Time-Varying Liquidity Risk and the Cross Section of Stock Returns

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This paper studies whether stock returns' sensitivities to aggregate liquidity fluctuations and the pricing of liquidity risk vary over time. We find that liquidity betas vary across two distinct states: one with high liquidity betas and the other with low betas. The high liquidity-beta state is short lived and characterized by heavy trade, high volatility, and a wide cross-sectional dispersion in liquidity betas. It also delivers a disproportionately large liquidity risk premium, amounting to more than twice the value premium. Our results are consistent with a model of liquidity risk in which investors face uncertainty about their trading counterparties' preferences. (*JEL* G12)

Recent studies find that there is a systematic component in the time-series variation of liquidity measures across stocks (Chordia, Roll, and Subrahmanyam, 2000; Hasbrouck and Seppi, 2001; and Huberman and Halka, 2001). While liquidity has long been regarded as a firm attribute with a negative effect on expected returns, the existence of liquidity commonality suggests that market-wide liquidity may also be an important risk factor in the cross section of stock returns. The risk view of liquidity has attracted much attention in recent years and has led to several studies confirming the pricing of liquidity risk, i.e., that stocks with greater return sensitivities to aggregate liquidity fluctuations—measured by their "liquidity betas"—earn higher expected returns (Pastor and Stambaugh, 2003; Acharya and Pedersen, 2005). Despite

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the growing interest, however, there is a paucity of research about how this pricing relation could change over time. As an attempt to address this issue, this paper examines whether liquidity betas and liquidity risk premium vary across different economic states.

We propose that changes in the prevailing level of preference uncertainty lead to time variation in liquidity betas and liquidity risk premium. Our approach is motivated by two facts about the actual trading environment: first, investors are unlikely to have complete knowledge about their trading counterparties' preferences, and second, they incur trading costs. The first point is emphasized by Gallmeyer, Hollifield, and Seppi (2005), who provide a theoretical model in which investors are asymmetrically informed about each other's preferences. This preference uncertainty potentially exposes the investors to resale price risk because they are *a priori* uncertain about the future asset demands of their trading counterparties. Investors' future preferences are, however, fully or partially revealed through their trades so that the resulting level of preference risk and its associated premium are endogenously determined. In particular, the authors construct examples in which elevated trading volume indicates periods of heightened preference risk, high risk premium, and a large price impact of trade.

We introduce illiquidity costs, modeled as the costs of selling, into their framework and first examine the effect of preference uncertainty on liquidity betas. When there is an unanticipated increase in the next period's illiquidity cost, it will also raise subsequent periods' costs due to persistence in illiquidity, which is widely documented in the empirical literature. The rise in future illiquidity costs will, in turn, decrease the next period's price because it is the sum of discounted future payoffs after cost. We argue that this future price decline, caused by the unit future illiquidity shock, will translate into a larger return decrease under higher preference uncertainty. This results from a lower current price that marginal investors require in order to compensate themselves for future resale price risk. Therefore, our first empirical hypothesis is that the return sensitivity to illiquidity shocks is negative and larger in magnitude, or equivalently, liquidity betas are positive and larger, when investors face a higher level of preference uncertainty.

The changing level of preference uncertainty will also affect liquidity risk premium. Liquidity risk is priced because illiquidity shocks make investors' consumptions volatile, and risk-averse investors dislike volatile consumptions; the more volatile their consumptions become, the more risk premium they will require. We show that preference uncertainty makes their consumptions more sensitive to the illiquidity shocks and hence more volatile, because the sensitivity is proportional to the trading volume. Intuitively, as investors accommodate larger sell trades from their counterparties under higher preference uncertainty, they will have to pay more illiquidity costs when they close out their positions in the future. This leads to our second hypothesis that the liquidity risk premium rises during times of high preference uncertainty.

We begin the empirical analysis by examining the dynamics of liquidity betas. Using two extreme size deciles as proxies for the most and the least liquidity-sensitive portfolios, we estimate a bivariate regime-switching model that allows both time-series and cross-sectional variations in liquidity betas. In support of our first hypothesis, we find that liquidity betas vary over time across two distinct states: one with high liquidity betas and the other with low liquidity betas. The transition from the low to the high liquidity-beta state is predicted by a rise in trading volume, which is consistent with preference uncertainty being the key state variable underlying the liquidity-beta dynamics. The high liquidity-beta state is associated with high volatility and preceded by a period of declining expectations about future market liquidity. The state is also less persistent and occurs less than one-tenth of the time, implying that it represents an "abnormal" trading regime. Furthermore, our analysis reveals new evidence on the cross-sectional variation in liquidity-beta dynamics; we find that the spread in liquidity betas across the two states is greater for small illiquid stocks than large liquid ones, indicating that illiquid stocks' liquidity betas are more sensitive to the state of preference uncertainty.

To test whether the liquidity risk premium also varies over time, we construct a conditional liquidity factor and examine its pricing in the cross section of stock returns. The conditional liquidity factor equals the liquidity factor during the high liquidity-beta state, and zero otherwise. It is thus designed to capture the additional impact of market liquidity on stock returns during periods of high preference uncertainty. Our asset pricing tests indicate that the premium on the conditional liquidity factor is statistically significant, controlling for the market, size, value, and momentum factors, as well as the illiquidity levels of test assets. The results are robust to a number of controls using additional risk factors, various stock characteristics, and different sets of test assets. Importantly, we find that the pricing of the conditional liquidity factor is significant not only statistically but also economically. The premium on its mimicking portfolio is 1.5% per month, more than twice the value premium in the sample. We argue that this premium is not unreasonably high, given that it is rendered only about one-tenth of the time.

Lastly, our framework allows us to study the relationship between illiquidity premium (premium on illiquid assets) and liquidity risk premium (premium on liquidity-sensitive assets). Consistent with previous findings, illiquid portfolios have higher average returns than liquid portfolios unconditionally; the mean excess-return spread between the two extreme liquidity-sorted decile portfolios is 0.48% per month. However, this illiquidity-return relationship exhibits a noticeable difference between the two liquidity-beta states. During the "normal" low liquidity-beta months, the relationship between illiquidity and return is virtually flat across the 10 portfolios. In contrast, during the "abnormal" high liquidity-beta months, the excess return strongly increases in the level of illiquidity. The mean spread between the two extreme deciles is a startling 5.4% per month. This implies that the illiquidity premium is rendered strongly and only in the high liquidity-beta state. Given our earlier finding that the conditional liquidity factor is significantly priced controlling for the illiquidity characteristic, the result suggests that the illiquidity premium is delivered primarily in the form of beta risk premium with respect to the liquidity factor during periods of high preference uncertainty.

To our knowledge, there are only a couple of empirical studies that investigate the dynamic nature of illiquidity premium or liquidity risk premium, but none of them does so in the cross section of stock returns. Longstaff (2004) finds large illiquidity premia in the Treasury bond prices and shows that they are correlated with market sentiment measures, such as changes in the Conference Board Consumer Confidence Index and in the amount of funds held in money-market mutual funds. Gibson and Mougeot (2004) document that the liquidity risk premium in the S&P 500 Index return has a time-varying component related to the probability of future recession. Our work adds to this line of research by shedding new light on the mechanism driving the joint dynamics of liquidity betas and liquidity risk premium.

The remainder of the paper is organized as follows. Section 1 presents a simple preference uncertainty model with illiquidity costs and establishes testable empirical hypotheses. Section 2 examines the dynamics of liquidity betas using a regime-switching model. Section 3 conducts asset pricing tests with a conditional liquidity factor, followed by a battery of robustness tests. It also discusses the characteristics of the high liquidity-beta state and the economic significance of the conditional liquidity risk premium. Section 4 concludes the paper.

1. Preference Uncertainty and Liquidity Risk

In the actual markets, investors are unlikely to have complete knowledge about each other's future preferences. Such preference uncertainty may arise from a variety of reasons, including stochastic risk aversion, temporary liquidity needs, endowment shocks, uncertain habits, and random market participation. A recent study by Gallmeyer, Hollifield, and Seppi (2005) demonstrates that in such an environment, trading becomes an important source of information from which investors can learn, either fully or partially, about the preferences driving their counterparties' future asset demands. Our goal is to understand how changes in the level of preference uncertainty affect liquidity betas and liquidity risk premium. To this end, we incorporate illiquidity costs, modeled as the costs of selling according to Acharya and Pedersen (2005), into the model of Gallmeyer, Hollifield, and Seppi (2005) and establish testable empirical hypotheses.

1.1 Preference uncertainty model with illiquidity costs

Following Gallmeyer, Hollifield, and Seppi (2005), we consider an economy in which investors are asymmetrically informed about each other's time preferences. The notation conforms to their work where possible. There are three dates, $t = 1, 2, 3$. At dates 1 and 2, the investors trade a default-free asset that pays a unit consumption good at date 3. For the sake of tractability, we model the single traded asset as a riskless asset representing the aggregate security market. As we will discuss later, however, the pricing implication of preference uncertainty extends to a setting with multiple stocks with risky cash flows. The total units of the asset outstanding are fixed at 1. There are two types of competitive risk-averse agents: long- and short-term investors. The long-term investors (denoted by subscript L) choose their asset holdings at dates 1 and 2 to maximize a three-period utility:

$$u(c_{L1}) + \delta_1 u(c_{L2}) + \delta_1\delta_2 u(c_{L3}), \tag{1}$$

where $u(c)$ is increasing, concave, and twice differentiable; $c_{Lt} > 0$ is their consumption at date t; and δ_1 and δ_2 are time-preference parameters. They are endowed with $\theta_{L0} > 0$ units of the asset prior to trading and $e_{L1} \geq 0$ and $e_{L2} > 0$ units of the consumption good at dates 1 and 2, respectively. Since there is no cash flow risk, no expectation is required in maximizing their utility.

Similarly, the short-term investors (denoted by subscript S) select their date-1 asset holdings to maximize an expected two-period utility:

$$v(c_{S1}) + \vartheta \boldsymbol{E}_1[v(c_{S2})], \tag{2}$$

where $v(c)$ is increasing, concave, and twice differentiable; $c_{St} > 0$ is their consumption at date t; ϑ is a time-preference parameter; and $\boldsymbol{E}_1[\cdot]$ denotes expectation given their information set at date 1. They are endowed with $\theta_{S0} = 1 - \theta_{L0} \geq 0$ units of the asset prior to trading and $e_{S1} > 0$ and $e_{S2} \geq 0$ units of the consumption good at dates 1 and 2, respectively. Since the short-term investors do not value consumption at date 3, they inelastically close out their positions at date 2. While their time-preference parameter, ϑ, is common knowledge, the short-term investors do not know the long-term investors' time-preference parameters, δ_1 and δ_2. This is the source of preference uncertainty in that it can prevent short-term investors from perfectly knowing their trading counterparties' future demands and hence the price at which they can trade the asset at date 2.

We introduce illiquidity costs to the above setting and assume that the seller of the asset must pay a random cost $\kappa_t > 0$ per unit asset sold at date t. Reflecting the empirical regularity that illiquidity costs are persistent, we model the dynamics of κ_t by an AR(1) process[1]:

$$\kappa_t = \alpha + \gamma\kappa_{t-1} + \mathcal{I}_t, \tag{3}$$

where $\alpha > 0$, $0 < \gamma < 1$, and $\mathcal{I}_t$ is an illiquidity shock with mean zero and a constant variance $\sigma^2_{\mathcal{I}}$, distributed independently of δ_1 and δ_2. We assume that

[1] Persistence in illiquidity is documented by Amihud (2002), Eckbo and Norli (2002), Jones (2002), Acharya and Pedersen (2005), and Bekaert, Harvey, and Lundblad (2006), among others.

the illiquidity cost is sufficiently small so that the selling price after the cost is $P_t - \kappa_t > 0$.

1.2 Pricing of liquidity risk under preference uncertainty

To illustrate how marginal investors incorporate future illiquidity costs into the pricing under possible preference uncertainty, we consider the case in which $\theta_{L0} = 1$ (i.e., the long-term investors hold the total units of the asset outstanding prior to the initial trading). The short-term investors will then accommodate sell orders from the long-term investors at date 1 and inelastically sell all of their asset holdings at date 2. To make the model tractable, we also make the following assumptions adopted from an analytic example of Gallmeyer, Hollifield, and Seppi (2005): both long- and short-term investors have a log utility, $u(c) = v(c) = \log(c)$; δ_2 is distributed binomially and takes a value $\delta_2^P = 1.5$ ("P" for a patient type of the long-term investors) or $\delta_2^I = 1.1$ ("I" for an impatient type) with equal probability; δ_1 is distributed uniformly conditional on δ_2, such that $\delta_1^P \sim U[0.95, 1.4]$ when $\delta_2 = \delta_2^P$ and $\delta_1^I \sim U[1.3, 1.55]$ when $\delta_2 = \delta_2^I$; and other parameters are set at $\vartheta = 1.03$, $e_{S1} = 1.4$, $e_{S2} = 0.6$, $e_{L1} = 0$, $e_{L2} = 1.0$, $\alpha = 0.01$, $\gamma = 0.8$, $\kappa_1 = 0.05$, and $\mathcal{I}_2 \sim U[-0.02, 0.02]$.[2]

Under these assumptions, Appendix A.1 solves for the equilibrium prices P_1 and P_2 at dates 1 and 2, respectively:

$$P_1 = \frac{\vartheta e_{S1} \boldsymbol{E}\left[\frac{P_2-\kappa_2}{e_{S2}+\frac{P_2-\kappa_2}{z}} | z, \kappa_1\right]}{1 + \frac{\vartheta}{z}\boldsymbol{E}\left[\frac{P_2-\kappa_2}{e_{S2}+\frac{P_2-\kappa_2}{z}} | z, \kappa_1\right]} \quad \text{and} \quad P_2 = \frac{e_{L2}}{\frac{1}{z} + \frac{1}{\delta_2}}, \tag{4}$$

where $z \equiv \delta_1\delta_2 = \frac{1}{1-\theta_{L1}} = \frac{1}{\theta_{S1}}$ with θ_{L1} and θ_{S1} representing the equilibrium asset holdings of the long- and short-term investors at date 1, and $\boldsymbol{E}[\cdot|z, \kappa_1]$ clarifies the conditioning information in $\boldsymbol{E}_1[\cdot]$. Note that z is revealed from the long-term investors' date-1 trades that the short-term investors accommodate. While P_2 is identical to the date-2 price obtained in Gallmeyer, Hollifield, and Seppi (2005), P_1 differs from theirs because it incorporates the expected date-2 selling cost κ_2; setting $\kappa_2 = 0$ restores the date-1 price in their no-cost model.

Panel A of Figure 1 depicts the date-1 prices with (P_1) and without ($P_{1,nc}$) the illiquidity cost along with the common expected price at date 2, $\boldsymbol{E}_1[P_2]$, plotted against the long-term investors' date-1 sell trades, $1 - \theta_{L1} = \frac{1}{z}$. First, we observe that P_1 is lower than $P_{1,nc}$, which confirms Amihud's (2002) argument that higher expected future illiquidity depresses the current price. Second, the figure illustrates the role of trading volume in revealing counterparty preferences. Given that $\frac{1}{\delta_1^P\delta_2^P} \sim U[0.48, 0.70]$ and $\frac{1}{\delta_1^I\delta_2^I} \sim U[0.58, 0.70]$, there is a

[2] As Gallmeyer, Hollifield, and Seppi (2005) note, the time-preference parameters need not be smaller than 1 in a finite-period model.

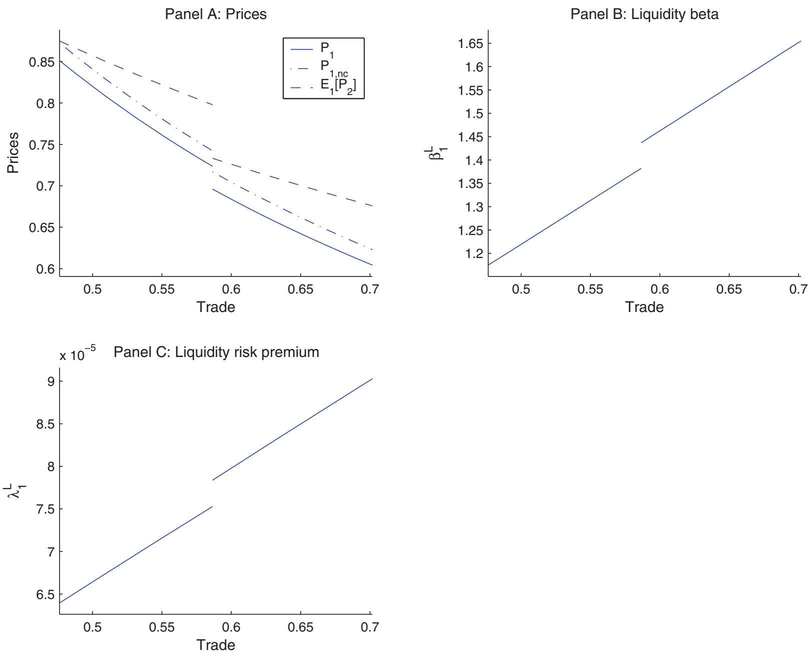

Figure 1
Pricing of liquidity risk under preference uncertainty
Panel A: Prices at date 1 with (P_1) and without $(P_{1,nc})$ the illiquidity cost and the common expected price at date 2, $E_1[P_2]$. Panel B: Liquidity beta, $\beta_1^{\mathcal{L}}$. Panel C: Liquidity risk premium, $\lambda_1^{\mathcal{L}}$. The long- and short-term investors have logarithmic preferences. Parameter values are: $\delta_2^P = 1.5$, $\delta_1^P \sim U[0.95, 1.4]$, $\delta_2^I = 1.1$, $\delta_1^I \sim U[1.3, 1.55]$, $\Pr(\delta_2^P) = \Pr(\delta_2^I) = 0.5$, $\vartheta = 1.03$, $e_{S1} = 1.4$, $e_{S2} = 0.6$, $e_{L1} = 0$, $e_{L2} = 1.0$, $\alpha = 0.01$, $\gamma = 0.8$, $\kappa_1 = 0.05$, and $\mathcal{I}_2 \sim U[-0.02, 0.02]$.

critical level of trading volume $0.58 \equiv \frac{1}{z'}$; volume below this level perfectly reveals that the long-term investors are of the patient type, while volume above it does not. This implies that trading volume is endogenously determined and serves as a proxy for the equilibrium level of preference uncertainty. Lastly, we can examine the impact of preference uncertainty on the equilibrium asset prices. The graphs to the left of the middle gaps correspond to the prices under low (in this particular example, no) preference uncertainty. In this case, P_2 is known and the short-term investors face no resale price risk. In contrast, the graphs to the right of the gaps show prices under high preference uncertainty. Because trade reveals counterparty preferences only partially in this region, the resale price P_2 is uncertain. In this partially revealing equilibrium, the short-term investors require additional price concession to hold the asset, which produces the observed gaps in prices.

From the first-order condition for the short-term investors' utility maximization problem, Appendix A.2 derives an asset pricing model of the form (Cochrane, 2005):

$$\boldsymbol{E}_1[R_2] = R_1^f + \beta_1^m \lambda_1^m, \tag{5}$$

where $\boldsymbol{E}_1[R_2]$ is the expected gross return after cost; R_1^f is the gross (shadow) risk-free rate; $\beta_1^m \equiv \frac{\mathrm{Cov}_1(R_2,m_2)}{\mathrm{Var}_1(m_2)}$ is the asset's beta with respect to the stochastic discounting factor (SDF) $m_2 = \vartheta\frac{c_{S1}}{c_{S2}}$, and $\lambda_1^m \equiv -\frac{\mathrm{Var}_1(m_2)}{\boldsymbol{E}_1(m_2)}$ is the price of risk.

To derive a linear factor pricing model, we linearize the SDF, m_2, with respect to the illiquidity shock, $\mathcal{I}_2$, by the first-order Taylor expansion:

$$m_2 = a_1 + b_1\mathcal{I}_2, \tag{6}$$

where both a_1 and b_1 are the functions of the short-term investors' information set at date 1.[3] Define the illiquidity beta $\beta_1^{\mathcal{I}} \equiv \frac{\mathrm{Cov}_1(R_2,\mathcal{I}_2)}{\mathrm{Var}_1(\mathcal{I}_2)}$ and rewrite Equation (5) in a factor-pricing form:

$$\boldsymbol{E}_1[R_2] = R_1^f + \beta_1^{\mathcal{I}}\lambda_1^{\mathcal{I}}, \tag{7}$$

where the illiquidity risk premium is

$$\lambda_1^{\mathcal{I}} \equiv -\frac{b_1}{a_1}\sigma_{\mathcal{I}}^2.$$

While the illiquidity beta here is interpreted as the aggregate return sensitivity to the aggregate illiquidity shock, we will discuss a case with multiple securities shortly.

Panel B of Figure 1 depicts the liquidity beta defined as $\beta_1^{\mathcal{L}} \equiv -\beta_1^{\mathcal{I}}$. We observe that $\beta_1^{\mathcal{L}}$ is higher in the region of high preference uncertainty, where a future price decline caused by a unit future illiquidity shock translates into a larger decrease in future return. This results from a lower current price that the short-term investors require in order to compensate themselves for future resale price risk (see Panel A). More formally, note that it is the proportional cost component, $\frac{\kappa_2}{P_1}$, of $R_2 = \frac{P_2-\kappa_2}{P_1}$ that contributes to the illiquidity beta. It is then straightforward to see that the illiquidity beta has a very simple form, $\beta_1^{\mathcal{I}} = -\frac{1}{P_1}$. Since the illiquidity beta is inversely related to the current price level, it exhibits a jump at the threshold volume level $\frac{1}{z'}$.

The changing level of preference uncertainty also affects the liquidity risk premium. Panel C of Figure 1 shows that the liquidity risk premium, defined as $\lambda_1^{\mathcal{L}} \equiv -\lambda_1^{\mathcal{I}}$, is again higher in the region of high preference uncertainty. Intuitively, the short-term investors' consumptions are more sensitive to the illiquidity shock under high preference uncertainty because the sensitivity is proportional to the trading volume. As the short-term investors accommodate larger sell trades from the long-term investors at date 1, they will have to pay more illiquidity costs to close out their positions at date 2; consequently, their consumptions become more volatile. Since they are risk averse and dislike

[3] An alternative linearization is to Taylor-expand the SDF with respect to both the illiquidity shock, $\mathcal{I}_2$, and the random time-preference parameter, δ_2. This will produce a preference uncertainty factor in addition to the illiquidity factor. Since we are interested in the effect of preference uncertainty on the pricing of liquidity risk, we do not take this approach.

volatile consumptions, they will require more premium for bearing the liquidity risk. To see this more formally, notice that the consumption growth is tied to the SDF, whose sensitivity to the illiquidity shock is measured by the b_1 coefficient in Equation (6). Appendix A.2 shows that b_1 is proportional to the size of sell orders from the long-term investors that the short-term investors accommodate at date 1, $1 - \theta_{L1} = \theta_{S1}$, which serves as a proxy for the level of preference uncertainty.

While we have been working with a single riskless asset, the model can be extended to a setting with multiple risky stocks in a straightforward manner. Assume that there are N stocks in addition to the long-term riskless asset. Stock j pays a random terminal dividend, x_{j3}, $j = 1, \ldots, N$, at date 3; x_{j3} is distributed independently of all the other random variables. Also, let $j = 0$ represent the riskless asset with a sure terminal payoff, $x_{03} \equiv 1$. No security pays an interim dividend. Supplies of all securities are normalized at 1. Under this setting, the illiquidity factor $\mathcal{I}_t$ is priced as long as it is systematic, which can be justified by the empirical evidence on commonality in illiquidity.[4] Specifically, assume that:

$$\boldsymbol{\kappa}_t = \boldsymbol{\alpha} + \gamma\boldsymbol{\kappa}_{t-1} + \boldsymbol{\xi}\mathcal{I}_t + \boldsymbol{\eta}_t, \tag{8}$$

where γ is a constant matrix; $\boldsymbol{\xi}$ is the loadings vector of asset costs on the illiquidity factor; $\boldsymbol{\eta}_t$ is a mean-zero vector of idiosyncratic illiquidity shocks that are distributed independently of all the other random variables; and bold symbols represent $N + 1$ vectors of the corresponding quantities. Following a procedure similar to the single security case, we may derive $\boldsymbol{E}_1[R_{j2}] = R_1^f + \beta_{j1}^{\mathcal{I}}\lambda_1^{\mathcal{I}}$, where security j's after-cost return is $R_{j2} \equiv \frac{P_{j2}-\kappa_{j2}}{P_{j1}}$, the illiquidity beta is $\beta_{j1}^{\mathcal{I}} \equiv \frac{Cov_1(R_{j2},\mathcal{I}_2)}{Var_1(\mathcal{I}_2)} = -\frac{\xi_j}{P_{j1}}$, and the illiquidity risk premium is $\lambda_1^{\mathcal{I}} \equiv -\frac{b_1}{a_1}\sigma_{\mathcal{I}}^2$. Thus, security j's illiquidity beta is proportional to its illiquidity-factor loading, ξ_j. The illiquidity betas can now vary across securities because of differences in their loadings and price levels. Note that ξ_j's are likely to be positive due to commonality in illiquidity. To the extent that this is true, illiquidity betas are again negative (liquidity betas are positive) and exhibit discontinuous jumps due to their dependence on prices. Next, $\lambda_1^{\mathcal{I}}$ retains the same form, but it can be shown that b_1 is now proportional to the loadings-adjusted trades, $\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\theta}_{S1}$, where the superscript "T" denotes vector transposition.[5] Thus, the liquidity risk premium again tends to be high when there are larger trades under high

[4] Commonality in illiquidity is documented by Chordia, Roll, and Subrahmanyam (2000); Huberman and Halka (2001); and to a different extent, Hasbrouck and Seppi (2001).

[5] Specifically, the short-term investors' consumptions at respective dates are $c_{S1} = e_{S1} + \boldsymbol{P}_1{}^{\mathrm{T}}(\boldsymbol{\theta}_{S0} - \boldsymbol{\theta}_{S1})$ and $c_{S2} = e_{S2} + (\boldsymbol{P}_2 - \boldsymbol{\kappa}_2)^{\mathrm{T}}\boldsymbol{\theta}_{S1}$. We can approximate the SDF as in Equation (6), where $a_1 \equiv \vartheta\boldsymbol{E}_1[\frac{e_{S1}+\boldsymbol{P}_1{}^{\mathrm{T}}(\boldsymbol{\theta}_{S0}-\boldsymbol{\theta}_{S1})}{e_{S2}+(\boldsymbol{P}_2-\boldsymbol{\alpha}-\gamma\boldsymbol{\kappa}_1-\boldsymbol{\eta}_t)^{\mathrm{T}}\boldsymbol{\theta}_{S1}}]$ and $b_1 \equiv \vartheta\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\theta}_{S1}\boldsymbol{E}_1[\frac{e_{S1}+\boldsymbol{P}_1{}^{\mathrm{T}}(\boldsymbol{\theta}_{S0}-\boldsymbol{\theta}_{S1})}{[e_{S2}+(\boldsymbol{P}_2-\boldsymbol{\alpha}-\gamma\boldsymbol{\kappa}_1-\boldsymbol{\eta}_t)^{\mathrm{T}}\boldsymbol{\theta}_{S1}]^2}]$. Here, we have ignored a possible factor structure in cash flows, x_{j3}. If there is one, it is straightforward to expand the SDF by the cash-flow factors, as well as $\mathcal{I}_2$. The equilibrium is now characterized by vectors $\boldsymbol{P}_1$, $\boldsymbol{P}_2$, and $\boldsymbol{\theta}_{S1} = \mathbf{1} - \boldsymbol{\theta}_{L1}$, whose $3N + 3$ elements are determined by the $3N + 3$ first-order conditions for the short- and long-term investors (the conditions for the latter are omitted for brevity). While there are multiple securities, their market prices and trades will collectively reveal the same single statistic, $z = \delta_1\delta_2$. Therefore, the equilibrium is not fully revealing in the region where the two types of the long-term investors are pooling.

preference uncertainty, and the trades of securities with higher illiquidity-factor loadings contribute more to the variation in the liquidity risk premium.

Changes in the level of preference uncertainty could be linked to economic or market events. For instance, Jackwerth (2000) documents a dramatic shift in the shape of risk-aversion functions around the 1987 crash. He finds that while they are generally positive and decreasing in wealth before the crash, these properties are partially reversed after it, a point that is hard to reconcile with standard assumptions made in economic theory. Just after such a crash, it may not be easy for investors to determine whether there is indeed such a shift in their counterparties' preferences, much less the degree of the shift. Thus, a major economic or market event can create a state of high preference uncertainty until it is resolved through trading activities and observing market prices.

The above analysis implies that both liquidity betas and liquidity risk premium are time varying when the level of preference uncertainty, proxied by trading volume, changes over time. These points are summarized in the following hypotheses.

Hypothesis 1. *(Time-varying liquidity beta) Liquidity betas are time varying. High (low) liquidity betas arise in the state of high (low) preference uncertainty. The transition from the low to the high preference uncertainty state is signaled by an elevated trading volume.*

Hypothesis 2. *(Time-varying liquidity risk premium) Liquidity risk premium is time varying and rises in the state of high preference uncertainty.*

We test these hypotheses empirically in the rest of the paper.

2. Time Variation in Liquidity Betas

Our hypotheses imply that both high liquidity betas and large liquidity risk premium arise simultaneously at times of high preference uncertainty. We therefore conduct the empirical analysis by first identifying states with different levels of liquidity betas using trading volume as a proxy for the level of preference uncertainty. We then examine whether the liquidity risk premium varies across these states in Section 3.

2.1 Methodology

Our first hypothesis calls for two important features of our empirical model: discontinuous variation in liquidity betas and the use of trading volume as a predictive variable for liquidity-beta states. These features can be readily accommodated by a Markov regime-switching model.

Specifically, we fit the following bivariate model to the excess returns of the smallest and largest size-decile portfolios:

$$\boldsymbol{r}_t = \boldsymbol{\alpha}_{s_t} + \boldsymbol{\beta}_{s_t}^{LIQ} LIQ_t + \boldsymbol{\varepsilon}_t, \tag{9}$$

where LIQ_t is a liquidity factor; $\boldsymbol{r}_t$, $\boldsymbol{\alpha}_{s_t}$, and $\boldsymbol{\beta}_{s_t}^{LIQ}$ are two-dimensional vectors of excess returns, intercepts, and liquidity betas, respectively; and $s_t = 1, 2$ represents the states. That is:

$$\boldsymbol{r}_t = \begin{pmatrix} r_{S,t} \\ r_{L,t} \end{pmatrix}, \quad \boldsymbol{\alpha}_{s_t} = \begin{pmatrix} \alpha_{S,s_t} \\ \alpha_{L,s_t} \end{pmatrix}, \quad \boldsymbol{\beta}_{s_t}^{LIQ} = \begin{pmatrix} \beta_{S,s_t}^{LIQ} \\ \beta_{L,s_t}^{LIQ} \end{pmatrix}, \tag{10}$$

with S and L denoting the smallest and largest deciles, respectively, of size-sorted portfolios [given by the value-weighted size-sorted portfolios of stocks listed on the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX) obtained from the Center for Research in Security Prices (CRSP) of the University of Chicago]. Note that this specification nests a standard multivariate linear regression model with constant coefficients if $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_2$ and $\boldsymbol{\beta}_1^{LIQ} = \boldsymbol{\beta}_2^{LIQ}$, and if other parameters to be introduced later are equal across the states.

The residual vector in Equation (9) is assumed to follow a bivariate Gaussian process with a state-dependent variance-covariance matrix:

$$\boldsymbol{\varepsilon}_t | s_t = \begin{pmatrix} \varepsilon_{S,t} \\ \varepsilon_{L,t} \end{pmatrix} \sim N(0, \boldsymbol{\Omega}_{s_t}), \quad \boldsymbol{\Omega}_{s_t} = \begin{pmatrix} \sigma_{S,s_t}^2 & \rho_{s_t}\sigma_{S,s_t}\sigma_{L,s_t} \\ \rho_{s_t}\sigma_{S,s_t}\sigma_{L,s_t} & \sigma_{L,s_t}^2 \end{pmatrix}. \tag{11}$$

This implies that returns are drawn from a mixture of bivariate normals with a binomial mixing variable. Thus, we allow the return volatility and correlation to vary across the two states.

Finally, we assume that the state transition is governed by a Markov switching probability:

$$\Pr(s_t = s | s_{t-1} = s; STOV_{t-1}) = \frac{\exp(c_{s_t} + d_{s_t} \cdot STOV_{t-1})}{1 + \exp(c_{s_t} + d_{s_t} \cdot STOV_{t-1})}, \quad s = 1, 2, \tag{12}$$

where $STOV$ is the detrended aggregate share turnover and d_{s_t} and c_{s_t} are scalars. Based on the preference uncertainty model, $STOV$, a measure of trading volume, is used as a state variable to help predict liquidity-beta states. The exponential transformation ensures that the transition probability always falls between 0 and 1. The construction of $STOV$ is explained in Section 2.3.

Several comments about the above formulation follow. First, we have allowed only two possible states, $s_t = 1$ or 2. This allows us to easily classify them as either a high or a low liquidity-beta state. More importantly, it is consistent with our preference uncertainty model in which there are two distinct equilibria—fully and partially revealing equilibria—corresponding to the low and high preference uncertainty states.[6]

Second, we use excess returns on the CRSP size-sorted portfolios as dependent variables. The use of the standard dataset facilitates the comparison of our results to the existing literature, such as Perez-Quiros and Timmermann (2000) and Amihud (2002). For instance, we will later show that the driving force behind our pricing results is distinct from the credit risk that the former authors emphasize. In addition, we note that our basic asset pricing results are robust to the use of illiquidity-sorted portfolios, specifically those sorted on Amihud's (2002) price-impact measure (results not reported).

Third, the model imposes common states only for the smallest and largest deciles for computational reasons. Our simple bivariate model with only one state variable already has 18 parameters, and the number of parameters increases quadratically with the number of assets; an n variate version of our regime-switching model with two states and a single state variable involves $n^2 + 5n + 4$ parameters. In a later section, we will provide suggestive evidence to support the use of the two extreme deciles; that is, liquidity betas in each of the two identified states vary almost monotonically across 10 portfolios sorted on illiquidity.

Fourth, Equation (9) does not control for factors typically used in asset pricing tests, especially the market return. Besides being parsimonious, this is appropriate and preferred for the purpose of state identification. For example, if we included the market return, the estimated liquidity betas would be those conditional on it. Rather, we know from the existing studies that unexpected liquidity shocks and the market return are correlated [see, e.g., Amihud (2002)], and we consider the level of the market return to be an important characteristic of the high liquidity-beta state.[7] Needless to say, however, we will rigorously control for known factors in our asset pricing tests.

Lastly, Equation (12) uses a measure of unsigned volume, *STOV*, to identify liquidity-beta states. The analytical example in Section 1 illustrated that a rise in preference uncertainty is signaled by increased sell orders from the long-term investors. Identifying these trades would require data containing both the sign of trades and the identity of traders. Order imbalance data will provide at least the former information, but its use will restrict the sample to a significantly shorter period. We therefore use unsigned volume, which may also include trades of investors who are not concerned with counterparty preferences. Such trades can have a very different effect on the level of liquidity. For example, volume caused by noise traders as in Kyle (1985) will decrease price impact

[6] It is important to model liquidity betas in discrete states. A natural alternative is to model liquidity betas as a continuous function of the lagged state variable, $STOV_{t-1}$, and include the term $(\beta_i^{LIQ} + \beta_i^{STOV} STOV_{t-1})LIQ_t$ in the return generating process. This specification resulted in an insignificant premium on the scaled factor, $STOV_{t-1} \cdot LIQ_t$, controlling for the size, book-to-market, and momentum factors, as well as the illiquidity-level characteristic.

[7] Nevertheless, we did estimate the system including the market return in Equation (9). The basic asset pricing results were not altered.

and improve liquidity. In contrast, large selling by the long-term investors in our model will typically lead to high price impact and hence low liquidity under plausible parameter values, as shown by Gallmeyer, Hollifield, and Seppi (2005). Despite the potential concern, we will later see that our high liquidity-beta months are preceded by periods of deteriorated expected market liquidity, suggesting that *STOV* adequately captures the level of preference uncertainty.

2.2 Construction of liquidity measure

Researchers have proposed several methods to create monthly liquidity series from standard datasets such as CRSP over periods long enough for asset pricing tests. We use Amihud's (2002) price-impact proxy as a measure of illiquidity costs as in Acharya and Pedersen (2005).

Following Amihud (2002), we first compute the price-impact measure for individual stocks, *PRIM*, as:

$$PRIM_{j,t} = \frac{1}{D_{j,t}} \sum_{d=1}^{D_{j,t}} \frac{|r_{j,d,t}|}{v_{j,d,t}}, \tag{13}$$

where $r_{j,d,t}$ and $v_{j,d,t}$ are the return and dollar volume (measured in millions), respectively, of stock j on day d in month t, and $D_{j,t}$ is the number of daily observations during that month.[8] We use ordinary common shares on NYSE and AMEX with $D_{j,t} \geq 15$ and the beginning-of-the-month price between \$5 and \$1,000. We exclude the NASDAQ stocks because their daily return and volume data are unavailable prior to November 1982 and their reported volumes are overstated due to the inclusion of inter-dealer trades.[9]

The aggregate price impact, *APRIM*, is simply the cross-sectional average of individual *PRIM*:

$$APRIM_t = \frac{1}{N_t} \sum_{j=1}^{N_t} PRIM_{j,t},$$

where N_t is the number of stocks included in month t, which ranges between 1282 and 2129 from August 1962 through December 2004. Although the above formulation is based on an equally weighted average, we obtain qualitatively similar asset pricing results when *APRIM* is computed on a value-weighted basis.[10] To extract innovations in *APRIM*, we fit the following modified AR(2)

[8] To avoid confusion with our liquidity factor, *LIQ*, and its variants to be introduced, we use the notation *PRIM* for Amihud's (2002) illiquidity ratio, $ILLIQ$.

[9] Also, including NASDAQ stocks in the construction of the liquidity series created a jump in its variability in November 1982 and ruined the estimation of our regime-switching model. As we will show later, however, including them in *test assets* does not alter the qualitative results of our asset pricing tests.

[10] The results are available upon request.

model using the whole sample[11]:

$$\left(\frac{mcp_{t-1}}{mcp_1}APRIM_t\right) = \alpha + \beta_1\left(\frac{mcp_{t-1}}{mcp_1}APRIM_{t-1}\right) + \beta_2\left(\frac{mcp_{t-1}}{mcp_1}APRIM_{t-2}\right) + \varepsilon_t, \tag{14}$$

where mcp_{t-1} is the total market capitalization at month $t-1$ of the stocks included in the month-t sample and mcp_1 is the corresponding value for the initial month, August 1962. In the above formulation, the ratio $\frac{mcp_{t-1}}{mcp_1}$ serves as a common detrending factor that controls for the time trend in *APRIM*.[12] Throughout the equation, the same factor is multiplied to the contemporaneous and lagged *APRIM* to capture only the innovations in illiquidity; multiplying lags of $\frac{mcp_{t-1}}{mcp_1}$ may introduce shocks induced mechanically by price changes. This adjustment is used by Pastor and Stambaugh (2003) and Acharya and Pedersen (2005).

The errors in Equation (14) are a measure of unexpected illiquidity shocks. We use the negative estimated residuals, $-\widehat{\varepsilon}_t$, as our liquidity measure, LIQ_t, which starts in October 1962. Panel A of Figure 2 plots the time series of *LIQ* with dotted vertical lines representing the NBER business cycle dates (each narrow band corresponds to a contraction). The time series is plotted from January 1965 through December 2004, which corresponds to the sample period for the estimation of the regime-switching model and asset pricing tests in later sections. Occasional plunges in *LIQ* coincide with major economic crises, such as the Penn Central commercial paper debacle of May 1970, the oil crisis of November 1973, the stock market crash of October 1987, the Asian financial crisis of 1997, and the Russian bond default of 1998.

2.3 Construction of volume measure

This section describes the construction of aggregate share turnover, *STOV*, used as the predictive variable in our regime-switching model. *STOV* is given by a trend-adjusted cross-sectional average of individual share turnovers, using the same sample stocks as in the *LIQ* construction. Specifically, we compute the raw aggregate share turnover for month t, $ATOV_t$, as the equally weighted average of individual stocks' share turnovers, which are defined as the time-series average of CRSP daily share volume divided by the number of shares outstanding during month t.[13] To control for the upward trend in *ATOV*, we

[11] *APRIM* is highly persistent and has first- and second-order autocorrelations of 0.878 and 0.858, respectively. The residuals from the modified AR(2) model and hence *LIQ*, the liquidity-shock measure to be defined below, have a first-order autocorrelation of only −0.020.

[12] *APRIM* has generally decreased over time with an exception from the late 1990s through the early 2000s when market volatility increased substantially. The declining trend is due to a steady increase in trading volume.

[13] Since the number of shares outstanding is recorded in units of 1000 in CRSP, $ATOV = 1$ implies that 0.1% of the outstanding shares are traded.

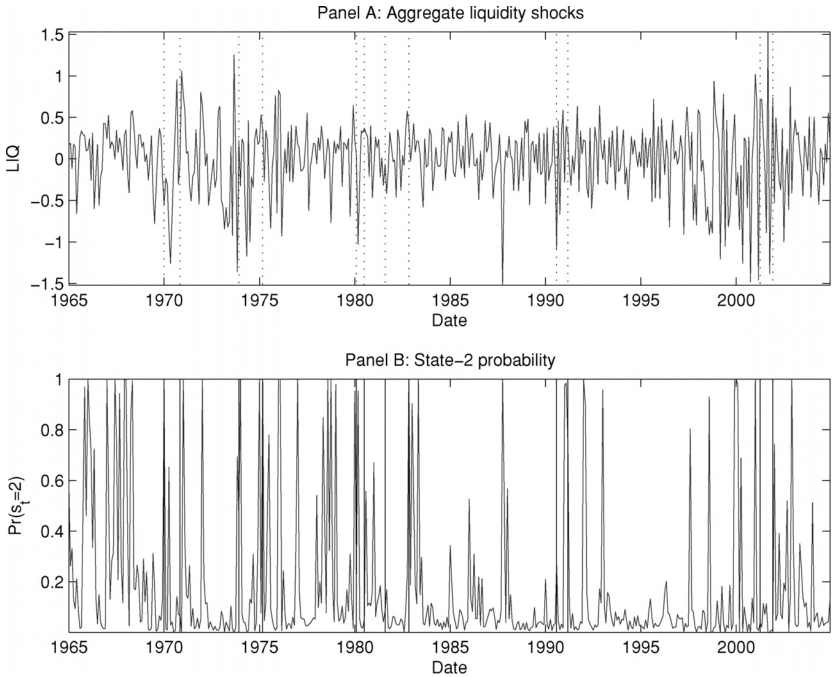

Figure 2
Time-series plots of aggregate liquidity shocks and the estimated probability of being in State 2
Panel A: The aggregate liquidity-shock series, *LIQ*, is given by the negative residuals from the modified AR(2) model for the aggregate Amihud (2002) price-impact proxy in Equation (14). Panel B: The probability of being in State 2 is calculated from the bivariate regime-switching model in Equations (9)–(12). The NBER business cycle dates are indicated by dotted vertical lines (each narrow band represents a recession). The sample period is from January 1965 to December 2004.

follow Eckbo and Norli (2002) and scale the series by a factor, $\frac{ATOV_1}{MATOV_t}$, where $ATOV_1$ is its initial value in July 1962 and $MATOV_t$ is the last 24-month moving average of $ATOV_t$. The detrended aggregate share turnover, $STOV$, is then given by $STOV_t = \frac{ATOV_1}{MATOV_t} ATOV_t$ and starts in July 1964.

2.4 Estimation of high liquidity-beta state

Table 1 reports the estimated parameters of the bivariate regime-switching model in Equations (9)–(12). The estimation is by maximum likelihood using a monthly sample from January 1965 through December 2004.

Consistent with Amihud (2002) and Acharya and Pedersen (2005), we observe significant positive relationships between liquidity shocks and contemporaneous returns, as indicated by the positive estimates of liquidity betas for the two extreme size-decile portfolios in both states ($\beta_{i,1}^{LIQ} > 0$ and $\beta_{i,2}^{LIQ} > 0$, $i = S, L$). In addition, we find that the liquidity betas of the two portfolios vary across two distinct states: State 1 with low liquidity betas and State 2 with high liquidity betas ($\beta_{i,1}^{LIQ} < \beta_{i,2}^{LIQ}$, $i = S, L$). The likelihood ratio tests strongly

Table 1
Estimated parameters of the bivariate regime-switching model

	Size: Smallest ($i = S$)		Largest ($i = L$)			
Parameters					**Common Parameters**	
$\alpha_{i,1}$	0.001	(0.41)	0.006	(3.01)	c_1	5.341 (3.38)
$\alpha_{i,2}$	0.056	(3.89)	−0.006	(−0.89)	c_2	0.365 (0.22)
$\beta_{i,1}^{LIQ}$	0.053	(8.15)	0.021	(4.58)	d_1	−22.750 (−2.44)
$\beta_{i,2}^{LIQ}$	0.192	(6.31)	0.085	(6.50)	d_2	− 3.324 (−0.36)
$\sigma_{i,1}$	0.046	(21.24)	0.035	(22.16)	ρ_1	0.565 (9.71)
$\sigma_{i,2}$	0.109	(10.05)	0.047	(9.02)	ρ_2	0.236 (1.97)
LR Tests					**Common LR Tests**	
$\sigma_{i,1} = \sigma_{i,2}$	115.63	(0.000)	0.42	(0.516)	$d_1 = d_2 = 0$	6.08 (0.048)
$\beta_{i,1}^{LIQ} = \beta_{i,2}^{LIQ}$	11.74	(0.001)	8.17	(0.004)	$\beta_{S,2}^{LIQ} - \beta_{S,1}^{LIQ} = \beta_{L,2}^{LIQ} - \beta_{L,1}^{LIQ}$	6.85 (0.009)
Max LK (Per Period)	1604.5 (3.34)					
Sample Period (*N Obs*)	196501:200412 (480)					

This table shows estimated parameters and corresponding t-statistics (in parentheses) of the bivariate regime-switching model,

$$\boldsymbol{r}_t = \boldsymbol{\alpha}_{s_t} + \boldsymbol{\beta}_{s_t}^{LIQ} LIQ_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t | s_t \sim N(0, \boldsymbol{\Omega}_{s_t}),$$

$$\Pr(s_t = s | s_{t-1} = s; STOV_{t-1}) = \frac{\exp(c_{s_t} + d_{s_t} \cdot STOV_{t-1})}{1 + \exp(c_{s_t} + d_{s_t} \cdot STOV_{t-1})}, \qquad s = 1, 2,$$

where $\boldsymbol{r}_t = \binom{r_{S,t}}{r_{L,t}}$ is the 2-by-1 vector of excess returns of the NYSE and AMEX smallest (S) and largest (L) size-decile portfolios; $\boldsymbol{\varepsilon}_t = \binom{\varepsilon_{S,t}}{\varepsilon_{L,t}}$ is the vector of residuals; $\boldsymbol{\alpha}_{s_t} = \binom{\alpha_{S,s_t}}{\alpha_{L,s_t}}$, $\boldsymbol{\beta}_{s_t}^{LIQ} = \binom{\beta_{S,s_t}^{LIQ}}{\beta_{L,s_t}^{LIQ}}$, and $\boldsymbol{\Omega}_{s_t} = \begin{pmatrix} \sigma_{S,s_t}^2 & \rho_{s_t}\sigma_{S,s_t}\sigma_{L,s_t} \\ \rho_{s_t}\sigma_{S,s_t}\sigma_{L,s_t} & \sigma_{L,s_t}^2 \end{pmatrix}$ are the state-dependent vectors of intercepts, liquidity betas, and the variance-covariance matrix of residuals, respectively; LIQ_t is the liquidity factor; $STOV_{t-1}$ is the lag of detrended aggregate share turnover; and $s_t = 1, 2$ represents states. The table also shows chi-square statistics and p-values (in parentheses) for the likelihood ratio tests (LR Tests) on various parameter restrictions; Max LK is the maximized log likelihood; and *N Obs* is the number of observations.

reject the null hypothesis that $\beta_{i,1}^{LIQ} = \beta_{i,2}^{LIQ}$ with p-values below 1% for both deciles. We therefore call States 1 and 2 the low and the high liquidity-beta states, respectively, and use these terms interchangeably in the rest of the paper.

Table 1 also shows that *STOV* contributes significantly to the identification of the liquidity-beta states. The restriction that $d_1 = d_2 = 0$ is rejected at the 5% level of significance. A significant negative d_1 coefficient on *STOV* implies that large abnormal volume tends to reduce the probability of staying in the low liquidity-beta state and consequently moves the economy to the high liquidity-beta state. This is consistent with the view that trading volume serves as a proxy for the level of preference uncertainty. Hence, in support of our first hypothesis, we find evidence that liquidity betas are time varying and that the transition from the low to the high liquidity-beta state is predicted by a rise in trading volume.

Furthermore, Table 1 documents that the two states are characterized by differences in volatility and persistence. The high liquidity-beta state has higher volatility than the low liquidity-beta state ($\sigma_{i,1} < \sigma_{i,2}, i = S, L$); the likelihood ratio test rejects the null of equal volatility ($\sigma_{i,1} = \sigma_{i,2}$) for the smallest decile. This is a manifestation of the well-known conditional heteroskedasticity in stock returns.[14] Moreover, the high liquidity-beta state is less persistent than the low liquidity-beta state. The average durations of the low and high liquidity-beta states evaluated at mean *STOV* are 8.3 and 1.7 months, respectively.[15] This point is graphically demonstrated in Panel B of Figure 2, which plots the estimated probability of being in the high liquidity-beta state. It is visually clear that whenever the probability approaches 1, it drops back quickly within a few months. The short duration of the high liquidity-beta state is consistent with the feature of an "abnormal" state. The sharp spikes in the probability coincide with some of the economic crises noted earlier, such as the stock market crash of October 1987 and the Russian bond default of August 1998. However, they also include non-crisis periods, and in fact many of them do not correspond to the LIQ plunges in Panel A. We will later observe that the high liquidity-beta state is a mixture of months with large positive and negative liquidity shocks accompanied by heavy trading volume.

A notable additional finding from Table 1 is the cross-sectional difference in liquidity-beta spreads. The spread between the two states is 0.139 (= 0.192 − 0.053) for the smallest decile and 0.064 (= 0.085 − 0.021) for the largest. The likelihood ratio test on the equality of the spreads, $\beta^{LIQ}_{S,2} - \beta^{LIQ}_{S,1} = \beta^{LIQ}_{L,2} - \beta^{LIQ}_{L,1}$, shows that the cross-sectional difference in the spreads is significant at the 1% level. This indicates that liquidity betas of small illiquid stocks exhibit greater variation across the high and low liquidity-beta states, suggesting that they are more sensitive to the state of preference uncertainty. Amihud (2002) and Acharya and Pedersen (2005) previously find that small illiquid stocks have higher liquidity betas than large liquid stocks on average. Our finding adds to theirs by documenting new evidence that such cross-sectional asymmetry in liquidity betas exhibits state-dependent time variation.

3. Time Variation in Liquidity Risk Premium

This section tests our second hypothesis by investigating whether the liquidity risk premium varies across the two liquidity-beta states.

3.1 Estimation of conditional liquidity risk premium

Consider the standard beta pricing model:

$$\boldsymbol{E}[r_{i,t}] = \alpha_i + \sum_{k=1}^{K} \beta_i^k \lambda^k, \tag{15}$$

[14] Vayanos (2004) provides a model that links the market volatility to illiquidity premium.

[15] The mean of *STOV* is $\overline{STOV} = 0.1472$ and 0.1979 for States 1 and 2, respectively (see Panel B(i) of Table 4). Using the parameter estimates from Table 1, $c_s + d_s \cdot \overline{STOV}$ is $1.992 \equiv f_1$ for State 1 and $-0.293 \equiv f_2$ for State 2. This gives $\frac{e^{f_1}}{1+e^{f_1}} = 0.880 \equiv p$ and $\frac{e^{f_2}}{1+e^{f_2}} = 0.427 \equiv q$. The average durations in the text are given by $\frac{1}{1-p}$ and $\frac{1}{1-q}$.

where $r_{i,t}$ is asset i's return; α_i is a constant; β_i^k is the loading of asset i on the kth factor; and λ^k is the risk premium for the kth factor. Cochrane (2005) shows the equivalence between a beta pricing model of the form in Equation (15) and a discounting factor model $\boldsymbol{E}[m_{t+1}(1 + r_{i,t+1})] = 1$, where the SDF, m_{t+1}, is a linear function of the K factors with constant coefficients. Since our model implies an SDF with state-dependent factor coefficients as in Equation (6), it can be modeled as a function of scaled factors. For parsimony, we scale only the liquidity factor and allow the liquidity betas to vary discontinuously across two distinct states. Specifically, we define a conditional liquidity factor:

$$CLIQ_t = I_t \cdot LIQ_t, \tag{16}$$

where I_t is an indicator variable that takes a value of 1 if the estimated probability of being in State 2 is higher than 0.75 in month t, and 0 otherwise.[16] We regard month t as being in the high liquidity-beta state if $I_t = 1$. Out of 480 months from January 1965 through December 2004, there are 43 such months. While the choice of the probability threshold is somewhat arbitrary, we will later confirm that the use of other reasonable levels between 0.5 and 1 leads to similar asset pricing results.

To understand the role of $CLIQ$, consider a simple return generating process:

$$r_{i,t} = \alpha_i + \beta_i^{MKT} MKT_t + \beta_i^{LIQ} LIQ_t + \beta_i^{CLIQ} CLIQ_t + \varepsilon_{i,t}, \tag{17}$$

where MKT is the excess return on the CRSP value-weighted portfolio. The above equation can be rewritten as:

$$r_{i,t} = \alpha_i + \beta_i^{MKT} MKT_t + \left(\beta_i^{LIQ} + \beta_i^{CLIQ} I_t\right) LIQ_t + \varepsilon_{i,t}. \tag{18}$$

The term $\beta_i^{CLIQ} I_t$ captures the time variation in the liquidity beta and makes the beta effectively β_i^{LIQ} in State 1 and $\beta_i^{LIQ} + \beta_i^{CLIQ}$ in State 2. Given the nature of State 2 as the high liquidity-beta state, we expect β_i^{CLIQ} to be positive. We call β_i^{CLIQ} the conditional liquidity beta.

We follow the standard Fama-MacBeth (1973) two-pass procedure to estimate the premia in Equation (15). We use the entire sample in the first-pass beta estimation; the rolling beta approach is redundant because the liquidity beta is effectively time varying as in Equation (18). Also, it is inappropriate in our study that makes use of an indicator variable.[17] To account for the possible errors-in-variables problem, we employ the correction for the standard errors proposed by Shanken (1992).

[16] The probability of being in State 2 is calculated as the standard probabilistic inference in Hamilton (1994, Section 22.4) using all observations. We use its indicator as a scaling variable because the transition probability is based on lagged *STOV* [see Equation (12)].

[17] Specifically, I_t can be 0 for some rolling periods, and consequently the regressor matrix can become singular. A subperiod test is not appropriate either because I_t has a limited number of non-zero entries (43 months), as noted earlier.

3.2 Characteristics of liquidity portfolios

We form our test portfolios by sorting NYSE and AMEX stocks on the book-to-market ratio (B/M) and the price-impact proxy (*PRIM*) using NYSE breakpoints.[18] This accords in spirit with the usual practice of forming test assets on the basis of size and B/M, while recognizing the fact that the size and illiquidity characteristics are cross-sectionally highly correlated. Balancing the desire to produce sufficient dispersion in liquidity betas and to represent as diverse cross sections as possible, we replace the sorting by size with that by *PRIM*. Later, however, we will explicitly include size in the sorting keys to make sure that this does not unduly favor the results. The portfolios are formed at the end of each year from 1964 through 2003, and the value-weighted monthly portfolio returns are calculated for the subsequent months from January 1965 through December 2004. Stocks are admitted to portfolios if they have the end-of-the-year prices between \$5 and \$1000 and more than 100 daily-return observations during the year.

We first look at the characteristics of the *PRIM*-sorted decile portfolios in Panel A of Table 2. Rank 1 represents the lowest *PRIM* (most liquid) and rank 10 the highest (most illiquid). Appendix A.3 describes the computation of portfolio characteristics. Consistent with our prior knowledge about illiquidity premium, the average raw return (r) generally increases with *PRIM*, and so does a return volatility measure (σ^r). A monotonic increase in *PRIM* and the absolute value of Pastor and Stambaugh's (2003) return-reversal measure ($Gamma$) indicates that we continue to get dispersion in illiquidity levels after portfolio formation. All the other characteristics exhibit expected near-monotonic relations with the *PRIM* ranking; more illiquid stocks tend to have more volatile *PRIM* (σ^{PRIM}), lower turnover ratio (*TOV*, although the relation is not strictly monotone), lower price (*PRC*), smaller market capitalization (Size), and greater B/M. These results suggest that it is important to control for the size and value factors as well as the illiquidity-level characteristic in our asset pricing test. The disproportionately large average number of stocks (N) in the most illiquid portfolio indicates that AMEX stocks are much less liquid than NYSE stocks in general.

The last two columns show the estimated portfolio betas with respect to *LIQ* and *CLIQ* in Equation (17). Both β^{LIQ} and β^{CLIQ} increase almost monotonically with *PRIM*. The two betas are negative for the most liquid portfolio because they are estimated conditional on *MKT* in the multiple regression; the adjustment for *MKT* reduces the loadings on the two liquidity factors since *MKT* and *LIQ* are positively correlated. Hence, the result that $\beta^{CLIQ} < \beta^{LIQ}$ for liquid portfolios is due to this conditioning and does not invalidate the nature of State 2 as the high liquidity-beta state. Importantly, the cross-sectional dispersion in β^{CLIQ} is much larger than the dispersion in β^{LIQ}; the difference between the two extreme *PRIM* deciles is 0.067 for β^{CLIQ}, as opposed to 0.034 for β^{LIQ}. Since β^{CLIQ} is

[18] We will later include NASDAQ stocks in our test portfolios and also use individual stocks as test assets.

Table 2
Characteristics of the liquidity-sorted portfolios

Panel A: *PRIM*10 portfolios

	r	σ^r	*PRIM*	σ^{PRIM}	*Gamma*	*TOV*	*PRC*	Size	B/M	*N*	β^{LIQ}	β^{CLIQ}
1	0.0089	0.0155	0.004	0.003	0.00005	1.91	72.8	13628	0.558	131.1	−0.0052	−0.0106
2	0.0096	0.0171	0.016	0.015	−0.00006	2.47	43.4	3074	0.719	130.7	0.0014	−0.0082
3	0.0106	0.0179	0.029	0.029	−0.00036	2.56	38.1	1661	0.720	133.1	0.0042	0.0031
4	0.0106	0.0185	0.050	0.053	−0.00051	2.54	36.0	1022	0.716	133.7	0.0079	−0.0010
5	0.0106	0.0189	0.080	0.089	−0.00116	2.38	34.3	674	0.722	135.7	0.0113	0.0121
6	0.0111	0.0194	0.123	0.139	−0.00234	2.30	30.8	467	0.738	140.7	0.0110	0.0191
7	0.0110	0.0200	0.188	0.220	−0.00360	2.07	28.9	329	0.751	146.5	0.0153	0.0234
8	0.0120	0.0207	0.294	0.349	−0.00646	1.89	26.0	227	0.802	156.7	0.0227	0.0271
9	0.0130	0.0214	0.511	0.619	−0.00942	1.74	24.7	151	0.863	185.5	0.0233	0.0361
10	0.0133	0.0238	1.567	1.899	−0.02523	1.54	19.4	61	0.949	431.8	0.0291	0.0564
10−1	0.0044	0.0083	1.564	1.895	−0.02529	−0.37	−53.4	−13568	0.391	300.7	0.0343	0.0670

Panel B: B/M5-*PRIM*5 portfolios

(i) r

		PRIM 1	2	3	4	5	5 − 1
B/M	1	0.0088	0.0088	0.0095	0.0100	0.0109	0.0021
	2	0.0087	0.0100	0.0094	0.0110	0.0118	0.0031
	3	0.0102	0.0114	0.0116	0.0100	0.0131	0.0028
	4	0.0107	0.0119	0.0123	0.0136	0.0143	0.0036
	5	0.0106	0.0155	0.0146	0.0145	0.0162	0.0056
	5-1	0.0018	0.0067	0.0050	0.0045	0.0053	

(ii) *PRIM*

		PRIM 1	2	3	4	5	5 − 1
B/M	1	0.005	0.038	0.098	0.239	0.915	0.910
	2	0.006	0.038	0.100	0.238	0.919	0.913
	3	0.007	0.036	0.097	0.230	0.989	0.981
	4	0.008	0.037	0.097	0.229	1.017	1.008
	5	0.010	0.036	0.095	0.229	1.159	1.148
	5 − 1	0.005	−0.001	−0.004	−0.011	0.244	

(*Continued on next page*.)

Table 2
Continued

Panel B: B/M5-*PRIM*5 portfolios

(iii) β^{LIQ}		*PRIM* 1	2	3	4	5	5 − 1
B/M	1	−0.005	0.007	0.015	0.024	0.034	0.039
	2	−0.003	0.007	0.011	0.018	0.030	0.033
	3	0.000	0.003	0.011	0.020	0.024	0.024
	4	0.000	0.003	0.008	0.013	0.022	0.023
	5	−0.001	0.008	0.008	0.016	0.023	0.024
	5−1	0.004	0.001	−0.007	−0.008	−0.011	

(iv) β^{CLIQ}		*PRIM* 1	2	3	4	5	5 − 1
B/M	1	−0.015	−0.011	0.005	0.019	0.027	0.042
	2	−0.010	0.004	0.017	0.026	0.047	0.057
	3	−0.001	0.004	0.019	0.020	0.048	0.048
	4	−0.005	0.005	0.012	0.024	0.054	0.058
	5	0.005	0.030	0.039	0.041	0.062	0.057
	5-1	0.021	0.040	0.034	0.022	0.035	

This table shows the average monthly postranking characteristics of the liquidity-sorted portfolios from January 1965 through December 2004. At the end of each year from 1964 to 2003, NYSE and AMEX stocks are sorted into portfolios using NYSE breakpoints. Panel A employs decile portfolios sorted on the Amihud (2002) price-impact proxy (*PRIM*), and Panel B uses 25 portfolios formed as the cross section of the book-to-market ratio (B/M) and *PRIM* quintiles. The following portfolio characteristics are calculated each month as the value-weighted cross-sectional averages of the corresponding member-stock characteristics: r is the CRSP monthly stock return; σ^r is the standard deviation of the CRSP daily return; *PRIM* is the average daily price-impact proxy; σ^{PRIM} is the standard deviation of the daily price-impact proxy; *Gamma* is the return-reversal measure given by the γ coefficient from the daily regression of Pastor and Stambaugh [2003, Equation (1)]; *TOV* is the average daily share turnover; and *PRC* is the CRSP month-end price. Size and B/M are computed following Fama and French (1993). N is the number of stocks in each portfolio. β^{LIQ} and β^{CLIQ} are the liquidity and conditional liquidity betas, respectively, calculated using excess portfolio returns as in Equation (17).

the beta spread between the two states [see Equation (18)], the dispersion in State-2 liquidity betas between the two extreme deciles is the sum of these two numbers, 0.101.

Panel B of Table 2 reports selected summary statistics for the 25 portfolios formed as the cross section of B/M and *PRIM* quintiles (rank 1 has the lowest value of each characteristic). In Panel B(i), the average raw return generally increases with both B/M and *PRIM*. Moreover, the illiquidity effect is stronger for value stocks (in the bottom rows) and the B/M effect is strong except for the most liquid stocks (except for the left-most column), with a caveat that we are perhaps partially picking up the size effect. Panel B(ii) confirms that there is dispersion in post-ranking illiquidity levels as measured by *PRIM*. The dispersion is slightly wider for value than growth stocks.

Panels B(iii) and (iv) of Table 2 report estimated portfolio liquidity betas from Equation (17). As expected, holding the B/M rank constant, illiquid stocks have higher β^{LIQ} and β^{CLIQ}. Again, the cross-sectional spread is wider for β^{CLIQ} and ranges between 0.042 and 0.058. Holding the *PRIM* rank constant, however, the two betas behave differently; for the most illiquid portfolio, β^{CLIQ} increases with B/M while the reverse is true for β^{LIQ}. Nevertheless, it is clear that the sum of the two betas exhibits a cross-sectional variation similar to that of β^{CLIQ}.

3.3 Main results

Using the 25 B/M-*PRIM*-sorted portfolios, we estimate factor premia each month by a cross-sectional regression in Equation (15). Table 3 reports the time-series means of the percentage premia and their *p*-values in parentheses

Table 3
Estimated premia

	1		2		3		4	
Constant	2.04***	(0.000)	0.69	(0.199)	0.36	(0.532)	0.38	(0.531)
MKT	−1.60***	(0.004)	− 0.14	(0.831)	0.15	(0.824)	0.12	(0.860)
LIQ	7.20	(0.186)	−11.57	(0.141)	−12.79	(0.245)	−11.05	(0.315)
CLIQ			12.49***	(0.001)	10.09***	(0.006)	10.23***	(0.008)
SMB					0.18	(0.532)	0.18	(0.555)
HML					0.52*	(0.077)	0.53*	(0.071)
UMD					1.16	(0.367)	1.12	(0.369)
PRIM							0.13	(0.183)
Sample Period					196501:200412			
N Obs					480			

This table shows estimated monthly percentage premia. Test assets are the 25 portfolios of NYSE and AMEX stocks formed as the cross section of quintiles sorted on the B/M ratio and the Amihud (2002) price-impact proxy. *MKT* is the excess return on the CRSP value-weighted portfolio; *LIQ* is the liquidity factor; *CLIQ* is the conditional liquidity factor given by the product of an indicator variable *I* and *LIQ*, where *I* takes the value of 1 if the probability of being in the high liquidity-beta state from the bivariate regime-switching model is higher than 0.75 and 0 otherwise; *SMB*, *HML*, and *UMD* are the size, book-to-market, and momentum factors, respectively; and *PRIM* is the lagged price-impact proxy of the test portfolios. The estimation follows the Fama-MacBeth (1973) two-pass procedure with the Shanken (1992) correction for standard errors. The estimated premia and *p*-values are reported in parentheses. *, **, *** represent significance at 10%, 5%, and 1%, respectively. *N Obs* is the number of observations.

with the Shanken (1992) correction for standard errors. Column 1 shows the simplest model including only *MKT* and *LIQ*. Consistent with existing studies, the *LIQ* premium is positive although insignificant. A significant positive constant and a negative *MKT* premium, however, suggest omitted factors. Column 2 introduces *CLIQ*, which corresponds to the model in Equation (17). While the *LIQ* premium changes its sign and remains insignificant, *CLIQ* carries a significantly positive premium. Moreover, both the constant and the *MKT* premium have now become insignificant. Column 3 adds the size (*SMB*), value (*HML*), and momentum (*UMD*) factors. Although *HML* enters significantly, *CLIQ* survives the inclusion of these factors commonly used in the asset pricing literature.

Acharya and Pedersen (2005) emphasize the importance of controlling for the level of illiquidity in estimating the liquidity risk premium. To accommodate this point, we include lagged portfolio *PRIM* as a characteristic of test assets on the right-hand side of Equation (15). Column 4 of Table 3 indicates that the *CLIQ* premium is robust to the inclusion of the illiquidity-level characteristic and is estimated to be 10.2% per unit beta. Note that *CLIQ* is a nontraded factor, and hence its premium is not directly comparable to others. We will later examine the economic significance of the *CLIQ* premium using a factor-mimicking portfolio. We will also control for other factors and firm characteristics in the robustness test section.

The significant positive premium on *CLIQ* supports our second hypothesis that the liquidity risk premium is time varying and is greater during periods of high preference uncertainty. Combined with our results from the previous section, we find that both high liquidity betas and a large liquidity risk premium arise simultaneously during periods of high trading volume.

3.4 Illiquidity premium and liquidity risk premium

This section examines the characteristics of the high liquidity-beta state with a particular focus on the state-dependent relationship between illiquidity premium (premium on illiquid assets) and liquidity risk premium (premium on liquidity-sensitive assets). Panel A of Table 4 reports average monthly excess returns of the *PRIM*-sorted decile portfolios by state. Consistent with the notion of illiquidity premium, the unconditional return increases with the level of illiquidity (in the column labeled "All"). This pattern, however, is not present in the low liquidity-beta state ("State 1"); the illiquidity-return relation is roughly flat with a zero-return spread between the least and the most liquid portfolios. In contrast, the return exhibits a strong monotonic relation with illiquidity in the high liquidity-beta state ("State 2"); the return spread between the two extreme deciles is striking at 5.4% per month. The return difference across the states ("State 2 − 1") is also monotonically increasing in illiquidity and

Table 4
Monthly means of the liquidity-sorted excess decile-portfolio returns and selected variables by state

Panel A: *PRIM*-sorted excess decile-portfolio returns

	All	State 1	State 2	State 2 − 1	*p*-Value
1	0.0037	0.0049	−0.0083	−0.0132	(0.053)
2	0.0046	0.0050	0.0000	−0.0050	(0.495)
3	0.0057	0.0058	0.0041	−0.0017	(0.827)
4	0.0057	0.0056	0.0064	0.0008	(0.915)
5	0.0056	0.0048	0.0142	0.0094	(0.235)
6	0.0062	0.0051	0.0166	0.0114	(0.161)
7	0.0062	0.0049	0.0195	0.0146	(0.076)
8	0.0071	0.0052	0.0259	0.0207	(0.014)
9	0.0080	0.0055	0.0334	0.0278	(0.001)
10	0.0085	0.0049	0.0453	0.0404	(0.000)
10 − 1	0.0048	0.0000	0.0536	0.0536	(0.000)

Panel B: Selected aggregate variables

B(i) All *LIQ*

	All	State 1	State 2	State 2 − 1	*p*-Value
LIQ	−0.0040	−0.0137	0.0945	0.1082	(0.121)
MKT	0.0045	0.0045	0.0042	−0.0003	(0.966)
STOV	0.1518	0.1472	0.1979	0.0507	(0.000)
RREV	−0.0308	−0.0287	−0.0518	−0.0231	(0.006)
SMB	0.0027	−0.0011	0.0414	0.0425	(0.000)
HML	0.0044	0.0036	0.0129	0.0093	(0.051)
UMD	0.0085	0.0094	−0.0006	−0.0100	(0.132)

B(ii) State 2^+ ($LIQ > 0$)

	State 2^+	State $2^+ - 1$	*p*-Value
LIQ	0.3756	0.3893	(0.000)
MKT	0.0347	0.0301	(0.000)
STOV	0.2145	0.0673	(0.000)
RREV	−0.0275	0.0012	(0.898)
SMB	0.0516	0.0527	(0.000)
HML	0.0123	0.0087	(0.121)
UMD	−0.0007	−0.0102	(0.183)

B(iii) State 2^- ($LIQ < 0$)

	State 2^-	State $2^- - 1$	*p*-Value
LIQ	−0.4877	−0.4740	(0.000)
MKT	−0.0589	−0.0634	(0.000)
STOV	0.1635	0.0163	(0.055)
RREV	−0.1020	−0.0734	(0.000)
SMB	0.0204	0.0215	(0.011)
HML	0.0142	0.0106	(0.184)
UMD	−0.0002	−0.0096	(0.363)

Panel A shows the mean excess returns of NYSE and AMEX decile portfolios sorted on the Amihud (2002) price-impact proxy (*PRIM*). Panel B shows the means of selected aggregate variables. Reported are the means for the full sample (in the column labeled "All") and in each state ("State 1" and "State 2"), the differences of the means between the two states ("State 2 − 1"), and the *p*-values of the differences in parentheses. Subpanels B(ii) and B(iii) divide State 2 into months with $LIQ > 0$ ("State 2^+") and months with $LIQ < 0$ ("State 2^-"), respectively. *LIQ* is the liquidity factor; *MKT* is the excess return on the CRSP value-weighted portfolio; *STOV* is the detrended aggregate share turnover; *RREV* is the Pastor and Stambaugh's (2003) aggregate return-reversal measure; and *SMB*, *HML*, and *UMD* are the size, book-to-market, and momentum factors, respectively. The sample period is from January 1965 to December 2004.

becomes significantly positive for the four most illiquid portfolios.[19] Overall, our analysis reveals that a large illiquidity premium arises only in the high liquidity-beta state. Moreover, since we find that the conditional liquidity risk premium is priced significantly controlling for the level of illiquidity (see Column 4 of Table 3), a large portion of what we know as the illiquidity premium appears to be delivered in the form of beta risk premium with respect to the conditional liquidity factor.

Next, we examine whether other variables of interest also differ across states. At a first glance at Panel B(i) of Table 4, State 2 seems to be associated with liquidity shocks (*LIQ*) that are only marginally higher than, and excess market returns (*MKT*) that are insignificantly different from, State 1. This inference, however, is misleading since State 2 is a mixture of relatively large positive and negative *LIQ* months accompanied by elevated trading volume. To confirm this point, subpanels B(ii) and (iii) further divide State 2 into months with positive (29 months, denoted as "State 2^{+}") and negative (14 months, "State 2^{-}") liquidity shocks, respectively (State 1 remains the same). We observe that State-2 months with positive and negative liquidity shocks, on average, experience large positive and negative excess market returns of 3.5% and −5.9%, respectively. This indicates that the high liquidity-beta state is not equivalent to liquidity crises.

Other characteristics of State 2 presented in Table 4 are easier to understand with the above point in mind. First, *STOV* is significantly higher regardless of the sign of *LIQ*, consistent with elevated trading volume signaling the high liquidity-beta state. Second, a significantly lower value of the aggregate return-reversal measure, *RREV*, accompanied by lower *MKT* and higher *STOV* as observed by Pastor and Stambaugh (2003), is present mainly during negative *LIQ* months.[20] Third, *SMB* is significantly higher in State 2 regardless of the sign of *LIQ*; as shown in Panel B(iii), *SMB* is positive even in State 2^{-} because small stocks still earn higher returns than large stocks during months with small negative *LIQ* (not shown). Fourth, *HML* is higher in State 2 than in State 1 [Panel B(i)], although the difference becomes marginal or insignificant when State 2 is broken down by the sign of *LIQ* [Panels B(ii) and (iii)]. Finally, *UMD* is generally lower in State 2 or in its breakdowns by the sign of *LIQ*, but the differences from State 1 are insignificant or at best marginal.

We emphasize that the dominance of positive liquidity shocks in State 2 does not imply high liquidity levels. To examine this point, we conduct block bootstraps and compare the mean levels of expected illiquidity (*EAPRIM*), given by the fitted values from the modified AR(2) model for *ARPIM* in Equation (14) surrounding State 2. Each sampling draws 43 blocks of *EAPRIM* with

[19] In contrast, the difference is significantly negative for the most liquid portfolio. As we will see shortly, this is because State 2 is a mixture of months with large positive and negative liquidity shocks accompanied by elevated trading volume, and the returns on liquid stocks turn out to be negative on average for these months.

[20] Similarly to *LIQ*, *RREV* is given by the residuals from a modified AR(2) model fitted to the equally weighted cross-sectional average of individual stocks' *Gamma* measures.

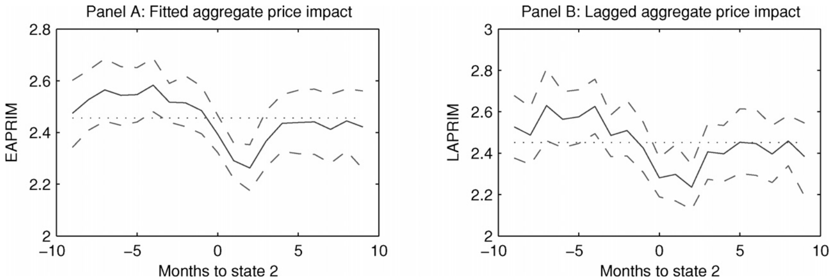

Figure 3
Bootstrap means of expected aggregate illiquidity measures around State 2
Panel A: *EAPRIM* is the fitted values from the modified AR(2) model for the aggregate Amihud (2002) price-impact proxy in Equation (14). Panel B: *LAPRIM* is the lagged aggregate price-impact proxy. The solid lines represent the bootstrap means, dashed lines the 95% confidence intervals, and dotted horizontal lines the block averages. Each sampling draws 43 blocks of an expected illiquidity measure with replacement, with each block starting at nine months prior to and ending at nine months after a high liquidity-beta month. Within each block, the difference between the measure and its block average is computed every month. The differences are then averaged across the 43 blocks month by month. This is repeated 5000 times, and the means and the 95% confidence intervals of the monthly differences across the resamplings are calculated.

replacement, with each block starting at nine months prior to and ending at nine months after a high liquidity-beta state. Within each block, we compute the difference between *EAPRIM* and its block average every month. The differences are then averaged across the 43 blocks month by month. This is repeated 5000 times, and the means and the 95% confidence intervals of the monthly differences across the resamplings are calculated.

Panel A of Figure 3 presents the bootstrap means of *EAPRIM* along with the corresponding confidence intervals. *EAPRIM* rises significantly above the block mean several months prior to the high liquidity-beta state, and then falls significantly below the mean a couple of months afterward before reverting back to the average level. Panel B shows that lagged *APRIM* (*LAPRIM*), another measure of expected illiquidity, also exhibits similar dynamics. Furthermore, the same pattern is observed for the lagged *PRIM* series of the *PRIM*-sorted decile portfolios, indicating the pervasiveness of this illiquidity variation around State 2 (results not reported). These findings suggest that investors may expect deteriorating future liquidity several months prior to the high liquidity-beta state.

3.5 Robustness tests

This section conducts a number of robustness tests on the conditional pricing of liquidity risk. We first control for additional factors and characteristics that are likely to be correlated with liquidity risk. We then examine if our pricing results hold for different test assets and with alternative probability thresholds for state identification. The effect of outliers is also analyzed.

3.5.1 Size and B/M characteristics. Daniel and Titman (1997) argue that the size and B/M premia arise because of such characteristics per se rather

Table 5
Robustness to various factors and characteristics

	1		2		3		4	
Constant	1.18*	(0.080)	0.15	(0.827)	0.53	(0.438)	0.68	(0.315)
MKT	−0.56	(0.408)	0.28	(0.690)	0.13	(0.864)	−0.16	(0.835)
LIQ	−13.54	(0.126)	−8.65	(0.470)	−16.33	(0.123)	−15.49	(0.225)
CLIQ	8.15**	(0.042)	8.97**	(0.031)	6.97*	(0.079)	10.68***	(0.007)
SMB			0.08	(0.813)	−0.01	(0.967)	0.10	(0.764)
HML			0.60**	(0.044)	0.37	(0.170)	0.57*	(0.065)
UMD	1.39	(0.176)	1.46	(0.223)	0.95	(0.279)	1.13	(0.395)
PRIM	0.11	(0.221)	0.13	(0.224)	0.22**	(0.032)	0.01	(0.959)
Size	−0.08*	(0.060)						
B/M	0.13	(0.246)						
CSMB			0.43	(0.441)				
CHML			0.09	(0.784)				
DSV					−0.70	(0.981)		
dVOL					−0.06	(0.597)		
IVAR							8.00	(0.421)
TOV							0.03	(0.673)
Sample Period	196501:200412		196501:200412		197101:199912		196501:200412	
N Obs	480		480		348		480	

This table shows results of the robustness tests to various factors and characteristics. Test assets are the 25 portfolios of NYSE and AMEX stocks formed as the cross section of quintiles sorted on the B/M ratio and the Amihud (2002) price-impact proxy. *MKT* is the excess return on the CRSP value-weighted portfolio; *LIQ* is the liquidity factor; *CLIQ* is the conditional liquidity factor given by the product of an indicator variable I and *LIQ*, where I takes the value of 1 if the probability of being in the high liquidity-beta state from the bivariate regime-switching model is higher than 0.75 and 0 otherwise; *SMB*, *HML*, and *UMD* are the size, book-to-market, and momentum factors, respectively; *PRIM* is the lagged price-impact proxy of the test portfolios; Size and B/M are the lagged market capitalization and book-to-market ratio, respectively, of the test portfolios; *CSMB* (*CHML*) is the conditional size (value) factor given by the product of I and *SMB* (*HML*); *DSV* and *dVOL* are the default and volatility factors, respectively; and *IVAR* and *TOV* are the lagged idiosyncratic variance and share turnover, respectively, of the test portfolios. The estimation follows the Fama-MacBeth (1973) two-pass procedure with the Shanken (1992) correction for standard errors. The estimated monthly percentage premia and p-values are reported in parentheses. *, **, *** represent significance at 10%, 5%, and 1%, respectively. *N Obs* is the number of observations.

than beta risks with respect to the factors. To control for this possibility, we replace *SMB* and *HML* with the lagged size and B/M characteristics of the test portfolios. The results are presented in Column 1 of Table 5. While lowered to about 8% in the presence of the significant size-characteristic effect, the *CLIQ* premium is still significant at the 5% level.

3.5.2 Conditional size and B/M factors. Recall from Panel B(i) of Table 4 that *SMB* and *HML* have higher returns in the high liquidity-beta state. This raises a concern as to whether the observed conditional liquidity effect is a conditional size or B/M effect in disguise. In fact, Guidolin and Timmermann (2005) find that size and value premia vary across economic regimes, while Perez-Quiros and Timmermann (2000) report that the small-firm premium increases sharply during late stages of most recessions. To address this concern, we construct conditional size and value factors as $CSMB_t = I_t \cdot SMB_t$ and $CHML_t = I_t \cdot HML_t$, where I_t is again the indicator variable for the high liquidity-beta state. Column 2 of Table 5 shows that the magnitude of the *CLIQ* premium is about 9% with a 5% significance level upon their inclusion. The premia on *CSMB* and *CHML* are insignificant, suggesting that the liquidity-beta states we identified are distinct from economic regimes underlying the variation in the size and value premia.

3.5.3 Default and aggregate volatility factors. It is sensible to suspect that liquidity risk is correlated with default risk because illiquid firms tend to be small firms that are relatively likely to default. Perez-Quiros and Timmermann (2000) find evidence of asymmetry in the small and large firms' exposures to credit risk across recession and expansion states. To control for the possible effect of default risk, we include Vassalou and Xing's (2004) default factor (*DSV*) in our test.[21] Recall also that our regime-switching model allowed volatility to vary across states. This may introduce a pseudo pricing effect due to the pricing of volatility risk (Ang et al., 2006). To account for this possibility, we additionally include an aggregate volatility factor (*dVOL*) given by the monthly first-order difference in market volatility (*VOL*), where *VOL* is defined as the standard deviation of the daily CRSP value-weighted returns (NYSE and AMEX stocks only) within each month.[22]

Column 3 of Table 5 shows that the *CLIQ* premium is robust to the inclusion of *DSV* and *dVOL*. Note that the availability of *DSV* restricts the sample period to January 1971 through December 1999 (348 observations). In this shorter sample period, the illiquidity characteristic, *PRIM*, is significantly priced. In a separate test, we also examined the robustness against the conditional versions of *DSV* and *dVOL*, constructed similarly to *CSMB* and *CHML*, but the *CLIQ* premium remained significant (results not shown). The robustness of *CLIQ* against the default risk is consistent with the distinction between preference risk and cash flow risk as discussed in Gallmeyer, Hollifield, and Seppi (2005).

3.5.4 Idiosyncratic volatility and volume characteristics. Since Amihud's (2002) price-impact measure is given by the ratio of absolute return to volume, another concern is that the *CLIQ* premium may simply proxy for the pricing of idiosyncratic volatility and/or trading volume, which are shown to predict returns or explain the cross-sectional variation in returns.[23] We therefore include lagged portfolio idiosyncratic variance (*IVAR*) constructed following Goyal and Santa-Clara [2003, Equation (2)] and lagged share turnover (*TOV*) as

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21 We thank Maria Vassalou for making the default factor available on her website.

22 Ang et al. (2006) construct a volatility factor using the *VIX* index from the Chicago Board Options Exchange. We do not use the index, since it restricts the sample to a substantially shorter period.

23 See, among others, Ang et al. (2006), Goyal and Santa-Clara (2003), and Spiegel and Wang (2005) for idiosyncratic volatility, and Brennan, Chordia, and Subrahmanyam (1998) and Lee and Swaminathan (2000) for volume.

additional characteristics.[24] Column 4 of Table 5 indicates that the *CLIQ* premium is again robust and significant in their presence.

3.5.5 Alternative test assets. We also check our results against alternative choices of test assets. We first replace the test portfolios with those formed as the cross section of size, B/M, *PRIM*, and/or *Gamma*, excluding or including NASDAQ stocks. We then extend the test assets to individual stocks.

We start by replacing *PRIM* with *Gamma* as the liquidity sorting key. Also, while illiquidity is cross-sectionally correlated with size as seen in Panel A of Table 2, we now explicitly use size in our sort; it is possible that the *SMB* premium has been insignificant because our previous test assets did not produce enough dispersion in size factor betas. Panel A of Table 6 shows results with the following alternative test assets: Column 1: 25 B/M5-*Gamma*5-sorted portfolios, Column 2: 27 size3-B/M3-*PRIM*3-sorted portfolios, and Column 3: 27 size3-B/M3-*Gamma*3-sorted portfolios.[25] Each of these sorting procedures includes one (il)liquidity measure to produce a cross section that varies in liquidity-risk loadings. The panel indicates that while the *CLIQ* premium drops to 4.5% when the size-B/M-*PRIM* triplets are used, it is still marginally significant at 10.5%. In the other two cases, the premium is estimated to be 8.8% and 6.6% with a 5% level of significance. *HML* is consistently priced with these test assets, which is similar to the findings of Pastor and Stambaugh (2003) and Acharya and Pedersen (2005). Interestingly, the significance of the illiquidity characteristic and the momentum factor varies depending on the choice of the liquidity sorting key; *PRIM* becomes significant when *Gamma* is used, whereas *UMD* becomes significant when *PRIM* is used instead.

Although we have been excluding NASDAQ stocks so far, we now include them in portfolio formation and present the results in Panel B of Table 6. Each column uses the same sorting keys as in Panel A with an addition of the 25 B/M5-*PRIM*5-sorted portfolios in Column 4. We find that the inclusion of NASDAQ stocks raises the *CLIQ* premium in all cases; the premium also becomes more significant except for Column 3, which is still significant at 8%.

Panel C of Table 6 uses individual stocks as test assets. Column 1 employs NYSE and AMEX stocks, and Column 2 additionally includes NASDAQ

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[24] Specifically, we construct a measure of total variance for portfolio i in month t as:

$$IVAR_{i,t} = \frac{1}{N_{i,t}} \sum_{j=1}^{N_{i,t}} [\sum_{d=1}^{D_{j,t}} r_{j,d,t}^2 + 2 \sum_{d=2}^{D_{j,t}} r_{j,d,t} r_{j,d-1,t}],$$

where $r_{j,d,t}$ and $D_{j,t}$ are defined as in Equation (13), and $N_{i,t}$ is the number of stocks in portfolio i. Goyal and Santa-Clara (2003) show that $IVAR_{i,t}$ can be interpreted as a measure of idiosyncratic risk under a certain factor structure in the cross section of returns.

[25] Column 1 uses independent sorts on B/M and *Gamma*. In Columns 2 and 3, 3-by-3 cross sections are formed first by independent sorts on size and B/M, and then within each cross section, stocks are sorted on either *PRIM* or *Gamma*. This method is used because fully independent three-dimensional sorts resulted in some portfolios (specifically, small and liquid, and large and illiquid ones) having missing observations in some months due to the cross-sectional correlation between size and illiquidity.

Table 6
Estimated premia with alternative test assets

Panel A: Alternative portfolios of NYSE and AMEX stocks

	1		2		3	
Constant	0.02	(0.981)	0.51	(0.291)	−0.20	(0.772)
MKT	0.40	(0.566)	−0.11	(0.850)	0.65	(0.390)
LIQ	−7.25	(0.353)	4.26	(0.602)	−7.09	(0.360)
CLIQ	8.81**	(0.049)	4.52	(0.105)	6.58**	(0.043)
SMB	−0.24	(0.435)	0.08	(0.743)	0.07	(0.785)
HML	0.52**	(0.049)	0.67***	(0.004)	0.52**	(0.029)
UMD	1.14	(0.258)	1.73**	(0.015)	0.79	(0.250)
PRIM	0.30**	(0.028)	0.05	(0.577)	0.17**	(0.045)

Panel B: Alternative portfolios of NYSE, AMEX, and NASDAQ stocks

	1		2		3		4	
Constant	0.69	(0.226)	0.89	(0.128)	1.14	(0.156)	0.78	(0.148)
MKT	−0.14	(0.823)	−0.31	(0.644)	−0.58	(0.490)	−0.24	(0.702)
LIQ	−10.58	(0.192)	−7.29	(0.494)	−17.37*	(0.084)	−9.88	(0.446)
CLIQ	9.33**	(0.027)	9.46**	(0.010)	7.16*	(0.073)	10.50***	(0.005)
SMB	0.16	(0.578)	0.27	(0.305)	0.31	(0.249)	0.30	(0.266)
HML	0.44*	(0.064)	0.61**	(0.016)	0.54**	(0.023)	0.51**	(0.037)
UMD	1.05	(0.177)	2.23**	(0.011)	0.66	(0.363)	0.50	(0.527)
PRIM	0.09	(0.297)	−0.03	(0.575)	0.06	(0.326)	0.04	(0.504)

Panel C: Individual stocks

	1		2	
Constant	1.72**	(0.011)	2.15***	(0.010)
MKT	−1.06	(0.193)	−1.46	(0.163)
LIQ	−20.66*	(0.082)	−30.84**	(0.048)
CLIQ	11.94**	(0.014)	12.44**	(0.010)
SMB	0.40	(0.224)	0.47	(0.246)
HML	0.53	(0.136)	0.67	(0.184)
UMD	1.19	(0.397)	1.37	(0.420)
PRIM	−0.09***	(0.000)	−0.09***	(0.000)

This table shows monthly percentage premia estimated using alternative test assets. *MKT* is the excess return on the CRSP value-weighted portfolio; *LIQ* is the liquidity factor; *CLIQ* is the conditional liquidity factor given by the product of an indicator variable I and *LIQ*, where I takes the value of 1 if the probability of being in the high liquidity-beta state from the bivariate regime-switching model is higher than 0.75, and 0 otherwise; *SMB*, *HML*, and *UMD* are the size, book-to-market (B/M), and momentum factors, respectively; and *PRIM* is the lagged Amihud (2002) price-impact proxy of the test portfolios. Panel A gives results for the 25 B/M5-*Gamma*5-sorted portfolios (Column 1), the 27 Size3-B/M3-*PRIM*3-sorted portfolios (Column 2), and the 27 Size3-B/M3-*Gamma*3-sorted portfolios (Column 3) of NYSE and AMEX stocks. *Gamma* is the individual-stock return-reversal measure given by the γ coefficient from the daily regression of Pastor and Stambaugh [2003, Equation (1)]. Panel B reports results using portfolios of NYSE, AMEX, and NASDAQ stocks. Columns 1–3 employ the portfolios using the same sorting keys in the corresponding columns in Panel A and Column 4 the 25 B/M5-*PRIM*5-sorted portfolios. Panel C gives results for individual stocks on NYSE and AMEX (Column 1) and on NYSE, AMEX, and NASDAQ (Column 2). The estimation follows the Fama-MacBeth (1973) two-pass procedure with the Shanken (1992) correction for standard errors. The sample period for all panels is from January 1965 to December 2004 (480 monthly observations). The estimated premia and p-values are reported in parentheses. *, **, *** represent significance at 10%, 5%, and 1%, respectively.

stocks. Since estimates of individual-stock betas are expected to be noisy, we follow Fama and French (1992) and assign portfolio betas to individual stocks. Specifically, we assign the betas of the B/M5-*PRIM*5 portfolios (corresponding to Column 3 of Table 3) to their component stocks. The *PRIM* characteristic is that of each individual stock. The panel shows the estimated *CLIQ* premium of 11.9% when the test assets exclude NASDAQ stocks and 12.4% including them. These numbers are similar in magnitude to what we have reported earlier and are significant almost at the 1% level. We do observe two differences, however. First, the *LIQ* premium is negative and significant, which implies that high liquidity-beta stocks earn significantly less premia than low liquidity-beta stocks during "normal" times. Second, the *PRIM*-characteristic premium is negative and highly significant, not positive; this might be due to the fact that *PRIM* measures for individual stocks are noisy.

3.5.6 Effect of outliers and threshold probability. Given the relatively small number of the high liquidity-beta months, the effect of outliers may be a concern. We therefore employ the following four methods to screen extreme observations of *LIQ*: (i) excluding the top five and the bottom five extreme values of *LIQ*; (ii) excluding the months immediately following those extreme *LIQ* months in (i); (iii) excluding the top and the bottom extreme values of *LIQ* within State 2; and (iv) excluding the top two and the bottom two extreme values of *LIQ* within State 2.

For brevity, the results are summarized here without a table.[26] After screenings (i) and (ii), the number of high liquidity-beta months only slightly dropped to 41 and 42, respectively. The exclusion of at most two high liquidity-beta months implies that they do not necessarily correspond to the months with extreme *LIQ*; recall that the important feature of the high liquidity-beta state is not only large absolute liquidity shocks, but also the company of highly elevated volume. By construction, screenings (iii) and (iv) drop two and four high liquidity-beta months, respectively. After these four screenings, the *CLIQ* premium from our benchmark model (corresponding to Column 4 of Table 3) ranged between 8.8% and 10.2%, all of which were significant.

Next, we also change the probability threshold for State 2 from 0.5 to 0.9 with an increment of 0.1. This exercise serves as an additional test against extreme observations since the number of high liquidity-beta months varies from 37 to as many as 62. Table 7 reports that the *CLIQ* premium is again robust and ranges between 8.8% and 13.2% with a 2% significance level throughout.

3.6 Economic significance of conditional liquidity risk premium

Lastly, we examine the economic significance of the conditional liquidity risk premium. To this end, we first construct a mimicking portfolio (Breeden, Gibbons, and Litzenberger, 1989) of the liquidity factor. We fit a linear regression for *LIQ* without an intercept:

$$LIQ_t = \boldsymbol{b}^{\mathrm{T}} \boldsymbol{r}_t^B + \varepsilon_t, \tag{19}$$

where $\boldsymbol{r}_t^B$ is the vector of excess returns on basis assets and $\boldsymbol{b}$ is a conforming vector of loadings. The return on the liquidity-factor mimicking portfolio (*LMP*)

[26] The results are available upon request.

Table 7
Estimated premia with alternative State-2 probability thresholds

Prob Threshold	0.5		0.6		0.7		0.8		0.9	
Constant	0.43	(0.497)	0.54	(0.388)	0.22	(0.723)	0.32	(0.597)	0.19	(0.742)
MKT	0.04	(0.961)	−0.07	(0.923)	0.29	(0.686)	0.17	(0.796)	0.29	(0.660)
LIQ	−12.24	(0.294)	−12.02	(0.295)	−9.94	(0.381)	−11.32	(0.301)	−11.02	(0.300)
CLIQ	13.23**	(0.013)	12.51**	(0.013)	12.10***	(0.009)	9.87***	(0.009)	8.81**	(0.016)
SMB	0.24	(0.450)	0.24	(0.443)	0.23	(0.465)	0.18	(0.555)	0.16	(0.597)
HML	0.49	(0.104)	0.50*	(0.097)	0.48	(0.113)	0.53*	(0.070)	0.51*	(0.073)
UMD	0.46	(0.736)	0.62	(0.642)	1.15	(0.376)	1.17	(0.344)	1.23	(0.306)
PRIM	0.10	(0.301)	0.09	(0.364)	0.11	(0.246)	0.13	(0.159)	0.15*	(0.099)
N State 2	62		51		47		41		37	
Sample Period					196501:200412					
N Obs					480					

This table shows monthly percentage premia estimated using alternative probability thresholds for State 2 ("Prob Threshold"). Test assets are the 25 portfolios of NYSE and AMEX stocks formed as the cross section of quintiles sorted on the B/M ratio and the Amihud (2002) price-impact proxy. *MKT* is the excess return on the CRSP value-weighted portfolio; *LIQ* is the liquidity factor; *CLIQ* is the conditional liquidity factor given by the product of an indicator variable *I* and *LIQ*, where *I* takes the value of 1 if the probability of being in the high liquidity-beta state from the bivariate regime-switching model is higher than 0.75 and 0 otherwise; *SMB*, *HML*, and *UMD* are the size, book-to-market, and momentum factors, respectively; and *PRIM* is the lagged price-impact proxy of the test portfolios. The estimation follows the Fama-MacBeth (1973) two-pass procedure with the Shanken (1992) correction for standard errors. The estimated premia and *p*-values are reported in parentheses. *, **, *** represent significance at 10%, 5%, and 1%, respectively. *N* State 2 is the number of months in State 2, and *N Obs* is the total number of observations.

is then constructed as:

$$LMP_t = \frac{1}{|\widehat{\boldsymbol{b}}^{\mathrm{T}}\mathbf{1}|}\widehat{\boldsymbol{b}}^{\mathrm{T}}\boldsymbol{r}_t^B, \tag{20}$$

where $\widehat{\boldsymbol{b}}$ is the vector of estimated loadings and $\mathbf{1}$ is the vector of ones. The division by $|\widehat{\boldsymbol{b}}^{\mathrm{T}}\mathbf{1}|$ simply normalizes the loadings so that they have a weight interpretation. Since each excess return is a zero investment portfolio, so is *LMP*. It holds a portfolio of stocks in basis assets financed by a short position in the one-month treasury bill (if $\widehat{\boldsymbol{b}}^{\mathrm{T}}\mathbf{1}$ is positive). We employ 25 liquidity-beta-sorted portfolios of NYSE, AMEX, and NASDAQ stocks as the basis assets. These basis assets are constructed by five-year rolling regressions. Every month starting in September 1967, we regress each excess individual-stock return on a constant, *MKT*, and *LIQ* using 60 monthly observations up to the current month. (The sample period for the first regression starts in October 1962, which matches the beginning of *LIQ*.) Then, stocks are sorted into 25 portfolios on the basis of their *LIQ* betas. The portfolio returns are measured in the next month from October 1967 through December 2004. We use this whole sample to estimate $\widehat{\boldsymbol{b}}$ and construct *LMP*.[27] The conditional liquidity-factor mimicking portfolio return is then given by:

$$CLMP_t = I_t \cdot LMP_t, \tag{21}$$

where I_t is the original indicator variable for the high liquidity-beta state with the 0.75 probability threshold.

Table 8 repeats the asset pricing tests in Table 3 with *LIQ* and *CLIQ* replaced by *LMP* and *CLMP*, respectively. The *CLMP* premium is significant controlling for the size, book-to-market, and momentum factors, as well as the illiquidity-level characteristic. Its monthly premium is 1.5% (Column 4), which is more than twice the estimated book-to-market premium of 0.6%. We argue that this premium is not unreasonably high, given that it is rendered less than one-tenth of the time.

Another way to evaluate the economic significance of *CLIQ* is to use the spread in estimated betas. Summarizing from all of the above analyses, the *CLIQ* premium appears to lie somewhere between 5% and 10%. From the bottom row in Panel B(i) of Table 2, the return spread between the value-illiquid and liquid portfolios is 0.0056. Since the *CLIQ* beta spread in Panel B(iv) of the same table is 0.057, assuming a conservative 5% premium, the return spread explained by the beta spread is $0.05 \times 0.057 = 0.00285$ or 51% of the return spread. If we assume a 7.5% premium, 76% of the return spread can be accounted for. If we repeat the same exercise with the *PRIM*-sorted

[27] Since we use the whole sample, *LMP* is not strictly a traded asset.

Table 8
Estimated premia with the liquidity-factor mimicking portfolio

	1		2		3		4	
Constant	2.05***	(0.000)	0.82*	(0.079)	0.68	(0.125)	0.55	(0.242)
MKT	−1.60***	(0.004)	−0.42	(0.470)	−0.31	(0.579)	−0.18	(0.747)
LMP	−1.03	(0.229)	−0.63	(0.479)	0.47	(0.720)	0.77	(0.550)
CLMP			2.08**	(0.013)	1.64**	(0.038)	1.49*	(0.071)
SMB					0.04	(0.883)	0.02	(0.953)
HML					0.62**	(0.022)	0.61**	(0.024)
UMD					0.25	(0.833)	0.20	(0.863)
PRIM							0.18*	(0.062)
Sample Period	196710:200412							
N Obs	447							

This table shows monthly percentage premia estimated using the liquidity-factor mimicking portfolio. Test assets are the 25 portfolios of NYSE and AMEX stocks formed as the cross section of quintiles sorted on the B/M ratio and the Amihud (2002) price-impact proxy. *MKT* is the excess return on the CRSP value-weighted portfolio; *LMP* is the return on the liquidity-factor mimicking portfolio; *CLMP* is the conditional liquidity-factor mimicking portfolio given by the product of an indicator variable *I* and *LMP*, where *I* takes the value of 1 if the probability of being in the high liquidity-beta state from the bivariate regime-switching model is higher than 0.75 and 0 otherwise; *SMB*, *HML*, and *UMD* are the size, book-to-market, and momentum factors, respectively; and *PRIM* is the lagged price-impact proxy of the test portfolios. The estimation follows the Fama-MacBeth (1973) two-pass procedure with the Shanken (1992) correction for standard errors. The estimated premia and *p*-values are reported in parentheses. *, **, *** represent significance at 10%, 5%, and 1%, respectively. *N Obs* is the number of observations.

decile portfolios in Panel A of Table 2, the explained percentage increases to 76% or higher.[28]

4. Conclusion

This paper has studied the dynamics of liquidity betas and liquidity risk premium based on a model of liquidity risk in which investors are asymmetrically informed about each other's preferences. We find that liquidity betas vary significantly over time across two distinct states: one with high liquidity betas and the other with low liquidity betas. The transition from the low to the high liquidity-beta state is predicted by a rise in trading volume, which proxies for heightened preference uncertainty. The high liquidity-beta state exhibits high volatility and a wide cross-sectional dispersion in liquidity betas, and is preceded by a period of declining expectations about future market liquidity. It is also short lived and occurs less than one-tenth of the time.

Using a conditional liquidity factor, we also document that the cross-sectional pricing of liquidity risk strengthens in the high liquidity-beta state. The conditional liquidity risk premium is statistically significant and economically large, and survives a variety of robustness tests. In addition, we find that the periods of high liquidity betas and large liquidity risk premium coincide with periods of high illiquidity premium. Since the conditional liquidity factor is significantly

[28] If the *CLIQ* premium is assumed to be 6.6% or more for Panel A, the percentage of the return spread explained by the *CLIQ* beta spread can exceed 100%. This is likely due to a large beta spread resulting from a one-dimensional sort; the *CLIQ* betas might be capturing loadings on other correlated risks, such as size and B/M as suggested in the same panel.

priced controlling for the level of illiquidity, the results suggest that the illiquidity premium is delivered primarily in the form of beta risk premium with respect to the liquidity factor during high liquidity-beta months.

We conclude by suggesting several directions for future research. First, our framework can be readily applied to study the dynamic pricing of liquidity risk in other financial markets, such as fixed income securities markets and international equity markets. Second, using order flow data, we can investigate how investors actually trade liquid and illiquid assets in the high liquidity-beta state that this paper has identified. Lastly, we can examine the multivariate dynamics of liquidity, volume, and volatility, and their implications on asset pricing. Our finding that the liquidity-return relationship is associated with variations in trading volume and volatility points to a close link among these variables.

Appendix A

A.1 Derivation of Equilibrium

This appendix derives an equilibrium of the preference uncertainty model set out in Section 1.1. Let θ_{Lt} and θ_{St} be the long- and short-term investors' asset holdings, respectively, at date $t \geq 1$. Under the assumptions in Section 1.2, the long-term investors sell $\theta_{L0} - \theta_{L1} > 0$ units of the asset at date 1 by paying a selling cost, $\kappa_1(\theta_{L0} - \theta_{L1})$, and buy back $\theta_{L2} - \theta_{L1} > 0$ units of the asset at date 2. The budget constraints imply that their consumptions are $c_{L1} = e_{L1} + (P_1 - \kappa_1)(\theta_{L0} - \theta_{L1})$, $c_{L2} = e_{L2} + P_2(\theta_{L1} - \theta_{L2})$, and $c_{L3} = 1 \cdot \theta_{L2}$. There is no selling cost at date 3 because no trading occurs at that time. Imposing market clearing, $\theta_{L2} = 1$, the first-order conditions for their utility maximization problem can be written as:

$$u'(e_{L1} + (P_1 - \kappa_1)(\theta_{L0} - \theta_{L1}))(P_1 - \kappa_1) = \delta_1 u'(e_{L2} + P_2(\theta_{L1} - 1))P_2, \quad \text{(A1)}$$

$$u'(e_{L2} + P_2(\theta_{L1} - 1))P_2 = \delta_2 u'(1). \quad \text{(A2)}$$

Substituting Equation (A2) into (A1) with $u'(c) = \frac{1}{c}$, $\theta_{L0} = 1$, and $e_{L1} = 0$, we see that $\frac{1}{1-\theta_{L1}} = \delta_1\delta_2 \equiv z$. Equation (A2) then immediately yields the expression for P_2 in Equation (4).

Similarly, the short-term investors buy $\theta_{S1} - \theta_{S0} > 0$ units of the asset at date 1 and sell $\theta_{S1} - \theta_{S2} = \theta_{S1} > 0$ units of the asset by paying a selling cost, $\kappa_2\theta_{S1}$, in order to close out their positions at date 2. Given their consumptions, $c_{S1} = e_{S1} + P_1(\theta_{S0} - \theta_{S1})$ and $c_{S2} = e_{S2} + (P_2 - \kappa_2)\theta_{S1}$, the first-order condition for their utility maximization problem is given by:

$$v'(e_{S1} + P_1(\theta_{S0} - \theta_{S1}))P_1 = \vartheta \boldsymbol{E}_1[v'(e_{S2} + (P_2 - \kappa_2)\theta_{S1})(P_2 - \kappa_2)]. \quad \text{(A3)}$$

Using $v'(c) = \frac{1}{c}$, $\theta_{S0} = 0$, and $\theta_{S2} = 0$ and denoting $\boldsymbol{E}_1[\cdot] = \boldsymbol{E}[\cdot|z, \kappa_1]$, we obtain the equilibrium date-1 price in Equation (4).

A.2 Derivation of Linear Factor Pricing Model

Next, we derive a beta representation of the model. Dividing the short-term investors' first-order condition in Equation (A3) by its left-hand side gives the pricing relation, $1 = \boldsymbol{E}_1[m_2 R_2] = Cov_1(m_2, R_2) + \boldsymbol{E}_1[m_2]\boldsymbol{E}_1[R_2]$, where $m_2 \equiv \vartheta \frac{v'(c_{S2})}{v'(c_{S1})} = \vartheta \frac{c_{S1}}{c_{S2}}$ is the SDF and $R_2 \equiv \frac{P_2 - \kappa_2}{P_1}$ is the gross second-period return after cost. Note that R_2 is an after-cost return; in a general multiperiod model, the price would have a discounted future-cost component, which would be linked to the contemporaneous illiquidity level due to its persistence. This component is not present in the date-2 price of our three-date model because there is no trading cost at date 3. To address this issue without introducing the mathematical complexity of a general multiperiod model, we opted to employ an after-cost return. This also implies that we do not consider a four-beta expression as in Acharya and Pedersen (2005) and focus on the pricing of risk due to the comovement between the (individual) asset return and the (aggregate) illiquidity shock.

Since the above pricing relation also holds for a risk-free rate, we can define a shadow gross risk-free rate as $R_1^f = \frac{1}{\boldsymbol{E}_1[m_2]}$; following the standard notation, we use the time subscript 1 because the second-period risk-free rate is known at date 1. Substituting this back to the pricing relation and manipulating, we obtain a beta pricing model as in Cochrane (2005):

$$\boldsymbol{E}_1[R_2] = R_1^f + \frac{Cov_1(R_2, m_2)}{Var_1(m_2)}\left[-\frac{Var_1(m_2)}{\boldsymbol{E}_1(m_2)}\right] \equiv R_1^f + \beta_1^m \lambda_1^m, \tag{A4}$$

where $\beta_1^m \equiv \frac{Cov_1(R_2, m_2)}{Var_1(m_2)}$ and $\lambda_1^m \equiv -\frac{Var_1(m_2)}{\boldsymbol{E}_1(m_2)}$.

Finally, we derive a linear factor pricing model. We can regard $\mathcal{I}_2$ as a (zero mean) illiquidity factor and write $m_2 = g(\mathcal{I}_2) \equiv \vartheta \frac{e_{S1} + P_1(\theta_{S0} - \theta_{S1})}{e_{S2} + (P_2 - \alpha - \gamma\kappa_1 - \mathcal{I}_2)\theta_{S1}}$. By the first-order Taylor expansion around zero, $m_2 = g(0) + g'(0)\mathcal{I}_2$, where:

$$g(0) = \vartheta \frac{e_{S1} + P_1(\theta_{S0} - \theta_{S1})}{e_{S2} + (P_2 - \alpha - \gamma\kappa_1)\theta_{S1}}, \tag{A5}$$

$$g'(0) = \vartheta\theta_{S1} \frac{e_{S1} + P_1(\theta_{S0} - \theta_{S1})}{[e_{S2} + (P_2 - \alpha - \gamma\kappa_1)\theta_{S1}]^2}. \tag{A6}$$

Since both $g(0)$ and $g'(0)$ are random under preference uncertainty, we approximate them by their expected values conditional on the information set at date 1, i.e., $m_2 = a_1 + b_1\mathcal{I}_2$, where $a_1 \equiv \boldsymbol{E}_1[g(0)]$ and $b_1 \equiv \boldsymbol{E}_1[g'(0)]$. Inspecting the expression for $g'(0)$ reveals that b_1 is proportional to $\theta_{S1} = 1 - \theta_{L1}$, the size of sell orders from the long-term investors that the short-term investors accommodate at date 1, as claimed in the main text. Defining illiquidity beta as $\beta_1^{\mathcal{I}} \equiv \frac{Cov_1(R_2, \mathcal{I}_2)}{Var_1(\mathcal{I}_2)} = \frac{1}{P_1}\frac{Cov_1(P_2 - \kappa_2, \mathcal{I}_2)}{Var_1(\mathcal{I}_2)} = -\frac{1}{P_1}$, we can then rewrite Equation (A4) as $\boldsymbol{E}_1[R_2] = R_1^f + \beta_1^{\mathcal{I}}\lambda_1^{\mathcal{I}}$, where $\lambda_1^{\mathcal{I}} \equiv -\frac{b_1}{a_1}\sigma_{\mathcal{I}}^2$.

A.3 Computation of Portfolio Characteristics

This appendix describes the computation of the portfolio characteristics summarized in Table 2. At the end of each year, preranking *PRIM* for each stock is computed in a similar fashion to Equation (13), where t is interpreted as a year instead of a month. The annual *PRIM* is used for portfolio formation. All the returns and characteristics in the table are time-series averages of postranking monthly portfolio returns and characteristics except for β^{LIQ} and β^{CLIQ}. These monthly portfolio returns and characteristics are calculated each month as value-weighted cross-sectional averages of member stocks' returns and characteristics except for size (which is a simple cross-sectional average) and the number of stocks.

Within each month, individual stocks' returns and characteristics are determined as follows: r and *PRC* are the return and the price, respectively, from the CRSP monthly stock file; σ^r is the standard deviation of the CRSP daily return; *PRIM* is the average daily Amihud (2002) price-impact proxy [see Equation (13)]; σ^{PRIM} is the standard deviation of the daily price-impact proxy; *Gamma* is the return-reversal measure given by the γ coefficient from the daily regression of Pastor and Stambaugh [2003, Equation (1)]; and *TOV* is the share turnover defined in Section 2.3. Size and B/M are computed using the CRSP monthly stock file and Compustat following Fama and French (1993), except that size (measured in millions of dollars) is recalculated monthly and B/M quarterly. B/M is lagged for six months and the latest available value is used.

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