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Essays

Financial econometrics: Past developments and future challenges

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Abstract

The field of financial econometrics has had a glamorous run during the life span of the *Journal of Econometrics*. This note provides a selective summary of the most important developments in the field over the past two decades, notably ARCH and GMM, along with a discussion of promising avenues for future research. © 2001 Elsevier Science S.A. All rights reserved.

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1. Introduction

The field of financial econometrics arguably constitutes one of the most active areas of research in econometrics today. It is rare that an issue of the *Journal of Econometrics* does not contain even a couple of articles within the area, and several recent *Annals* issues have been devoted to specific topics in the field.[1]

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[1] A cursory look at the traditional econometrics journals – *Econometrica*, *Econometric Theory*, *Journal of Applied Econometrics*, *Journal of Business and Economic Statistics*, *Journal of Econometrics*, and *Review of Economics and Statistics* – severely underestimates the scope of the field, as many of the important econometric advances are now also published in the premier finance journals – the *Journal of Finance*, the *Journal of Financial Economics*, and the *Review of Financial Studies* – as well as a host of other empirically oriented finance journals.

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This has not always been the case. Until 20 years ago, most empirical finance papers relied on – by statistical and econometric standards – fairly simplistic data analytical tools. Meanwhile, the rapid acceleration in computer power, the increased availability of high quality data for a range of financial instruments, along with the development and adaptation of more sophisticated econometric techniques have dramatically changed the field. Importantly, these advances have not been restricted to academia, but have profoundly influenced the modern day practice of finance and investment management in particular, and this close affiliation has further helped to fuel many new developments.

Needless to say, any attempt at a comprehensive overview of such a rapidly growing field – let alone one restricted to a few pages – is doomed at the outset (Campbell et al. (1997) provide a broad textbook treatment, while Bollerslev and Hodrick (1995) and Pagan (1996) offer more eclectic surveys). Instead, let me invoke the usual disclaimer, that this note provides a highly selective summary of what I personally consider to be the two most influential developments over the life span of the *Journal*, along with a discussion of some promising avenues for future research closely related to my own interests.

2. Past developments

In chronicling the development in financial econometrics over the past two decades time-varying volatility models, in the form of ARCH and stochastic volatility formulations, and robust methods-of-moments-based estimation procedures, such as GMM, stand out as milestones.[2] Much subsequent work in the field have clearly been influenced by both of these innovations. Nonetheless, at the risk of masking the close connection between the different themes, I will structure my discussion of past and future research around these two, originally independent, conceptions.

2.1. Time-varying volatility

Uncertainty plays a central role in financial economics. Although volatility clustering, i.e., the tendency for large (small) price changes to be followed by other large (small) price changes, was documented as early as Mandelbrot (1963)

[2] Coincidently, the paper on the ARCH model by Engle (1982) and the paper on GMM by Hansen (1982) were published in the same issue of *Econometrica*. Moreover, neither paper offered any application directly related to finance – Engle's paper presents an ARCH model for quarterly UK inflation, while Hansen's paper is purely theoretical. Engle et al. (1987) and Hansen and Singleton (1982) in subsequent issues of *Econometrica* did, of course, utilize the ARCH-M model and GMM procedure in estimating time-varying term premia and a consumption based asset pricing model, respectively.

and Fama (1965), it was not until Engle (1982) and the advent of the ARCH and GARCH (Bollerslev, 1986) models that financial econometricians started to seriously model this phenomenon. Since then a burgeoning empirical literature has developed, and we now have a much better understanding of the salient distributional features of daily and lower frequency speculative returns (surveys include Bollerslev et al., 1992, 1994; Engle, 1995; Ghysels et al., 1996; Shephard, 1996). Noteworthy empirical findings include strongly persistent volatility dependencies (Bollerslev and Engle, 1993), spillovers and linkages across different assets and markets (Engle et al., 1990), asymmetries or leverage effects in both volatilities and correlations (Nelson, 1991), to mention but a few. The past decade has also produced important theoretical results regarding the statistical properties of the most popular univariate ARCH and stochastic volatility models (Drost and Nijman, 1993; Nelson, 1990), along with the development of MLE, QMLE, Bayesian, and adaptive estimation procedures (e.g., Geweke, 1989; Bollerslev and Wooldridge, 1992). We now know that even when misspecified, ARCH models may serve as consistent filters and forecasters for the continuous-time stochastic volatility diffusions often employed in the theoretical asset pricing literature (Nelson, 1992). Not withstanding these developments, several challenging questions related to the proper modeling of ultra high-frequency data, longer-run dependencies, and large dimensional systems remain.

2.2. Flexible estimation procedures

The notion of instrumental variables estimation in econometrics dates back at least to Sargan (1958). However, the GMM procedure in Hansen (1982) offers the first distribution-free estimation framework for the type of multi-period non-linear moment conditions that often arise from partial equilibrium restrictions and moreover, readily accommodates the pronounced volatility clustering documented in the extant ARCH literature. It is not an exaggeration to say that, along with the stochastic discount factor approach (Hansen and Richard, 1987), GMM has served as a cornerstone in the empirical asset pricing literature over the past two decades (for a more comprehensive discussion of the basic GMM approach see, e.g., Hall, 1992; Hamilton, 1994). From an econometric point of view, noteworthy subsequent contributions include algorithms for practical estimation of the weighting matrix (Gallant, 1987; Newey and West, 1987; Andrews, 1991), guidance on the optimal choice of instruments (Hansen, 1985), along with extensions of the theory to situations in which the moments may have to be computed by numerical simulation techniques (Pakes and Pollard, 1989; Duffie and Singleton, 1993). The indirect inference procedure proposed by Gourieroux et al. (1993) and the efficient methods of moments (EMM) procedure developed by Gallant and Tauchen (1996, 1998) also rely on the basic GMM framework for the estimation of general model structures, by utilizing the parameter estimates or the score function from an auxiliary model to define a set of operational moment conditions (see, e.g., Tauchen, 1997). Such estimation procedures are likely to play a pivotal role in some of the future research directions discussed below.

3. Interesting current directions and challenges for future research

3.1. Time-varying volatility: High-frequency data, long-memory, heavy tails, and large dimensional systems

High-frequency, or tick-by-tick, data have recently become available for a range of different financial instruments and markets (see, e.g., Goodhart and O'Hara, 1997). While such data obviously contain very useful information about a host of market microstructure issues, it has become clear this data may also hold important information about longer-run interdaily phenomena. Meanwhile, the mere size of the data bases, often involving millions of observations, presents a number of practical problems related to data verification, storage, and numerical manipulations.

More important, many of the inference procedures routinely used in the literature for analyzing daily or lower frequency observations are ill suited for modeling high-frequency data (e.g., Hasbrouck, 1996; Engle, 2000). Specifically, the ARCH and stochastic volatility models discussed above are based on the assumption of equally distant discretely sampled observations. However, with high-frequency financial time series the distance between observations vary importantly through time – sometimes the market is very active and prices change very rapidly while at other times there may be large gaps between successive observations. This feature naturally suggests the use of marked point processes, or continuous time methods in which the sampling frequency is determined by some notion of time deformation, as in the mixture-of-distributions hypothesis originally advocated by Clark (1973) and Tauchen and Pitts (1983), and extended by Andersen (1996) among others. In fact, the autoregressive conditional duration (ACD) model in Engle and Russell (1998) is explicitly motivated by the objective of modeling the times between high-frequency observations. Of course, other hazard models could be employed. Additionally, financial markets exhibit strong periodic dependencies across the trading day – typically trading volume and volatility are highest at the open and toward the close of the day – and a failure to account for this may seriously distort the inference (see, e.g., Andersen and Bollerslev, 1997b; Andersen et al., 2000b). Also, the actual prices and bid-ask spreads tend to cluster at discrete support points. This is largely immaterial when analyzing interdaily time series, but the continuous unbounded support employed in traditional time series models obviously results in a mis-specified high-frequency model. The ordered probit model in Hausman et al. (1992), the multinomial model in Russell and Engle (1998), the

BIN model in Rydberg and Shephard (1998), and the non-linear state space model in Hasbrouck (1999), all afford ways in which to deal with discreteness. Not withstanding much recent progress, the formulation of a workable dynamic time series model which readily accommodates all of the high-frequency data features, yet survives under temporal aggregation, remains elusive.

Alternatively, it may be that for many interesting questions, simple summary measures constructed from the high-frequency data may efficiently aggregate the information, possibly avoiding the need for specialized modeling and/or inference procedures. The model-free realized, or integrated, volatility measures defined by the summation of the high-frequency squared returns, advocated in a series of recent papers by Andersen et al. (1999, 2000c) and Bollerslev and Wright (2000), provide a case-in-point. The intraday high-low range is another example.

There is generally no evidence for long-memory dependencies in the mean of asset returns (Lo, 1991). However, several recent studies have argued that the apparent long-run dependence in financial market volatility may be conveniently modeled by long-memory, or fractionally integrated, processes (e.g., Ding et al., 1993; Baillie et al., 1996). These empirical findings have in turn stimulated a renewed interest in the development and refinement of inference procedures for long-memory processes (see, e.g., Robinson, 1995; Robinson and Henry, 1999; Phillips, 1999 and the references therein). This research program has also begun to clarify the close connection between the notions of long-memory volatility dependencies employed in the empirical literature and alternative models with multiple volatility components and/or structural breaks (see, for instance, Andersen and Bollerslev, 1997a; Chen et al., 1999; Diebold and Inoue, 1999; Engle and Lee, 1999; Gallant et al., 1999; Park, 1999). Nonetheless, it is certainly possible that from a pragmatic perspective, the assumption of long-memory may yield the most accurate empirical out-of-sample volatility forecasts (Bollerslev and Mikkelsen, 1999).

Most practical risk management decisions inherently rely on such out-of-sample volatility forecasts – typically for a large number of assets. Unfortunately, the multivariate models proposed in the existing ARCH and stochastic volatility literature do not readily lend themselves to the analysis of large dimensional systems. Moreover, the imposition of long-memory-type volatility dependencies, possibly allowing for fractional cointegration in the variance (Robinson and Marinucci, 1998), presents other problems. Multivariate latent factor structures (Diebold and Nerlove, 1989), and/or Bayesian procedures (Chib et al., 1999) may prove particularly convenient in addressing these questions. Stock and Watson (1999), in a different context, also offer some potentially useful ideas for future work along these lines. Alternatively, for actively traded assets, the realized volatility measures discussed above could be modeled directly using standard multivariate time series techniques (Andersen et al., 2000d).

Oftentimes it is the tail probabilities – in particular the probability of a big market correction – that is of the greatest practical concern when monitoring financial market risks. Most of the existing statistical work on tail index estimation has been based on i.i.d. assumptions (see, e.g., Embrechts et al., 1997). Explicitly modeling tail probability clustering constitute another interesting area for future research (Engle and Manganelli, 1999; Chernozhukov, 2000, discuss the use of quantile regression techniques in this context). The scaling laws for the distribution of the absolute returns implied by the long-memory volatility dependencies may be helpful in the construction of more accurate tail probability predictions by appropriately extrapolating from intraday high-frequency price movements (Mandelbrot et al., 1997; Müller et al., 1998; Starica, 1999 offer some intriguing suggestions along these lines).

3.2. Flexible estimation procedures: Continuous-time models and risk-neutral pricing kernels

Continuous time methods and no-arbitrage arguments figure prominently in the theoretical asset pricing literature. Meanwhile, some of the most influential contributions to date have been derived under very restrictive, and arguably unrealistic, assumptions about the process for the underlying state variable(s), e.g., the celebrated Black–Scholes option pricing formula and the one-factor CIR model for the term structure of interest rates. In order to rectify this problem, a number of recent studies have proposed more realistic continuous time processes, explicitly allowing for time varying volatility in the state variable(s). Leading examples include the Hull and White (1987) and Heston (1993) stochastic volatility option pricing formula and the exponential-affine class of term structure models in Duffie and Kan (1996) and Dai and Singleton (2000).

Although much recent progress has been made, the prevailing estimation methods for continuous time models remain highly specialized and difficult to implement for all but the specially trained econometrician (an incomplete list of alternative estimation procedures includes Lo, 1988; Hansen and Scheinkman, 1995; Aït-Sahalia, 1996, 1998; Gallant and Tauchen, 1996; Stanton, 1997; Bandi and Phillips, 1999; Chacko and Viceira, 1999; Singleton, 1999). As such, the development of new simpler estimation procedures is likely to continue to play an active role in the literature over the coming years (see, e.g., Bollerslev and Zhou, 2000; Alizadeh et al., 1999 for some recent suggestions using realized and intraday high–low volatility measures, respectively). More important, this research program will hopefully also result in new more realistic continuous time model structures that are capable of explaining some of the high-frequency data features discussed above, yet still permit the use of standard theoretical no-arbitrage-based hedging and pricing arguments. In this regard, the added complication provided by non-continuous jump components may turn out to be important for many markets (recent evidence along these lines includes Bates, 1996; Johannes et al., 1999; Andersen et al., 2000a). The use of Levy driving process, as in Barndorff-Nielson and Shephard (1999), may prove especially convenient for accommodating such jumps.

Recent research on the link between the probability distributions of actual asset prices and the corresponding risk-neutral probability distributions implied by derivatives prices has just started to deliver important new insights on the way in which the market prices risk (see, e.g., Chernov and Ghysels, 2000; Duffie et al., 2000; Pan, 1999). As discussed in Aït-Sahalia and Lo (2000), by utilizing the risk-neutral distributions as opposed to the actual distributions when forecasting and evaluating future asset payoffs, this line of research holds the promise of better risk management practices. However, much remains to be done. For instance, the existence of multiple simultaneous pricing errors, as implicit in Black–Scholes style implied volatility smiles and smirks, is difficult to reconcile within the complete market's framework (see Renault, 1997 and Clement et al., 2000 for further discussion along these lines). A similar problem arises in the empirical analysis of no-arbitrage-based models for the term structure of interest rates. This raises important econometric, as well as theoretical questions concerning the proper treatment of the panel data structure in options and interest rate data.

4. Conclusion

The field of financial econometrics has had a glamorous run during the life span of the *Journal of Econometrics*. Out-of-sample forecasting is always marred with difficulties, and simply extrapolating the future vitality of the field based on past observations does not necessarily result in optimal predictions. However, the multitude of interesting and challenging research questions, some of which are discussed above, set the stage for an equally exciting future.

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