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Periodic market closure and trading volume

A model of intraday bids and asks*

William A. Brock
University of Wisconsin, Madison, WI 53706, USA

Allan W. Kleidon
Stanford University, Stanford, CA 94305-5015, USA

This paper examines the effect of periodic stock market closure on transactions demand and volume of trade, and consequently bid and ask prices. We extend Merton (1971) to show that transactions demand at open and close is greater and less elastic than at other times of the trading day. In response, a market maker such as an NYSE specialist may effectively price discriminate by charging a higher price to transact at these periods of peak demand. Our predictions of periodic demand with high volume and concurrent wide spreads are consistent with empirical evidence, while the predictions of current information based models are not.

1. Introduction

The open and the close of trading in a stock market that is open during the day and closed overnight seem to be natural candidates for investigation. While the information on which the asset prices are based evolves continuously over the entire period, there is an abrupt change from a regime of continuous trading to one of zero trading. How does this affect trading behavior at the transition points, namely open and close?

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It has been known for some time that there are abrupt changes in the characteristics at open and close of stock markets such as the New York Stock Exchange (NYSE). Most obviously, these markets have institutionalized differences in trading at these times. On the NYSE, for example, trades can be designated to be made 'at the opening', in which any part of an order that is unfilled after the opening is cancelled, or at the close, called 'market-on-close', whereby orders are designated to be executed as close as possible to the end of trading. The opening itself is a call market as opposed to the subsequent continuous auction, and recently the NYSE has extended trading hours to allow many trades to be carried out at the closing price (see section 3.2 below for institutional details). Accompanying these institutional distinguishing characteristics are high volumes at open and close.

While it seems natural to us to attempt to link these trading characteristics to the fact that the open and close are the points of discontinuity between continuous and zero trading opportunities, there is little research on this issue. In the primary paper which has attempted to explain the high NYSE volume at open and close, Admati and Pfleiderer (1988) present a theory of trade clustering that can occur at arbitrary times in the trading day, although their concluding remarks refer to the possible role of open and close as unique clustering points.

The approach in this paper is to directly examine the implications of the discontinuity in trading regimes represented by the open and close. The main paper on portfolio holdings in continuous markets is Merton (1971), and most work that examines optimal trading in the presence of transactions costs is based on Merton's results. However, he assumes a continuous market with no periodic closures. In section 2, we extend his model to allow for the periodic market closures that are the focus of this paper. The major implications of the extension are increased and less elastic demand to trade at the open and the close, for two main reasons. First, the accumulation of overnight information in the absence of an opportunity to trade means that portfolios at the open have in general deviated from optimal holdings, resulting in opening trade to reestablish optimal portfolios. Second, in preparation for an overnight nontrading period, the optimal portfolios at the close will differ from those that are optimal during the continuous trading interval.

Optimal portfolios at close will certainly differ from those during the preceding continuous trading interval if the stochastic dynamics during the close differ from those during the open period, as documented by French and Roll (1986). Even if the stochastic dynamics during the close are the same as during the open, it seems that except for special cases (where a type of 'self-averaging' in the stochastic dynamics leads to the same portfolio proportions during the close as during the open), there will be an abrupt change in the optimal portfolios as the closing time is passed. Moreover, changes in optimal portfolios in advance of a closed overnight period are consistent with the trading floor observation that many short-term traders attempt to close out positions at day's end to avoid the large overnight risk.

Our model of periodic market closure implies higher transactions demand at open and close, which is consistent with the empirical phenomenon of higher volume at those times. Section 3 examines the reaction of the specialist market maker to such increased transactions demand. We extend the model of Garman (1976) (who coined the term 'market microstructure') to account for periodic changes in transactions demand. We provide mild conditions under which periods of high demand for the purchase (sale) of securities will lead to higher asks (bids), so that the high transactions demand at open and close will result in higher bid–ask spreads than over the rest of the day.[1] The primary intuition for this result is that a monopolist specialist is able to price discriminate, and hence charges a higher transactions price in periods of high transactions demand. The result can also occur under competition if the fixed costs of supplying liquidity are high relative to marginal costs, as argued by Grossman and Miller (1988).

The empirical evidence on volume and spreads is examined in section 4, and the results support the implications of our models of periodic demand and market marker reaction. In particular, NYSE volume is concentrated at open and close, and is accompanied by wider bid–ask spreads at these times. Moreover, direct tests of our models provide empirical support; for example, Gerety and Mulherin (1991) relate periodic volume at open and close to expected and unexpected overnight volatility, which they tie directly to our model of periodic market closure. However, the evidence appears at odds with the information model of Admati and Pfleiderer (1988), which requires lower transactions costs to explain high volume. We compare our results with those from the information economics tradition, and find that extant information models can reconcile high volume and spreads only by relying on the sort of periodic demand that we model here.

Section 5 outlines potential extensions to this work, and section 6 contains concluding remarks.

2. Periodic trading demand shifts: Open and close of trading

We discuss several possible reasons for increased transactions demand at the open and close of trading. Our primary arguments, given in section 2.1, consider the effect of a periodic inability to trade. We discuss related

[1]Although our results in this context are most easily stated in terms of spreads, we view the fundamental economic variables as bid and ask prices. For example, we allow for the possibility of different shifts in demand for transactions services for sellers than for buyers; although the resulting sequence of bids and asks will define a spread that may change over the trading interval, this is not the primary variable. Grossman and Miller (1988) also deemphasize the spread as the primary variable, in an analysis that is consistent with ours in many respects.

literature on optimal trading in the presence of transactions costs and a continuous market, extend the arguments to periodic market closures, and note possible additional extensions to the model. The conclusion reached in section 2.1 is that typically there will be greater demand to trade at open and close than at other times the exchange is open. Section 2.2 discusses secondary considerations, namely the ability to trade on an alternate market when the primary market is closed, the filling of unexecuted orders at close, and the effect of judging some public traders on criteria that depend on how closely their trade prices match closing prices.

2.1. *Inability to trade when the market is closed*

Assume that the calendar day is divided into two periods. The first, extending over an interval of length T, allows continuous trading of securities; the second interval of length N allows no trading. Adopting the convention that the first day of trading extends over $[0, T]$, the first closed period is over $(T, T+N)$. Trading resumes the next day over $[T+N, 2T+N]$, and so on.

Consider first the open of trading the next day, at time $T+N$. Information arriving while the market is closed is likely to have induced trading had the market been open. If traders wish to be at the same position at the open of trade the next day as they would have been had the market been open overnight, they will have to execute their net overnight trades at the first trade the next day. This provides a natural explanation for the high opening volume, assuming that net trades over about 17 hours are larger than the trades typically made when continuous trading is possible.

The effect of a closed market on trading prior to the close is more difficult to determine. One possibility is that investors' optimal overnight portfolios would change even if they could trade continuously, because of overnight changes in price behavior. French and Roll (1986) show that the variance rate overnight differs significantly from that during the day (although it is an open question as to how the rate would differ if continuous 24-hour trading were possible). Miller (1989) claims that short-sellers wish to close out their positions at day end because of the typical overnight rise in price as settlement is delayed by one day, and short-term day traders try to achieve a net-zero overnight position to avoid settlement.[2]

More generally, however, optimal portfolios may change overnight simply because continuous trading is not possible, so that the distributions of returns faced over the discrete closed period are different from those over continu-

[2]Presumably, if settlement were calculated at a particular instant but trading were otherwise continuous overnight, such traders would desire zero inventory or short positions at just that instant.

ous intervals. This is an alternative explanation for a desire by traders to close positions by day's end: potential exposure over a long period when the market is closed is greater than when one has the option to close out a position continuously.

We now formalize these arguments.

2.1.1. Continuously open markets and transactions costs

The relevant literature on optimal trading with transactions costs takes the original model of Merton (1971), which assumes a continuous time diffusion process for prices and zero transactions costs, and modifies the results to account for costs of trading. For proportional transactions costs, the essence of the solution is that there exists a region around the optimal 'Mertonian' portfolio, and trade occurs only if portfolio proportions move outside this region.[3]

2.1.2. Periodic market closure: Extension of Merton (1971)

The problem we consider differs because of periodic market closure. Appendix 1 extends Merton's (1971) model to this alternate regime. The primary result is that periodic closure implies that demand to trade will in general be stronger and relatively inelastic at open and close compared to other times during the trading day, for two reasons. First, even if optimal portfolio proportions (i.e., the desired fraction of wealth allocated to each security) were constant at all times, the investor will want to trade at the open because overnight price changes will imply changes in the number of shares held to maintain the (assumed constant) portfolio weights. In the context of time stationary log normal returns processes over the open period and HARA utility [i.e., $u(C) = C^a/a$], the investor's desired proportions are constant during the open period (appendix 1, Remark 4). More generally, if the optimal proportions change during the open period, they change continuously. Even if the desired proportions over both open and closed periods were constant, there will in general be unusually high trade at open since the stochastic processes describing stock prices continue to evolve overnight. The price at the close will typically differ from the price at the following open, implying a change in the number of shares required to maintain the optimal proportions (appendix 1, Remarks 2 and 4).

[3]See Constantinides (1986), Davis (1988), Koo (1991), Magill and Constantinides (1976), Taksar, Klass, and Assaf (1988), and especially Davis and Norman (1990). See also Dumas and Luciano (1991). With transactions costs and portfolio management fees proportional to portfolio values, Duffie and Sun (1990) show that the portfolio problem for an investor with constant relative risk aversion is equivalent to a stochastic discrete time portfolio selection problem at equally spaced transaction times.

Second, the optimal weights for the overnight period (which will ideally be achieved at the previous close of trading) need not be the same as those over the time the exchange is open. The most obvious cause is changes in the price processes over the open and close of trading, as documented, for example, by French and Roll (1986). If the optimal weights are constant over the day but change for the overnight period, they change only across the open period to the closed period and vice versa. If they change (continuously) over the day, they may change discontinuously over the close. The investor's portfolio proportions during the closed period are determined by optimal choice right before the close. Changes in the price processes overnight will be sufficient to change the optimal weights overnight (appendix 1, Remarks 2, 4, and 6). Since the sample paths of diffusion processes are continuous, abrupt changes in desired portfolio proportions at the open and at the close will give a spike in demand for trading at the open and the close (appendix 1, Remarks 7 and 9).

Our analysis of the optimal choice at close in appendix 1 provides details of how to calculate the optimal portfolio proportions [eqs. (A.11) through (A.16)]. In general, the amount of the abrupt change at close seems difficult to quantify analytically, although our analytical methods can pinpoint the expected change for potential numerical analysis (which is beyond the scope of this paper). An interesting unresolved issue is whether our arguments for increased volume at open and close of trade apply if asset characteristics do not change over the open and closed periods. The question raises issues dealing with the role of the market in investor's portfolio choice. Kreps (1982) and Duffie and Huang (1985) have shown that continuous trading can induce Pareto optimality in markets that look quite incomplete, by analogy with the perfect hedge that continuous trading provides in the Black–Scholes option pricing model. Clearly there are different opportunities to adjust portfolios to provide optimal hedging in the two trading environments, and it would not be surprising to find that the optimal portfolio during the open period with potential continuous adjustment across assets differs from the optimal portfolio during the closed period.

Additional work, especially within general equilibrium models, is warranted. However, the evidence that the conditional moments of stocks are not constant over both periods is sufficient for our conclusion, namely that the extension of Merton (1971) to a regime of periodic market closure implies that demand to trade will in general be stronger and relatively inelastic at open and close.

2.1.3. Periodic market closure: Transactions costs and extensions

A similar result occurs in models with transactions costs, where trade is postponed until portfolio proportions exceed a region around the zero-cost optimal portfolio. At open, there will be stronger demand to trade than during most of the day since there has been no opportunity to rebalance the portfolio as soon as the nontrading region is violated during the long overnight period. At close, an abrupt change in the Mertonian solution changes the nontrading region, so that some portfolio positions that would be tolerable in a regime of continuous future trading will be suboptimal if there is no further opportunity to trade for many hours.

Our extension of Merton (1971) shows that the desired volume of trade increases at the open and at the close, which is the primary goal of our analysis. A more realistic general equilibrium model with multiple traders and assets, transactions costs and equilibrium movement in bid–ask spreads would be satisfying, but we have not been able to work out an analytically tractable general equilibrium model of this form. A representative agent model such as in Cox, Ingersoll, and Ross (1985a) begs the question of volume, since in equilibrium all shares must be held and there will be no trade.

To deal with this issue, consider an extension of Merton (1971) to $H > 1$ investors, where each investor, h, has a utility of the form $U_h = u(C_h, X_h) = C_h^a X_h / a$. To keep the notation manageable assume that there is just one risky asset driven by $\mathrm{d}Z$ and one risk-free asset with rate of return r. Here h's 'taste shock', X_h, is a diffusion process,

$$\mathrm{d}X_h = A_h\,\mathrm{d}t + B_h\,\mathrm{d}Z_h, \qquad \mathrm{E}_t\,\mathrm{d}Z_h\,\mathrm{d}Z = e_h\,\mathrm{d}t.$$

Think of the 'taste shock' as an analytically tractable way of representing changes in investor h's needs due to shifts, for example, in fortunes of other activities not explicitly represented in the model.

Our basic point is that the optimal portfolio proportions over the open and closed periods will differ in this case as in the case dealt with above. It may be possible to derive a closed form solution. It can be shown [see Cox, Ingersoll, and Ross (1985b, p. 389)] that, for each investor, the state valuation function $J(W, X)$ is multiplicatively separable in W, X and is isoelastic in W for U of the multiplicatively separable isoelastic form in the consumption argument. This is proved for the open period. A similar type of result may be had for the closed period. Using this fact one can show that the portfolio proportions, w_h, of h's wealth that she desires to hold in the risky asset do not depend on W_h. The optimal proportion does depend upon X_h as well as the risk-free rate, the expected return and standard deviation of return on the risky asset as well as the covariance e_h. The analytically tractable formula for w_h can be used to solve for the demand $N(h)$ for shares of the risky asset by h. Letting the supply of risky shares be $N = 1$, we can use the aggregate demand for shares of the risky asset summing to unity and the aggregate demand for riskless bonds summing to zero to solve for the equilibrium demands for shares. Since the risky asset price is fixed by the constant returns technology as in Cox, Ingersoll, and Ross (1985a, b), the equilibrating variable is the risk-free rate.

In principle it should be possible to work out the movement of equilibrium volume of trade in shares over time across the H investors. There is a motive for trade in this model because the taste shocks are all different and each investor desires to hedge against his taste shock. In any event, at the close, there will be an abrupt change in the demand for trade because the optimal portfolio proportions will abruptly change for each of the H investors. Hence volume of trade will abruptly change, and similarly at the open.

An alternative model would motivate trade by assuming that investors have different risk tolerances [see, for example, Dumas (1989)]. This would fit well into the institutional observation that short-sellers and day traders avoid the overnight exposure of their positions by typically closing them out at the end of the day's trading, then reestablishing postions the next day. In the colorful language of the futures and options markets, traders 'do not go to bed with an unhedged position' precisely to avoid the potential exposure overnight when there is no opportunity to quickly adjust the portfolio.

We do not work out the algebra of these models here since we only want to make the point that in a heterogeneous agent model where equilibrium trading occurs, the demand for trading services will increase at the open and at the close. Placing a bid–ask spread in these models would be a building block in a theory of the equilibrium bid–ask spread. Such a theory would be analytically satisfying, but is beyond our ability at this point.[4] Hence we choose the tack in section 3 below of extending current models of the bid–ask spread to account for periodic shifts in trading demand.

2.2. Secondary considerations: Open and close

2.2.1. Ability to trade elsewhere

We briefly consider how the previous analysis may change if it is possible to trade on other markets during the closed period $(T, T+N)$. This is possible in some stocks, with the advent of access to international markets such as Tokyo or London.

Assume initially that the alternate markets extend the period of potential continuous trading to $[0, T+N)$; that is, there is no need for an investor to decide at T on portfolio holdings that cannot be changed over $(T, T+N)$. Ignoring transactions costs, investors would seek to use the additional markets to hold optimal portfolios at all times. Exactly how this would translate into trading volume on the primary market would depend on the trading

[4]See also the discussion in Davis and Norman (1990) concerning the analytic difficulties in extending the model.

rules of the alternate markets. For example, if there were literally zero transactions costs, it may be optimal to liquidate the entire portfolio on one market and reinvest in the next; this would lead to increased volume at times of open and close of trading across markets. An alternative strategy may be one of not changing the portfolio on the primary market, and making incremental changes on other markets, which would require an absence of restrictions on short sales.

In the presence of transactions costs, investors make similar trade-offs between sub-optimal portfolios and trading costs as are made when no overnight trading is possible. Depending on the costs of trading on alternate markets, it may be optimal for some (particularly small) investors to remain within one market, which effectively puts them in the position examined in the previous section with a truly closed market.[5] The effect on trading for those investors for whom it pays to trade continuously – e.g., institutional trading where the fixed costs of maintaining world-wide accounts can be more effectively amortized – will depend on the institutional features of the various markets (e.g., how much of the portfolio is liquidated to accommodate sales in alternate markets?) and the extent of trading at open and close if the opportunity to continue trading had not existed.

2.2.2. Nonportfolio trading demand

There are at least two institutional reasons for strong trading demand at certain times of the day other than the general portfolio considerations discussed in section 2.1 above. First, brokers are given orders to execute at their discretion over the trading day. As close approaches, the need to fill any remaining orders clearly increases. Second, there will be differential demands across trading times if investors receive payoffs that depend on the time of day at which trading occurs. One example is if passive portfolio managers are judged by how closely their portfolio matches some benchmark portfolio, e.g., an index such as the S&P 500. Since the values of such benchmark portfolios are typically based on closing prices, the divergence between the portfolio manager's results and the benchmark can be reduced by timing the former's trades as nearly as possible to the close of trading. More generally, a fund manager may be evaluated on how much was paid to acquire the fund's portfolio which is currently valued at closing prices; this again provides an incentive to trade at the close.[6]

[5]The costs of transacting in different markets can be subtle. For example, settlement procedures may differ, making it difficult to compare trades across markets – see, e.g., 'International Equities: Acts of Settlement' in *The Economist*, September 17, 1988, pp. 91–92.

[6]If the compensation contract were based *solely* on the difference between the price paid and the closing price, the portfolio manager may be willing to pay a high price for the privilege of transacting at the price used to calculate the benchmark, i.e., would have very inelastic demand for transactions at the last trade of the day.

A related argument concerns sales and redemptions of shares in mutual funds, which are typically based on the net asset value per share, with the underlying securities in the fund's portfolio valued at closing prices. Fund managers have an incentive to execute transactions in the underlying securities at the same prices at which the fund is valued. Again, this implies inelastic demand for transactions at the close of trade.

There is evidence that this argument is valid, resulting in high demand for trades at market close. The *Wall Street Journal* (September 21, 1988) reports trades that are 'lined up ahead of time in New York' and then 'booked in London's over-the-counter market'. It continues:

> Perhaps the biggest attraction to London program trading is that it can all be pegged to New York Stock Exchange closing prices. Many big institutions – particularly mutual funds – like to trade at closing prices because these are what they use to value their portfolios. But dealing at the actual close of trading can be surprisingly tough, given how may big investors all try to buy or sell in the final minute.

The NYSE has recently responded to such demand for trades at the close by extending the hours of trading, with a specific period, 4:15 p.m. to 5:00 p.m., during which trades at the (4:00 p.m.) closing price may be executed.[7]

3. Bids and asks with periodic changes in transactions demand

The last section shows that periodic market closure results in periodic changes in the demand for transactions services. In particular, there will be increased and less elastic transactions demand at the open and the close, the natural breaks in trading regimes. We now examine the reaction of a monopolist market maker who stands ready to trade at a posted bid and ask. This problem was first examined by Garman (1976), and in section 3.1 we extend his model to include our findings of periodic demand changes at open and close. The result is that (under mild conditions) the market maker responds by setting a higher ask in periods of high demand to buy stocks, and lower bids in periods of high demand to sell. Increased transactions demand to both buy and sell stocks at the open and close implies that the ask rises, the bid falls, and the bid–ask spread widens at those times. The intuition is

[7]See *Wall Street Journal*, May 22, 1991. Note that the possibility of demand by portfolio managers for trading at the close, coupled with a secular increase in the value of such portfolios, leads to the possibility of higher demand for trading services on the buy side than on the sell side at close of trading. This implies an increased probability of a closing trade at the ask rather than at the bid, as documented by Harris (1989) and Terry (1986), which is consistent with the increase in average measured price at the close of trade [see Wood, McInish, and Ord (1985) and Harris (1986)].

natural: since the conditions for price discrimination are satisfied in the stock market, a profit-maximizing market maker will extract some surplus from those with stronger and less elastic demand to trade at open and close.

Section 3.2 provides institutional background for the NYSE opening and closing procedures, that links our analysis in 3.1 to the empirical evidence on bid–ask spreads in section 4. Section 3.3 takes this institutional setting and shows that the assumption of a monopolist market maker is not crucial, since a similar result can occur if the market maker faces some competition but has greater market power (due to institutional features) at open and close, or even if the market is fully competitive.

3.1. Periodic demand shifts and increased bid–ask spreads

3.1.1. Basic model

We follow Garman (1976) in assuming a monopolist market maker who faces stochastic buy and sell orders that are Poisson distributed in time. The market maker is assumed to maximize expected profit per unit time, and we employ Garman's (p. 265) assumption that the market maker seeks zero inventory drift, i.e., seeks to equate arrival rates of buy and sell orders rather than require a positive drift over time to avoid failure.[8] We define the arrival rates at time t as potentially dependent on t as well as on a vector z of variables other than (either bid or ask) price p. The arrival rates of sell orders are designated $\lambda_B(p,t,z)$, where B denotes that public sell orders will be transacted at the bid price of the market maker. The corresponding public buy orders are given by $\lambda_A(p,t,z)$,[9] and we assume that $\partial\lambda_A/\partial p < 0$ and $\partial\lambda_B/\partial p > 0$. These are supply and demand functions at each instant t in the trading day $[0,T]$; that is, they represent the expected order flow, as a function of price, expressed as a Poisson instantaneous rate. Corresponding to these general supply and demand functions at each time $t \in [0,T]$ are equilibrium bid and ask prices at time t, which we denote B_t and A_t.

If the arrival rates are stationary over some interval t_i as in Garman's assumption D-1 (p. 263), we define the arrival rates of buy orders as $\lambda_A(p,t,z) = \lambda_{Ai}$ for all $t \in t_i$. Asks and bids for $t \in t_i$ are defined as $A_t = Ai$ and $B_t = Bi$. Thus, two intervals in which the demand rates differ have arrival rates λ_{A1} and λ_{A2} and asks of $A1$ and $A2$. To illustrate the setting, fig. 1 replicates Garman's fig. 2 using our notation, for a trading interval t_1 over

[8]This could be justified by the additional assumption that the market maker has essentially infinite inventories [Garman (1976, p. 265)], or more generally as in Ho and Stoll (1981, p. 52) by assuming that the relevant time horizon is sufficiently short relative to collateral and the characteristics of the price process as to make the explicit avoidance of the possibility of bankruptcy unnecessary.

[9]Garman writes the demand function as λ_B for buy; we write the same demand function as λ_A for ask, as do Ho and Stoll (1981).

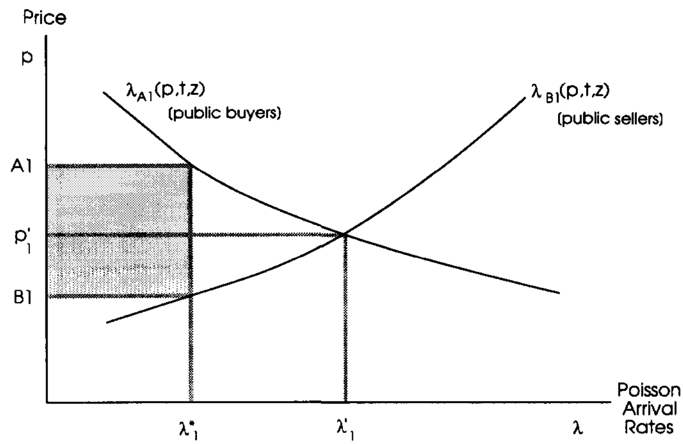

Fig. 1. Market maker's optimal expected bid, ask, and arrival rate, assuming zero inventory drift.

which the supply and demand rates are assumed constant. The market maker seeks to maximize expected profits while keeping the arrival rates of supply and demand equal, i.e., holding expected inventory constant. The optimal arrival rates are given as $\lambda^*_{A1} = \lambda^*_{B1} = \lambda^*_1$. The bid (ask) at all times in the interval t_1 is equal to $B1$ ($A1$).

At this stage, we have imposed no structure on possible changes in the supply and demand functions over subsequent intervals in the day, i.e., $t_2, t_3, \ldots$. However, for any given interval t_i, the optimal Ai, Bi and the resulting λ^*_i are determined in the same fashion as for fig. 1. Clearly over the day there is the possibility of marked changes in quantity λ^*_i and spreads $S_t \equiv A_t - B_t$ (here $Ai - Bi$ over the interval t_i), given a sequence of nonconstant demand and supply functions.[10]

3.1.2. Periodic changes in transactions demand

Appendix 2 gives conditions under which increased transactions demand results in increased volume and wider bid–ask spreads. For convenience we adopt Garman's assumption (D-6) of zero costs of production, so that the market maker equates the marginal cost of purchasing shares (determined from the supply function) with the marginal revenue from reselling them. Fig. 2 shows a fixed supply function $B(\lambda)$[11] with corresponding marginal cost

[10]Ho and Stoll (1981) derive the optimal spread for a risk-averse market maker which contains as the first term the spread from the risk-neutral monopolist, i.e., the spread $A1 - B1$ in fig. 1 (fn. 4, p. 58). Constant risk premia imply the same changes in spreads as for the risk-neutral case. See also Amihud and Mendelson (1980).

[11]For notational convenience, we write $B(\lambda)$ for $B(\lambda, t, z)$, the inverse function of $\lambda_B(p, t, z)$.

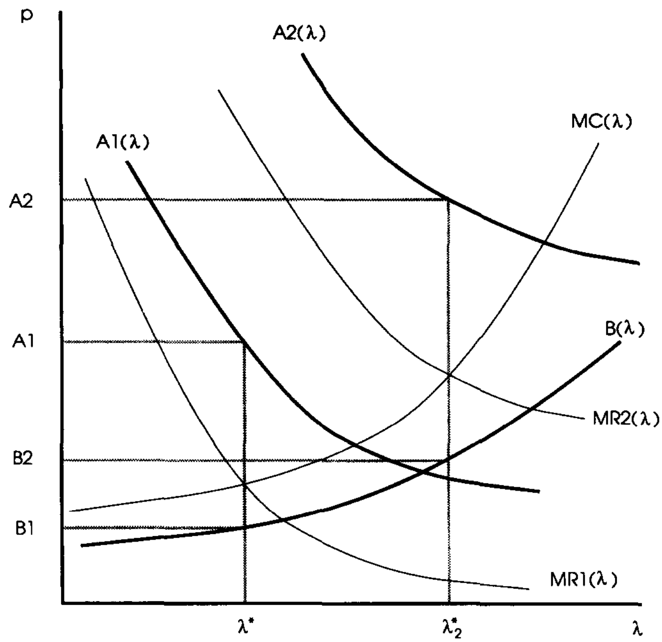

Fig. 2. Increased bid–ask spread resulting from increased demand and nonchanging supply.

$MC(\lambda)$, together with original demand and marginal revenue given by $A1(\lambda)$ and $MR1(\lambda)$ and increased demand and marginal revenue $A2(\lambda)$ and $MR2(\lambda)$.

Under the mild Assumptions A(i) to A(iii),[12] appendix 2 shows that this increased demand with nonchanging supply results in increased quantity, a higher ask, and a wider bid–ask spread. Similar analysis applies for an increase in supply.

Three observations complete our discussion. First, there may be binding capacity constraints in periods of peak demand if fixed costs make it uneconomic to provide the capacity to cope with high arrival rates over short intervals. Indeed, if there is great demand for a particular trade, such as the last trade of the day, it may not be possible to avoid a binding capacity constraint within the continuous trading model of this section.[13] If a capacity

[12]Assumption A(i) implies increased optimal quantity with the increase in demand, A(ii) requires the new demand to be no more elastic than implied by parallel shifts in demand, and A(iii) requires the sum of (absolute) slopes of marginal cost and marginal revenue functions to exceed that for demand and supply functions.

[13]As noted in section 2.2 above, the NYSE has recently changed its trading procedures at close from the previous continuous trading as modelled here to allow for an extended interval after regular hours during which many trades may be made at the 'closing' price.

constraint is violated, then a monopolist market maker will charge market clearing prices given the increased trading demand, which will again imply wider spreads than in periods of lower demand.

Second, if the market maker's response to increased demand results in higher trading costs, some of the increase may spill over to adjacent periods if there is sufficient substitutability for trades across periods. For example, a trader wishing to avoid overnight exposure may close out a position prior to the last trading instant, if this means a transaction at a higher bid or lower ask than during the period of greatest transactions demand.

Third, we emphasize that the argument in this section applies separately to bids and asks, if there is a change in transactions demand on one side of the market. For example, if over some interval the demand to buy stock increased but the supply remained constant, the analysis in appendix 2 predicts a greater increase in the ask than in the bid (and consequently a wider spread). Our current focus on open and close suggests that the increased transactions demand would be for both purchases and sales, but this is not essential to the analysis in this section. Complicated spread changes can in principle emerge over the trading day, as bids and asks respond to asymmetric changes in supply or demand, and spillover from peak periods induces additional changes in spreads.

3.2. Implications for open and close on the NYSE

3.2.1. NYSE opening procedure

The NYSE has a call market to open trade, in which the specialist matches buy and sell orders and makes up any imbalance from his own inventory, in contrast to subsequent continuous double-sided auction trading until (and including) close.[14] While the model of section 3.1 with posted bids and asks is not directly applicable to the NYSE opening, the specialist has extensive discretion in setting the opening price, which will be determined by the order imbalance faced by the specialist. Although there is only one price and no posted bid and ask, the opening price will in effect be either the bid or the ask of the specialist, and the previous analysis applies. Stoll and Whaley (1990) apply techniques such as Roll (1984) to estimate the implied specialist spread in opening prices (see section 4 below).

Our analysis of demand for trading services also applies to bids and asks set after the initial opening price. To link our model to those bids and asks, we need information on demand at the initial trades after opening. First, there may be spillover of trades if the specialist exploits the high demand at

[14]See Schwartz (1988, p. 54). Note also that after trading opens, imbalances of buy and sell orders beyond the normal capacity of the specialist can result in a trading halt in which a call market similar to the open of trade is employed.

open by high (implicit) spreads. Second, if the opening procedure disposes of accumulated overnight demand, it may take some time before portfolio positions established at opening are sufficiently suboptimal to warrant subsequent trade. On the other hand, traders who condition their orders on the opening price may wait until after the opening to place their order, or if they do submit limit orders at the opening, they may be sufficiently far from the opening price as to be unfilled and hence cancelled. It is possible that such demand may result in above average demand for early trades.[15]

3.2.2. *NYSE closing procedure*

We assume that the NYSE closing market corresponds to the model in section 2, and that an increase in demand to trade at the last instant of trading leads to an increase in the ask and a decrease in the bid. Again, there may be spillover effects where high volume at the close is spread over earlier trades, as would be an increase in the spread.

In fact, the situation on the NYSE at close is more complicated than our model.[16] The NYSE Constitution and Rules (1989, Rule 13, ¶2013) defines an 'At the Close Order' as a 'market order which is to be executed at or as near to the close as practicable'. The usual procedure on high volume days is that at 3:30 p.m., i.e., one half hour prior to close, the specialist matches market-on-close orders that have been received at that time, either by hand or using the automatic trading system (Opening Automated Report System, or OARS) if volume is high. If there is an imbalance of orders, no further market-on-close orders may be submitted on the excess side, although orders on the other side may be submitted up to close. For large volume stocks, more than one round of matching may occur, possibly at different prices, although all trades executed at one time execute at the same price. For example, at the first round, all orders which are matched will cross at the same price; but at the final close, the remaining orders and any new orders

[15]Informed traders have an incentive to condition on the opening price and trade in the first couple of trades after opening. They are given an option by the market mechanism once the opening price is set. If the opening price is too low, they can buy with the knowledge that the price will not immediately jump beyond one eighth because the specialist is judged on price continuity (a fair and orderly market) and seeks to defend the initial opening price. Consequently, although the specialist may not set a biased opening price given information generally available at the open, it is not surprising if *ex post* there are runs in either direction at the start of trade as the knowledge of informed traders is incorporated in prices. Contrast Miller (1989), who argues that the requirement of price continuity means that large overnight price changes (either good or bad news) are not incorporated into the opening price, but extend over several trades at the start of the day. However, the opening mechanism specifically allows for different procedures if there would be large order imbalances at opening prices within one eighth or one quarter (depending on the price) of the previous close, and the specialist loses if order imbalances are knowingly filled at incorrect prices.

[16]The following description is based on conversations with James Shapiro, Director, Economic Research, NYSE, and applies prior to August 1990; our data in section 4 are for 1987.

that are permitted may result in a closing price that differs from that in the first round. There is no guarantee of the price at which market-on-close orders will execute; depending on the time of execution and the state of the book at that time, it could be at a bid, ask, or within the spread. Regular trading in market and limit orders occurs until close; for example, traders wishing to submit market orders after 3:30 p.m. will not be able to submit market-on-close orders if they are on the wrong side of an order imbalance, and must instead submit regular market orders. For other than high volume expiration days (i.e., the twelve days on which multiple index contracts expire), market-on-close buy and sell orders can trade at different prices (e.g., the ask and bid, respectively), but not more than one eighth point apart.

The closing market thus shares some characteristics of both a call and a continuous auction market. To the extent that traders know in advance which side of the market they will be on, they face an alternative to submitting regular market orders at close, i.e., they can submit market-on-close orders; however, there is no guarantee of better execution by doing so. Traders submitting regular market orders at the close of trade, either by necessity or choice, face the market we model here. Although we assume that our model is an adequate description of the NYSE close, further study may be warranted concerning the effects of the actual procedure.

3.2.3. *Implications for NYSE asks and bids*

Assuming increased periodic demand for transactions services at the open and close of trade on both the buy and sell side, the model in section 3.1 implies increases in the ask and decreases in the bid. We also expect to see spillover effects, with optimal increases in both transactions demand and

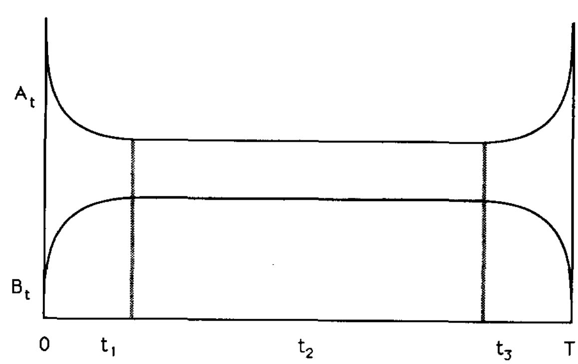

Fig. 3a. Predicted asks and bids, with increased transactions demand at open and close.

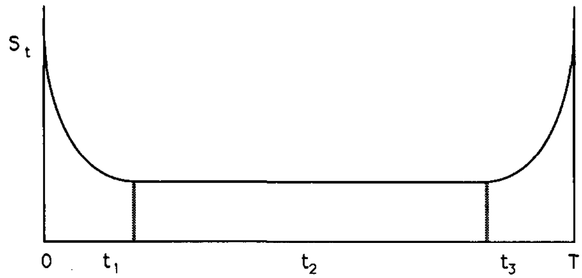

Fig. 3b. Predicted bid–ask spread, with increased transactions demand at open and close.

spread in intervals immediately after open and prior to close.[17] Asymmetry in demand changes may result in asymmetric changes in asks and bids.

These predictions about asks and bids are shown in fig. 3, which shows the predicted sequence of asks and bids over the trading day $[0, T]$ (fig. 3a) and the predicted sequence of the spread (fig. 3b). We assume three periods of interest: t_1, comprising the open of trade (and adjacent spillover); t_3, the close; and t_2, the complement, which in this case extends for most of the trading day $[0, T]$. Fig. 3 assumes symmetric changes on both bid and ask sides and equal changes at open and close. As discussed above, such symmetry need not occur, and open and close changes need not be equal. Fig. 3 also assumes that there is no average drift in asks or bids over the day. We provide evidence confirming our predictions in section 4 below.

3.3. Competition

Our analysis assumes a monopolist market maker, which is more reasonable for a specialist on the NYSE than for, say, NASDAQ market makers. NYSE specialists also face some degree of competition from limit orders and floor traders. We now argue that our basic results, in which periodic transactions demand affects bids and asks, are robust to changes in the assumption of a monopolist market maker.

First, NYSE specialists have different amounts of potential market power at different times of the day. The NYSE opening mechanism described above shows that specialists have a considerable amount of discretion in the setting of the opening price at which all opening market and limit orders are executed. Since there is no institutional ability to set opening limit orders conditional on the opening price, competition from limit orders and floor traders is necessarily temporally limited since the specialist's opening price is

[17]Discreteness of prices will tend to mask such spillover.

not known until the actual open. Even assuming that public traders or floor traders are prepared to submit limit orders inside the specialist's opening quotes, there will be some delay between the open of trade and when these limit orders are submitted.[18]

Similarly, in general the specialist has access to more information concerning market-on-close orders than do public traders. Moreover, since the specialist has discretion over the extent to which his own trading will give a closing trade on the buy or sell side, public limit order placement becomes increasingly risky since a position taken at close cannot be reversed overnight. This gives the market maker relatively less competition at close of trade.

However, the result given here of increased spreads in the face of increased transactions demand can obtain even if there were full competition for market makers (although nonzero prices for NYSE specialists' seats suggest that not all trading profits are competed away). If it were costless to provide infinite market making services, free competition and the resulting zero profit condition would drive bids and asks to the single price p'_1 in fig. 1. The question of changes in the spread would not arise, since the spread would not exist.[19] If there were periodic increases in trading demand, extra resources would costlessly flow to that time of day. However, since it is not costless to provide market making services, a discussion of competitive pricing must allow market makers to cover their costs. Fixed costs, including the opportunity cost of providing a presence in the market, may comprise the major production costs. If so, the usual problems in the definition of the marginal cost of one element of a jointly produced product arise here. It may be impossible, given the market technology, to perform one trade without simultaneously producing the ability to perform another. Consequently the procedure of equating 'marginal cost' with price is not helpful in determining equilibrium price or output.

Although price competition may not provide an equilibrium under these conditions, an alternative [discussed in Kyle (1984)] is Cournot competition with freedom of entry, which leads to pricing that covers fixed costs (and may lead to profits for existing market makers but which are insufficient to induce new entry). Price discrimination, with higher spreads in periods of increased, inelastic demand for transactions services, may be required to cover the costs of the market maker.[20]

In short, our analysis of periodic transactions demand over the trading day and consequent increased volume and spreads at open and close is not restricted to a monopolistic market maker.

[18]Personal discussion with NYSE specialists verifies that the reason given for a narrowing of spreads over time after the open is the arrival of public orders between the opening quotes.

[19]Cohen et al. (1981) discuss equilibrium spreads in the presence of transactions costs.

[20]While traders who need not maintain an active presence at the market at all times may submit public limit orders, they may bear substantial costs, e.g., of becoming informed, relative to changes in spreads over very short intervals such as the last trade.

4. Empirical evidence on volume and bids and asks

Volume on the NYSE is roughly U-shaped over the trading day. Jain and Joh (1988) report the most detailed published work of which we are aware. They examine intraday volume on an hourly basis, finding for 1979–1983 that average trading volume 'is highest during the first hour of the day for each day of the week...[and] declines monotonically until the fourth hour but increases again in the fifth and sixth hours' (p. 271). Similar results are found by Foster and Viswanathan (1989). McInish and Wood (1987, pp. 16–17) cite Wood, McInish, and Ord (1985) as showing that 'frequency of trading is high at the beginning of trading, declines to a low point at midday and then rises through the close of trading'.

Increased volume at open and close is consistent with our model of periodic transactions demand at open and close. We now examine whether these increased volumes are accompanied by wider bid–ask spreads. Empirical evidence on spreads is given in section 4.1, and as predicted by our analysis in section 3, spreads are wider at open and close.[21] Section 4.2 presents related evidence, including Foster and Viswanathan (1989), Stoll and Whaley (1990), and direct tests of our model by Gerety and Mulherin (1991), Harris, McInish, and Chakravarty (1991), and McInish (1989). Section 4.3 contrasts the correspondence between our predictions and the evidence on volume and spreads with the apparent inability of current information based models to account simultaneously for volume and spreads at open and close without resort to the type of periodic behavior that is the basis of our model.

4.1. NYSE bid–ask spreads: Empirical results

McInish and Wood (1988) find that, for all NYSE stocks for five months of 1987, the bid–ask spread (as a percentage of price) has a U-shape with particularly high spreads in the first minutes of trading, declining over about 15 minutes to a level which lasts until the last few minutes of the day. Porter (1988, p. 68) finds that proportional spreads are 'widest in the morning, narrow to midday, and widen again to the close of trading' (from which he concludes that 'variances calculated with transactions prices will vary systematically throughout the day'). Jaffe and Patel (undated) find a U-shaped intraday bid–ask spread, 'falling sharply in the first few minutes of the day and rising sharply in the last few minutes' up to close (p. 10).[22]

[21]When we first developed these predictions in 1987, there was, to our knowledge, no evidence available on bid–ask spreads at open and close.

[22]See also Sirri (1989). Marsh and Rock (1986) also examine bid–ask data, and conclude that 'movements over time in the spread seem to be little more than a by-product of the market maker's strategy for adjusting prices in response to order imbalances' (pp. 3–4). Their results provide valuable evidence concerning alternate models of how market makers react to temporary inventory changes, but they do not address the systematic spread patterns of interest here.

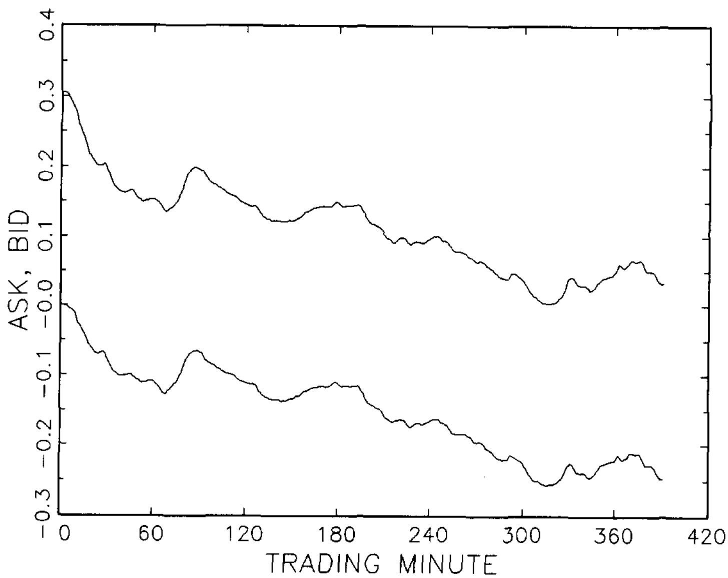

Fig. 4a. Average ask and bid (opening bid standardized to zero) of Standard and Poor's 500 stocks trading on the NYSE, October 1–15, 1987, from 9:30 a.m. to 4:00 p.m.

We provide additional evidence on NYSE spreads. The data comprise bids and asks for the 462 stocks in Standard and Poor's 500 that are traded on the NYSE for the period October 1 to October 15, 1987.[23] A record of minute-by-minute bids and asks is created. The first quote is recorded in the minute in which it occurs; this and subsequent quote changes unambiguously define the quote at the beginning of subsequent minutes. Fig. 4 presents the average across all companies and days of bids, asks, and spread by minute, from 9:30 a.m. to 4:00 p.m. (the opening bid is standardized to zero). Prior to all stocks opening for trade, the average for each minute is computed across only those stocks which had started trading; thereafter all had outstanding quotes until close.

The average decline of bid and asks shown in fig. 4a reflects the average decline in the market over this period. Of greater interest is the U-shaped pattern in the spread in fig. 4b.[24] Fig. 5 shows that the U-shaped pattern of

[23]The individual security quote data were kindly provided by the Securities and Exchange Commission and were carefully checked for errors. All corrections indicated by the transactions codes were checked and carried out. Further, after flagged corrections were made, all consecutive quote changes of at least five percent (absolute) were hand checked. In some cases, they were correct quote changes for low priced stocks. Obvious errors were corrected; all other records were deleted.

[24]To see if the decrease at open were affected by nonsynchronous openings, the subset of stocks that opened at 9:30 a.m. was examined, with similar results.

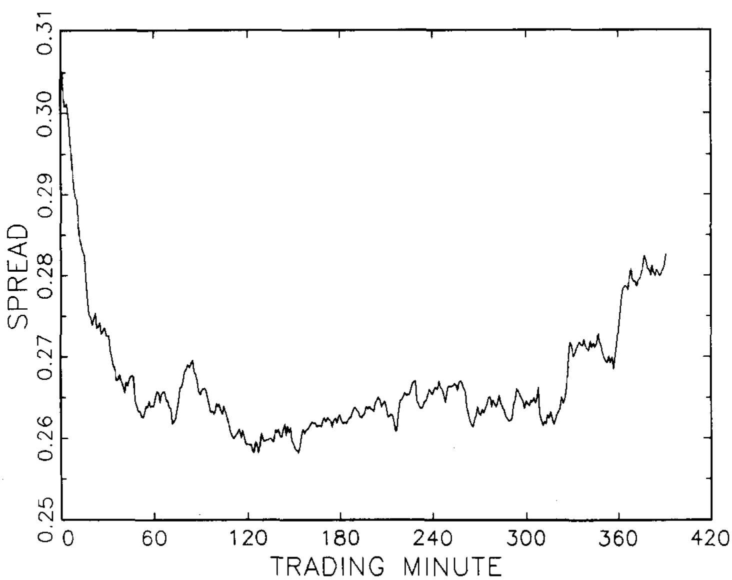

Fig. 4b. Average spread, 9:30 a.m. to 4:00 p.m., for stocks in fig. 4a.

spreads over the trading day is not restricted to one subset of stocks. Fig. 5a gives results for quintiles of stocks based on bid–ask average at the start of the period. For example, for the lowest priced stocks (quintile 1), the average spread is approximately 0.24 at open, declines to about 0.20 during the day, then rises to about 0.22 at close. Although the spread varies across different priced stocks, with the higher priced stocks having higher (absolute) spreads, the basic U-shape over the trading day is seen in all deciles. Figs. 5b and 5c present similar evidence for quintiles ranked (in ascending order) on opening market value of equity and beta, respectively. The basic U-shape is constant across all groups.

4.2. Related evidence on the periodic demand model

Section 4.1 shows that the primary direct implications of the periodic demand model, namely that increased volume and wider bid–ask spreads will accompany the open and close of trading, are supported empirically. Other evidence is also relevant.

Stoll and Whaley (1990) estimate the spread implicit in opening prices, and conclude that the implied cost of immediacy is significantly higher at open than at close, which they attribute in part to temporary price deviations at open induced by the exploitation of market power by the specialist. This

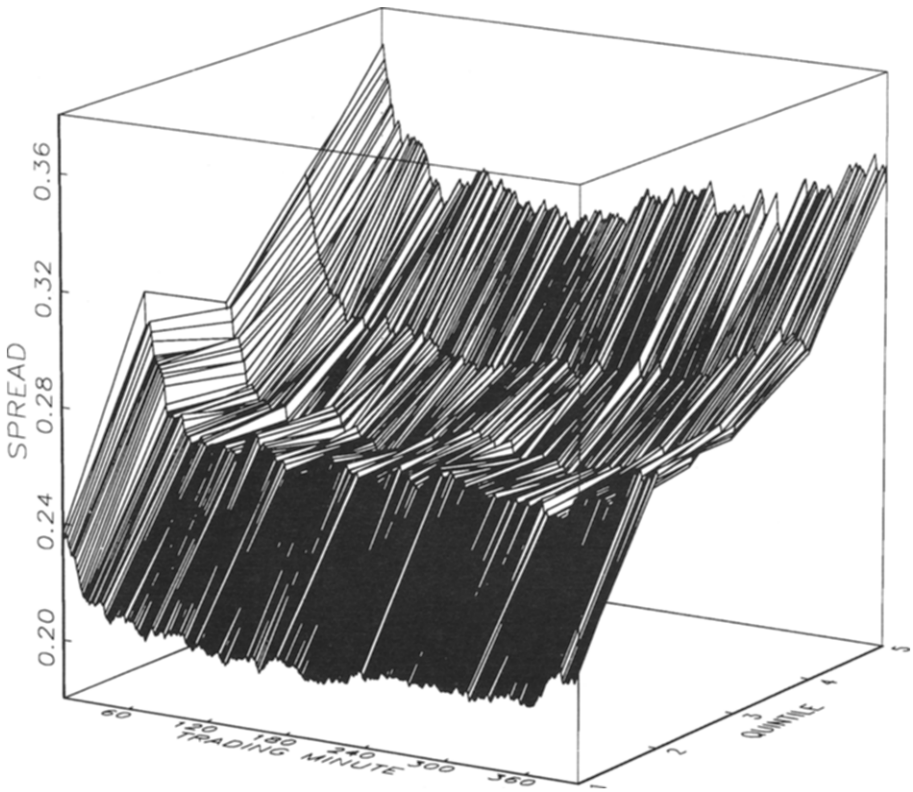

Fig. 5a. Average spreads, 9:30 a.m. to 4:00 p.m., for quintiles of stocks in fig. 4a, ranked (in ascending order) on bid–ask average.

argument is similar to ours in section 3, although they do not motivate the periodic volume as we do in section 2. However, since they do not examine intraday data, they cannot compare spreads throughout the day. Foster and Viswanathan (1989), following Glosten and Harris (1988), model the spread as comprising fixed cost, adverse selection cost, and inventory cost components, and find that the fixed cost component (which includes the transactions costs we model) is highest at open and close. They regard the increase in volume and trading costs at the close as 'puzzling'.

Harris, McInish, and Chakravarty (1991) and McInish (1989) explicitly test predictions based on the periodic market closure model of this paper. McInish finds support in open and close increases in spreads, and the behavior of bids and asks separately. Harris, McInish, and Chakravarty test our prediction that increased transactions demand will raise the ask and

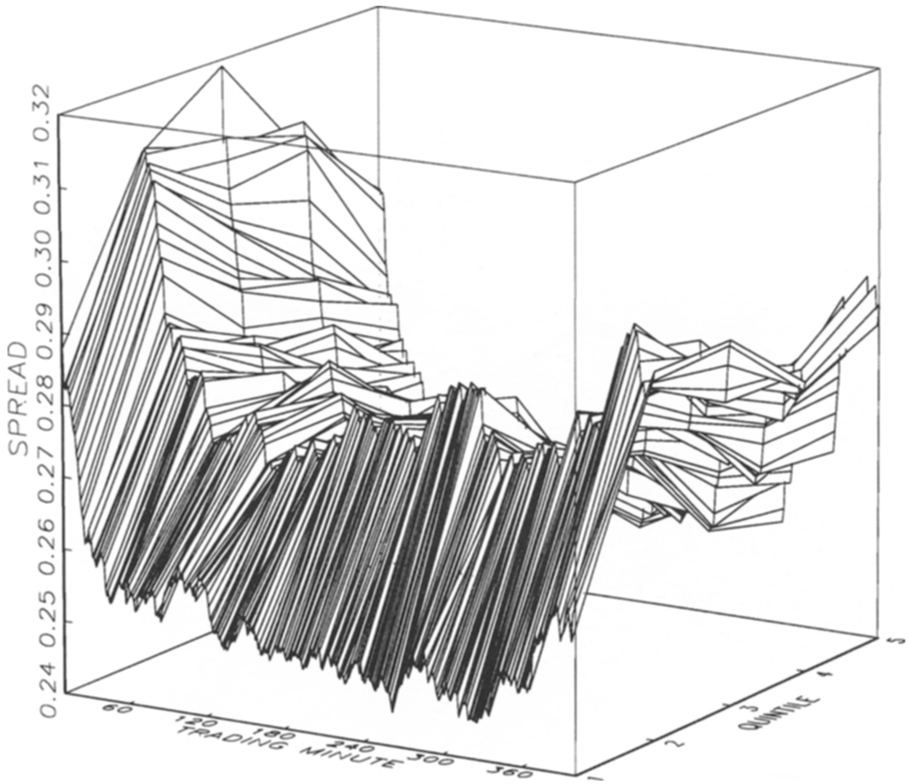

Fig. 5b. Average spreads, 9:30 a.m. to 4:00 p.m., for quantiles of stocks in fig. 4a, ranked (in ascending order) on market value of equity.

lower the bid, using trade and quote data for IBM in 1988, and conclude (p. 24) that the data 'strongly support the Brock and Kleidon hypothesis'.

Gerety and Mulherin (1991) also explicitly test the model of periodic market closure we develop in this paper. Rather than examining the results of section 3 on spreads, they test the fundamental results in section 2 that relate periodic trading at open to larger portfolio rebalancing due to an overnight period in which trading bands may be greatly violated, and periodic trading at open and close to systematic reallocation of portfolios across investors because of the inability to trade overnight. They find that opening volume is positively correlated with overnight unexpected volatility, 'confirming the intuition that the overnight accrual of information leads to trading activity at the open' (p. 15). They also find that trading at both one day's close and the following day's open are positively correlated with expected overnight volatility, 'consistent with the argument that much of the clustering

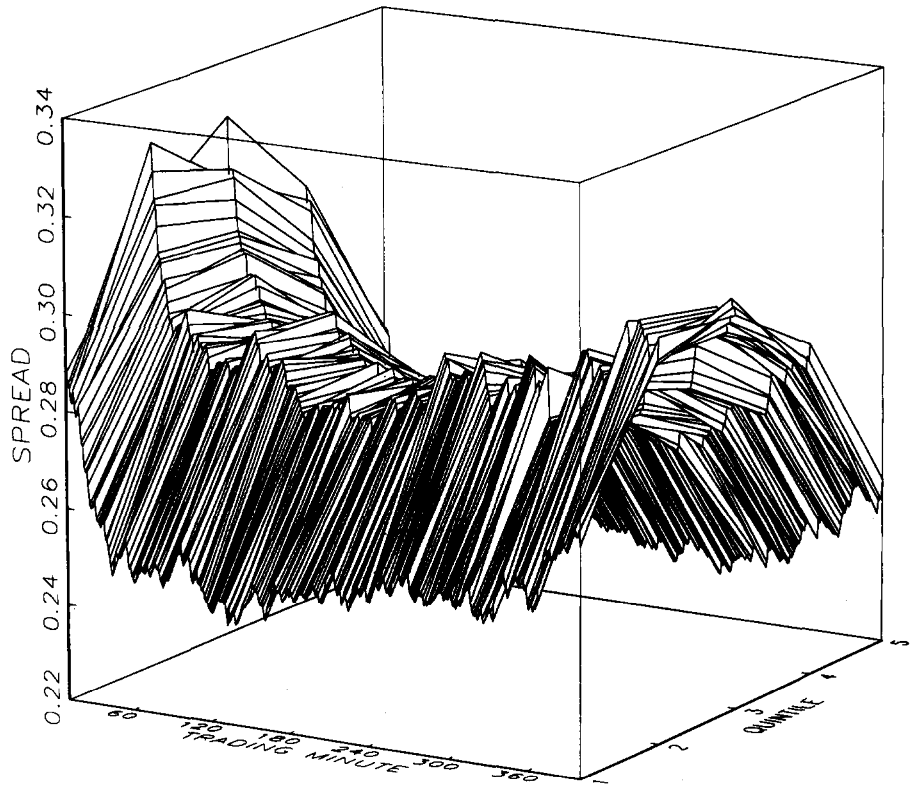

Fig. 5c. Average spreads, 9:30 a.m. to 4:00 p.m., for quintiles of stocks in fig. 4a, ranked (in ascending order) on beta.

of trading around the open and the close is due to the desire of investors to exchange the exposure to price changes when the market is closed'.

4.3. Information based models

The previous section provides a substantial body of evidence in support of the periodic transactions demand model of this paper, which predicts high volume and simultaneous wide spreads at open and close. An alternative information economics model currently used to analyse market microstructure also attempts to explain the high volume at open and close of trade. However, the main paper on periodic volume, Admati and Pfleiderer (1988), derives high volume because of *low* transactions costs at open and close that attract traders who have some discretion over when to trade. Subrahmanyam

(1989) modifies the assumptions of Admati and Pfleiderer to produce high trading costs at open and close, but in so doing loses the essential feature of Admati and Pfleiderer's model that produces high volume. The apparent solution to the dilemma relies on periodic demand by nondiscretionary liquidity traders which is not motivated within the information models.

We now discuss these alternative theories in more detail.

Bagehot (1971) argues that the market maker loses in trades with better informed traders, so that trades with uninformed liquidity traders must make sufficient profit to cover those losses plus costs. This notion is formalized in subsequent work, including Copeland and Galai (1983), Kyle (1984, 1985), Glosten and Milgrom (1985), Foster and Viswanathan (1989, 1990), Admati and Pfleiderer (1988, 1989), and Subrahmanyam (1989).

A model of endogenous trading volume is provided by Admati and Pfleiderer (1988), who extend Kyle (1984). They assume three types of agents: informed traders, who will trade only on terms advantageous to them, given their superior information; discretionary liquidity traders, who must trade over a given day, but who choose when to trade during the day on the basis of trading costs, i.e., they trade in those periods of lowest cost; and nondiscretionary liquidity traders who must trade at a given time during the day regardless of cost. Trading costs, in this model, arise solely because of the activity of the informed, whose profits are paid by the uninformed liquidity traders.

Given their assumptions, Admati and Pfleiderer show that it is possible to obtain concentrations of volume at arbitrary trading times because in equilibrium these high volume periods attract both informed traders and discretionary liquidity traders. The informed are attracted because there will be more uninformed liquidity traders behind whom they can camouflage their trades. The discretionary liquidity traders are attracted because, in this model, the increased activity of informed traders implies sufficiently increased competition among them that the cost of trading to the uninformed is *lowered* relative to other periods. Admati and Pfleiderer relate their results to observed empirical behavior, especially to volume and variance at the open and close of a day's trading on the NYSE, and 'show that the patterns that have been observed empirically can be explained in terms of the optimizing decisions of these traders' (p. 4).

The key result, namely that increased activity by informed traders lowers the costs to the uninformed who must pay the price of the presence of the informed, is not intuitively obvious. Subrahmanyam (1989) builds on the model of Admati and Pfleiderer, and shows that their result depends on the assumption of risk-neutral informed traders. If the informed traders are risk-averse, then increased activity on their part can increase the trading costs of liquidity traders, which leads discretionary liquidity traders to avoid periods of high informed participation, and which in turn breaks the concentration relied on in Admati and Pfleiderer's equilibrium.[25] Subrahmanyam (1989, p. 18) cites Foster and Viswanathan (1989) as showing that the adverse selection component of bid–ask spreads is highest at the beginning of the day, which 'contrasts with the model of Admati and Pfleiderer [1988], which predicts that spreads should be *lowest* at the beginning of the day' (emphasis in original). The evidence in section 4.1 above is similarly inconsistent with the key result in their model, since spreads follow the same U-shape as volume; highest volume is associated with highest, not lowest, costs.[26]

Subramanyam interprets this result as consistent with his extension of Admati and Pfleiderer to the case of risk-averse informed traders, since then more trading by informed traders results in lower market liquidity and higher costs. To do this, he requires (p. 17) the additional assumption that 'more individuals are informed at the beginning of the day than at other times during the day'. For this model to be a full explanation of the relation between spreads and volume, he presumably requires that the informed also trade more heavily at close.

More important, his model implies that liquidity traders who can time their trades will avoid high cost, high volume periods (p. 17). This makes it difficult to explain the observed high volume in terms of discretionary liquidity traders and informed traders, since the former will avoid the high-cost open and close, and the number of informed traders must be 'sufficiently small' (p. 18) for the result to go through. Further, if the increased volume were due to a very large increase in the number of informed (sufficient to both offset the departure of discretionary liquidity traders and account for the total increase in volume), one wonders who takes the other side of their trades, especially since they receive correlated signals.

Presumably the burden falls, once again, on the luckless nondiscretionary liquidity traders who in these models have zero elasticity of demand and must trade at these times regardless of price or cost. That is, an implication of the empirical results on spreads and volume is that within extant information

[25]This result would conform with McInish and Wood's (1988, p. 12) claim that information models would predict an increase in the spread as informed traders are attracted to periods of high volume; however, they attribute this prediction to Admati and Pfleiderer, who in fact reach the opposite conclusion.

[26]The model of Admati and Pfleiderer (1988) and Subrahmanyam (1989) assumes sequential batch auctions rather than the continuous auctions associated with bids and asks on the NYSE. However, Admati and Pfleiderer regard their results as applying to the volume behavior on the NYSE, and Subrahmanyam explicitly equates the costs in Admati and Pfleiderer (which are the same as in his model) with bid–ask spreads. We follow this approach. Further, as Grossman and Miller (1988, p. 628) point out, transactions costs should be measured by the difference between the price paid now versus the price expected to be paid by waiting; but if the average spread falls after opening and rises at close, one would expect *a priori* that a given liquidity trader would expect higher transactions costs in such periods.

models, there must be an increase in nondiscretionary liquidity trading at open and close which is sufficient to offset the departure of discretionary traders in the face of higher transactions costs. Admati and Pfleiderer (1988, p. 34) conjecture that the orders of nondiscretionary traders may cluster around open and close because of market closure; however, this is not part of their formal model, and they rely on such periodic demand as simply a timing catalyst for their endogenous clustering that requires low trading costs at open and close to attract discretionary traders. Although we regard the assumption of zero elasticity of demand as too extreme, we do believe that the source of high volume at open and close resides outside the equilibrium behavior of agents as derived in these information models. That is precisely our model of periodic market closure in section 2 above, which together with the market maker response of section 3 reconciles high volume and wide spreads at open and close of trade.

5. Extensions

Aside from additional empirical testing and the potential model extensions outlined in section 2.1 above, there are two main areas in which this paper can be extended. First, we have concentrated on the market mechanism of the NYSE. It will be useful to compare alternate market mechanisms, and obtain predictions about the behavior of bids and asks in different settings. These include overseas markets and other U.S. markets.

Second, although we restrict primary attention in this paper to intraday behavior on the NYSE, the model applies to other periodic changes in trading demand across time. These include holidays, weekends, different times of the month, and different times of the year (particularly around year end), each of which has been examined in empirical papers detailing apparent excess stock returns (see Ariel (1987, 1988) [holidays, times of month], French (1980) and Gibbons and Hess (1981) [weekends], Keim (1983) and Roll (1983) [year end]). These anomalies may all be effected by the phenomena we describe, combined with different proportions of trades at the bid or ask corresponding to differential demands for purchase or sale of securities. For example, Keim (1989) documents that there is a concentration of closing trades at the bid and ask respectively at the end of one year and the beginning of the next, which accounts for about half of the turn of the year effect. If market makers are able to exploit the well-documented increase in tax-related sales around year end, resulting in a lowering of the bid and an increase in the ask before and after year end respectively as in our model [see Clark, McConnell, and Singh (1990) for consistent evidence], then more of the apparent anomaly is accounted for. In general, any argument that implies periodic shifts in transactions demand has implications for bids and asks via the model in section 3.

6. Conclusions

The primary contributions of this paper are twofold. First, we examine whether the open and close of trading are special because they are the points of discontinuity between a continuous trading regime and one in which trading is not possible. Following models of trading volume in the presence of transactions costs, we extend Merton (1971) to our case of periodic market closure, and find that in general there will be strong and inelastic demand for trade at the open and close of the market, as traders seek to achieve optimal portfolio proportions or transfer overnight risk.

Second, we extend Garman (1976) to model how bids and asks respond to these periodic changes in demand for transactions services. Under mild conditions, increased transactions demand will result in increased volume and wider bid–ask spreads. The empirical evidence is consistent with our predictions of high volume and concurrent wide spreads, and direct tests of our arguments support both the periodic demand model and the implications for bids and asks.

This empirical support for our approach is important, because the main alternative model, based on information economics, is currently unable to derive concurrent high volume and wide spreads in terms of the optimal equilibrium behavior of the model's traders. Alternate versions either imply high volume and low spreads at open and close, or else can explain high spreads but cannot simultaneously account for high volume without recourse to the type of periodic transactions demand that we develop in this paper. However, the information based models treat such periodic behavior as coming from nondiscretionary liquidity traders whose motives lie outside the model.

The underlying ideas in the information literature are crucial to understanding some trading decisions and empirical phenomena. However, there are good reasons to believe that not all decisions are based on factors that are considered in such models. For example, it is well documented that tax considerations affect the desire to purchase or sell securities around the turn of the year, and concentrations of buying or selling occur at that time [see, e.g., Roll (1983) and Constantinides (1984)].

Similarly, our model of periodic trading demand at open and close implies that there are sound reasons that are outside the currently popular information models for trading at these points of market discontinuity. Future work will doubtless shed more light on the types of problems for which these alternative approaches are most fruitful.

Appendix 1

This appendix extends Merton (1971) to periodic market closures.

Let $J_1(W)$ be the value of wealth W starting at open and $J_2(W)$ the value of wealth W starting at close; that is,

$$J_1(W_0) = \max\{\mathrm{E}g_1(W_0, W_T) + \mathrm{e}^{-\rho T}\mathrm{E}J_2(W_T)\}, \tag{A.1}$$

$$J_2(W_T) = \max\{\mathrm{E}g_2(W_T, W_{T+N}) + \mathrm{e}^{-\rho N}\mathrm{E}J_1(W_{T+N})\}, \tag{A.2}$$

where

g_1 = value of going from open with wealth W_0 to close with wealth W_T,
g_2 = value of going from close with wealth W_T to open with wealth W_{T+N},
T = length of open (or 'first') period,
N = length of closed (or 'second') period,
ρ = the discount factor,
and E denotes the expectation operator.

Rewrite (A.1), (A.2) as follows:

$$J_1(W_0) = \max\left\{\mathrm{E}\int_0^T \mathrm{e}^{-\rho t}(c_1^a/a)\,\mathrm{d}t + \mathrm{e}^{-\rho T}\mathrm{E}J_2(W_t)\right\}, \tag{A.3}$$

subject to

$$\mathrm{d}W = W\left(\sum w_{1j}(\alpha_j\,\mathrm{d}t + \beta_j\,\mathrm{d}Z_j)\right) - c_1\,\mathrm{d}t,$$

$$W(0) = W_0, \qquad \sum w_{1j} = 1,$$

$$\mathrm{d}P_j/P_j = \alpha_j\,\mathrm{d}t + \beta_j\,\mathrm{d}Z_j, \qquad \mathrm{d}Z_i\,\mathrm{d}Z_j = \rho_{ij}\,\mathrm{d}t, \tag{A.4}$$

where $c_1(t)$ and $w_{1j}(t)$ are consumption and portfolio weights over the open (first) period, α_j and β_j are time-stationary over the open period, and using the notation '$\mathrm{d}Z_i\,\mathrm{d}Z_j$' as short for the correct '$\mathrm{E}_t\,\mathrm{d}Z_i\,\mathrm{d}Z_j$'; and

$$J_2(W_T) = \max\Bigg\{\theta(N)c_2^a/a + \mathrm{e}^{-\rho N}\mathrm{E}J_1\left(W_T\sum w_{2j}\frac{P_j(T+N)}{P_j(T)} - c_2N\right)\Bigg\}, \tag{A.5}$$

where

$$\theta(N) = \int_0^N \mathrm{e}^{-\rho t}\,\mathrm{d}t,$$

$$W(T+N) = \sum N_j(T) P_j(T+N) - c_2 N$$

$$= W_T \sum w_{2j} \frac{P_j(T+N)}{P_j(T)} - c_2 N,$$

$$w_{2j} \equiv \frac{N_j(T) P_j(T)}{\sum N_i(T) P_i(T)} \equiv \frac{N_j(T) P_j(T)}{W_T}. \tag{A.6}$$

Remark 1: We assume the individual consumes at a constant rate c_2 during the 'night'. This assumption can be relaxed. Compounding terms in (A.6) are neglected for simplicity, which does not change the conclusion we are after, viz., volume changes abruptly at the close and at the open.

Remark 2: The univariate process for stock j is given by

$$\frac{P_j(T+N)}{P_j(T)} = \mathrm{e}^{(\alpha_j - \beta_j^2/2)N + \beta_j \tilde{Z}_j(N)}, \qquad \tilde{Z}_j(N) \overset{\mathrm{d}}{\approx} \mathrm{N}(0, N),$$

where α_j and β_j over the closed period are not necessarily equal to those over the open. More generally $\underset{\sim}{P}$ is vector log normally distributed [Merton (1971)].

To solve the open period problem (A.3) for closed form put

$$M(W_t, t; T) \equiv \max \mathrm{E}_t \left\{ \int_t^T \mathrm{e}^{-\rho s} \frac{c_1^a(s)}{a}\,\mathrm{d}s + \mathrm{e}^{-\rho(T-t)} J_2(W(T)) \right\},$$

and conjecture

$$M(W_t, t; T) = b_1(t;T) W_t^a\,\mathrm{e}^{-\rho(T-t)} \equiv b_1 W_t^a\,\mathrm{e}^{-\rho(T-t)}. \tag{A.7}$$

Then

$$M(W_0, 0; T) = J_1(W_0), \qquad M(W(T), T; T) = J_2(W(T)).$$

Remark 3: J_1, J_2 depend on (N, T). We suppress this in the notation.

Using the price process (A.4) implies that M satisfies:

$$-M_t = \max \left(\mathrm{e}^{-\rho t} u(c_1) + M_W \frac{\overline{\mathrm{d}W}}{\mathrm{d}t} + \tfrac{1}{2} M_{WW} \frac{\overline{\mathrm{d}W}^2}{\mathrm{d}t} \right),$$

$$\frac{\overline{\mathrm{d}W}}{\mathrm{d}t} = W\left(\sum w_{1j}\alpha_j\right) - c_1, \qquad \frac{\overline{\mathrm{d}W}^2}{\mathrm{d}t} = W^2 \sum \sum w_{1i} w_{1j} \beta_i \beta_j \rho_{ij}.$$

For our case of $u(c) = c^a/a$,

$$-M_t = M_W^{(a/a-1)} \frac{1-a}{a} \mathrm{e}^{(\rho t/a-1)} + M_W W A + M_{WW} W^2 B, \tag{A.8}$$

where

$$A = \sum \overline{w}_{1j}\alpha_j, \qquad B = \tfrac{1}{2}\sum\sum \overline{w}_{1i}\overline{w}_{1j}\beta_i\beta_j\rho_{ij},$$

and where $(\overline{w}_{1i})$ solves Merton's equation

$$\begin{aligned}&\frac{\partial}{\partial w_{1i}}\Big[M_W W\Big(\sum w_{1j}\alpha_j\Big) + \tfrac{1}{2}M_{WW}W^2\Big(\sum\sum w_{1i}w_{1j}\beta_i\beta_j\rho_{ij}\Big)\\ &+\lambda\Big(1-\sum w_{1j}\Big)\Big] = 0.\end{aligned} \tag{A.9}$$

Remark 4: For $\mathrm{d}Z_i\,\mathrm{d}Z_j = 0\cdot \mathrm{d}t$ for $i \neq j$ and $1\cdot \mathrm{d}t$ for $i = j$, we get

$$\overline{w}_{1j} = \frac{\alpha_j - \alpha_I}{\beta_j^2}\tau, \qquad \tau \equiv -\frac{M_W}{M_{WW}W} = \frac{1}{1-a},$$

with I risk-free. More generally, see Ingersoll (1987, p. 274, (12)).

To solve explicitly for the state valuation function, substitute (A.7) into (A.8), conjecture $J_2(W(T)) = b_2(T)W(T)^a$ to get (by cancelling)

$$\begin{aligned}&\rho b_1 - \dot{b}_1 = (ab_1)^{(a/a-1)}\frac{(1-a)}{a} + ab_1(A + (a-1)B),\\ &b_1(T) = b_2(T),\end{aligned} \tag{A.10}$$

where $\dot{b}_1 \equiv \mathrm{d}b_1/\mathrm{d}t$. This is periodic (by construction) over $[0,T]$, $[T+N, 2T+N]$, $[2T+2N, 3T+2N], \ldots$.

Remark 5: (A.10) is an ODE for $b_1(t)$ of Bernoulli form which can be solved by transform methods [Malliaris and Brock (1982, p. 177)].

Now let us search for closed form solutions of the closed period problem (A.5) [recall that $b_1(T+N) = b_1(0)$] of the form:

$$\begin{aligned}J_2(W_T) &= b_2(T)W_T^a\\ &\equiv \max_{c_2, w_2}\Bigg\{\theta(N)\frac{c_2^a}{a} + b_1(T+N)\mathrm{e}^{-\rho N}\\ &\quad\times \mathrm{E}\left[W_T\sum w_{2j}\frac{P_j(T+N)}{P_j(T)} - c_2 N\right]^a\Bigg\}.\end{aligned} \tag{A.11}$$

Setting up a Lagrangian by adding the term $\lambda(1-\sum w_{2j})$, one can show $\exists!$ solution of the form $\lambda(W)=\lambda_2^* W^a$, $\{w_{2j}\}$ independent of W, $c_2(W)=c_2^* W$.[27] So $J_2(W_T)=b_2(T)W_T^a$ 'works'. Note, using $c_2=c_2^* W$, that $b_2(T)$ relates to $b_1(T+N)=b_1(0)$ through

$$b_2(T)W_T^a=\theta(N)\frac{(c_2^*)^a}{a}W_T^a+b_1(0)\mathrm{e}^{-\rho N}$$
$$\times \mathrm{E}\left[\sum \frac{w_{2j}P_j(T+N)}{P_j(T)}-c_2^* N\right]^a W_T^a. \tag{A.12}$$

Differentiating (A.12),

$$\frac{\partial}{\partial c_2^*}=0:\quad \theta(N)(c_2^*)^{a-1}W_T^a+b_1(0)\mathrm{e}^{-\rho N}aW_T^a$$
$$\times \mathrm{E}\left[\left(\sum \frac{w_{2j}P_j(T+N)}{P_j(T)}-c_2^* N\right)^{a-1}(-N)\right]=0, \tag{A.13}$$

$$\frac{\partial}{\partial w_{2j}}=0:\quad b_1(0)\mathrm{e}^{-\rho N}aW_T^a$$
$$\times \mathrm{E}\left[\left(\sum \frac{w_{2j}P_j(T+N)}{P_j(T)}-c_2^* N\right)^{a-1}\frac{P_j(T+N)}{P_j(T)}\right]=\lambda. \tag{A.14}$$

Put $w_{2i}=w_{2i}^*$, $i=1,2,\dots,I$, and $\lambda=\lambda_2^* W^a$ into (A.14) to get

$$ab_1(0)\mathrm{e}^{-\rho N}\mathrm{E}\left\{\left[\sum_i w_{2i}^*\frac{P_i(T+N)}{P_i(T)}-c_2^* N\right]^{a-1}\frac{P_j(T+N)}{P_j(T)}\right\}=\lambda_2^*, \tag{A.15}$$

because W_T^a cancels off both sides. Now for each λ_2 solve (A.15), $j=1,2,\dots,I$, to get $w_{2j}(\lambda_2)$, $j=1,2,\dots,I$. Choose λ_2^* so that

$$\sum w_{2j}(\lambda_2^*)=1.$$

[27]The uniqueness of the solution assumes, as is standard, that there are no redundant assets. The quantities c_2^*, λ_2^* depend upon time, but since the functions $\lambda(\cdot), c(\cdot)$ are separable in time and wealth we suppress time in the notation for convenience.

Put $w_{2j}^* = w_{2j}(\lambda_2^*)$, and insert into (A.11) to get

$$
\begin{aligned}
J_2(W_T) &= b_2(T) W_T^a \\
&= \theta(N)\frac{(c_2^*)^a}{a} W_T^a + b_1(0) \\
&\quad \times \mathrm{E}\left\{ \mathrm{e}^{-\rho N} W_T^a \left(\sum w_{2j}^* \frac{P_j(T+N)}{P_j(T)} - c_2^* N \right)^a \right\}, \\
b_2(T) &= \theta(N)\frac{c_2^*(b_1(0))^a}{a} + b_1(0) \\
&\quad \times \mathrm{E}\left\{ \mathrm{e}^{-\rho N} \left(\sum w_{2j}^*(b_1(0)) \frac{P_j(T+N)}{P_j(T)} - c_2^*(b_1(0)) N \right)^a \right\},
\end{aligned}
\tag{A.16}
$$

where $c_2^*(b_1(0))$ and $\{w_{2j}^*(b_1(0))\}$ solve (A.13), (A.14), (A.15).

Remark 6: Since the price process parameters α_j and β_j are not constrained to equal those over the open trading period, the optimal weights for this problem will differ from those over the open period.

To solve for the valuation function we may write, using (A.16) and (A.10),

$$
b_2(T) = \varphi_T(b_1(0)) = b_1(T), \qquad b_1(0) = \varphi_T^{-1}(b_1(T)),
$$

where $\varphi_t(x)$ solves (A.10) with $b = x$ at $t = 0$.

Implications: On $[0,T)$, $[T+N, 2T+N)$, $[2T+2N, 3T+2N), \dots$, i.e., open periods, proportions are $\{\bar{w}_{1j}\}_{j=1}^{I}$ so $\forall t \in \Gamma$, $\Gamma = [0,T) \cup [T+N, 2T+N) \cup \dots$, we have $\forall j$:

$$
N_j(t) = \frac{\bar{w}_{1j} W(t)}{P_j(t)},
$$

$$
\begin{aligned}
\frac{\mathrm{d}W}{W} &= \sum \bar{w}_{1j}(\alpha_j\,\mathrm{d}t + \beta_j\,\mathrm{d}Z_j) - \bar{c}_1\,\mathrm{d}t \\
&= \sum \left(\bar{w}_{1j}\alpha_j - \bar{c}_1\right)\mathrm{d}t + \sum \bar{w}_{1j}\beta_j\,\mathrm{d}Z_j,
\end{aligned}
\tag{A.17}
$$

where $c(W) \equiv \bar{c}W$. From (A.17) over open periods one can compute $\mathrm{d}N_j(t)$ and $\sum \mathrm{d}N_j$ by stochastic calculus. On $\hat{t} \in \Psi \equiv [T, T+N) \cup [2T+N, 2T+2N) \cup \dots$,

$$N_j(\hat{t}) = \frac{w_{2j}^* W(\hat{t})}{P_j(\hat{t})}, \qquad W(\hat{t}) = W(\hat{T}) \sum w_{2j}^* \frac{P_j(\hat{t})}{P_j(\hat{T})},$$

where $\hat{T}$ = time at close of trade, and $\hat{t} = \hat{T}$ plus time since close.

Remark 7, Result: Notice that volume of trade in shares of j as measured by $|\mathrm{d}N_j|$ abruptly changes at opens and closes. This is because w_j changes abruptly at open and close, whereas the ratio of wealth to P_j changes continuously.

Remark 8: The same type of derivation shows that during open periods [c.f. (A.9)]

$$w_{1j} = \frac{\alpha_j - \alpha_I}{\beta_j^2} \tau,$$

even when α_j, β_j are not constant and τ is not constant. Hence, when the 'Sharpe Indices' $(\alpha_j - \alpha_I)/\beta_j^2$ change a lot, w_{1j} changes a lot and you get high volume.

Remark 9: At open/close transactions demand should be relatively *inelastic* as well as strong. The urge to change w_{1j} is especially strong at open/close/open.

Appendix 2

This appendix provides conditions under which increased transactions demand with higher volume implies wider bid–asks spreads. We assume that in fig. 2 above the increase in demand for the security from $A1(\lambda)$ to $A2(\lambda)$, with nonchanging supply $B(\lambda)$, is such that:

Assumption A(i): $MR2(\lambda^*) > MR1(\lambda^*)$;

Assumption A(ii): $\dfrac{\partial A2(\lambda)}{\partial \lambda} \leq \dfrac{\partial A1(\lambda)}{\partial \lambda}$, for all λ;

Assumption A(iii): $\dfrac{\partial A2(\lambda)}{\partial \lambda} + \lambda \dfrac{\partial^2 A2(\lambda)}{\partial \lambda^2} < \dfrac{\partial B(\lambda)}{\partial \lambda} + \lambda \dfrac{\partial^2 B(\lambda)}{\partial \lambda^2}$, for all λ.

Assumption A(i) states that $MR2(\lambda^*)$, the value of $MR2(\lambda)$ evaluated at λ^*, exceeds $MR1(\lambda^*)$, which ensures that the increased demand is such that the new optimal quantity λ_2^* is greater than the original optimum λ^*.

A(ii) assumes that the (absolute) slope of the new demand is greater than or equal to that of the original demand at all quantities, which allows for parallel shifts in demand, or for new demand to be less elastic than that implied by parallel shifts. This assumption is fairly weak, given the arguments in section 2 for increased, less elastic demand at particular trading times.

Assumption A(iii) implies that the sum of (absolute) slopes of $MR2(\lambda)$ and $MC(\lambda)$ exceeds the sum of (absolute) slopes of $A2(\lambda)$ and $B(\lambda)$, as follows. Using obvious notation,

$$MC = \frac{\partial(B\lambda)}{\partial\lambda} = B + \lambda\frac{\partial B}{\partial\lambda}, \qquad \frac{\partial MC}{\partial\lambda} = 2\frac{\partial B}{\partial\lambda} + \lambda\frac{\partial^2 B}{\partial\lambda^2}.$$

Similarly,

$$\frac{\partial MR2}{\partial\lambda} = 2\frac{\partial A2}{\partial\lambda} + \lambda\frac{\partial^2 A2}{\partial\lambda^2}.$$

Now A(iii) implies

$$\frac{\partial B}{\partial\lambda} - \frac{\partial A2}{\partial\lambda} < \left(2\frac{\partial B}{\partial\lambda} + \lambda\frac{\partial^2 B}{\partial\lambda^2}\right) - \left(2\frac{\partial A2}{\partial\lambda} + \lambda\frac{\partial^2 A2}{\partial\lambda^2}\right),$$

that is,

$$\frac{\partial B}{\partial\lambda} - \frac{\partial A2}{\partial\lambda} < \frac{\partial MC}{\partial\lambda} - \frac{\partial MR2}{\partial\lambda}.$$

We note that A(iii) is also fairly mild; for example, it is clearly satisfied by linear demand and supply functions, since then

$$\frac{\partial MC}{\partial\lambda} = 2\frac{\partial B}{\partial\lambda}, \qquad \frac{\partial MR2}{\partial\lambda} = 2\frac{\partial A2}{\partial\lambda};$$

but conditions weaker than linear demand and supply also satisfy A(iii).

Lemma. Given assumptions A(i) and A(ii), $A2(\lambda^*) - A1(\lambda^*) \geq MR2(\lambda^*) - MR1(\lambda^*)$, *i.e., at the original optimal quantity* λ^*, *the increase in demand implies a greater increase in the ask price than the corresponding increase in marginal revenue.*

Proof.

$$MR = A + \lambda \frac{\partial A}{\partial \lambda}.$$

Hence

$$MR2(\lambda^*) - MR1(\lambda^*) = A2(\lambda^*) - A1(\lambda^*) + \lambda^* \left[\frac{\partial A2}{\partial \lambda} - \frac{\partial A1}{\partial \lambda} \right].$$

The result follows immediately from A(ii). ■

Proposition 1. Given assumptions A(i), A(ii), and A(iii), the spread widens when demand increases with nonchanging supply, that is, $(A2 - B2) > (A1 - B1)$.

Proof. The Lemma considers the original optimum λ^*. Now consider quantities $\hat{\lambda} > \lambda^*$. We know $A2(\hat{\lambda}) < A2(\lambda^*)$ and $B(\hat{\lambda}) > B(\lambda^*)$, since we assume throughout that $\partial B(\lambda)/\partial\lambda > 0$ and $\partial A(\lambda)/\partial\lambda < 0$. Hence the spread narrows relative to that at λ^*, i.e.,

$$A2(\lambda^*) - B(\lambda^*) > A2(\hat{\lambda}) - B(\hat{\lambda}).$$

But by A(iii), the difference between $MR2$ and MC narrows more quickly, i.e.,

$$\begin{aligned}&[A2(\lambda^*) - B(\lambda^*)] - [A2(\hat{\lambda}) - B(\hat{\lambda})]\\ &< [MR2(\lambda^*) - MC(\lambda^*)] - [MR2(\hat{\lambda}) - MC(\hat{\lambda})].\end{aligned}$$

This holds for all $\hat{\lambda} > \lambda^*$, in particular, $\lambda_2^* > \lambda^*$. Hence

$$\begin{aligned}&[A2(\lambda^*) - B(\lambda^*)] - [A2(\lambda_2^*) - B(\lambda_2^*)]\\ &< [MR2(\lambda^*) - MC(\lambda^*)] - [MR2(\lambda_2^*) - MC(\lambda_2^*)].\end{aligned}$$

But since λ_2^* is the equilibrium quantity, we know $MR2(\lambda_2^*) = MC(\lambda_2^*)$, and similarly, $MR1(\lambda^*) = MC(\lambda^*)$. Hence

$$\begin{aligned}[A2(\lambda^*) - B(\lambda^*)] - [A2(\lambda_2^*) - B(\lambda_2^*)] &< MR2(\lambda^*) - MR1(\lambda^*)\\ &\leq A2(\lambda^*) - A1(\lambda^*),\end{aligned}$$

by the Lemma. That is, $A1(\lambda^*) - B(\lambda^*) < A2(\lambda_2^*) - B(\lambda_2^*)$. ■

Proposition 2. Given assumptions A(i) to A(iii), the ask rises when demand increases with nonchanging supply, that is, $A2(\lambda_2^*) > A1(\lambda^*)$.

Proof. At $\lambda_2^* > \lambda^*$, $B(\lambda_2^*) > B(\lambda^*)$. But $A1(\lambda^*) - B(\lambda^*) < A2(\lambda_2^*) - B(\lambda_2^*)$, by Proposition 1. Hence $A2(\lambda_2^*) > A1(\lambda^*)$. ■

Propositions 1 and 2 give an increased spread and an increased ask for a nonchanging supply function and increased demand. The arguments are similar for an increase in supply.

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