What captures liquidity risk? Order based versus trade based liquidity measures

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Abstract

Is the effect of liquidity risk on asset prices sensitive to the choice of liquidity proxy? In our study we achieve three main results. First, we extract three factors from a broad range of trade- and order based liquidity variables, and find that an order based liquidity factor is not related to realized returns while a trade based liquidity factor tied to information risk is. Second, we find evidence that the factor related to information risk also explain expected returns when we estimate the theoretical liquidity-adjusted CAPM. Third, when we estimate a liquidity-augmented factor model, the Fama-French model does well in pricing stocks relative to each individual liquidity factor, but a CAPM model augmented with all our liquidity factors outraces the Fama-French model.

JEL Codes: G12; G14

Keywords: Market microstructure; Common factor; Asset pricing; Liquidity factor; high frequency liquidity

1 Introduction

A main obstacle for reaching robust conclusions about the relationship between asset prices and liquidity is the fact that we lack a clean definition of liquidity as well as an unambiguous way of measuring it, see for instance Amihud et al (2005) and Cochrane (2005). In this paper we investigate how sensitive the price effects of liquidity are to the choice of empirical liquidity proxy, and whether certain aspects of market liquidity are more relevant for asset prices than others.

Liquidity is difficult to define because it comprises several interrelated dimensions including the cost per share of liquidity, the volume that can be traded at a given price, how quickly a given volume can be traded, and how easy it is to trade at a minimal price impact. This multi-dimensionality of liquidity has led to a broad range of empirical measures, but no single measure captures all dimensions. To reduce this problem, we construct common liquidity factors using dimension-reduction techniques. In this way we try to extract a few "deep" variables that capture the fundamental variations in liquidity. This exercise is particularly valuable since the proxies used in the previous literature are sometimes narrow, or may not capture all the dimensions of liquidity that investors actually value, see Hodrick and Moulton (2003).

Another difficulty is the close interrelation between liquidity and price discovery. The coordination function of the market is what is commonly referred to as liquidity, but the price discovery function may also have implications for asset prices in the form of a premium for holding information risk, see O'Hara [2003]. We argue that the distinction between liquidity and price discovery can be linked to the distinction between order based (pre-trade) and trade based (post-trade) liquidity measures. Order based measures reflect ex ante available liquidity, while trade based measures reflect ex post realizations of liquidity demands. Hence, although the two classes of liquidity measures are closely related, order based measures are better suited to capture how well the coordination mechanism of the market is working, while trade based measures are more related to price discovery. Using the absolute value of net order flow as a proxy for price discovery (information arrival), we find strong empirical support for this claim.

When we pool a diversified set of order- and trade based liquidity measures in a common factor analysis, we identify three common liquidity factors which we show can be interpreted as one order based factor, one trade based factor, and one factor reflecting return volatility and information risk (which we refer to as the Amihud factor).

To explore the relationship between the different liquidity factors and asset prices, we first estimate the liquidity-adjusted CAPM derived by Acharya and Pedersen [2005] for each factor. The main result from the estimation is that only the Amihud factor is related to expected returns. Thus, the coordination quality of markets does not seem to be reflected in expected returns. This suggests that the risk premium found for liquidity most likely reflect a compensation for information risk. This is important since many studies on the role of liquidity risk in asset pricing use versions of Amihud's illiquidity measure.

To study possible joint effects of the liquidity factors, and to explore the relationship between the liquidity factors and traditional risk factors, we also estimate and test several liquidity-augmented factor models. [Preliminary result. This part of the analysis is unfinished: In a cross section of stocks listed at the Oslo Stock Exchange, the CAPM augmented with all our liquidity factors prices out the SMB and HML factors. However, SMB and HML add information for models that include less than all three liquidity factors. There is mixed evidence on which liquidity factors receive premia. Finally, there is strong evidence of parameter instability.]

The remainder of the paper is organized as follows. Section 2 summarizes the relevant literature. Section 3 describes the data sample, discuss the concept of liquidity, present some descriptive statistics for the liquidity variables, and present the results from the common factor analysis. We then investigate the relationship between the common liquidity factor and asset returns in sections 5. Section 6 concludes.

2 Literature

A few papers investigate the characteristics of different liquidity measures. Holl and Winn [1995] calculate the correlation structure for 25 measures of liquidity using transactions data from the Australian Stock Exchange (ASE) in 1995. Only those measures that are similar by design are found to be correlated, indicating that different measures of liquidity capture different characteristics of assets. Using data from the Jakarta Stock Exchange (JSE), Aitken and Comerton-Fordre [2003] divide liquidity measures into trade and order based measures and find little correlation between the two categories. By examining changes in the liquidity measures within each category before and after an economic crisis on the JSE, the authors provide evidence that order based measures provide a better proxy for liquidity than trade based measures. Neither of these studies consider liquidity in an asset pricing perspective nor do they investigate shared variation in liquidity measures.

There is a large literature on the relationship between liquidity and asset pricing.[1] Papers investigating empirically the question whether liquidity affect the risk of holding an asset include Chordia et al. [2000], Huberman and Halka [2001], Hasbrouck and Seppi [2001], Amihud (2002), Eckbo and Norli (2002), Pastor and Stambaugh [2003], Korajczyk and Sadka [2006], and Chen [2006].

Three papers stand out as especially close to our work; Hasbrouck and Seppi [2001], Chen [2006], and Korajczyk and Sadka [2006]. Hasbrouck and Seppi [2001] analyze liquidity measures, returns, and order flows using a similar methodology as we do and find little evidence of a common factor in liquidity. However, their approach differs from ours in a few important aspects. They study whether a part of the variation in one separate liquidity measure is common across a small subsets of firms. Then they study whether this common part is related to common parts of variation in returns and order flows using a canonical correlation analysis. We search for the shared variance across many different liquidity measures to obtain a proxy for liquidity which captures the latent dimensions of liquidity. We then investigate whether these common factors are related to cross-sectional differences in realized returns.

Chen [2006] constructs common liquidity factor across different liquidity measures in the same way as we do. She finds that exposure to the first principal component is priced. She also distinguishes liquidity effects from volatility effects, and documents that stock market liquidity is priced in bond markets. Korajczyk and Sadka [2006] construct common liquidity factors both within single liquidity measures (similar to the Hasbrouck and Seppi [2001] approach) and across different liquidity measures. They document that common liquidity across different measures is a robust priced factor, while common liquidity factors within single measures are not. Our work is different from Chen [2006] and Korajczyk and Sadka [2006] in that we investigate whether the relationship between common liquidity factors and asset prices is different for trade based and order based factors, and whether certain dimensions of liquidity are more relevant for asset prices than others.

3 Empirical measures of liquidity

The challenge of measuring liquidity comprises a data availability problem, a multi-dimensionality problem, and a problem of separating liquidity from price discovery. In this section, we first provide a short description of our data sample. We then discuss how to capture the different di-

[1] Good surveys of the literature include O'Hara [2003], Madhavan (2003), Biais et al (2004), and Amihud et al (2005).

mensions of liquidity, how to approach the problem of separating liquidity from price discovery, and present some descriptive statistics for a large set of liquidity measures. Finally, we present the main results from running a common factor analysis on the set of liquidity variables.

3.1 The data sample

We have access to almost five years of rich high-frequency data from the period February 1999 through 2004. The data is provided to us by the Oslo Stock Exchange (OSE) in Norway.[2] Norway is a member of the European Economic Area, and its equity market is among the 30 largest world equity markets by market capitalization.[3] At the end of 2005, 219 companies were listed at the OSE with a total market value of NOK 1 403.3 billion. Since January 1999, the OSE has operated a fully automated computerized trading system similar to the public limit order book systems in Paris, Stockholm, and Toronto.

Our data sample is unique in that it enables us to construct a wide range of liquidity measures from the actual sequence of trades and orders. We know every order and trade that occurred during the sample period. The order data include all order submissions, deletions and amendments of existing orders. We also know whether the order is a buy or a sell order. Thus, for each security in the data, we are able to reconstruct the full order book at any point in time. Every trade is linked to the underlying orders through an order ID. Thus, if a large order is executed against many smaller orders resulting in several smaller trades, we can trace each executed part back to the initial order.

To remove very illiquid securities and securities that only have a short listing period in the data sample, we filter the sample as follows. First, each firm is required to have been listed for the entire data sample period. In addition, the firm must have been traded on at least 80 percent of the days when the Oslo Stock Exchange is open for trading (1539 days). This reduces the sample to a total of 42 securities. To remove outliers from the reduced data sample, we check for erroneous order submissions. This is done for the largest orders submitted across all firms on each day. If we see that a large order is immediately canceled or amended to a significantly lower volume, we correct the volume in the initial submission. In addition, we remove all odd-lot trades and orders and all trades reported as off market trades.

[2] We obtained the data directly from the exchange's surveillance system. The SMARTS© system is the core of the exchange's surveillance operations. Through access to the SMARTS© database, we obtained all the information on orders and trades in the market

[3] Source is FIBV (International Federation of Stock exchanges).

3.2 The challenge of measuring liquidity

Capturing the many aspects of liquidity Liquidity is often vaguely defined as "an ability to trade large quantities quickly at low cost with little price impact". From this definition we can extract a cost dimension, a quantity dimension (how much can you trade at a given cost), a time dimension (how quickly can you trade a given quantity at a given cost), and a resiliency dimension (how good is your ability to trade at minimal price impact).[4] For the task of estimating the effects of liquidity on asset prices, a major problem has been to find empirical measures that can capture all of these aspects.

We try to overcome this problem in the following way. First, we calculate a large set of liquidity variables that according to the literature should capture the different dimensions of liquidity. Second, we use common factor techniques to extract a small number of factors to account for the inter correlations among these variables.

Separating liquidity from price discovery As pointed out by O'Hara [2003], financial markets has two closely related functions; one is to coordinate buyers and sellers and the other is price discovery. The coordination role of the market is carried out by liquidity providers - either designated market makers or traders themselves through a limit order book. Inherent in the process of matching buyers and sellers lies the price discovery process, by which prices come to reflect relevant information. The coordination role of the market is what is commonly referred to as "liquidity", however, a liquid market also require high quality of price discovery: in a liquid situation, both coordination and price discovery work well with a minimum of frictions, while in an illiquid situation, traders have a hard time trying to buy or sell and there is typically high uncertainty about fundamental asset values.[5] An important implication of this is that we do not know whether a relationship between asset prices and liquidity come from liquidity risk or information risk. Consider for instance the spread, which is a widely used measure of liquidity. The spread is a good measure of liquidity in the sense that spreads are low when the market functions well, and high when the market functions poor. The size of the spread is however determined by both a coordination cost component and an adverse selection cost component.

To address this problem we split liquidity measures into order based and trade based measures. Order based liquidity measures - such as quoted spread and depths - are based on

[4]see Harris [1990]

[5]Matching of buyers and sellers is not likely to be easy in a market characterized by large uncertainty about fundamental asset values. Similarly, if prices are to be close to efficient prices and revert quickly (resilient) towards efficient prices after liquidity shocks or information events, the coordination role of the market must function well.

pre-trade information, while trade based liquidity measures - such as turnover and trading volume - are based on post-trade information. Order based information should be more updated with respect to the available liquidity than data from realized trades. Hence, we argue that order based liquidity measures should provide better information about the expected coordination quality of the market than trade based measures. Trade based measures obviously also contain information about the efficiency of the coordination process since when coordination is poor one would also expect the trading activity to be low. However, these measures should be more closely related to price discovery as they reflect ex post realizations of liquidity demands from impatient traders, where the impatience can be related to private information as well as funding needs. Trade based measures should therefore give more information about the current direction of liquidity demand (and potentially the direction of information) than order based measures.[6]

3.3 Descriptive statistics

Our set of variables is picked out based on two criteria. First, we want the variables to capture all dimensions of liquidity, that is cost, quantity, immediacy, and resiliency. Second, since we are interested in the distinction between order based and trade based liquidity measures, we seek both trade- and order based measures along the different dimensions. The resulting variables comprise five trade based measures and nine order based measures. Trade based measures include the number of trades, the trading volume in shares, turnover, the number of seconds between trades, and Amihud's illiquidity ratio. Order based measures include quoted spread, relative quoted spread[7], the depth at the inner quotes, the number of submitted limit orders, the fraction of all orders that are limit orders, the time between submitted limit orders, the order book symmetry, and two measures of the order book slope.

All variables are first calculated for each security on each trading date. To avoid biases due to intra-day trading patterns, we split the trading date into 6 hourly intervals. Except for the trading frequency, the share volume and the illiquidity ratio, all measures are first averaged within each interval and then averaged over the 6 intervals to get a daily average. Then, a cross sectional average is calculated for each date. The cross sectional average represents the market wide realization of the liquidity variable on each date. Table 1 provides some descriptive

[6] Aitken and Comerton-Fordre [2003] argue that order based measures provide the best proxy for liquidity because trade based measures may wrongly indicate that liquidity is good during turbulent periods.

[7] We do not include the effective spread since it is actually a hybrid between a trade based and an order based measure. Moreover, effective spreads are highly correlated with quoted spreads, especially in a limit order market where there are no price improvements.

statistics for the liquidity measures. A detailed description of how the measures are calculated is provided in the Appendix.

[Table 1 about here.]

At year-end 2004, the average market cap of the sample firms was NOK 7.46 billion with the largest firm having a market value of NOK 124 billion and the smallest firm having a market value of NOK 104 millions (these numbers are not shown in the table). Over the sample period, there were on average 4494 trades in the firms each day, and the average depth at the inner quotes was 8893 shares. Measured by the quoted spread, the average cost of trading over the period was NOK 1.37, or 2.28 percent of the midpoint prices. A median number of seconds between trades of 2718 (over 45 minutes) reflects that some firms in the sample were traded quite infrequently.[8]

One important criterion for determining the appropriateness of factor analysis is that the variables are sufficiently correlated. A rule of thumb is that a substantial number of the correlation coefficients should be greater than 0.30. In the correlation matrices in Table 2, correlations greater than 0.3 are in bold. Visual inspection of the matrix indicate that a factor analysis is appropriate. Note also that most of the order based and trade based measures within the same liquidity dimensions exhibit fairly high correlations.

[Table 2 about here.]

We argue that (ex post) trade based liquidity measures are more related to price discovery than (ex ante) order based liquidity measures. To investigate this claim empirically, we need a proxy variable for information arrivals. Looking to the market microstructure literature, the natural candidate for this is net order flow.[9] Table 3 shows the correlation structure between returns, volatility, net order flow, and the absolute value of net order flow for our sample firms as well as the correlation coefficients between these variables and our set of liquidity variables.

[Table 3 about here.]

First of all, we notice a strong contemporaneous relationship between returns and net order flow, and also between volatility and the absolute value of net order flow. These results indicate

[8] Note that the numbers in the table describe the distribution of the daily average over all firms. Hence, many firms were considerably more actively traded than these numbers suggest.

[9] The market microstructure literature on foreign exchange shows a strong relationship between returns and net order flow, see for instance Evans and Lyons (200?), Bjnnes and Rime (200?).

that the strong explanatory power of net order flow on returns which is documented for the foreign exchange markets also apply for stock markets. Note also that the illiquidity ratio is almost perfectly correlated with daily volatility. The results with respect to the correlations between net order flow and the two types of liquidity measures are quite striking. All the trade based liquidity measures have a notable correlation with the absolute value of net order flow, while order based measures, except the frequency of limit orders, have not.[10]

3.4 Common liquidity factors

In this section we use factor analysis to reduce the set of liquidity variables into a small number of liquidity factors. A short discussion of the main design issues involved in the analysis and some robustness tests of the results from the analysis are provided in Appendix B.

Table 4 summarizes the main results from the estimation of a common factor model on the full set of liquidity variables described in section 3. The table shows rotated factor loadings for three extracted factors as well as the final estimates of shared variance among the variables. A rule of thumb frequently used is that factor loadings greater than 0.30 in absolute value are significant. These loadings are marked gray in the table. We also report Kaiser's Measure of Sampling Adequacy (MSA), both overall and for the individual variables.[11] In general, values of MSA greater than 0.8 are considered good, while values less than 0.5 are unacceptable. The table shows an overall MSA of 0.78, and individual MSA numbers that varies from a minimum of 0.61 for the fraction of all orders that are limit orders to a maximum of 0.89 for the quoted spread. Hence, the set of liquidity variables seems well suited to factor analysis.

[Table 4 about here.]

The first common factor (Factor 1) explains 39 percent of total shared variance among the variables, while the second (Factor 2) and the third factor (Factor 3) explain respectively 36 and 25 percent. Factor 1 is mainly an order based quantity measures, however, it also have significant negative loadings on quoted spread and the illiquidity ratio. Factor 2 capture both trade- and order based measures of quantity and immediacy. Factor 3 has significant loadings on all liquidity dimensions from both types of measures. However, three variables stick out;

[10] The correlation coefficients between liquidity measures and net order flow are much lower because net order flow frequently changes its sign.

[11] The underlying assumption of factor analysis is that there exists a number of unobserved latent variables that account for the correlations among the observed variables, such that if the latent variables were held constant, the partial correlations among the observed variables would be small. Kaiser's Measure of Sampling Adequacy is a summary measure of how small the partial correlations are relative to the ordinary correlations.

Amihud's price impact measure (largest significant split loading), the relative spread (only significant loading), and the symmetry of the order book (only significant loading).[12] The two first of these variables are typically related to private information.

We also estimate one common factor model where we include trade based liquidity variables only (model B), and one common factor model where we include order based variables only (model C). The results from these estimations are summarized in table 12 and table 13 in appendix B.[13] Model B has one common factor related to the quantity and immediacy dimensions of liquidity. Model C has two common factors. The first factor explains 68 percent of the shared variance and is mainly related to quantity variables. The second factor is a bit hard to interpret because it is loaded from variables representing three different liquidity dimensions. Based on the factor analysis, we extract daily score series that represent the daily realizations of the common factors in the three models. Table 5 presents the correlation matrix for the factor score series.

[Table 5 about here.]

Some interesting patterns emerge from the factor score correlation matrix. Even when we pool all the liquidity variables together in Model A, we extract one order based factor (Factor 1 in Model A has a correlation coefficient of 0.96 with the first factor in the order based model), and one trade based factor (Factor 2 in Model A has a correlation coefficient of 0,93 with the factor in the trade based model). The order based factor reflects the quantity (or depth) dimension of liquidity, while the trade based factor is related to both quantity and immediacy. The third factor in Model A, which we suggested could be related to information risk, is negatively related to the "hard to interpret" factor in the order based model (Factor 2 in Model C). Note that Factor 1 and Factor 2 are liquidity variables, i.e. they increase in value when liquidity increase, whereas Factor 3 is an illiquidity variable, i.e. it increases in value when liquidity decreases. Table 6 shows how the score series for the three factors are correlated with volatility, net order flow, market return and relative spread.

[Table 6 about here.]

The correlation coefficients provide further support to our factor interpretation. Factor 1 has a low correlation with net order flow, as was the case for most of the order based liquidity

[12] A split loading means that a variable has multiple significant loadings. Ideally, we would like to see a single significant loading for each variable on only one factor.

[13] Appendix B also include the results from a robustness test of the factor analysis where we split the sample in two and re-estimate all the models.

variables. Factor 2 has a fairly high correlation with the absolute value of order flow, as was the case for most of the trade based liquidity variables. Factor 3 is highly correlated with volatility and relative spread. The high correlation with volatility comes from the significant loading to Amihud's illiquidity measure. Together with a high correlation with the relative spread and a moderate correlation with the absolute value of net order flow, this factor seems more related to information risk than factor 2. To sum up, the common factor analysis have provides us with three orthogonal liquidity factors; one order based factor, one trade based factor, and one factor reflecting return volatility and information risk (hereafter referred to as the Amihud factor).

4 Asset pricing with different proxies for liquidity risk

Existing empirical evidence on the role of liquidity risk in asset pricing largely supports the existence of a liquidity risk premium.[14] The main aim of this paper is to study how sensitive these results are to the choice of an empirical liquidity proxy. In this section, we first describe how time-series for market-wide and firm specific liquidity are created from the liquidity factors found in section 3. We then employ two different frameworks to explore the relationship between these liquidity variables and asset returns:

- First, we estimate the theoretical *liquidity-adjusted* CAPM derived by Acharya and Pedersen [2005]. In this model liquidity risk affect asset prices through three different channels reflecting the covariances between firm specific return, firm specific illiquidity, market return, and market illiquidity.

- Second, we estimate different versions of an empirical *liquidity-augmented* factor model. That is, we try to explain the cross-section of stock returns by different multi-factor models that includes liquidity factors as well as traditional risk factors (market, size, and value). This framework is similar to the one used by for instance Pastor and Stambaugh [2003] and Liu [2006].

By estimating the liquidity-adjusted CAPM, we can study the liquidity factors within the context of a theoretical model where the implications of liquidity on asset prices are exactly derived and easy to interpret. To investigate differences between the liquidity factors, we estimate the model separately for each of them. To study possible joint effects of the liquidity factors, and to explore the relationship between the liquidity factors and other risk factors found to explain

[14] The empirical evidence is substantial, see for example Chordia et al. [2000], Hasbrouck and Seppi [2001], Pastor and Stambaugh [2003], Acharya and Pedersen [2005], and Liu [2006].

returns in the empirical asset pricing literature, we also estimate several liquidity-augmented factor models. These models are empirical in nature and make no attempt to explain why different factors should affect returns. This is the approach followed by most empirical work on this subject. Moreover, we do not know what constitute the correct pricing model.

4.1 Liquidity variables

The common factor analysis suggest that the joint variation in our set of liquidity variables are best captured by three orthogonal factors; one order based factor, one trade based factor, and the Amihud factor. In order to estimate and test the relationship between these factors and asset returns, we need time series of both market-wide liquidity and firm specific liquidity. Natural candidates for the market-wide liquidity variables are the score realizations from the factor analysis, i.e.

$$s_t^M(j) = \sum_{k^M} \omega(j)^{k^M} k_t^M \tag{1}$$

where $s_t^M(j)$ is the market-wide score realization of liquidity factor $j \in \{1, 2, 3\}$ at time t, k^M are the normalized market-wide liquidity variables underlying the factor analysis, and $\omega(j)^{k^M}$ are the standardized scoring coefficients of the liquidity variables k^M for factor j. We calculate the firm specific liquidity variables in a similar way using the standardized scoring coefficients from the factor analysis and normalized firm specific liquidity variables, i.e.

$$s_t^i(j) = \sum_{k^i} \omega(j)^{k^M} k_t^i \tag{2}$$

where $s_t^i(j)$ is the score realization of liquidity factor j for firm i at time t, and k_t^i are the normalized firm specific liquidity variables.

4.2 The liquidity-adjusted CAPM

Acharya and Pedersen [2005] argue that the CAPM in a frictionless economy must translate into a CAPM in net returns in a real economy with illiquidity costs. Rewriting the CAPM in net returns in terms of gross returns gives the conditional liquidity-adjusted CAPM, reproduced here from Proposition 1 in their paper,

$$
\begin{aligned}
E_t(r^i_{t+1}) \quad = \quad & r^f_t + E_t(c^i_{t+1}) + \lambda_t \frac{cov_t(r^i_{t+1}, r^M_{t+1})}{var_t(r^M_{t+1} - c^M_{t+1})} + \lambda_t \frac{cov_t(c^i_{t+1}, c^M_{t+1})}{var_t(r^M_{t+1} - c^M_{t+1})} \\
& - \lambda_t \frac{cov_t(r^i_{t+1}, c^M_{t+1})}{var_t(r^M_{t+1} - c^M_{t+1})} - \lambda_t \frac{cov_t(c^i_{t+1}, r^M_{t+1})}{var_t(r^M_{t+1} - c^M_{t+1})} \qquad (3)
\end{aligned}
$$

where r^f is the risk free return, r^i (r^M) is the return of firm i (the market), c^i (c^M) is a measure of the illiquidity of firm i (the market) and $\lambda_t = E_t(r^M_{t+1} - c^M_{t+1} - r^f_t)$ is the risk premium. The model suggests that investors are faced with three different liquidity risks. First, investors want to be compensated for holding stocks that become illiquid when the market becomes illiquid, i.e. the return increases with the covariance between the firm's illiquidity and the market illiquidity. Second, investors are willing to accept a lower return on an asset with a high return in times of market illiquidity, i.e. the return decreases with the covariance between a firm's return and the market illiquidity. Finally, investors are willing to accept a lower expected return on a stock that is liquid in a down market, i.e. the return decreases with the covariance between a stock's illiquidity and the market return. To estimate and test the model, Acharya and Pedersen derive an unconditional version of it, assuming constant conditional covariances of innovations in illiquidity and returns, reproduced here from equations (12)-(16) in their paper

$$
E(r^i_t - r^f_t) = E(c^i_t) + \lambda\beta^{1i} + \lambda\beta^{2i} - \lambda\beta^{3i} - \lambda\beta^{4i} \qquad (4)
$$

where

$$
\beta^{1i} = \frac{cov(r^i_t, r^M_t - E_{t-1}(r^M_t))}{var(r^M_t - E_{t-1}(r^M_t) - [c^M_t - E_{t-1}(c^M_t)])} \qquad (5)
$$

$$
\beta^{2i} = \frac{cov(c^i_t - E_{t-1}(c^i_t), c^M_t - E_{t-1}(c^M_t))}{var(r^M_t - E_{t-1}(r^M_t) - [c^M_t - E_{t-1}(c^M_t)])} \qquad (6)
$$

$$
\beta^{3i} = \frac{cov(r^i_t, c^M_t - E_{t-1}(c^M_t))}{var(r^M_t - E_{t-1}(r^M_t) - [c^M_t - E_{t-1}(c^M_t)])} \qquad (7)
$$

$$
\beta^{4i} = \frac{cov(c^i_t - E_{t-1}(c^i_t), r^M_t - E_{t-1}(r^M_t))}{var(r^M_t - E_{t-1}(r^M_t) - [c^M_t - E_{t-1}(c^M_t)])} \qquad (8)
$$

Estimation

We estimate three versions of the liquidity-adjusted CAPM; one where we adjust for illiquidity computed from the order based factor, one where we adjust for illiquidity computed from the trade based factor, and one where we adjust for illiquidity computed from the Amihud factor. In essentials our estimation follows the procedure described in Acharya and Pedersen [2005].[15]

- First, we translate our liquidity variables into measures of illiquidity costs. In the model, illiquididy is defined as the per share cost of selling. We therefore scale the liquidity variables with the relative half spread. More specifically, we first run the regression,

$$S_t^i = \hat{\alpha}^i + \hat{\beta}^i s_t^i \tag{9}$$

  where S_t^i is the daily percentage half spread of firm i averaged over week t and s_t^i is the score realization of firm i for week t. We then construct firm i's illiquidity measure, c_t^i, based on the estimate of the average cost of trading from the regression model (α^i), the coefficient relating changes in the liquidity variable to changes in the cost of trading (β^i), and the ratio of the volatility between the half spread and the score realization i.e.

$$c_t^i = (\hat{\beta}^i s_t^i + \hat{\alpha}^i)\frac{\sigma(S^i)}{\sigma(s^i)} \tag{10}$$

  where $\sigma(S^i)$ is the volatility of the half spread and $\sigma(s^i)$ is the volatility of the score realization. Hence, α is used to match the liquidity variable to the first moment of the half spread, while the volatility ratio $\frac{\sigma(S^i)}{\sigma(s^i)}$ is used to match the variable to the second moment of the half spread. Finally, β is used to convert variations in the liquidity variable into variations in illiquidity costs.[16] The market illiquidity, c_t^M, is constructed similarly.

- Second, we form a market portfolio for each week t over our entire sample period based on all sample stocks. We also form four test portfolios for each week w during the sample period based on each stock's illiquidity in month $w-1$, computed as the average over month $m-1$ of the stock's daily illiquidities.[17] Similarly, we form four liquidity risk portfolios by

[15] Some differences follow from the limitations of our data sample. To reduce the problem with a short time period, we use weekly instead of monthly observations when we estimate the betas and test the model. This gives us 301 time series observations. In addition, when we form our test portfolios, we sort on the average daily illiquidity over the previous month (not previous year).

[16] β is negative for the trade and order based liquidity variables and positive for the Amihud variable.

[17] Note that by adding the estimated constant term from the regression model in (9) to our illiquidity measure, it becomes quite similar to the relative half spread for all three versions of the model.

ranking the stocks each month based on the average daily standard deviation of illiquidity over the previous month. We then compute returns and illiquidity costs for each portfolio for each week and the weekly returns on the market portfolio.

- Third, we estimate the innovations in market illiquidity and market returns and the innovations in illiquidity for the four test portfolios using an AR(4) specification.
- Finally, based on portfolio returns, innovations in portfolio illiquidity, innovations in market returns, and innovations in market illiquidity, we estimate the betas and the model by running cross sectional regressions.

Properties of liquidity sorted portfolios

Table 7 shows some properties of the four illiquidity portfolios for each of the liquidity factors.[18] The four betas are computed as defined in Equations (5)-(8) in Acharya and Pedersen [2005] using all weekly return and illiquidity observations for each portfolio and for an equally weighted market portfolio.

[Table 7 about here.]

Acharya and Pedersen estimate their model using a version of Amihud's illiquidity measure. Our model based on the Amihud factor should therefore produce properties of the illiquidity portfolios that are quite similar to what they find. The magnitude of the beta estimates is in fact quite similar in the two models. However, since they have a larger sample of firms from a more liquid stock exchange, the variation in their estimates is larger and our portfolios matches best the most illiquid half of their portfolios. We also find (for all models) that the most illiquid stocks (based on previous illiquidity) have the highest variation in illiquidity and the smallest market cap.

An interesting finding in Acharya and Pedersen [2005] is that illiquid stocks also have higher return volatility and higher liquidity risk (measured by all three liquidity betas). Using the Amihud factor, we get similar results for β^2 and β^4 but not for β^3. An unsystematic pattern for β^3 (and β^1) risk may be explained by the relationship between illiquidity and return volatility in our sample; the most liquid portfolio also includes the most volatile stocks. This pattern should affect β^3 but not β^2 and β^4 since only β^3 includes firm specific returns.

[18] We find very similar results when we sort the portfolios by the variation in illiquidity.

Having established that there is a good correspondence between our results and the results in Acharya and Pedersen [2005], we are interested in studying whether there are any differences across the three liquidity factors. For the order based factor, the importance of β^3 (higher return sensitivity to market liquidity) seems to be a bit lower than for the other models. Also, the negative relationship between illiquidity and β^2 (meaning that commonality in liquidity with the market is highest for the most liquid stocks) is opposite from the other two models. Based on the Amihud factor, the relationship between illiquidity and β^4 increases monotonically over the portfolios and have a much larger variation than when we base portfolio construction on the two other factors. This is also the only factor for which we find a monotonically increasing relationship between illiquidity and weekly returns.

As is also emphasized in Acharya and Pedersen [2005], there is an inherent severe collinearity problem in their model caused by high correlations between the betas. This means that we cannot use the empirical results to distinguish separate effects of the individual betas.

Results

To determine whether the choice of liquidity variable is important for the existence of a liquidity risk premium, we estimate the Acharya and Pedersen [2005] model by a cross-sectional regression with the excess returns of our test portfolios on pre-estimated betas that are re-estimated each year. Admittedly, the period over which the model is estimated is relatively short. Thus, the results will not provide any reliable test of the model. However, the main purpose of the exercise is to examine whether the order-based, trade-based and Amihud variables contain similar information about expected returns or whether the there are differences among the three types of liquidity measures along this dimension.

Table 8 provides the results from the cross sectional regressions on portfolios that differ in their liqudity attributes. The betas are estimated for each year and portfolio as in eq.5-eq.8. In the theoretical model the liquidity cost incur once every period such that κ=1. However, since we are estimating the model for weekly periods, this is much shorter than a typical investor's holding period. Thus, following Acharya and Pedersen, in the models where the $E(c_t^p)$ has a superscript ^{a}, we scale the liquidity cost by the average turnover for all stocks in the sample, which is equal to 19 months (or 76 weeks). Since the average investor only incur the illiquidity cost once during his holding period, we proxy the estimation period cost to be 1/76th of the average cost of a trade. To accomodate a fixed κ, we treat the net return in models 1, 4 and 7 as $E(r_t^p - r^f) - \kappa E(c_t^p)$. In models 2, 5 and 8 we estimate κ as a free parameter, and in models 3 and 6 we ignore transaction costs. Also, the theoretical model restricts the risk premia related to each beta to be the same, i.e. $\lambda^1 = \lambda^2 = \lambda^3 = \lambda^4$. To facilitate the model-implied constraint, Acharya and Pedersen construct a net beta which is the sum of the estimated betas for each portfolio. Following their approach, Model 1, in all three panels, estimates the liquidity adjusted CAPM with the model restriction of a single λ and a fixed transaction cost scaled to match the weekly estimation period. This is the estimation that is the most consistent with the theoretical model, and avoids the severe collinearity problem related to estimating the different risk premia separately.

[Table 8 about here.]

Examining the estimation results for Model 1 across the three liquidity variables, we see that the liquidity adjusted beta is significant and positive only in model 1c. This is the setup where we construct portfolios based on the Amihud factor. The R-squared suggests that the model is a poor fit to the data, but it is difficult with the small sample size to ascribe this to the model or noise. However, comparing the fit to the unadjusted CAPM which is estimated in model 3c, we see that the liquidity-adjusted CAPM in model 1c improves the explanatory power relatively much. Thus, although the unadjusted CAPM manages to explain some of the cross sectional return differences across portfolios, the liquidity adjusted CAPM does better.

The results in models 1a and 1b shows no systematic relationship between the portfolio returns and the liquidity adjusted beta. As a preliminary conclusion this suggests that there is no priced liquidity risk connected to order-based and trade-based liquidity variables, at least when we assume that the theoretical model of Acharya and Pedersen [2005] is correct.

If we believe the estimated coefficient in Model 1c, we can calculate the return premium associated with portfolios with the highest liquidity risk. Our estimate in model 1c suggests that λ=2.30. By using the β-estimates for portfolio 1 and 4 in table 7 we can calculate the difference in the annualized expected return between the two portfolios attributed to their differences in liquidity betas. For β^2 the model estimate of λ suggests that the return premium between portfolio 1 and 4 associated with β^2 risk is about 0.56% per year. The expected return difference related to β^3 is about 1% and the return difference associated with β^4 is almost 6%. This seems a bit high, at least compared to the results for the US in Acharya and Pedersen [2005]. However, on must take into consideration that the value weighted market return on the OSE over the estimation period was more than 20% per year on average. More interestingly, the ranking of the return premiums associated with the liquidity betas are the same as in Acharya and Pedersen [2005].[19]

4.3 Liquidity-augmented factor models

The liquidity-adjusted CAPM has only one risk premium and one factor.[20] Hence, we can estimate the model and study what proxy for liquidity that fits the model best, but we have to believe that prices are actually determined by the liquidity-adjusted CAPM factor only. This is at odds with the evidence from a huge empirical asset pricing literature. In this section, we take a different view and ask whether the cross section of stock returns can be explained by an empirical factor model that includes one or more of our liquidity factors in *addition* to the CAPM beta and other risk factors found to explain returns in the empirical literature. We construct test-assets and risk factors for each of the three liquidity factors using the following three measures of liquidity risk:

- covariance between firm specific and market-wide liquidity, i.e. $cov(s_t^i, s_t^M)$. This measure is similar in spirit to β^2 in the liquidity-adjusted CAPM.
- covariance between firm specific return and market-wide liquidity, i.e. $cov(r_t^i, s_t^M)$. This measure is similar in spirit to β^3 in the liquidity-adjusted CAPM.
- covariance between firm specific liquidity and market returns, i.e. $cov(s_t^i, r_t^M)$. This measure is similar in spirit to β^4 in the liquidity-adjusted CAPM.

By utilizing a similar cross sorting method as is used for the construction of the Fama-French factor, we reduce the collinearity problem discussed above among the liquidity risk measures. We are interested in answering two questions.[21] First, can the standard Fama French factors, SMB and HML, price portfolios that are sorted on the basis of the these measures of liquidity risk? Second, how do our liquidity risk factors compare to standard factors, in terms of pricing a cross section of firms? We discuss the results for each question, in turn.

(In this version of the paper we have only finished the last set of asset pricing tests for the first measure of liquidity risk.)

[19] Acharya and Pedersen [2005] calculates the premium associated with β^2 to be 0.08% per year, β^3 to 0.16% per year and β^4 to be about 0.8% per year for the US market over a much longer sample period.

[20] $\beta^{net} = \beta^1 + \beta^2 - \beta^3 - \beta^4$, see equation (24) in Acharya and Pedersen [2005]

[21] The asset pricing tests on Norwegian data have to be interpreted with care, since the data comprises only 71 observations for each portfolio. Consequently, in the linear pricing models, inclusion of more than 3 factors leads to highly overidentified or collinear systems. We therefore only use pricing models with 3 or fewer factors.

Constructing test-assets and liquidity risk factors

Test-assets are "factor-mimicking" portfolios constructed to have different degrees of liquidity risk. We create the mimicking portfolios by sorting firms into quartiles based on the liquidity risk measures described above. Based on the factor-mimicking portfolios, we construct liquidity risk factors by computing the difference in the risk adjusted return between the high liquidity risk (mimicking) portfolio and the low liquidity risk (mimicking) portfolio. Table 9 shows monthly returns on the factor mimicking portfolios and the liquidity risk factors.

[Table 9 about here.]

Positive and significant liquidity risk factors (P4-P1 in the table) for all the three liquidity proxies suggest that there is a positive relationship between returns and the risk of holding stocks that become illiquid when the market becomes illiquid. However, to get more reliable results on this matter, we must conduct formal asset pricing tests.

Can standard factors price our liquidity portfolios?

Not ready yet

How does liquidity compare to standard suspects?

We now consider the second question from above, and compare the pricing performance of our liquidity factors to that of other factors. For this purpose, we use publicly traded firms from the Oslo Stock Exchange.[22]

Table 10 shows estimated factor loadings and risk premia when we sort portfolios based on the covariance between firm specific and market-wide liquidity. All factors receive significant loadings, except for the order based factor (F1) when it is in the comprehensive model of CAPM plus all liquidity factors. All liquidity factors receive significant positive premia in all specifications.

[Table 10 about here.]

The asset pricing results are in Table 11. The J-test is Hansen's (1982) test of the overidentifying restrictions of each model. The HJ distance of Hansen and Jagannathan (1997) measures the maximum annualized pricing error for each model. The SupLM test of Andrews (1993)

[22] The total sample is 50 stocks, comprising all the stocks that were available for the full sample period, February 1999 to December 2004. The total number of monthly observations is 71.

measures whether the model parameters are stable over time. The Wald test assesses whether the coefficient on all the factors is equal to zero. Finally, the delta-J test of Newey and West (1987) examines whether SMB and HML have additional ability to explain asset prices, relative to each alternative model.[23]

[Table 11 about here.]

The J-test of overidentifying restrictions cannot reject any of the models. However, the more discriminating HJ distance is very large in all cases, and indicates that none of the models is in the set of true pricing models. The Wald test cannot reject the nonzero coefficients on the factors, and the delta-J test indicates that SMB and HML have incremental power in all models except where the CAPM is combined with all liquidity factors.

[23]These tests are standard in asset pricing literature. For more details on these tests, see Cochrane (2001).

5 Conclusion

There is currently little research on how best to define and test liquidity, based on theoretical and statistical reasons. This makes it difficult to agree on the asset pricing implications of liquidity, since many of the liquidity measures proposed in the literature are only modestly correlated with each other. In this paper we utilize a common factor approach to extract fundamental liquidity factors from a large set of liquidity variables. We then estimate a theoretical asset pricing model and conduct asset pricing tests on liquidity portfolios in a liquidity-augmented factor model. It is valuable to summarize our work in terms of the liquidity factors, and the asset pricing results.

With regard to liquidity factors, we construct factors from a broad range of trade- and order based measures, and discover that order based liquidity cannot explain return differences while certain measures of trade based liquidity can. We argue that the difference between trade- and order based measures reflects the two inherent functions of the market; coordination of buyers and sellers and price discovery. Order based measures naturally reflect coordination quality, while trade based measures are contaminated by realizations of liquidity demands due to private information or funding needs. This is supported by a difference in the relationship between the various liquidity measures and net order flow. Our results suggest that research using trade based liquidity might only capture effects related to information risk or funding needs.

With regard to asset pricing results, we document three main findings. First, when we estimate a liquidity-adjusted CAPM, we find that although the unadjusted CAPM manages to explain some of the cross sectional return differences across portfolios, the liquidity adjusted CAPM does better, as long as we use a liquidity proxy related to information risk and return volatility. When we use an order based liquidity proxy, we find no systematic relationship between portfolio returns and the liquidity adjusted beta. Second, when we test a liquidity-augmented factor model, we find that although the Fama-French model does well in pricing stocks relative to each individual liquidity factor, a CAPM model augmented with all our liquidity factors outraces the Fama-French model. Third, there is strong evidence of parameter instability in the liquidity-augmented factor model.

In sum, to the best of our knowledge, we are the first to construct a comprehensive set of liquidity factors, based on microstructure measures. Using these factors, we document that the effect of liquidity on asset prices is sensitive to the choice of empirical liquidity proxy.

What are the relevance and importance of our results? These results are interesting and important, since the portfolios that we form can be easily utilized by moderately sophisticated investors. Consequently, investors may (in principle) be able to hedge specific types of liquidity risk by holding our factors. In light of the third finding, it is evidently valuable to know whether we can estimate or forecast structural breaks in liquidity.

A Calculation of liquidity variables

The appendix describes how the various liquidity measures used in the common factor estimation are calculated. To make the data discrete and have a common time frame we use hourly windows, starting from 10:00 am until 16:00 pm. Thus, we have 6 one hour intervals during each trading day. If not otherwise stated, the measures are first average within each interval, and then averaged over these intervals to get a daily average. For simplicity, summing and averaging operators as well as security and time indicators are suppressed in the equations. The variables are presented in alphabetical order.

`Depth at the inner quotes` is defined as the average share volume at the best quotes. The measure is calculated as

$$\frac{v^a + v^b}{2}$$

where v^a is the share-volume at the best ask price and v^b is the share-volume at the best bid price.

`Effective spread` is calculated for each trade as

$$2|p^k - \frac{p^a + p^b}{2}|$$

where p^k is the trade price, and p^b and p^a are respectively the best bid and ask prices at the time of the trade.

`Fill time` is defined as the number of seconds it takes to (fully) fill an order.

`Liquidity ratio` measures how large (log) share-volume is needed to move the price by 1%, i.e, it is the inverse of the illiquidity measure proposed by Amihud [2002]. The measure is calculated per day for each firm, using close to close returns as

$$\frac{\log(V)}{|r| * 100)}$$

where V is the total daily share volume, and r is the open to close return.

`Normalized price slope` is the `Price slope` normalized relative to the number of issued shares in the firm, i.e

$$\texttt{Price slope}/NOSH$$

where `Price slope` is defined below and $NOSH$ is the number of issued shares in the firm.

`Order book symmetry` measures the symmetry of the limit order book. It is calculated as the difference between the ask and bid slopes over the first 6 ticks of the order book divided by the added 6 tick slopes (to ensure that the measure is equal to 1 and -1 if one side in the order book is empty), i.e

$$(\frac{v_6^a - v_0^a}{p_6^a - p_0^a} - \frac{v_6^b - v_0^b}{p_0^b - p_6^b})/(\frac{v_6^a - v_0^a}{p_6^a - p_0^a} + \frac{v_6^b - v_0^b}{p_0^b - p_6^b})$$

where v_0^b is the share volume at the best bid quote, v_6^b is the cumulative share volume 6 ticks away from the best bid quote, v_0^a is the share volume at the best ask quote, v_6^a is the cumulative share volume 6 ticks away from the best ask quote, p_0^b and p_0^a are respectively the best bid and ask quotes, and p_6^b and p_6^a are respectively the bid and ask quotes 6 ticks away from the best quotes.

`Price slope` is defined as the average of bid and ask slopes over the first 6 ticks of the order book computed relative to the price levels in the book, i.e

$$\left\{\frac{v_6^a - v_0^a}{p_6^a - p_0^a} + \frac{v_6^b - v_0^b}{p_0^b - p_6^b}\right\}/2$$

where v_0^b is the share volume at the best bid quote, v_6^b is the cumulative share volume 6 ticks away from the best bid quote, v_0^a is the share volume at the best ask quote, v_6^a is the cumulative share volume 6 ticks away from the best ask quote, p_0^b and p_0^a are respectively the best bid and ask quotes, and p_6^b and p_6^a are respectively the bid and ask quotes 6 ticks away from the best quotes.

`Quoted spread` is the difference between the best ask and bid quotes,

$$p^a - p^b$$

where p^a is the best ask quote and p^b is the best bid quote.

`Relative effective spread` is calculated as the effective spread divided by the bid-ask midpoint price,

$$\frac{2|p^k - \frac{p^a+p^b}{2}|}{(p^a + p^b)/2}$$

where p^k is the trade price, p^a is the best ask price, and p^b is the best bid price.

`Relative quoted spread` is calculated as the quoted spread divided by the bid-ask midpoint price,

$$\frac{p^a - p^b}{(p^a + p^b)/2}$$

where p^a is the best ask price, and p^b is the best bid price.

`Seconds between trades` is defined as the average number of seconds between trade executions.

`Tick slope` is defined as the average slope of the bid and ask side of the order book, where the slopes are measured over the 6 first ticks of the book and computed relative to the number of ticks away from the best quotes,

$$\left\{ \frac{v_6^a - v_0^a}{6} + \frac{v_6^b - v_0^b}{6} \right\} /2$$

where v_0^b is the share volume at the best bid quote, v_6^b is the cumulative share volume 6 ticks away from the best bid quote, v_0^a is the share volume at the best ask quote, v_6^a is the cumulative share volume 6 ticks away from the best ask quote.

`Trades per order` is calculated by tracking each order from its initial submission into the system (accounting for subsequent amendments) and counting the number of trades that are required for the order to be fully filled.

`Trading frequency` is measured as the total number of trade executions across all firms during the day.

`Trading volume` is measured as the total number of shares traded in the security during the day.

`Turnover` is measured as the trading volume divided by the number of issued shares in the company.

B Factor analysis

Factor analysis comprises a family of statistical techniques concerned with the reduction of a set of observable variables in terms of a small number of latent factors.[24] In this appendix, we first discuss a few details on the research design and methodology. We then present the main results from running factor models separately on trade based and order based liquidity measures. Finally, the robustness of the factor analysis results is discussed based on a comparison of the results with the results from running all three models on two sub-period samples.

B.1 Some design issues

The two main factor methods are common factors and principal components. In a common factor analysis it is assumed that the variance can be decomposed in two parts: common variance that is shared by other variables in the model, and unique variance that is unique to a particular variable, including an error component. As the name suggests, a common factor analysis focus on the common variance of the observed variables. Specific variation and the error term are excluded from the analysis. In a principal component analysis, no distinction is made between common and unique variance. Here, the objective is to account for the maximum portion of variance present in the original set of variables with a minimum number of composite components. Thus, while both techniques are widely used for the same purpose (data reduction), they are quite different in terms of the underlying assumptions.

We intend to identify the latent dimensions that explain why different liquidity variables are correlated with each other. Thus, we want to extract a small number of factors to account for the intercorrelations among our observed liquidity variables. This objective calls for a common factor rather than a principal component analysis. Moreover, the scarce amount of prior knowledge we have about the composition of the variance of different liquidity measures also speaks for the common factor method.

A second design issue concerns how many factors to extract. While there exist several criteria or empirical guidelines for determining this number, the decision is ultimately subjective. We look at several criteria before we decide on the number of common liquidity factors. First, we use a version of the latent root criterion which says that the latent root (or eigenvalue) of the factors should exceed the average of the initial communality estimates. Second, we check that the number of factors lies close to the "elbow" of the scree plot.[25] We also require a minimum

[24] For a detailed discussion of factor analysis, see for example Hair et al. [1998].
[25] A scree plot is a plot of eigenvalues against corresponding factor numbers.

of three observed variables for each factor expected to emerge.

Factor rotation is a way to simplify the rows and columns of the factor matrix to make the factors interpretable. There are two main methods; orthogonal and oblique. In an orthogonal rotation, the axes are maintained at 90 degrees, while in an oblique rotation, there is no such restriction, meaning that the factors can be correlated with each other. The orthogonal methods are most widely used, although the oblique rotation is more flexible and also more realistic, since important underlying dimensions are not necessarily uncorrelated.[26] Since we want to use the factors for later regression analysis, it is most convenient for us to work with uncorrelated factors constructed using an orthogonal factor rotation.

B.2 A factor model on trade based measures

In model B, we extract one common factor. The overall MSA of 0.71 indicate that the model is acceptable. Expect for the illiquidity ratio, the indiviual MSA numbers are also within the acceptable range. The factor is related to the quantity and immediacy dimensions of liquidity.

[Table 12 about here.]

B.3 A factor model on order based measures

In Model C we extract two common factors. Factor 1 explains 68 percent of the total shared variance among the variables, and is mainly driven by variables related to the quantity dimension of liquidity. Factor 2 is driven by variables related to the cost and immediacy dimensions of liquidity. The model is acceptable based on the MSA criterion. The overall MSA is 0.74, and the individual MSA numbers varies from a minimum of 0.56 for relative spread to a maximum of 0.82 for the NOK spread.

[Table 13 about here.]

B.4 Robustness of the factor analysis results

To check the robustness of our factor analysis results, we run the models on two sub-samples; one from February 1999 through December 2001 (723 observations) and one from January 2002 through December 2004 (752 observations). For Model B and Model C, the sub-period results

[26] On the other hand, the analytical procedures for performing orthogonal rotation are better developed than the procedures for oblique rotations.

are provided in the two lower panels of table 12 and 13 respectively. Sub-period results for Model A is provided in table 14 below.

[Table 14 about here.]

The number of factors extracted from the three models are identical for all sample periods; that is we get three factors for model A (all variables), one factor for model B (trade based variables), and two factors for model C (order based variables). With the exception of Model C over the first half of the sample period, factor analysis is justified for all models according to the MSA criterion. The estimates of shared variance also seem quite robust to the choice of sample period. Several individual factor loadings do change depending on the sample period, in size and/or in sign. The total variance explained by the factors also varies somewhat across the sample periods (i.e. the numbering of the factors changes).[27] However, as illustrated in table 15, where the relationships between the factors for each sample period are summarized, the overall factor structure is quite stable across sample periods.

[Table 15 about here.]

One first thing to note from table 15 is that the interpretation of the three factors in Model A is similar for all three sample periods: one factor is mainly representing the quantity (or depth) dimension of liquidity, one factor is mainly representing a combination of the quantity and immediacy dimensions, and one factor has significant loadings on variables thought to capture asymmetric information. Note also the high stability of the one factor in the trade based model. For the first half of the sample period, the MSA for the order based model is hardly acceptable (0.55). We therefore disregard this model in the comparison. For the second half of the sample period, the order based factors have a similar interpretation as the order based factors in the full sample version of the model. The table also illustrates that the correlations between the factors are quite robust to the choice of sample period. For all periods (except the first half of the sample for model B), we have that:

- The joint quantity and immediacy factor in Model A has a high positive correlation with the trade based factor and a fairly high negative correlation with the order based factor related to all dimensions.

[27] If we look at Model C for instance, we can see that factor 1 in the full sample model (which is mainly related to quantity) is similar to factor 2 in second half sample model.

- The quantity factor in Model A has a high positive correlation with the order based quantity factor.
- The information related factor in Model A has a high correlation with the order based factor related to all dimensions, though the sign of the correlation coefficient is different for the two sample periods.

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Table 1 Descriptive statistics

The table presents means, medians, standard deviations, and maximum and minimum values for daily cross-sectional averages of our set of liquidity variables. A description of how the liquidity variables are calculated is provided in Appendix A.

	Mean	Median	STD	Max	Min
Trade based measures					
Trade frequency	4494	4434	1779	12030	604
Trading volume	10455041	9352699	5432928	41891165	1397510
Turnover	0.26	0.24	0.12	0.98	0.05
Sec. between trades	2770	2718	738	5126	965
Illiquidity ratio	0.57	0.55	0.20	2.41	0.22
Order based measures					
Frequency of limit orders	4343	4273	1717	11965	734
Depth inner quotes	8893	8623	2268	25304	3811
Tick slope	4893	4546	1673	10830	1822
Price slope	53899	37859	50709	313468	6956
Fraction of limit orders	0.66	0.66	0.03	0.77	0.56
Quoted spread	1.37	1.34	0.34	2.37	0.70
Relative quoted spread	2.28%	2.20 %	0.70 %	6.53 %	0.97 %
Inter-quote time	1096	1071	257	2404	549
Book symmetry	0.06	0.06	0.08	0.28	-0.20

Table 2 Correlation structure

	Tfreq.	Tvol.	Turnover	Ttime	ILR	LO freq.	Depth	Tslope	Pslope	LO frac.	Spread	Rel. Spr.	IQtime
Trade based													
Trading frequency	1.00												
Trading volume	0.62	1.00											
Turnover	0.72	0.67	1.00										
Sec between trades	-0.48	-0.19	-0.49	1.00									
Illiquidity ratio	0.16	0.15	0.02	0.13	1.00								
Order based													
Limit order frequency	0.80	0.62	-0.10	-0.39	-0.10	1.00							
Depth inner quotes	0.03	0.51	-0.16	0.05	-0.16	0.26	1.00						
Tick slope	0.26	0.61	-0.34	-0.17	-0.34	0.54	0.76	1.00					
Price slope	0.11	0.67	0.01	0.08	0.01	0.29	0.62	0.64	1.00				
Limit order fraction	-0.10	0.08	-0.17	0.21	-0.33	0.36	0.26	0.39	0.32	1.00			
Spread	-0.10	-0.41	-0.21	0.11	0.43	-0.50	-0.59	-0.75	-0.55	-0.57	1.00		
Relative Spread	-0.16	0.05	-0.27	0.48	0.68	-0.33	-0.06	-0.35	0.13	-0.16	0.40	1.00	
Inter-quote time	-0.42	-0.20	-0.43	0.76	0.21	-0.48	-0.02	-0.26	0.00	-0.10	0.25	0.49	1.00
Book symmetry	-0.01	-0.20	0.00	-0.15	-0.24	-0.02	-0.15	-0.03	-0.25	-0.06	0.04	-0.29	-0.11

vided in Appendix A

Table 3 Correlation with returns, volatility, and order flow

The table presents the correlation structure of returns, volatility, and net order flow as well as the correlation coefficients between these variables and our set of trade and order based liquidity variables.

	Return	Volatility	Net order flow	|Net order flow|
Return	1.00			
Volatility	-0.03	1.00		
Net order flow	0.64	0.11	1.00	
|Net order flow|	0.06	0.40	0.20	1.00
Trade based measures				
Trade frequency	0.06	0.28	0.15	0.44
Trading volume	0.11	0.25	0.18	0.26
Turnover	0.21	0.17	0.19	0.29
Sec. between trades	-0.08	0.00	-0.02	-0.20
Illiquidity ratio	-0.06	0.96	0.08	0.32
Order based measures				
Frequency of limit orders	0.05	0.00	0.11	0.26
Depth inner quotes	0.11	-0.14	0.05	-0.06
Tick slope	0.16	-0.29	0.09	-0.02
Price slope	0.08	0.02	0.04	0.01
Fraction of limit orders	-0.02	-0.37	-0.01	-0.17
Quoted spread	-0.08	0.40	0.00	0.09
Relative quoted spread	-0.14	0.60	-0.02	0.05
Inter-quote time	-0.12	0.09	-0.06	-0.15
Book symmetry	-0.24	-0.23	-0.29	-0.05

Table 4 Results from the estimation of a common factor model on all liquidity measures

The table presents the main results from a common factor model estimated on 5 trade based liquidity variables and 9 order based liquidity variables. MSA is Kaiser's measure of sampling adequacy. The factors are rotated orthogonally using the Varimax method. Grey cells indicate a factor loading above 0.30.

Model A	**MSA**	**Shared variance**	**Rotated factor loadings Factor 1**	**Factor 2**	**Factor 3**
Trade based measures					
Trade frequency	0.70	0.81	0.00	0.89	0.09
Trading volume	0.72	0.89	0.53	0.67	0.41
Turnover	0.85	0.67	0.15	0.81	0.02
Sec. between trades	0.72	0.63	0.14	-0.67	0.41
Illiquidity ratio	0.76	0.62	-0.34	0.11	0.70
Order based measures					
Frequency of limit orders	0.73	0.67	0.39	0.71	-0.11
Depth inner quotes	0.82	0.58	0.75	0.09	0.12
Tick slope	0.85	0.85	0.85	0.33	-0.12
Price slope	0.80	0.69	0.74	0.14	0.34
Fraction of limit orders	0.61	0.35	0.54	-0.14	-0.19
Quoted spread	0.89	0.79	-0.84	-0.14	0.27
Relative quoted spread	0.74	0.80	-0.18	-0.27	0.84
Inter-quote time	0.75	0.55	-0.03	-0.59	0.45
Book symmetry	0.87	0.15	-0.14	0.01	-0.36
Overall MSA	0.76				
Shared variance explained			3.47	3.45	2.14
% of total shared variance			38	38	24

Table 5 Relationship between factor models

The table presents the correlation matrix between the factor scores from three factor models. The three factors in Model A are extracted from all liquidity variables. The factor in Model B is extracted from trade based liquidity variables only, and the two factors in Model C are extracted from order based liquidity variables only.

	Model B (trade based)	**Model C** (order based)	
	Factor 1	Factor 1	Factor 2
Model A	(quantity, immediacy)	(quantity)	(all dimensions)
Factor 1 (quantity)	0.18	0.96	0.11
Factor 2 (quantity, immediacy)	0.93	0.19	-0.48
Factor 3 (information?)	-0.11	0.14	-0.83

Table 6 Correlation between factor scores, order flow and volatility

The table shows correlations between the three score series for the market-wide factors and volatility, net order flow and absolute net order flow. We also show the correlation of the factors with market returns and relative spread. All correlations are significant at the 1 % level.

	Factor 1	Factor 2	Factor 3
Volatility	-0.34	0.27	**0.70**
Net order flow	0.04	0.17	0.09
|Net order flow|	-0.14	**0.41**	0.18
EW market return	0.09	0.13	-0.06
VW market return	0.07	0.09	-0.04
relative spread	-0.18	-0.28	**0.88**

Table 7 Properties of illiquidity portfolios under different measures of illiquidity

The table presents descriptive statistics for the four betas in the liquidity-adjusted CAPM derived by Acharya and Pedersen [2005]. The betas are estimated based on three different liquidity factors for a set of four portfolios sorted on firm specific illiquidity (c_t^i). The most liquid (illiquid) stocks are sorted into portfolio 1(4). Following Acharya and Pedersen [2005], the beta estimates are multiplied by 100. The table also reports the mean and standard deviation of the illiquidity measure, the mean and standard deviation of the percentage weekly portfolio return, the MCAP of the portfolios in million NOK, and the mean and standard deviation of the percentage relative half-spread.

	Portfolio 1	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**
Order based factor				
β^1	95.2	79.7	67.1	69.1
β^2	0.02	0.02	0.08	0.09
β^3	-1.28	-0.90	-0.68	-0.56
β^4	-1.86	-0.97	-1.46	-3.92
Mean illiquidity (%)	0.21	0.69	1.30	3.34
σ(illiquidity) (%)	0.36	0.20	0.35	0.95
Weekly return (%)	0.62	0.35	0.38	0.61
σ(weekly return) (%)	3.66	3.24	2.72	3.14
MCAP (mill. NOK)	16523	4607	1925	1085
Relative half spread (%)	0.51	0.73	1.21	2.86
σ(relative half spread) (%)	0.16	0.28	0.39	1.15
Trade based factor				
β^1	94.4	79.0	66.6	68.5
β^2	0.41	0.20	0.26	0.59
β^3	-1.44	-1.21	-1.23	-1.26
β^4	-1.54	-1.41	-2.19	-3.50
Mean illiquidity (%)	0.26	0.73	1.32	3.38
σ(illiquidity) (%)	0.63	0.24	0.35	0.85
Weekly return (%)	0.31	0.56	0.51	0.60
σ(weekly return) (%)	3.60	3.06	2.90	3.12
MCAP (mill. NOK)	17179	4182	1874	834
Relative half spread (%)	0.46	0.70	1.24	2.93
σ(relative half spread) (%)	0.20	0.21	0.41	1.15
Amihud factor				
β^1	95.7	80.1	67.5	69.4
β^2	0.04	0.10	0.13	0.51
β^3	-1.00	-1.16	-0.82	-1.86
β^4	-0.28	-0.75	-1.76	-5.23
Mean illiquidity (%)	0.35	0.75	1.25	3.04
σ(illiquidity) (%)	0.12	0.24	0.38	1.14
Weekly return (%)	0.35	0.49	0.53	0.61
σ(weekly return)(%)	3.43	3.21	2.80	3.29
MCAP (mill. NOK)	18124	3238	1723	972
Relative half spread (%)	0.36	0.75	1.25	2.98
σ(relative half spread) (%)	0.08	0.21	0.36	1.19

Table 8 Expected returns and liquidity sorted portfolios

The table reports the estimated coefficients from various specifications of the model,

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1\beta^{1,p} + \lambda^2\beta^{2,p} + \lambda^3\beta^{3,p} + \lambda^4\beta^{4,p} + \lambda \ + beta^{net,p}$$

We use weekly data for the period 1999-2005, and the betas are re-estimated for each year for each portfolio (p). The portfolios are constructed each week based on firms' average liquidity variable the previous week. Panel (a) shows the results when the portfolios are based on the order based factor. In panel (b) the portfolios are constructed based on the trade based factor and in panel (c) the portfolios are based on the Amihud factor. The $\beta^{net,p}$ is when we impose the model implied constraint that the risk premia of the different betas are the same. The estimated models' R^2 are in percentage terms. ** denote that the estimated coefficient is significantly different from zero at the 1% level, and superscript ^{a} in the $E(c_t^p)$ column denotes that the liquidity cost is scaled by the average turnover for all stocks in the sample.

(a) Order based factor

	Constant	$E(c^p)$	β_1^p	β_2^p	β_3^p	β_4^p	β_{net}^p	R^2 (%)
Model 1a	-0.29	0.01^a					0.75	0.15
Model 2a	0.21	-0.12					0.39	0.34
Model 3a	-0.13		0.58					0.09
Model 4a	-0.05	0.01^a	-5.24				5.86	0.34
Model 5a	0.55	-0.13	-6.08				6.26	0.59
Model 6a	0.00		-5.32				5.89	0.33
Model 7a	0.52	0.01^a	0.29	-351.66**	42.29**	-9.94		1.46
Model 8a	1.60**	-0.25**	-0.43	-305.02**	59.63**	-12.85**		2.10

(b) Trade based factor

	Constant	$E(c^p)$	β_1^p	β_2^p	β_3^p	β_4^p	β_{net}^p	R^2 (%)
Model 1b	1.21**	0.01^a					-1.00	0.23
Model 2b	2.51**	-0.26					-2.10**	1.15
Model 3b	0.86		-0.53					0.06
Model 4b	0.31	0.01^a	11.09**				-11.60**	1.50
Model 5b	1.66**	-0.28**	11.67**				-13.34**	2.55
Model 6b	0.37		11.12**				-11.68**	1.53
Model 7b	0.03	0.01^a	-0.58	-53.85	-46.06**	-5.15		3.93
Model 8b	1.12	-0.23**	-1.44**	-70.21	-43.71**	-9.95		4.64

(c) Amihud factor

	Constant	$E(c^p)$	β_1^p	β_2^p	β_3^p	β_4^p	β_{net}^p	R^2 (%)
Model 1c	-1.67**	0.01^a					2.30**	0.98
Model 2c	-1.54**	-0.02					2.21**	0.96
Model 3c	-1.26**		1.83**					0.54
Model 4c	-0.34	0.01^a	-15.25**				16.26**	2.35
Model 5c	-0.33	0.01	-15.23**				16.23**	2.32
Model 6c	-0.30		-15.19**				16.17**	2.31
Model 7c	-0.94	0.01^a	1.82**	27.64	48.60**	-0.04		3.38
Model 8c	-0.76	-0.13	1.73**	68.13	49.23**	-2.04		3.49

Table 9 Monthly returns on portfolios sorted on $cov(c_t^i, c_t^M)$

The table shows monthly returns on portfolios sorted on the covariance between firms specific and market-wide liquidity, $cov(c_t^i, c_t^M)$. etc.

	Portfolio 1	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Diff.** P4-P1	**t-test** (P4-P1=0)
Sorted on $cov(c_t^i, c_t^M)$						
Order based factor	1.58	1.17	1.72	3.12	1.53	3.68
Trade based factor	1.17	0.92	1.92	3.57	2.40	4.99
Amihud factor	1.16	1.04	2.36	3.03	1.87	4.08

Table 10 Factor loadings and risk premia for risk factors constructed from the covariance between firm specific and market-wide liquidity

This table presents the estimated factor loadings and risk premia for risk factors constructed from the covariance between firm specific and market-wide liquidity. Robust t-statistics are written below, in square braces. F1, F2 and F3 denote the Order based factor, the Trade based factor, and the Amihud factor, respectively.

				Risk Factors			
Model	Constant	Market	SMB	HML	F1	F2	F3
Factor loadings							
CAPM and F1	1.50	13.16			-24.68		
	[39.58]	[11.42]			[-6.24]		
CAPM and F2	1.61	15.24				-28.13	
	[45.13]	[14.31]				[-15.06]	
CAPM and F3	1.52	22.42					-58.88
	[29.37]	[13.30]					[-14.04]
CAPM and All Factors	1.69	18.80			-9.70	-15.70	-25.50
	[45.67]	[13.48]			[-1.82]	[-4.37]	[-4.02]
Risk Premia							
CAPM and F1	-0.02			0.00			
	[-11.15]			[5.37]			
CAPM and F2	-0.02				0.00		
	[-11.53]				[7.65]		
CAPM and F3	-0.04					0.01	
	[-10.78]					[10.25]	
CAPM and All Factors	-0.03			0.01	0.01	0.00	
	[-11.97]			[8.20]	[13.41]	[5.60]	

Table 11 Asset pricing tests

This table reports results of asset pricing tests on the Oslo Stock Exchange firms. Robust p-values are in round brackets. The J-test is Hansen's (1982) overidentifying restriction test. HJ-distance refers to the distance metric of Hansen and Jagannathan (1997). Wald-p is the result from a Wald test that the factor coefficients are zero in a linear pricing kernel. The delta-J test of Newey and West (1987) assesses whether the inclusion of HML and SMB improves model fit. The supLM test of Andrews (1993) is a test of parameter stability. * denotes significance at the 1 percentage level.

	Test statistics				
Model	J-test	HJ-distance	Wald-p	delta-J	SupLM
CAPM and F1	17.66	1.44	0.00	94.38	17.68*
	(0.99)	(0.00)	(0.00)	(0.00)	
CAPM and F2	17.33	1.44	0.00	18.66	17.68*
	(0.99)	(0.00)	(0.00)	(0.00)	
CAPM and F3	17.72	1.40	0.00	137.63	17.68*
	(0.99)	(0.00)	(0.00)	(0.00)	
CAPM and All Factors	17.41	1.40	0.00	4.63	22.49*
	(0.99)	(0.00)	(0.00)	(0.10)	

Table 12 Results from a factor analysis on trade based liquidity measures

The table presents main results from a common factor model where the variables set includes trade based liquidity measures only. MSA is Kaiser's measure of sampling adequacy.

Model B	**MSA**	**Shared variance**	**Factor loadings**
Trade frequency	0.74	0.74	0.86
Trading volume	0.70	0.50	0.69
Turnover	0.72	0.81	0.90
Sec. between trades	0.64	0.23	-0.48
Illiquidity ratio	0.40	0.01	0.09
Overall MSA/Shared variance	0.70	2.27	
Sub-period results			
February 1999 - December 2001			
Trade frequency	0.76	0.83	0.91
Trading volume	0.71	0.74	0.86
Turnover	0.79	0.71	0.85
Sec. between trades	0.73	0.31	-0.55
Illiquidity ratio	0.82	0.09	0.30
Overall MSA/Shared variance	0.75	2.68	
January 2002 - December 2004			
Trade frequency	0.74	0.84	0.92
Trading volume	0.64	0.48	0.69
Turnover	0.73	0.82	0.91
Sec. between trades	0.63	0.13	-0.36
Illiquidity ratio	0.49	0.00	0.04
Overall MSA/Shared variance	0.68	2.28	

Table 13 Results from a factor analysis on order based liquidity measures

The table presents the main results from a factor model where the variable set includes order based liquidity measures only. MSA is the Kaiser's measure of sampling adequacy. The factors are rotated according to an orthogonal rotation method (Varimax in SAS).

Model C	MSA	Shared variance	Rotated factor loadings Factor 1	Factor 2
Frequency of limit orders	0.78	0.43	0.49	-0.43
Depth inner quotes	0.74	0.61	0.77	0.13
Tick slope	0.74	0.85	0.90	-0.22
Price slope	0.77	0.69	0.78	0.28
Fraction of limit orders	0.78	0.25	0.47	-0.15
Quoted spread	0.82	0.75	-0.82	0.30
Relative quoted spread	0.56	0.65	-0.16	0.79
Inter-quote time	0.71	0.39	-0.16	0.60
Book symmetry	0.70	0.13	-0.17	-0.32
Overall MSA/Shared variance	0.74	4.75		
Shared variance explained			3.22	1.53
% of total shared variance			68	32

Sub-period results

February 1999 - December 2001	MSA	Shared variance	Factor 1	Factor 2
Frequency of limit orders	0.43	0.26	0.29	-0.42
Depth inner quotes	0.51	0.42	0.62	0.16
Tick slope	0.57	0.84	0.89	-0.20
Price slope	0.74	0.53	0.71	-0.18
Fraction of limit orders	0.56	0.15	0.28	0.26
Quoted spread	0.68	0.39	-0.62	-0.01
Relative quoted spread	0.27	0.10	0.00	0.31
Inter-quote time	0.53	0.85	-0.16	0.91
Book symmetry	0.59	0.02	-0.17	-0.05
Overall MSA/Shared variance	0.55	3.55		
Shared variance explained			2.28	1.27
% of total shared variance			64	36

January 2002 - December 2004	MSA	Shared variance	Factor 1	Factor 2
Frequency of limit orders	0.82	0.32	-0.57	0.04
Depth inner quotes	0.69	0.62	-0.08	0.78
Tick slope	0.64	0.90	-0.56	0.77
Price slope	0.65	0.55	0.28	0.68
Fraction of limit orders	0.77	0.16	-0.21	-0.34
Quoted spread	0.79	0.53	0.70	-0.19
Relative quoted spread	0.69	0.90	0.93	0.18
Inter-quote time	0.85	0.47	0.65	0.17
Book symmetry	0.86	0.21	-0.41	-0.19
Overall MSA/Shared variance	0.73	4.64		
Shared variance explained			2.71	1.92
% of total shared variance			58	42

Table 14 Factor analysis results from sub-period samples - Model A

The table presents the main results from running a factor model on all liquidity variables over two sub-periods of the data sample. MSA is the Kaiser's measure of sampling adequacy. The factors are rotated according to an orthogonal rotation method (Varimax in SAS).

Model A		**Shared**	**Rotated factor loadings**		
Sub-period results	**MSA**	**variance**	**Factor 1**	**Factor 2**	**Factor 3**
February 1999 - December 2001					
Trade based measures					
Trade frequency	0.70	0.89	0.94	0.02	0.12
Trading volume	0.87	0.76	0.84	0.18	0.20
Turnover	0.79	0.64	0.79	0.00	-0.15
Sec. between trades	0.73	0.76	-0.68	0.19	0.50
Illiquidity ratio	0.71	0.49	0.33	-0.31	0.54
Order based measures					
Frequency of limit orders	0.66	0.72	0.76	0.24	0.25
Depth inner quotes	0.61	0.41	-0.14	0.62	0.05
Tick slope	0.64	0.76	0.19	0.86	-0.02
Price slope	0.74	0.56	0.29	0.67	0.15
Fraction of limit orders	0.71	0.29	-0.30	0.29	0.34
Quoted spread	0.74	0.46	0.06	-0.67	0.06
Relative quoted spread	0.74	0.67	-0.17	-0.13	0.80
Inter-quote time	0.72	0.53	-0.61	0.15	0.37
Book symmetry	0.72	0.04	-0.04	-0.08	-0.19
Overall MSA/shared variance	0.71	8.03			
Shared variance explained			4.00	2.39	1.64
% of total shared variance			50	30	20
January 2002 - December 2004					
Trade based measures					
Trade frequency	0.67	0.92	0.00	0.94	0.21
Trading volume	0.81	0.87	0.37	0.57	0.64
Turnover	0.89	0.75	-0.10	0.76	0.41
Sec. between trades	0.78	0.50	0.48	-0.52	0.06
Illiquidity ratio	0.85	0.68	0.83	0.03	0.04
Order based measures					
Frequency of limit orders	0.66	0.75	-0.28	0.82	0.02
Depth inner quotes	0.78	0.57	0.00	0.06	0.76
Tick slope	0.78	0.86	-0.41	0.28	0.80
Price slope	0.75	0.58	0.36	-0.05	0.67
Fraction of limit orders	0.48	0.22	-0.31	-0.16	-0.31
Quoted spread	0.89	0.57	-0.68	-0.18	-0.28
Relative quoted spread	0.84	0.90	0.90	-0.28	0.09
Inter-quote time	0.82	0.55	0.55	-0.50	0.18
Book symmetry	0.91	0.23	-0.46	0.07	-0.13
Overall MSA/shared variance	0.77	9.00			
Shared variance explained			3.32	3.16	2.52
% of total shared variance			37	35	28

Table 15 Relationship between factor models under different sample periods

The table presents the correlation matrix between the factor scores from three factor models estimated over three different sample periods. Model A is estimated from all liquidity variables, model B is estimated from trade based liquidity variables only, and model C is estimated from order based liquidity variables only. The top panel shows the results from estimating the models over the period from February 1999 through December 2001, while the mid panel shows the results from estimating the models over the period from January 2002 through December 2004. Results from estimating the models over the full sample period (February 1999-December 2004) is reproduced from table 5 in the bottom panel.

	Model B (trade based) Factor 1	Model C (order based) Factor 1	Factor 2
Feb1999-Dec2001			
Model A	(quantity, immediacy)	(quantity, spread)	(quantity, immediacy)
1 (quantity, immediacy)	0.97	0.08	-0.66
2 (quantity)	0.05	0.97	-0.02
3 (information?)	0.05	0.14	0.40
Jan2002-Dec2004			
Model A	(quantity, immediacy)	(all dimensions)	(quantity)
1 (information?)	0.02	0.90	0.10
2 (quantity, immediacy)	0.91	-0.38	0.08
3 (quantity)	0.35	-0.12	0.97
Feb1999-Dec2004			
Model A	(quantity, immediacy)	(quantity)	(all dimensions)
1 (quantity)	0.18	0.96	0.11
2 (quantity, immediacy)	0.93	0.19	-0.48
3 (information?)	-0.11	0.14	-0.83