

The pricing of systematic liquidity risk: Empirical evidence from the US stock market

Rajna Gibson ^{a,*}, Nicolas Mougeot ^{b,1}

^a *ISB – Swiss Banking Institute, University of Zurich, 14 Plattenstrasse, Zurich 8032, Switzerland*

^b *European Quantitative Research, Citigroup Smith Barney, Citigroup Center, Canada Square, Canary Wharf, London, E14 5LB, UK*

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Abstract

In this study, we examine whether aggregate market liquidity risk is priced in the US stock market. We define a bivariate Garch (1,1)-in-mean specification for the market portfolio excess returns and the changes in the standardized number of shares in the S&P 500 Index, the aggregate market liquidity proxy. The findings, based on monthly data, suggest that systematic liquidity risk is priced in the US over the period January 1973–December 1997. The liquidity premium represents a non-negligible, negative and time-varying component of the total market risk premium whose magnitude is not influenced by the October'87 Crash.

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1. Introduction

Liquidity is a fundamental concept in finance which can, broadly speaking, be defined as the time and cost which are associated with the liquidation (or purchase) of a given quantity of financial securities. Liquidity thus refers to both the time and costs associated with the transformation of a given position into cash and vice versa. Typically, continuous-time arbitrage or equilibrium asset pricing models ignore liquidity since the cost and time required to transfer financial wealth into cash is assumed to

* Corresponding author. Tel.: +41-1-634-29-69; fax: +41-1-634-49-03.

E-mail addresses: rgibson@isb.unizh.ch (R. Gibson), nicolas.mougeot@citigroup.com (N. Mougeot).

¹ Tel.: +44-20-7986-4247.

be nil and since trading is often ruled out by these equilibrium asset pricing models. Yet, in practice, recent financial crises (such as in Asia or in Russia) and the debacle of the LTCM hedge fund suggest that at times of tight credit and market conditions, liquidity can decline and even temporarily dry out. This leads investors to aggressively bid for the safest, that is, the most liquid securities, which raises their price relative to the one of their less liquid counterparts. **If market liquidity evolves randomly, securities or portfolios that covary more with liquidity, should offer a lower liquidity risk premium. We ask ourselves whether market liquidity risk is priced and whether the omission of stochastic market liquidity shocks may explain the market risk premium's lack of significance reported in former empirical studies (see Table 1 in Scruggs (1998)).** We attempt to test the latter conjecture and to further characterize the importance, magnitude and variations of the systematic liquidity premium as a component of the total expected excess market rate of return.

We focus on a broader definition of systematic liquidity in order to examine whether long term – in our case, monthly – random movements in market liquidity affect stock prices to the extent that their returns covary with changes in market liquidity. Such a relationship is often implicitly assumed. For instance, when used to explain the small firms effect, or to justify the higher expected returns of less liquid financial instruments such as hedge funds.

Recently, a number of studies have examined the presence of commonality in individual stocks' liquidity measures. Hasbrouck and Seppi (1999) look at the 30 constituent stocks from the Dow Jones Industrial Index and conclude, on the basis of principal component and canonical correlation analyses, that the source of commonality in intra-daily liquidity measures for these stocks is rather small. Chordia et al. (2000) reach a distinct conclusion however after examining the sources of commonality in the changes of several daily liquidity measures for 1169 US stocks during the year 1992. Using a market model for liquidity, they find that common market and industry influences on individual stock's liquidity measures such as their quoted spreads or depth are significant and material. In particular, they find that a stock's bid and ask spread is negatively related to the aggregate level of market trading. They interpret this result as being consistent with a diminution in inventory risk resulting from greater market trading. Their findings are however less supportive of common factors driving asymmetric information based stock trading. Thus, their results can explain common liquidity factors influence on stocks' expected returns through increased average trading costs. Huberman and Halka (1999) also explore the commonality in liquidity, using the depth as well as the bid–ask spread as proxies for the liquidity of 240 US traded stocks. Their findings are similar to the results of Chordia et al. (2000), and they attribute commonality in stocks' liquidity to the presence of noise traders. These studies have left open the question as to whether illiquidity is a systematic risk factor, in which case stocks that are more sensitive to unexpected market illiquidity shocks, should offer higher expected returns. An exception is to be found in Pastor and Stambaugh (2001) who introduce a market-wide liquidity measure and show that cross-sectional expected stock returns are related to fluctuations in aggregate liquidity. Along the same lines, the recent study by Amihud (2002) introduces a new measure of illiquidity defined as the ratio of a stock's

absolute daily return over its daily trading volume (in dollars) and applies it to NYSE stocks traded during the period 1964–1997. He tests whether expected market liquidity has a positive effect on *ex ante* stock excess returns and whether unexpected market illiquidity has a negative effect on contemporaneous stock returns. The empirical results support the conjectured hypotheses.

By examining whether aggregate market liquidity risk is priced in a time-series framework, we intend to complement the latter stream of recent literature on commonality in stocks' liquidity risk measures. For that purpose, we examine the significance and magnitude of systematic liquidity risk pricing for an actively traded well-diversified US stock portfolio, that is the S&P 500 stock market index.

Two important difficulties are related with the concept of aggregate market liquidity risk. First, one needs to define a proxy for the state variable describing aggregate market liquidity and second to specify a joint stochastic process for the latter and the excess returns of the market portfolio. While several candidate variables have emerged in the market microstructure literature to measure liquidity (for instance, Kyle's λ (1985), the bid–ask spread, the effective spread or the market depth), they are essentially intended as proxies of the liquidity of individual stocks. Furthermore, these measures are primarily suited to study the cross-sectional and time-series determinants of liquidity over short-term horizons. We need a proxy for longer horizons market-wide liquidity shocks. For that purpose, we chose to define the market liquidity as the number of traded shares in the S&P 500 Index during a month. The changes in the state variable are represented by the monthly relative changes in the number of traded shares in the S&P 500 Index. Recent empirical evidence tends to support the choice of the latter liquidity risk proxy. Indeed, Chordia et al. (2000) findings suggest that the number of shares traded may be considered as a measure capturing the sources of commonality in market-wide liquidity arising from aggregate inventory risk. They also conclude that greater market-wide volume leads to reduced bid–ask spread measures.

We further assume that the market excess returns and the liquidity state variable jointly follow a bivariate Garch (1,1)-in-mean process with possibly time-varying unitary market and liquidity risk premia in the general specification of the model. In the latter, the unitary liquidity and market risk premia are driven by a set of instrumental variables that capture business cycles effects on investors' risk aversion. The model is tested in its general form and in various nested specifications over the period January 1973–December 1997.

The structure of the paper is the following: Section 2 provides a heuristic justification for the sign of the liquidity risk premium coefficient. Section 3 further describes the methodology and the data used to conduct the empirical tests. Section 4 discusses the main empirical results on the pricing of liquidity risk in the S&P 500 excess returns both under a constant and a time-varying specification of the unitary liquidity risk premium coefficient. Section 5 examines the stability of the results and the role of the October'87 Crash on the pricing of systematic liquidity risk. Section 6 discusses the economic significance of the systematic liquidity risk premium. In Section 7, we conclude by emphasizing the main findings, limitations and research questions raised by this study.

2. The sign of the systematic liquidity risk premium

This section introduces a “heuristic” framework in which the sign of the systematic liquidity risk premium is discussed. Unfortunately, although the relationship between a stock’s expected return and its liquidity measures is well documented in the market microstructure literature, there is no theoretical model that is directly applicable to study the effect of aggregate market liquidity risk on expected stock returns. In the spirit of Chen et al. (1986), we assume that a pre-specified two-factor model is driving securities returns in a continuous-time economy. We assume that these two factors in the economy are the market factor and the systematic liquidity factor. In that context, we are interested in testing whether the following equation characterizing stock market expected excess returns is satisfied:

$$E_{t-1}(r_{M,t}) = \lambda_{M,t}\sigma_{M,t}^2 + \lambda_{ML,t}\sigma_{ML,t}, \quad (1)$$

where $r_{M,t}$ denotes the stock market excess return, $\sigma_{M,t}^2$ its variance and $\sigma_{ML,t}$ is the instantaneous covariance between the excess market return and the liquidity of the market. $\lambda_{M,t}$ is the unitary price of market risk while $\lambda_{ML,t}$ is the unitary price of liquidity risk.

Assuming that $\sigma_{ML,t}$ is positive (which will be discussed in the empirical section), we thus conjecture that the sign of the systematic liquidity risk premium $\lambda_{ML,t}$ should be negative. This means that if investors value future immediacy, they should raise the current prices of securities (and portfolios) that covary more with market liquidity and thus offer them an improved investment opportunity set (in terms of their higher cash proceeds from their higher returns) in the future.

Notice that in his study, Amihud (2002) finds a positive relationship between expected excess stock returns and the expected illiquidity of the stock market. In our study, the liquidity proxy is defined as the standardized number of shares traded in the S&P 500 Index during a month. It is negatively related to the new measure of “illiquidity” of Amihud, defined as the ratio of a stock’s absolute daily returns over its daily trading volume. A higher value of our proxy implies greater market liquidity rather than greater market illiquidity. Thus, the negative relationship between expected excess market returns and the expected liquidity of the stock market conjectured in our study is consistent with the positive one obtained by Amihud.

3. Methodology and data analysis

3.1. The bivariate Garch (1,1)-in-mean excess return generating model

In order to empirically assess the significance and the sign of the liquidity risk premium, we assume that the excess market returns $r_{M,t}$ and the relative changes of the market liquidity proxy $r_{L,t}$ follow a bivariate Garch (1,1)-in-mean, of the form:

$$\begin{aligned} r_{M,t} &= \mu_M + \lambda_M \hat{\sigma}_{M,t}^2 + \lambda_{ML} \hat{\sigma}_{ML,t} + \varepsilon_{1,t}, \\ r_{L,t} &= \mu_L + \varepsilon_{2,t}, \\ H_t &= \Omega + A' \varepsilon_{t-1} \varepsilon_{t-1}' A + B' H_{t-1} B \end{aligned} \quad (2)$$

with

$$H_t = \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{pmatrix} \quad \text{and} \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix},$$

H_t is the variance–covariance matrix of the excess market returns and of the market liquidity changes. A , B and Ω are constrained to be constant symmetric matrices. These specifications are a compromise between the need to rely on a very general model and to maintain a parsimonious representation. Increasing the number of parameters tends to increase the difficulties associated with the empirical estimation. A constant μ_M is often included in this equation, in order to take into account market imperfections, such as transaction costs. λ_M is the unitary market risk premium and λ_{ML} , the unitary liquidity risk premium. The quadratic form of the volatility process H_t ensures positive values for the volatility of the excess market returns and market liquidity changes. Contrarily to the E-Garch specification for the conditional variance retained by Scruggs (1998), we model the covariance as a free variable, by choosing the BEKK variance specification of Engle and Kroner (1995). While in the E-Garch specification, the covariance depends directly on the variances through a constant correlation, the BEKK model does not assume any specific *ex ante* relation between the variances and the covariances.

This general model allows us to test various nested models. In particular, we test a bivariate Garch (1,1)-in-mean with a time-varying liquidity premium.

3.2. Estimation procedure

All the models proposed in Section 2 are conditionally heteroskedastic. Thus, the quasi-maximum likelihood (QML) method of Bollerslev and Wooldridge (1992) seems appropriate to estimate the parameters. The method assumes that the residuals are normally distributed. Under the assumption of conditional normality, the log-likelihood can be written as follows:

$$\ln L(\theta) = -\frac{T}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \ln(\det(H_t(\theta))) - \frac{1}{2} \sum_{t=1}^T \varepsilon_t'(\theta) H_t^{-1}(\theta) \varepsilon_t(\theta), \quad (3)$$

where θ denotes the parameters to be estimated. If the first two moments are correctly specified, the estimators given by QML are still consistent even if the hypothesis of normality of the residuals is not satisfied due to the skewness and/or to the leptokurtosis. The residuals have to be corrected by using the asymptotic robust residuals proposed by Bollerslev and Wooldridge, in order to calculate robust t -statistics. We use the Berndt et al. (1974) algorithm to estimate the different specifications, assuming that the initial residuals are equal to zero and taking the sample variances and covariances as initial values for h_{11} , h_{12} and h_{22} .

3.3. Data description

We use monthly data covering the period from January 1973–December 1997, for a total of 300 observations. The market excess return is calculated as the difference between the continuously compounded return of the Standard and Poors 500 composite stock index (S&P 500) and the yield on a one-month treasury bill. The data has been downloaded from Datastream. The choice of the liquidity proxy is rather delicate. On the one side, there exist many liquidity measures at the level of an individual stock, and one could think about extending them at the level of the market portfolio. For instance, one could construct an equally – or market value – weighted aggregate bid–ask spread or market depth measure. On the other side, we are interested in a measure of long run systematic changes in market liquidity, in contrast to a market microstructure based definition of intra-daily liquidity measures for individual stocks. Thus, we need a liquidity risk proxy that accounts for systematic shocks or commonalities in liquidity risk induced by aggregate inventory shocks rather than a measure of short-term illiquidity induced by asymmetric information. This already rules out several classical market microstructure liquidity measures that also account for a stock's illiquidity component associated with asymmetric information. This also suggests that we should use a market portfolio-based liquidity measure as opposed to a liquidity measure defined at the individual stock level. Indeed, we wish to abstract from diversifiable liquidity risk (see Chordia et al., 2000).

Based on the above considerations, we decided to use the monthly relative changes in the number of shares traded in the S&P 500 as a proxy for the changes in market liquidity. The monthly number of shares traded is standardized by the number of index constituents stocks to take into account changes in the composition of the S&P 500 stock index.

Let us notice that we also examined but decided not to use two other potential proxies for systematic liquidity: the dollar monthly volume of S&P 500 shares traded and a more complex proxy to measure the price impact of a given trade. This second measure, very close to the one defined by Amihud (2002) at the single stock level, was defined as the monthly excess market return, divided by the total number of shares traded during the month. We decided not to use these two measures in light of data availability regarding the former measure and of the multi-collinearity problem both measures of systematic liquidity would have induced when testing Eq. (2). Indeed, both measures are functions of the market excess returns (or its price component) and we believe that this feature may affect the significance of the statistical results.

Descriptive data on the excess monthly returns of the S&P 500 stock index and on aggregate liquidity monthly relative changes are reported in Table 1. We can observe, based on the results of the Jarque–Bera test, that the two series are not normally distributed. The augmented Dickey–Fuller test (of order 4) shows that the series are stationary.

Since the series are not normally distributed, we test whether they display Arch effects. Arch effects are present in series exhibiting leptokurtosis, i.e. fatter tails than a Gaussian distribution. To test for Arch effects, we regress the market excess returns and the aggregate changes in liquidity on a constant and then apply the Lagrange

Table 1

Descriptive statistics of the market excess returns and liquidity relative changes

Series	Mean	Standard deviation	Jarque–Bera	Augmented Dickey–Fuller
Market excess returns	0.0071	0.0431	251.4090	–7.1398***
Liquidity returns	0.0129	0.1589	108.8094	–10.2444***

This table reports basic monthly statistics for the excess monthly returns on the S&P 500 composite index and the liquidity relative changes. The liquidity variable is defined as the dollar monthly number of shares traded in the S&P 500 Index standardized by the number of index constituent stocks. The sample period extends from January 1973–December 1997. (***) indicates that the test is significant at 1% level. The critical levels are –3.4543 for 1%, –2.8715 for 5% and –2.5720 for 10%.

Table 2

Arch tests

Series	Obs \times R^2	Prob.
Market excess returns	7.246	0.007
Liquidity returns	0.027	0.869

The table reports the Lagrange multiplier test for the market excess returns and the liquidity relative changes. The liquidity variable is defined as the dollar monthly number of shares traded in the S&P 500 Index standardized by the number of index constituent stocks. The sample period extends from January 1973–December 1997. The test statistic is computed as the number of observations times the R -square from the test regression, which is defined as the squared residuals regressed on a constant and lagged squared residuals up to order p . The test asymptotically follows a Chi-square (p). The above results are calculated with $p = 4$.

multiplier test on the residuals while allowing for four lags. The results of the Lagrange multiplier test are reported in Table 2.

The market excess returns display Arch effects whereas the aggregate liquidity relative changes do not. The presence of Arch effects in the market excess returns suggests that the model takes into account well-documented empirical characteristics of stock market index excess returns. However, the absence of Arch effects in the aggregate liquidity relative changes does not reject the econometric specification since we are primarily interested in the covariance between changes in aggregate liquidity and market excess returns. We also test for the presence of a trend in the liquidity relative changes that may affect our results. We therefore perform a simple linear regression of the relative changes in liquidity on a constant plus a trend. Following the results reported in Table 3, we reject the presence of a trend in the aggregate liquidity proxy.²

Fig. 1 exhibits the moving average covariance and correlation between the market excess returns and the market liquidity relative changes. The numbers reported in Fig. 1 are calculated using the last 100 observations. The moving average covariance between the two series is almost always positive but appears to be downward sloping, gradually tending towards zero. We will test if this feature of the data has an

² We have also performed a regression without a constant and obtained similar insignificant results.

Table 3

Linear regression of the liquidity relative changes on a constant and a trend

Variable	Coefficient
Constant	0.006 (0.26)
Trend	9E-05 (0.74)
Adjusted R^2	0.0018
Durbin–Watson stat.	2.28

The table shows the results of a linear regression of the liquidity relative changes on a constant and a trend. The liquidity variable is defined as the dollar monthly number of shares traded in the S&P 500 Index standardized by the number of index constituent stocks. The sample period is from January 1973–December 1997.

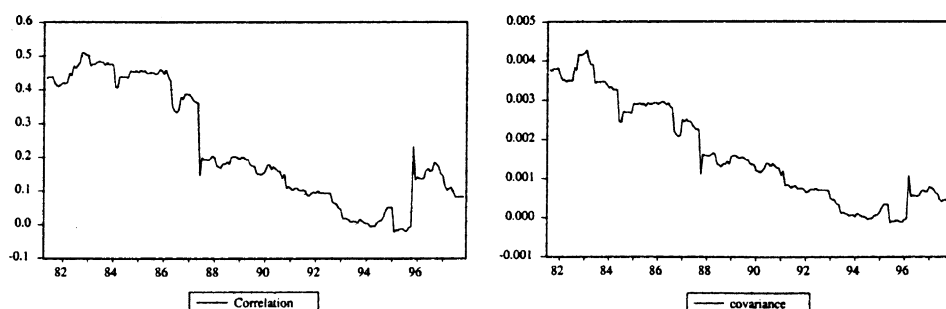


Fig. 1. Moving average covariance and moving average correlation between market excess returns and aggregate liquidity changes.

impact on the results by testing the bivariate Garch (1,1) model over two different sub-sample periods. Since the estimates reproduced in Fig. 1 are calculated using a moving window of 100 months, the two jumps correspond to the two dates when the October'87 market crash event respectively enters and exits from the moving window. This reduces the meaningfulness of the moving average variance or covariance estimates analysis since they depend upon the chosen length of the moving window. Indeed, the second jump is purely artificial and we therefore examine, in Section 4, the conditional variances and covariances that naturally do not exhibit such patterns.

Let us finally describe the three instrumental variables which were chosen, to characterize the evolution of the time-varying unitary liquidity and market risk premia. We have selected three financial as well as non-financial instrumental variables. Indeed, many empirical studies which consider a time-varying risk premium, use financial instrumental variables such as the default premium, the slope of the term structure of interest rate, the dividend yield or the variation of the 1-month T-Bill rate (see De Santis and Gerard, 1997; Dumas and Solnik, 1995) but other macroeco-

conomic variables related to the business cycle also represent candidates to model the evolution of the risk premia. Dumas (1994), for instance, employs several economic indices provided by the NBER and the OECD to capture the market risk premium evolution over time.

We define as our first instrumental variable, the Experimental Recession Index (Rec), provided by NBER and calculated regularly by Stock and Watson (1989). The Experimental Recession Index is an estimate of the probability that the economy will be in a recession phase six months from the computation date of the index. It provides an estimation of agents' beliefs on the future state of the economy. A value of 0.40 means that there is a probability of 40% that the economy enters in recession six months later. The index is computed using the following variables: (i) industrial production, (ii) real personal income total, less transfer payments, (iii) real manufacturing and trade sales total, (iv) total employee-hours in non-agricultural establishments, (v) housing authorizations, (vi) real manufacturers unfilled orders: durable goods industries (smoothed), (vii) trade-weighted index of nominal exchange rates between the US and the UK, Germany, France, Italy, and Japan (smoothed), (viii) number of people working part-time in non-agricultural industries because of slack work (smoothed), (ix) yield on a constant-maturity portfolio of 10-year US Treasury bonds (smoothed), (x) spread between the interest rate on 3-month commercial paper and the interest rate on 3-month US Treasury bills and (xi) spread between the yield on constant-maturity portfolio of 10-year US Treasury bonds and the yield on 1-year US Treasury bonds. Hence, the information contained in this recession index is parsimoniously synthesized while economically meaningful.

The second instrumental variable is the variation of the default premium (VarDef), which is itself calculated as the annualized yield-to-maturity of the Moody's BAA index in excess of the 1-month T-bill rate. The level of the default premium is often taken as an instrumental variable (see De Santis and Gerard (1997), for instance) but following an augmented Dickey–Fuller unit root test, this series appears to be non-stationary. Hence, we choose to use its first variation instead of the default premium itself. The third instrumental variable is the monthly change in the slope of the term structure of interest rates (VarTerm). The term structure slope is calculated as the difference between the yield-to-maturity on long-term US government bonds (with maturities over 10 years) and the 1-month T-bill rate.

Table 4 reports the results of the augmented Dickey–Fuller tests for each instrumental variable. It shows that the Experimental Recession Index, the variation of the

Table 4
Stationarity tests for the instrumental variables

Series	Augmented Dickey–Fuller
Experimental Recession Index	–3.663***
Change in the default premium	–8.323***
Change in the slope of the term structure	–9.533***

In the table above (***) indicate that the test is significant at 1% level. The levels of significance are –3.4543 for 1%, –2.8715 for 5%, and –2.5720 for 10%. The tests are performed based on monthly data for each of the three instrumental variables over the period January 1973–December 1997.

term structure and the variation of the default premium are all stationary at the 1% confidence level.

4. Is market liquidity risk priced?

4.1. Empirical evidence with constant market and liquidity risk premia

The first empirical model is based on the specification stated in Eq. (1), translated into a BEKK formulation, as given by Eq. (2) with constant unitary market and liquidity risk premia:

$$\begin{aligned} r_{M,t} &= \mu_M + \lambda_M \hat{\sigma}_{M,t}^2 + \lambda_{ML} \hat{\sigma}_{ML,t} + \varepsilon_{1,t}, \\ r_{L,t} &= \mu_L + \varepsilon_{2,t}, \\ H_t &= \Omega + A' \varepsilon_{t-1} \varepsilon'_{t-1} A + B' H_{t-1} B \end{aligned} \quad (4)$$

with

$$H_t = \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{pmatrix} \quad \text{and} \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}.$$

The results for the mean equation are reported in Table 5.³

Looking at the parameters of the bivariate Garch (1,1)-in-mean excess return generating model displayed in Table 5, we can see that the estimated liquidity risk premium λ_{ML} is significantly negative. Its sign corroborates the theoretical analysis provided in Section 2. Risk averse agents require lower expected market returns for securities whose returns covary positively with our liquidity proxy variable. This proxy variable is negatively related to the new measure of “illiquidity” of Amihud (2002), defined as the ratio of a stock’s absolute daily returns over its daily trading volume. A higher value of $\hat{\sigma}_{ML,t}$ implies greater covariance of the market excess returns with market “liquidity” rather than with market “illiquidity”. Thus, the negative relationship between expected excess market returns and the expected liquidity of the stock market conjectured in our study is consistent with the positive one obtained by Amihud.

The results also show that the introduction of the liquidity proxy as a state variable fails to yield a significant and positive value for the unitary market risk premium λ_M . The estimate of λ_M is positive but insignificant. This result is consistent with the one found by French et al. (1987) when testing for a positive relationship between the expected risk premium of common stocks and their predicted variance. Applying a univariate Garch (1,2)-in-mean to the monthly CRSP value-weighted index excess returns, they found a positive but insignificant relation between the excess market return and its variance. They also call for analysing whether other variables might

³ The estimates of the conditional variance equation parameters for all the models tested in the paper can be obtained from the authors upon request. Let us note that they all imply a stationary process for the corresponding variance–covariance matrices.

Table 5

Bivariate Garch (1,1)-in-mean excess market return model with constant risk premia

Parameter	General model
μ_M	0.013* (1.48)
λ_M	3.325 (0.88)
λ_{ML}	-7.04*** (-2.96)
μ_L	0.013* (1.48)
Log-likelihood	1232.584

$$r_{M,t} = \mu_M + \lambda_M \hat{\sigma}_{M,t}^2 + \lambda_{ML} \hat{\sigma}_{ML,t} + \varepsilon_{1,t},$$

$$r_{L,t} = \mu_L + \varepsilon_{2,t},$$

$$H_t = \Omega + A' \varepsilon_{t-1} \varepsilon_{t-1}' A + B' H_{t-1} B.$$

In this table (***), (**), (*) indicate that the test is significant at respectively 1%, 5%, and 10% level. The robust *t*-statistics are given in parentheses. $r_{M,t}$ is defined as the monthly excess return on the S&P 500 composite index and $r_{L,t}$ is defined as the monthly relative change in the liquidity state variable. The liquidity variable is defined as the dollar monthly number of shares traded in the S&P 500 Index standardized by the number of index constituent stocks. The sample period is from January 1973–December 1997.

affect the expected market risk premium. The lack of significance of the positive relationship between the expected risk premium and volatility (or variance) has since then been widely discussed but there is no consensus regarding its explanation. Scruggs (1998, Table 1) provides a summary of previous empirical studies and demonstrates that, depending on the database, the sample period and the introduction of a state variable characterizing changes in the investment opportunity set, the sign and significance of the unitary market risk premium can be substantially altered.

Fig. 2 displays the evolution of the expected market excess return as provided by the coefficient estimates reported in Table 5. The mean of the monthly series is 9.22E-04, significantly different from zero,⁴ and corresponds to an average annualized return of 1.112%. What is more puzzling is that the expected market excess return is often negative. However, such a trend in the time-series evolution of the US market expected excess returns was also reported in other empirical studies (Scruggs (1998), for instance finds negative excess market returns but of a smaller magnitude).

Fig. 3 displays the evolution of the conditional variances of the aggregate liquidity proxy relative changes and the market excess returns and their conditional covariance and correlation. The plot of the conditional variance of the excess market returns exhibits several distinct periods of volatility clustering, i.e. periods when the conditional variance is high and persistent. The pattern of the variance of the relative

⁴ It is significant when we test it by a simple *t*-test, given by $(\sqrt{N}(\bar{x} - m)/s)$, where \bar{x} is the sample mean, s is the sample standard deviation and N the number of observations. The resulting *t*-stat is 3.95.

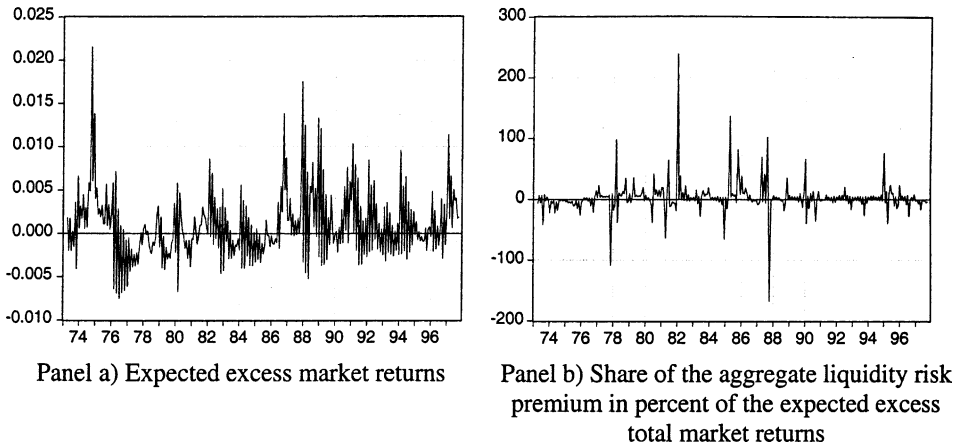


Fig. 2. Panel (a) displays the evolution of the expected excess market returns generated by the bivariate Garch (1,1)-in-mean excess market return model with constant risk premia and Panel (b) displays the variations of the market liquidity risk premium expressed as a fraction of the total expected excess market returns (i.e. $(\lambda_{ML}\hat{\sigma}_{ML,t}/r_{M,t})$).

changes in the aggregate liquidity proxy variance appears at first sight perplexing, compared with the other series. In fact, it is consistent with the previous results from the test for Arch effects reported in Section 3. We found that the liquidity proxy does not display Arch effects. Thus, it is not surprising to see that it displays a monotonically decreasing conditional variance. The conditional correlation and covariance are everywhere positive. The conditional covariance is slightly downward sloping and displays a strong decay only at the time of the October'87 market crash, when the stock market return had fallen while the systematic liquidity proxy increased.

4.2. Introducing time-varying liquidity and market risk premia

4.2.1. Time-varying liquidity risk premium

In this section, we relax the assumption of a constant unitary liquidity risk premium. We allow the liquidity risk premium to vary with the three instrumental variables previously described, i.e. the Experimental Recession Index (Rec), the variation of the slope of the term structure (VarTerm) and the variation of the default premium (VarDef). Since no theoretical model has so far offered any guidance relative to the sign of the market liquidity risk premium, we do not impose a functional form that would constrain its sign. Instead, we assume a linear relation between the liquidity risk premium and the instrumental variables plus a constant term. Formally, the model tested can be written as follows:

$$\begin{aligned}
 r_{M,t} &= \mu_M + \lambda_M \hat{\sigma}_{M,t}^2 + \lambda_{ML,t} \hat{\sigma}_{ML,t} + \varepsilon_{1,t}, \\
 r_{L,t} &= \mu_L + \varepsilon_{2,t}, \\
 H_t &= \Omega + A' \varepsilon_{t-1} \varepsilon'_{t-1} A + B' H_{t-1} B
 \end{aligned} \tag{5}$$

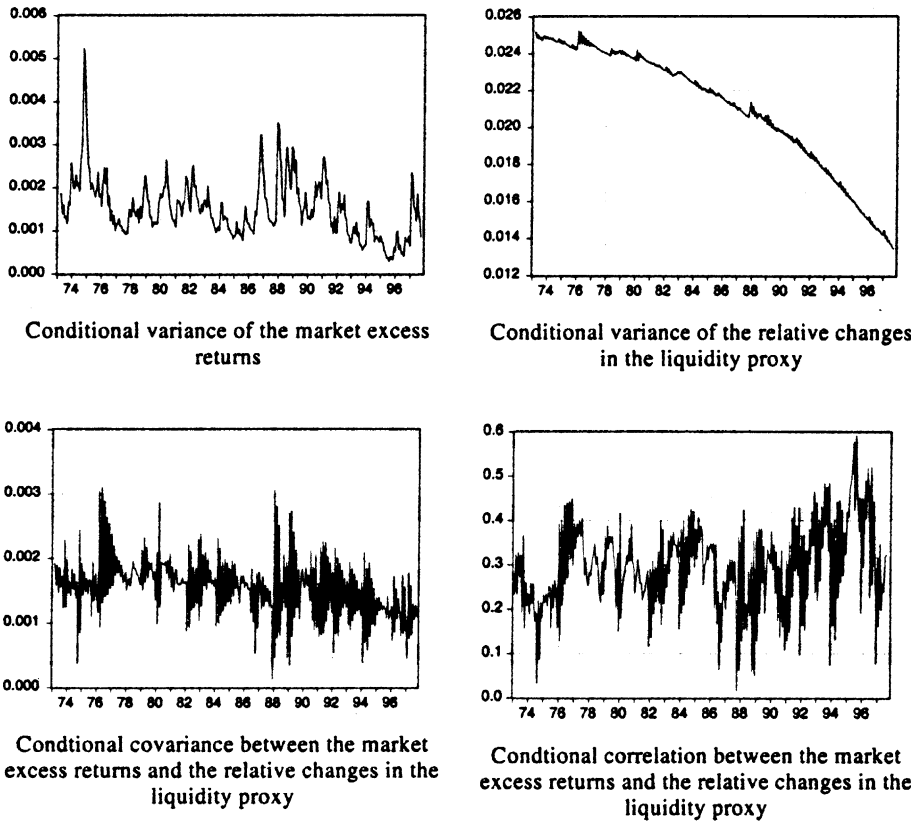


Fig. 3. Conditional variances, covariances, and correlation of the market excess return and the liquidity proxy relative changes. Panel (a) displays the conditional variance of the market excess returns. Panel (b) displays the conditional variance of the relative changes in the liquidity proxy. Panel (c) shows the conditional covariance between the market excess returns and the relative changes in the liquidity proxy. Panel (d) shows the conditional correlation between the market excess returns and the relative changes in the liquidity proxy.

with

$$H_t = \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{pmatrix} \quad \text{and} \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix},$$

where the time-varying unitary liquidity risk premium $\lambda_{ML,t}$ is simply a linear combination of instrumental variables Z_t , added to a constant term:

$$\lambda_{ML,t} = k_0 + k'Z_t. \quad (6)$$

Table 6 reports the parameter estimates of the bivariate Garch (1,1)-in-mean model with a time-varying liquidity risk premium. Columns 2–4 display the estimates when the liquidity risk premium depends on only one instrumental variable. In the univariate liquidity risk premium specification, we can see that the Experimental

Table 6

Bivariate Garch (1,1)-in-mean excess market returns generating model with a time-varying liquidity risk premium

Parameter	I	II	III	IV
μ_M	0.005 (0.94)	0.007* (1.41)	0.008** (1.72)	-0.007 (-0.74)
λ_M	5.043 (1.10)	3.304 (0.85)	1.559 (0.41)	1.839 (0.24)
λ_{ML}	-5.789** (-2.32)	-6.872*** (-2.86)	-6.412*** (-2.62)	-9.848*** (-2.75)
c_1	-9.171** (-1.92)	—	—	-17.605*** (-2.93)
c_2	—	1819.964 (0.90)	—	—
c_3	—	—	6019.241* (1.59)	4063.909* (1.95)
μ_L	0.013* (1.53)	0.013* (1.48)	0.014* (1.62)	0.015** (1.77)
Log-likelihood	1235.142	1232.883	1233.701	1236.091
L-R test	5.116**	0.598	2.234	7.020**

$$r_{M,t} = \mu_M + \lambda_M \hat{\sigma}_{M,t}^2 + (\lambda_{ML} + c_1 \text{Rec}_t + c_2 \text{VarTerm}_t + c_3 \text{VarDef}_t) \hat{\sigma}_{ML,t} + \varepsilon_{1,t},$$

$$r_{L,t} = \mu_L + \varepsilon_{2,t},$$

$$H_t = \Omega + A' \varepsilon_{t-1} \varepsilon'_{t-1} A + B' H_{t-1} B.$$

In this table (***), (**), (*) indicate that the test is significant at respectively 1%, 5%, and 10% level. The robust *t*-statistics are given in parentheses. $r_{M,t}$ is defined as the monthly excess return on the S&P 500 composite index and $r_{L,t}$ is defined as the monthly relative change in the liquidity state variable. The liquidity variable is defined as the dollar monthly number of shares traded in the S&P 500 Index standardized by the number of index constituent stocks. The sample period extends from January 1973–December 1997. Model (I) uses the Experimental Recession Index as instrumental variable, model (II) the variation of the term structure slope, model (III) the variation of the default premium and model (IV) the Experimental Recession Index and the variation of the default premium.

Recession Index is the only instrumental variable which presents a coefficient significant at the 5% level whereas the coefficients attached to the variation of the term structure as well as the variation of the default premium are insignificant. The last row of Table 6 reports the likelihood ratio test (L-R test), based on which we can assess if the introduction of a new instrumental variable has a significant impact on the likelihood. We see that the introduction of the Experimental Recession Index as an instrumental variable improves significantly (at 5%) the likelihood. On the other side, the variation of the default premium has a negligible effect on the likelihood.

We next examine if the joint introduction of the Experimental Recession Index and the variation of the default premium is required. The estimates for the two instrumental variables model are provided in column 5 of Table 6. According to the

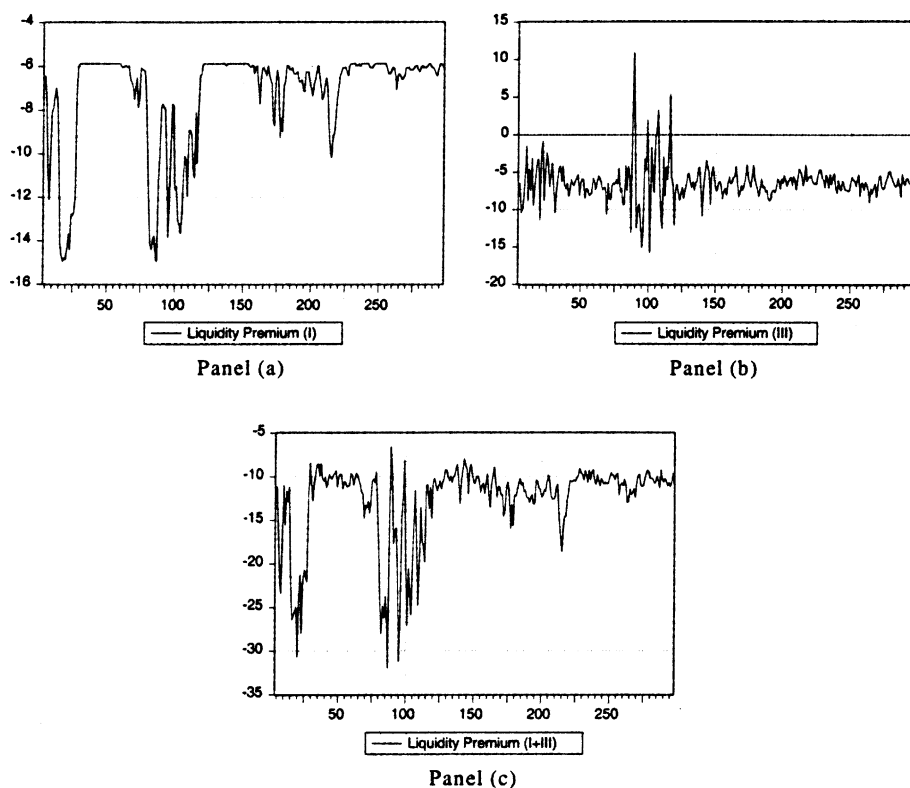


Fig. 4. Time-varying liquidity premium as a function of the Experimental Recession Index in Panel (a), the variation of the default premium in Panel (b) and the recession index and the variation of the default premium in Panel (c).

likelihood ratio test, the constrained model being the constant specification, the joint introduction of the Experimental Recession Index and of the variations of the default premium seems to improve the results.

Fig. 4 illustrates the unitary liquidity risk premium as a linear function of the Experimental Recession Index in Panel (a), of the variation of the default premium in Panel (b) and of the Experimental Recession Index and the variation of the default premium taken together in Panel (c). The surprising shape of the unitary liquidity risk premium in the first graph is due to the dynamics of the Experimental Recession Index and to the linear specification retained for the time-varying liquidity risk premium. The third graph takes into account the Experimental Recession Index as well as the variation of the default premium and leads to a smoother premium. In each graph, two periods are characterized by a higher sensitivity of the expected excess market returns to the covariance between the market liquidity and the excess market returns (or a higher liquidity risk premium in absolute value). The first one is in 1974 and the second one in 1979. They match with the two oil crashes, and define two periods with a high recession probability. It appears natural that agents should be more

concerned about market liquidity during recession periods. Recessions can adversely affect both their financial and their human capital. They may thus become more pre-occupied with their ability to transform their financial investments proceeds into cash.

5. Further analysis of the liquidity risk premium

The significance of the systematic liquidity risk premium as well as of the other results reported in Tables 5 and 6 suggest that one should care about market liquidity risk. This section is devoted to test the stability of the previous results based on the general bivariate Garch (1,1)-in-mean specification with constant market and liquidity risk premia.⁵ Hence, we first perform the QML estimation for the two sub-samples presented in Section 4, and subsequently analyse the impact of the October'87 Crash on the results.⁶

5.1. Sub-sample analysis

We first examine the stability of the unitary liquidity risk premium. In Table 7, we therefore report the estimates for the general bivariate Garch (1,1)-in-mean model of market excess returns over the full as well as over the two sub-sample periods.

The unitary market risk premium is positive during the first period and negative during the second one.⁷ The systematic liquidity risk premium is always negative: its estimated value is equal to -14.666 , significant at the 5% level, during the first period, and equal to -16.40 , significant at the 10% level, during the second period. Hence, even if the results can be affected by the reduced sample size, the coefficients reported in Table 7 suggest that the results are fairly robust across time.

Finally, one may question whether our results are affected by the presence in the sample of the October'87 Crash event and whether the presence of a significant and negative liquidity risk premium is entirely driven by the crash. Thus, in the next section, we examine the effect of the October'87 Crash on the full sample as well as on the second sub-sample parameter estimates.

5.2. The role of the October'87 Crash on the risk premia

The October'87 Crash might have an impact on our estimates since it translates into unexpected jumps in the excess market returns as well as in the liquidity proxy

⁵ We do not test the stability of the empirical results for the general bivariate model with time-varying risk premia due to the number of parameters that had to be estimated in a constrained data sample.

⁶ We also investigated whether asymmetric components in the conditional variance equation should also be considered. We tested the model using the asymmetric BEKK formulation proposed by Kroner and Ng (1998). The asymmetric model is only significant at the 10% confidence level, and does not affect the sign nor the significance of the liquidity risk premium.

⁷ These results are furthermore consistent with the estimates of the market risk premium obtained with the univariate Garch (1,1)-in-mean model discussed in Section 6.

Table 7

Sub-sample estimates for the bivariate Garch (1,1)-in-mean excess market return model with constant risk premia

Parameter	Full sample	First sub-sample 1973:01–1985:07	Second sub-sample 1985:07–1997:12
μ_M	0.013* (1.48)	0.033** (1.57)	0.027 (0.81)
λ_M	3.325 (0.88)	2.935 (0.72)	-1.13 (-0.12)
λ_{ML}	-7.44*** (-2.96)	-14.666** (-1.73)	-16.40* (-1.38)
μ_L	0.013* (1.48)	0.022* (1.57)	0.001* (0.02)

$$r_{M,t} = \mu_M + \lambda_M + \hat{\sigma}_{M,t}^2 + \lambda_{ML}\hat{\sigma}_{ML,t} + \varepsilon_{1,t},$$

$$r_{L,t} = \mu_L + \varepsilon_{2,t},$$

$$H_t = \Omega + A'e_{t-1}e'_{t-1}A + B'H_{t-1}B.$$

In this table (***), (**), (*) indicate that the test is significant at respectively 1%, 5%, and 10% level. The robust *t*-statistics are given in parentheses. $r_{M,t}$ is defined as the monthly excess return on the S&P 500 composite index and $r_{L,t}$ is defined as the monthly relative change in the liquidity state variable. The liquidity variable is defined as the dollar monthly number of shares traded in the S&P 500 Index standardized by the number of index constituent stocks. The full sample period extends from January 1973–December 1997.

Table 8

Impact of the 1987 stock market crash on the unitary liquidity risk premium

Parameter	Full sample	Second sub-sample 1985:07–1997:12
μ_M	0.008** (1.64)	0.001 (0.10)
λ_M	2.902 (0.77)	12.260 (0.99)
λ_{ML}	-7.531*** (-3.01)	-17.035** (-2.27)
μ_L	0.012* (1.38)	0.015* (1.26)

$$r_{M,t} = \mu_M + \lambda_M + \hat{\sigma}_{M,t}^2 + \lambda_{ML}\hat{\sigma}_{ML,t} + \varepsilon_{1,t},$$

$$r_{L,t} = \mu_L + \varepsilon_{2,t},$$

$$H_t = \Omega + A'e_{t-1}e'_{t-1}A + B'H_{t-1}B.$$

In this table (***), (**), (*) indicate that the test is significant at respectively 1%, 5%, and 10% level. The robust *t*-statistics are given in parentheses. $r_{M,t}$ is defined as the monthly excess return on the S&P 500 composite index and $r_{L,t}$ is defined as the monthly relative change in the liquidity state variable. The liquidity variable is defined as the dollar monthly number of shares traded in the S&P 500 Index standardized by the number of index constituent stocks. Both the full sample and second sub-sample period estimates are computed without the observations for $r_{M,t}$ and $r_{L,t}$ pertaining to October and November 1987.

that are not captured by the bivariate Garch (1,1)-in-mean excess market return generating model. Furthermore, Garch processes can be viewed as an approximation of stochastic volatility models which themselves do not account for jumps (see Nelson, 1990). In order to test the effect of the October'87 stock market crash, we estimate the general bivariate Garch (1,1)-in-mean model with constant risk premia during the sample period while excluding the months of October and November 1987. October is excluded for obvious reasons since it is the month when the crash occurred. November is also excluded since after having increased by 45% during October, the liquidity proxy has decreased by 50% in November. It became more stable thereafter. In Table 8, the estimate of the unitary systematic liquidity risk premium is still negative and significant at the 1% level when we exclude the two months surrounding the October'87 Crash. Furthermore, the magnitude of the coefficient estimate is comparable to the one found in Table 7 when we include the Crash. Hence, following the results displayed in Table 8, we can say that the introduction of the October'87 Crash has no major impact on the sign or on the magnitude of the liquidity risk premium.

6. Economic significance of the systematic liquidity risk premium

In this section, we discuss the economic significance of the systematic liquidity risk premium. We therefore compare the average equity premium estimated with and without the introduction of systematic liquidity risk. For this purpose, we start by estimating a univariate Garch (1,1)-in-mean specification for the excess market returns. More formally

$$\begin{aligned} r_{M,t} &= \lambda_0 + \lambda_M \hat{\sigma}_{M,t}^2 + \varepsilon_t, \\ \hat{\sigma}_{M,t}^2 &= \alpha_0 + \alpha_1 \hat{\sigma}_{M,t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 \end{aligned} \quad (7)$$

with α_0 , α_1 and α_2 positive constants and $\alpha_1 + \alpha_2 < 1$ in order to guarantee a positive and stationary variance process. Table 9 reports the estimates for the univariate Garch (1,1)-in-mean specification of the market excess returns with a constant market risk premium. We run the QML estimation over the full sample as well as over two equal length sub-sample periods (from 1973:01 to 1985:07 and from 1985:07 to 1997:12). Over the full sample period, the unitary market risk premium is positive but insignificant. Nevertheless, its t -statistic is so small that the positivity of λ_M is questionable. The results observed during each sub-sample period are contradictory. Indeed, the market risk premium is positive and significant during the first half of the period as the theory predicts it to be. However, there is no empirical evidence of a positive relation between expected market returns and their conditional variance during the second sub-sample period. This could partially be explained by the presence of the October'87 stock market crash during the second period given that a univariate Garch (1,1)-in-mean model of excess returns is not well suited to handle market crashes.

Contrarily to the expected market excess returns displayed in Fig. 2 for the general bivariate Garch (1,1)-in-mean excess market returns generating model described

Table 9

Univariate Garch (1,1)-in-mean market excess returns generating model with a constant market risk premium

Parameter	Full sample	First sub-sample 1973:01–1985:07	Second sub-sample 1985:07–1997:12
λ_0	0.001 (0.16)	−0.031** (1.96)	0.007 (1.11)
λ_M	0.533 (0.16)	15.19** (1.82)	−1.818 (−0.44)
α_0	6^{E-05} (1.28)	2^{E-04**} (1.48)	2^{E-05} (0.72)
α_1	0.882*** (21.60)	0.769*** (6.81)	0.892*** (19.17)
α_2	0.091*** (2.71)	0.089** (1.82)	0.114** (1.94)

$$r_{M,t} = \lambda_0 + \lambda_M \hat{\sigma}_{M,t}^2 + \varepsilon_t,$$

$$\hat{\sigma}_{M,t}^2 = \alpha_0 + \alpha_1 \hat{\sigma}_{M,t-1}^2 + \alpha_2 \varepsilon_{t-1}^2.$$

In this table (***), (**), (*) indicate that the test is significant at respectively 1%, 5%, and 10% level. The robust *t*-statistics are given in parentheses. $r_{M,t}$ is defined as the monthly excess return on the S&P 500 composite index. The full sample period extends from January 1973–December 1997.

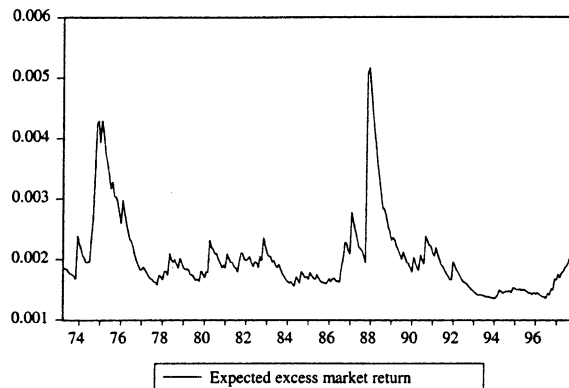


Fig. 5. Expected excess returns of the S&P 500 Index. This figure displays the monthly expected market excess returns of the S&P 500 Index based on the estimates reported in Table 9.

in Table 5, the univariate Garch (1,1)-in-mean model generates a strictly positive expected market excess return as shown in Fig. 5. This is mainly due to the positive relationship between the expected market excess return with its variance, and to the presence of a positive constant λ_0 in the specification of Eq. (7). As a consequence, the mean of the expected market excess return as estimated by the univariate Garch (1,1)-in-mean model is equal to 2.465% per annum and is significantly different from zero (with a *t*-stat of 46.11).

However, this estimated premium just considers the excess market returns' time series expected by investors if they could effectively buy or sell financial assets at any given point in time. Under liquidity risk, investors may not be able to liquidate their positions at the right moment and/or could incur a price impact and thus would be willing to accept lower expected returns for securities whose returns have higher liquidity betas.

Under systematic liquidity risk, the agent would use the bivariate Garch (1,1)-in-mean model (Table 5) and would obtain estimates of the expected market excess returns as displayed in Fig. 2. The average annualized equity premium given by the bivariate Garch (1,1)-in-mean model is only equal to 1.112%. Thus, taking into account systematic liquidity reduces the size of the equity premium that risk averse investors may require by 55%. Hence, the equity premium becomes very small when systematic liquidity is taken into account. Moreover, this significant reduction in the equity premium is corroborated by the strong statistical significance of the estimates of the systematic liquidity premium obtained in this study.

7. Conclusion

In this study, we rely on a bivariate Garch (1,1)-in-mean specification for the stock market excess returns in order to examine whether systematic liquidity risk is priced and whether the sign of the unitary liquidity risk premium is negative. The bivariate Garch (1,1)-in-mean specification is tested on monthly excess market returns of the S&P 500 Index during the period January 1973–December 1997. We use the number of shares traded in the S&P 500 per month as a proxy measure for aggregate liquidity. Overall, the results suggest that liquidity risk is indeed priced during the entire as well as over sub-periods in the US. The sign of the liquidity risk premium is significantly negative and time-varying. Furthermore, according to these preliminary results, the unitary market risk premium becomes insignificant within the general bivariate Garch (1,1)-in-mean model with constant risk premia. According to our results, systematic liquidity risk dominates market risk and is insensitive to the introduction of extreme liquidity events such as the October'87 Crash.

It is important to stay aware of the joint hypothesis testing implications of this preliminary investigation into systematic liquidity risk pricing on the US stock market. Indeed, our results are clearly influenced by the specification of the excess market return generating model and of the time-varying unitary liquidity risk premium retained and ultimately by the systematic liquidity proxy variable chosen. It is interesting to mention that using a different market "illiquidity" risk measure, Amihud (2002) finds a positive relationship between expected market illiquidity and expected stock returns. The latter is consistent with our findings given our specific proxy for systematic "liquidity".

The theoretical implications of these results are also worth exploring since they raise questions about the potential biases in traditional asset pricing models that ignore systematic liquidity risk. As pointed out by Chordia et al. (2000), commonality in liquidity risk is still an emerging stream of theoretical and empirical literature. The

empirical results found in this study should be extended along several dimensions to deepen our understanding of the origins, the time-series properties and the international cross-effects of systematic liquidity risk pricing. Finally, these empirical results emphasize the need to develop inter-temporal asset pricing models that endogenize agents' need to hedge against inter-temporal systematic liquidity shocks. This would allow us to reconsider and perhaps explain some empirical anomalies within a rational asset pricing framework that accounts for the pricing of systematic liquidity risk.

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