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Trading Mechanisms in Securities Markets

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ABSTRACT

This paper analyzes price formation under two trading mechanisms: a continuous *quote-driven* system where dealers post prices before order submission and an *order-driven* system where traders submit orders before prices are determined. The order-driven system operates either as a *continuous auction*, with immediate order execution, or as a *periodic auction*, where orders are stored for simultaneous execution. With free entry into market making, the continuous systems are equivalent. While a periodic auction offers greater price efficiency and can function where continuous mechanisms fail, traders must sacrifice continuity and bear higher information costs.

THERE IS A REMARKABLE diversity in the method by which trading is accomplished around the world and across assets. For example, a trader on the International Stock Exchange (London) can obtain price quotations before trading, and order execution at those prices is generally assured. By contrast, some stocks on smaller European exchanges can be traded only once a day and orders must be irrevocably submitted before prices are determined. Yet, we know little about how differences in trading designs affect price formation. This paper examines and contrasts the process of price formation under different forms of market organization when information is imperfect and traders act strategically.

Understanding the relationship between trading structures and price behavior is important for theoretical and applied reasons. From a theoretical viewpoint, the extensive literature on rational expectations suggests that prices efficiently aggregate information when trading is organized as an auction with large numbers of traders. Yet, most securities markets in the

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United States are not organized as auction markets, but rely instead upon market makers to provide liquidity by buying or selling on demand. This is the case even for active securities where it is feasible to maintain a high degree of continuity by conducting auctions at frequent intervals. Models of the trading process that incorporate institutional detail and strategic behavior may exhibit equilibria quite distinct from that of a frictionless Walrasian market, the traditional economic benchmark. Furthermore, recent empirical research suggests that market structure has important effects on properties of asset prices.[1] Finally, the securities industry is in the process of rapid structural changes generated by intermarket competition, innovations in communications technology, and the proliferation of new financial instruments. Understanding the relationship between market structure and price formation is necessary to evaluate the impact of these changes and to guide public policy.

The crucial function of a trading mechanism is to transform the latent demands of investors into realized transactions. The key to this transformation is *price discovery*, the process of finding market clearing prices. No two trading mechanisms are alike in the performance of price discovery; they differ in the types of orders permitted, the times at which trading can occur, the quantity and quality of market information made available to investors at the time of order submission, and the reliance upon market makers to provide liquidity.[2]

An important distinction is often made between *continuous* and *periodic* mechanisms. In a continuous market an investor's order is executed immediately upon submission. A continuous trading system is characterized by a sequence of bilateral transactions at (possibly) different prices. In a periodic system, however, investors' orders are accumulated for simultaneous execution at a pre-determined time. The periodic system (commonly referred to as a *call auction* or *batch market*) is characterized by a set of multilateral transactions at one price.

Another useful distinction is between *quote-driven* and *order-driven* trading mechanisms. In a quote-driven system such as NASDAQ or the International Stock Exchange (London), investors can obtain firm price quotations from market makers prior to order submission.[3] This mechanism is also known as a *continuous dealer market* because an investor need not wait for order execution, but instead trades immediately with a market maker.

[1] See for example, Amihud and Mendelson (1987), Stoll and Whaley (1990), Amihud, Mendelson, and Murgia (1990), and Amihud and Mendelson (1991).

[2] A number of excellent papers analyze this diversity. Cohen, Maier, Schwartz, and Whitcomb (1986) provide a comprehensive survey of international trading structures and their relative merits. Stoll (1990) discusses the principles underlying various trading arrangements. Harris (1990) examines a number of policy issues in the design of trading protocols.

[3] Market makers' bid and ask quotations typically depend on the size of the order. In London, for example, dealers quote price schedules. Other examples include the market for foreign exchange and the secondary market for U.S. Treasury bills.

By contrast, in an order-driven system, investors submit their orders for execution through an auction process. Order-driven mechanisms can operate either as continuous systems or as periodic systems. In the first type, known as a *continuous auction*, investors submit orders for immediate execution by dealers on an exchange floor or against existing limit orders submitted by public investors or dealers. The system is continuous, since orders are executed upon arrival, but operates as an auction because the price is determined multilaterally.[4] The second type of order-driven system is known as a *periodic auction*, where the orders of investors are stored for execution at a single market clearing price.[5]

Most trading mechanisms are complex hybrids of these three systems. For example, the NYSE opens with an auction or batch market, and then switches to a dealer market. For some thickly traded stocks, the market has been described as a *continuous double auction*. These compound mechanisms can be understood only through the analysis of their simpler component structures.

Previous studies of market mechanisms have examined the differences between continuous and periodic trading when traders have fixed reservation prices.[6] This paper differs from this line of research by modeling trading as a game between strategic traders with rational expectations. We compare and contrast a quote-driven system with an order-driven system that can operate as a continuous auction or as a batch market. These mechanisms differ in many respects, most importantly in the sequence of order submission and the amount of market information made available to traders.

We show that equilibrium may not exist in continuous mechanisms (i.e., the quote-driven system and the continuous auction) unless there is a minimum amount of noninformation trading. With free entry into market making, the quote-driven system is equivalent to the continuous auction mechanism. By contrast, the periodic auction aggregates information efficiently and is more robust to problems of information asymmetry in that it can operate where continuous markets fail. The trade-off comes in the form of

[4] Examples of continuous auction systems include the à la criée or open-outcry system for active stocks on the Paris Bourse, the Swiss Option and Financial Futures Exchange (SOFFEX), the Frankfurt Stock Exchange, the Toronto Stock Exchange's Computer Assisted Trading System (CATS), the Tokyo Stock Exchange's Computer Assisted Routing and Execution System (CORES), and 'crowd trading' in U.S. futures markets. Many continuous auction systems are proprietary automated systems, where order submission is electronic. See Domowitz (1990) for a description of the mechanics of automated trading.

[5] Periodic systems are used to open many continuous markets such as the New York Stock Exchange (NYSE) and Tokyo Stock Exchange. In addition, many European stock exchanges operate batch markets, although these systems are being supplanted by continuous trading systems. In the U.S., by contrast, new proprietary periodic systems such as the *Wunsch auction* are complementing the operation of continuous markets.

[6] See, for example, Garbade and Silber (1979), Ho, Schwartz, and Whitcomb (1985), Pithyachariyakul (1986), Mendelson (1987), and Gammill (1990).

loss of continuity of trading and the costs of gathering market information that would otherwise be revealed through price quotations. The Bayes-Nash equilibrium for the auction game is shown to have a close analogy in the classical notion of a Walrasian auction, but the two equilibria are quite distinct. Finally, we derive several testable hypotheses regarding the time-series properties of prices and the cross-sectional determinants of bid-ask spreads and price volatility.

The rest of the paper proceeds as follows. In Section II, we set up the basic framework, and in Section III we analyze a continuous market where dealers post prices before trading occurs. In Section IV we model an order-driven mechanism that can operate as a continuous auction or as a periodic mechanism. We derive a number of comparative statics results that yield testable implications. Section V compares the operation of these mechanisms, and relates our game-theoretic market model to classical ideas of equilibrium. Section VI summarizes the paper. All proofs are in the appendix.

I. The Model

The model is based on a framework familiar in the rational expectations literature. There are two assets, cash and a single risky asset with a stochastic liquidation value, denoted by $\tilde{v}$, which is realized after time 1. There are two types of agents in this model. The first type, termed 'traders' (indexed by i) enter the market according to an exogenous stochastic process at calendar times $\{t_i\}_{i=1}^{\infty}$ in the interval (0, 1). Thus, trader i is the trader who trades at time t_i. The second type of agent, termed 'dealers' or 'market makers' provide liquidity by trading with the first group of traders. We will examine two different forms of market organization: a quote-driven system and an order-driven system that operates either as a continuous auction system or a periodic auction mechanism.

We first describe the objectives and information of the agents in the model and then discuss the specifics of the trading arrangements. Let q_i represent the order quantity of trader i who arrives at time t_i with the convention that $q > 0$ denotes a trader purchase and $q < 0$ a trader sale. Denote by p_i the security's price. For simplicity, we assume that each trader trades only once.[7] In a dynamic model, each trader's strategy would depend on the probability of repeat trading not only for herself, but for all other traders as well, greatly complicating the analysis.[8]

[7] This assumption could be motivated by the Prisoner's Dilemma nature of competition among traders with private information. Since all traders possess private information, a trader who fails to fully exploit his information when he gets a chance to trade runs the risk that his information will already be impounded in prices when he next obtains an opportunity to trade.

[8] See, for example, Seppi (1990) and Leach and Madhavan (1990) who provide dynamic models where agents act strategically.

Traders maximize the expected utility of final period wealth. Each trader is assumed to have a negative exponential utility function:

$$u(W_{1i}) = -e^{-\rho W_{1i}}$$

where W_{1i} is the final period wealth of trader i, and $\rho > 0$ is the coefficient of absolute risk aversion. Trader i's initial endowment is described by the vector (x_i, c_{0i}), where x_i is the initial inventory of risky securities and c_{0i} represents cash holdings. Endowments of the risky asset are distributed normally across traders with mean 0 and precision (the reciprocal of the variance) ψ.[9] Since traders are risk averse, variation in asset endowments generates portfolio hedging trade which is not information motivated. The amount of this 'liquidity' trading is inversely related to the value of ψ.

For trader i, final period wealth, $\tilde{W}_{1i}$, is a random variable given by:

$$\tilde{W}_{1i} = (q_i + x_i)\tilde{v} + c_{0i} - p_i q_i. \tag{1}$$

Let Φ_i represent the information set of trader i at time t_i.[10] If $\tilde{W}_{1i}$ is normally distributed conditional upon Φ_i, then maximizing expected utility is equivalent to maximizing:

$$E[\tilde{W}_{1i} \mid \Phi_i] - \left(\frac{\rho}{2}\right)\mathrm{Var}[\tilde{W}_{1i} \mid \Phi_i] \tag{2}$$

where $E[\cdot \mid \Phi_i]$ and $\mathrm{Var}[\cdot \mid \Phi_i]$ represent the conditional expectation and variance operators relative to Φ_i.

We turn now to the determinants of traders' information sets. At time 0, the risky security is known to be distributed normally with mean μ and precision τ. This is public information and is known to both dealers and traders. In addition to public information, trader i $(i = 1, \cdots, N)$ observes the realization of a random variable, $\tilde{y}_i = v + \tilde{\epsilon}_i$ where $\tilde{\epsilon}_i$ is white noise and v is the time 1 value of the risky asset. We assume that $\tilde{\epsilon}_i$ is independently normally distributed with mean 0 and precision θ. The realized value of the random variable $\tilde{y}_i$ is denoted y_i. The private information of trader i is the pair (x_i, y_i). Then, trader i's *prior* distribution of $\tilde{v}$ is normal with mean y_i and precision θ. Of course, the trader's order strategy depends on his or her *posterior* beliefs, which take into account the information contained in prices. We discuss this in more detail later. The distributional assumptions concerning the information structure are common in models of this type, but unfortunately permit prices to be negative.[11] A possible source for diverse prior

[9] The assumption of a zero mean is without loss of generality. We also assume the security is 'widely' held so that information on x_i does not convey information about x_j, for $i \neq j$.

[10] Formally, the information set is the σ-algebra generated by public and private information.

[11] The probability of this occurrence could be made arbitrarily small by assuming the variance of the risky asset is small relative to its mean.

beliefs may be differences in access to market information as described by Working (1958):

> The amount of pertinent information potentially available to traders in most modern markets is far beyond what any one trader can both acquire and use to good effect. Circumstance and inclination lead different traders to seek out and use different sorts of information. In short, traders are forced to engage in a sort of informal division of labor in their use of available information. Using different information, different traders must find themselves often of different opinions, one buying at the same time that another sells, even though all may stand at an equal level of intelligence, steadiness in judgement, and quantity of information at their command (pp. 188–199).

The assumption of independent signals can be relaxed to allow correlation among conditional expectations, but at considerable notational cost.

Turning now to the dealers, we assume that there are $M > 2$ competitive dealers. These dealers are assumed to be risk-neutral and their objective is to maximize expected profits, subject to competitive constraints. Unlike traders, dealers do not receive private information signals; they know the prior distribution of $\tilde{v}$, which is public information, and, over time, make inferences about the security's value from the order flow. Since the signals dealers receive from order flow will depend on the way in which trading is accomplished, we must first discuss the specifics of the trading arrangements. Before doing so, however, it is useful to define a measure of information quality at the start of the trading day. Define Υ as:

$$\Upsilon \equiv \frac{(\theta + \tau)\theta}{\tau}. \tag{3}$$

The measure Υ has an important role in our analysis; it can be thought of as a proxy for the initial degree of information asymmetry. A security with pronounced information asymmetries has (relatively) large values of θ and low values of τ, implying Υ is large. Conversely, for securities where information asymmetries are inconsequential, τ is large and θ is small, implying Υ is small. We now address the specifics of market organization, and then discuss the various performance measures used to assess these regimes.

A. The Quote-Driven Mechanism

We consider first the quote-driven mechanism. The model of the quote-driven system is based on the single-period model of Glosten (1989), who contrasts a monopolistic market maker system with a competitive market maker system. Trading is accomplished through M competitive, risk-neutral market makers who take the opposite side of all transactions. The critical feature of this mechanism is that it is quote-driven; market makers provide bid-ask quotations to traders on demand, and can revise their quotes only after a transaction is complete. Strict price priority prevails, so that the market maker with the lowest ask price or the highest bid price is matched with the trader.

We model the dealer mechanism as a two-stage game. In the first stage, the dealers determine their quoted prices and in the second stage the trader chooses his or her order given the quoted prices. Price competition forces the expected profits of market makers on each trade to zero. Rational dealers set prices so that the price for a given order size is an unbiased estimate of the asset's value given initial beliefs, the trading history, and the information provided by the size of the order, i.e., quoted prices are 'regret-free' or ex post rational.[12] Since order size is variable, this discussion implies that the quoted prices are contingent on order size. Formally, at time t_i, market makers determine a quotation schedule $p_i(\cdot)$, then observe q_i, the order placed by trader i, after which market makers can revise their quotation schedules for trader $i+1$, i.e., choose $p_{i+1}(\cdot)$. In setting $p_i(\cdot)$, market makers use the information contained in the trading history, $h_i = \{(p_1, q_1), \cdots, (p_{i-1}, q_{i-1})\}$, and their prior information (μ, τ). Let Φ_d^i represent the pre-trade information set of market makers at time t_i.[13] The quote-driven (continuous dealer) mechanism in each period is described as a game $\Gamma_c = (\{p_i(\cdot), q_i(y_i, x_i)\})$ characterized by the players and their strategies.

Definition 1: At time t_i, equilibrium for the quote-driven mechanism Γ_c is a differentiable price quotation function, $p_i : \Re \to \Re_+$, and a corresponding demand, q_i, such that:

(a) Each market maker ($j = 1, \cdots, M$) quotes a price schedule that ensures non-negative expected profits on each transaction

$$\left(E[\tilde{v} \mid q_i, \Phi_d^i] - p_i(q_i)\right)(-q_i) \geq 0.$$

(b) Strict price priority prevails, i.e., there does not exist another function, p_i^o, satisfying condition (a) such that:

$$qp_i^o(q) \leq qp_i(q) \text{ for all } q \in \mathscr{R}$$

and

$$qp_i^o(q) < qp_i(q) \text{ for some } q \in \mathscr{R}$$

(c) Trader i maximizes expected utility given $p_i(\cdot)$:

$$q_i \in \operatorname{argmax}_{\{q_i\}}\{E[u(\tilde{W}_{1i}(q_i)) \mid \Phi_i, p_i\}.$$

[12] Glosten and Milgrom (1985) use this condition to demonstrate the existence of an information-based bid-ask spread. In their model, order size is fixed.

[13] Formally, this set is the σ-algebra generated by the history h_i and public information.

Condition (a) is the requirement that market makers earn nonnegative expected profits given post-trade information, while condition (b) requires that the quotation function p_i not be dominated by another function, p_i^o, with lower ask prices and/or higher bid prices. Together, these conditions imply that market makers earn zero expected profits on every trade. Finally, condition (c) states that each trader maximizes the expected utility of final period wealth given the price schedule posted by dealers. Definition 1 implies that equilibrium is a fixed point in the space continuous functions.

B. The Order-Driven Mechanism

An alternative to the quote-driven continuous dealer system is an order-driven system where traders submit orders to an exchange for execution by floor traders or dealers. This type of system includes continuous auctions, where orders are executed immediately, as well as periodic auctions where orders are accumulated for simultaneous execution at a single market clearing price.[14]

Unlike a posted-price system, the transaction price in an auction mechanism is not known at the time of order submission. We argue, however, that traders can effectively condition their beliefs on the price. Rational investors know that the equilibrium price reveals information, so that the demand schedule they submit is the set of price-quantity combinations such that the quantity demanded at each price is the desired order quantity conditional upon that particular price clearing the market.[15] In this respect, the equilibrium corresponds to the rational expectations model of Kyle (1989), with the difference that liquidity trading is endogenous in our model.[16] Consider an auction with N traders and M dealers. We denote the vector of trader demand functions by $\vec{q} = (q_1(\cdot), \cdots, q_N(\cdot))$, and the vector of dealer demands by $\vec{d} = (d_1(\cdot), \cdots, d_M(\cdot))$, with the usual sign convention. Let $\vec{q}_{-1} = (q_1, \cdots, q_{i-1}, q_{i+1}, \cdots, q_N)$, and similarly define $\vec{d}_{-j}$. We model the order-driven process as a game $\Gamma_a = (\{q_i\}_{i=1}^N, \{d_j\}_{j=1}^M)$ indexed by the players and their strategy functions.

Definition 2: A Bayes-Nash equilibrium for the order-driven mechanism Γ_a consists of a vector of strategy functions, $\vec{d}$, for dealers $j = 1, \cdots, M$, a vector of strategy functions, $\vec{q}$, for traders $i = 1, \cdots, N$, and a price, p^*, such that:

[14] Batch markets have been examined by Ho, Schwartz, and Whitcomb (1985), Mendelson (1982), and Mendelson (1987). In these models traders do not condition on prices or act strategically.

[15] This is analogous to our construction of the dealer quotation schedule using a sequential rationality argument.

[16] In Kyle's model (1989), there is exogenous noise trading in addition to trading by investors and speculators. In our paper, liquidity trading takes the form of portfolio adjustment for hedging purposes. The endogeneity of liquidity trading is critical to our argument about the viability of trading systems.

(a) $\sum_j^M d_j(p^*) + \sum_i^N q_i(p^*, y_i, x_i) = 0$,
(b) $q_i(p^*, y_i, x_i) \in \operatorname{argmax} E_i[u(\tilde{W}_{1i} \mid p^*, (\vec{q}_{-i}, \vec{d}))]$,
(c) $d_j(\cdot) \in \operatorname{argmax}_{\{d_j\}}\{E_j[(\tilde{v} - p)d_j(p) \mid p^*, (\vec{q}, \vec{d}_{-j})]\}$, subject to non-negativity.

Condition (a) requires that the market clear in equilibrium, while condition (b) requires that the strategy of trader i maximize expected utility given the equilibrium price and the strategy functions of other agents. Finally, condition (c) requires that the strategy (demand schedule submitted) of a dealer be a best-response to the strategies of other dealers and traders. The differences in the two systems lie in the sequence of trading, which leads to differences in the information provided to players and the strategic nature of the game. In the order-driven system, traders do not know the price until after the market clears, whereas in the quote-driven system, traders know the execution price for their order. In the order-driven system, competition among dealers takes the form of competition in demand schedules rather than pure price competition, as in a quote-driven system. Before turning to the analysis of the different systems, we first discuss a number of performance measures used to evaluate trading systems.

C. Performance Measures

To compare the different trading mechanisms considered here we will examine a number of objective measures of 'market quality' that have been the focus of recent policy debates. Of course, there are innumerable potentially reasonable 'yardsticks,' but we will focus on those measures that are well known and readily quantified. Our objective is to provide criteria that can be used to empirically validate our hypotheses as well as guide policy discussions without making value judgements as to which attributes are preferred. The performance measures we consider are the following.

C.1. The Bid-Ask Spread Measures the Costs of Trading

The bid-ask spread is explicitly defined in a quote-driven system, since it is just the difference between the price for a buy order and the price for a sell order. However, even in an auction system with one price, an analogous measure or *effective* bid-ask spread can be constructed because buy orders raise prices while sell orders lower prices. Let $p(q)$ represent the price in a particular mechanism as a function of order quantity, q, where we assume $p(\cdot)$ is continuous. For an order quantity, q, the ask price is defined as $p(\mid q \mid)$ and bid price is $p(-\mid q \mid)$. Then, the *effective bid-ask spread* for an order q is a function $s_i(q) = p_i(\mid q \mid) - p_i(-\mid q \mid)$. In intraday transactions data, bid-ask quotations are typically good only for certain order sizes. Accordingly, it is important to distinguish between the effective spread and the quoted spread. In the case of a quote-driven system, we define a *bid-ask quotation* for one round lot by the pair $(p_i^a, p_i^b) \equiv (p_i(1), p_i(-1))$.

C.2. Closely Related to the Definition of the Quoted Bid-Ask Spread is the Notion of Market Depth, Which Measures the Sensitivity of Prices to Order Flow

Kyle (1985) defined market depth as the volume required to generate a unit price change. Following Kyle, the *market depth* at volume q is $\nabla = [p'(q)]^{-1}$, assuming that the price functional is differentiable. Market depth is a measure of liquidity, with higher depth implying more liquidity, i.e., greater ability to absorb order flow without large changes in price.

C.3. Price Efficiency Measures the Extent to Which Prices Reflect Public Information About the Asset's Value

A mechanism is *semi-strong form efficient* if $p_i = E[\tilde{v} \mid p_i]$. If prices reflect all available information, including private information, they are *strong-form efficient*. Strong-form efficiency corresponds to a fully revealing rational expectations equilibrium.

C.4. Price Variance, i.e., Var$[\tilde{p}_i]$, *Measures the Volatility of Prices*

A related issue concerns how closely prices reflect the underlying asset value, which can be measured by the predictive or forecast error $e_i \equiv (v - p_i)$.

C.5. A Trading System Is Robust if It Can Function under a Variety of Economic Conditions

We show in this paper that differences in trading arrangements imply differences in the robustness to asymmetric information, in a sense that is formally defined below.

II. Equilibrium in the Quote-Driven Mechanism

Our first objective is to characterize equilibrium in the quote-driven (continuous dealer) system. Proposition 1 shows that a sufficient condition for the existence of a unique equilibrium is that the information parameter Υ be bounded above.

Proposition 1 (The Quote-Driven System): *If* $\Upsilon < \dfrac{\rho^2}{\psi}$, *equilibrium exists at time* $t_i (i = 1, 2, \cdots)$. *At time* t_i *equilibrium is linear*:

(a) The demand function is:

$$q_i = \frac{E[\tilde{v} \mid \Phi_i] - p_{i-1} - \alpha_i x_i}{\alpha_i + 2\lambda_i}.$$

(b) The price quotation schedule is:

$$p_i(q_i) = p_{i-1} + \lambda_i q_i$$

where λ_i *and* α_i *are constants defined in the appendix and* $p_0 = \mu$.

The sequence of the game, where dealers 'move first,' means that dealers cannot offer a single price to traders in equilibrium. Rather, they offer a price schedule such that each price on this schedule is the expected value of the asset given the size of the order. In turn, the trader selects a particular point on this schedule taking into account the effect of order quantity at the margin on the price of the entire trade. In addition, the trader conditions on any information revealed by the quotation schedule posted by the dealer. As a result, the dealer is at an informational disadvantage relative to traders. Intuition suggests that if the degree of information asymmetry is sufficiently great, this disadvantage may be so severe that dealers cannot make non-negative expected profits.

Proposition 1 shows that this is indeed the case. Equilibrium exists in every period if the information asymmetry parameter Υ (where $\Upsilon = (\theta^2/\tau) + \theta$) is less than a critical bound which is related to the noninformation motives for trade. We denote this upper bound by $\Upsilon^c \equiv \rho^2/\psi$. The higher the coefficient of risk aversion, ρ, the greater the upper bound Υ^c. Similarly, Υ^c is an increasing function of the variance of initial endowments of the risky asset, ψ^{-1}. Clearly, if there is insufficient liquidity trading, because ρ is low or ψ is high, dealers may be unable to open markets without suffering expected losses. If markets cannot open in the first period they cannot open in any subsequent periods and there is no way for information to be aggregated over time. We refer to this phenomenon as market failure. Proposition 1 shows that a continuous system may not be viable in periods of severe information asymmetry (i.e., Υ is high because τ is low relative to θ) or when there is little liquidity trading and Υ^c is low. Possible violations may be at the start of the trading day or immediately preceding the public revelation of new information. We will show below that these problems can be overcome by a switch to an alternative trading system.

A. Price Dynamics

Recall that the predictive (forecast) error $e_i = (v - p_i)$. Proposition 2 shows that prices are efficient with respect to public information, but that this result is consistent with autocorrelation in the predictive errors.

Proposition 2 (Price Dynamics): *If equilibrium exists*:

(a) *Transaction prices follow a martingale, i.e.,* $E[\tilde{p}_{i+1} \mid p_i] = p_i$, *and prices are semi-strong form efficient, i.e.,* $E[v \mid \Phi_d^{i+1}] = p_i$.

(b) *The predictive errors are positively correlated:*

$$E[\tilde{e}_{i+1} \mid e_i] = \eta_i e_i$$

where $0 < \eta_i < 1$ *for all* i

(c) *The effective spread is an increasing function of order size,* $s(q_i) = 2\lambda_i \mid q_i \mid$. *Further, the quoted bid-ask spread decreases over the day:*

$$\left(p_i^a - p_i^b\right) < \left(p_{i-1}^a - p_{i-1}^b\right).$$

The martingale property of prices is present in models of competitive dealer markets such as Glosten and Milgrom (1985) and Easley and O'Hara (1987). Part (b) shows that although prices follow a martingale (relative to post-trade public information Φ_d^{i+1}), the predictive errors are still positively correlated, so that any initial mispricing is partly carried forward. This is quite consistent with part (a), which says that prices follow a martingale with respect to post-trade public information. The parameter η_i measures the informational efficiency of the market, i.e., the rate of convergence of transaction prices to the full-information price. High values of η_i are associated with less rapid convergence, since errors persist longer. Since $\eta_i < 1$ for all i, any mispricing at time t_i exerts a diminishing influence on future prices. Prices converge in the limit to the full information value.

The model provides some insights into the nature of the bid-ask spread implicit in the price-quotation schedule. First, observe that the size and placement of the spread are informative, and form a sufficient statistic for the entire history of trading. Since the $E[\tilde{v} \mid \Phi_d^i] = (p_i^a + p_i^b)/2$, the mid-quote provides a point estimate of the asset's value given pre-trade public information. The precision of this estimate can be inferred from the spread since τ_i is a monotonically decreasing in $(p_i^a - p_i^b)$. The size of the bid-ask quotation, therefore, is linked to the variability of observed transaction prices.

Second, the linearity of the quotation schedule implies the entire quotation schedule can be inferred from a single bid-ask quotation. For a given order, the effective bid-ask spread, $s_i(q)$, is strictly increasing in q, and the quoted bid-ask spread, $(p_i^a - p_i^b)$ is strictly decreasing with the number of trades.[17] The result suggests that the components of volume (order size and the frequency of trading) have opposite effects on the bid-ask spread. A security whose trading volume is composed of a few large trades will have a wider effective spread than a security with identical volume composed of many small-sized trades. This point has been ignored in previous empirical studies of the cross-sectional determinants of bid-ask spreads which use total transaction or dollar volume as an independent variable. In these studies, transaction volume and bid-ask spreads are inversely related. Proposition 2 implies that in a cross-sectional regression of bid-ask spreads on average order size (relative to daily volume) and trading frequency (per unit time), the coefficient of trading frequency will have a negative sign and the coefficient of order size will have a positive sign.

Third, spreads narrow over the day because each trade reduces the information asymmetry between dealers and traders. Evidence for a decline in spreads over the day is provided by McInish and Wood (1988) who analyze intraday bid-ask spreads for NYSE stocks for five months of 1987. McInish and Wood group the sample into quintiles by trading frequency and find that

[17] A similar prediction for order quantity arises from the model of Ho and Stoll (1983) because of the dynamic inventory control policies pursued by market makers. Easley and O'Hara (1987) consider a model with two order sizes and show that the spread is strictly higher for the larger order size.

bid-ask spreads decline rapidly after opening within each quintile. The decline is most evident in the first 15 minutes of trading, after which spreads remain stable, possibly because the discreteness of prices generates a minimum spread.

The model makes other time-series predictions. From the proposition, we can write $p_i = p_{i-1} + \lambda_i q_i$. Given data on transaction prices and signed order flow, this equation can be estimated directly. Glosten and Harris (1988) estimate a similar model (correcting price discreteness) using time-series data for a sample NYSE stocks. In the estimation, they assume λ_i is a constant. Glosten and Harris then use three-stage least squares to examine the cross-sectional determinants of the estimated parameters. They find that λ is a decreasing function of trading frequency and an increasing function of the percentage of shares held by insiders (a proxy for information asymmetry), findings consistent with our predictions. Similarly, Madhavan and Smidt (1991) estimate a model of intraday price formation using inventory data obtained from a NYSE specialist and find that λ is significantly positive. They show that this reflects the effects of perceived information asymmetries, not transaction or inventory carrying costs. Proposition 2, however, implies that if an intraday pricing equation such as the Glosten-Harris of Madhavan-Smidt model is estimated by trading hour, the estimated hourly coefficients λ should decrease over the course of the day. This procedure could be carried out on a daily basis in order to isolate any 'day-of-the-week' effects, providing an alternative test for the model of Foster and Vishwanathan (1990).

III. Equilibrium in the Order-Driven Mechanism

Having established the equilibrium for the quote-driven system we now consider the order-driven mechanism that takes two forms: a continuous auction and a batch market.

A. The Continuous Auction Mechanism

Consider a continuous auction, where an order is executed upon arrival by the 'trading crowd' of dealers who are present on the exchange floor.[18] The continuous auction is a special case of the order-driven mechanism Γ_a, with $N = 1$. Let d_j^i be the demand of dealer j in time t_i. We represent this mechanism by the game Γ_{ca}.

Proposition 3 (The Continuous Auction): *If* $\Upsilon < \Upsilon^a \equiv \left(1 - \dfrac{2}{M}\right)\dfrac{\rho^2}{\psi}$, *there exists an equilibrium for the continuous auction mechanism* Γ_{ca} *at time* t_i *where*:

[18] It is convenient to think of orders being price-contingent, but whether traders submit limit orders or market orders has no effect on the equilibrium outcome, as shown in the appendix.

(a) *Dealer j's* ($j = 1, \cdots, M$) *strategy function is linear:*

$$d_j^i(p_i) = \gamma_i(p_{i-1} - p_i) - \gamma_i \zeta_{i-1} q_{i-1}.$$

(b) *The trader's demand function is:*

$$q_i(p_i) = \frac{E[\tilde{v} \mid \Phi_i] - p_i - \alpha_i x_i}{\alpha_i + \zeta_i}.$$

(c) *The equilibrium price is:*

$$p_i^* = \frac{[M\gamma_i(\alpha_i + \zeta_i) + (1 - \delta_i)](p_{i-1} - \zeta_{i-1}q_{i-1}) + \delta_i y_i - \alpha_i x_i}{M\gamma_i(\alpha_i + \zeta_i) + 1}.$$

where α_i, γ_i, δ_i, *and* ζ_i *are constants described in the appendix and* $p_0 = \mu$. *Otherwise, if* $\Upsilon > \Upsilon^a$, *there does not exist a symmetric linear equilibrium.*

Proposition 3 demonstrates the existence of equilibrium under general conditions. We require $M > 2$ for a linear equilibrium.[19] As before, equilibrium is linear, but there are significant differences from the equilibrium for the quote-driven mechanism Γ_c. We discuss these differences next, but note in passing that the proposition has an important empirical implication for studies of market making. From part (a) of Proposition 3, dealers sell when prices rise and buy when prices fall (i.e., $\gamma > 0$), so that a time series regression of dealer activity against price changes would suggest that dealers 'stabilize' the market. However, this behavior arises from profit maximization, not from a desire to create a societal benefit.

Proposition 4 (Continuous Markets): *If the number of dealers M is finite, then:*

(a) *If an equilibrium exists for the quote-driven mechanism* Γ_c, *then it exists for the continuous auction mechanism* Γ_{ca}.
(b) *Prices in the continuous auction system do not follow a martingale (i.e.,* $E[\tilde{p}_{i+1} \mid p_i] \neq p_i$, *a.s.), and prices are not semi-strong form efficient (i.e.,* $E[\tilde{v} \mid p_i, \Phi_d^i] \neq p_i$, *a.s.).*
(c) *The unconditional variability of prices in the continuous auction system is higher than in the continuous dealer system. However, the trading history is equally informative, i.e.,* $E[\tilde{v} \mid h_i]$ *is the same in both mechanisms.*

Proposition 4 demonstrates that continuous trading is more robust to problems of asymmetric information if organized as a dealer system. If both mechanisms are viable, prices in the auction system are not efficient and are more volatile than in the dealer system. Intuitively, in the dealer system,

[19] It is possible that other equilibria exist (e.g., mixed strategy equilibria) but throughout the paper we will focus on linear equilibria to facilitate the comparisons across different mechanisms. One may also argue that the linear equilibrium is the most 'natural' equilibrium given the computational burden facing agents in the economy.

price competition between dealers to quote bid and ask prices eliminates the 'wedge' between the transaction price and the expected value of the asset that is the source of dealers' expected profits. In the auction system, the price is determined simultaneously so that each player has some influence on price. Strategic behavior distorts prices, inducing inefficiency and making the system more sensitive to the problems of information asymmetry. However, the trading history in both cases is equally informative since rational traders can 'undo' the distortion due to strategic behavior in the continuous auction system when making inferences about the security's value.

When are the two mechanisms equivalent? Proposition 5 shows that with free entry into market making, the equilibria for the two mechanisms coincide.

Proposition 5 (Equivalence): *For any time period t_ι, as $M \to \infty$, the equilibrium price-quantity pair (p_i, q_i) for the continuous auction Γ_{ca} converges to the equilibrium price-quantity pair of the quote-driven mechanism Γ_c.*

With this limiting proposition, we can derive a number of interesting comparative statics results for both markets, assuming free entry into market making.

Proposition 6 (Comparative Statics): *If there is free entry into market making and if $\Upsilon < \Upsilon^c$ then:*

(a) The quoted bid-ask spread $(p_i^a - p_i^b)$ increases in θ and ψ and decreases in ρ and τ. Further, as $\Upsilon \to \Upsilon^c$, bid-ask spreads become arbitrarily large. Market depth ∇ decreases in θ and ψ and increases in ρ and τ.
(b) In any period, price variability is decreasing in θ, τ, and ψ and is increasing in ρ. Over the course of the day, price variability and the conditional variance of $\tilde{v}$ given public information is monotonically decreasing.

The comparative statics results of Proposition 6 shows that the quality of market maker and trader signals are critical determinants of market depth and the size of the bid-ask spread. The greater the precision of the trader's signal, θ, the wider the spread, since traders have better information than before and a greater revision in prices is needed for any order size. Similarly, the greater the precision of the market maker's signal, τ, the smaller are spreads, since this reduces information asymmetry. A decrease in the precision of risky asset endowments, ψ, implies more liquidity trading so that order quantity conveys less information and requires a smaller revision in beliefs, decreasing spreads. Finally, spreads decline monotonically as the coefficient of risk aversion, ρ, increases, as this implies more liquidity trade.

When we consider the efficiency of pricing, however, the results are altered. Higher precision information signals always reduce price variance whether they are public or private. The higher the precision of asset endowments, the lower the amount of noninformation trading, and the more accurate the pricing. The discussion above suggests that price efficiency and the size of the bid-ask spread are related in a complex manner. A security with high price volatility may have narrower bid-ask spreads than a security whose volatility is low because the quantity of available information is poor in the first case, while in the second, despite high quality signals, the ratio of public to private information is low. The results imply that price volatility and spreads decline with the number of traders, a prediction which is testable by regressing price volatility on the average number of trades per unit time in a cross-section of assets. Having examined the nature of quote-driven and order-driven continuous systems, we turn our attention to periodic systems.

B. The Periodic Auction

We now consider the general case for the order-driven mechanism Γ_a, where the market only clears after $N > 1$ traders have submitted their orders. The next proposition shows that if the number of traders in the auction system Γ_a is sufficiently large, the periodic mechanism provides less variable prices and is more robust than the continuous system.

Proposition 7 (The Periodic Auction): *There exists N such that:*

(a) *The auction mechanism Γ_a is viable in economies where continuous mechanisms fail. The equilibrium strategy functions of traders and dealers are linear in price.*
(b) *In a particular periodic auction, the price p_N^a, is semi-strong form efficient and over a sequence of periodic auctions, the auction prices follow a martingale.*
(c) *The price in the auction mechanism Γ_a is less variable than each of the corresponding sequence of N prices in the continuous mechanism Γ_c.*

Corollary 1: *As the number of traders participating in a given auction grows large, prices converge to the strong form efficient price, i.e.,* $\lim_{N\to\infty} p_N^a = v$.

Proposition 7 shows that a large enough auction can provide more efficient prices than a continuous market. In a periodic system, all traders observe a noisy estimate of their aggregate information, in addition to public and private information signals. The more traders participating in the auction, the more efficient the price is as a signal of asset value. Further, the system can function in economies where continuous systems fail. The disadvantage of the periodic system is that it does not provide for continuous trading. Rather, investors must wait until pre-specified times for order execution.[20]

Corollary 1 shows that prices are strong form efficient in the limit, i.e., the prices are fully revealing. Another interesting case of Proposition 7 concerns the limiting equilibrium as the quality of public information becomes extremely poor, i.e., as $\tau \to 0$. Since $\lim_{\tau\to 0} \Upsilon = \infty$ our previous analysis of continuous systems (both the quote-driven system Γ_c and the continuous

[20] See Garbade and Silber (1979) who provide a model where there exists an optimal time between market clearings that minimizes investors' liquidity risk.

auction system Γ_{ca}) implies that there does not exist an equilibrium for τ sufficiently small. Intuitively, for any $\theta > 0$, the degree of information asymmetry is increasing as τ grows small, so dealers condition more and more on order flow. This implies the volume of liquidity trading goes to zero, so that dealers cannot make nonnegative expected profits, and M goes to zero. Intuition suggests that a periodic auction can function without the presence of intermediaries with the N traders sharing risk among themselves. The following proposition establishes that this intuition is correct, provided the quality of private information signals, θ, is bounded above.

The next proposition establishes a link between the strategic rational expectations equilibrium of this paper and the classical description of an auction market. In the classical Walrasian auction, traders act competitively and have fixed reservation prices. The equilibrium price is found by computing the zero of the aggregate excess demand function. Proposition 8 establishes that the price in the periodic auction game Γ^a is the *same* price that would prevail in the classical framework, but that the two equilibria are quite distinct.

Proposition 8 (Market Clearing without Dealers): *If* $\theta < \left(1 - \frac{2}{N}\right)\frac{\rho^2}{\psi}$, *there exists a linear equilibrium for the auction mechanism* Γ_a *with* $\tau = 0$ *where*:

(a) The equilibrium strategy functions of traders are linear in price and dealers do not trade. The price is efficient and price volatility is a decreasing function of the number of traders N, the precision of signals θ*, and is increasing in the amount of liquidity trading, as measured by endowment variation,* ψ^{-1}.

(b) The Walrasian equilibrium is defined for all $\theta > 0$. *The price in the auction mechanism* Γ_a *is equal to the price that would prevail in a Walrasian auction. However, the volume of trade in the auction mechanism* Γ_a *is strictly lower than in the Walrasian auction.*

Note that in Proposition 8, existence requires that the precision of signals, θ, be bounded above. The bound is $\left(1 - \frac{2}{N}\right)\frac{\rho^2}{\psi}$, which, for large N, converges to the bound Υ^c. So market failure may occur even if the number of traders is large. This is directly attributable to the fact that traders in the game are strategic; the Walrasian equilibrium exists for all $\theta > 0$, even if we extend the model to allow traders to have rational expectations. From a policy viewpoint, Proposition 8 implies that an auction system can function even in a situation where public information is so poor that dealers choose to withdraw from market making activities.

Part (b) shows the Walrasian price equals the periodic auction price. Intuitively, a trader's *reservation price*, at which demand equals zero, is the same in both mechanisms because traders view the auction price as distributed symmetrically about their prior mean. Since an auction mechanism operates by averaging traders' reservation prices, both mechanisms yield the same price. However, the equilibria are distinct because strategic traders in the periodic system trade less to reduce their price impact, implying a smaller trading volume.

IV. Continuous versus Periodic Trading

In this section, we consider the relative merits of continuous versus periodic trading systems and the implications for public policy. We also discuss the effect of explicitly introducing trading costs into the model.

A. Circuit Breakers and Trading Halts

Propositions 1 and 5 shows that market failure can occur in a continuous market with free entry into market making if the degree of information asymmetry Υ, exceeds the bound, $\Upsilon^c = \frac{\rho^2}{\psi}$. In this case, dealers cannot make non-negative expected profits on the first trade and on all subsequent trades. The upper bound Υ^c measures the noninformation motives for trading; it increases with risk aversion, ρ, and the variance of initial endowments of the risky asset, ψ^{-1}. For some securities there may be periods when θ is high relative to τ, so that Υ exceeds the critical bound and a continuous system may not be viable. Possible violations may be at the start of the trading day or immediately preceding the public revelation of new information.

Our analysis of the continuous system implies that trading could be restarted by increasing the ratio of public to private information. Casual observation suggests many devices and procedures to supply dealers with information on current market conditions when market failure is likely. For example, many continuous markets open (or re-open following a trading halt) with a call auction to allow the assimilation of new information.

However, our results show that continuous trading cannot always be restarted with a public information signal. To see this, note that as $\tau \to \infty$, $\Upsilon \downarrow \theta$, so that when $\Upsilon^c < \theta$, a continuous market is not viable even if the quality of public information is very high. The results suggest that proposals to reduce the market stress in continuous systems with 'circuit breakers' (trading halts triggered by large price movements) can exacerbate the original problem. Once trading is halted, it may be difficult or impossible to restart the process. Proposition 7 suggests a possible solution is to switch to a periodic trading mechanism in times of market stress, since this system can operate even if dealers choose not to make markets.

A related issue concerns the timing of the switch. In most markets, trading halts are triggered by large price movements. However, if these movements are warranted by changes in fundamentals, a trading halt will reduce market efficiency. Proposition 6 suggests a superior trigger. The proposition establishes that the quoted bid-ask spread becomes arbitrarily large as the measure of degree of information asymmetry Υ approaches from below the level at which the market fails. This suggests initiating a halt or a mechanism switch when the quoted bid-ask spread exceeds a critical level based on trading volume and historical spreads.

Empirical validation of these hypotheses is hampered by the difficulty in comparing trading structures across different markets and securities. Amihud and Mendelson (1987) examine NYSE stock returns from open-to-open, where trading is organized as an auction market, and close-to-close, where prices are determined in a continuous dealer market. They find significant differences in the two returns distribution, noting that the open-to-open returns variance is generally higher than the close-to-close returns variance. Stoll and Whaley (1990) confirm these findings, attributing them to NYSE opening practices. However, it is not clear whether these results reflect differences in the trading mechanism or the effect of the preceding overnight trading halt. Two recent papers attempt to resolve this issue using evidence from what comes close to controlled experiments.

Amihud and Mendelson (1991) use data for 50 stocks traded on Tokyo Stock Exchange, where a periodic auction, the *Itayose*, is used to open the two trading sessions of the day, during which trading is accomplished through a continuous mechanism, the *Zaraba*. Although the daily returns from the opening auction transaction are more volatile than the returns from the continuous market, this is not the case for the returns from the auction mechanism used to open the afternoon session. This suggests that the higher volatility at the opening found in studies of the U.S. market primarily reflects the preceding non-trading interval rather than the auction mechanism itself. Amihud and Mendelson, (1991) find that the mid-day auction "may well exhibit the least volatility and most efficient value discovery process" consistent with Proposition 7. They also establish that "a sequence of recent transaction prices facilitates value discovery and eases traders' inference on the current value of the security," as predicted by Proposition 6(b). In a similar study of 12 stocks traded on the Milan Stock Exchange, Amihud, Mendelson, and Murgia (1990) conclude that a call auction mechanism "provides a more efficient value discovery process for opening the trading day" than a continuous system. Further, they find that over the course of the day, "...investors correct perceived errors or noise in the prices set at the [opening] call," as predicted by Proposition 2(b).

B. Trading Costs and the Choice of Market Design

Although our discussion has emphasized the importance of asymmetric information, the choice of market design may be affected by other factors, especially the costs of operating the system. In this section, we consider the impact of introducing a fixed cost of providing market making services into the model. We focus on fixed costs since variable (order execution) costs appear to be relatively small in comparison.[21] Certainly, in models that exhibit linear equilibria (as is the case here) it is straightforward to incorpo-

[21] Glosten and Harris (1988) and Stoll (1989) decompose the bid-ask spread into its component parts and find that variable costs are a small portion of the spread.

rate a constant per share variable cost, without altering our qualitative results. Suppose now that there are a large number of potential dealers who can enter the market if they choose to do so. Each dealer must bear a fixed cost $F > 0$ at time 0 in order to trade.

The cost F may include the opportunity costs of capital, the costs of exchange membership, as well as fixed costs associated with order routing, processing, and settlement. Formally, we modify the model to incorporate a pre-game round where market makers enter the market until further entry would lead to negative expected profits. Thus, M is determined endogenously.

Proposition 9 (Trading Costs): *If there are fixed costs to providing continuous market making services, the quote-driven system* Γ^{c} *can function only with a monopolistic market maker. However, the continuous auction mechanism* Γ^{ca} *can support multiple market makers provided the fixed costs of dealers are sufficiently small.*

Proposition 9 demonstrates that a monopolistic market maker system is the only form of a quote-driven system that is viable when there are fixed costs to market making. Intuitively, pure price competition implies prices are set on the basis of marginal costs, which are below average costs. Without regulations to ensure a minimum bid-ask spread, the equilibrium number of dealers, if any, is one. The continuous auction Γ^{ca} does not suffer from such problems since expected trading profits are positive if the number of dealers is finite; in fact, the fixed costs determine the equilibrium number of market makers. The result provides some insight into the nature of exchange mandated price discreteness, as well as the persistence of quasi-monopolistic market making regimes. The periodic system Γ^{a}, of course, can function without dealers, but can impose high submission costs on traders, requiring either the physical presence of traders on the exchange floor or the costly submission of written demand schedules.

A related issue concerns the trading costs borne by investors. Without empirical evidence, it is difficult to assess the relative magnitudes of these costs, but the model provides some theoretical guidance. Traders' information costs in a dealer market are lower because traders need only solicit the dealer's bid-ask quotes. These quotes are sufficient statistics for the entire trading history. By contrast, in an auction market, investors must collect past trading information, which imposes private costs directly and social costs through duplication. Thus, there is no unambiguous ranking of mechanisms by their operating costs. The determination of which system has lower operating costs must be resolved empirically.

V. Conclusions

This paper models the process of price discovery under two alternative forms of market organization: a quote-driven system and an order-driven system. The quote-driven system relies on competitive dealers to post prices before orders are submitted while the order-driven system requires all participants to submit their orders before prices are determined. The quote-driven system is a continuous system since orders are executed upon submission. The order-driven system can operate either as a continuous system, with immediate execution on the exchange floor, or as a periodic system, where orders are stored for simultaneous execution. Trading is modeled as a game where order quantities and beliefs are determined endogenously and players act strategically. We showed that a quote-driven system provides greater price efficiency than a continuous auction system. However, with free entry into market making, the equilibria of the two mechanisms coincide.

We demonstrated that a periodic trading mechanism can function where a continuous market fails. This occurs because pooling orders for simultaneous execution can overcome the problems of information asymmetry that cause failure in a mechanism where trading takes place sequentially. However, a periodic system cannot provide immediate order execution. Further, a periodic system imposes higher costs for traders who must collect market information instead of observing price quotations. If a continuous market fails, it cannot re-open unless the degree of information asymmetry is lowered. Consequently, proposals to halt trading in times of market stress may actually exacerbate the original problem, possibly leading to market failure. Our results suggest a better alternative to a trading halt would be a switch to an auction market since this system can operate when dealers refuse to make markets. Such a switch would allow the public to observe a common information signal (the auction price) that may allow continuous trading to re-open at a future date. Casual observation suggests that thickly traded securities are generally traded in continuous markets, whereas thinly traded securities are traded in periodic auction systems. To the extent that information asymmetry is inversely related to the market value and trading activity, the model is consistent with the observation. Our results provide a partial explanation for differences in trading structures across markets and assets.

Appendix

Proof of Proposition 1: The proposition is a straightforward extension of Glosten's single-period model. We provide a detailed proof here for the sake of completeness. The price at time t_i is a function of the order quantity of the traders, and we write $p_i = p_i(q)$. Using (1), we see:

$$E\left[\tilde{W}_{1i} \mid \Phi_i\right] = \left(q_i + x_i\right) E\left[\tilde{v} \mid \Phi_i\right] + c_{0i} - p\left(q_i\right) q_i \tag{A1}$$

and

$$\operatorname{Var}\left[\tilde{W}_{1i} \mid \Phi_i\right] = \operatorname{Var}\left[\tilde{v} \mid \Phi_i\right]\left(q_i + x_i\right)^2. \tag{A2}$$

To simplify the notation, define $r_i \equiv E[\tilde{v} \mid \Phi_i]$ and $\alpha_i \equiv \rho \operatorname{Var}[\tilde{v} \mid \Phi_i]$. Expected utility maximization in equation (2) implies that q_i, solves:

$$r_i - p_i(q_i) - p_i'(q_i) q_i - \alpha_i q_i - \alpha_i x_i = 0. \tag{A3}$$

The second order condition is:

$$-p_i''(q_i)q_i - 2p_i'(q_i) - \alpha_i < 0. \tag{A4}$$

Suppose at time t_i, trader i places an order for q_i securities, given $p_i(q)$, the market maker's quotation function. Define $\hat{r}_i$ as follows:

$$\hat{r}_i(q_i) \equiv p_i(q_i) + p_i'(q_i)q_i + \alpha_i q_i. \tag{A5}$$

At time t_i, suppose the prior distribution of $\tilde{v}$ given public information (including any information revealed through the quotation schedule itself) is a normal distribution with mean μ_i and precision τ_i. Applying a basic proposition from statistical decision theory (see, e.g., DeGroot (1970)), the trader's posterior mean of $\tilde{v}$ given the signal, y_i, is:

$$r_i = \frac{\theta y_i + \tau_i \mu_i}{(\theta + \tau_i)}. \tag{A6}$$

The posterior precision of this estimate is $(\theta + \tau_i)$ so, by definition, $\alpha_i = \dfrac{\rho}{(\theta + \tau_i)}$. Define $z_i(q_i)$ by:

$$z_i(q_i) = \frac{\hat{r}_i(q_i)(\tau_i + \theta) - \mu_i \tau_i}{\theta}. \tag{A7}$$

Using the definition of $\hat{r}(q)$, we see that $\hat{r}_i(q_i) = r_i - \alpha_i x_i$. Substituting this and equation (A6) into equation (A7), we obtain

$$z_i(q_i) = \frac{\theta y_i - \alpha_i x_i(\tau_i + \theta)}{\theta}. \tag{A8}$$

Using the definition of α_i, $z_i = y_i - (\rho/\theta)x_i$. Recall $y_i = v + \epsilon_i$ so we can write:

$$z_i = v + \epsilon_i - x_i\left(\frac{\rho}{\theta}\right). \tag{A9}$$

Using equation (A9), we see that, from a market maker's perspective, z_i is drawn from a normal distribution with mean v and precision ω, where:

$$\omega \equiv \frac{\theta^2}{\left(\theta + \dfrac{\rho^2}{\psi}\right)}. \tag{A10}$$

The parameter ω is a critical constant, reappearing in our analysis of the order-driven mechanism. Note that ω is time-independent. As $\mathrm{Cov}(z_i, \mu_i) = 0$, we see that the market maker's posterior distribution of $\tilde{v}$ has mean:

$$E[\tilde{v} \mid q_i, \Phi_d^i] = \frac{\omega z_i + \tau_i \mu_i}{(\omega + \tau_i)} \tag{A11}$$

and precision $(\omega + \tau_i)$. To solve for the equilibrium quotation schedule, we express z_i in terms of q_i by substituting equation (A5) into (A7):

$$z_i = \frac{(\tau_i + \theta)(p_i(q) + p_i'(q)q + \alpha_i q) - \mu_i \tau_i}{\theta}. \tag{A12}$$

Substituting (A12) into equation (A11) and using the zero expected profits condition $p_i(q) = E[\tilde{v} \mid q_i, \Phi_d^i]$, we obtain:

$$p_i(q) = \frac{\omega}{(\omega + \tau_i)}\left[\frac{(\tau_i + \theta)}{\theta}(p_i(q) + p_i'(q)q + \alpha_i q) - \frac{\tau_i \mu_i}{\theta}\right] + \frac{\tau_i \mu_i}{(\omega + \tau_i)}.$$

Some algebraic manipulation yields the following expression:

$$qp_i'(q) = \kappa_i(p_i(q) - \mu_i) - \alpha_i q \tag{A13}$$

a first-order differential equation in q, where $\kappa_i \equiv \dfrac{\tau_i(\theta - \omega)}{\omega(\theta + \tau_i)}$ is a constant. The solution to (A13) is given by

$$p_i(q) = \mu_i + \left[\frac{\alpha_i}{\kappa_i - 1}\right]q + \mathrm{K}\,\mathrm{sign}(q) \mid q \mid^{\kappa_i} \tag{A14}$$

for $\kappa_i \neq 1$, where K is an arbitrary constant of integration. If $\kappa_i = 1$, the solution to (A13) is:

$$p_i(q) = \mu + \mathrm{K}q - \alpha_i q \ln(\mid q \mid).$$

We have derived the general functional form of the quotation function. The next step in the proof is to show that the linear function is the unique function satisfying the conditions of Definition 1.

Consider first the general case, where $\kappa_i \neq 1$. It is conventional to define $\lambda_i \equiv \alpha_i/(\kappa_i - 1)$. Suppose $\kappa_i < 1$ (i.e., $\lambda_i < 0$) and $\mathrm{K} < 0$. Then, $p(q) < p(-q)$ for all $q > 0$, an arbitrage opportunity that will be eliminated by interdealer trading. Now suppose $\mathrm{K} > 0$. Using (A14), the second order condition (A4) is:

$$\kappa_i(\kappa_i - 1)\mathrm{K} \mid q \mid^{\kappa_i - 1} + 2\lambda_i + 2\kappa_i \mathrm{K} \mid q \mid^{\kappa_i - 1} > -\alpha_i.$$

Inspection of the second order condition shows there exist some q^* and q_*, where $q^* > 0 > q_*$, such that the second order condition (A4) is violated if q does not lie in the interval (q^*, q_*), so that this cannot be an equilibrium.

Now assume $\kappa_i > 1$ (equivalently $\lambda_i > 0$) and $\mathrm{K} < 0$. The previous argument applies directly, so that $\mathrm{K} \geq 0$. If $\mathrm{K} > 0$, then the second order condition (A4) is always satisfied. However, the family of curves defined by equation (A14) is bounded from below (above) in the positive (negative) orthant by the linear equilibrium, and using condition (b) of Definition 1, $\mathrm{K} = 0$. Hence, if $\kappa_i \neq 1$, then $\mathrm{K} = 0$ in equilibrium, and the equilibrium is unique and linear in every period. Finally, the analysis of the case where $\kappa_i = 1$ is treated exactly like the case of $\kappa_i < 1$. The linear equilibrium is the unique equilibrium if $\kappa_i > 1$.

At time t_{i+1}, market makers' posterior distribution of v, given q_i, is normal if the initial prior is normal, because the normal distribution is closed under sampling (a conjugate family). This distribution becomes the prior distribution in determining $p_{i+1}(q)$. At time t_{i+1}, the dealer's prior distribution of $\tilde{v}$ is normal with mean μ_{i+1}. By definition, $p_i(q_i)$ is the posterior mean of $\tilde{v}$ given q_i:

$$\mu_{i+1} = p_i(q_i). \tag{A15}$$

and the posterior precision is:

$$\tau_{i+1} = \tau_i + \omega \tag{A16}$$

where ω is defined in (A10). In the first period, market makers have a normal prior distribution with mean μ and precision τ. Hence, by induction, the proposition holds, if equilibrium exists in every period. It remains to show that if the equilibrium condition is met in the first period, it will be satisfied in all subsequent periods.

For $\kappa_i > 1$ (equivalently, when $\lambda_i > 0$), we see that $\tau_i(\rho^2/\psi - \theta) > \theta^2$. From (A16), τ_i is increasing in i, so that if $\tau\rho^2/\psi > \theta(\theta + \tau)$ at time 1 then equilibrium exists in all subsequent periods. So, from equations (A14), (A15), and the definition of λ_i, the unique equilibrium at time t_i is linear. Then

$$p_i(q_i) = p_{i-1}(q_{i-1}) + \lambda_i q_i \tag{A17}$$

where $p_0 = \mu$ and λ_i is given by:

$$\lambda_i = \frac{\rho\theta}{\dfrac{\tau_i\rho^2}{\psi} - \theta(\theta + \tau_i)}. \tag{A18}$$

Let $\Upsilon^c = \dfrac{\rho^2}{\psi}$. Clearly, if $\Upsilon < \Upsilon^c$, the unique equilibrium is given by (A17) and (A18).

Proof of Proposition 2: (a) Note from proposition 1 that:

$$p_{i+1} = p_i + \lambda_{i+1} q_{i+1}$$

so that it is sufficient to show:

$$E[\tilde{q}_{i+1} \mid p_i] = 0.$$

From (A3), q_{i+1} is:

$$q_{i+1} = \frac{(r_{i+1} - p_i) - \alpha_{i+1} x_{i+1}}{(\alpha_{i+1} + 2\lambda_{i+1})}. \tag{A19}$$

Now

$$E[\tilde{r}_{i+1} - p_i \mid p_i] = E\left[\frac{\tilde{y}_{i+1}\theta + p_i\tau_{i+1}}{\theta + \tau_{i+1}} - p_i \mid p_i\right]$$
$$= \frac{\theta}{\theta + \tau_{i+1}}\left(E[\tilde{v} + \tilde{\epsilon}_{i+1} \mid p_i] - p_i\right).$$

Since $p_i = E[\tilde{v} \mid p_i]$ and $E[\tilde{\epsilon}_{i+1} \mid p_i] = E[\tilde{x} \mid p_i] = 0$, the result follows immediately.

(b) Since $e_{i+1} = v - p_{i+1}$, we obtain using (A17):

$$E[\tilde{e}_{i+1} \mid e_i] = -\lambda_{i+1}E[\tilde{q}_{i+1} \mid e_i] + e_i.$$

From the definition of q_i, we obtain:

$$E[\tilde{e}_{i+1} \mid e_i] = \eta_i e_i$$

where

$$\eta_i = 1 - \frac{\lambda_{i+1}}{(\alpha_{i+1} + 2\lambda_{i+1})} \cdot \frac{\theta}{(\theta + \tau_{i+1})}.$$

Clearly, $0 < \eta_i < 1$. Thus the predictive errors are positively correlated.

(c) The parameter λ_i depends on i through τ_i. Comparative statics shows that λ_i is a decreasing function of τ_i. Since τ_i is strictly increasing in i, it follows from (A18) that $\lambda_i < \lambda_{i-1}$, hence $s_i(1) < s_{i-1}(1)$.

Proof of Proposition 3: The proof constructs the Bayes-Nash equilibrium for the continuous auction mechanism Γ_{ca} by solving for a trader's best response to the conjectured strategies adopted by other traders and then shows the conjectures are consistent. Suppose at time t_i, the prior distribution of $\tilde{v}$, based on public information, is normal with mean μ_i and precision τ_i, where $\mu_1 = \mu$ and $\tau_1 = \tau$. Suppose trader i, who arrives at time t_i, conjectures that dealers (indexed by $j = 1, \cdots, M$) adopt strategy functions of the form:

$$d_j^i(p_i) = \gamma_i(\mu_i - p_i). \tag{A20}$$

We will show this conjecture is correct in equilibrium, and that the conjectured demand function satisfies all the conditions of the Definition 2. Using part (a) of Definition 2, we see that in equilibrium:

$$M\gamma_i(\mu_i - p_i) + q_i = 0. \tag{A21}$$

Using (A21) the price can be written as a function of q_i:

$$p_i = \mu_i + \zeta_i q_i \tag{A22}$$

where $\zeta_i \equiv \dfrac{1}{M\gamma_i}$. We turn now to the functional form of the demand schedule submitted by trader i. From equation (A6), it follows that:

$$E[\tilde{v} \mid \Phi_i] = \delta_i y_i + (1 - \delta_i)\mu_i \tag{A23}$$

where $\delta_i = \theta/(\tau_i + \theta)$ is a constant. From the utility maximization condition (A3), the demand schedule $q_i(p)$ given the price functional (A22) is:

$$q_i(p) = \frac{E[\tilde{v} \mid \Phi_i] - p_i - \alpha_i x_i}{(\alpha_i + \zeta_i)}. \tag{A24}$$

where we define $\alpha_i = \rho/(\tau_i + \theta)$.

Substituting (A23) and equation (A22) into equation (A24) and simplifying, we can express the optimal demand of trader i as:

$$q_i = \frac{\delta(y_i - \mu_i) - \alpha_i x_i}{\alpha_i + 2\zeta_i}. \tag{A25}$$

This equation shows that if the trader has rational expectations and correctly conjectures the price functional, he can submit a market order that is equivalent to the limit order, so that order form is irrelevant.

Consider now the strategic decision of a dealer, say dealer m. From (A21) and the market clearing condition, we obtain:

$$(M-1)\gamma_i(\mu_i - p_i) + q_i + d_m^i(p_i) = 0. \tag{A26}$$

It follows that for dealer m the price functional is given by:

$$p_i = (\mu_i + \beta_i q_i) + \beta_i d_m \tag{A27}$$

where we define $\beta_i \equiv \dfrac{1}{(M-1)\gamma_i}$. The expected profits of dealer m at time t_i are given by:

$$\pi_i = (E[\tilde{v} \mid p_i] - p_i) d_m^i(p_i) \tag{A28}$$

where the conditional expectation reflects rational expectations on the part of the dealer. Substituting (A27) into (A28), and solving for the optimal demand, we obtain:

$$d_m^i(p) = \frac{E[\tilde{v} \mid p_i] - \mu_i - \beta_i q_i}{2\beta_i}. \tag{A29}$$

Now consider the conditional expectation of the dealer. The dealer has rational expectations, and learns from the market clearing price and order submitted. Noted that a dealer who knows his own order quantity and the price knows q_i since $q_i = -Md_m$; under the conjectured strategies the trader's order is absorbed equally by the M dealers. From equation (A25), observing q_i is equivalent to observing:

$$\hat{y}(q_i) = \mu_i + \frac{\alpha_i + 2\zeta_i}{\delta_i} q_i. \tag{A30}$$

From equation (A25), we can express the signal as:

$$\hat{y}(q_i) = y_i - \left(\frac{\alpha_i}{\delta_i}\right) x_i. \tag{A31}$$

So, $\hat{y}$ is the minimum variance unbiased estimator of the private information of trader i a dealer can make given the order quantity q_i. Since $\alpha_i / \delta_i = \rho / \theta$, the signal is distributed normally with mean v and precision ω, where ω is given by (A10). Using Bayes' rule, we obtain:

$$E[\tilde{v} \mid p_i] = \chi_i \hat{y}(q_i) + (1 - \chi_i)\mu_i \tag{A32}$$

where $\chi_i = \dfrac{\omega}{\omega + \tau_i}$. Substituting (A27) into (A29), we can write the desired order quantity of dealer m as:

$$d_m^i(p) = \frac{E[\tilde{v} \mid p] - p}{\beta_i}. \tag{A33}$$

Using (A32) and (A30), (A33) can be written as:

$$d_m^i(p) = \frac{\mu_i - p + C_i q_i}{\beta_i} \tag{A34}$$

where $C_i \equiv \dfrac{\chi_i(\alpha_i + 2\zeta_i)}{\delta_i}$ is a constant. Since $q_i = -Md_m^i$, we obtain:

$$d_m^i(p) = \frac{\mu_i - p}{MC_i + \beta_i} \tag{A35}$$

which has the conjectured form, with $\gamma_i = (MC_i + \beta_i)^{-1}$. Since both β_i and C_i depend on γ_i, we must verify that γ_i is well-defined and satisfies the second order conditions for a maximum. This requires that $\gamma_i > 0$. Only then are the proposed strategies an equilibrium. Substituting in the definitions of C_i and β_i, we see that γ_i satisfies:

$$\frac{1}{\gamma_i} = M\chi_i\left(\frac{\rho}{\theta} + \frac{2}{M\delta_i\gamma_i}\right) + \frac{1}{\gamma_i(M-1)}. \tag{A36}$$

The solution is:

$$\gamma_i = \frac{\theta(\omega + \tau_i)}{M\omega\rho}\left[1 - \frac{2\omega(\theta + \tau_i)}{\theta(\omega + \tau_i)} - \frac{1}{M-1}\right]. \tag{A37}$$

Given that $M > 2$, for $\gamma_i > 0$ we require (using equation (A37)) that:

$$\frac{\theta(\theta + \tau)}{\tau} < \left(1 - \frac{2}{M}\right)\frac{\rho^2}{\psi}. \tag{A38}$$

If the inequality in (A38) is satisfied, $\gamma_i > 0$ and equilibrium is well-defined. Note that if γ_1 exists, then equilibrium exists in all subsequent periods since $\tau_i = \tau + (i - 1)\omega$. The conjugate property of the normal distribution ensures that the prior distribution is in fact normal. The prior is given by $\mu_i = p_{i-1} - \zeta_{i-1} q_{i-1}$, so the strategies and price functional can be easily expressed in the form stated in the proposition.

Proof of Proposition 4: (a) From equation (A38), equilibrium for the continuous auction system exists only if γ_1 exists. Define by Υ^a the right-hand side of (A38), so that $\Upsilon^a < \Upsilon^c$, implying the quote-driven system is viable in economies where the order-driven mechanism does not possess an equilibrium.

(b) To show that prices are not efficient (in a semi-strong form sense), suppose to the contrary that $p_i = E[\tilde{v} \mid p_i]$. Using (A33), this implies that $d_j^i = 0$ for all $j = 1, \cdots, M$. This implies that $q_i = 0$ for all $i = 1, \cdots, N$, or equivalently that:

$$\delta_i(\tilde{y}_i - \tilde{\mu}_i) - \alpha_i \tilde{x}_i = 0.$$

Since the probability of this event is (almost surely) zero, we obtain a contradiction. From equation (A33) we obtain:

$$p_i = E[\tilde{v} \mid \Phi_d^i, q_i] + \zeta_i q_i \tag{A39}$$

which shows that prices are not efficient.

(c) To distinguish the prices in the two mechanisms, at time t_i, let p_i^{ca} denote the price in the continuous auction Γ_{ca} and denote by p_i^c the price in the quote-driven system Γ_c. Now, using equation (A32) and the definition of χ, we see that the first term in equation (A39) is the price that prevails in time t_i in the continuous dealer mechanism, i.e., $E[\tilde{v} \mid \Phi_d^i, q_i] = p_i^c$. Accordingly, we can write the continuous auction price as:

$$p_i^{ca} = p_i^c + \zeta_i q_i \tag{A40}$$

Note that the quantity q_i differs from the quantity traded in the continuous dealer market because of differences in market liquidity as measured by ζ_i. Equation (A40) shows that the trading history under a continuous auction contains the information necessary to recover the unbiased dealer market prices; both histories are equally informative. Taking the unconditional variance of p_i^{ca} in (A40), we obtain:

$$\mathrm{Var}[\tilde{p}_i] = \mathrm{Var}[\tilde{p}_i^c] + 2\zeta_i \mathrm{Cov}(\tilde{p}_i^c, \tilde{q}_i) + \zeta_i^2 \mathrm{Var}[\tilde{q}_i].$$

Since the covariance term is positive, the variance of price at times $\{t_i\}$ in the continuous auction is higher than the price in the continuous dealer market.

Proof of Proposition 5: By definition, $\zeta = \dfrac{1}{\gamma M}$, and taking limits we obtain:

$$\lim_{M \to \infty} \frac{1}{\gamma M} = \frac{\rho\theta}{\dfrac{\tau_i \rho^2}{\psi} - \theta(\theta + \tau_i)}. \tag{A41}$$

From (A18), this limit is the parameter λ_i, showing that with an infinite number of traders, the existence condition (A38) for the game Γ_{ca} coincides with the condition for the game Γ_c. The equilibria of the two mechanisms coincide only in the limit. Note that as $M \to \infty$, $\gamma \to 0$, so that $d_m(p) \to 0$, i.e., each dealer's trade size becomes arbitrarily small, so the allocation of q_i across dealers differs from the market maker system.

Proof of Proposition 6: (a) A change in the parameters ρ, τ, ψ, and θ affects the quoted bid-ask spread through λ_i, and we consider each of these in turn. Since market depth is $\nabla = \lambda^{-1}$, spreads and liquidity are inversely related.

1. The effect of a change in ρ on λ_i is given by:

$$\frac{\partial \lambda_i}{\partial \rho} = \lambda_i \left[\frac{1}{\rho} - \frac{2\tau_i \lambda_i}{\psi\theta} \right] \tag{A42}$$

which is always negative in equilibrium. To demonstrate this, assume to the contrary that an increase in ρ increases λ_i. Then, it follows that $(1/\rho) > 2\lambda_i \tau_i/(\theta\psi)$, or $\psi/(\rho\tau_i) > 2(\lambda_i/\theta)$. Substituting the expression for λ_i in equation (A18), we obtain $1 > \dfrac{2\rho^2\tau_i}{\tau_i\rho^2 - \psi\theta(\theta + \tau_i)}$, which is a contradiction.

2. The effect of a change in τ_i on λ_i is given by:

$$\frac{\partial \lambda_i}{\partial \tau_i} = \lambda_i \left[\frac{\theta - \dfrac{\rho^2}{\psi}}{\rho\theta} \right] \tag{A43}$$

which is always negative since if equilibrium exists, $\rho^2 > \psi\theta$. Similarly, it is straightforward to show that λ_i is increasing in θ and in ψ.

(b) Zero ex post expected profits implies that the transaction price at time t_i is the posterior mean of $\tilde{v}$ given public information, i.e., that $p_i = \mu_{i+1}$. Now the prior distribution of $\tilde{v}$ at time t_{i+1} has mean μ_{i+1}, where μ_{i+1} is normally distributed with mean v and precision τ_{i+1}. Since traders have rational expectations, this distribution is correct. This implies that price volatility, conditional on public information at time t_{i-1} is:

$$\mathrm{Var}[\tilde{p}_i] = \tau_{i+1}^{-1} = (\tau + i\omega)^{-1}.$$

Clearly, the variance of price is decreasing in τ. Since ω is an increasing function of θ, it follows that higher private information quality implies more accurate pricing. Similarly, since ω is increasing in ψ, more noise trading means greater price variability. The opposite is true with the coefficient of risk aversion ρ, since intuitively greater risk aversion is associated with relatively less liquidity trading and hence more accurate pricing. Since $\mathrm{Var}[e_i] = \mathrm{Var}[v - p_i]$, these results also hold for the pricing error.

Proof of Proposition 7: (a) Consider an auction N traders. If $\tau > 0$, open entry for dealers implies the equilibrium price is the conditional expectation of $\tilde{v}$ given public information and the price itself, although the finite number of traders will act strategically. Formally, the market is efficient in that:

$$p = E[\tilde{v} \mid p]. \tag{A44}$$

The proof constructs the Bayes-Nash equilibrium by solving for a trader's best response to the conjectured strategies adopted by other traders and then shows the conjectures are consistent. Suppose trader i, $(i = 1, \cdots, N)$ conjectures that all other traders (indexed by $h = 1, \cdots, i-1, i+1, \cdots, N$.) adopt strategy functions of the form:

$$q_h(p) = a_0\mu + a_1 y_h - a_2 x_h - bp \tag{A45}$$

for $h \neq i$, where p is the batch market price and $b > 0$ is a constant. Note that (y_h, x_h) is the private information of trader h and is not known to trader i. In this mechanism, the orders are submitted directly to dealers, so that each dealer observes the net order flow $Q \equiv \Sigma_i q_i$. Then, a dealer can form a statistic $\hat{y}^N(Q)$ defined as:

$$\hat{y}^N(Q) = \frac{Q + Nbp - Na_0\mu}{Na_1}. \tag{A46}$$

From (A45), the statistic $\hat{y}$ is simply:

$$\hat{y}^N(Q) = \frac{\Sigma_i y_i - \left(\dfrac{a_2}{a_1}\right)\Sigma_i x_i}{N} \tag{A47}$$

This statistic is distributed normally with mean v and with precision denoted by $N\pi$, where π is the precision of $\tilde{y} - \dfrac{a_2}{a_1}\tilde{x}$. Accordingly, using (A44), the equilibrium price can be written as:

$$p = \xi\mu + (1 - \xi)\hat{y}^N \tag{A48}$$

where we define $\xi \equiv \tau/(\tau + N\pi)$. Rearranging, we obtain:

$$p = \frac{a_1\xi - a_0(1-\xi)}{a_1 - (1-\xi)b}\mu + \frac{1-\xi}{N(a_1 - (1-\xi)b)}Q. \tag{A49}$$

We turn now to the construction of the demand function of trader i, where $i = 1, \cdots, N$. From (A49), the price functional faced by trader i takes the form:

$$p = \left(\phi_1\mu + \phi_2\sum_{h\neq i} q_h\right) + \phi_2 q_i \tag{A50}$$

where $\phi_1 \equiv \frac{a_1\xi - a_0(1-\xi)}{a_1 - (1-\xi)b}$ and $\phi_2 \equiv \frac{1-\xi}{N(a_1 - (1-\xi)b)}$. Utility maximization implies that:

$$q_i(p) = \frac{E[\tilde{v} \mid y_i, p] - p - \alpha' x_i}{\alpha' + \phi_2} \tag{A51}$$

where $\alpha' \equiv \rho \operatorname{Var}[\tilde{v} \mid y_i, p]$. The conditional expectation depends on price, so to ensure the conjectures are correct and equilibrium exists, we must write this out in full. Given the market clearing price, trader i can compute:

$$\sum_{h \neq i} q_h = \frac{p - \phi_1\mu}{\phi_2} - q_i$$

Now, when $N \geq 2$, trader i can form the statistic:

$$\tilde{v}_i \equiv \frac{\phi_2^{-1}(p - \phi_1\mu) - q_i - (N-1)\mu a_0 + (N-1)bp}{(N-1)a_1}. \tag{A52}$$

Note that $\hat{v}_i = (N-1)^{-1}[\sum_{h\neq 1} y_h - (a_2/a_1)\sum_{h\neq i} x_h]$ is a sufficient statistic for the information content of the market clearing price. By construction, $\hat{v}_i$ is distributed normally with mean v and precision $(N-1)\pi$, independently of y_i and x_i. Now define:

$$\delta_0 = \frac{\tau}{\tau + \theta + (N-1)\pi}$$

$$\delta_1 = \frac{\theta}{\tau + \theta + (N-1)\pi}$$

$$\delta_2 = \frac{(N-1)\pi}{\tau + \theta + (N-1)\pi}.$$

Then, using Bayes' rule, we can write the conditional expectation of trader i as follows:

$$E[\tilde{v} \mid \Phi_i, p] = \delta_0\mu + \delta_1 y_i + \delta_2 \hat{v}_i. \tag{A53}$$

Substituting (A53) into (A51), and rearranging, we see that q_i has the conjectured form where the coefficients solve:

$$a_0 = \frac{\delta_0 - \delta_2[(N-1)a_1\phi_1\phi_2]^{-1} - \delta_2 a_0 a_1^{-1}}{\alpha' + \phi_2 + \delta_2[(N-1)a_1]^{-1}} \tag{A54}$$

$$a_1 = \frac{\delta_1}{\alpha' + \phi_2 + \delta_2[(N-1)a_1]^{-1}} \tag{A55}$$

$$a_2 = \frac{\alpha'}{\alpha' + \phi_2 + \delta_2\left[(N-1)a_1\right]^{-1}} \tag{A56}$$

$$b = \frac{-\delta_2\left[(N-1)a_1\phi_2\right]^{-1} - \delta_2 a_1^{-1} b + 1}{\alpha' + \phi_2 + \delta_2\left[(N-1)a_1\right]^{-1}}. \tag{A57}$$

The system consists of four equations in nine unknowns, a_0, a_1, a_2, b, ϕ_1, ϕ_2, δ_0, δ_1, and δ_2. The parameters ϕ_1 and ϕ_2 are functions of the a's and b while the δ's depend on π which in turn depends on $\frac{a_2}{a_1}$. To ensure a closed-form solution, we must express the ratio a_2/a_1 in terms of the original parameters of the model, ρ, ψ, θ, τ, and N.

From the definition of a_1 and a_2, we have:

$$\frac{a_2}{a_1} = \frac{\alpha'}{\delta_1} = \left(\frac{\rho}{\tau + \theta + (N-1)\pi}\right) \Bigg/ \left(\frac{\theta}{\tau + \theta + (N-1)\pi}\right) = \frac{\rho}{\theta}.$$

Hence, $\pi = \omega$, where ω was defined in equation (A10). Using this result, we can directly determine the values of δ_0, δ_1, and δ_2. Now consider, ϕ_1. If the market clears without dealer participation ($Q = 0$), by symmetry, the base price should be μ, which implies that $\phi_1 = 1$. Alternatively, since ϕ_1 does not depend on N, setting $N = 1$ should correspond to the limiting equilibrium of the continuous auction game Γ_{ca} as shown by Proposition 5, which implies that $\phi_1 = 1$. This implies that in equation (A49) $b = a_0 + a_1$. Substituting in the equation for ϕ_2, the solution can be obtained from the system (A54)–(A57) by first solving simultaneously for a_1 and b and then computing the remaining parameters. A sufficient condition for these coefficients to be well-defined is $\theta < \frac{(N-2)\rho^2}{N\psi}$. In the proof of Proposition 8 we construct an example where continuous markets fail but an auction market is viable.

(b) Using (A44), we see the auction mechanism Γ_a is semi-strong form efficient. This condition implies that in a sequence of periodic auctions, indexed by t, with N_t traders in each auction, the auction prices follow a martingale. To show this, note that

$$E[\tilde{p}_{t+1} \mid p_t] = E[E[\tilde{v} \mid p_{t+1}] \mid p_t] = E[\tilde{v} \mid p_t] = p_t.$$

(c) From equation (A48), the unconditional price variability is:

$$\mathrm{Var}[\tilde{p}_i] = \xi^2\tau^{-1} + (1-\xi)^2 \mathrm{Var}[y^N(\tilde{Q})] \tag{A58}$$

which is simply $(\tau + N\omega)^{-1}$. Now consider a sequence of N transactions in the quote-driven mechanism Γ_c. Using Proposition 1, the variance of these prices is monotonically decreasing over time as market makers learn. Consequently, of this sequence, p_N has the lowest variance which is $(\tau + N\omega)^{-1}$. It follows that the unconditional variance of the auction price is (weakly) less than the unconditional variance of the entire sequence of corresponding prices $p_1, \cdots, p_N$ in the dealer market.

The corollary follows from substituting equation (A47) into (A48), and noting that as $N \to \infty$, $\xi \to 0$ and $\hat{y}_N \to v$.

Proof of Proposition 8: (a) Consider the limiting case where $\tau = 0$, i.e., there is no public information.

In this case, $M = 0$, since dealers cannot make expected profits if they absorb any net order flow. The proof is a straightforward repetition of the proof technique from Proposition 7 where equation (A44) is replaced with the market clearing condition $Q(p) = 0$, and we set $a_0 = 0$. We now summarize the results of this exercise: The demand function of trader i is linear of the form:

$$q_i = a_1(y_i - p) - a_2 x_i \tag{A59}$$

where

$$a_1 = \frac{\bar{a}\theta}{(N-1)\omega + \theta}$$

$$a_2 = \frac{\bar{a}\rho}{(N-1)\omega + \theta}$$

and $\bar{a}$ is a constant given by:

$$\bar{a} = \frac{((N-1)\omega + \theta)[(N-2)\theta - 2(N-1)\omega]}{(N-1)\rho\theta}$$

and ω was defined in (A10). From this, the equilibrium price is:

$$p^* = \frac{1}{N}\sum_i^N \left[y_i - \left(\frac{\rho}{\theta}\right) x_i \right]. \tag{A60}$$

Equilibrium exists if $\theta < \dfrac{\rho^2(N-2)}{\phi N}$. This argument demonstrates the auction system can function in markets where a continuous system is not defined. Since $E[\tilde{v}] = p$, the price is efficient. The variance of the price is $\mathrm{Var}[\tilde{p}^*] = \dfrac{1}{N\theta} + \left(\dfrac{\rho}{\theta}\right)^2 \dfrac{1}{N\psi}$, and the comparative statics results follow immediately.

(b) In the classical model, traders ignore their effect on price and correspondingly ignore the information contained in price. Utility maximization yields the trader's demand function:

$$q_i(p) = \frac{\theta(y_i - p)}{\rho} - x_i$$

where we have used the fact that the trader i's unconditional beliefs regarding the asset's value have mean y_i and precision θ. Setting $\Sigma_i q_i(p) = 0$, the equilibrium price is given by equation (A60), the same price when traders have rational expectations and act strategically. The equilibria are distinct, however, in that the quantities traded differ. The quantities demanded coincide only if $a_1 = \theta/\rho$ and $a_2 = 1$, but this is not the case. Write the demand function (A59) as:

$$q_i = a_2\left[\frac{\theta}{\rho}(y_i - p) - x_i\right].$$

Since the term in brackets is the demand in the classical Walrasian auction and $a_2 < 1$, the volume of trade is smaller in the auction system Γ_a than in the Walrasian auction.

Proof of Proposition 9: In the quote-driven system, if $M \geq 2$, pure price competition implies that $p_i = E[\tilde{v} \mid q_i, \Phi_d^i]$. This implies that expected trading profits are zero, and any positive fixed costs are not recoverable. Only if $M = 1$ can a dealer hope to cover his fixed costs, and the equilibrium corresponds to that of Glosten (1989). In the continuous auction system expected dealer profits are given by (A28), and we define by $\pi(M)$ the expected profits of a dealer as a function of M. It is readily verified that for finite M, $\pi(M) > 0$, provided equilibrium exists. Further, it can be shown that $\pi'(M) < 0$. So, if F is sufficiently small, there exists a $M^* > 1$ such that $\pi(M^*) \geq F$ and $\pi(M^* + 1) < F$.

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