

# Market Liquidity and Funding Liquidity

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We provide a model that links an asset's market liquidity (i.e., the ease with which it is traded) and traders' funding liquidity (i.e., the ease with which they can obtain funding). Traders provide market liquidity, and their ability to do so depends on their availability of funding. Conversely, traders' funding, i.e., their capital and margin requirements, depends on the assets' market liquidity. We show that, under certain conditions, margins are destabilizing and market liquidity and funding liquidity are mutually reinforcing, leading to liquidity spirals. The model explains the empirically documented features that market liquidity (i) can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) is subject to "flight to quality," and (v) co-moves with the market. The model provides new testable predictions, including that speculators' capital is a driver of market liquidity and risk premiums.

Trading requires capital. When a trader (e.g., a dealer, hedge fund, or investment bank) buys a security, he can use the security as collateral and borrow against it, but he cannot borrow the entire price. The difference between the security's price and collateral value, denoted as the margin or haircut, must be financed with the trader's own capital. Similarly, short-selling requires capital in the form of a margin; it does not free up capital. Therefore, the total margin on all positions cannot exceed a trader's capital at any time.

Our model shows that the funding of traders affects—and is affected by—market liquidity in a profound way. When funding liquidity is tight, traders

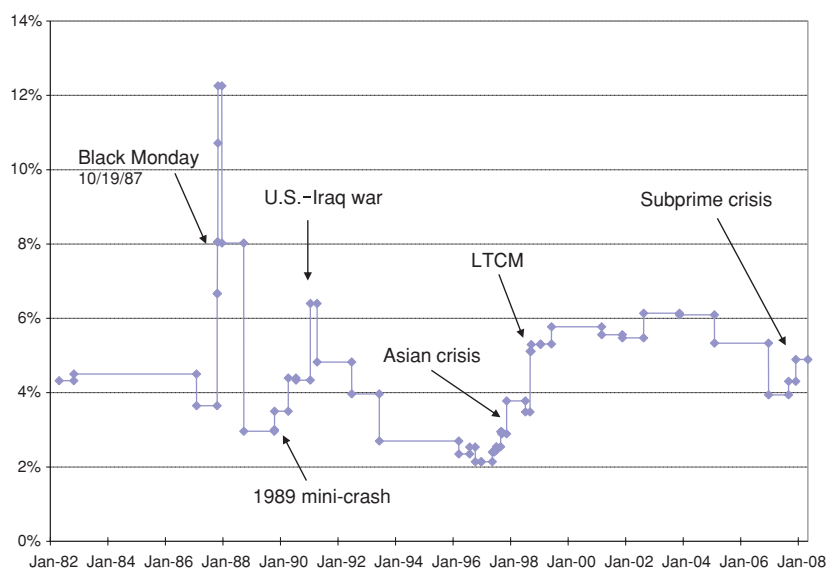
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become reluctant to take on positions, especially “capital intensive” positions in high-margin securities. This lowers market liquidity, leading to higher volatility. Further, under certain conditions, low future market liquidity increases the risk of financing a trade, thus increasing margins. Based on the links between funding and market liquidity, we provide a unified explanation for the main empirical features of market liquidity. In particular, our model implies that market liquidity (i) can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) is subject to “flight to quality,” and (v) co-moves with the market. The model has several new testable implications that link margins and dealer funding to market liquidity: We predict that (i) speculators’ (mark-to-market) capital and volatility (as, e.g., measured by VIX) are state variables affecting market liquidity and risk premiums; (ii) a reduction in capital reduces market liquidity, especially if capital is already low (a nonlinear effect) and for high-margin securities; (iii) margins increase in illiquidity if the fundamental value is difficult to determine; and (iv) speculators’ returns are negatively skewed (even if they trade securities without skewness in the fundamentals).

Our model is similar in spirit to Grossman and Miller (1988) with the added feature that speculators face the real-world funding constraint discussed above. In our model, different customers have offsetting demand shocks, but arrive sequentially to the market. This creates a temporary order imbalance. Speculators smooth price fluctuations, thus providing market liquidity. Speculators finance their trades through collateralized borrowing from financiers who set the margins to control their value-at-risk (VaR). Since financiers can reset margins in each period, speculators face funding liquidity risk due to the risk of higher margins or losses on existing positions. We derive the competitive equilibrium of the model and explore its liquidity implications. We define market liquidity as the difference between the transaction price and the fundamental value, and funding liquidity as speculators’ scarcity (or shadow cost) of capital.

We first analyze the properties of margins, which determine the investors’ capital requirement. We show that margins can increase in illiquidity when margin-setting financiers are unsure whether price changes are due to fundamental news or to liquidity shocks, and volatility is time varying. This happens when a liquidity shock leads to price volatility, which raises the financier’s expectation about future volatility, and this leads to increased margins. Figure 1 shows that margins did increase empirically for S&P 500 futures during the liquidity crises of 1987, 1990, 1998, and 2007. More generally, the October 2007 IMF Global Stability Report documents a significant widening of the margins across most asset classes during the summer of 2007. We denote margins as “destabilizing” if they can increase in illiquidity, and note that anecdotal evidence from prime brokers suggests that margins often behave in this way. Destabilizing margins force speculators to de-lever



**Figure 1**  
**Margins for S&P 500 futures**

The figure shows margin requirements on S&P 500 futures for members of the Chicago Mercantile Exchange as a fraction of the value of the underlying S&P 500 index multiplied by the size of the contract. (Initial or maintenance margins are the same for members.) Each dot represents a change in the dollar margin.

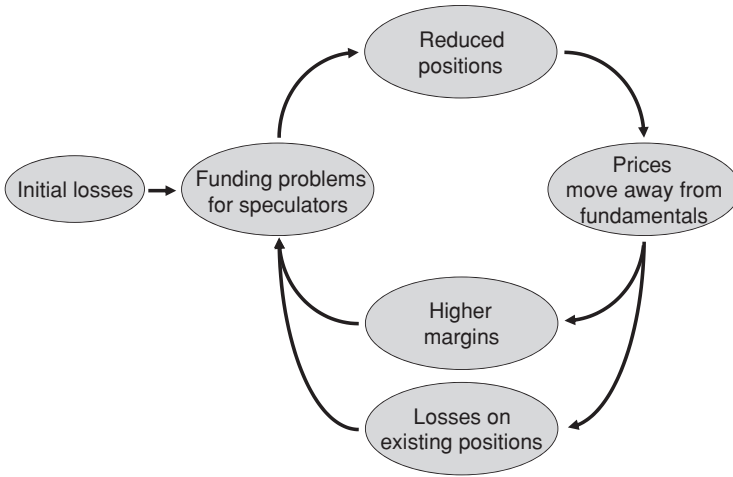
their positions in times of crisis, leading to pro-cyclical market liquidity provision.<sup>1</sup>

In contrast, margins can theoretically decrease with illiquidity and thus can be “stabilizing.” This happens when financiers know that prices diverge due to temporary market illiquidity and know that liquidity will be improved shortly as complementary customers arrive. This is because a current price divergence from fundamentals provides a “cushion” against future adverse price moves, making the speculators’ position less risky in this case.

Turning to the implications for market liquidity, we first show that, as long as speculators’ capital is so abundant that there is no risk of hitting the funding constraint, market liquidity is naturally at its highest level and is insensitive to marginal changes in capital and margins. However, when speculators hit their capital constraints—or risk hitting their capital constraints over the life of a trade—then they reduce their positions and market liquidity declines. At that point prices are more driven by funding liquidity considerations rather than by movements in fundamentals, as was apparent during the quant hedge fund crisis in August 2007, for instance.

When margins are destabilizing or speculators have large existing positions, there can be multiple equilibria and liquidity can be *fragile*. In one equilibrium,

<sup>1</sup> The pro-cyclical nature of banks’ regulatory capital requirements and funding liquidity is another application of our model, which we describe in Appendix A.2.



**Figure 2**  
**Liquidity spirals**

The figure shows the loss spiral and the margin/haicut spiral.

markets are liquid, leading to favorable margin requirements for speculators, which in turn helps speculators make markets liquid. In another equilibrium, markets are illiquid, resulting in larger margin requirements (or speculator losses), thus restricting speculators from providing market liquidity. Importantly, any equilibrium selection has the property that small speculator losses can lead to a discontinuous drop of market liquidity. This “sudden dry-up” or fragility of market liquidity is due to the fact that with high levels of speculator capital, markets must be in a liquid equilibrium, and, if speculator capital is reduced enough, the market must eventually switch to a low-liquidity/high-margin equilibrium.<sup>2</sup> The events following the Russian default and LTCM collapse in 1998 are a vivid example of fragility of liquidity since a relatively small shock had a large impact. Compared to the total market capitalization of the U.S. stock and bond markets, the losses due to the Russian default were minuscule but, as Figure 1 shows, caused a shiver in world financial markets. Similarly, the subprime losses in 2007–2008 were in the order of several hundred billion dollars, corresponding to only about 5% of overall stock market capitalization. However, since they were primarily borne by levered financial institutions with significant maturity mismatch, spiral effects amplified the crisis so, for example, the overall stock market losses amounted to more than 8 trillion dollars as of this writing (see Brunnermeier 2009).

Further, when markets are illiquid, market liquidity is highly sensitive to further changes in funding conditions. This is due to two liquidity spirals, as illustrated in Figure 2. First, a *margin spiral* emerges if margins are increasing

<sup>2</sup> Fragility can also be caused by asymmetric information on the amount of trading by portfolio insurance traders (Gennotte and Leland 1990), and by losses on existing positions (Chowdhry and Nanda 1998).

in market illiquidity. In this case, a funding shock to the speculators lowers market liquidity, leading to higher margins, which tightens speculators' funding constraint further, and so on. For instance, Figure 1 shows how margins gradually escalated within a few days after Black Monday in 1987. The subprime crisis that started in 2007 led to margin increases at the end of August and end of November 2007 for the S&P futures contract. For other assets, margins and haircuts widened significantly more (see, for example, IMF Global Stability Report, October 2007). The margin spiral forces traders to de-lever during downturns and recently, Adrian and Shin (2009) found consistent evidence for investment banks. Second, a *loss spiral* arises if speculators hold a large initial position that is negatively correlated with customers' demand shock. In this case, a funding shock increases market illiquidity, leading to speculator losses on their initial position, forcing speculators to sell more, causing a further price drop, and so on.<sup>3</sup> These liquidity spirals reinforce each other, implying a larger total effect than the sum of their separate effects. Paradoxically, liquidity spirals imply that a larger shock to the customers' demand for immediacy leads to a reduction in the provision of immediacy during such times of stress. Consistent with our predictions, Mitchell, Pedersen, and Pulvino (2007) find significant liquidity-driven divergence of prices from fundamentals in the convertible bond markets after capital shocks to the main liquidity providers, namely convertible arbitrage hedge funds. Also, Garleanu, Pedersen, and Potoshman (2008) document that option market makers' unhedgeable risk is priced, especially in times following recent losses.

In the cross section, we show that the ratio of illiquidity to margin is the same across all assets for which speculators provide market liquidity. This is the case since speculators optimally invest in securities that have the greatest expected profit (i.e., illiquidity) per capital use (determined by the asset's dollar margin). This common ratio is determined in equilibrium by the speculators' funding liquidity (i.e., capital scarcity). Said differently, a security's market illiquidity is the product of its margin and the shadow cost of funding. Our model thus provides a natural explanation for the *commonality* of liquidity across assets since shocks to speculators' funding constraint affect all securities. This may help explain why market liquidity is correlated across stocks (Chordia, Roll, and Subrahmanyam 2000; Hasbrouck and Seppi 2001; and Huberman and Halka 2001), and across stocks and bonds (Chordia, Sarkar, and Subrahmanyam 2005). In support of the idea that commonality is driven at least in part by our funding-liquidity mechanism, Chordia, Roll, and Subrahmanyam (2005) find that "money flows . . . account for part of the commonality in stock and bond market liquidity." Moreover, their finding that

<sup>3</sup> The loss spiral is related to the multipliers that arise in Grossman (1988); Kiyotaki and Moore (1997); Shleifer and Vishny (1997); Chowdhry and Nanda (1998); Xiong (2001); Kyle and Xiong (2001); Gromb and Vayanos (2002); Morris and Shin (2004); Plantin, Sapra, and Shin (2005); and others. In Geanakoplos (2003) and in Fostel and Geanakoplos (2008), margins increase as risk increases. Our paper captures the margin spiral—i.e., the adverse feedback loop between margins and prices—and the interaction between the margin and loss spirals. Garleanu and Pedersen (2007) show how a risk management spiral can arise.

“during crisis periods, monetary expansions are associated with increased liquidity” is consistent with our model’s prediction that the effects are largest when traders are near their constraint. Acharya, Schaefer, and Zhang (2008) document a substantial increase in the co-movement among credit default swaps (CDS) during the GM/Ford rating downgrade in May 2005 when dealer funding was stretched. Coughenour and Saad (2004) provide further evidence of the funding-liquidity mechanism by showing that the co-movement in liquidity among stocks handled by the same NYSE specialist firm is higher than for other stocks, commonality is higher for specialists with less capital, and decreases after a merger of specialists.

Next, our model predicts that market liquidity declines as fundamental volatility increases, which is consistent with the empirical findings of Benston and Hagerman (1974) and Amihud and Mendelson (1989).<sup>4</sup> Further, the model can shed new light on “flight to quality,” referring to episodes in which risky securities become especially illiquid. In our model, this happens when speculators’ capital deteriorates, which induces them to mostly provide liquidity in securities that do not use much capital (low-volatility stocks with lower margins), implying that the liquidity differential between high-volatility and low-volatility securities increases. This capital effect means that illiquid securities are predicted to have more liquidity risk.<sup>5</sup> Recently, Comerton-Forde, Hendershott, Jones, Moulton and Seasholes (2008) test these predictions using inventory positions of NYSE specialists as a proxy for funding liquidity. Their findings support our hypotheses that market liquidity of high-volatility stocks is more sensitive to inventory shocks and that this is more pronounced at times of low funding liquidity. Moreover, Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) document empirical evidence consistent with flight to liquidity and the pricing of this liquidity risk.

Market-making firms are often net long in the market. For instance, Ibbotson (1999) reports that security brokers and speculators have median market betas in excess of one. Therefore, capital constraints are more likely to be hit during market downturns, and this, together with the mechanism outlined in our model, helps to explain why sudden liquidity dry-ups occur more often when markets decline. Further, capital constraints affect the liquidity of all securities, leading to co-movement as explained above. The fact that this effect is stronger in down markets could explain that co-movement in liquidity is higher during large negative market moves, as documented empirically by Hameed, Kang, and Viswanathan (2005).

<sup>4</sup> The link between volatility and liquidity is shared by the models of Stoll (1978); Grossman and Miller (1988); and others. What sets our theory apart is that this link is connected with margin constraints. This leads to testable differences since, according to our model, the link is stronger when speculators are poorly financed, and high-volatility securities are more affected by speculator wealth shocks—our explanation of flight to quality.

<sup>5</sup> In Vayanos (2004), liquidity premiums increase in volatile times. Fund managers become effectively more risk-averse because higher fundamental volatility increases the likelihood that their performance falls short of a threshold, leading to costly performance-based withdrawal of funds.

Finally, the very risk that the funding constraint becomes binding limits speculators' provision of market liquidity. Our analysis shows that speculators' optimal (funding) risk management policy is to maintain a "safety buffer." This affects initial prices, which increase in the covariance of future prices with future shadow costs of capital (i.e., with future funding illiquidity).

Our paper is related to several literatures.<sup>6</sup> Traders rely both on (equity) investors and counterparties, and, while the limits to arbitrage literature following Shleifer and Vishny (1997) focuses on the risk of investor redemptions, we focus on the risk that counterparty funding conditions may worsen. Other models with margin-constrained traders are Grossman and Vila (1992) and Liu and Longstaff (2004), which derive optimal strategies in a partial equilibrium with a single security; Chowdhry and Nanda (1998) focus on fragility due to dealer losses; and Gromb and Vayanos (2002) derive a general equilibrium with one security (traded in two segmented markets) and study welfare and liquidity provision. We study the endogenous variation of margin constraints, the resulting amplifying effects, and differences across high- and low-margin securities in our setting with multiple securities. Stated simply, whereas the above-cited papers use a fixed or decreasing margin constraint, say, \$5000 per contract, we study how market conditions lead to changes in the margin requirement itself, e.g., an increase from \$5000 to \$15,000 per futures contract as happened in October 1987, and the resulting feedback effects between margins and market conditions as speculators are forced to de-lever.

We proceed as follows. We describe the model (Section 1) and derive our four main new results: (i) margins increase with market illiquidity when financiers cannot distinguish fundamental shocks from liquidity shocks and fundamentals have time-varying volatility (Section 2); (ii) this makes margins destabilizing, leading to sudden liquidity dry-ups and margin spirals (Section 3); (iii) liquidity crises simultaneously affect many securities, mostly risky high-margin securities, resulting in commonality of liquidity and flight to quality (Section 4); and (iv) liquidity risk matters even before speculators hit their capital constraints (Section 5). Then we outline how our model's new testable predictions may be helpful for a novel line of empirical work that links measures of speculators' funding conditions to measures of market liquidity (Section 6). Section 7 concludes. Finally, we describe the real-world funding constraints for the main liquidity providers, namely market makers, banks, and hedge funds (Appendix A), and provide proofs (Appendix B).

<sup>6</sup> Market liquidity is the focus of market microstructure (Stoll 1978; Ho and Stoll 1981, 1983; Kyle 1985; Glosten and Milgrom 1985; Grossman and Miller 1988), and is related to the limits of arbitrage (DeLong et al. 1990; Shleifer and Vishny 1997; Abreu and Brunnermeier 2002). Funding liquidity is examined in corporate finance (Shleifer and Vishny 1992; Holmström and Tirole 1998, 2001) and banking (Bryant 1980; Diamond and Dybvig 1983; Allen and Gale 1998, 2004, 2005, 2007). Funding and collateral constraints are also studied in macroeconomics (Aiyagari and Gertler 1999; Bernanke and Gertler 1989; Fisher 1933; Kiyotaki and Moore 1997; Lustig and Chien 2005), and general equilibrium with incomplete markets (Geanakoplos 1997, 2003). Finally, recent papers consider illiquidity with constrained traders (Attari, Mello, and Ruckes 2005; Bernardo and Welch 2004; Brunnermeier and Pedersen 2005; Eisfeldt 2004; Morris and Shin 2004; Weill 2007).

## 1. Model

The economy has  $J$  risky assets, traded at times  $t = 0, 1, 2, 3$ . At time  $t = 3$ , each security  $j$  pays off  $v^j$ , a random variable defined on a probability space  $(\Omega, \mathcal{F}, \mathcal{P})$ . There is no aggregate risk because the aggregate supply is zero and the risk-free interest rate is normalized to zero, so the fundamental value of each stock is its conditional expected value of the final payoff  $v_t^j = E_t[v^j]$ . Fundamental volatility has an autoregressive conditional heteroscedasticity (ARCH) structure. Specifically,  $v_t^j$  evolves according to

$$v_{t+1}^j = v_t^j + \Delta v_{t+1}^j = v_t^j + \sigma_{t+1}^j \varepsilon_{t+1}^j, \quad (1)$$

where all  $\varepsilon_t^j$  are i.i.d. across time and assets with a standard normal cumulative distribution function  $\Phi$  with zero mean and unit variance, and the volatility  $\sigma_t^j$  has dynamics

$$\sigma_{t+1}^j = \underline{\sigma}^j + \theta^j |\Delta v_t^j|, \quad (2)$$

where  $\underline{\sigma}^j, \theta^j \geq 0$ . A positive  $\theta^j$  implies that shocks to fundamentals increase future volatility.

There are three groups of market participants: “customers” and “speculators” trade assets while “financiers” finance speculators’ positions. The group of customers consists of three risk-averse agents. At time 0, customer  $k = 0, 1, 2$  has a cash holding of  $W_0^k$  bonds and zero shares, but finds out that he will experience an endowment shock of  $\mathbf{z}^k = \{z^{1,k}, \dots, z^{J,k}\}$  shares at time  $t = 3$ , where  $\mathbf{z}$  are random variables such that the aggregate endowment shock is zero,  $\sum_{k=0}^2 z^{j,k} = 0$ .

With probability  $(1 - a)$ , all customers arrive at the market at time 0 and can trade securities in each time period 0, 1, 2. Since their aggregate shock is zero, they can share risks and have no need for intermediation.

The basic liquidity problem arises because customers arrive sequentially with probability  $a$ , which gives rise to order imbalance. Specifically, in this case customer 0 arrives at time 0, customer 1 arrives at time 1, and customer 2 arrives at time 2. Hence, at time 2 all customers are present, at time 1 only customers 0 and 1 can trade, and at time 0 only customer 0 is in the market.

Before a customer arrives in the marketplace, his demand is  $\mathbf{y}_t^k = 0$ , and after he arrives he chooses his security position each period to maximize his exponential utility function  $U(W_3^k) = -\exp\{-\gamma W_3^k\}$  over final wealth. Wealth  $W_t^k$ , including the value of the anticipated endowment shock of  $\mathbf{z}^k$  shares, evolves according to

$$W_{t+1}^k = W_t^k + (\mathbf{p}_{t+1} - \mathbf{p}_t)'(\mathbf{y}_t^k + \mathbf{z}^k). \quad (3)$$

The vector of total demand shock of customers who have arrived in the market at time  $t$  is denoted by  $Z_t := \sum_{k=0}^t \mathbf{z}^k$ .



The early customers' trading need is accommodated by speculators who provide liquidity/immediacy. Speculators are risk-neutral and maximize expected final wealth  $W_3$ . Speculators face the constraint that the total margin on their position  $\mathbf{x}_t$  cannot exceed their capital  $W_t$ :

$$\sum_j (x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-}) \leq W_t, \quad (4)$$

where  $x_t^{j+} \geq 0$  and  $x_t^{j-} \geq 0$  are the positive and negative parts of  $x_t^j = x_t^{j+} - x_t^{j-}$ , respectively, and  $m_t^{j+} \geq 0$  and  $m_t^{j-} \geq 0$  are the dollar margin on long and short positions, respectively. The institutional features related to this key constraint for different types of speculators like hedge funds, banks, and market makers are discussed in detail in Appendix A.

Speculators start out with a cash position of  $W_0$  and zero shares, and their wealth evolves according to

$$W_t = W_{t-1} + (\mathbf{p}_t - \mathbf{p}_{t-1})' \mathbf{x}_{t-1} + \eta_t, \quad (5)$$

where  $\eta_t$  is an independent wealth shock arising from other activities (e.g., a speculator's investment banking arm). If a speculator loses all his capital,  $W_t \leq 0$ , he can no longer invest because of the margin constraint (4), i.e., he must choose  $x_t = 0$ . We let his utility in this case be  $\varphi_t W_t$ , where  $\varphi_t \geq 0$ . Limited liability corresponds to  $\varphi_t = 0$ , and a proportional bankruptcy cost (e.g., monetary, reputational, or opportunity costs) corresponds to  $\varphi_t > 0$ . We focus on the case in which  $\varphi_2 = 1$ , that is, negative consumption equal to the dollar loss in  $t = 2$ . We discuss  $\varphi_1$  in Section 5. Our results would be qualitatively the same with other bankruptcy assumptions.

We could allow the speculators to raise new capital as long as this takes time. Indeed, the model would be the same if the speculators could raise capital only at time 2 (and in this case we need not assume that the customers' endowment shocks  $z^j$  aggregate to zero). Hence, in this sense, we can view our model as one of "slow moving capital," consistent with the empirical evidence of Mitchell, Pedersen, and Pulvino (2007).

Each financier sets the margins to limit his counterparty credit risk. Specifically, each financier ensures that the margin is large enough to cover the position's  $\pi$ -value-at-risk (where  $\pi$  is a non-negative number close to zero, e.g., 1%):

$$\pi = \Pr(-\Delta p_{t+1}^j > m_t^{j+} \mid \mathcal{F}_t), \quad (6)$$

$$\pi = \Pr(\Delta p_{t+1}^j > m_t^{j-} \mid \mathcal{F}_t). \quad (7)$$

Equation (6) means that the margin on a long position  $m^+$  is set such that price drops that exceed the amount of the margin only happen with a small probability  $\pi$ . Similarly, Equation (7) means that price increases larger than the margin on a short position only happen with small probability. Clearly, the margin is larger for more volatile assets. The margin depends on

financiers' information set  $\mathcal{F}_t$ . We consider two important benchmarks: "informed financiers," who know the fundamental value and the liquidity shocks  $\mathbf{z}$ ,  $\mathcal{F}_t = \sigma\{\mathbf{z}, \mathbf{v}_0, \dots, \mathbf{v}_t, \mathbf{p}_0, \dots, \mathbf{p}_t, \eta_1, \dots, \eta_t\}$ , and "uninformed financiers," who only observe prices,  $\mathcal{F}_t = \sigma\{\mathbf{p}_0, \dots, \mathbf{p}_t\}$ . This margin specification is motivated by the real-world institutional features described in Appendix A. Theoretically, Stiglitz and Weiss (1981) show how credit rationing can be due to adverse selection and moral hazard in the lending market, and Geanakoplos (2003) considers endogenous contracts in a general-equilibrium framework of imperfect commitment.

We let  $\Lambda_t^j$  be the (signed) deviation of the price from fundamental value:

$$\Lambda_t^j = p_t^j - v_t^j, \quad (8)$$

and we define our measure of market illiquidity as the absolute amount of this deviation,  $|\Lambda_t^j|$ . We consider competitive equilibria of the economy:

**Definition 1.** An *equilibrium* is a price process  $\mathbf{p}_t$  such that (i)  $\mathbf{x}_t$  maximizes the speculators' expected final profit subject to the margin constraint (4); (ii) each  $\mathbf{y}_t^k$  maximizes customer  $k$ 's expected utility after their arrival at the marketplace and is zero beforehand; (iii) margins are set according to the VaR specification (6–7); and (iv) markets clear,  $\mathbf{x}_t + \sum_{k=0}^2 \mathbf{y}_t^k = 0$ .

**Equilibrium.** We derive the optimal strategies for customers and speculators using dynamic programming, starting from time 2, and working backwards. A customer's value function is denoted  $\Gamma$  and a speculator's value function is denoted  $J$ . At time 2, customer  $k$ 's problem is

$$\Gamma_2(W_2^k, p_2, v_2) = \max_{y_2^k} -E_2[e^{-\gamma W_3^k}] \quad (9)$$

$$= \max_{y_2^k} -e^{-\gamma(E_2[W_3^k] - \frac{\gamma}{2} \text{Var}_2[W_3^k])}, \quad (10)$$

which has the solution

$$y_2^{j,k} = \frac{v_2^j - p_2^j}{\gamma(\sigma_3^j)^2} - z^{j,k}. \quad (11)$$

Clearly, since all customers are present in the market at time 2, the unique equilibrium is  $p_2 = v_2$ . Indeed, when the prices are equal to fundamentals, the aggregate customer demand is zero,  $\sum_k y_2^{j,k} = 0$ , and speculators also has a zero demand. We get the customer's value function  $\Gamma_2(W_2^k, p_2 = v_2, v_2) = -e^{-\gamma W_2^k}$ , and the speculator's value function  $J_2(W_2, p_2 = v_2, v_2) = W_2$ .

The equilibrium before time 2 depends on whether the customers arrive sequentially or simultaneously. If all customers arrive at time 0, then the simple arguments above show that  $p_t = v_t$  at any time  $t = 0, 1, 2$ .

We are interested in the case with sequential arrival of the customers such that the speculators' liquidity provision is needed. At time 1, customers 0 and 1 are present in the market, but customer 2 has not arrived yet. As above, customer  $k = 0, 1$  has a demand and value function of

$$y_1^{j,k} = \frac{v_1^j - p_1^j}{\gamma(\sigma_2^j)^2} - z^{j,k} \quad (12)$$

$$\Gamma_1(W_1^k, p_1, v_1) = -\exp \left\{ -\gamma \left[ W_1^k + \sum_j \frac{(v_1^j - p_1^j)^2}{2\gamma(\sigma_2^j)^2} \right] \right\}. \quad (13)$$

At time 0, customer  $k = 0$  arrives in the market and maximizes  $E_0[\Gamma_1(W_1^k, p_1, v_1)]$ .

At time  $t = 1$ , if the market is perfectly liquid so that  $p_1^j = v_1^j$  for all  $j$ , then the speculators are indifferent among all possible positions  $x_1$ . If some securities have  $p_1 \neq v_1$ , then the risk-neutral speculators invest all his capital such that his margin constraint binds. The speculators optimally trade only in securities with the highest expected profit per dollar used. The profit per dollar used is  $(v_1^j - p_1^j)/m_1^{j+}$  on a long position and  $-(v_1^j - p_1^j)/m_1^{j-}$  on a short position. A speculators' shadow cost of capital, denoted  $\phi_1$ , is 1 plus the maximum profit per dollar used as long as he is not bankrupt:

$$\phi_1 = 1 + \max_j \left\{ \max \left( \frac{v_1^j - p_1^j}{m_1^{j+}}, \frac{-(v_1^j - p_1^j)}{m_1^{j-}} \right) \right\}, \quad (14)$$

where the margins for long and short positions are set by the financiers, as described in the next section. If the speculators are bankrupt,  $W_1 < 0$ , then  $\phi_1 = \phi_1$ . Each speculator's value function is therefore

$$J_1(W_1, p_1, v_1, p_0, v_0) = W_1 \phi_1. \quad (15)$$

At time  $t = 0$ , the speculator maximizes  $E_0[W_1 \phi_1]$  subject to his capital constraint (4).

The equilibrium prices at times 1 and 0 do not have simple expressions but we can characterize their properties, starting with a basic result from which much intuition derives:

**Proposition 1 (market and funding liquidity).** *In equilibrium, any asset  $j$ 's market illiquidity  $|\Lambda_1^j|$  is linked to its margin  $m_1^j$  and the common funding illiquidity as measured by the speculators' marginal value of an extra dollar  $\phi_1$ :*

$$|\Lambda_1^j| = m_1^j(\phi_1 - 1), \quad (16)$$

where  $m_1^j = m_1^{j+}$  if the speculator is long and  $m_1^j = m_1^{j-}$  otherwise. If the speculators have a zero position for asset  $j$ , the equation is replaced by  $\leq$ .

We next go on to show the (de-)stabilizing properties of margins, and then we further characterize the equilibrium connection between market liquidity and speculators' funding situation, and the role played by liquidity risk at time 0.

## 2. Margin Setting and Liquidity (Time 1)

A key determinant of speculators' funding liquidity is their margin requirement for collateralized financing. Hence, it is important to determine the margin function,  $m_1$ , set by, respectively, informed and uninformed financiers. The margin at time 1 is set to cover a position's value-at-risk, knowing that prices equal the fundamental values in the next period 2,  $p_2 = v_2$ .

We consider first informed financiers who know the fundamental values  $v_1$  and, hence, price divergence from fundamentals  $\Lambda_1$ . Since  $\Lambda_2 = 0$ , they set margins on long positions at  $t = 1$ , according to

$$\begin{aligned}\pi &= \Pr(-\Delta p_2^j > m_1^{j+} \mid \mathcal{F}_1) \\ &= \Pr(-\Delta v_2^j + \Lambda_1^j > m_1^{j+} \mid \mathcal{F}_1) \\ &= 1 - \Phi\left(\frac{m_1^{j+} - \Lambda_1^j}{\sigma_2^j}\right),\end{aligned}\quad (17)$$

which implies that

$$\begin{aligned}m_1^{j+} &= \Phi^{-1}(1 - \pi)\sigma_2^j + \Lambda_1^j \\ &= \bar{\sigma}^j + \bar{\theta}|\Delta v_1^j| + \Lambda_1^j,\end{aligned}\quad (18)$$

where we define

$$\bar{\sigma}^j = \underline{\sigma}^j \Phi^{-1}(1 - \pi), \quad (19)$$

$$\bar{\theta}^j = \theta^j \Phi^{-1}(1 - \pi). \quad (20)$$

The margin on a short position can be derived similarly and we arrive at the following surprising result:

**Proposition 2 (stabilizing margins and the cushioning effect).** *When the financiers are informed about the fundamental value and knows that prices will equal fundamentals in the next period,  $t = 2$ , then the margins on long and short positions are, respectively,*

$$m_1^{j+} = \max\{\bar{\sigma}^j + \bar{\theta}^j|\Delta v_1^j| + \Lambda_1^j, 0\}, \quad (21)$$

$$m_1^{j-} = \max\{\bar{\sigma}^j + \bar{\theta}^j|\Delta v_1^j| - \Lambda_1^j, 0\}. \quad (22)$$

The more prices are below fundamentals  $\Lambda_1^j < 0$ , the lower is the margin on a long position  $m_1^{j+}$ , and the more prices are above fundamentals  $\Lambda_1^j > 0$ , the lower is the margin on a short position  $m_1^{j-}$ . Hence, in this case illiquidity reduces margins for speculators who buy low and sell high.

The margins are reduced by illiquidity because the speculators are expected to profit when prices return to fundamentals at time 2, and this profit “cushions” the speculators from losses due to fundamental volatility. Thus, we denote the margins set by informed financiers at  $t = 1$  as *stabilizing margins*.

Stabilizing margins are an interesting benchmark, and they are hard to escape in a theoretical model. However, real-world liquidity crises are often associated with increases in margins, not decreases. To capture this, we turn to the case of a in which financiers are uninformed about the current fundamental so that he must set his margin based on the observed prices  $p_0$  and  $p_1$ . This is in general a complicated problem since the financiers need to filter out the probability that customers arrive sequentially, and the values of  $z_0$  and  $z_1$ . The expression becomes simple, however, if the financier’s prior probability of an asynchronous arrival of endowment shocks is small so that he finds it likely that  $p_t^j = v_t^j$ , implying a common margin  $m_1^j = m_1^{j+} = m_1^{j-}$  for long and short positions in the limit:

**Proposition 3 (destabilizing margins).** *When the financiers are uninformed about the fundamental value, then, as  $a \rightarrow 0$ , the margins on long and short positions approach*

$$m_1^j = \bar{\sigma}^j + \bar{\theta}^j |\Delta p_1^j| = \bar{\sigma}^j + \bar{\theta}^j |\Delta v_1^j + \Delta \Lambda_1^j|. \quad (23)$$

*Margins are increasing in price volatility and market illiquidity can increase margins.*

Intuitively, since liquidity risk tends to increase price volatility, and since uninformed financiers may interpret price volatility as fundamental volatility, this increases margins.<sup>7</sup> Equation (23) corresponds closely to real-world margin setting, which is primarily based on volatility estimates from past price movements. This introduces a procyclicality that amplifies funding shocks—a major criticism of the Basel II capital regulation. (See Appendix A.2 for how banks’ capital requirements relate to our funding constraint.) Equation (23) shows that illiquidity increases margins when the liquidity shock  $\Delta \Lambda_1^j$  has the same sign as the fundamental shock  $\Delta v_1^j$  (or is greater in magnitude), for example, when bad news and selling pressure happen at the same time. On the other hand, margins are reduced if the nonfundamental  $z$ -shock counterbalances a fundamental

<sup>7</sup> In the analysis of time 0, we shall see that margins can also be destabilizing when price volatility signals future liquidity risk (not necessarily fundamental risk).

move. We denote the phenomenon that margins can increase as illiquidity rises by *destabilizing margins*. As we will see next, the information available to the financiers (i.e., whether margins are stabilizing or destabilizing) has important implications for the equilibrium.

### 3. Fragility and Liquidity Spirals (Time 1)

We next show how speculators' funding problems can lead to liquidity spirals and fragility—the property that a small change in fundamentals can lead to a large jump in illiquidity. We show that funding problems are especially escalating with uninformed financiers (i.e., destabilizing margins). For simplicity, we illustrate this with a single security  $J = 1$ .

#### 3.1 Fragility

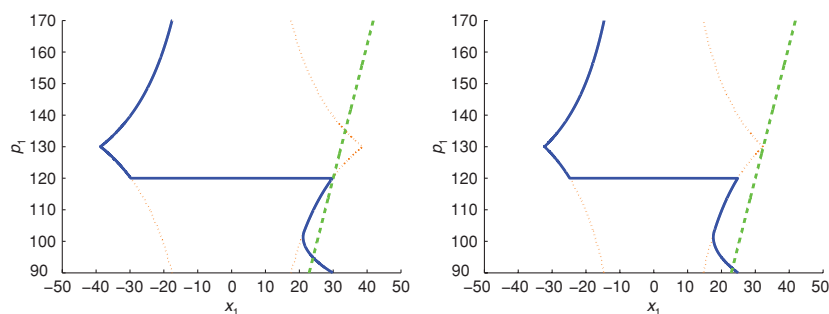
To set the stage for the main fragility proposition below, we make a few brief definitions. Liquidity is said to be *fragile* if the equilibrium price  $p_t(\eta_t, v_t)$  cannot be chosen to be continuous in the exogenous shocks, namely  $\eta_t$  and  $\Delta v_t$ . Fragility arises when the excess demand for shares  $x_t + \sum_{k=0}^1 y_1^k$  can be non-monotonic in the price. While under “normal” circumstances, a high price leads to a low total demand (i.e., excess demand is decreasing), binding funding constraints along with destabilizing margins (margin effect) or speculators' losses (loss effect) can lead to an increasing demand curve. Further, it is natural to focus on *stable* equilibria in which a small negative (positive) price perturbation leads to excess demand (supply), which, intuitively, “pushes” the price back up (down) to its equilibrium level.

**Proposition 4 (fragility).** *There exist  $\underline{x}, \underline{\theta}, \underline{a} > 0$  such that:*

- (i) *With informed financiers, the market is fragile at time 1 if speculators' position  $|x_0|$  is larger than  $\underline{x}$  and of the same sign as the demand shock  $Z_1$ .*
- (ii) *With uninformed financiers the market is fragile as in (i) and additionally if the ARCH parameter  $\theta$  is larger than  $\underline{\theta}$  and the probability,  $a$ , of sequential arrival of customers is smaller than  $\underline{a}$ .*

**Numerical example.** We illustrate how fragility arises due to destabilizing margins or dealer losses by way of a numerical example. We consider the more interesting (and arguably more realistic) case in which the financiers are uninformed, and we choose parameters as follows.

The fundamental value has ARCH volatility parameters  $\sigma = 10$  and  $\theta = 0.3$ , which implies clustering of volatility. The initial price is  $p_0 = 130$ , the aggregate demand shock of the customers who have arrived at time 1 is  $Z_1 = z_0 + z_1 = 30$ , and the customers' risk aversion coefficient is  $\gamma = 0.05$ . The speculators have an initial position of  $x_0 = 0$  and a cash wealth of  $W_1 = 900$ . Finally, the financiers use a VaR with  $\pi = 1\%$  and customers learn their endowment shocks sequentially with probability  $a = 1\%$ .



**Figure 3**  
**Speculator demand and customer supply**

This figure illustrates how margins can be destabilizing when financiers are uninformed and the fundamentals have volatility clustering. The solid curve is the speculators' optimal demand for  $a = 1\%$ . The upward sloping dashed line is the customers' supply, that is, the negative of their demand. In panel A, the speculators experience a zero wealth shock,  $\eta_1 = 0$ , while in panel B they face a negative wealth shock of  $\eta_1 = -150$ , otherwise everything is the same. In panel A, perfect liquidity  $p_1 = v_1 = 120$  is one of two stable equilibria, while in panel B the unique equilibrium is illiquid.

Panel A of Figure 3 illustrates how the speculators' demand  $x_1$  and the customers' supply (i.e., the negative of the customers' demand as per Equation (12)) depend on the price  $p_1$  when the fundamental value is  $v_1 = 120$  and the speculators' wealth shock is  $\eta_1 = 0$ . Customers' supply is given by the upward sloping dashed line since, naturally, their supply is greater when the price is higher. Customers supply  $Z_1 = 30$  shares, namely the shares that they anticipate receiving at time  $t = 3$ , when the market is perfectly liquid,  $p_1 = v_1 = 120$  (i.e., illiquidity is  $|\Delta_1| = 0$ ). For lower prices, they supply fewer shares.

The speculators' demand,  $x_1$ , must satisfy the margin constraints. It is instructive to consider first the simpler limiting case  $a \rightarrow 0$  for which the margin requirement is simply  $m = \bar{\sigma} + \bar{\theta}|\Delta p_1| = 2.326(10 + 0.3|\Delta p_1|)$ . This implies that speculators demand  $|x_1| \leq W_1/(\bar{\sigma} + \bar{\theta}|\Delta p_1|)$ . Graphically, this means that their demand must be inside the "hyperbolic star" defined by the four (dotted) hyperbolas (that are partially overlaid by a solid demand curve in Figure 3). At the price  $p_1 = p_0 = 130$ , the margin is smallest and hence the constraint is most relaxed. As  $p_1$  departs from  $p_0 = 130$ , margins increase and speculators become more constrained—the horizontal distance between two hyperbolas shrinks.

After establishing the hyperbolic star, it is easy to derive the demand curve for  $a \rightarrow 0$ : For  $p_1 = v_1 = 120$ , the security's expected return is zero and each speculator is indifferent between all his possible positions on the horizontal line. For price levels  $p_1 > v_1$  above this line, the risk-neutral speculators want to short-sell the asset,  $x_1 < 0$ , and their demand is constrained by the upper-left side of the star. Similarly, for prices below  $v_1$ , speculators buy the asset,  $x_1 > 0$ , and their demand is limited by the margin constraint. Interestingly, the speculators' demand is upward sloping for prices below 120. As the price

declines, the financiers' estimate of fundamental volatility, and consequently of margins, increase.

We now generalize the analysis to the case where  $a > 0$ . The margin setting becomes more complicated since uninformed financiers must filter out to what extent the equilibrium price change is caused by a movement in fundamentals  $\Delta v_1$  and/or an occurrence of a liquidity event with an order imbalance caused by the presence of customers 0 and 1, but not customer 2. Since customers 0 and 1 want to sell ( $Z_1 = 30$ ), a price increase or modest price decline is most likely due to a change in fundamentals, and hence the margin setting is similar to the case of  $a = 0$ . This is why speculators' demand curve for prices above 100 almost perfectly overlays the relevant part of the hyperbolic star in Figure 3. However, for a large price drop, say below 100, financiers assign a larger conditional probability that a liquidity event has occurred. Hence, they are willing to set a lower margin (relative to the one implying the hyperbolic star) because they expect the speculator to profit as the price rebounds in period 2—hence, the cushioning effect discussed above reappears in the extreme here. This explains why the speculators' demand curve is backward bending only in a limited price range and becomes downward sloping for  $p_1$  below roughly 100.<sup>8</sup>

Panel A of Figure 3 shows that there are two stable equilibria: a perfect liquidity equilibrium with price  $p_1 = v_1 = 120$  and an illiquid equilibrium with a price of about 94 (and an uninteresting unstable equilibrium with  $p_1$  just below 120).

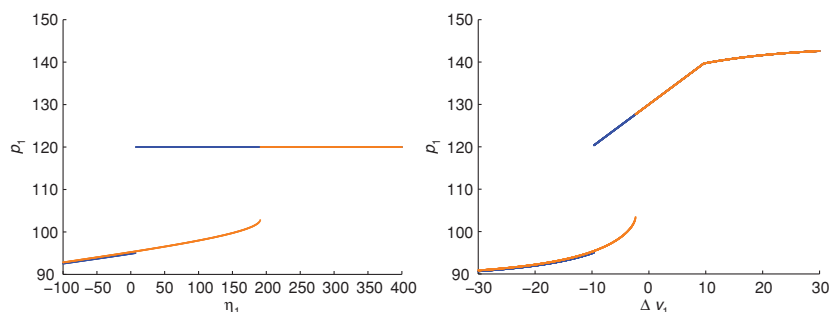
Panel B of Figure 3 shows the same plot as panel A, but with a negative wealth shock to speculators of  $\eta_1 = -150$  instead of  $\eta_1 = 0$ . In this case, perfect liquidity with  $p_1 = v_1$  is no longer an equilibrium since the speculators cannot fund a large enough position. The unique equilibrium is highly illiquid because of the speculators' lower wealth and, importantly, because of endogenously higher margins.

This “disconnect” between the perfect-liquidity equilibrium and the illiquid equilibrium and the resulting fragility is illustrated more directly in Figure 4. Panel A plots the equilibrium price correspondence for different exogenous funding shocks  $\eta_1$  (with fixed  $\Delta v_1 = -10$ ) and shows that a marginal reduction in funding cannot always lead to a smooth reduction in market liquidity. Rather, there must be a level of funding such that an infinitesimal drop in funding leads to a discontinuous drop in market liquidity.

The dark line in Figure 4 shows the equilibrium with the highest market liquidity and the light line shows the equilibrium with the lowest market liquidity. We note that the financiers' filtering problem and, hence, the margin function depend on the equilibrium selection. Since the margin affects the

<sup>8</sup> We note that the cushioning effect relies on the financiers' knowledge that the market will become liquid in period  $t = 2$ . This is not the case in the earlier period 0, though. In an earlier version of the paper, we showed that the cushioning effect disappears in a stationary infinite horizon setting in which the “complementary” customers arrive in each period with a constant arrival probability.





**Figure 4**  
**Fragility due to destabilizing margins**

The figure shows the equilibrium price as a function of the speculators' wealth shock  $\eta_1$  (panel A) and of fundamental shocks  $\Delta v_1$  (panel B). This is drawn for the equilibrium with the highest market liquidity (light line) and the equilibrium with the lowest market liquidity (dark line). The margins are destabilizing since financiers are uninformed and fundamentals exhibit volatility clustering. The equilibrium prices are discontinuous, which reflects *fragility* in liquidity since a small shock can lead to a disproportionately large price effect.

speculators' trades, the equilibrium selection affects the equilibrium outcome everywhere—prices are slightly affected even outside the  $\eta$  region ( $v_1$  region) with fragility.

Panel B of Figure 4 plots the equilibrium price correspondence for different realizations of the fundamental shock  $\Delta v_1$  (with fixed  $\eta_1 = 0$ ) and shows the same form of discontinuity for adverse fundamental shocks to  $v_1$ . The discontinuity with respect to  $\Delta v_1$  is most easily understood in conjunction with panel A of Figure 3. As  $\Delta v_1$  falls, the horizontal line of speculator demand shifts downward, and the customer supply line moves downward. As a result, the perfect liquidity equilibrium vanishes. Panel B of Figure 4 also reveals the interesting asymmetry that negative fundamental shocks lead to larger price movements than corresponding positive shocks (for  $Z_1 := z_0 + z_1 > 0$ ). This asymmetry arises even without a loss effect since  $x_0 = 0$ .

Fragility can also arise because of shocks to customer demand or volatility. Indeed, the market can also be suddenly pushed into an illiquid equilibrium with high margins due to an increase in demand and an increase in volatility. Paradoxically, a marginally larger demand for liquidity by customers can lead to a drastic reduction of liquidity supply by the speculators when it pushes the equilibrium over the edge.

While the example above has speculators with zero initial positions,  $x_0 = 0$ , it is also interesting to consider  $x_0 > 0$ . In this case, lower prices lead to losses for the speculators, and graphically this means that the constraints in the “hyperbolic star” tighten (i.e., the gap between the hyperbolas narrows) at low prices. Because of this “loss effect,” the discontinuous price drop associated with the illiquid equilibrium is even larger.

In summary, this example shows how destabilizing margins and dealer losses give rise to a discontinuity in prices, which can help to explain the sudden market

liquidity dry-ups observed in many markets. For example, Russia's default in 1998 was in itself only a trivial wealth shock relative to global arbitrage capital. Nevertheless, it had a large effect on liquidity in global financial markets, consistent with our fragility result that a small wealth shock can push the equilibrium over the edge.

### 3.2 Liquidity Spirals

To further emphasize the importance of speculators' funding liquidity, we now show how it can make market liquidity highly sensitive to shocks. We identify two amplification mechanisms: a "margin spiral" due to increasing margins as speculator financing worsens, and a "loss spiral" due to escalating speculator losses.

Figure 2 illustrates these "liquidity spirals." A shock to speculator capital ( $\eta_1 < 0$ ) forces speculators to provide less market liquidity, which increases the price impact of the customer demand pressure. With uninformed financiers and ARCH effects, the resulting price swing increases financiers' estimate of the fundamental volatility and, hence, increases the margin, thereby worsening speculator funding problems even further, and so on, leading to a "margin spiral." Similarly, increased market illiquidity can lead to losses on speculators' existing positions, worsening their funding problem and so on, leading to a "loss spiral." Mathematically, the spirals can be expressed as follows:

**Proposition 5.** (i) *If speculators' capital constraint is slack, then the price  $p_1$  is equal to  $v_1$  and insensitive to local changes in speculator wealth.*  
(ii) (**Liquidity spirals**) *In a stable illiquid equilibrium with selling pressure from customers,  $Z_1, x_1 > 0$ , the price sensitivity to speculator wealth shocks  $\eta_1$  is*

$$\frac{\partial p_1}{\partial \eta_1} = \frac{1}{\frac{2}{\gamma(\sigma_2)^2} m_1^+ + \frac{\partial m_1^+}{\partial p_1} x_1 - x_0} \quad (24)$$

*and with buying pressure from customers,  $Z_1, x_1 < 0$ ,*

$$\frac{\partial p_1}{\partial \eta_1} = \frac{-1}{\frac{2}{\gamma(\sigma_2)^2} m_1^- + \frac{\partial m_1^-}{\partial p_1} x_1 + x_0}. \quad (25)$$

A **margin/haicut spiral** arises if  $\frac{\partial m_1^+}{\partial p_1} < 0$  or  $\frac{\partial m_1^-}{\partial p_1} > 0$ , which happens with positive probability if financiers are uninformed and  $a$  is small enough. A **loss spiral** arises if speculators' previous position is in the opposite direction as the demand pressure,  $x_0 Z_1 > 0$ .

This proposition is intuitive. Imagine first what happens if speculators face a wealth shock of \$1, margins are constant, and speculators have no inventory

$x_0 = 0$ . In this case, the speculator must reduce his position by  $1/m_1$ . Since the slope of each of the two customer demand curves is<sup>9</sup>  $1/(\gamma(\sigma_2)^2)$ , we get a total price effect of  $1/(\frac{2}{\gamma(\sigma_2)^2}m_1)$ .

The two additional terms in the denominator imply amplification or dampening effects due to changes in the margin requirement and to profit/losses on the speculators' existing positions. To see that, recall that for any  $k > 0$  and  $l$  with  $|l| < k$ , it holds that  $\frac{1}{k-l} = \frac{1}{k} + \frac{l}{k^2} + \frac{l^2}{k^3} + \dots$ ; so with  $k = \frac{2}{\gamma(\sigma_2)^2}m_1$  and  $l = -\frac{\partial m_1^\pm}{\partial p_1}x_1 \pm x_0$ , each term in this infinite series corresponds to one loop around the circle in Figure 2. The total effect of the changing margin and speculators' positions amplifies the effect if  $l > 0$ . Intuitively, with  $Z_1 > 0$ , then customer selling pressure is pushing down the price, and  $\frac{\partial m_1^+}{\partial p_1} < 0$  means that as prices go down, margins increase, making speculators' funding tighter and thus destabilizing the system. Similarly, when customers are buying,  $\frac{\partial m_1^-}{\partial p_1} > 0$  implies that increasing prices leads to increased margins, making it harder for speculators to short-sell, thus destabilizing the system. The system is also destabilized if speculators lose money on their previous position as prices move away from fundamentals.

Interestingly, the total effect of a margin spiral together with a loss spiral is greater than the sum of their separate effects. This can be seen mathematically by using simple convexity arguments, and it can be seen intuitively from the flow diagram of Figure 2.

Note that spirals can also be "started" by shocks to liquidity demand  $Z_1$ , fundamentals  $v_1$ , or volatility. It is straightforward to compute the price sensitivity with respect to such shocks. They are just multiples of  $\frac{\partial p_1}{\partial \eta_1}$ . For instance, a fundamental shock affects the price both because of its direct effect on the final payoff and because of its effect on customers' estimate of future volatility—and both of these effects are amplified by the liquidity spirals.

Our analysis sheds some new light on the 1987 stock market crash, complementing the standard culprit, portfolio insurance trading. In the 1987 stock market crash, numerous market makers hit (or violated) their funding constraint:

"By the end of trading on October 19, [1987] thirteen [NYSE specialist] units had no buying power," —SEC (1988, chap. 4, p. 58)

While several of these firms managed to reduce their positions and continue their operations, others did not. For instance, Tompane was so illiquid that it was taken over by Merrill Lynch Specialists and Beauchamp was taken over by Spear, Leeds & Kellogg (Beauchamp's clearing broker). Also, market makers outside the NYSE experienced funding troubles: the Amex market makers Damm Frank and Santangelo were taken over; at least 12 OTC market makers ceased operations; and several trading firms went bankrupt.

<sup>9</sup> See Equation (12).

These funding problems were due to (i) reductions in capital arising from trading losses and defaults on unsecured customer debt, (ii) an increased funding need stemming from increased inventory, and (iii) increased margins. One New York City bank, for instance, increased margins/haircuts from 20% to 25% for certain borrowers, and another bank increased margins from 25% to 30% for all specialists (SEC, 1988, pp. 5–27 and 5–28). Other banks reduced the funding period by making intraday margin calls, and at least two banks made intraday margin calls based on assumed 15% and 25% losses, thus effectively increasing the haircut by 15% and 25%. Also, some broker-dealers experienced a reduction in their line of credit and—as Figure 1 shows—margins at the futures exchanges also drastically increased (SEC 1988 and Wigmore 1998). Similarly, during the ongoing liquidity and credit crunch, the margins and haircuts across most asset classes widened significantly starting in the summer of 2007 (see IMF Global Stability Report, October 2007).

In summary, our results on fragility and liquidity spirals imply that during “bad” times, small changes in underlying funding conditions (or liquidity demand) can lead to sharp reductions in liquidity. The 1987 crash exhibited several of the predicted features, namely capital-constrained dealers, increased margins, and increased illiquidity.

#### 4. Commonality and Flight to Quality (Time 1)

We now turn to the cross-sectional implications of illiquidity. Since speculators are risk-neutral, they optimally invest all their capital in securities that have the greatest expected profit  $|\Lambda^j|$  per capital use, i.e., per dollar margin  $m^j$ , as expressed in Equation (14). That equation also introduces the shadow cost of capital  $\phi_1$  as the marginal value of an extra dollar. The speculators’ shadow cost of capital  $\phi_1$  captures well the notion of funding liquidity: a high  $\phi$  means that the available funding—from capital  $W_1$  and from collateralized financing with margins  $m_1^j$ —is low relative to the needed funding, which depends on the investment opportunities deriving from demand shocks  $z^j$ .

The market liquidity of all assets depends on the speculators’ funding liquidity, especially for high-margin assets, and this has several interesting implications:

**Proposition 6.** *There exists  $c > 0$  such that, for  $\theta^j < c$  for all  $j$  and either informed financiers or uninformed with  $a < c$ , we have:*

- (i) **Commonality of market liquidity.** *The market illiquidities  $|\Lambda|$  of any two securities,  $k$  and  $l$ , co-move,*

$$\text{Cov}_0(|\Lambda_1^k|, |\Lambda_1^l|) \geq 0, \quad (26)$$

and market illiquidity co-moves with funding illiquidity as measured by speculators' shadow cost of capital,  $\phi_1$ ,

$$\text{Cov}_0[|\Lambda_1^k|, \phi_1] \geq 0. \quad (27)$$

- (ii) **Commonality of fragility.** *Jumps in market liquidity occur simultaneously for all assets for which speculators are marginal investors.*
- (iii) **Quality and liquidity.** *If asset  $l$  has lower fundamental volatility than asset  $k$ ,  $\underline{\sigma}^l < \underline{\sigma}^k$ , then  $l$  also has lower market illiquidity,*

$$|\Lambda_1^l| \leq |\Lambda_1^k|, \quad (28)$$

if  $x_1^k \neq 0$  or  $|Z_1^k| \geq |Z_1^l|$ .

- (iv) **Flight to quality.** *The market liquidity differential between high- and low-fundamental-volatility securities is bigger when speculator funding is tight, that is,  $\underline{\sigma}^l < \underline{\sigma}^k$  implies that  $|\Lambda_1^k|$  increases more with a negative wealth shock to the speculator,*

$$\frac{\partial |\Lambda_1^l|}{\partial(-\eta_1)} \leq \frac{\partial |\Lambda_1^k|}{\partial(-\eta_1)}, \quad (29)$$

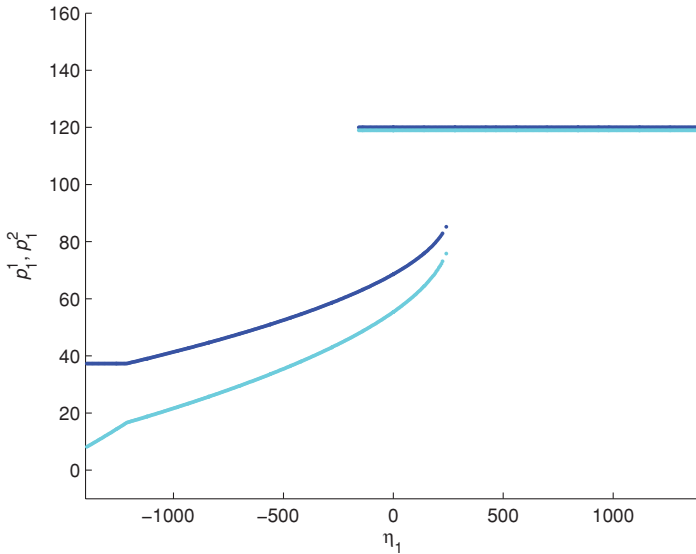
if  $x_1^k \neq 0$  or  $|Z_1^k| \geq |Z_1^l|$ . Hence, if  $x_1^k \neq 0$  or  $|Z_1^k| \geq |Z_1^l|$  a.s., then

$$\text{Cov}_0(|\Lambda_1^l|, \phi_1) \leq \text{Cov}_0(|\Lambda_1^k|, \phi_1). \quad (30)$$

**Numerical example, continued.** To illustrate these cross-sectional predictions, we extend the numerical example of Section 3 to two securities. The two securities only differ in their long-run fundamental volatility:  $\bar{\sigma}^1 = 7.5$  and  $\bar{\sigma}^2 = 10$ . The other parameters are as before, except that we double  $W_1$  to 1800 since the speculators now trade two securities, the financiers remain uninformed, and we focus on the simpler limited case with  $a \rightarrow 0$ .

Figure 5 depicts the assets' equilibrium prices for different values of the funding shock  $\eta_1$ . First note that as speculator funding tightens and our funding illiquidity measure  $\phi_1$  rises, the market illiquidity measure  $|\Lambda_1^j|$  rises for both assets. Hence, for random  $\eta_1$ , we see our commonality in liquidity result  $\text{Cov}_0[|\Lambda_1^k|, |\Lambda_1^l|] > 0$ .

The "commonality in fragility" cannot directly be seen from Figure 5, but it is suggestive that both assets have the same range of  $\eta_1$  with two equilibrium prices  $p_1^j$ . The intuition for this result is the following. Whenever funding is unconstrained, there is perfect market liquidity provision for all assets. If funding is constrained, then it cannot be the case that speculators provide perfect liquidity for one asset but not for the other, since they would always have an incentive to shift funds toward the asset with non-perfect market



**Figure 5**  
**Flight to quality and commonality in liquidity**

The figure plots the prices  $p_1^j$  of assets 1 and 2 as functions of speculators' funding shocks  $\eta_1$ . Asset 1 (darker curve) has lower long-run fundamental risk than asset 2 (lighter curve),  $\bar{\sigma}^1 = 7.5 < 10 = \bar{\sigma}^2$ .

liquidity. Hence, market illiquidity jumps for both assets at exactly the same funding level.

Our result relating fundamental volatility to market liquidity ("quality and liquidity") is reflected in  $p_1^2$  being below  $p_1^1$  for any given funding level. Hence, the high-fundamental-volatility asset 2 is always less liquid than the low-fundamental-volatility asset 1.

Figure 5 also illustrates our result on "flight to quality." To see this, consider the two securities' relative price sensitivity with respect to  $\eta_1$ . For large wealth shocks, market liquidity is perfect for both assets, i.e.,  $p_1^1 = p_1^2 = v_1^1 = v_1^2 = 120$ , so in this high range of funding, market liquidity is insensitive to marginal changes in funding. On the lower branch of the graph, market illiquidity of both assets increases as  $\eta_1$  drops since speculators must take smaller stakes in both assets. Importantly, as funding decreases,  $p_1^2$  decreases more steeply than  $p_1^1$ , that is, asset 2 is more sensitive to funding declines than asset 1. This is because speculators cut back more on the "funding-intensive" asset 2 with its high margin requirement. Speculators want to maximize their profit per dollar margin,  $|\Lambda^j|/m^j$ , and therefore  $|\Lambda^2|$  must be higher than  $|\Lambda^1|$  to compensate speculators for using more capital for margin.

Both price functions exhibit a kink around  $\eta = -1210$ , because, for sufficiently low funding levels, speculators put all their capital into asset 2. This is because the customers are more eager to sell the more volatile asset 2, leading to more attractive prices for the speculators.

## 5. Liquidity Risk (Time 0)

We now turn attention to the initial time period,  $t = 0$ , and demonstrate that (i) funding liquidity risk matters even before margin requirements actually bind; (ii) the pricing kernel depends on future funding liquidity,  $\phi_{t+1}$ ; (iii) the conditional distribution of prices  $p_1$  is skewed due to the funding constraint (inducing fat tails *ex ante*); and (iv) margins  $m_0$  and illiquidity  $\Lambda_0$  can be positively related due to liquidity risk even if financiers are informed.

The speculators' trading activity at time 0 naturally depends on their expectations about the next period and, in particular, the time 1 illiquidity described in detail above. Further, speculators risk having negative wealth  $W_1$  at time 1, in which case they have utility  $\phi_t W_t$ . If speculators have no dis-utility associated with negative wealth levels ( $\phi_t = 0$ ), then they go to their limit already at time 0 and the analysis is similar to time 1.

We focus on the more realistic case in which the speculators have dis-utility in connection with  $W_1 < 0$  and, therefore, choose not to trade to their constraint at time  $t = 0$  when their wealth is large enough. To understand this, note that while most firms legally have limited liability, the capital  $W_t$  in our model refers to pledgable capital allocated to trading. For instance, Lehman Brothers' 2001 Annual Report (p. 46) states:

"The following must be funded with cash capital: Secured funding 'haircuts,' to reflect the estimated value of cash that would be advanced to the Company by counterparties against available inventory, Fixed assets and goodwill, [and] Operational cash ..."

Hence, if Lehman suffers a large loss on its pledgable capital such that  $W_t < 0$ , then it incurs monetary costs that must be covered with its unpledgable capital like operational cash (which could also hurt Lehman's other businesses). In addition, the firm incurs non-monetary cost, like loss in reputation and in goodwill, that reduces its ability to exploit future profitable investment opportunities. To capture these effects, we let a speculator's utility be  $\phi_1 W_1$ , where  $\phi_1$  is given by the right-hand side of Equation (14) both for positive and negative values of  $W_1$ . With this assumption, equilibrium prices at time  $t = 0$  are such that the speculators do not trade to their constraint at time  $t = 0$  when their wealth is large enough. In fact, this is the weakest assumption that curbs the speculators' risk taking since it makes their objective function linear. Higher "bankruptcy costs" would lead to more cautious trading at time 0 and qualitatively similar results.<sup>10</sup>

<sup>10</sup> We note that risk aversion also limits speculators' trading in the real world. Our model based on margin constraints differs from one driven purely by risk aversion in several ways. For example, an adverse shock that lowers speculator wealth at  $t = 1$  creates a profitable investment opportunity that one might think partially offsets the loss—a natural "dynamic hedge." Because of this dynamic hedge, in a model driven by risk-aversion, speculators (with a relative-risk-aversion coefficient larger than one) increase their  $t = 0$  hedging demand, which in turn, lowers illiquidity in  $t = 0$ . However, exactly the opposite occurs in a setting with capital constraints. Capital constraints prevent speculators from taking advantage of investment opportunities in  $t = 1$  so they cannot exploit this "dynamic hedge." Hence, speculators are reluctant to trade away the illiquidity at  $t = 0$ .

If the speculator is not constrained at time  $t = 0$ , then the first-order condition for his position in security  $j$  is  $E_0[\phi_1(p_1^j - p_0^j)] = 0$ . (We leave the case of a constrained time-0 speculator for Appendix B.) Consequently, the funding liquidity,  $\phi_1$ , determines the pricing kernel  $\phi_1/E_0[\phi_1]$  for the cross section of securities:

$$p_0^j = \frac{E_0[\phi_1 p_1^j]}{E_0[\phi_1]} = E_0[p_1^j] + \frac{Cov_0[\phi_1, p_1^j]}{E_0[\phi_1]}. \quad (31)$$

Equation (31) shows that the price at time 0 is the expected time-1 price, which already depends on the liquidity shortage at time 1, further adjusted for liquidity risk in the form of a covariance term. The liquidity risk term is intuitive: The time-0 price is lower if the covariance is negative, that is, if the security has a low payoff during future funding liquidity crises when  $\phi_1$  is high.

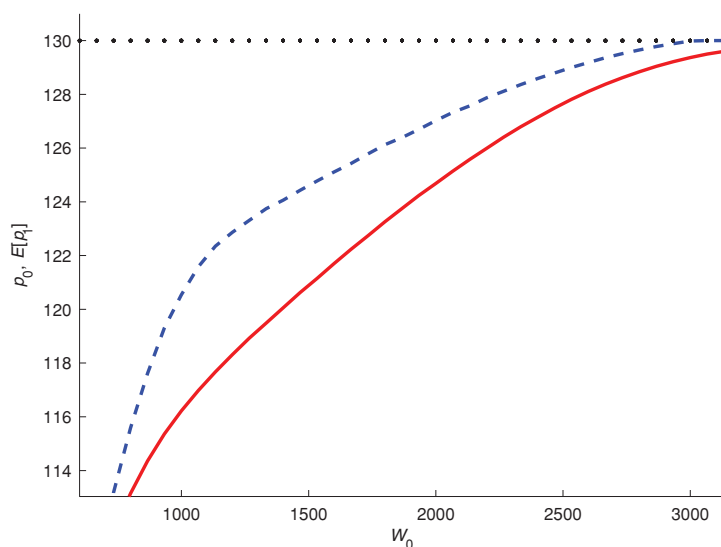
An illustration of the importance of funding-liquidity management is the “LTCM crisis.” The hedge fund Long Term Capital Management (LTCM) had been aware of funding liquidity risk. Indeed, they estimated that in times of severe stress, haircuts on AAA-rated commercial mortgages would increase from 2% to 10%, and similarly for other securities (HBS Case N9-200-007(A)). In response to this, LTCM had negotiated long-term financing with margins fixed for several weeks on many of their collateralized loans. Other firms with similar strategies, however, experienced increased margins. Due to an escalating liquidity spiral, LTCM could ultimately not fund its positions in spite of its numerous measures to control funding risk, it was taken over by fourteen banks in September 1998. Another recent example is the funding problems of the hedge fund Amaranth in September 2006, which reportedly ended with losses in excess of USD 6 billion. The ongoing liquidity crisis of 2007–2008, in which funding based on the asset-backed commercial paper market suddenly eroded and banks were reluctant to lend to each other out of fear of future funding shocks, provides a nice out-of-sample test of our theory.<sup>11</sup>

**Numerical example, continued.** To better understand funding liquidity risk, we return to our numerical example with one security,  $\eta_1 = 0$  and  $a \rightarrow 0$ . We first consider the setting with uninformed financiers and later turn to the case with informed financiers.

Figure 6 depicts the price  $p_0$  and expected time-1 price  $E_0[p_1]$  for different initial wealth levels,  $W_0$ , for which the speculators’ funding constraint is not binding at  $t = 0$ . The figure shows that even though the speculators are unconstrained at time 0, market liquidity provision is limited with prices below the fundamental value of  $E_0[v] = 130$ . The price is below the fundamental for two reasons: First, the expected time-1 price is below the fundamental value because of the risk that speculators cannot accommodate the customer selling pressure at that time. Second,  $p_0$  is even below  $E_0[p_1]$ , since speculators

<sup>11</sup> See Brunnermeier (2009) for a more complete treatment of the liquidity and credit crunch that started in 2007.





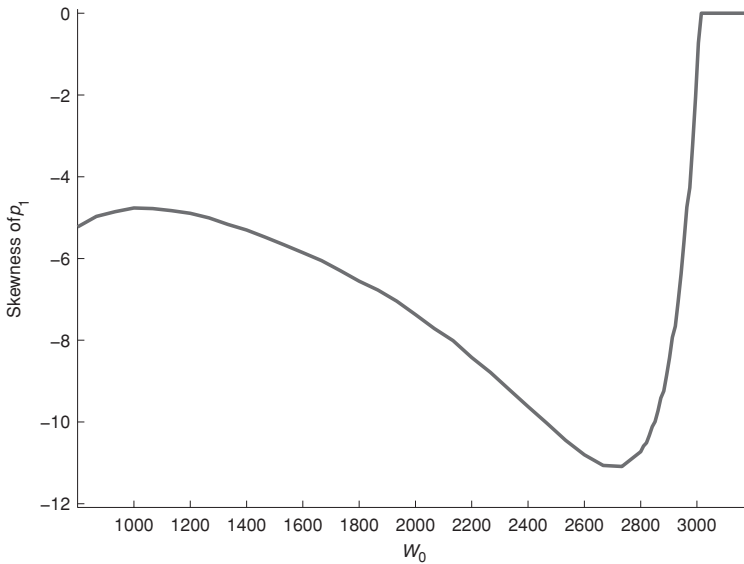
**Figure 6**  
**Illiquidity at time 0**

This graph shows the price  $p_0$  at time 0 (solid line), the expected time-1 price  $E_0[p_1]$  (dashed line), and the fundamental value  $E_0[v] = 130$  (dotted line) for different levels of speculator funding  $W_0$ . The price  $p_0$  is below the fundamental value due to illiquidity, in particular, because of customer selling pressure and the risk that speculators will hit their capital constraints at time 1, even though speculators are not constrained at time 0 for the depicted wealth levels.

face liquidity risk: Holding the security leads to losses in the states of nature when speculators are constrained and investment opportunities are good, implying that  $\text{Cov}[\phi_1, p_1] < 0$ . The additional compensation for liquidity risk is  $\frac{\text{Cov}_0[\phi_1, p_1']}{E_0[\phi_1]}$ , as seen in Equation (31), which is the difference between the solid line  $p_1$  and the dashed  $E_0[p_1]$ .

The funding constraint not only affects the price level, it also introduces skewness in the  $p_1$ -distribution conditional on the sign of the demand pressure. For  $Z_1 > 0$ , speculators take long positions and, consequently, negative  $v_1$ -shocks lead to capital losses with resulting liquidity spirals. This amplification triggers a sharper price drop than the corresponding price increase for positive  $v_1$ -shocks. Figure 7 shows this negative skewness for different funding levels  $W_0$ . The effect is not monotone—zero dealer wealth implies no skewness, for instance.

When customers want to buy (not sell as above), and funding constraints induce a positive skewness in the  $p_1$ -distribution. The speculator's return remains negatively skewed, as above, since it is still its losses that are amplified. This is consistent with the casual evidence that hedge fund return indexes are negatively skewed, and it can help explain why FX carry trade returns are negatively skewed (see Brunnermeier, Nagel, and Pedersen 2009). It also suggests that from an *ex ante* point of view (i.e., prior to the realization of  $Z_1$ ), funding constraints lead to higher kurtosis of the price distribution (fat tails).



**Figure 7**  
**Conditional price skewness**

The figure shows the conditional skewness of  $p_1$  for different funding levels  $W_0$ . While the funding constraint is not binding at time 0, it can become binding at time 1, leading to large price drops due to liquidity spirals. Price increases are not amplified, and this asymmetry results in skewness.

Finally, we can also show numerically that unlike at time  $t = 1$ , margins can be positively related to illiquidity at time 0, even when financiers are fully informed.<sup>12</sup> This is because of the liquidity risk between time 0 and time 1. To see this, note that if we reduce the speculators' initial wealth  $W_0$ , then the market becomes less liquid in the sense that the price is further from the fundamental value. At the same time, the equilibrium price in  $t = 1$  is more volatile and thus equilibrium margins at time 0 can actually increase.

## 6. New Testable Predictions

Our analysis provides a theoretical framework that delivers a unified explanation for a host of stylized empirical facts. Our analysis further suggests a novel line of empirical work that tests the model at a deeper level, namely, its prediction that speculator funding is a driving force underlying these market liquidity effects.

First, it would be of interest to empirically study the determinants of margin requirements (e.g., using data from futures markets or from prime brokers). Our model suggests that both fundamental volatility and liquidity-driven volatility affect margins (Propositions 2 and 3). Empirically, fundamental volatility can

<sup>12</sup> The simulation results are available upon request from the authors.

be captured using price changes over a longer time period, while the sum of fundamental and liquidity-based volatility can be captured by short-term price changes as in the literature on variance ratios (see, for example, Campbell, Lo, and MacKinlay 1997). Our model predicts that, in markets where it is harder for financiers to be informed, margins depend on the total fundamental and liquidity-based volatility. In particular, in times of liquidity crises, margins increase in such markets, and, more generally, margins should co-move with illiquidity in the time series and in the cross section.<sup>13</sup>

Second, our model suggests that an exogenous shock to speculator capital should lead to a reduction in market liquidity (Proposition 5). Hence, a clean test of the model would be to identify exogenous capital shocks, such as an unconnected decision to close down a trading desk, a merger leading to reduced total trading capital, or a loss in one market unrelated to the fundamentals of another market, and then study the market liquidity and margin around such events.

Third, the model implies that the effect of speculator capital on market liquidity is highly nonlinear: a marginal change in capital has a small effect when speculators are far from their constraints, but a large effect when speculators are close to their constraints—illiquidity can suddenly jump (Propositions 4 and 5).

Fourth, the model suggests that a cause of the commonality in liquidity is that the speculators' shadow cost of capital is a driving state variable. Hence, a measure of speculator capital tightness should help explain the empirical co-movement of market liquidity. Further, our result "commonality of fragility" suggests that especially sharp liquidity reductions occur simultaneously across several assets (Proposition 6(i)–(ii)).

Fifth, the model predicts that the sensitivity of margins and market liquidity to speculator capital is larger for securities that are risky and illiquid on average. Hence, the model suggests that a shock to speculator capital would lead to a reduction in market liquidity through a spiral effect that is stronger for illiquid securities (Proposition 6(iv)).

Sixth, speculators are predicted to have negatively skewed returns since, when they hit their constraints, they incur significant losses because of the endogenous liquidity spirals, and, in contrast, their gains are not amplified when prices return to fundamentals. This leads to conditional skewness and unconditional kurtosis of security prices (Section 5).

## 7. Conclusion

By linking funding and market liquidity, this paper provides a unified framework that explains the following stylized facts:

<sup>13</sup> One must be cautious with the interpretation of the empirical results related to changes in Regulation T since this regulation may not affect speculators but affects the demanders of liquidity, namely the customers.

- (1) Liquidity suddenly dries up; we argue that fragility in liquidity is in part due to destabilizing margins, which arise when financiers are imperfectly informed and the fundamental volatility varies.
- (2) Market liquidity and fragility co-moves across assets since changes in funding conditions affects speculators' market liquidity provision of all assets.
- (3) Market liquidity is correlated with volatility, since trading more volatile assets requires higher margin payments and speculators provide market liquidity across assets such that illiquidity per capital use, i.e., illiquidity per dollar margin, is constant.
- (4) Flight to quality phenomena arise in our framework since when funding becomes scarce speculators cut back on the market liquidity provision especially for capital intensive, i.e., high margin, assets.
- (5) Market liquidity moves with the market since funding conditions do.

In addition to explaining these stylized facts, the model also makes a number of specific testable predictions that could inspire further empirical research on margins. Importantly, our model links a security's market illiquidity and risk premium to its margin requirement (i.e. funding use) and the general shadow cost of funding.

Our analysis also suggests policy implications for central banks. Central banks can help mitigate market liquidity problems by controlling funding liquidity. If a central bank is better than the typical financiers of speculators at distinguishing liquidity shocks from fundamental shocks, then the central bank can convey this information and urge financiers to relax their funding requirements—as the Federal Reserve Bank of New York did during the 1987 stock market crash. Central banks can also improve market liquidity by boosting speculator funding conditions during a liquidity crisis, or by simply stating the intention to provide extra funding during times of crisis, which would loosen margin requirements as financiers' worst-case scenarios improve.

## **Appendix A: Real-World Margin Constraints**

A central element of our paper is the capital constraints that the main providers of market liquidity face. In this section, we review the institutional features that drive the funding constraints of securities firms such as hedge funds, banks' proprietary trading desks, and market makers.

### **A.1 Funding requirements for hedge funds**

We first consider the funding issues faced by hedge funds since they have relatively simple balance sheets and face little regulation. A hedge fund's capital consists of its equity capital supplied by the investors, and possible long-term debt financing that can be relied upon during a potential funding crisis. The investors can withdraw their capital at certain times so the equity is not locked into the firm indefinitely as in a corporation,<sup>14</sup> but, to ensure funding, the withdrawal is subject to initial lock-up periods and general redemption notice periods before specific redemption dates (typically at least a month, often several months or even years). Also, hedge funds use a variety of

<sup>14</sup> A few hedge funds have in fact raised some amount of permanent equity capital.

other contractual arrangements to manage their funding liquidity: “Side pocket” determines that a proportion of each investor’s capital, for example, 10%, can only be redeemed when the designated assets (e.g., a privately held firm) are sold. A “gate” limits the fraction of the total capital that can leave the fund during any redemption period. Individual investors’ redemptions are typically prorated in case of excess demand for outflows. “Withdrawal suspensions” (or *force majeure* terms) temporarily suspend withdrawals completely.

Hedge funds usually do not have access to unsecured debt financing, but a few large hedge funds have managed to obtain medium-term bank loans, a guaranteed line of credit,<sup>15</sup> or even issued bonds (see, for example, *The Economist* 1/27/2007, p. 75).

The main sources of leverage for hedge funds are (i) collateralized borrowing financed through the repo market; (ii) collateralized borrowing financed by the hedge fund’s prime broker(s); and (iii) implicit leverage using derivatives, either exchange traded or over the counter (OTC). Real-world financing contracts are complex, opaque (i.e., negotiated privately and hence unobservable to an outsider), different across market participants, and change over time, so our description is somewhat stylized and we discuss some caveats below. Nevertheless, all three forms of financing are based on the same general principle, which we describe first, and then we outline a few specific issues.

The guiding principle for margin setting on levered positions is that the hedge fund’s counterparty should be relatively immune to the hedge fund’s possible losses. In particular, if a hedge fund buys at time  $t$  a long position of  $x_t^j > 0$  shares of a security  $j$  at price  $p_t^j$ , it has to come up with  $x_t^j p_t^j$  dollars. The security can, however, be used as collateral for a new loan of, say,  $l_t^j$  dollars. The difference between the price of the security and the collateral value is denoted as the margin requirement  $m_t^{j+} = p_t^j - l_t^j$ . Hence, this position uses  $x_t^j m_t^{j+}$  dollars of the fund’s capital. The collateralized funding implies that the cash use depends on margins, not notional amounts. The margins are typically set so as to make the loan almost risk-free for the counterparty, that is, such that it covers the largest possible price drop with a certain degree of confidence (i.e., it covers the VaR).<sup>16</sup> Hence, if the price drops and the hedge fund defaults, the counterparty can still recover its loan by selling the security.

Similarly, if the hedge fund wants to sell short a security,  $x_t^j < 0$ , then the fund asks one of its brokers to locate a security that can be borrowed, and then the fund sells the borrowed security. Duffie, Gârleanu, and Pedersen (2002) describe in detail the institutional arrangements of shorting. The broker keeps the proceeds of the short sale and, additionally, requires that the hedge fund posts a margin  $m_t^{j-}$  that covers the largest possible price increase with a certain degree of confidence (in case the hedge fund defaults when the price increases, in which case the broker needs enough cash to buy the security back at a higher price).

This stylized description of collateralized financing portrays well the repo market for fixed-income securities (e.g., government and corporate bonds) and the prime brokerage that banks offer hedge funds for financing equities and convertible bonds, among other things. However, these forms of financing have different implementation. Prime brokerage is an ongoing service provided by banks in which they finance a whole portfolio of the hedge funds’ securities on an ongoing basis (as well as providing other services), whereas in the repo market, a hedge fund will often get bids from multiple counterparties each time they make a new repo transaction. The portfolio nature of the prime brokerage business means that the prime broker can take diversification among securities into account and therefore lower the margin using so-called cross-margining, as we describe further below.

<sup>15</sup> A line of credit may have a “material adverse change” clause or other covenants subject to discretionary interpretation of the lender. Such covenants imply that the line of credit may not be a reliable source of funding during a crisis.

<sup>16</sup> An explicit equation for the margin is given by Equation (6) in Section 1. Often brokers also take into account the delay between the time a failure by the hedge fund is noticed, and the time the security is actually sold. Hence, the margin of a one-day collateralized loan depends on the estimated risk of holding the asset over a time period that is often set as five to ten days.

As an aside, margins on U.S. equities are in principle subject to Regulation T, which stipulates that non-brokers/dealers must put down an initial margin (down payment) of 50% of the market value of the underlying stock, both for new long and short positions. Hedge funds can, however, get around Regulation T in various ways and therefore face significantly lower stock margins. For example, their prime broker can organize the transaction offshore or as a total return swap, which is a derivative that is functionally equivalent to buying the stock.

With derivatives, the principle is similar, although the hedge fund does not “borrow against” the security, it must simply post margins to enter into the derivative contract in the first place. Suppose, for instance, that a hedge fund buys an OTC forward contract. The forward contract initially has a market value of zero (so in this sense the contract has leverage built in), but this does not mean that you can buy the forward without cash. To enter into the forward contract, which obviously has risk, the hedge fund must post margins corresponding to the largest adverse price move with a certain confidence. To ease netting long and short positions, unwinding, and other things, many OTC derivatives are structured using standardized swaps provided by the International Swaps and Derivatives Association (ISDA).

For exchange-traded derivatives such as futures and options, a hedge fund trades through a clearing broker (sometimes referred to as a futures clearing merchant). The exchange requires margins from the broker, and these margins are set using the same principle as described above, that is, the margin is set to make the exchange almost immune to losses and hence riskier contracts have larger margins. The broker, in turn, typically passes the margin requirement on to the hedge fund. Sometimes, the broker requires higher margins from the hedge fund or lower margins (the latter is considered granting the hedge fund a risky loan, usually at an interest rate spread). While the broker margins are opaque as mentioned above, the exchange margins are usually publicly available. Figure 1 depicts the exchange margins charged by the CME for the S&P 500 futures contract.

A hedge fund must finance all of its positions, that is, the sum of all the margin requirements on long and short positions cannot exceed the hedge fund's capital. In our model, this is captured by the key Equation (4) in Section 1.

At the end of the financing period, time  $t + 1$ , the position is “marked-to-market,” which means that the hedge fund is credited any gains (or pays any losses) that have occurred between  $t$  and  $t + 1$ , that is, the fund receives  $x_t^j(p_{t+1}^j - p_t^j)$  and pays interest on the loan at the funding rate. If the trade is kept on, the broker keeps the margin to protect against losses going forward from time  $t + 1$ . The margin can be adjusted if the risk of the collateral has changed, unless the counterparties have contractually fixed the margin for a certain period.

Instead of posting risk-free assets (cash), a hedge fund can also post other risky assets, say asset  $k$ , to cover its margin on position, say  $x^j$ . However, in this case, a “haircut,”  $h_t^k$ , is subtracted from asset  $k$ 's market value to account for the riskiness of the collateral. The funding constraint becomes  $x_t^j m_t^j \leq W_t - x_t^k h_t^k$ . Moving the haircut term to the left-hand side reveals that the haircut is equivalent to a margin, since the hedge fund could alternatively have used the risky security to raise cash and then used this cash to cover the margins for asset  $j$ . We therefore use the terms “margins” and “haircuts” interchangeably.

We have described how funding constraints work when margins and haircuts are set separately for each security position. As indicated earlier, it is, however, increasingly possible to “cross-margin” (i.e., to jointly finance several positions). This leads to a lower total margin if the risks of the various positions are partially offsetting. For instance, much of the interest rate risk is eliminated in a “spread trade” with a long position in one bond and a short position in a similar bond. Hence, the margin/haircut of a jointly financed spread trade is smaller than the sum of the margins of the long and short bonds. For a strategy that is financed jointly, we can reinterpret security  $j$  as such a strategy. Prime brokers compete (especially when credit is as loose as in early 2007) by, among other things, offering low margins and haircuts, a key consideration for hedge funds, which means that it has become increasingly easy to finance more and more strategies jointly. It is by now relatively standard to cross-margin an equity portfolio or a portfolio of convertible bonds, and so-called cross-product-margining, which attempts to give diversification benefits across asset classes,

is becoming more common although it is associated with some issues that make some hedge funds avoid it.<sup>17</sup> In the extreme, one can imagine a joint financing of a hedge fund's total position such that the "portfolio margin" would be equal to the maximum portfolio loss with a certain confidence level. Currently, it is often not practical to jointly finance a large portfolio with all the benefits of diversification. This is because a large hedge fund finances its trades using several brokers; both a hedge fund and a broker can consist of several legal entities (possibly located in different jurisdictions); certain trades need separate margins paid to exchanges (e.g., futures and options) or to other counterparties of the prime broker (e.g., securities lenders); prime brokers may not have sufficiently sophisticated models to evaluate the diversification benefits (e.g., because they do not have enough data on the historical performance of newer products such as CDOs); and because of other practical difficulties in providing joint financing. Further, if the margin requirement relies on assumed stress scenarios in which the securities are perfectly correlated (e.g., due to predatory trading, as in Brunnermeier and Pedersen 2005), then the portfolio margin constraint coincides with position-by-position margins.

## A.2 Funding requirements for commercial and investment banks

A bank's capital consists of equity capital plus its long-term borrowing (including credit lines secured from commercial banks, alone or in syndicates), reduced by assets that cannot be readily employed (e.g., goodwill, intangible assets, property, equipment, and capital needed for daily operations), and further reduced by uncollateralized loans extended by the bank to others (see, for example, Goldman Sachs's 2003 Annual Report). Banks also raise money using short-term uncollateralized loans, such as commercial paper and promissory notes, and, in the case of commercial banks, demand deposits. These sources of financing cannot, however, be relied on in times of funding crisis since lenders may be unwilling to continue lending, and therefore this short-term funding is often not included in measures of capital.

The financing of a bank's trading activity is largely based on *collateralized* borrowing. Banks can finance long positions using collateralized borrowing from corporations, other banks, insurance companies, and the Federal Reserve Bank, and can borrow securities to short-sell from, for instance, mutual funds and pension funds. These transactions typically require margins that must be financed by the bank's capital, as captured by the funding constraint in Equation (4).

The financing of a bank's trading is more complicated than that of a hedge fund, however. For instance, banks may negotiate zero margins with certain counterparties, and banks can often sell short shares held in-house, that is, held in a customer's margin account (in "street name") such that the bank does not need to use capital to borrow the shares externally. Further, a bank receives margins when financing hedge funds (i.e., the margin is negative from the point of view of the bank). In spite of these caveats, in times of stress, banks face margin requirements and are ultimately subject to a funding constraint in the spirit of Equation (4). Bear Stearns's demise is a vivid reminder that banks' funding advantage from clients' margin accounts can quickly evaporate. In March of 2008, Bear Stearns's clients terminated their brokerage relationships and ran on the investment bank. Only an orchestrated merger with JPMorgan Chase avoided a bankruptcy.

Banks must also satisfy certain regulatory requirements. Commercial banks are subject to the Basel Accord, supervised by the Federal Reserve System for U.S. banks. In short, the Basel Accord of 1988 requires that a bank's "eligible capital" exceeds 8% of the "risk-weighted asset holdings," which is the sum of each asset holding multiplied by its risk weight. The risk weight is 0% for cash and government securities, 50% for mortgage-backed loans, and 100% for all other assets. The requirement posed by the 1988 Basel Accord corresponds to Equation (4) with margins of 0%, 4%, and 8%, respectively. In 1996, the accord was amended, allowing banks to measure market risk using an internal model based on portfolio VaRs rather than using standardized risk weights. To

<sup>17</sup> For instance, cross-product margining means that the broker effectively can move extra cash from one margin account to cover a loss elsewhere, even if the hedge may dispute the loss. Also, collecting all positions with one broker may mean that the hedge fund cannot get a good pricing on the trades, e.g., on repos, and may expose the hedge fund to predatory trading.

outmaneuver the Basel Accord, banks created a shadow banking system, which allowed them to off-load assets to off-balance sheet vehicles like SIVs and conduits. For details, see Brunnermeier (2009).

Broker-speculators in the United States, including banks acting as such, are subject to the Securities and Exchange Commission's (SEC's) "net capital rule" (SEC Rule 15c3-1). This rule stipulates, among other things, that a broker must have a minimum "net capital," which is defined as equity capital plus approved subordinate liabilities minus "securities haircuts" and operational charges. The haircuts are set as security-dependent percentages of the market value. The standard rule requires that the net capital exceeds at least  $6\frac{2}{3}\%$  (15:1 leverage) of aggregate indebtedness (broker's total money liabilities) or alternatively 2% of aggregate debit items arising from customer transactions. This constraint is similar in spirit to Equation (4).<sup>18</sup> As of August 20, 2004, SEC amended the net capital rule for Consolidated Supervised Entities (CSEs) such that CSEs may, under certain circumstances, use their internal risk models to determine whether they fulfill their capital requirement (SEC Release No. 34-49830).

### A.3 Funding requirements for market makers

There are various types of market-making firms. Some are small partnerships, whereas others are parts of large investment banks. The small firms are financed in a similar way to hedge funds in that they rely primarily on collateralized financing; the funding of banks was described in Section A2.

Certain market makers, such as NYSE specialists, have an obligation to make a market; and a binding funding constraint means that they cannot fulfill this requirement. Hence, avoiding the funding constraint is especially crucial for such market makers.

Market makers are in principle subject to the SEC's net capital rule (described in Section A2), but this rule has special exceptions for market makers. Hence, market makers' main regulatory requirements are those imposed by the exchange on which they operate. These constraints are often similar in spirit to Equation (4).

## Appendix B: Proofs

### Proof of Propositions 1–3

These results follow from the calculations in the text.

### Proof of Proposition 4

We prove the proposition for  $Z_1 > 0$ , implying  $p_1 \leq v_1$  and  $x_1 \geq 0$ . The complementary case is analogous. To see how the equilibrium depends on the exogenous shocks, we first combine the equilibrium condition  $x_1 = -\sum_{k=0}^1 y_1^k$  with the speculator funding constraint to get

$$m_1^+ \left( Z_1 - \frac{2}{\gamma(\sigma_2)^2} (v_1 - p_1) \right) \leq b_0 + p_1 x_0 + \eta_1, \quad (\text{B1})$$

that is,

$$G(p_1) := m_1^+ \left( Z_1 - \frac{2}{\gamma(\sigma_2)^2} (v_1 - p_1) \right) - p_1 x_0 - b_0 \leq \eta_1. \quad (\text{B2})$$

For  $\eta_1$  large enough, this inequality is satisfied for  $p_1 = v_1$ , that is, it is a stable equilibrium that the market is perfectly liquid. For  $\eta_1$  low enough, the inequality is violated for  $p_1 = \frac{2v_1}{\gamma(\sigma_2)^2} - Z_1$ , that is, it is an equilibrium that the speculator is in default. We are interested in intermediate values

<sup>18</sup> Let  $L$  be the lower of  $6\frac{2}{3}\%$  of total indebtedness or 2% of debit items and  $h^j$  the haircut for security  $j$ ; then the rule requires that  $L \leq W - \sum_j h^j x^j$ , that is,  $\sum_j h^j x^j \leq W - L$ .



of  $\eta_1$ . If the left-hand side  $G$  of (B2) is increasing in  $p_1$ , then  $p_1$  is a continuously increasing function of  $\eta_1$ , implying no fragility with respect to  $\eta_1$ .

Fragility arises if  $G$  can be decreasing in  $p_1$ . Intuitively, this expression measures speculator funding needs at the equilibrium position, and fragility arises if the funding need is greater when prices are lower, that is, further from fundamentals. (This can be shown to be equivalent to a non-monotonic excess demand function.)

When the financiers are informed, the left-hand side  $G$  of (B2) is

$$(\bar{\sigma} + \bar{\theta}|\Delta v_1| + p_1 - v_1) \left( Z_1 + \frac{2}{\gamma(\sigma_2)^2}(p_1 - v_1) \right) - p_1 x_0 - b_0. \quad (\text{B3})$$

The first product is a product of two positive increasing functions of  $p_1$ , but the second term,  $-p_1 x_0$ , is decreasing in  $p_1$  if  $x_0 > 0$ . Since the first term does not depend on  $x_0$ , there exists  $\underline{x}$  such that, for  $x_0 > \underline{x}$ , the whole expression is decreasing.

When the financier is uninformed, we first show that there is fragility for  $a = 0$ . In this case, the left-hand side of (B2) is

$$G^0(p_1) := (\bar{\sigma} + \bar{\theta}|\Delta p_1|) \left( Z_1 + \frac{2}{\gamma(\sigma_2)^2}(p_1 - v_1) \right) - p_1 x_0 - b_0. \quad (\text{B4})$$

When  $p_1 < p_0$ ,  $\bar{\theta}|\Delta p_1| = \theta(p_0 - p_1)$  decreases in  $p_1$  and, if  $\bar{\theta}$  is large enough, this can make the entire expression decreasing. (Since  $\bar{\theta}$  is proportional to  $\theta$ , this clearly translates directly to  $\theta$ .) Also, the expression is decreasing if  $x_0$  is large enough.

Finally, on any compact set of prices, the margin function converges uniformly to (23) as  $a$  approaches 0. Hence,  $G$  converges uniformly to  $G^0$ . Since the limit function  $G^0$  has a decreasing part, choose  $p_1^a < p_1^b$  such that  $\varepsilon := G^0(p_1^a) - G^0(p_1^b) > 0$ . By uniform convergence, choose  $\underline{a} > 0$  such that for  $a < \underline{a}$ ,  $G$  differs from  $G^0$  by at most  $\varepsilon/3$ . Then we have

$$G(p_1^a) - G(p_1^b) = G^0(p_1^a) - G^0(p_1^b) + [G(p_1^a) - G^0(p_1^a)] - [G(p_1^b) - G^0(p_1^b)] \quad (\text{B5})$$

$$\geq \varepsilon - \frac{\varepsilon}{3} - \frac{\varepsilon}{3} = \frac{\varepsilon}{3} > 0, \quad (\text{B6})$$

which proves that  $G$  has a decreasing part.

It can be shown that the price cannot be chosen continuously in  $\eta_1$  when the left-hand side of (B2) can be decreasing.

## Proof of Proposition 5

When the funding constraint binds, we use the implicit function theorem to compute the derivatives. As above, we have

$$m_1^+ \left( Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1) \right) = b_0 + p_1 x_0 + \eta_1. \quad (\text{B7})$$

We differentiate this expression to get

$$\frac{\partial m_1^+}{\partial p_1} \frac{\partial p_1}{\partial \eta_1} \left( Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1) \right) + m_1^+ \frac{2}{\gamma(\sigma_2)^2} \frac{\partial p_1}{\partial \eta_1} = \frac{\partial p_1}{\partial \eta_1} x_0 + 1, \quad (\text{B8})$$

which leads to Equation (24) after rearranging. The case of  $Z_1 < 0$  (i.e., Equation (25)) is analogous.

Finally, spiral effects happen if one of the last two terms in the denominator of the right-hand side of Equations (24) and (25) is negative. (The total value of the denominator is positive by definition of a stable equilibrium.) When the speculator is informed,  $\frac{\partial m_1^+}{\partial p_1} = 1$  and  $\frac{\partial m_1^-}{\partial p_1} = -1$  using Proposition 2. Hence, in this case, margins are stabilizing.

If the speculators are uninformed and  $\alpha$  approaches 0, then using Proposition 3, we find that  $\frac{\partial m_1^+}{\partial p_1} = \frac{\partial m_1^+}{\partial \Lambda_1}$  approaches  $-\bar{\theta} < 0$  for  $v_1 - v_0 + \Lambda_1 - \Lambda_0 < 0$  and  $\frac{\partial m_1^-}{\partial p_1} = \frac{\partial m_1^-}{\partial \Lambda_1}$  approaches  $\bar{\theta} > 0$  for  $v_1 - v_0 + \Lambda_1 - \Lambda_0 > 0$ . This means that there is a margin spiral with positive probability. The case of a loss spiral is immediately seen to depend on the sign on  $x_0$ .

### Proof of Proposition 6

We first consider the equation that characterizes a constrained equilibrium. When there is selling pressure from customers,  $Z_1^j > 0$ , it holds that

$$|\Lambda_1^j| = -\Lambda_1^j = v_1^j - p_1^j = \min \left\{ (\phi_1 - 1)m_1^{j+}, \frac{\gamma(\sigma_2^j)^2}{2} Z_1^j \right\}, \quad (\text{B9})$$

and if customers are buying,  $Z_1^j < 0$ , we have

$$|\Lambda_1^j| = \Lambda_1^j = p_1^j - v_1^j = \min \left\{ (\phi_1 - 1)m_1^{j-}, \frac{\gamma(\sigma_2^j)^2}{2} (-Z_1^j) \right\}. \quad (\text{B10})$$

We insert the equilibrium condition  $x_1^j = -\sum_k y_1^{j,k}$  and Equation (12) for  $y_1^{j,k}$  into the speculators' funding condition to get

$$\begin{aligned} \sum_{Z_1^j > \frac{2(\phi_1 - 1)m_1^{j+}}{\gamma(\sigma_2^j)^2}} m_1^{j+} \left( Z_1^j - \frac{2(\phi_1 - 1)m_1^{j+}}{\gamma(\sigma_2^j)^2} \right) + \sum_{-Z_1^j > \frac{2(\phi_1 - 1)m_1^{j-}}{\gamma(\sigma_2^j)^2}} m_1^{j-} \left( -Z_1^j - \frac{2(\phi_1 - 1)m_1^{j-}}{\gamma(\sigma_2^j)^2} \right) \\ = \sum_j x_0^j p_1^j + b_0 + \eta_1, \end{aligned} \quad (\text{B11})$$

where the margins are evaluated at the prices solving Equations (B9)–(B10). When  $\phi_1$  approaches infinity, the left-hand side of Equation (B11) becomes zero, and when  $\phi_1$  approaches one, the left-hand side approaches the capital needed to make the market perfectly liquid. As in the case of one security, there can be multiple equilibria and fragility (Proposition 4). On a stable equilibrium branch,  $\phi_1$  increases as  $\eta_1$  decreases. Of course, the equilibrium shadow cost of capital  $(\phi_1 - 1)$  is random since  $\eta_1, \Delta v_1^1, \dots, \Delta v_1^J$  are random. To see the commonality in liquidity, we note that  $|\Lambda^j|$  is increasing in  $\phi_1$  for each  $j = k, l$ . To see this, consider first the case  $Z_1^j > 0$ . When the financiers are uninformed,  $\alpha = 0$ , and  $\theta^j = 0$ , then,  $m_1^{j+} = \bar{\sigma}^k$ , and, therefore, Equation (B9) shows directly that  $|\Lambda_1^j|$  increases in  $\phi_1$  (since the minimum of increasing functions is increasing). When financiers are informed and  $\theta^j = 0$  then  $m_1^{j+} = \bar{\sigma}^k + \Lambda_1^j$ , and, therefore, Equation (B9) can be solved to be  $|\Lambda_1^j| = \min \left\{ \frac{\phi_1 - 1}{\phi_1} \bar{\sigma}^j, \frac{\gamma(\sigma_2^j)^2}{2} Z_1^j \right\}$ , which increases in  $\phi_1$ . Similarly, Equation (B10) shows that  $|\Lambda^j|$  is increasing in  $\phi_1$  when  $Z_1^j < 0$ .

Now, since  $|\Lambda^j|$  is increasing in  $\phi_1$  and does not depend on other state variables under these conditions,  $\text{Cov}(|\Lambda^k(\phi)|, |\Lambda^l(\phi)|) > 0$  because any two functions that are both increasing in the same random variable are positively correlated (the proof of this is similar to that of Lemma 1 below). Since  $|\Lambda^j|$  is bounded, we can use dominated convergence to establish the existence of  $c > 0$  such that part (i) of the proposition applies for any  $\theta^j, \alpha < c$ .

To see part (ii) of the proposition, note that, for all  $j$ ,  $|\Lambda^j|$  is a continuous function of  $\phi_1$ , which is locally insensitive to  $\phi_1$  if and only if the speculator is not marginal on security  $j$  (i.e., if the second term in Equation (B9) or (B10) attains the minimum). Hence,  $|\Lambda^j|$  jumps if and only if  $\phi_1$  jumps.

To see part (iii), we write illiquidity using Equations (B9)–(B10) as

$$|\Lambda_1^j| = \min \left\{ (\phi_1 - 1)m_1^{j, \text{sign}(Z_1^j)}, \frac{\gamma(\sigma_2^j)^2}{2}|Z_1^j| \right\}. \quad (\text{B12})$$

Hence, using the expression for the margin, if the financier is uninformed and  $\theta^j = a = 0$ , then

$$|\Lambda_1^j| = \min \left\{ (\phi_1 - 1)\bar{\sigma}_1^j, \frac{\gamma(\sigma_2^j)^2}{2}|Z_1^j| \right\} \quad (\text{B13})$$

and, if the financiers are informed and  $\theta^j = 0$ , then

$$|\Lambda_1^j| = \min \left\{ \frac{\phi_1 - 1}{\phi_1} \bar{\sigma}_1^j, \frac{\gamma(\sigma_2^j)^2}{2}|Z_1^j| \right\}. \quad (\text{B14})$$

In the case of uninformed financiers as in Equation (B13), we see that, if  $x_1^k \neq 0$ ,

$$|\Lambda_1^k| = (\phi_1 - 1)\bar{\sigma}_1^k > (\phi_1 - 1)\bar{\sigma}_1^l \geq |\Lambda_1^l| \quad (\text{B15})$$

and, if  $|Z_1^k| \geq |Z_1^l|$ ,

$$|\Lambda_1^k| = \min \left\{ (\phi_1 - 1)\bar{\sigma}_1^k, \frac{\gamma(\sigma_2^k)^2}{2}|Z_1^k| \right\} > \min \left\{ (\phi_1 - 1)\bar{\sigma}_1^l, \frac{\gamma(\sigma_2^l)^2}{2}|Z_1^l| \right\} = |\Lambda_1^l|. \quad (\text{B16})$$

Since  $\Lambda^k$  and  $\Lambda^l$  converge to these values as  $\theta^j, a$  approach zero, we can choose  $c$  so that inequality holds for  $\theta^j, a$  below  $c$ . With informed financiers, it is seen that  $|\Lambda_1^k| \geq |\Lambda_1^l|$  using similar arguments.

For part (iv) of the proposition, we use that

$$\frac{\partial |\Lambda_1^j|}{\partial(-\eta_1)} = \frac{\partial |\Lambda_1^j|}{\partial \phi_1} \frac{\partial \phi_1}{\partial(-\eta_1)}. \quad (\text{B17})$$

Further,  $\frac{\partial \phi_1}{\partial(-\eta_1)} \geq 0$  and, from Equations (B13)–(B14), we see that  $\frac{\partial |\Lambda_1^k|}{\partial \phi_1} \geq \frac{\partial |\Lambda_1^l|}{\partial \phi_1}$ . The result that  $\text{Cov}(\Lambda^k, \phi) \geq \text{Cov}(\Lambda^l, \phi)$  now follows from Lemma 1 below.

**Lemma 1.** *Let  $X$  be a random variable and  $g_i, i = 1, 2$ , be weakly increasing functions of  $X$ , where  $g_1$  has a larger derivative than  $g_2$ , that is,  $g_1'(x) \geq g_2'(x)$  for all  $x$  and  $g_1'(x) > g_2'(x)$  on a set with nonzero measure. Then*

$$\text{Cov}[X, g_1(X)] > \text{Cov}[X, g_2(X)]. \quad (\text{B18})$$

*Proof.* For  $i = 1, 2$  we have

$$\text{Cov}[X, g_i(X)] = E[(X - E[X])g_i(X)] \quad (\text{B19})$$

$$= E \left[ (X - E[X]) \left( \int_{E[X]}^X g_i'(y) dy \right) \right]. \quad (\text{B20})$$

The latter expression is a product of two terms that always have the same sign. Hence, this is higher if  $g_i'$  is larger. ■

**Liquidity Risk (Time 0).** Section 5 focuses on the case of speculators who are unconstrained at  $t = 0$ . When a speculator's problem is linear and he is constrained at time 0, then he invests only

in securities with the highest expected profit per capital use, where profit is calculated using the pricing kernel  $\phi_1^i / E_0[\phi_1^i]$ . In this case, his time-0 shadow cost of capital is

$$\phi_0^i = E_0[\phi_1^i] \left\{ 1 + \max_j \left( \frac{E_0[\frac{\phi_1^i}{E_0[\phi_1^i]} p_1^j] - p_0^j}{m_0^+}, -\frac{E_0[\frac{\phi_1^i}{E_0[\phi_1^i]} p_1^j] - p_0^j}{m_0^-} \right) \right\}. \quad (B21)$$

## References

- Abreu, D., and M. K. Brunnermeier. 2002. Synchronization Risk and Delayed Arbitrage. *Journal of Financial Economics* 66:341–60.
- Acharya, V. V., and L. H. Pedersen. 2005. Asset Pricing with Liquidity Risk. *Journal of Financial Economics* 77:375–410.
- Acharya, V. V., S. Schaefer, and Y. Zhang. 2008. Liquidity Risk and Correlation Risk: A Clinical Study of the General Motors and Ford Downgrade of 2005. Working Paper, London Business School.
- Adrian, T., and H. S. Shin. 2009. Liquidity and Leverage. *Journal of Financial Intermediation*.
- Aiyagari, S. R., and M. Gertler. 1999. “Overreaction” of Asset Prices in General Equilibrium. *Review of Economic Dynamics* 2:3–35.
- Allen, F., and D. Gale. 1998. Optimal Financial Crisis. *Journal of Finance* 53:1245–84.
- Allen, F., and D. Gale. 2004. Financial Intermediaries and Markets. *Econometrica* 72:1023–61.
- Allen, F., and D. Gale. 2005. Financial Fragility, Liquidity, and Asset Prices. *Journal of the European Economic Association* 2:1015–48.
- Allen, F., and D. Gale. 2007. *Understanding Financial Crises*. Clarendon Lectures in Economics. Oxford, UK: Oxford University Press.
- Amihud, Y., and H. Mendelson. 1989. The Effects of Beta, Bid-Ask Spread, Residual Risk, and Size on Stock Returns. *Journal of Finance* 44:479–86.
- Attari, M., A. S. Mello, and M. E. Ruckes. 2005. Arbitraging Arbitrageurs. *Journal of Finance* 60:2471–511.
- Benston, G. J., and R. L. Hagerman. 1974. Determinants of the Bid-Ask Spread in the Over-the-Counter Market. *Journal of Financial Economics* 1:353–64.
- Bernanke, B., and M. Gertler. 1989. Agency Costs, Net Worth, and Business Fluctuations. *American Economic Review* 79:14–31.
- Bernardo, A. E., and I. Welch. 2004. Liquidity and Financial Markets Run. *Quarterly Journal of Economics* 119:135–58.
- Brunnermeier, M. K. 2009. Deciphering the Liquidity and Credit Crunch 2007–08. *Journal of Economic Perspectives*. 2009, Issue 1.
- Brunnermeier, M. K., S. Nagel, and L. H. Pedersen. 2009. Carry Trades and Currency Crashes, in Daron Acemoglu, Kenneth Rogoff, and Michael Woodford (eds.), *NBER Macroeconomics Annual 2008*, vol. 23. Cambridge, MA: MIT Press.
- Brunnermeier, M. K., and L. H. Pedersen. 2005. Predatory Trading. *Journal of Finance* 60:1825–63.
- Bryant, J. 1980. A Model of Reserves, Bank Runs, and Deposit Insurance. *Journal of Banking and Finance* 4:335–44.
- Campbell, J. Y., A. W. Lo, and A. C. MacKinlay. 1997. *The Econometrics of Financial Markets*. Princeton, NJ: Princeton University Press.

- Chordia, T., R. Roll, and A. Subrahmanyam. 2000. Commonality in Liquidity. *Journal of Financial Economics* 56:3–28.
- Chordia, T., A. Sarkar, and A. Subrahmanyam. 2005. An Empirical Analysis of Stock and Bond Market Liquidity. *Review of Financial Studies* 18:85–129.
- Chowdhry, B., and V. Nanda. 1998. Leverage and Market Stability: The Role of Margin Rules and Price Limits. *Journal of Business* 71:179–210.
- Comerton-Forde, C., T. Hendershott, C. M. Jones, P. C. Moulton, and M. S. Seasholes. 2008. Time Variation in Liquidity: The Role of Market Maker Inventories and Revenues. Working Paper, Columbia University.
- Coughenour, J. F., and M. M. Saad. 2004. Common Market Makers and Commonality in Liquidity. *Journal of Financial Economics* 73:37–69.
- DeLong, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann. 1990. Noise Trader Risk in Financial Markets. *Journal of Political Economy* 98:703–38.
- Diamond, D., and P. Dybvig. 1983. Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy* 91:401–19.
- Duffie, D., N. Gârleanu, and L. H. Pedersen. 2002. Securities Lending, Shorting, and Pricing. *Journal of Financial Economics* 66:307–39.
- Eisfeldt, A. 2004. Endogenous Liquidity in Asset Markets. *Journal of Finance* 59:1–30.
- Fisher, I. 1933. The Debt-Deflation Theory of Great Depression. *Econometrica* 1:337–57.
- Fostel, A., and J. Geanakoplos. 2008. Leverage Cycles and The Anxious Economy. *American Economic Review* 98:1211–44.
- Gârleanu, N., and L. H. Pedersen. 2007. Liquidity and Risk Management. *The American Economic Review (Papers & Proceedings)* 97:193–97.
- Gârleanu, N., L. H. Pedersen, and A. Potesman. 2008. Demand-Based Option Pricing. *Review of Financial Studies*.
- Geanakoplos, J. 1997. Promises, Promises in W.B. Arthur, S. Durlauf and D. Lane (eds.), *The Economy as an Evolving Complex System II* 1997: 285–320. Addison-Wesley, Reading MA.
- Geanakoplos, J. 2003. Liquidity, Default and Crashes: Endogenous Contracts in General Equilibrium, in Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky (eds.), *Advances in Economics and Econometrics: Theory and Applications II, Eighth World Congress*, vol. 2, pp. 170–205. Econometric Society Monographs. Cambridge, UK: Cambridge University Press.
- Gennotte, G., and H. Leland. 1990. Market Liquidity, Hedging, and Crashes. *American Economic Review* 80:999–1021.
- Glosten, L. R., and P. R. Milgrom. 1985. Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders. *Journal of Financial Economics* 14:71–100.
- Gromb, D., and D. Vayanos. 2002. Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs. *Journal of Financial Economics* 66:361–407.
- Grossman, S. J. 1988. An Analysis of the Implications for Stock and Futures Price Volatility of Program Trading and Dynamic Hedging Strategies. *Journal of Business* 61:275–98.
- Grossman, S. J., and M. H. Miller. 1988. Liquidity and Market Structure. *Journal of Finance* 43:617–33.
- Grossman, S. J., and J.-L. Vila. 1992. Optimal Dynamic Trading with Leverage Constraints. *Journal of Financial and Quantitative Analysis* 27:151–68.
- Hameed, A., W. Kang, and S. Viswanathan. 2005. Asymmetric Comovement in Liquidity. Mimeo, Duke University.

- Hasbrouck, J., and D. Seppi. 2001. Common Factors in Prices, Order Flows, and Liquidity. *Journal of Financial Economics* 59:383–411.
- Ho, T. S. Y., and H. R. Stoll. 1981. Optimal Dealer Pricing under Transactions and Return Uncertainty. *Journal of Financial Economics* 9:47–73.
- Ho, T. S. Y., and H. R. Stoll. 1983. The Dynamics of Dealer Markets under Competition. *Journal of Finance* 38:1053–74.
- Holmström, B., and J. Tirole. 1998. Private and Public Supply of Liquidity. *Journal of Political Economy* 106:1–39.
- Holmström, B., and J. Tirole. 2001. LAPM: A Liquidity-Based Asset Pricing Model. *Journal of Finance* 56:1837–67.
- Huberman, G., and D. Halka. 2001. Systematic Liquidity. *Journal of Financial Research* 24:161–78.
- Ibbotson. 1999. *Cost of Capital Quarterly, 1999 Yearbook*, 2nd ed. Chicago: Ibbotson Associates, 1976.
- International Monetary Fund. 2007. *Global Financial Stability Report*. October.
- Kiyotaki, N., and J. Moore. 1997. Credit Cycles. *Journal of Political Economy* 105:211–48.
- Kyle, A. S. 1985. Continuous Auctions and Insider Trading. *Econometrica* 53:1315–35.
- Kyle, A. S., and W. Xiong. 2001. Contagion as a Wealth Effect. *Journal of Finance* 56:1401–40.
- Liu, J., and F. A. Longstaff. 2004. Losing Money on Arbitrages: Optimal Dynamic Portfolio Choice in Markets with Arbitrage Opportunities. *Review of Financial Studies* 17:611–41.
- Lustig H., and Y. Chien. 2005. The Market Price of Aggregate Risk and Wealth Distribution. NBER Working Paper 11132.
- Mitchell, M., L. H. Pedersen, and T. Pulvino. 2007. Slow Moving Capital. *American Economic Review (Papers & Proceedings)* 97:215–20.
- Morris, S., and H. Shin. 2004. Liquidity Black Holes. *Review of Finance* 8:1–18.
- Pastor, L., and R. F. Stambaugh. 2003. Liquidity Risk and Expected Stock Returns. *Journal of Political Economy* 111:642–85.
- Plantin, G., H. Sapra, and H. S. Shin. 2005. Marking-to-Market: Panacea or Pandora's Box? *Journal of Accounting Research* 46:435–60.
- Shleifer, A., and R. W. Vishny. 1992. Liquidation Values and Debt Capacity: A Market Equilibrium Approach. *Journal of Finance* 47:1343–66.
- Shleifer, A., and R. W. Vishny. 1997. The Limits of Arbitrage. *Journal of Finance* 52:35–55.
- Stiglitz, J. E., and A. Weiss. 1981. Credit Rationing in Markets with Imperfect Information. *American Economic Review* 71:393–410.
- Stoll, H. R. 1978. The Supply of Dealer Services in Securities Markets. *Journal of Finance* 33:1133–51.
- Vayanos, D. 2004. Flight to Quality, Flight to Liquidity, and the Pricing of Risk. Working Paper, London School of Economics.
- Weill, P.-O. 2007. Leaning Against the Wind. *Review of Economic Studies* 74:1329–54.
- Wigmore, B. 1998. Revisiting the October 1987 Crash. *Financial Analysts Journal* 54:36–48.
- Xiong, W. 2001. Convergence Trading with Wealth Effects: An Amplification Mechanism in Financial Markets. *Journal of Financial Economics* 62(2): 247–92.