M,t+1}\right]\left(Cov_t\left[r_{i,t+1}, r_{M,t+1}\right] - Cov_t\left[L_{i,t+1}, L_{M,t+1}\right]\right) + A_t E_t\left[r_{M,t+1} - L_{M,t+1}\right] + B_t E_t\left[\left(r_{M,t+1} - L_{M,t+1}\right)^2\right] \tag{19}$$

$$\text{Or } E_t\left[R_{i,t+1}\right] - R_z = A_t E_t\left[r_{M,t+1}^*\right] + B_t E_t\left[r_{M,t+1}^{*2}\right] \tag{20}$$

[9] See Harvey and Siddique (2000a, 2000b) for more detailed discussions on the relation between the stochastic discount factor and the marginal rate of substitution.

[10] See Appendix for the derivation.

where

$$R_z = R_F + E_t\left[L_{i,t+1}\right] + 2E_t\left[L_{M,t+1}\right]\left(Cov_t\left[r_{i,t+1}, r_{M,t+1}\right] - Cov_t\left[L_{i,t+1}, L_{M,t+1}\right]\right);$$

$$A_t = A1_t + A2_t + A3_t;$$

$$B_t = B1_t + B2_t + B3_t;$$

$$A1_t = \left(\frac{Var_t\left[r^{*2}_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right]Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2}\right)Cov_t\left[r_{i,t+1}, r_{M,t+1}\right];$$

$$A2_t = \left(\frac{Var_t\left[r^{*2}_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right]Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2} + 2\right)Cov_t\left[L_{i,t+1}, L_{M,t+1}\right];$$

$$A3_t = \left(\frac{-Skew_t\left[r^*_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right]Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2}\right)Cov_t\left[r_{i,t+1}, r^2_{M,t+1}\right];$$

$$B1_t = \left(\frac{-Skew_t\left[r^*_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right]Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2}\right)Cov_t\left[r_{i,t+1}, r_{M,t+1}\right];$$

$$B2_t = \left(\frac{-Skew_t\left[r^*_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right]Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2}\right)Cov_t\left[L_{i,t+1}, L_{M,t+1}\right];$$

$$B3_t = \left(\frac{Var_t\left[r^*_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right]Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2}\right)Cov_t\left[r_{i,t+1}, r^2_{M,t+1}\right].$$

Eq. (20) is an empirically testable restriction imposed on the cross section of expected asset returns by the asset pricing model incorporating skewness and liquidity. Here $A1_t$ and $B1_t$ are analogous to the beta in the standard CAPM. $A2_t$ and $B2_t$ are analogous to the liquidity beta in the LCAPM of Acharya and Pedersen (2005). $A3_t$ and $B3_t$ are analogous to the gamma in the three moments CAPM of Kraus and Litzenberger (1976).

5. Conclusion

This paper derives a liquidity-adjusted conditional two-moment capital asset pricing model (CAPM) and a liquidity-adjusted conditional three-moment CAPM respectively based on the theory of stochastic discount factor. The liquidity-adjusted conditional two-moment CAPM shows that a security's conditional expected excess return consists of three parts: its conditional expected liquidity cost, the systemic risk premium and the liquidity risk premium. The liquidity-adjusted conditional three-moment CAPM shows that a security's conditional expected excess return depends on its conditional expected liquidity cost, the conditional covariance between its return and the market return, the conditional covariance between its liquidity cost and the market liquidity cost, and the conditional coskewness of its return and the market return.

Acknowledgments

This research is supported by China Natural Science Foundation under Grant No. 70673116, Huifeng Finance Institute of Peking University, the 985 Project, China Social Science Foundation under Grant No. 08AJL007 and No. 09CJY083, Fujian Social Science Project under Grant No. 2011B170, Guangdong Natural Science Foundation and Guangdong Social Science Project.

Appendix 1

This appendix presents the proofs of Eqs. (9) and (18) respectively.

Proof of Eq. (9).

Assume $m_{t+1} = a_t + b_t R^*_{M,t+1}$. (a1)

Taking expectations for Eq. (a1) and notice that $E_t[m_{t+1}] = \frac{1}{R_F}$, we have

$$a_t = \frac{1}{R_F} - b_t E_t\left[R^*_{M,t+1}\right] \tag{a2}$$

$E_t[R_{M,t+1}{}^*m_{t+1}] = 1$ can be expressed as

$$E_t\left[\left(R_{M,t+1} - L_{M,t+1}\right)\right] = \frac{1}{E_t\left[m_{t+1}\right]} - \frac{Cov_t\left[m_{t+1}, \left(R_{M,t+1} - L_{M,t+1}\right)\right]}{E_t\left[m_{t+1}\right]} \tag{a3}$$

Substituting R_F for $\frac{1}{E_t[m_{t+1}]}$ in Eq. (a3), we have

$$E_t\left[\left(R_{M,t+1} - L_{M,t+1}\right)\right] = R_F - R_F Cov_t\left[m_{t+1}, \left(R_{M,t+1} - L_{M,t+1}\right)\right]. \tag{a4}$$

Substituting Equation $m_{t+1} = a_t + b_t R_{M,t+1}{}^*$ into Eq. (a4), we have

$$b_t = -\frac{E_t\left[R_{M,t+1} - R_F - L_{M,t+1}\right]}{Var_t\left[R_{M,t+1} - L_{M,t+1}\right]R_F}. \tag{a5}$$

Substituting $E_t[m_{t+1}] = \frac{1}{R_F}$, Eqs. (a1) and (a5) into the first-order condition equation $E_t\left[\left(R_{i,t+1} - L_{i,t+1}\right)\right] = \frac{1}{E_t[m_{t+1}]} - \frac{Cov_t\left[m_{t+1}, \left(R_{i,t+1} - L_{i,t+1}\right)\right]}{E_t[m_{t+1}]}$, we have

$$E_t\left[R_{i,t+1} - L_{i,t+1} - R_F\right] = \frac{Cov_t\left[\left(R_{M,t+1} - L_{M,t+1}\right), \left(R_{i,t+1} - L_{i,t+1}\right)\right]}{Var_t\left[R_{M,t+1} - L_{M,t+1}\right]}E_t\left[R_{M,t+1} - R_F - L_{M,t+1}\right]. \tag{a6}$$

Notice that $cov(R_i, L_i) = 0$, then we have

$$E_t\left[R_{i,t+1}\right] - R_F = E_t\left[L_{i,t+1}\right] + \frac{Cov_t\left[R_{M,t+1}, R_{i,t+1}\right]}{Var_t\left[R_{M,t+1} - L_{M,t+1}\right]}E_t\left[R_{M,t+1} - R_F - L_{M,t+1}\right] + \frac{Cov_t\left[L_{M,t+1}, L_{i,t+1}\right]}{Var_t\left[R_{M,t+1} - L_{M,t+1}\right]}E_t\left[R_{M,t+1} - R_F - L_{M,t+1}\right]. \tag{a7}$$

Q.E.D.

Proof of Eq. (18).

Assume $m_{t+1} = a_t + b_t R^*_{M,t+1} + c_t R^{*2}_{M,t+1}$. (b1)

According to the definition of m_{t+1}, for risk free asset, market portfolio and the asset which has return of $R^{*2}_{M,t+1}$, we have

$$\begin{cases} 1 = E_t\left[R^*_{M,t+1} m_{t+1}\right] \\ 1 = E_t\left[R^{*2}_{M,t+1} m_{t+1}\right] \\ 1 = E_t\left[m_{t+1}\right] R_F \end{cases}$$

$\Rightarrow$

$$\begin{pmatrix} E_t\left[R^*_{M,t+1}\right] & E_t\left[R^{*2}_{M,t+1}\right] & E_t\left[R^{*3}_{M,t+1}\right] \\ E_t\left[R^{*2}_{M,t+1}\right] & E_t\left[R^{*3}_{M,t+1}\right] & E_t\left[R^{*4}_{M,t+1}\right] \\ 1 & E_t\left[R^*_{M,t+1}\right] & E_t\left[R^{*2}_{M,t+1}\right] \end{pmatrix} \begin{pmatrix} a_t \\ b_t \\ c_t \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1/R_F \end{pmatrix}$$

$\Rightarrow$

$$\begin{cases} a_t = \dfrac{1}{R_F} - b_t E_t\left[R^*_{M,t+1}\right] - c_t E_t\left[R^{*2}_{M,t+1}\right] \\ b_t = -\dfrac{E_t\left[r^*_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right] R_F} - \dfrac{c_t Skew_t\left[r^*_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right]} \\ c_t = \dfrac{Var_t\left[r^*_{M,t+1}\right]\left(R_F - 1 - E_t\left[R^{*2}_{M,t+1}\right]\right) + Skew_t\left[r^*_{M,t+1}\right] E_t\left[r^*_{M,t+1}\right]}{\left(Var_t\left[r^*_{M,t+1}\right] Var_t\left[R^{*2}_M\right] - Skew^2_t\left[r^*_{M,t+1}\right]\right) R_F} \end{cases}.$$

Inserting a_t, b_t, c_t and Eq. (b1) into the following first-order condition equation $E_t\left[R^*_{i,t+1}\right] = \frac{1}{E_t[m_{t+1}]} - \frac{Cov_t\left[m_{t+1}, R^*_{i,t+1}\right]}{E_t[m_{t+1}]}$, after arrangements we have

$$E_t\left[r^*_{i,t+1}\right] = \lambda_{1,t} Cov_t\left[r^*_{i,t+1}, r^*_{M,t+1}\right] + \lambda_{2,t} Cov_t\left[r^*_{i,t+1}, r^{*2}_{M,t+1}\right] \tag{b2}$$

where

$$\lambda_{1,t} = \frac{Var_t\left[r^{*2}_{M,t+1}\right] E_t\left[r^*_{M,t+1}\right] - Skew_t\left[r^*_{M,t+1}\right] E_t\left[r^{*2}_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right] Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2};$$

$$\lambda_{2,t} = \frac{Var_t\left[r^*_{M,t+1}\right] E_t\left[r^{*2}_{M,t+1}\right] - Skew_t\left[r^*_{M,t+1}\right] E_t\left[r^*_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right] Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2}.$$

Plugging $r^*_{i,t+1} = R_{i,t+1} - R_F - L_{i,t+1} = r_{i,t+1} - L_{i,t+1}$ and $r^*_{M,t+1} = R_{M,t+1} - R_F - L_{M,t+1} = r_{M,t+1} - L_{M,t+1}$ into Eq. (b2) yields the following result:

$$\begin{aligned} E_t\left[r_{i,t+1}\right] &= E_t\left[L_{i,t+1}\right] + \lambda_{1,t} Cov_t\left[\left(r_{i,t+1} - L_{i,t+1}\right), \left(r_{M,t+1} - L_{M,t+1}\right)\right] \\ &\quad + \lambda_{2,t} Cov_t\left[\left(r_{i,t+1} - L_{i,t+1}\right), \left(r_{M,t+1} - L_{M,t+1}\right)^2\right] \\ &= E_t\left[L_{i,t+1}\right] + \lambda_{1,t} Cov_t\left[r_{i,t+1}, r_{M,t+1}\right] + \lambda_{1,t} Cov_t\left[L_{i,t+1}, L_{M,t+1}\right] \\ &\quad - 2E_t\left(L_{M,t+1}\right) Cov_t\left[r_{i,t+1}, r_{M,t+1}\right] + \lambda_{2,t} Cov_t\left[r_{i,t+1}, r^2_{M,t+1}\right] \\ &\quad - \lambda_{2,t} Cov_t\left[L_{i,t+1}, L^2_{M,t+1}\right] + 2E_t\left(r_{M,t+1}\right) Cov_t\left[L_{i,t+1}, L_{M,t+1}\right] \\ &= E_t\left[L_{i,t+1}\right] + \left(\lambda_{1,t} - 2E_t\left(L_{M,t+1}\right)\right) Cov_t\left[r_{i,t+1}, r_{M,t+1}\right] \\ &\quad + \left(\lambda_{1,t} + 2E_t\left(r_{M,t+1}\right)\right) Cov_t\left[L_{i,t+1}, L_{M,t+1}\right] + \lambda_{2,t} Cov_t\left[r_{i,t+1}, r^2_{M,t+1}\right] \\ &= E_t\left[L_{i,t+1}\right] + \lambda^*_{1,t} Cov_t\left[r_{i,t+1}, r_{M,t+1}\right] + \lambda^*_{2,t} Cov_t\left[L_{i,t+1}, L_{M,t+1}\right] \\ &\quad + \lambda^*_{3,t} Cov_t\left[r_{i,t+1}, r^2_{M,t+1}\right] \end{aligned}$$

where

$$\lambda^*_{1,t} = \frac{Var_t\left[r^{*2}_{M,t+1}\right] E_t\left[r^*_{M,t+1}\right] - Skew_t\left[r^*_{M,t+1}\right] E_t\left[r^{*2}_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right] Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2} + 2E_t\left[L_{M,t+1}\right];$$

$$\lambda^*_{2,t} = \frac{Var_t\left[r^{*2}_{M,t+1}\right] E_t\left[r^*_{M,t+1}\right] - Skew_t\left[r^*_{M,t+1}\right] E_t\left[r^{*2}_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right] Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2} + 2E_t\left[r_{M,t+1}\right];$$

$$\lambda^*_{3,t} = \frac{Var_t\left[r^*_{M,t+1}\right] E_t\left[r^{*2}_{M,t+1}\right] - Skew_t\left[r^*_{M,t+1}\right] E_t\left[r^*_{M,t+1}\right]}{Var_t\left[r^*_{M,t+1}\right] Var_t\left[r^{*2}_{M,t+1}\right] - \left(Skew_t\left[r^*_{M,t+1}\right]\right)^2}.$$

Q.E.D

References

Acharya, Viral V., Pedersen, L.H., 2005. Asset pricing with liquidity risk. Journal of Financial Economics 77, 375–410.
Amihud, Y., 2002. Illiquidiy and stock returns: cross-section and time-series effects. Journal of Financial Markets 5, 31–56.
Amihud, Y., Mendelson, H., 1986. Asset pricing and the bid–ask spread. Journal of Financial Economics 17, 223–249.
Amihud, Y., Mendeleson, H., 1989. The effect of beta, bid-ask spread, residual risk and size on stock returns. Journal of Finance 44, 479–486.
Amihud, Y., Mendelson, H., Wood, R., 1990. Liquidity and the 1987 stock market crash. Journal of Portfolio Management 65–69 (Spring).
Arrow, K., 1971. Essay in the Theory of Risk-Bearing. Markham Publishing Co., Chicago.
Avramov, D., Chao, J., Chordia, T., 2002. Hedging against liquidity risk and short sale constraints. Working Paper. University of Maryland.
Baker, M., Stein, J.C., 2004. Market liquidity as a sentiment indicator. Journal of Financial Markets 7, 271–299.
Barber, B.M., Odean, T., 2002. Online investors: do the slow die first? Review of Financial Studies 15, 455–487.
Black, F., 1972. Capital market equilibrium with restricted borrowing. Journal of Business 45, 444–455.
Black, F., Jensen, M.C., Scholes, M., 1972. The capital asset pricing model: Some empirical tests. In: Jensen, M.C. (Ed.), Studies in the Theory of Capital Markets. Praeger, New York.
Brennan, M.J., Subrahmany, A., 1996. Market microstructure and asset pricing: on the compensation for illquidity in stock returns. Journal of Financial Economics 41, 441–464.
Brennan, M.J., Chordia, T., Subrahmanyam, A., 1998. Alternative factor specifications, security characteristics, and the cross-section of expected stock returns. Journal of Financial Economics 49, 345–373.
Brunnermier, Markus, Pedersen, Lasse H., 2009. Market liquidity and funding liquidity. Review of Financial Studies 22, 2201–2238.
Chen, L., Li, S., Wang, J., 2011. Liquidity, Skewness and Stock Returns: Evidence from Chinese Stock Market. Asia-Pacific Financial Markets 18 (4), 405–427.
Chalmers, J.M.R., Kadlec, G.B., 1998. An empirical examination of the amortized spread. Journal of Financial Economics 48 (2), 159–188.
Chordia, T., Roll, R., Subrahmanyam, A., 2000. Commonality in liquidity. Journal of Financial Economics 56, 3–28.
Chordia, T., Subrahmanyam, A., Anshuman, V.R., 2001. Order imbalance and individual stock returns. Journal of Financial Economics 59, 3–32.
Chunhachinda, P., Dandapani, K., Hamid, S., Prakash, A.J., 1997. Portfolio selection and skewness: evidence from international stock markets. Journal of Banking and Finance 23, 143–167.
Datar, V.T., Naik, N.Y., Radcliffe, R., 1998. Liquidity and asset returns: an alternative test. Journal of Financial Markets 1, 203–219.
Eckbo, E.B., Norli, O., 2005. Liquidity risk, leverage and long-run IPO returns. Journal of Corporate Finance 11, 1–35.
Eleswarapu, V.R., 1997. Cost of transacting and expected returns in the Nasdaq market. Journal of Finance 52, 2113–2127.
Emilios, C.G., Giouvris, E., 2009. Systematic liquidity and excess returns: evidence from the London Stock Exchange. Review of Accounting and Finance 8 (3), 279–307.
Fama, E.F., French, K.R., 1992. The cross-section of expected stock returns. Journal of Finance 47, 427–465.
Fama, E.F., MacBeth, J., 1973. Risk, returns, and equilibrium: empirical tests. Journal of Political Economy 81, 607–636.
Friend, I., Westerfield, R., 1980. Co-skewness and capital asset pricing. Journal of Finance 35, 897–913.
Gibbons, M.R., 1982. Multivariate test of financial models: a new approach. Journal of Financial Economics 10, 3–27.
Gibson, R., Mougeot, N., 2004. The pricing of systematic liquidity risk: empirical evidence from the US stock market. Journal of Banking and Finance 28, 157–178.
Harvey, C.R., Siddique, A., 1999. Autoregressive conditional skewness. Journal of Financial and Quantitative Analysis 34, 465–487.
Harvey, C.R., Siddique, A., 2000a. Time-varying conditional skewness and the market risk premium. Research in Bank and Finance 1, 25–58.

Harvey, C.R., Siddique, A., 2000b. Conditional skewness in asset pricing tests. Journal of Finance 55, 1263–1295.

Hasbrouk, J., Seppi, D.J., 2001. Common factors in prices, order flows, and liquidity. Journal of Financial Economics 59, 383–411.

Haugen, R.A., Baker, N., 1996. Commonality in the determinants of expected stock returns. Journal of Financial Economics 41, 401–439.

He, Z., Kryzanowski, L., 2006. The cross section of expected returns and amortized spreads. Review of Pacific Basin Financial Market and Policies 9, 1–41.

Huang, Jiekun, 2010. Dynamic liquidity preferences of mutual funds. Working Paper. NUS Business School, National University of Singapore.

Huberman, G., Halka, D., 2001. Systematic liquidity. Journal of Financial Research 24, 161–178.

Jacoby, G., Fowler, D.J., Gottesman, A.A., 2000. The capital asset pricing model and the liquidity effect: a theoretical approach. Journal of Financial Markets 3, 69–81.

Jones, C.M., 2001. A century of stock market liquidity and trading costs. Working Paper. Graduate School of Business, Columbia University.

Kenz, P.J., Ready, M.J., 1997. On the robustness of size and book-to-market on cross-sectional regressions. Journal of Finance 111 (4), 1355–1382.

Kothari, S.P., Shanken, J., Sloan, R., 1995. Another look at the cross-section of expected stock returns. Journal of Finance 50, 185–224.

Kraus, A., Litzenberger, R.H., 1976. Skewness preference and the valuation of risk assets. Journal of Finance 31, 1085–1100.

Lee, A., Moy, R.L., Lee, C.F., 1996. A multivariate test of the covariance-co-skewness restriction for the three moments CAPM. Journal of Economics and Business 48, 515–523.

Lim, K.G., 1989. A new test of three-moment capital asset pricing model. Journal of Financial Quantitative Analysis 24, 205–216.

Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Review of Economics and Statistics 47, 13–37.

Marshall, B.R., Young, M.R., 2003. Liquidity and stock returns in pure order-driven markets: evidence from the Australian stock market. International Review of Financial Analysis 12, 173–188.

Martinez, M.A., Nieto, B., Rubio, G., Tapia, M., 2003. Asset pricing and systematic liquidity risk: an empirical investigation of the spanish stock market. International Review of Economics and Finance 11, 81–105.

Merton, R.C., 1987. A simple model of capital market equilibrium with incomplete information. Journal of Finance 42, 483–510.

Mossin, J., 1966. Equilibrium in a capital asset market. Econometrica 34, 768–783.

Pastor, L., Stambaugh, R., 2003. Liquidity risk and expected stock returns. Journal of Political Economy 111, 642–685.

Prakash, A.J., Chang, C.H., Pactwa, T.E., 2003. Selecting a portfolio with skewness: recent evidence from US, European, and Latin American equity markets. Journal of Banking and Finance 27, 1375–1390.

Sharpe, W., 1964. Capital asset prices: a theory of market equilibrium under conditions of risk. Journal of Finance 19, 425–442.

Yang, J., Zhou, Y., Wang, Z., 2010. Conditional coskewness in stock and bond markets: time series evidence. Management Science 56 (11), 2031–2049.