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# Disclosure, Liquidity, and the Cost of Capital

DOUGLAS W. DIAMOND and ROBERT E. VERRECCHIA\*

#### ABSTRACT

This paper shows that revealing public information to reduce information asymmetry can reduce a firm's cost of capital by attracting increased demand from large investors due to increased liquidity of its securities. Large firms will disclose more information since they benefit most. Disclosure also reduces the risk bearing capacity available through market makers. If initial information asymmetry is large, reducing it will increase the current price of the security. However, the maximum current price occurs with some asymmetry of information: further reduction of information asymmetry accentuates the undesirable effects of exit from market making.

This paper studies the causes and consequences of a security's liquidity, especially the effect of future liquidity on the security's current price—equivalently the effect on its required expected rate of return, its cost of capital. Under conditions that we identify, reducing information asymmetry reduces the cost of capital. Under other (less typical) conditions, this reduced information asymmetry can have the opposite effect. We use public disclosure of information as the means of changing information asymmetry, but the points are more general.

Our model is related to those of Kyle (1985), Glosten and Milgrom (1985), and Admati and Pfleiderer (1988). They assume that there is unlimited risk bearing capacity devoted to market making, which implies that changes in future liquidity never influence a security's cost of capital. In contrast, we develop a model of trade in an illiquid market with limited risk bearing capacity of risk-averse market makers and examine the effects of private information on the incentives of market makers to provide risk bearing

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<sup>1</sup>Given the assumption, one could even close the secondary market for an arbitrarily long time in the future without influencing the current price. As a security becomes less liquid in these models, more of the security is permanently held by the market makers who do not care about liquidity.

capacity. We show how limited risk bearing capacity of market makers interacts with the effects of private information in the determination of security prices. Some may find the assumption of risk-averse market makers objectionable, on the grounds that widely-held corporations could enter into market making. However, the rapid quotation of prices requires skill and effort, and agency theory teaches us that even if the ultimate owners were corporations, much risk would optimally be imposed on the individuals making markets. Alternatively, a corporation market maker could set limits on the positions that its agents could take. We do not model this agency problem; however, Diamond and Verrecchia (1982) and Holmström (1982) develop related ideas.

One motivation for studying the risk bearing capacity in market making is that it is the primary factor discussed in recent investigations into financial market volatility and the October 1987 market crash (e.g., Brady Commission (1988), Grossman and Miller (1988), and NYSE (1990)). Another is the large literature focusing on the effects of market maker inventory when there is no private information and market makers are risk averse, for example Ho and Stoll (1981). The only related model we are aware of that incorporates entry decisions by risk-averse market makers and the effects of liquidity is Grossman and Miller (1988), which disallows private information.

We provide a framework to analyze effects of changes in information asymmetry on security pricing, while providing cross-sectional predictions about differences in sensitivity of security prices to information asymmetry. Although the points are more general, we study this by examining the effects of corporate disclosures that reduce information asymmetry. As a result, our analysis also complements that in Diamond (1985), which shows that stockholder welfare can be improved in perfectly competitive and fully liquid markets by disclosing data that reduce information asymmetry among investors. The models of Diamond and Verrecchia (1981) and Verrecchia (1982) are used in that paper.

Disclosure improves future liquidity of a firm's securities (reduces price impact), and this reduces the cost of capital for the firm, with the reduction in cost of capital larger for larger firms.<sup>2</sup> When one allows an endogenous amount of risk bearing capacity in market making, increased disclosure causes some market makers to exit. Despite this exit, disclosure generally reduces a firm's cost of capital. We also identify situations, however, where reduced information asymmetry is not desirable and raises the cost of capital, because reduced information asymmetry leads to rapid exit from market making.

We do not explicitly introduce a firm that sells new shares to the public, but because the increased future liquidity generally increases the current

<sup>&</sup>lt;sup>2</sup>Private information does not lead to increased physical transactions costs but instead gives rise to a transfer of wealth across investors. If, instead, costs were the cause of illiquidity, increasing these costs would necessitate an increased expected return to induce investors to hold the security. On this point, see Amihud and Mendelson (1986).

price, a firm selling shares will then benefit from the improved future liquidity.<sup>3</sup>

We introduce two large institutional traders subject to future liquidity shocks that may force them to trade a particular random amount in the future. Alternatively, they may receive private information in the future. We argue that the traders who are directly concerned about liquidity and its relation to information asymmetry are large traders. Private information causes illiquidity that is roughly proportional to the amount of information revealed by a trade. Because there is little information content in a small trade, the price impact of a small trade (the bid-ask spread) must be primarily related to other factors. Similarly, the limited risk bearing capacity of market makers is only relevant for trades that are large relative to that capacity.

An investor is large if he or she would, under some circumstances, take a very large position in a security. A highly risk-averse investor would not take a large position. We make large traders large by limiting their risk aversion (motivated by the idea that they invest on behalf of many individuals, as for an insurance company that might have a liquidity need to meet insurance claims). We take this to the limit by assuming that institutional investors are risk neutral. Our results hold if they are risk averse but not so risk averse that their desired trades have negligible impact on the market price.

Another group of traders receives no relevant information or liquidity shocks and observes all public information and the order imbalance for trading the security. This group includes the explicit market makers in the security and other traders who compete with the market makers such as NYSE floor traders, and upstairs block positioners. We subsequently add a group of small, price-taking investors who do not observe the order flow in the security.

The larger the total size of the firm the more the security price depends on how broad a market it attracts. For large firms it is important to attract large holdings from institutional investors who make large trades and are thus concerned about future liquidity. If large positions from these investors are not attracted, then the current price will have to fall substantially to attract large positions from the smaller group of investors who are not as concerned about liquidity. For small firms, attracting only those less concerned with liquidity will have a minimal effect on the current security price because the size of the position per investor is not large even when divided over a small number of investors. The effect of breadth of market attracted is larger for large firms, and as a result our model identifies these firms as receiving the largest benefit from reduced information asymmetry. If there is

<sup>3</sup>An informed firm's choice of date to sell shares may reveal bad news about itself, for well-known reasons. We analyze the effect of a change in *future* information asymmetry on the price the firm would be able to obtain *today*. If one examined a date on which a firm offers its shares, one would interpret the prior information available to traders as conditional on the firm choosing to offer shares.

a cost to increased disclosure, for example one that is proportional to firm size (or less than proportional), then larger firms will disclose more information.

Of additional significance is the amount of information asymmetry that would occur without disclosure. When informed investors have precise information they place large orders, which requires that market makers take large positions. Market makers adjust the price at which they take the other side of these positions to reflect their information content but in addition charge a risk premium for taking them. This premium is paid by all large investors, even those without private information. Given a fixed number of market makers, large traders would be better off with reduced information asymmetry and facing a more liquid market as a result. However, reduced information asymmetry reduces the volatility of order imbalances, causing some market makers to exit. If there is little information asymmetry, the welfare of large traders and their willingness to take large positions depend primarily on the number of market makers. Large traders are better off, and security prices are higher, with a small positive amount of information asymmetry than none at all. However, with large information asymmetry, the effect of disclosure in reducing order imbalances dominates the effect of exit. In this case, disclosure makes large traders better off and raises security prices.

Our results on the beneficial effects of disclosure that reduce information asymmetry are derived after removing an unrealistic benefit of disclosing information to the public: the reduction of future risk by early disclosure. In Section II, we limit the effect of disclosure to influencing the distribution of order imbalances caused by informed traders. Simply changing the timing of future resolution of uncertainty should have no effect on the pricing of securities (see Ross (1989)).

The paper is organized as follows. Section I describes the large traders in the model and describes the effects of future liquidity of a security on traders' current desired holdings, taking the liquidity of the prices quoted by market makers as a given. Section II characterizes the competition between a given number of market makers and the amount of liquidity that they provide. A full characterization of the model is provided in the Appendix. To make the discussion more readable, our text focuses on what we call a *long run equilibrium*, and also uses a limiting argument that focuses attention on disclosures that have almost no effect on the future risk of owning the asset. Section III examines the welfare effects of disclosure. All results are extended in Section IV to the case of an endogenous number of market makers. Section V briefly discusses the effect of adding a fourth group of traders: a large group of small competitive price takers who are not market makers. Section VI concludes the paper.

# I. A Model of Liquidity

# A. The Role of Large Information and Large Liquidity Traders

Actions by large traders who anticipate the possibility of future liquidity needs are the essential elements of our model. We consider two (ex ante

identical) large *institutional* traders at three dates. At date 3 the per share value of the firm is revealed to be  $\bar{v}$ . Let  $Q_0$  represent the total number of shares outstanding of the firm, assumed to be positive: thus,  $\bar{v}Q_0$  is the total value of the firm. At date 2 one of the traders needs to obtain liquidity, and the other receives some private information. At date 1 each trader has an ex ante probability of one-half of being the trader who needs liquidity (i.e., becomes a liquidity trader) versus an ex ante probability of one-half of being the trader who receives some private information (i.e., becomes informed).

For convenience we adopt the following conventions in describing uncertainty. All unknown, random elements have a normal distribution, are mutually independent, and are designated with a tilde over the symbol (e.g., the per share value of the firm is uncertain until date 3 and therefore is represented by  $\tilde{v}$  at dates 1 and 2). The realization of a random variable is designated with a bar over the symbol (e.g., the per share value of the firm is  $\bar{v}$  at date 3). Finally, the variance of the random variable is represented by the symbol alone (e.g., the variance of  $\tilde{v}$  is v).

The investor who becomes a liquidity trader at date 2 finds out that he or she must trade to a final holding of  $\bar{\eta}$  shares. The distribution of this random quantity  $\tilde{\eta}$  is normal with mean 0 and variance  $\eta$ . The distribution of  $\tilde{\eta}$ , and the fact that this amount of liquidity will be needed at date 2 with probability one-half is known by both traders at date 1. This in turn influences the trade he or she makes at this date. The required final liquidity trader's holding of  $\bar{\eta}$  is a liquidity need because we assume that the mean of the shock is zero, and this implies, on average, that the entire position must be sold. The holdings desired at date 1 for any price are influenced by their effect on the required trade that must be made if a liquidity shock is received at date 2. For example, if the holding going into date 2 is large, then a large sell order must be put in, and this would push down the price, in part because it could be mistaken for an information-based trade. If a large trader does not receive a liquidity shock at date 2, he or she becomes an information trader who receives private information about the per share value of the stock and trades optimally based on it at that time.

#### B. The Disclosure of Information

The per share value of the firm at date 3 is  $\tilde{v}=\bar{v}$ , where  $\tilde{v}$  is determined as the sum of three mutually independent, random variables,  $\tilde{u}$ ,  $\tilde{\delta}$ , and  $\tilde{\Delta}$ , that is,  $\tilde{v}=\tilde{u}+\tilde{\delta}+\tilde{\Delta}$ . Each of  $\tilde{u}$ ,  $\tilde{\delta}$ , and  $\tilde{\Delta}$  has a normal distribution with means m, 0, and 0, respectively, and variances u,  $\delta$ , and  $\Delta$ , respectively. The realization of  $\bar{v}$  becomes known to all market participants at date 3. The informed trader, and possibly the firm, learns  $\tilde{\delta}$  at date 2. The firm can disclose any or all of its information about  $\tilde{\delta}$  before trade occurs at date 2. Let the disclosure by the firm about the informed trader's private information be  $\tilde{x}=\tilde{\delta}+\tilde{\varepsilon}$ , where  $\tilde{\varepsilon}$  has a normal distribution with mean 0 and variance  $\varepsilon$ . At date 2 the firm can also disclose information about  $\tilde{\Delta}$ ;  $\tilde{\Delta}$  is information possessed by the firm that is unknown to anyone else before date 3 unless it is disclosed. The disclosure about  $\tilde{\Delta}$  is  $\tilde{y}=\tilde{\Delta}+\tilde{\xi}$ , where  $\tilde{\xi}$  has a normal

distribution with mean 0 and variance  $\xi$ . The realization of  $\tilde{u}$  is unknown to all parties at date 2.

The disclosure can be interpreted as a choice of an accounting technique or a committed policy of making earnings or other forecasts. The announcement must be somewhat verifiable to make it contain information.

# C. Trades on the Basis of a Given Price Function

To motivate our subsequent analysis, this section analyzes the trade selected at date 2 by the two (large) traders after they have learned whether they need liquidity or have received private information. Given the final holdings implied by these trades, we compute the effect of a quantity of the security held going into date 2 on the expected date 3 consumption by taking the expected value of final consumption before a trader finds out if he or she will need liquidity or will be able to trade on private information. When we later analyze trade at date 1, this *indirect utility function* will give the value of security holdings at the end of date 1 and show how this depends on the liquidity of the market. To facilitate the discussion, we assume that the traders take date-2 security prices as an exogenous linear function of total quantity traded. The determination of an endogenous price function is studied in Section II.

Our discussion here of an informed trader facing a linear price function parallels the discussion in Kyle (1985). We label the two (large) traders a and b. Let  $D_{at}$  and  $D_{bt}$  represent holdings by a and b of the security at date t, respectively, and let  $d_{at} = D_{at} - D_{at-1}$  and  $d_{bt} = D_{bt} - D_{bt-1}$  denote the trades executed by a and b for the security at date t, respectively. If trader a becomes the liquidity trader at date 2, he or she will need to trade to a final holding of  $\bar{\eta}$ , so the trade at that date is for  $d_{a2} = \bar{\eta} - D_{a1}$ . If trader b becomes the information trader, he or she chooses a trade of  $d_{b2}$ , such that the expected marginal cost of the trade is equal to the expected value of the security based on what he or she knows at that time. The total order imbalance, or total net demand, at date 2 is then given by  $d_{T2} = d_{a2} + d_{b2}$ .

Suppose that the price function of the market makers at date 2 is of the linear form  $P_2 = \Phi + \lambda d_{T2}$ . If trader a becomes the liquidity trader at date 2, he or she will need to trade to a final holding of  $\bar{\eta}$ , so the trade executed at that date is  $d_{a2} = \bar{\eta} - D_{a1}$ . Suppose, alternatively, that trader a becomes an informed trader at date 2. The date-3 wealth of an informed trader is

$$\overline{W}_{a3} = (d_{a2} + D_{a1})\overline{v} - d_{a2}\overline{P}_{2}.$$

Taking the expectation over  $\tilde{W}_{a3}$  at date 2 yields

$$E\big[\,\tilde{W}_{a3}\big] \,=\, \big(\,d_{a2}\,+\,D_{a1}\big)\,E\big[\,\tilde{v}\,|\,\bar{b},\,\bar{y}\,\big] \,-\, d_{a2}\big(\Phi\,+\,\lambda \big\{\,-\,D_{b1}\,+\,d_{a2}\big\}\,\big),$$

where  $E[\tilde{v} \mid \bar{b}, \bar{y}]$  is the expectation of the value of the security at date 3 (i.e.,  $\tilde{v}$ ) conditional on what an informed trader knows at date 2 (i.e.,  $\bar{b}$  and  $\bar{y}$ ), and  $E[\tilde{P}_2] = \Phi + \lambda(-D_{b1} + d_{a2})$  since the informed trader does not observe total

net demand at date 2 when choosing his or her own trade.<sup>4</sup> Consequently, the trade executed by trader a to maximize expected profit at date 2, conditional on him being the informed trader, is

$$d_{a2} = (1/2 \lambda) (E[\tilde{v} | \bar{\delta}, \bar{y}] - \Phi) + \frac{1}{2}D_{b1}.$$

The informed trader equates the expected marginal cost of the security to its expected value, given his or her information. Note that the trade executed by the risk-neutral informed trader is linear in  $E[\tilde{v} \mid \bar{\delta}, \bar{y}]$ , limited only by  $\lambda$ , the trade's price impact, and does not depend on  $D_{a1}$ , the trader's initial holding.

If the large trader who becomes informed were assumed instead to be risk averse, the risk aversion would limit the size of the trade made on the basis of information regardless of the price impact. A very risk-averse trader would not choose a large final holding of a risky asset and would be further unwilling to make a large market order trade at a highly variable price at date 2. If risk aversion grows without bound, the trade is not limited by its price impact because for a very small trade there is almost no price impact. To model large institutional investors who choose large trades, we assume that large traders are risk neutral.

Consider, now, trader a's expected utility conditional on being a liquidity trader. The date 3 wealth of a liquidity trader is  $\overline{W}_{a3} = \overline{\eta}\overline{v} - (\overline{\eta} - D_{a1})\overline{P}_2$ . Given the anticipated action of the informed trader,

$$\overline{W}_{a3} = \overline{\eta}\overline{v} - (\overline{\eta} - D_{a1})(\Phi + \lambda \left[ \left\{ \left( E \left[ \overline{v} \mid \overline{\delta}, \overline{y} \right] - \Phi \right) / 2\lambda \right\} + \frac{1}{2}D_{a1} + \left\{ \overline{\eta} - D_{a1} \right\} \right] \right),$$

provided that  $\lambda \neq 0$ . Taking an expectation over  $\tilde{W}_{a3}$  at date 1 yields

$$E[\tilde{W}_{a3}] = \frac{1}{2}D_{a1}(\Phi + E[\tilde{v}]) - \lambda(\eta + \frac{1}{2}D_{a1}^2).$$

Finally, consider trader a's unconditional value of holding  $D_{a1}$ , going into date 2, before it is known whether he or she is to become informed or suffer a liquidity shock. The expected utility of trader a with holding  $D_{a1}$ , denoted by  $\Pi$ , is

$$\begin{split} \Pi &= q E \big[ \big( d_{a2} + D_{a1} \big) E \big[ \tilde{v} \, | \, \bar{\delta}, \, \bar{y} \big] - d_{a2} \big( \Phi + \lambda \{ -D_{b1} + d_{a2} \} \big) \big] \\ &+ \big( 1 - q \big) \big( \frac{1}{2} D_{a1} \big\{ \Phi + E \big[ \, \tilde{v} \big] \big\} - \lambda \big\{ \eta + \frac{1}{2} D_{a1}^2 \big\} \big). \end{split}$$

<sup>4</sup>The informed trader also knows  $\bar{x}$  at date 2, but  $\bar{x}$  is redundant in *his* valuation of  $\tilde{v}$ , since the informed trader observes  $\bar{\delta}$  directly. For example,  $E[\tilde{v} | \bar{\delta}, \bar{x}, \bar{y}] = m + \bar{\delta} + (\Delta/\Delta + \xi)\bar{y}$ , which is independent of  $\bar{x}$ .

 $^5$ To see the effect of risk aversion, suppose that a trader had a (negative) exponential utility function for wealth at date 3 given by  $-\exp(-\tau \tilde{W}_{a3})$ , where  $\tau$  is trader a's level of absolute risk aversion. Then, the trade executed when trader a is informed at date 2 is given by

$$d_{a2} = \frac{E\left[\left.\tilde{v}\,\right|\,\bar{\delta},\,\bar{y}\right] - \Phi + \lambda D_{b1} - \tau \operatorname{Var}\left[\left.\tilde{v}\,\right|\,\bar{\delta},\,\bar{y}\right] D_{a1}}{2\,\lambda + \tau \operatorname{Var}\left[\left.\tilde{v}\,\right|\,\bar{\delta},\,\bar{y}\right] + \tau \lambda^2 \eta}\,.$$

If  $\tau$  becomes large, the trade becomes independent of  $E[\bar{v} | \bar{\delta}, \bar{y}]$ , and the liquidity of the market does not influence the extent of trade based on information (the parameter  $\lambda$  then enters only though its effect on the volatility of the date 2 price).

Note, however, that  $E[E[\tilde{v} | \bar{\delta}, \bar{y}]] = E[\tilde{v}]$ . The marginal utility from a small increase in date 1 holding  $D_{a1}$  (i.e., the derivative with respect to  $D_{a1}$ ) is

$$\partial \Pi / \partial D_{a1} = qE[\tilde{v}] + (1 - q)(\frac{1}{2}\{\Phi + E[\tilde{v}]\} - \lambda D_{a1}),$$

noting that  $\partial d_{a2}/\partial D_{a1}=0$ , where q is the probability of becoming an informed trader versus a liquidity trader, which in our model is one-half (i.e.,  $q=\frac{1}{2}$ ). Therefore, the marginal value of holding  $D_{a1}$  at date 1 is

$$\partial \Pi / \partial D_{a1} = \tfrac{3}{4} E \big[ \, \tilde{v} \big] \, + \tfrac{1}{4} \Phi \, - \tfrac{1}{2} \lambda D_{a1}.$$

This shows how imperfect liquidity of the market in the future (i.e., a high value of  $\lambda$  at date 2) reduces the value of a current position in the security for a trader subject to liquidity shocks.

This argument ignores the effect of traders' decisions on  $\Phi$  and  $\lambda$ , as well as the possibility of trade at date 1.7 To understand their role, we analyze endogenous current and future liquidity in the next section.

#### **II. Price Formation**

## A. Market Maker Competition

In the next sections, we derive expressions for the price of the security at dates 1 and 2 through a series of lemmas. The formal mechanism through which price formation is implemented is Bertrand-style competition.8 Competing market makers quote prices (that depend on aggregate trade quantity) to investors who submit market orders. Each investor takes account of his or her trade's effect on prices paid when deciding how much to trade. Market makers are not subject to liquidity shocks and do not acquire private information about fundamentals of the security. There are N > 2 risk-averse market makers who each have constant absolute risk aversion with coefficient  $\rho_{MM}$ . If the market makers end up sharing the risk of market making by each taking an equal part of large orders, the aggregate risk aversion of the set of market makers is  $(\rho_{MM}/N) \equiv \rho$ . Initially, we take N, the number of market makers, and as a result  $\rho$ , the aggregate risk aversion, as exogenous. We later make the number of market makers endogenous and abstract from the effect of a change in number of market makers on the amount of competition between market makers.

<sup>7</sup>The trader will choose a trade at date 1 that will equate the expected marginal cost of

acquiring  $D_{a1}$  to  $\partial \Pi/\partial D_{a1}$ .

There are many equilibria to the game of competition among risk-averse market makers, but this is not our focus. Other ways of modeling competition between market makers who arrive at a price include: giving all the bargaining power to the sellers such that each market maker receives his or her certainty equivalent; or modeling Cournot-style competition of a Nash equilibrium in demand curves as in Kyle (1989). The model's qualitative results do not depend on which of the mechanisms described here are used.

 $<sup>^6</sup>$ We use the expression q throughout the discussion to point out the effect of there being some ex ante probability at date 1 of a large trader being either a liquidity trader or an informed trader. However, our analysis is confined to the case in which q is one-half and should only be interpreted more generally with caution.

To abstract from changes in the amount of competition between market makers, we assume that each market maker's risk aversion is high, implying that large risk bearing capacity always leads to a very large number of competing market makers. For concreteness, a brute force assumption that determines the same equilibrium even with a finite number of market makers is as follows. Assume that traders' market orders are handled by a nonstrategic broker who chooses a set of market makers with whom to trade. The broker's rule is to ask each of the N market makers for a quote for a fraction 1/N of the aggregate order imbalance. The broker divides the order imbalance equally across all N market makers if they quote the same price. If  $M \leq N$  market makers are tied for the best price (highest if ask, lowest if bid) for 1/Nth of the aggregate order imbalance, then the broker trades with the M market makers; a fraction M/N of each trade is filled, and this is a binding commitment. Given this commitment, all market makers quote the same price, determined as follows. The decision as to which market makers to use is based on their quote for 1/Nth of the order imbalance. A symmetric Nash equilibrium is for each to quote a price such that at that price for a given quantity, the market maker prefers that quantity to all other given the price. A deviation to a lower price for a positive net demand (more buy than sell orders) would have the market maker supply a larger quantity than desired at that price because his or her desired quantity supplied is increasing in price. Deviation to a higher price would attract zero quantity. With a negative net demand (more sell than buy orders), deviation to a higher price would attract a larger quantity, but the desired quantity at that price is lower, while a lower price attracts zero quantity.

Market makers compete for the marginal unit of each order imbalance, and earn surplus on any inframarginal units. Therefore, they are better off with large, volatile order imbalances.

### B. Endogenous Market Liquidity at Date 2

Our first lemma concerns the price of the security at date 2. Let  $P_{i2}$  represent the price for the security quoted by the market maker i at date 2 and  $P_{i1}$  the price at which he or she transacts at date 1. Note that  $\tilde{d}_{T2}$  represents the aggregate order imbalance observed by all market makers at date 2. Market maker i's wealth at date 3, on the basis of receiving an amount  $d_{Ti2}$  of the total aggregate order imbalance at date 2 and an amount  $d_{Ti1}$  of the total aggregate order imbalance at date 1, is  $\tilde{W}_i$ , where  $\tilde{W}_i$  is defined by

$$\tilde{W}_{i} = (Q_{i1} - d_{Ti2})\tilde{v} + P_{i2}d_{Ti2} + P_{i1}d_{Ti1},$$

and  $Q_{i1}$  is market maker *i*'s holdings of the security at date 1.9 Therefore, market maker *i*'s expected utility at date 2, conditional on observing the

<sup>9</sup>This means that  $Q_{i1}$  is the difference between market maker *i*'s holdings of the security at date 0 (i.e., his endowment) minus that part of the total order imbalance he assumes at date 1 (i.e.,  $d_{Ti1}$ ).

aggregate order imbalance and the information the firm disseminates about itself,  $\tilde{x} = \bar{x}$  and  $\tilde{y} = \bar{y}$  is

$$EU_{i2} = E \left[ -\exp(-N\rho \tilde{W}_i) \mid \overline{d}_{T2}, \, \overline{x}, \, \overline{y} \right].$$

Note that  $\tilde{d}_{T2}$  is uncertain until it is observed because it depends on  $\tilde{\eta}$  and the realizations of  $\tilde{\delta}$ ,  $\tilde{x}$ , and  $\tilde{y}$  since these are observed directly by the informed trader. Furthermore, even though market makers observe  $\tilde{d}_{T2}$ , they cannot distinguish a liquidity shock from an informed trade.

With the competition between market makers described in Section II.A, the holdings of each market maker are those desired by each market maker at the price that is quoted. This implies that given his or her inventory holdings from date 1 and the observed order imbalance at date 2, the market maker quotes a price such that the final holdings are the optimum given that price as a parameter, and each order is divided equally among the market makers. Market maker i's quote for 1/Nth of the order imbalance at date 2 is: $^{10}$ 

$$P_{i2} = E[\bar{v} \mid \bar{x}, \bar{y}, \bar{d}_{T2}] + N\rho \operatorname{Var}[\bar{v} \mid \bar{x}, \bar{y}, \bar{d}_{T2}](d_{Ti2} - Q_{i1}).$$

Symmetry across market makers implies that in equilibrium all market makers have  $Q_{i1} \equiv (Q_1/N)$  and  $d_{Ti2} \equiv (d_{T2}/N)$ . Thus,  $P_{i2} \equiv P_2$ , where

$$P_2 = E\big[\,\tilde{\boldsymbol{v}}\,|\,\,\bar{\boldsymbol{x}},\,\bar{\boldsymbol{y}},\,\bar{\boldsymbol{d}}_{T2}\,\big] \,+\,\rho\,\mathrm{Var}\big[\,\tilde{\boldsymbol{v}}\,|\,\,\bar{\boldsymbol{x}},\,\bar{\boldsymbol{y}},\,\bar{\boldsymbol{d}}_{T2}\,\big]\big(\,\bar{\boldsymbol{d}}_{T2}\,-\,Q_1\big).$$

The price at date 2 depends on the order imbalance  $\overline{d}_{T2}$  for two reasons. The first is the information content of the orders (the effect on  $E[\tilde{v} \mid \bar{x}, \bar{y}, \bar{d}_{T2}]$ ), while the second is the change in the risk premium needed to hold the order imbalance ( $\rho$  times the variance of  $\tilde{v}$ ). Let  $\lambda = \partial P_2 / \partial \bar{d}_{T2}$  denote the slope of the date-2 price function; it is the sum of the two components  $\partial E[\tilde{v} \mid \bar{x}, \bar{y}, \bar{d}_{T2}] / \partial \bar{d}_{T2} + \rho \theta$ , where  $\theta \equiv \mathrm{Var}[\tilde{v} \mid \bar{x}, \bar{y}, \bar{d}_{T2}]$ . This discussion implies our first result.

Lemma 1: The price of the security at date 2 is given by

$$P_2 = m + (\delta/\delta + \varepsilon)\bar{x} + (\Delta/\Delta + \xi)\bar{y} + \lambda \bar{d}_{T2} - (\lambda - \rho\theta)E[\tilde{d}_{T2}] - \rho\theta Q_1,$$

where

$$\theta = u + \frac{4\lambda^2 \eta \delta \varepsilon}{\delta \varepsilon + 4\lambda^2 \eta (\delta + \varepsilon)} + \frac{\Delta \xi}{\Delta + \xi},$$

and \(\lambda\) solves

$$\lambda = \frac{2\lambda\delta\varepsilon}{\delta\varepsilon + 4\lambda^2\eta(\delta + \varepsilon)} + \rho\theta.$$

<sup>10</sup>The expression for the price quoted by market maker i at date 2,  $P_{i2}$ , is determined by requiring that market maker i's marginal expected utility for an additional unit of  $d_{Ti2}$  be zero, that is,  $\partial E U_{i2} / \partial d_{Ti2} = 0$ .

Proof: See the Appendix.

If there were unlimited risk bearing capacity of market makers (and  $\rho$  were zero), this would reduce to the Kyle (1985) model. Limited risk bearing capacity implies more price impact of a trade ( $\lambda$  is larger) because of the temporary price change needed to induce market makers to take the other side of the transaction.

#### C. Price and Demands at Date 1

In this section we broadly characterize the price of the security at date 1 and the demand for the security by traders at that time, leaving details for the Appendix. First, consider the price market maker i quotes for a share  $d_{Ti1}$  of the total order imbalance at date 1, where  $d_{T1}$  represents the aggregate order imbalance, or total net demand, at date 1. This requires computing the market maker i's expected utility as of date 1 and then determining the price for the security he or she quotes at date 1. Recall that  $P_{i1}$  is the price quoted for the security by market maker i at date 1. As argued above, in a symmetric equilibrium all market makers starting from the same initial conditions quote the same price and have the same beliefs. This implies that given the observed order imbalance at date 1, the market maker quotes a price such that his or her final holdings are optimal given that price as a parameter, and each order is divided equally among market makers.  $^{11}$ 

A formal analysis of this problem yields the following lemma.

Lemma 2: The price of the security at date 1 is

$$P_1 = m - \gamma_1 (Q_0 - D_{a1} - D_{b1}) + \gamma_2 E[\tilde{d}_{T2}],$$

where  $\gamma_1$  and  $\gamma_2$  are endogenously determined, nonnegative parameters that are described in detail in the Appendix, and the expression  $Q_0 - D_{a1} - D_{b1}$  (which equals  $Q_1$ ) represents that quantity of the risky security held by market makers at date 1.

Proof: See the Appendix.

Lemma 2 implies that the asset price at date 1 increases as either traders' demands for the asset at date 1 increase (i.e.,  $D_{a1} + D_{b1}$ ), or the expected total order imbalance at date 2 increases (i.e.,  $E[\tilde{d}_{T2}]$ ).

Because the price quoted by market makers at date 1 depends on the holdings of the security by traders a and b at that time, we analyze this next. Let  $Q_a$  and  $Q_b$  represent the initial holdings of the security by traders a and b, respectively, (i.e., their endowments): we assume that  $Q_a$  and  $Q_b$ 

<sup>&</sup>lt;sup>11</sup>The expression for the price quoted by market maker i at date 1,  $P_{i1}$ , is determined by requiring that market maker i's marginal expected utility for an additional unit of  $d_{Ti1}$  be zero, that is,  $\partial EU_{i1}/\partial d_{Ti1}=0$ , where  $EU_{i1}$  describes market maker i's expected utility at date 1 (see the proof of Lemma 2 in the Appendix).

are both positive. At date 1, trader a's expected utility is given by

$$\begin{split} EU_{a1} &= qE \big[\, \tilde{v}\, \tilde{\eta} \, + \, \tilde{P}_2 \big(\, D_{a1} \, - \, \tilde{\eta} \big) \\ &\quad + P_1 \big(\, Q_a \, - \, D_{a1} \big) \, | \, \text{trader } \, a \, \text{becoming a liquidity trader} \big] \\ &\quad + \big(1 \, - \, q \big) \, E \big[\, \tilde{v} \, \big(\, \tilde{d}_{a2} \, + \, D_{a1} \big) \\ &\quad - \, \tilde{P}_2 \, \tilde{d}_{a2} \, + \, P_1 \big(\, Q_a \, - \, D_{a1} \big) \, | \, \text{trader } \, a \, \text{becoming an informed trader} \big] \,, \end{split}$$

where  $q = \frac{1}{2}$  is the ex ante probability of trader a becoming a liquidity trader and  $(1 - q) = \frac{1}{2}$  is the probability of trader a becoming an informed trader (and vice versa for trader b).

Note, however, that from Lemma 2 the price at date 1 will depend on the holdings of the traders at date 0 (i.e.,  $Q_a$  and  $Q_b$ ) through  $D_{a1}$  and  $D_{b1}$ . For example, if traders held a much larger quantity of the asset than their desired long-run level, they would sell some, depressing the price in the illiquid market. If they held less than desired, they would do the reverse, pushing up the price. Because the market is illiquid, there would be a period of adjustment before the market achieved a long-run equilibrium.

To focus on the long-run effects of disclosure and liquidity, we suppose that date 0 holdings represent a *long-run equilibrium* where each trader is willing to hold that quantity, given forecasts of current and future prices. That is, each trader will be holding the position such that there is no trade at date 1. This determines a price and allocation that supports the price. This long-run equilibrium also greatly simplifies the exposition of the model.

A long-run equilibrium at date 1 is equivalent to setting  $Q_a = D_{a1}$ , and  $Q_b = D_{b1}$ , in the solution for  $D_{a1}$  and  $D_{b1}$  (see the discussion in the Appendix). This, in turn, implies that  $D_{a1}$  and  $D_{b1}$  are linear in  $Q_0$  alone, and, in addition, identical since traders a and b are symmetric economic agents.

Lemma 3: The long-run equilibrium, date-1 holdings of the security by traders a and b are  $D_{a1} \equiv D_{b1} \equiv D_1$ , where  $D_1 = \Gamma Q_0$ , and the expression for  $\Gamma$  is given in the Appendix.

*Proof*: See the Appendix.

Finally, we apply the results of Lemmas 1 through 3 to disclosures of information that have almost no effect on the future risk of the security. This is done for the following reasons that are discussed in the introduction. For most disclosures, an asset will be about as risky after a disclosure as it was before. The risk bearing potential and the dispersion of liquidity needs of the entire economy and of market makers are not likely to be changing over time (or at least not rapidly declining). Therefore, simply changing the timing of resolution of uncertainty should have almost no effect on the pricing of securities.

Furthermore, there is a built-in asymmetry between the time periods: investors receive liquidity and information shocks only on date 2. Disclosing a very large amount of information about an asset on date 1 could, in principle, eliminate (or greatly reduce) all uncertainty about its value. If it

were feasible to disclose information at date 1 that resolved all future uncertainty, this would make the asset riskless and perfectly liquid at date 2 and would be the optimal policy. The large risk-neutral investors would hold all of the asset at date 1, with no risk premium, and sell at date 2 into a perfectly liquid market. The interpretation of this odd result is that if there is a date where there are few liquidity needs, then it would be desirable to disclose all types of information on that date, to make the security less risky and thus more liquid in the future. This would not be a useful result. See Ross (1989) for a complete markets model with public information that shows that the current security price is unaffected by the timing of future resolution of uncertainty.

In the discussion below we suggest a limit that simplifies the analysis of the model and eliminates the ability for the firm to change its future total risk by current disclosures, while allowing there to be risk. This limit makes the variance of the residual component of uncertainty that neither the informed trader nor the firm currently know large relative to current knowledge, so current disclosures have a vanishingly small effect on the future conditional variance of the asset's return. The limit is taken by making the variance of the residual component grow without bound but letting the aggregate risk aversion of market makers divided by this variance approach a finite bound. The information known by the informed trader of the firm has a finite variance, and thus its disclosure has no effect on the conditional variance of returns or the risk premium that market makers demand to hold the security at date 2. This limits the effect of disclosure to changing the distribution of orders to market makers and removes some benefits of disclosure.

Specifically, let the aggregate risk aversion of market makers, i.e.,  $\rho$ , become small relative to the variance of one period's announcements. This means that, in aggregate, market makers are approximately risk neutral with regard to current disclosures of information. In particular, we assume that  $\rho = r/u$ , where r is initially a fixed parameter (later r is endogenously determined by an entry condition). Market makers in aggregate become asymptotically risk neutral over uncertainty that is finitely variable (as u becomes large). However,  $\rho v = \rho(u + \delta + \Delta) \rightarrow r$  as  $u \rightarrow \infty$ . This implies that market makers remain risk averse with regard to  $\tilde{v}$  when  $\tilde{u}$  is unboundedly variable. We refer to r as aggregate market makers' asymptotic risk aversion.

This implies the following result.

Proposition 1: When aggregate market makers' risk aversion approaches a limit while the variance of  $\tilde{u}$  becomes unboundedly large, such that  $\rho v \rightarrow r$ ,

 $<sup>^{12}</sup>$ Recall that the firm's per share value at date 3 is  $\bar{v}=\bar{u}+\bar{\delta}+\bar{\Delta}$ , at which time it becomes known to all market participants. That element of firm value represented by  $\tilde{\delta}$  is disclosed to the informed trader at date 2 and partially revealed to all other market participants at date 2 through the signal  $\tilde{x}$ . That element represented by  $\tilde{\Delta}$  is partially revealed through the signal  $\tilde{y}$ . The element  $\tilde{u}$ , which we refer to as the residual component, is unknown throughout.

traders' long-run equilibrium demand for the risky asset in the first period becomes

$$D_1=\frac{2r}{5r+\lambda}Q_0,$$

and the long-run equilibrium price of the risky asset adjusts to

$$P_{1} = m - \frac{r\lambda(2\lambda^{2} + 7\lambda r + 7r^{2})}{2(\lambda + r)^{2}(\lambda + 5r)}Q_{0}$$

where m is the unconditional value of the security and  $Q_0$  is the total number of shares of the firm.

Proof: See the Appendix.

We briefly outline the properties of the price of the security at date 1,  $P_1$ , through a series of corollaries.

Corollary 1: The price of the security at date 1,  $P_1$ , increases as the market becomes more liquid, holding aggregate market maker risk aversion r fixed.

**Proof**: Observe that

$$\left( \partial P_1 / \partial \lambda \right) = -\frac{1}{2} Q_0 \left( \lambda + r \right)^{-3} \left( \lambda + 5r \right)^{-2} \left( 7 \lambda^3 r^2 + 23 \lambda^2 r^3 + 35 \lambda r^4 + 35 r^5 \right) < 0.$$
 Q.E.D.

Corollary 2: The price of the security at date 1,  $P_1$ , increases as aggregate market maker risk aversion r decreases, holding liquidity fixed.

**Proof**: Observe that

$$\left( \partial P_1 / \partial r \right) = -\frac{1}{2} Q_0 (\lambda + r)^{-3} (\lambda + 5r)^{-2} (2\lambda^5 + 12\lambda^4 r + 36\lambda^3 r^2 + 42\lambda^2 r^3) < 0.$$
 Q.E.D.

The significance of Corollary 2 will become clearer in Section IV, when r becomes endogenous.

Liquidity, which decreases in  $\lambda$ , is independent of disclosure of information that is not privately observed (information about  $\tilde{\Delta}$ ). However, liquidity does depend on the disclosure of privately known information  $\tilde{\delta}$ . Providing additional disclosure on  $\tilde{\delta}$ , through an increase in the precision of  $\tilde{x}$  (as implied by decreasing  $\varepsilon$ ), increases  $P_1$ .

Corollary 3: For fixed number of market makers, the long-run equilibrium price of the security at date 1,  $P_1$ , increases with more disclosure about private information  $\tilde{\delta}$ . Formally,  $(\partial P_1/\partial \varepsilon) < 0$ .

*Proof*: Note that for the case of asymptotic risk aversion, the liquidity parameter  $\lambda$  solves  $\lambda = (2 \lambda \delta \varepsilon / \delta \varepsilon + 4 \lambda^2 \eta [\delta + \varepsilon]) + r$ . Taking the derivative of

 $\lambda$  with respect to  $\varepsilon$  yields

$$\frac{\partial \lambda}{\partial \varepsilon} = \frac{\left(\delta^2/\delta + \varepsilon\right) \lambda \left(\lambda + r\right)}{2(\lambda + r)\delta \varepsilon + r\left(4\lambda^2 \eta \left[\delta + \varepsilon\right] + \delta \varepsilon\right)},$$

which is positive for finite-valued parameters. From Corollary 1,  $P_1$  increases as  $\lambda$  decreases, implying that  $P_1$  increases as  $\varepsilon$  decreases. Q.E.D.

The higher security price at date 1 (lower cost of capital) occurs because the increased disclosure of privately known information makes the market more liquid at date 2, which induces large traders to take bigger positions at date 1 for any fixed price. This increased demand due to attracting a broader market increases the price at date 1, reducing the cost of capital. All firms benefit from increased disclosure that reduces information asymmetry. The next result shows that larger firms, with more total risk to be borne by investors, get the largest per share benefit (percentage increase in value) from the broader institutional market attracted by this disclosure.

Corollary 4: The increase in per share price from increased disclosure increases as the size of the firm increases.

*Proof*: From the definition of  $P_1$  given in Proposition 1 and Corollaries 1 and 3,  $(\partial^2 P_1/\partial Q_0 \partial \varepsilon) = (Q_0)^{-1}(\partial P_1/\partial \varepsilon) < 0$ . Q.E.D.

Note that if there exists a cost to disclosure that is proportional to the size of the firm, this result still holds. In this event, smaller sized firms may choose not to disclose at all. However, among firms that disclose, larger firms disclose more information. Any fixed cost of disclosure reinforces this result.

#### III. The Effect of Disclosure on Traders' Welfare

The increase in price caused by disclosure of private information makes the firm selling shares to the public better off. An issue yet to be addressed is how disclosure affects the welfare of traders who may already own some shares of the firm. Disclosing small amounts of information not privately known (information about  $\Delta$ ) has no effect on liquidity or welfare. However, information about what an informed trader knows (information about  $\delta$ ) does affect welfare through its influence on liquidity. Note that each of the two large traders has a probability of one-half of being either the informed or the liquidity trader at date 2. Therefore, each trader can be thought of as representative of the average, and consequently we examine the effects of disclosure on the welfare of the representative trader. Clearly, if one trader had free and exclusive access to private information about the security, disclosing it before he or she could trade on it would not improve his or her welfare. Finally, note that in this section we assume that the number of market makers is fixed. In the next section, we let the number of market makers adjust in response to changed disclosure policies.

The first result concerns the effect of an unanticipated change in the disclosure of private information, after date 1 but before date 2. An

unanticipated change means that traders have already acquired any shares that the firm offers to the public and begin at a long-run equilibrium for the old level of disclosure (that is, their holdings of the risky asset at date 1 equals  $D_1$ , as given in the statement of Proposition 1). In this circumstance more disclosure about private information  $\tilde{\delta}$  (as implied by a decrease in  $\varepsilon$ ) boosts traders' welfare.

Proposition 2: An unanticipated increase in disclosure about privately known information  $\tilde{\delta}$ , thereby reducing information asymmetry at date 2, makes traders better off.

Proof: See the Appendix.

By way of explaining Proposition 2, we contrast it with Proposition 3 below, a result for the long-run equilibrium effect of an anticipated disclosure of private information. In Proposition 2 the traders have already purchased or sold shares of the asset (up to a long-run equilibrium) before the unanticipated disclosure; their holdings of the asset at date 1 are  $D_1$ . Therefore, they benefit directly from the better liquidity at date 2, without having the changed price of the shares at date 1 affect the costs of acquiring them. Proposition 3 allows the possibility that the traders initially hold an arbitrary quantity of shares (including, possibly, few or none) at date 0. Consequently and possibly, part of traders' and market makers' holdings of the asset at date 1 is acquired at price  $P_1$ . The improved future liquidity raises the price of the shares market participants purchase from the firm, allowing the firm to capture most of the gains. Subtracting the change in price at date 1 times the difference between their initial and their equilibrium holdings can result in traders capturing fewer rents than with less disclosure.

Proposition 3: There are two sufficient conditions such that a policy of increased disclosure of private information increases the long-run equilibrium welfare of large traders. First, independent of the level of information asymmetry that prevails at date 2, large traders' date 0 holdings as a fraction of the total supply of the security are above a certain threshold (which is less than 1). Second, independent of large traders' endowments at date 0, information asymmetry is above a certain threshold. Increased disclosure always makes any fixed number of market makers worse off.

**Proof:** See the Appendix.

The proof of Proposition 3 in the Appendix provides more detail about the nature of the thresholds alluded to in the statement of the result. For example, a sufficient condition (that is not necessary) on endowed holdings for traders at date 0, such that traders benefit from reduced information asymmetry, is that their initial holdings equal the long-run equilibrium quantity for the lower level of disclosure. That is, they are existing shareholders as opposed to new buyers who acquire all of their shares at the higher date-1 prices induced by disclosure.

To explain Proposition 3, note that reduced asymmetry of private information reduces the transfer of wealth across large investors at date 2 (which has essentially no effect on ex ante welfare) and reduces the expected quantity of trade with the market maker that the sum of the two large investors make at date 2. This reduced trading at date 2 implies that the liquidity premium paid to risk-averse market makers is reduced, and this makes traders better off. Because there is less information contained in a given order imbalance. prices respond less to a given imbalance. Thus, liquidity is increased. In addition, the reduction in private information effectively leads to more competition between traders and the market makers in providing liquidity at date 2 for the expected portion of the liquidity needs at date 2. This reduces the absolute value of the order imbalance at date 2 and provides risk-averse market makers with holdings of smaller absolute value. For example, if there is expected selling pressure from some investors (the trader who receives a liquidity shock), other investors (the informed trader who does not receive the liquidity shock) will put in buy orders that offset some of the expected selling, leading to a smaller acquisition by market makers at date 2. If these other investors have substantial private information, they trade on the basis of it, reducing the extent to which they offset the order imbalance from liquidity traders. This requires traders to rely more on market makers' services. 13

The cost of using the market makers' services is the liquidity premium they charge, and this is large if the order imbalance at date 2 is large and variable. This cost arises because large trades generate a temporary component in security returns. The order imbalance is large and variable when informed investors have very significant private information ( $\delta$  high) and little of it is disclosed ( $\varepsilon$  high). In this case, reducing the asymmetry by disclosing more private information (reduced  $\varepsilon$ ) reduces the risk premium at date 2 sufficiently to make large traders better off, even if all of their holdings are acquired at the higher date 1 price that the security commands with the increased disclosure. If there is a lower level of information asymmetry before the increased disclose ( $\varepsilon$  or  $\delta$  small, implying  $\lambda \rightarrow r$ ), then risk bearing is not as inefficient at date 2. In this case, large traders are worse off if all of their holdings are acquired at the higher date-1 prices that disclosure induces. If we imagine that there is a firm outside the model that sells all of the shares at the date-1 price, it captures all of the gain. Competition between the two large traders at date 1 prevents them from benefiting from the small improvement in risk bearing efficiency induced by the disclosure if they are not current holders of the security.

In summary, disclosure that reduces information asymmetry reduces the magnitude of the order imbalance at date 2 and as a result the risk of the

<sup>&</sup>lt;sup>13</sup>More generally, if there are other uninformed investors who do not observe the order flow but do compete with market makers by placing limit orders, they compete more effectively given reduced private information. This reinforces the effect of disclosure leading to more liquidity through reduced information asymmetry.

positions taken by market makers. This reduces the compensation paid to market makers for bearing the risk of providing liquidity, which makes existing holders better off.

# IV. Endogenous Entry into Market Making

# A. Analyzing the Effect of Disclosure on Price and Welfare

To this point we assume that the risk bearing capacity of market makers is given: market maker risk aversion is r. Disclosure of information that changes information asymmetry will influence the risk and return associated with making part of the market for a security. More disclosure of private information reduces the risk that market makers bear in providing liquidity, by making the absolute value of order imbalances smaller. This, in turn, causes fewer market makers to enter than with lower levels of disclosure.

The effect of changed information asymmetry on the entry decision of market makers is interesting in itself, and this effect is important in understanding the full effect of changes in private information disclosure on endogenous liquidity and the cost of capital.

We assume that there is a cost associated with being a market maker, for example the cost of observing the order flow and observing all public information continuously. This implies that there is a minimum associated level of expected utility required to induce someone to be one of the security's market makers. <sup>14</sup> The cost must be incurred to be a market maker for any and all of the periods in the model. <sup>15</sup> Note that this implies that the number of market makers does not change period-by-period. When disclosure is unanticipated, Proposition 2 will then apply even when the number of market makers is endogenous.

There are N potential market makers, and  $N \leq N$  can choose to enter the market making for this security. We assume that we start from a long-run equilibrium level of expected utility that each market maker achieves with some number N for a given level of disclosure and hold this level of expected utility constant as we vary the firm's disclosure policy. We again work with the limiting case where aggregate risk aversion vanishes for a single period's announcement risk but approaches a limit for the firm's larger long-run uncertainty. The amount of risk bearing capacity (number of market makers) as a proportion of the long-run risk  $\tilde{u}$  of the security depends on N/N, the proportion of the total number of potential market makers who enter. In any case, the number that turn out to enter, N, is large relative to the variance of the date-1 announcements. We ignore integer constraints on N, and assume  $N \to \infty$ .

<sup>15</sup>This implies that changing the rate of disclosure of public information would not influence the entry decision in a long horizon stationary equilibrium, just as it does not in the limiting asymptotic risk aversion model we use here.

<sup>&</sup>lt;sup>14</sup>For purposes of discussing endogenous entry into market making, we assume that market makers are endowed with none of the risky asset at date 0. This implies that if their equilibrium date 1 holdings are  $Q_1$ , the cost to them is  $P_1Q_1$ .

Recall that the aggregate asymptotic risk aversion of the group of market makers is given by r. There is an inversely proportional relation between the number of market makers (who divide each order imbalance equally), and risk aversion r. For example, let  $N_{\varepsilon}$  and  $r_{\varepsilon}$  represent the changes in the number of market makers and the aggregate risk aversion, respectively, that occur with less public disclosure of private information (increasing  $\varepsilon$  implies less disclosure). When a change in disclosure affects entry decisions, then the proportional change in aggregate asymptotic risk aversion  $(r_{\varepsilon}/r)$  is equal to -1 times the proportional change in the number of market makers,  $(N_{\varepsilon}/N)$ , induced by a change in disclosure.

Proposition 4: Increased disclosure of private information  $\tilde{\delta}$  causes exit among market makers, improves liquidity (decreases  $\lambda$ ), and raises the level of aggregate market maker risk aversion r.

Proof: See the Appendix.

The number of market makers is endogenously determined by holding the expected utility of each active market maker fixed. More disclosure for any fixed number of market makers reduces the expected utility of each one, inducing exit (a reduced number of market makers), which, in turn, increases the equilibrium value of r, the aggregate asymptotic risk aversion of market makers. The reduced information asymmetry leads to a smaller variability of the order imbalance at date 2, because the informed trader takes a smaller position. In the extreme, for sufficiently little private information, the informed trader offsets a portion (approaching 100%) of the expected selling of the liquidity trader and competes with the market maker in providing some of the liquidity at date 2. These effects work to reduce the expected utility of each market maker for a fixed total number because market makers compete to hold the marginal unit of order imbalance at date 2. Improved liquidity (lower  $\lambda$ ) implies that the value to a market maker of the marginal unit of the order imbalance is not as far below the value of the average unit in the imbalance. 17

<sup>16</sup> Formally, select parameters of the variance of  $\tilde{u}$  such that the variance of  $\tilde{u}$  is proportional to the number of potential market makers  $\sigma_u^2$ . This implies that  $\rho v = \rho_{MM} \sigma_u^2 (N/N)$ , where we continue to assume that  $\lim \rho v \to r$ . Thus,

$$\frac{\partial}{\partial \varepsilon} \rho v = -\rho_{MM} \sigma_u^2 (N/N) (N_{\varepsilon}/N).$$

This, in turn, implies that

$$\lim_{N\to\infty}\frac{\partial}{\partial\varepsilon}\rho v=\lim_{N\to\infty}-\rho_{MM}\sigma_u^2(N/N)(N_\varepsilon/N)\to-r(N_\varepsilon/N),$$

or

$$r_{\varepsilon} = -r(N_{\varepsilon}/N).$$

 $<sup>^{17}</sup>$ This relation between marginal and average is essentially the same as consumer surplus.

Corollaries 1 and 2 showed that  $P_1$  was decreasing in  $\lambda$  and increasing in r. Disclosure that reduces information asymmetry increases liquidity (reduces  $\lambda$ ). But it also causes market makers to exit, thereby decreasing their risk bearing capacity; this, in turn, translates into an increase in r. Proposition 5 describes the net influence of these countervailing effects of disclosure on the security price at date 1,  $P_1$ .

Proposition 5: With endogenous entry into market making and information asymmetry above a certain threshold, the long-run equilibrium price of the security at date 1,  $P_1$ , increases with a small increase in information about  $\tilde{\delta}$ . For information asymmetry below a certain threshold,  $P_1$  decreases with a small increase in disclosure. The maximum value of  $P_1$  occurs with a small, but positive, amount of information asymmetry.

Proof: See the Appendix.

The reduction in private information when there is little initial information asymmetry causes a large reduction in private information based orders, causing the order imbalances at date 2 to decline. This makes market makers worse off because declining order imbalances reduce the expected utility of any fixed number of market makers, inducing exit. Exit, in turn, raises both r and the fraction of each order imbalance handled by each trader. Prices are reduced because there is rapid exit of market makers. In addition, when there is little information asymmetry (low  $\varepsilon$  or  $\delta$ ), there is maximal competition between traders and market makers for the expected component of liquidity trades at date 2, the expected net trade at date 2 approaches 0. Here, the utility per market maker is relatively insensitive to the number of market makers, and there must be a large rate of exit (implying a large rate of increase in r) to keep per market maker expected utility constant when increased disclosure reduces the expected utility of being a market maker.

A result that parallels Corollary 4 when the number of market makers is endogenous is the following. The percentage impact of disclosure on firm value is again larger for larger firms.

Corollary 5: With endogenous entry into market making, the absolute value of the per share price change due to increased disclosure increases with firm size.

*Proof*: From the proof to Proposition 5, note that

$$\left(\partial \left[ \left| \partial P_1 / \partial \varepsilon \right| \right] / \partial Q_0 \right) = \left( Q_0 \right)^{-1} \left| \partial P_1 / \partial \varepsilon \right| > 0.$$

Q.E.D.

Corollary 5 specifies an absolute value because reduced information asymmetry can reduce current prices; for small amounts of information asymmetry, value maximizing firms will not disclose even if it is costless to do so, and this is especially true for larger firms. Costs of disclosure reinforce this result. Alternatively, if information asymmetry is sufficiently large such that costless disclosure increases price, then the per share price increase is

increasing in firm size. This again implies that larger firms will disclose more than smaller ones, and there can be a minimum firm size for which there is some disclosure that increases its current stock price.

This leads to a final result concerning the effect of more disclosure of private information on traders' welfare when market making is endogenous.

Proposition 6: Suppose that traders hold an arbitrary quantity of shares at date 0 and there exists endogenous entry into market making. Independent of the level of information asymmetry that prevails across the market at date 2, a sufficient condition such that a policy of disclosure of private information increases the long-run equilibrium welfare of large traders is that the total supply of the security,  $Q_0$ , is above a certain threshold.

Proof: See the Appendix.

If  $Q_0$  is large, the security is the claim to a large firm with substantial total supply. Attracting the risk bearing ability of large traders will make a large difference to the cost and efficiency of risk bearing for a large firm but not for a very small firm with  $Q_0$  near 0. Therefore, large firms will choose a disclosure policy that attracts large investors. Because there are not perfect substitutes for the risk bearing services of large investors, the investors will be made better off by this disclosure policy.

To understand Proposition 6 in the context of Proposition 3 (our parallel welfare result with a fixed number of market makers), note that making  $Q_0$  very small while holding the number of market makers constant makes the expected order imbalance at date 2 arbitrarily small. This is similar to the situation when there is little information asymmetry ( $\varepsilon \to 0$  or  $\delta \to 0$ ) as discussed in regard to Proposition 4. For expected utility per market maker to remain fixed (the entry condition) when  $Q_0$  is small, r must be large and N/N small and the utility of each market maker relatively insensitive to a given rate of exit (change in N/N). Increased disclosure (a reduction in  $\varepsilon$ ) makes market makers worse off for given r, and thus a large increase in r (drop in N/N) is required to hold per market maker welfare constant. This implies a large elasticity of market maker exit to reductions in  $\varepsilon$ . Consequently, increased information asymmetry (higher  $\varepsilon$ ) may actually make traders better off and increase the price of the security at date 1 by enhancing depth of market making due to increased entry of market makers.

Our interpretation of this result is that in situations where market makers have a difficult time breaking even, face substantial competition, and have high entry costs (thereby implying that there is little entry into market making in the security), a small amount of information asymmetry is better for stockholders and firms than none. The higher level of information asymmetry causes rapid entry into market making. A situation in which information asymmetry is beneficial, because it causes a very large increase in the number of market makers, must be regarded as the exception rather than the rule. The next section suggests several sensible extensions of our model that tend to work against it being a common occurrence.

It is worth reviewing the interpretation of the changes in information asymmetry induced by disclosure in our model. We examine the effect of changes in the known amount of information asymmetry on the expected utility associated with making a market in a security and then on the decision to enter the activity of market making. If there is more asymmetry of information (less public disclosure), market makers quote less liquid prices to take account of the increased information revealed by a given order imbalance. Despite this reduced liquidity, there are larger order imbalances when there is more information asymmetry. Thus, information asymmetry leads to greater expected utility for any fixed number of market makers because they compete for the marginal share of each order and earn some surplus on the inframarginal shares traded. On the other hand, if there were more private information than market makers anticipated when they quoted prices, this secret increase in information asymmetry would make market makers worse off because they would overpay when they buy and undercharge when they sell.

# B. Generalizing the Model: Other Effects on the Rate of Entry

Our result that increased information asymmetry may be beneficial by inducing sufficiently rapid entry of market makers may be weakened by some obvious generalizations of the model. Two features that tend to reverse or attenuate the result are: the assumed constancy of the variance of the final holdings of the liquidity trader, for differing levels of date-2 liquidity; and the assumption that the entry condition for market makers is a fixed level of expected utility per market maker, independent of the number of market makers who enter market making in this security.

If liquidity traders allocated their date 2 trades across holdings partly based on the liquidity of each holding, then  $\eta$  would be a decreasing function of  $\lambda$ .<sup>18</sup> It is then possible that disclosure that increased liquidity would increase liquidity trade faster than it reduced informed trade. Sufficiently elastic supply of liquidity trade to changes in liquidity could make disclosure increase the order imbalances, inducing entry. This effect would tend to prevent increased information asymmetry from being beneficial. Alternatively, the larger the amount of inelastic demand for trading a given security, the more likely that a small amount of information asymmetry leads to a higher security price than no information asymmetry at all.

If potential market makers faced varying opportunity costs of becoming market makers, then the expected utility needed to induce entry would be an increasing function of the number of market makers who enter because the lowest opportunity cost market makers would enter first. This reduces the rate of exit of market makers as the rents of each market maker decrease and makes the endogenous market maker case more like the exogenous market

<sup>&</sup>lt;sup>18</sup>It is difficult to calibrate the magnitude of this because a need for liquidity generally will make binding some constraints on selling short.

maker case. This effect attenuates the effects of endogenous exit and makes disclosure that reduces information asymmetry look attractive to more types of firms.

# V. The Role of Competitive Traders

By only explicitly considering market makers and large institutional traders subject to liquidity and information shocks in our model, our results may suggest that market makers assume an unrealistically large position in the risky asset at date 1. To give a simple example of how the model works with other traders who are neither market makers nor large traders, consider a large group of G risk-averse, competitive traders (who are price takers). They do not observe the current order flow. They take the price  $P_1$  of the risky asset at date 1 as given and maintain their holdings of the asset until the value of the firm is revealed at date 3. Each individual in this syndicate has constant absolute risk aversion of  $\rho_{CPT}$ . We suppose that as G and u become large,

$$\frac{\rho_{CPT}}{G}(u+\delta+\Delta)\to t.$$

In other words (as in the case of market makers), the syndicate evidences aggregate asymptotic risk aversion of t over the future uncertainty associated with holding the asset.

The holdings  $D_1^c$  of the competitive traders at date 1, as a function of the date-1 price, are  $D_1^c = (m-P_1)/t$ . This then implies that the holdings of the market makers plus that of the two large traders equal  $Q_0 - D_1^c \equiv \hat{Q}$ . Replacing  $Q_0$  with  $\hat{Q}$  allows all of the model's results to be reinterpreted in the presence of a group of competitive traders. The qualitative results are unchanged. Improved liquidity leads to a broader market that allows less reliance on the price takers and the market maker at date 1. If the firm is small ( $Q_0$  small), then the cost of capital is low even if the large traders hold none of the security. A nonbinding lower bound on the price at date 1 (an upper bound on the cost of capital) is the price that would induce the competitive traders to hold the entire quantity of the security,  $P_1 > m - tQ_0$ , because at that price both the market maker and the large traders would want to be long.

The other change introduced by competitive traders concerns the interpretation of the meaning of the variance of the information on date 3 when the uncertainty about the value of the security is resolved. Loosely stated, this is now interpreted as the variability of the information arriving before market makers can place that increment to their holdings over the long-run equilibrium level into the hands of other (competitive) traders.

$$Q_1 = Q_0([\lambda + r]/[\lambda + 5r]).$$

<sup>&</sup>lt;sup>19</sup>At date 1, market makers' (collective) holdings of the risky asset are:

The introduction of competitive traders is intended to show that our results are robust and do not depend on there being only large traders and market makers. The price takers absorb part of the supply of the asset and put an upper bound on the firm's cost of capital; the lowest security price is the one in which all of the stock is held by the competitive traders. Consequently, introducing competitive traders more fully into the model does not qualitatively influence our results.

#### VI. Conclusion

Policies that reduce asymmetry of information will increase the liquidity of the market for a firm's securities. This induces large institutional traders who anticipate making future large trades to take larger current positions. The reduced information asymmetry increases the competition with market makers and reduces the volatility of future order imbalances, leading to exit of market makers. This exit of market makers increases the temporary component of future security returns.

If the firm is large enough, then to have a low cost of capital it must attract large positions from institutional investors. In addition, for any fixed amount of information asymmetry, a large firm will have larger order imbalances because the equilibrium holdings of large traders subject to liquidity shocks are increasing in the size of the firm. Provided there is some information asymmetry, these large order imbalances attract more market makers, implying that the largely institutional holders of large firms both need the liquidity provided by market makers and provide some trading activity to attract them. For smaller firms, institutional investors will choose smaller positions, leading to smaller trades if they liquidate and lower compensation to market makers. The cost of capital for these smaller firms is not as dependent on large positions by large traders, so they disclose less information. The small number of market makers they attract in any case limits both the attractiveness of the stock to big investors and the amount that they profit from the information.

We use a model with a single firm. Our results on firm size hold for multiple firms of various sizes if the returns are distributed independently or if it is possible for individuals and market makers to hedge systematic risks in a liquid futures market. For example, suppose that there are common factors in firm returns and that no one has private information about these factors. Then, liquid futures markets for each factor allow these types of risks to be hedged and these risks to be removed when market makers take large positions. In this case, where the private information is firm specific, the interpretation of our result that breadth of market is especially important for large firms is that large firms have a large amount of firm-specific risk which cannot be hedged.

Another interpretation of our results is that in addition to being consistent with voluntary disclosure of public information by large firms, it explains the preferential disclosure of additional information by large firms at analysts'

meetings. A large firm can increase its current stock price by revealing some low-precision information only to large institutional investors and can make the timing of the release of this *private information* known to the public. This gives the institutional investors an incentive to make additional trades, implying a higher equilibrium number of market makers. This additional depth of market making induces the large traders to take larger positions on dates where they have no private information (despite the higher dependence of price on order imbalances as compared to fully symmetric information) because the profits that they make from trading on information offset some of their losses when instead they place large liquidity-motivated trades.

The effect of information asymmetry on entry into market making, and the result that a little information asymmetry can be better than none, may prove useful in future studies of other firm policies that have been explained in terms of attempting to change information asymmetry. Some of the relevant papers in this area are Barclay and Smith (1988) and Brennan and Thakor (1990) that study dividend policy as an attempt to minimize informed trade and Leland (1990) that studies insider trade.

# **Appendix**

Proof of Lemma 1: Market makers conjecture that  $\tilde{v}$ ,  $\tilde{x}$ ,  $\tilde{y}$ , and  $\tilde{d}_{T2}$  have a multivariate normal distribution. In particular, imagine that at date 2 trader a becomes a liquidity trader and b an informed trader. Market makers conjecture that the informed trade at date 2, which we represent by  $\tilde{d}_{b2}$ , is a linear function of those things observed by the informed trader plus a constant:

$$\tilde{d}_{b2} = \alpha_1 \tilde{\delta} + \alpha_2 \tilde{x} + \alpha_3 \tilde{y} + \alpha_0,$$

where all of the coefficients  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  are conjectures on the part of market makers. (Below, we determine values for these coefficients such that market makers' conjectures are self-fulfilling.) This implies that  $(\tilde{v}, \, \tilde{x}, \, \tilde{y}, \, \tilde{d}_{T2})$  has the following covariance structure:

$$\tilde{\tilde{x}} \begin{bmatrix} u+\delta+\Delta & \delta & \Delta & (\alpha_1+\alpha_2)\delta+\alpha_3\Delta \\ \delta & \delta+\varepsilon & 0 & \alpha_1\delta+\alpha_2(\delta+\varepsilon) \\ \Delta & 0 & \Delta+\xi & \alpha_3(\Delta+\xi) \\ (\alpha_1+\alpha_2)\delta+\alpha_3\Delta & \alpha_1\delta+\alpha_2(\delta+\varepsilon) & \alpha_3(\Delta+\xi) & (\alpha_1+\alpha_2)^2\delta+(\alpha_2)^2\varepsilon \\ & & & +(\alpha_3)^2(\Delta+\xi)+\eta \end{bmatrix}.$$

Using standard formulae concerning the computation of conditional expectations from a population of normal distributions, define  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  by

$$E[\tilde{v} \mid \bar{x}, \tilde{y}, \bar{d}_{T2}] = m + \beta_1 \bar{x} + \beta_2 \bar{y} + \beta_3 (\bar{d}_{T2} - E[\tilde{d}_{T2}]),$$

and

$$\theta \equiv \operatorname{Var} \left[ \, \tilde{\boldsymbol{v}} \, | \, \, \bar{\boldsymbol{x}}, \, \bar{\boldsymbol{y}}, \, \bar{\boldsymbol{d}}_{T2} \, \right].$$

It can be shown that  $\beta_1 = (\delta/\delta + \varepsilon)$ , and  $\beta_2 = (\Delta/\Delta + \xi)$ , but  $\beta_3$  and  $\theta$  are more complicated expressions that depend directly on market makers' conjectures about  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$ . This implies that

$$\begin{split} P_2 &= E\big[\,\widetilde{v}\,|\,\,\overline{x},\,\overline{y},\,\overline{d}_{T2}\,\big] + \rho\,\mathrm{Var}\big[\,\widetilde{v}\,|\,\,\overline{x},\,\overline{y},\,\overline{d}_{T2}\,\big]\big(\,\overline{d}_{T2} - Q_1\big),\\ &= m + \beta_1\overline{x} + \beta_2\,\overline{y} + \beta_3\big(\,\overline{d}_{T2} - E\big[\,\widetilde{d}_{T2}\,\big]\,\big) + \rho\theta\big(\,\overline{d}_{T2} - Q_1\big),\,\mathrm{and}\\ &= m + \big(\delta/\delta + \varepsilon\big)\,\overline{x} + \big(\Delta/\Delta + \xi\big)\,\overline{y} + \lambda\overline{d}_{T2} - \beta_3E\big[\,\widetilde{d}_{T2}\,\big] - \rho\theta\,Q_1, \end{split}$$

where

 $\lambda$  is defined by  $\lambda = \beta_3 + \rho \theta$ .

To determine  $\lambda$ , as well as  $\beta_3$  and  $\theta$ , consider the market makers' conjectures about the informed trader's behavior, specifically, the coefficients  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$ . The trader who becomes informed observes  $\tilde{\delta} = \bar{\delta}$ ,  $\tilde{x} = \bar{x}$ , and  $\tilde{y} = \bar{y}$  before he or she submits his or her demand order at date 2. However, the trader does not observe total net demand, which implies that he or she cannot infer the liquidity trader's shock  $\tilde{\eta}$ . Consequently, he or she treats  $\tilde{P}_2$  as a random variable since the actual price quoted for the security at date 2 depends on market makers' observation of the aggregate order imbalance. From the discussion in Section I.C. concerning the trade executed by the informed trader at date 2, we know that

$$\begin{split} \overline{d}_{b2} &= (1/2\lambda) \left( E \left[ \tilde{v} \mid \bar{\delta}, \, \bar{y} \right] - \Phi \right) + \frac{1}{2} D_{a1} \\ &= (1/2\lambda) \left( \bar{\delta} - \left\{ \delta/\delta + \varepsilon \right\} \bar{x} \right) + \frac{1}{2} D_{a1} + \left( \beta_3/2\lambda \right) E \left[ \tilde{d}_{T2} \right] + \left( \rho\theta/2\lambda \right) Q_1, \end{split}$$

where  $E[\tilde{v} \mid \bar{\delta}, \bar{y}] = m + \bar{\delta} + (\Delta/\Delta + \xi)\bar{y}$ . Alternatively, the trade executed by trader a if he becomes informed is

$$\bar{d}_{a2} = \left(1/2\lambda\right)\left(\bar{\delta} - \left\{\delta/\delta + \varepsilon\right\}\bar{x}\right) + \frac{1}{2}D_{b1} + \left(\beta_3/2\lambda\right)E\left[\tilde{d}_{T2}\right] + \left(\rho\theta/2\lambda\right)Q_1.$$

Therefore, from the perspective of market makers,  $ilde{d}_{T2}$  is a random variable defined by

$$\tilde{d}_{T2} = q(\tilde{\eta} - D_{a1} + \tilde{d}_{b2}) + (1 - q)(\tilde{d}_{a2} + \tilde{\eta} - D_{b1}),$$

where q is the probability of trader a becoming a liquidity trader versus b becoming a liquidity trader (i.e.,  $q = \frac{1}{2}$ ). This implies

$$E[\tilde{d}_{T2}] = -\frac{1}{2}(\lambda/\lambda + \rho\theta)(D_{a1} + D_{b1}) + (\rho\theta/\lambda + \rho\theta)Q_1.$$

Furthermore, reconciling the expressions for demand with the specialist's

original conjecture about the informed trader's demand yields:

$$\begin{split} &\alpha_0 = \frac{1}{2} \big( \lambda/\lambda + \rho \theta \big) \big( D_{a1} + D_{b1} \big) + \big( \rho \theta/\lambda + \rho \theta \big) Q_1, \\ &\alpha_1 = \big( 1/2 \, \lambda \big), \\ &\alpha_2 = - \big( \delta/2 \, \lambda \big[ \, \delta + \varepsilon \big] \big), \\ &\alpha_3 = 0, \\ &\theta = u + \frac{4 \lambda^2 \eta \delta \varepsilon}{\delta \varepsilon + 4 \lambda^2 \eta \big( \delta + \varepsilon \big)} + \frac{\Delta \xi}{\Delta + \xi}, \text{and} \\ &\beta_3 = \frac{2 \lambda \delta \varepsilon}{\delta \varepsilon + 4 \lambda^2 \eta \big( \delta + \varepsilon \big)}, \end{split}$$

where  $\lambda$  is determined as the solution to  $\lambda = \beta_3 + \rho\theta$ . Note that solving for  $\lambda$  is equivalent to determining a positive, real-valued root to a third-order polynomial.<sup>20</sup> Q.E.D.

*Proof of Lemma 2:* The price of the security at date 1 is

$$P_1 = m - \gamma_1(Q_0 - D_{a1} - D_{b1}) + \gamma_2 E[\tilde{d}_{T2}],$$

where

$$\begin{split} \gamma_1 &= \rho v + \left(\rho \delta \varepsilon / 2 \lambda \left[\delta + \varepsilon\right]\right) \gamma_2, \\ \gamma_2 &= -\rho \theta \frac{\left(\rho \delta \varepsilon / 2 \lambda \left[\delta + \varepsilon\right]\right) + k}{\left(1 + \rho \left[\lambda \eta - \left\{\delta \varepsilon / 4 \lambda \left[\delta + \varepsilon\right]\right\}\right]\right)}, \end{split}$$

 $k \equiv -\frac{1}{2}(\lambda + 2\,\rho\theta\,/\lambda + \rho\theta)$ , and  $Q_0$  represents the total supply of the security at date 0 (i.e., the total endowment). To establish this, note that at date 1 the market maker anticipates three sources of uncertainty at date 2: the value of the security at date 3,  $\tilde{v}$ ; the total order imbalance at date 2,  $\tilde{d}_{T2}$ ; and the price at which the security trades at date 2,  $\tilde{P}_2$ . Furthermore, market maker i's wealth at date 3 is

$$\begin{split} \tilde{W}_i &= \big( Q_{i1} - \tilde{d}_{iT2} \big) \tilde{v} + \tilde{P}_2 \tilde{d}_{Ti2} + P_{i1} d_{Ti1} \text{ and} \\ &= N^{-1} \big( \big[ Q_1 - \tilde{d}_{T2} \big] \tilde{v} + \tilde{P}_2 \tilde{d}_{T2} + P_{i1} N d_{Ti1} \big). \end{split}$$

The tri-variate  $(\tilde{v}, \tilde{d}_{T2}, \tilde{P}_2)$  has a normal distribution with mean  $(m, E[\tilde{d}_{T2}],$ 

 $<sup>^{20}</sup>$ It is a simple exercise to show that there exists only one root because the third-order polynomial is an increasing function of  $\lambda$ . Therefore,  $\lambda$  is unique.

 $E[\tilde{P}_2]$ ) and covariance matrix C, where

$$C = \begin{bmatrix} u + \delta + \Delta & \frac{1}{2\lambda} \frac{\delta \varepsilon}{\delta + \varepsilon} & \frac{\delta^2}{\delta + \varepsilon} + \frac{\Delta^2}{\Delta + \xi} + \frac{1}{2} \frac{\delta \varepsilon}{\delta + \varepsilon} \\ \frac{1}{2\lambda} \frac{\delta \varepsilon}{\delta + \varepsilon} & \frac{1}{4\lambda^2} \frac{\delta \varepsilon}{\delta + \varepsilon} + \eta & \frac{1}{4\lambda} \frac{\delta \varepsilon}{\delta + \varepsilon} + \lambda \eta \\ \frac{\delta^2}{\delta + \varepsilon} + \frac{\Delta^2}{\Delta + \xi} + \frac{1}{2} \frac{\delta \varepsilon}{\delta + \varepsilon} & \frac{1}{4\lambda} \frac{\delta \varepsilon}{\delta + \varepsilon} + \lambda \eta & \frac{\delta^2}{\delta + \varepsilon} + \frac{\Delta^2}{\Delta + \xi} + \frac{1}{4} \frac{\delta \varepsilon}{\delta + \varepsilon} + \lambda^2 \eta \end{bmatrix},$$

This implies that the market maker i's expected utility at date 1 is

$$egin{aligned} EU_{i1} &= Eig[ \, EU_{i2} ig] \, = Eig[ \, Eig[ -\expig( -N
ho ilde{W}_iig) \, | \, ar{x}, \, ar{y}, \, ar{d}_{T2} ig] ig] \, ext{and} \ &= - \, | \, I + 
ho C\Omega \, |^{-1/2} \! \expig( rac{1}{2}
ho Vig[ \, 
ho^{-1}I + \, C\Omega ig]^{-1}CV' \ &+ 
hoig[ -mQ_1 + ig\{ m - Eig[ \, ilde{P}_2 ig] ig\} Eig[ \, ilde{d}_{T2} ig] \, - P_{i1}Nd_{Ti1} ig] ig), \end{aligned}$$

where I is the identity matrix, V is a  $1 \times 3$  vector defined by  $V = (E[\tilde{d}_{T2}] - Q_1, m - E[\tilde{P}_2], - E[\tilde{d}_{T2}])$ , and  $\Omega$  is a  $3 \times 3$  matrix defined by

$$\Omega = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Note also that all market makers observe total net demand  $d_{T1}$ . Therefore, each market maker interprets a shift in  $d_{T1}$  as implying a change of k in  $E[\tilde{d}_{T2}]$ ; that is,  $(\partial E[\tilde{d}_{T2}]/\partial d_{T1}) = k$ .  $P_1$  is determined by requiring that each market maker's marginal expected utility for an additional unit of the total order imbalance at date 1 is zero; this implies  $P_1 = m - \gamma_1 Q_1 + \gamma_2 E[\tilde{d}_{T2}]$ , where  $\gamma_1$  and  $\gamma_2$  are given as in the statement of Lemma 2. Finally, note that  $Q_1 = Q_0 - D_{a1} - D_{b1}$ , where  $Q_0$  represents the total endowment of the asset at date 0. Q.E.D.

*Proof of Lemma 3:* The long-run equilibrium, date-1 holdings of the security by traders a and b are  $D_{a1} \equiv D_{b1} \equiv D_1 = \Gamma Q_0$ , where

$$\Gamma = \left[ \frac{\gamma_1 - \frac{\rho \theta}{\lambda + \rho \theta} \gamma_2 - \frac{1}{2} \frac{\rho \theta}{\lambda + \rho \theta} (\lambda - \beta k + \rho \theta)}{2\gamma_1 + 2\gamma_2 k + \frac{1}{2} (\lambda + \beta k - \rho \theta) - \frac{1}{2\lambda} (\beta k - \rho \theta)^2} \right]. \tag{1}$$

This result relies on the following argument. When trader a becomes a liquidity trader,

$$\tilde{P}_2 = m + (\delta/\delta + \varepsilon)\tilde{x} + (\Delta/\Delta + \xi)\tilde{y} + \lambda(\tilde{d}_{b2} + \tilde{\eta} - D_{a1}) - \beta_3 E[\tilde{d}_{T2}] - \rho\theta Q_1,$$
 whereas when trader a becomes an informed trader,

$$\tilde{P}_2 = m + (\delta/\delta + \varepsilon)\tilde{x} + (\Delta/\Delta + \xi)\tilde{y} + \lambda(\tilde{d}_{a2} + \tilde{\eta} - D_{b1}) - \beta_3 E[\tilde{d}_{T2}] - \rho\theta Q_1.$$

Trader a's demand for the security at date 1 is determined by maximizing  $EU_{a1}$  with respect to  $D_{a1}$ . This yields a  $D_{a1}$  that is linear in: trader a's endowment of the security,  $Q_a$ ; the demand of trader b at date 1,  $D_{b1}$ ; and the total number of shares of the firm,  $Q_0$ . Performing a similar calculation for trader b yields a  $D_{b1}$  that is linear in  $Q_b$ ,  $D_{a1}$ , and  $Q_0$ . Therefore, treating  $Q_0$ ,  $Q_a$ ,  $Q_b$ , as exogenous constants, traders' demands  $D_{a1}$  and  $D_{b1}$  are linear in  $Q_0$ ,  $Q_a$ ,  $Q_b$ . First, determine  $D_{a1}$  such that  $\partial EU_{a1}/\partial D_{a1}=0$ . This implies (where we drop the subscript 3 in the parameter  $\beta_3$  for notational convenience):

$$\begin{split} D_{a1} \big( 2\gamma_1 + 2\gamma_2 k + \big[ 1 - q \big] \big[ \lambda + \beta k - \rho \theta \big] - \big[ q/2\lambda \big] \big[ \beta k - \rho \theta \big]^2 \big) \\ &= Q_a \big( \gamma_1 + \gamma_2 k \big) \\ &- D_{b1} \big( \gamma_1 + \gamma_2 k + \frac{1}{2} \big[ 1 - q \big] \big[ \beta k - \rho \theta \big] \\ &- \big[ q/2\lambda \big] \big[ \beta k - \rho \theta \big] \big[ \lambda + \beta k - \rho \theta \big] \big) \\ &+ Q_0 \big( \gamma_1 - \big[ \rho \theta / \lambda + \rho \theta \big] \big[ \gamma_2 + \{ 1 - q \} \lambda - q \{ \beta k - \rho \theta \} \big] \big), \end{split}$$

where  $k=-\frac{1}{2}(\lambda+2\rho\theta/\lambda+\rho\theta)$ . In this calculation we assume that  $\partial E[\tilde{d}_{T2}]/\partial D_{a1}=k$  and  $\partial P_1/\partial D_{a1}=\gamma_1+\gamma_2 k$ ; that is, when trader a increases his or her demand at date 1 by one unit,  $(D_{a1}+D_{b1}-Q_0)$  increases by one unit and  $E[\tilde{d}_{T2}]$  increases by k units, where k results from the expression for  $E[\tilde{d}_{T2}]$  given in the proof of Lemma 1. The long-run equilibrium, in combination with the fact that traders a and b are symmetric, implies that  $D_{a1}=D_{b1}=Q_a\equiv D_1$ . Making these substitutions in the relation above yields the expression for  $D_1$  given in equation (1). Q.E.D.

*Proof of Proposition 1:* The long-run equilibrium price of the security at date 1 is  $P_1$ , where

$$P_{1} = m + \gamma_{1}(2D_{1} - Q_{0}) - \gamma_{2}\left(\frac{\lambda}{\lambda + \rho\theta}D_{1} + \frac{\rho\theta}{\lambda + \rho\theta}[2D_{1} - Q_{0}]\right), \quad (2)$$

and  $D_1$  is given as in equation (1). This results from the expression for  $P_1$  given in Lemma 2, the expression for  $E[\tilde{d}_{T2}]$  given in the *Proof of Lemma 1*, and the expression for  $D_1$  given in Lemma 3. To see how the expressions in equations (1) and (2) imply those in the statement of Proposition 1, note that  $\gamma_1 \to r$ ,  $\gamma_2 \to \frac{1}{2} r([\lambda + 2r]/[\lambda + r])$ ,  $k \to -\frac{1}{2} ([\lambda + 2r]/[\lambda + r])$ , and  $\rho\theta \to r$ , as  $u \to \infty$ . Substituting these relations into equations (1) and (2) implies the result. Q.E.D.

<sup>&</sup>lt;sup>21</sup>This problem is well-defined in that it can be shown that  $EU_{a1}$  is a concave function of  $D_{a1}$ .

<sup>22</sup>In arriving at this relation, we use the expression for  $E[\tilde{d}_{T2}]$  given in the proof of Lemma 1, which, in turn, is linear in  $D_{a1}$ ,  $D_{t1}$ , and  $Q_0$ .

Proof of Proposition 2: The expected utility of a trader at date 1 reduces to:

$$\begin{split} EU_{a1} &= \tfrac{1}{2} E \big[ \big( \tilde{v} - \tilde{P}_2 \big) \tilde{d}_{a2} \big] - \tfrac{1}{2} E \big[ \tilde{P}_2 \tilde{\eta} \big] + E \big[ \tilde{P}_2 \big] D_1 \\ &= - \tfrac{1}{2} \bigg( \lambda \eta - \frac{1}{4 \lambda} \frac{\delta \varepsilon}{(\delta + \varepsilon)} \bigg) + m D_1 \\ &+ \tfrac{1}{2} r \big( E \big[ \tilde{d}_{T2} \big] - Q_1 \big) \bigg( D_1 - E \big[ \tilde{d}_{T2} \big] + \frac{r}{\lambda} \big\{ E \big[ \tilde{d}_{T2} \big] - Q_1 \big\} \bigg) \\ &= - \tfrac{1}{2} \bigg( \lambda \eta - \frac{1}{4 \lambda} \frac{\delta \varepsilon}{(\delta + \varepsilon)} \bigg) + m D_1 - \frac{r \lambda}{\left(\lambda + r\right)^2} \big( D_1 + Q_1 \big) (\lambda D_1 - r Q_1 \big). \end{split}$$

The next step is to consider the derivative of  $EU_{a1}$  with respect to  $\varepsilon$ . Note, however, that in this calculation  $D_1$  and  $Q_1$  are treated as constant since we hold endowments at date 1 fixed, and each is positive. Let  $\lambda_{\varepsilon} \equiv (\partial \lambda / \partial \varepsilon)$ . Then, it can be shown that:

$$egin{aligned} rac{\partial E U_{a1}}{\partial \, arepsilon} &= \, - rac{1}{2} r igg( \eta \, - \, rac{1}{4 \, \lambda^2} \, rac{\delta \, arepsilon}{\left( \delta \, + \, arepsilon 
ight)} igg) \lambda_{arepsilon} \ &- rac{r^2 ig( D_1 + Q_1 ig)}{ig( \lambda + r ig)^3} ig( ig[ \lambda - r ig] Q_1 + 2 \, \lambda D_1 ig) \lambda_{arepsilon} &< 0, \end{aligned}$$

since

$$\eta - \frac{1}{4\lambda^2} \frac{\delta \varepsilon}{\left(\delta + \varepsilon\right)} > 0, \text{ and } \lambda - r > 0,$$

from the fact that the equilibrium condition governing  $\lambda$  can be stated as  $\lambda(4\lambda^2\eta[\delta+\varepsilon]-\delta\varepsilon)=r(4\lambda^2\eta[\delta+\varepsilon]+\delta\varepsilon)$ , which implies that  $4\lambda^2\eta(\delta+\varepsilon)-\delta\varepsilon>0$  and  $\lambda>r$ , and the fact that  $\lambda_\varepsilon>0$ , which follows from Corollary 3. Q.E.D.

*Proof of Proposition 3:* First, consider traders' expected utility. The expected utility of a trader at date 0 reduces to:

$$egin{aligned} EU_{a1} &= rac{1}{2} Eig[ig( ilde{v} - ilde{P}_2ig) ilde{d}_{a2}ig] - rac{1}{2} Eig( ilde{P}_2 ilde{\eta}ig] + Eig( ilde{P}_2ig]D_1 - P_1(D_1 - Q_a) \ &= \Psi_1 + \Psi_2, \end{aligned}$$

where

$$\Psi_1 = -\frac{1}{2} \left( \lambda \eta - \frac{1}{4\lambda} \frac{\delta \varepsilon}{(\delta + \varepsilon)} \right),$$

and

$$\Psi_2 = \lambda r^2 Q_0^2 ([\lambda + r][\lambda + 5r])^{-2} (\lambda^2 + 5\lambda r + 10r^2) + P_1 Q_a.$$

From the *Proof of Proposition 2*, we already know that the first expression in the expression for  $EU_{a1}$  above,  $\Psi_1$ , decreases as  $\varepsilon$  increases. Therefore, it only remains to deal with the second expression,  $\Psi_2$ . The derivative of  $\Psi_2$  with respect to  $\varepsilon$  is:

$$(\partial \Psi_2 / \partial \varepsilon) = -r^2 Q_0^2 ([\lambda + r][\lambda + 5r])^{-3}$$
$$(\lambda^4 + 4\lambda^3 r + 15\lambda^2 r^2 + 10\lambda r^3 - 50r^4) \lambda_{\varepsilon} + (\partial P_1 / \partial \varepsilon) Q_a,$$

where

$$(\partial P_1/\partial \varepsilon) = -\frac{1}{2}Q_0(\lambda + r)^{-3}(\lambda + 5r)^{-2}(7\lambda^3r^2 + 23\lambda^2r^3 + 35\lambda r^4 + 35r^5)\lambda_{\varepsilon}.$$

Note that statements about the degree of information asymmetry are equivalent to statements about the relation between  $\lambda$  and r. This is because the exogenous parameters  $\varepsilon$  and  $\delta$  are measures of the degree of information asymmetry that exists between the informed trader and the market: as  $\varepsilon$  or  $\delta$  increases, information asymmetry increases. From the equilibrium condition governing  $\lambda$ ,  $\lambda(4\lambda^2\eta[\delta+\varepsilon]-\delta\varepsilon)=r(4\lambda^2\eta[\delta+\varepsilon]+\delta\varepsilon)$ , which implies that  $\lambda\to r$  as either  $\varepsilon$  or  $\delta\to 0$ . But since  $\lambda_\varepsilon>0$  from Corollary 3 (and similarly,  $\lambda_\delta>0$  since in the equilibrium condition governing  $\lambda$ ,  $\varepsilon$  and  $\delta$  assume identical roles),  $\lambda$  becomes large relative to r as either  $\varepsilon$  or  $\delta$  increases. This means statements about illiquidity being large relative to aggregate risk aversion (i.e.,  $\lambda$  large relative to r) are equivalent to assuming a large degree of information asymmetry (i.e.,  $\varepsilon$  and/or  $\delta$  large in absolute value). Therefore, two sufficient conditions that imply  $(\partial\Psi_2/\partial\varepsilon)<0$  are:  $\lambda$  is large relative to r (e.g.,  $\lambda>2r$ ) for any arbitrary  $Q_a\geq 0$ ; or  $Q_a\geq (D_1/5)=(2r/5[\lambda+5r])Q_0$  for any arbitrary  $\lambda>r$ .

Finally, consider a market maker's expected utility. At date 0, each market maker's expected utility is given by

$$\begin{split} EU_{i1} &= -\exp\Bigl(-\rho\Bigl[\,mQ_1 + \tfrac{1}{2}r\bigl\{E\bigl[\,\tilde{d}_{T2}\bigr]\bigr\}^2 - \tfrac{1}{2}rQ_1^2 - P_1Q_1\Bigr]\Bigr) \\ &= -\exp\Bigl(-\rho\Bigl[\,\tfrac{1}{2}\lambda rQ_0^2\bigl\{\bigl(\lambda + r\bigr)\bigl(\lambda + 5r\bigr)\bigr\}^{-2}\bigl\{\lambda^3 + 5\lambda^2 r + 9\lambda r^2 + r^3\bigr\}\Bigr]\Bigr), \end{split}$$

making the appropriate substitutions. Differentiating this expression with respect to  $\varepsilon$ , and holding the number of market makers constant, yields

$$\lambda_{\varepsilon} \Big( \frac{1}{2} r Q_0^2 \Big[ \{ \lambda + r \} \{ \lambda + 5r \} \Big]^{-3} \Big[ 7 \lambda^4 r + 32 \lambda^3 r^2 + 72 \lambda^2 r^3 + 84 \lambda r^4 + 5r^5 \Big] \Big) \\ \exp \Big( -\rho \Big[ \frac{1}{2} \lambda r Q_0^2 \{ (\lambda + r)(\lambda + 5r) \}^{-2} \{ \lambda^3 + 5 \lambda^2 r + 9 \lambda r^2 + r^3 \} \Big] \Big) > 0.$$

Thus, each market maker's expected utility decreases as more information is disclosed. Q.E.D.

*Proof of Proposition 4:* In this result we determine  $(N_{\varepsilon}/N)$ ,  $\lambda_{\varepsilon}$ , and  $r_{\varepsilon}$ , where throughout this discussion  $r_{\varepsilon} = -r(N_{\varepsilon}/N)$ . At date 0, each market

maker's expected utility is given by

$$\begin{split} EU_{i1} &= -\exp\left(-\rho \Big[\, mQ_1 + \frac{1}{2}r \big\{ E \big[\, \tilde{d}_{T2} \big] \big\}^2 - \frac{1}{2}rQ_1^2 - P_1Q_1 \big]\right) \\ &= -\exp\left(-\rho \Big[\frac{1}{2}\lambda rQ_0^2 \big\{ (\lambda + r)(\lambda + 5r) \big\}^{-2} \big\{ \lambda^3 + 5\lambda^2 r + 9\lambda r^2 + r^3 \big\} \big]\right), \end{split}$$

making the appropriate substitutions. Holding  $EU_{i1}$  fixed as  $\varepsilon$  changes is equivalent to requiring that

$$\begin{split} &- \big(N_{\varepsilon}/N\big) \Big(\frac{1}{2} \lambda r Q_0^2 \big[ \{\lambda + r\} \{\lambda + 5r\} \big]^{-2} \big[ \lambda^3 + 5\lambda^2 r + 9\lambda r^2 + r^3 \big] \Big) \\ &- r_{\varepsilon} \Big(\frac{1}{2} \lambda^2 Q_0^2 \big[ \{\lambda + r\} \{\lambda + 5r\} \big]^{-3} \big[ \lambda^4 + 4\lambda^3 r + 12\lambda^2 r^2 + 8\lambda r^3 - 33r^4 \big] \Big) \\ &+ \lambda_{\varepsilon} \Big(\frac{1}{2} r Q_0^2 \big[ \{\lambda + r\} \{\lambda + 5r\} \big]^{-3} \\ & \big[ 7\lambda^4 r + 32\lambda^3 r^2 + 72\lambda^2 r^3 + 84\lambda r^4 + 5r^5 \big] \Big) = 0. \end{split}$$

Solving this expression for  $(N_{\epsilon}/N)$  yields

$$\frac{N_{\varepsilon}}{N} = \frac{7\lambda^4 r + 32\lambda^3 r^2 + 72\lambda^2 r^3 + 84\lambda r^4 + 5r^5}{2\lambda^5 + 15\lambda^4 r + 56\lambda^3 r^2 + 88\lambda^2 r^3 + 18\lambda r^4 + 5\lambda r^5} \frac{\lambda_{\varepsilon}}{\lambda}.$$
 (3)

Next, consider the relation that governs  $\lambda$ ,  $\lambda(4\lambda^2\eta[\delta+\varepsilon]-\delta\varepsilon)=r(4\lambda^2\eta[\delta+\varepsilon]+\delta\varepsilon)$ . Differentiating this expression with respect to  $\varepsilon$  and allowing N to be endogenous yields

$$\lambda_{\varepsilon} = \frac{\left(\delta^{2}/\delta + \varepsilon\right)\lambda(r+\lambda) - r\lambda\left(4\lambda^{2}\eta[\delta + \varepsilon] + \delta\varepsilon\right)\left(N_{\varepsilon}/N\right)}{2(\lambda + r)\delta\varepsilon + r\left(1 + \left[z_{1}/z_{2}\right]\right)\left(4\lambda^{2}\eta[\delta + \varepsilon] + \delta\varepsilon\right)}.$$
 (4)

Combining equation (4) with equation (3) above yields

$$\lambda_{\varepsilon} = \frac{\left(\delta^{2}/\delta + \varepsilon\right)\lambda(\lambda + r)}{2(\lambda + r)\delta\varepsilon + r\left(1 + \left[z_{1}/z_{2}\right]\right)\left(4\lambda^{2}\eta[\delta + \varepsilon] + \delta\varepsilon\right)},\tag{5}$$

where  $z_1 = 7\lambda^4 r + 32\lambda^3 r^2 + 72\lambda^2 r^3 + 84\lambda r^4 + 5r^5$ ,  $z_2 = 2\lambda^5 + 15\lambda^4 r + 56\lambda^3 r^2 + 88\lambda^2 r^3 + 18\lambda r^4 + 5\lambda r^5$ . Note that this last expression implies that  $(N_{\varepsilon}/N) > 0$  and  $r_{\varepsilon} < 0$ . In particular,

$$r_{\varepsilon} = -\frac{\left(\delta^{2}/\delta + \varepsilon\right)r(\lambda + r)\left(z_{1}/z_{2}\right)}{2\delta\varepsilon(\lambda + r) + r\left(1 + \left[z_{1}/z_{2}\right]\right)\left(4\lambda^{2}\eta\left[\delta + \varepsilon\right] + \delta\varepsilon\right)}.$$
 (6)

Q.E.D.

Proof of Proposition 5: Taking the derivative of  $P_1$  with respect to  $\varepsilon$  yields

$$(\partial P_1/\partial \varepsilon) = -\frac{1}{2}Q_0(\lambda + r)^{-3}(\lambda + 5r)^{-2}(7\lambda^3r^2 + 23\lambda^2r^3 + 35\lambda r^4 + 35r^5)\lambda_{\varepsilon}$$

$$-\frac{1}{2}Q_0(\lambda + r)^{-3}(\lambda + 5r)^{-2}(2\lambda^5 + 12\lambda^4r + 36\lambda^3r^2 + 42\lambda^2r^3)r_{\varepsilon}.$$

Substituting the expressions for  $\lambda_{\varepsilon}$  and  $r_{\varepsilon}$  given in (5) and (6), respectively, (see the Proof of Proposition 4), some tedious calculations indicate that  $(\partial P_1/\partial \varepsilon)$  is negative whenever  $\lambda$  is large relative to r (e.g.,  $\lambda > 4r$ ) and positive when  $\lambda$  is small relative to r (e.g.,  $r < \lambda < 3r$ ). Note that here, as well as in the case in which the number of market makers is fixed (see the discussion in the Proof of Proposition 3), statements about the degree of information asymmetry are equivalent to statements about the relation between  $\lambda$  and r. From the equilibrium condition governing  $\lambda$ ,  $\lambda \rightarrow r$  as either  $\varepsilon$  or  $\delta \rightarrow 0$ . But since  $\lambda_{\varepsilon} > 0$  and  $r_{\varepsilon} < 0$  from Proposition 4 (and similarly,  $\lambda_{\delta} > 0$  and  $r_{\delta} < 0$  since in the equilibrium condition governing  $\lambda$ ,  $\varepsilon$  and  $\delta$  assume identical roles),  $\lambda$  becomes large relative to r as either  $\varepsilon$  or  $\delta$  increases. This means that results concerning  $\lambda$  large relative to r are equivalent to results concerning a large degree of information asymmetry (i.e.,  $\varepsilon$  and/or  $\delta$  large in absolute value). Q.E.D.

Proof of Proposition 6: The expected utility of a trader at date 0 reduces to:

$$\begin{split} EU_{a1} &= \frac{1}{2} E \big[ \big( \tilde{v} - \tilde{P}_2 \big) \tilde{d}_{a2} \big] - \frac{1}{2} E \big[ \tilde{P}_2 \tilde{\eta} \big] + E \big[ \tilde{P}_2 \big] D_1 - P_1 (D_1 - Q_a) \\ &= \Psi_1 + \Psi_2, \end{split}$$

where the expressions for  $\Psi_1$  and  $\Psi_2$  are given in the *Proof of Proposition 3*. Essentially, we show below that  $\Psi_1$  increases, while  $\Psi_2$  decreases, as  $\varepsilon$  increases (less disclosure). Therefore, whether  $EU_{a1}$  increases or decreases depends on whether the increase in  $\Psi_1$  offsets the decline in  $\Psi_2$ . But this, in turn, is influenced by the (exogenous) parameter of total quantity of the risky asset  $Q_0$ ; as  $Q_0$  increases it gives more effect to the role of  $\Psi_2$  and less to that of  $\Psi_1$  ( $\Psi_1$  does not depend on  $Q_0$ ). Furthermore, a sufficiently large (absolute) quantity of  $Q_0$  can offset the role of  $\Psi_1$  (for any fixed, finite parameters  $\delta$ ,  $\varepsilon$ ,  $\eta$ , etc.), thereby ensuring that traders' expected utilities increase with more disclosure ( $\varepsilon$  decreases). To substantiate some of our claims, consider, first,  $\Psi_1$ . Here, where the number of market makers is endogenous,

$$\begin{split} \frac{\partial \Psi_1}{\partial \varepsilon} &= -\frac{1}{8\lambda^2} \left( \left[ 4\lambda^2 \eta + \frac{\delta \varepsilon}{\delta + \varepsilon} \right] \lambda_{\varepsilon} - \left[ \frac{\delta}{\delta + \varepsilon} \right]^2 \lambda \right), \\ &= -\frac{1}{8\lambda} \frac{\delta^2}{\left(\delta + \varepsilon\right)^2} \left( \frac{(\lambda + r)(4\lambda^2 \eta [\delta + \varepsilon] + \delta \varepsilon)}{2(\lambda + r)\delta \varepsilon + r(1 + [z_1/z_2](4\lambda^2 \eta [\delta + \varepsilon] + \delta \varepsilon)} - 1 \right), \end{split}$$

where  $\lambda_{\varepsilon}$  is as defined in equation (A3). This expression is positive. To see this, note that:

$$\begin{split} &(\lambda+r)\big(4\lambda^2\eta\big[\delta+\varepsilon\big]+\delta\varepsilon\big)-\big(\lambda+r)2\delta\varepsilon-r\big(1+\big[\,z_1/z_2\big]\big)\big(4\lambda^2\eta\big[\delta+\varepsilon\big]+\delta\varepsilon\big)\\ &=\big(\lambda+r\big)\big(4\lambda^2\eta\big[\delta+\varepsilon\big]-\delta\varepsilon\big)-r\big(1+\big[\,z_1/z_2\big]\big)\big(4\lambda^2\eta\big[\delta+\varepsilon\big]+\delta\varepsilon\big)\\ &=\big(\big[\,\lambda+r\big]\big[\,r/\lambda\big]-r\big[1+\big\{z_1/z_2\big\}\big]\big)\big(4\lambda^2\eta\big[\delta+\varepsilon\big]+\delta\varepsilon\big)\\ &=r\big(\big[\,r/\lambda\big]-\big[\,z_1/z_2\big]\big)\big(4\lambda^2\eta\big[\delta+\varepsilon\big]+\delta\varepsilon\big)<0, \end{split}$$

from the definitions of  $z_1$  and  $z_2$  (see the *Proof of Proposition 4*). Thus,  $(\partial \Psi_1 / \partial \varepsilon) > 0$ . Consider, now, the behavior of  $\Psi_2$  as  $\varepsilon$  changes:

$$\begin{split} \frac{\partial \Psi_2}{\partial \varepsilon} &= -r^2 Q_0^2 \big( \big[ \lambda + r \big] \big[ \lambda + 5r \big] \big)^{-3} \big( \lambda^4 + 4\lambda^3 r + 15\lambda^2 r^2 + 10\lambda r^3 - 50r^4 \big) \lambda_{\varepsilon} \\ &+ \lambda^2 r Q_0^2 \big( \big[ \lambda + r \big] \big[ \lambda + 5r \big] \big)^{-3} \big( 2\lambda^3 + 15\lambda^2 + 10\lambda r^2 + 95r^3 \big) r_{\varepsilon} + \big( \partial P_1 / \partial \varepsilon \big) Q_a, \end{split}$$

where

$$\begin{split} \left( \partial P_1 / \partial \varepsilon \right) &= - \tfrac{1}{2} Q_0 \big( \lambda + r \big)^{-3} \big( \lambda + 5r \big)^{-2} \big( 7 \lambda^3 r^2 + 23 \lambda^2 r^3 + 35 \lambda r^4 + 35 r^5 \big) \lambda_{\varepsilon} \\ &- \tfrac{1}{2} Q_0 \big( \lambda + r \big)^{-3} \big( \lambda + 5r \big)^{-2} \big( 2 \lambda^5 + 12 \lambda^4 r + 36 \lambda^3 r^2 + 42 \lambda^2 r^3 \big) r_{\varepsilon}. \end{split}$$

We claim that  $(\partial \Psi_2/\partial \varepsilon) < 0$  for all  $\lambda$ , r, and  $Q_a$ . To see this, consider two situations involving the choice of  $Q_a$  for any given  $\lambda$ , r pair. First, suppose there exists a  $\lambda$ , r pair such that  $(\partial P_1/\partial \varepsilon) < 0$ ; then the highest value  $(\partial \Psi_2/\partial \varepsilon)$  could assume would be at  $Q_a = 0$ . But at  $Q_a = 0$ , one can show that  $(\partial \Psi_2/\partial \varepsilon) < 0$  for any choice of  $\lambda$  and r. Alternatively, suppose there exists a  $\lambda$ , r pair such that  $(\partial P_1/\partial \varepsilon) > 0$  (which can happen from Proposition 4); then the highest value  $(\partial \Psi_2/\partial \varepsilon)$  could assume would be at  $Q_a = Q_0$ . But at  $Q_a = Q_0$ , one can show that  $(\partial \Psi_2/\partial \varepsilon) < 0$  for any choice of  $\lambda$  and r. This proves the claim. Finally, note that  $\Psi_2$  is weighted by  $Q_0$ , whereas  $\Psi_1$  is not. Thus, a sufficiently large  $Q_0$  implies that the decline in  $\Psi_2$  dominates the increase in  $\Psi_1$ . Q.E.D.

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