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The Impact of Liquidity Risk: A Fresh Look*

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ABSTRACT

This paper takes a fresh look at the importance of liquidity risk using a comprehensive liquidity measure, weighted spread, in a Value-at-Risk (VaR) framework. The weighted spread measure extracts liquidity costs by order size from the limit order book. Using a unique, representative data set of 160 German stocks over 5.5 years, we show that liquidity risk is an important risk component. Actually, liquidity risk is increasing the total price risk by over 25%, even at 10-day horizons and for liquid blue chip stocks and especially in larger, yet realistic order sizes beyond €1 million. When correcting for liquidity risk, it is commonly assumed that liquidity risk can be simply added to price risk. Our empirical results show that this is not correct, as the correlation between liquidity and price is non-perfect and total risk is thus overestimated.

I. INTRODUCTION

Liquidity as the ease of trading an asset has lately received considerable attention in the academic world and in practice. From a risk management perspective, liquidity risk is the potential loss due to the time-varying cost of trading. Many risk management systems assume that a position can be bought or sold without significant cost if the liquidation horizon is long enough.

Several authors have exemplary evidence that this myth might not hold true in intraday settings. Francois-Heude and Van Wynendaele (2001) find a 2–21% contribution of intraday liquidity impact in one stock over 4 months. Giot and Grammig (2005) show that 30-minute intraday liquidity-adjusted Value-at-Risk (VaR) is 11–30% for three large stocks over 3 months. In a 7-month sample of 60 stocks, Angelidis and Benos (2006) estimate that liquidity risk constitutes 11% of the total intraday VaR in low capitalization stocks. Lei and Lai (2007) reveal a

* We would like to thank Deutsche Börse AG for providing the 'Xetra Liquidity Measure (XLM),' weighted spread for the major German stock indices. We are grateful to Tobias Berg, the participants of the Munich Finance Seminars and the German Finance Association Annual Meeting 2009 for helpful comments. The usual caveats apply. Comments or questions are highly welcome. An earlier version of this paper was available under the title 'Why and How to Integrate Liquidity Risk into a VaR-Framework' on October 30, 2008.

30% total intraday risk contribution by liquidity in 41 small-price stocks over 12 months.

Further exemplary results show that liquidity risk might also be non-significant in more illiquid markets. At daily horizon, Bangia et al. (1999) find an underestimation of the total VaR by 25–30% in emerging market currencies when looking at bid–ask spread liquidity. Le Saout (2002) estimates for 41 stocks over 28 months that the bid–ask spread liquidity component can represent 50% of the total daily risk for illiquid stocks.

However, is liquidity risk also significant when looking at standard 10-day horizons and more perfect, liquid markets? Price risk increases the longer the forecasting horizon; liquidity is a one-time cost independent of the horizon. Thus, the liquidity risk component in total risk will be smaller for longer horizons. As a consequence, the liquidity risk might be negligible in liquid stock markets at standard horizons and neglect by risk frameworks could be justified. In addition, can the assumption of insignificant liquidity risk be refuted on a more representative basis beyond above small samples?

Above authors use different types of modeling approaches to introduce a liquidity risk adjustment, each justified mainly by the type of data available. What generally remains unjustified is the precise way of adjusting for liquidity. Bangia et al. (1999) and Angelidis and Benos (2006) simply add liquidity risk to price risk, implicitly assuming a perfect correlation between liquidity and price.[1] Berkowitz (2000), Cosandey (2001) and Francois-Heude and Van Wynendaele (2001) assume zero correlation. So which assumption holds true? This has been empirically untested so far.[2]

In this paper, we have a fresh look at those two questions on a representative basis: Is liquidity risk significant in more liquid markets at standard horizons beyond intraday and how should liquidity risk be integrated, does correlation play a role?

Similar to Giot and Grammig (2005), we use weighted spread as the liquidity measure. Weighted spread measures the liquidity cost of a specific order size as the average spread in the limit order book weighted by individual limit order sizes.[3] It generalizes the approach of Bangia et al. (1999) beyond the quoted spread and is a precise price-impact measure when immediately transacting against the limit order book. While Giot and Grammig (2005) look at intraday liquidity risk in a small sample, we analyze horizons beyond intraday on a representative basis.

For our empirical analysis, we use – as far as we know – the most representative sample of daily weighted spread available to academia. We

1 Giot and Gramming circumvent the problem of liquidity–price correlation by modeling *t*-distributed net-returns, i.e. returns net of liquidity costs.

2 Critique brought forward by Francois-Heude and Van Wynendaele (2001), Angelidis and Benos (2006), Loebnitz (2006), Lei and Lai (2007) and Jorion (2007).

3 This measure corresponds to the CRT by Irvine et al. (2000). Similar measures are used in other contexts by Coppejans et al. (2001), Gomber and Schweickert (2002), Gomber et al. (2004) and Domowitz et al. (2005).

analyze a data set of the Xetra Liquidity Measure (XLM) for 160 stocks over 5.5 years. Usual samples are restricted to few stocks over few months, because weighted spread has to be calculated manually from the whole intraday order book, which is highly computational extensive. Our sample is available from Deutsche Börse.

With the help of this sample, we empirically analyze the magnitude of liquidity impact at standard, larger than intraday horizons. We also clarify whether tail correlation between price and liquidity costs is perfect or not.

The remainder of the paper is organized as follows. Section II defines liquidity, the liquidity measure and liquidity risk and discusses the situations when our approach is valid. In Section III, we describe our empirical data set and the empirical results. Section IV summarizes and concludes.

II. THEORETICAL FRAMEWORK AND ASSUMPTIONS

In Section II.A, we first define liquidity from a cost perspective, characterize the situational assumptions in which our framework can be applied and describe our empirical liquidity measure. In Section II.B, we introduce our risk estimation approach as well as a risk decomposition to distill structural insights.

A. Liquidity cost framework

i. Definition of liquidity

To make the assumptions of our analysis explicit, we define liquidity as precise as possible. We use a cost definition of liquidity, which takes a practical, concrete investor's perspective. We define illiquidity as the cost of trading an asset relative to fair value (cp. Dowd 2001, p. 187ff.; Buhl 2004; Amihud and Mendelson 2006). Fair value is assumed to be the mid-point of the bid–ask spread. Extending from Amihud and Mendelson (2006), we distinguish three components of the relative liquidity cost $L_t(q)$ in percent of the mid-price[4] for an order quantity q at time t

$$L_t(q) := T(q) + PI_t(q) + D_t(q) \tag{1}$$

where $T(q)$ are direct trading costs, $PI_t(q)$ is the price impact versus mid-price due to the size of the position and $D_t(q)$ are delay costs if a position cannot be traded immediately.

Direct trading costs $T(q)$ include exchange fees, brokerage commissions and transaction taxes.[5] They will be neglected for further analysis as they are very small and not relevant in a risk setting where time variation is of interest.[6]

4 Mid-price is the mid-point of the bid–ask spread.

5 Also called explicit or deterministic transaction costs, cp. Loebnitz (2006, p. 18f.).

6 On the Xetra system of the Deutsche Börse, for example, institutional traders pay only around 0.5 bp as transaction fee, cp. Deutsche Boerse (2008, p. 6ff.).

Delay costs $D_t(q)$ comprise costs for searching a counter party and the cost imposed on the investor due to the bearing price risk and price-impact risk during the execution delay.[7] We assume that both forced and deliberate delays are not relevant.

As we look at asset positions, which are continuously tradable during crises, forced delay is not relevant. This means that no (or very few) zero trading days occur in crises and the position size is not larger than the market depth. Scanning our data, zero trading days appear to occur mainly in calmer market periods, which, we hypothesize, happens because tumbling market prices attract traders, which in turn ensures continuous trading.

We also assume that a deliberate, strategic delay has no significant benefit, i.e. we assume that positions can be equally good when instantly liquidated against the limit order book. This also neglects liquidation via limit instead of market orders as well as up-floor or over-the-counter trading. So, we neglect any (potential) effect of optimal trading strategies, which balance the increased price risk of delay against the reduced liquidity cost by trading smaller quantities (cp., e.g., Almgren and Chriss 1999, 2000; Almgren 2003; Bertsimas and Lo 1998; and others). In our view, this is a reasonable assumption in four cases. When we adopt the worst-case perspective of impatient traders, a common risk assumption, potential benefits are consciously neglected. Benefits are also non-existent if informational content of our trade is too high. The trader wants to trade immediately on an informational advantage, which would be revealed by trading more slowly or dissolve over time. Adverse informational effects are also possible, i.e. trading more slowly could have price effects because the market assumes informational advantage, which is not present in reality.[8] Immediate liquidation is fair, too, if liquidity prices are efficient and a trader's risk aversion is greater or equal to that of the market.[9] In this case, the marginal gain from lower liquidity costs by delaying a transaction balances the marginal loss due to higher price risk. Finally, optimal trading strategies might not be feasible in times of market stress,[10] because the optimization parameters are not stable or strategic trading is not always possible.

As direct trading costs and delay costs are argued to be insignificant in our analysis, we focus on price impact $PI_t(q)$ only, the difference between the transaction price and the mid-price resulting from imperfectly elastic demand and supply curves. For small volumes this is the bid–ask spread, but for larger volumes the price impact is larger. Thus, if markets are fairly liquid, positions are not too large and we adopt a worst-case perspective, the total liquidity cost

7 Almgren (2003) calls price impact risk 'trading enhanced risk.'

8 Technically expressed as high permanent price impact rendering optimal trading strategies useless.

9 If liquidity costs are too high, liquidity providers will enter with limit orders, because liquidity costs, i.e. their profits, will compensate for the additional risk during the delay until the limit order is executed. If liquidity costs are too low, market orders and withdrawn limit orders will deplete the order book, because nobody is willing to take price risk during delay.

10 A point raised in Jarrow and Protter (2005, p. 9).

can be fairly measured with the price impact from immediate execution ($L(q)=PI(q)$).

ii. The weighted spread liquidity measure

We have obtained our liquidity data from the Xetra system of the Frankfurt Stock Exchange covering the bulk of stock transactions in Germany (cp. Deutsche Boerse 2005). Xetra is an electronic trading platform by Deutsche Börse, which is among the top 10 largest stock exchanges in the world. Trading starts with an opening auction at 09:00 hours, is interrupted by an intraday auction around 13:00 hours and ends with a closing auction finished at 17:00 hours. In between, trading is continuous. An electronic order book collects all limit and market orders from market participants and matches them on price, followed by time priority. The order book is anonymous, but visible to all market participants. However, traders can also submit invisible, 'iceberg' orders to trade large volumina, where traded volume is only revealed up to a certain size and a similar order of equal size will be initiated once the first limit order is transacted. For illiquid stocks, market makers post bid and ask quotes up to a prespecified minimum quotation volume (cp. Deutsche Boerse 2004).

We measure price impact with the XLM. XLM is a weighted spread measure that provides the liquidity cost of a round trip (CRT) of size q compared with its fair value at the mid-price. The Xetra system automatically calculates XLM from the visible and invisible part of the limit order book, i.e. including 'iceberg' orders. Mathematically, XLM is defined as follows. The weighted bid price $b_t(n)$ for selling n number of shares is calculated as

$$b_t(v) = \frac{\sum_i b_{i,t} n_{i,t}}{n} \tag{2}$$

where $b_{i,t}$ and $n_{i,t}$ are the bid prices in € and bid volumes of individual limit orders at time t sorted by price priority. The individual limit order volume adds up to n shares, $\sum_i n_i = n$. The weighted ask price is calculated analogously. XLM is then calculated as the weighted spread in basis points (bp) for predefined order sizes q

$$XLM(q) = \frac{a_t(n) - b_t(n)}{P_{\text{mid}}} 100 \tag{3}$$

where P_{mid} is the mid-price of the quoted (minimum) spread and $q = nP_{\text{mid}}$ is the size of the position measured in euro-mid-price value.

Graphically, XLM is the area between the bid and the ask curve in the limit order book up to the order size q divided by the mid-price value (see Figure 1). XLM calculates the price impact of an order of size q in basis points. It can also be seen as the relative liquidity discount for a round trip of an order of size q.[11] XLM is an *ex-ante* measure, because it calculates the cost from committed liquidity in the order book – including hidden 'iceberg orders' – and neglects any hidden liquidity (cp. Irvine et al. 2000, p. 4).

11 Gomber and Schweickert (2002) provide further theoretical background.

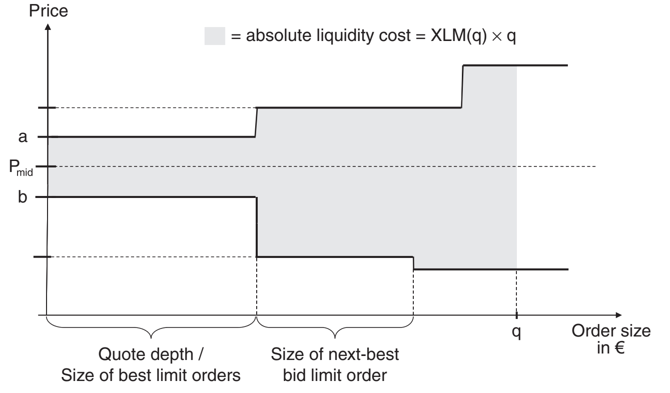

Figure 1 XLM as the area between the limit order curves.
This figure shows a graphical representation of the order book; P_{mid} is the mid-price of the bid–ask spread, a is the ask price, b is the bid price, q is the size of the position in € mid-price value, XLM(q) is the weighted spread measuring *ex-ante* liquidity cost for a round trip of size q.

Liquidity cost $L(q)$ is then estimated from a transaction perspective. As a per-transaction figure has much more practical meaning than a per-round-trip figure, we assume that the order book is symmetrical on average.[12] Therefore, we can calculate the price impact per transaction under the situational assumptions outlined in Section II.A.i as

$$L(q) = PI(q) = \frac{XLM(q)}{2}. \tag{4}$$

In contrast to other price-impact proxies, measure (4) is a precise measure of the *ex-ante*, order size differentiated liquidity cost at and beyond the bid–ask spread depth.[13] However, it is important to note that this liquidity cost measure increases computational complexity because the price-impact curve must be estimated, at least with a liquidity cost vector. In addition, concrete position sizes must be interpolated between vector entries. Nevertheless, additional computations are limited as long as weighted spread is provided by the exchange, like in the case of XLM, and not calculated manually from the intraday order book.

There are important similarities and differences between XLM and the quoted bid–ask spread. The quoted spread is the simplest version of an *ex-ante* liquidity measure, but is valid only up to the quoted depth. XLM is its natural generalization, because it extends beyond best bid–ask prices to the rest of the

12 Liquidity cost estimation could gain further precision if exchanges would provide buy-side- and sell-side-weighted half-spread data.

13 Up to now, it been empirically impossible to distill precise price impact measures for single assets from *ex-post* transaction data (cp. Amihud 2002; Pastor and Stambaugh 2003).

order book. The bid–ask spread is the minimum weighted spread (for small order sizes). However, spread is usually not measured for constant order sizes q, because quoted depth differs between stocks. Spread also has upper bounds regulated by the exchange protocol if there is market maker coverage.[14] This is a first indicator that liquidity-cost dynamics will be different when moving beyond the minimum bid–ask spread.

B. Liquidity risk framework

i. Measurement approach

We want to calculate liquidity risk estimates as precise as possible. Therefore, we use the historical, empirical distribution instead of a parametric approach to estimate percentiles. This approach is possible due to our large sample and has the advantage that we do not have to make any assumption regarding the distribution of liquidity. This is important because liquidity distributions are often far from normal.[15] The development of a suitable parametrization approach is left to future research.

To use percentiles of the historical distribution, we have to rely on the full sample period, because short samples do not have enough observations to finely estimate percentiles. We also deliberately accept that risk might be different at different estimation periods. As a robustness test, we later look into time variation in a parametric framework to test whether those drawbacks have any significant impact (see Section III.D.ii).

Similarly, we measure risk *ex-post* and not *ex-ante*. This avoids any distortion through a specific forecasting method, which is similarly a point left for future development.

ii. Definition of risk measures

Before we turn to defining liquidity risk, we start with the definition of price risk. We use standard risk statistics, against which we measure the impact of liquidity risk.

Price and return are described in the usual framework of

$$P_{\text{mid},t} = P_{\text{mid},t-\Delta t} \exp(r_{t,\Delta t})$$

where P_{mid} is defined as the mid-price $P_{\text{mid},t} = \frac{a_t + b_t}{2}$ with a_t and b_t being the (best) ask- and bid price at time t, respectively. $r_{t,\Delta t}$ is the Δt-period continuous mid-price return at time t, i.e. $r_{t,\Delta t} = \ln(P_{\text{mid},t}/P_{\text{mid},t-\Delta t})$. We adopt a traditional approach from a VaR perspective and define price risk as the relative VaR at the $(1-\alpha)$% confidence level over the horizon Δt

$$VaR^{\alpha,\Delta t}_{\text{price}} = 1 - \exp(r^{\alpha}_{t,\Delta t}) \tag{5}$$

14 On Xetra illiquid stocks, defined by XLM and past volume, are covered by market makers. If the stock is not covered, the bid–ask spread corresponds to the minimum spread in the order book.

15 Cp. Stange and Kaserer (2008) for a detailed discussion of the properties of XLM.

where $r^{\alpha}_{t,\Delta t}$ is the α percentile of Δt-period return distribution. Consequently, VaR_{price} measures the maximum percentage loss over the period Δt with a confidence of $(1-\alpha)\%$.

Analogously, we measure the total risk including the liquidity risk. To calculate the impact of liquidity, we define the Δt-period *net return* in t as the sum of the continuous mid-price return and the liquidity discount converted to a continuous value

$$\text{rnet}_{t,\Delta t}(q) = r_{t,\Delta t} + l_t(q) = \ln\left(P_{\text{mid},t}/P_{\text{mid},t-\Delta t}\right) + \ln(1 - L_t(q)). \tag{6}$$

Note the difference of (6) to net-price returns, i.e. $\ln\left(\left[P_{\text{mid},t}\times(1 - L_t(q))\right]/\left[P_{\text{mid},t-\Delta t} \times (1 - L_{t-\Delta t}(q))\right]\right)$. Using net returns instead of net-price returns, we implicitly assume that the liquidity cost of entering a position has already been properly accounted for. While the quantified difference might not be large, we believe that using net returns is conceptually closer to reality.

Price is then calculated as

$$P_{\text{net},t}(q) = P_{\text{mid},t-\Delta t}\exp(r_{t,\Delta t} + l_t(q)) \tag{7}$$

where $P_{\text{net},t}(q)$ is the achievable transaction price.

The Δt-period *liquidity-adjusted total risk* is then defined in a VaR framework as the empirical α percentile of the net-return distribution.

$$VaR^{\alpha,\Delta t}_{\text{total}}(q) = 1 - \exp(\text{rnet}^{\alpha}_{t,\Delta t}(q)) \tag{8}$$

VaR_{total} is the maximum percentage loss due to mid-price risk and liquidation cost over the period Δt with a confidence of $(1-\alpha)\%$. This specification covers the real dynamics of the net return on a certain stock position. It is practical but also more general than existing approaches in the following ways:

1 We use a more precise liquidity measure than most papers by covering more aspects of liquidity. Specifically, we account for the impact of order size on liquidity. This extends the approach of Bangia et al. (1998, 1999), where liquidity costs of any order size is proxied for with the bid–ask spread. As the spread is valid only for very small order sizes, it is insufficient, which will be demonstrated in the empirical section. The XLM measure is also more precise than the ones used in Berkowitz (2000), Francois-Heude and Van Wynendaele (2001) or Angelidis and Benos (2006), because it directly measures liquidity costs in the limit order book instead of proxying for it.

2 As we take empirical percentiles instead of a parametric method, we avoid any distributional assumption, especially on liquidity cost, such as in Giot and Grammig (2005). Our approach will capture the non-normality of the distribution as well, which is made possible by our large sample size. Figure 2 shows that the empirical net-return distributions can be far from normal.

3 Our approach takes percentiles of the net-return distribution and does not treat price risk and liquidity separately. We look at the dynamics of net returns that combines the mid-price-return dynamics and liquidity cost dynamics. Instead of adding distribution percentiles of liquidity and price risk separately, we acknowledge that liquidity cost and mid-price might not

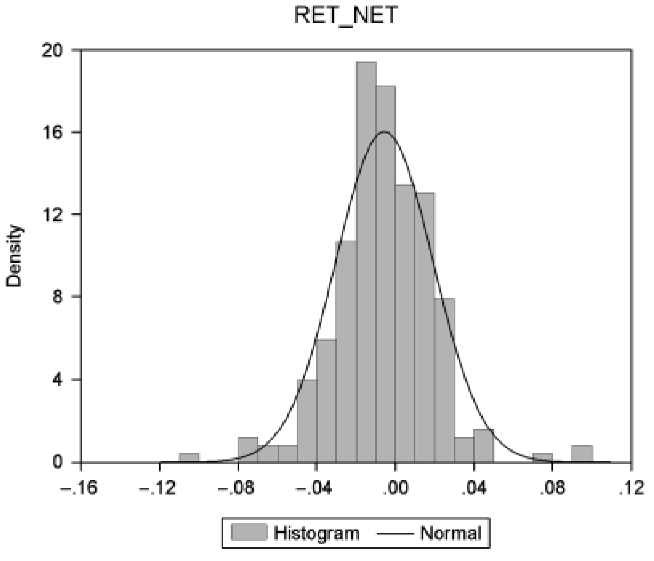

Figure 2 Exemplary net-return histogram and fitted normal distribution for the Comdirect stock in 2007.
This figure shows the exemplary histogram of net-return of the Comdirect stock for trade-size of €100 thsd. and normal distribution based on its mean and variance in 2007.

be perfectly correlated. Although it is possible that large liquidity discounts and low prices coincide, this must not be the case.

iii. Risk decomposition
To uncover the structure of the liquidity impact, we decompose total risk into its components. We define relative liquidity impact $\lambda(q)$ as

$$\lambda(q) = \frac{VaR_{\text{total}}(q) - VaR_{\text{price}}}{VaR_{\text{price}}}. \tag{9}$$

$\lambda(q)$ is the maximum percentage loss due to the liquidity in relation to price risk. It can be interpreted as the error made when ignoring liquidity. It is therefore a measure of the relative significance of liquidity in the risk management context. In addition, it can be used as a scaling factor with which price risk would need to be adjusted in order to correctly account for liquidity. We measure it relative to price risk, because absolute liquidity impact has little meaning by itself for our type of analysis.

In order to uncover the effect of tail correlation between liquidity and price, we define liquidity cost risk as the relative worst liquidity cost

$$VaR^{\alpha}_{\text{liquidity}}(q) = 1 - \exp(l^{\alpha}_{t,\Delta t}(q)) \tag{10}$$

with $l^{\alpha}_{t,\Delta t}$ being the empirical percentile of the continuous liquidity discount. This is the maximum percentage loss due to liquidity cost at a $(1-\alpha)$% confidence level.

We can now apply a further decomposition of total risk and define the correlation factor $\kappa(q)$ as a residual of

$$VaR_{\text{total}}(q) = VaR_{\text{price}} + VaR_{\text{liquidity}}(q) + \kappa(q) \times VaR_{\text{liquidity}}(q). \tag{11}$$

Naturally, this is just a further decomposition of the liquidity impact

$$\lambda(q) = \frac{VaR_{\text{liquidity}}}{VaR_{\text{price}}}(1 + \kappa(q)). \tag{12}$$

$\kappa(q)$ measures the tail correlation factor between mid-price return and liquidity cost, the proportion of liquidity risk, that is diversified away due to tail correlation. In this definition, the correlation factor is always non-positive, $\kappa(q) \leq 0$. If tail correlation is perfect, $\kappa(q)$ is zero and worst mid-prices and worst liquidity costs can be added to get the total risk as is, for example, done in Bangia et al. (1999). If there is some diversification between cost and price, $\kappa(q)$ will become negative.

The liquidity impact $\lambda(q)$ contains the following conceptual components. First, it contains the mean liquidity discount for the position of size q – in contrast to other approaches. This is suitable as positions are usually valued at mid-prices already neglecting mean liquidity costs. Second, it includes negative deviations from the mean cost as measured by volatility and higher moments. Third, possible diversification effects between price and liquidity are included and liquidity risk is reduced. If liquidity cost and mid-prices have a less than perfect, negative tail correlation ($\kappa(q) < 0$), a liquidity risk estimate based on the α percentile of the liquidity cost distribution as in (10) will be incorrectly higher than that when based on the net-return distribution as in (9).

iv. Interpretation of time horizon

The time horizon in the VaR framework is usually the time required to orderly liquidate an asset. It is differentiated between asset classes but usually assumed constant within one asset class such as stocks (cp., e.g., Jorion 2001, p. 24).

We would like to stress that in the framework presented above, the time horizon Δt gets a more specific interpretation than usual. If we assume, for example, a standard 10-day period ($\Delta t = 10$), the total risk measure (8) calculates a 10-day risk forecast, which is the time required for the management to decide and react. At day 10, the stock position will be instantly liquidated.

This interpretation is consistent with a general view on 'orderly liquidation,' where the time required comprises the management reaction time as well as the liquidation time. It stands, however, in slight contrast to a more narrow view of 'orderly liquidation' that the time horizon of 10 days represents the period during which a position is continuously liquidated.

Both interpretations are, however, valid in certain situations. In a situation where very large positions can be liquidated without much time pressure, a continuous liquidation over a certain time period is valid. This is also the situation where optimal trading strategies can be applied to maximize the net sales proceeds. In our framework, we are looking at a situation characterized in II.A.i, which justifies instant liquidation. If we look at impatient traders or equivalently at the worst case, we do not allow for mitigation of some of the liquidity cost by allowing continuous liquidation. In such a case, 'orderly liquidation' needs to be more generally defined and our approach is suitable.

III. EMPIRICAL RESULTS

In the empirical part, Section III.A describes our data set and Section III.B provides some market background to our analysis. Section III.C presents our empirical results and Section III.D includes our robustness tests.

A. Description of data

Our sample consists of 5.5 years of daily XLM data (July 2002–January 2008) for all 160 stocks in the four major German stock indices (DAX, MDAX, SDAX andf TecDAX).[16] Therefore, in total, we cover a market capitalization of approximately €1.2 trillion (as of January 2008), which represents the largest part of the market capitalization in Germany. As far as we know, this is the most representative sample on weighted spread available to academia.

We received XLM data for all days, where a stock was included in one of the four indices.[17] Daily values are calculated by Xetra as the equal-weighted average of all available by-minute data points.[18] XLM(q) comprises for each day the weighted spread for 10 standardized order sizes q. Standardized order size reach from €25,000 to €5 million in the DAX and from €10,000 to €1 million in all other indices. In addition to XLM data, we obtained the day-closing bid–ask spread s at the Xetra trading system from Datastream.

Three stocks were excluded from the analysis due to missing XLM or Datastream data.[19] We also had to eliminate 408 XLM observations, where liquidity data were available outside the standardized volume class structure described above, to ensure that our estimates remain representative in each

16 The DAX contains the 30 largest publicly listed companies in Germany (by free-float market volume), the MDAX includes the subsequent 70 largest before March 24, 2003 and 50 largest thereafter and the SDAX includes the following 50 largest. The TecDAX, introduced during the sample period on March 24, 2003, comprises the 30 largest technology stocks.

17 Therefore, our sample is non-constant containing 275 different stocks, but only 160 stocks at one point in time.

18 This comprises a maximum of 1060 measurements during continuous trading.

19 Procon Multimedia (in SDAX between October 2002 and March 2003) and Medisana (in SDAX between December 2002 and March 2003). Data could not be obtained for Sparks Networks (in SDAX between June 2004 and December 2005), because it was not available in Datastream anymore.

volume class.[20] These exclusions left 99.9% or 323,670 of the stock days in the sample.

In total, our remaining sample contains 1.8 million observations for the 1424 trading days. We divide our total sample into four sub-samples, each containing the stocks of one index.

B. Market background

As background to our analysis, Table 1 summarizes market conditions during the sample period. Markets were bullish in the largest part of the sample period. We also captured the downturns in the second half of 2002 and the first month of 2008. Because of beginning and end-of-period declines, the overall return was rather average at 8% p.a. Naturally, market capitalization increased similar to returns. Market capitalization is several times larger in the DAX than in all other indices. MDAX contained the second largest average market capitalization stocks, followed by TecDAX and SDAX. Volatility exhibited a similar, but reversed pattern than returns. Because of the bullish period, our sample is probably rather positively biased.

Daily transaction volume increased considerably during the sample period, which is already a plausible indicator for improving liquidity. Transaction volume was largest in the DAX; in the other indices it was several magnitudes smaller. Contrary to the general positive trend, transaction volume in the TecDAX remained steady after its initiation in 2003 and exhibits a level slightly lower than the MDAX. SDAX transaction volume was again several times smaller than that in MDAX or TecDAX. The high diversity in transaction volumes underlines the representativeness of our sample.

C. Liquidity impact and its components

In this section, we analyze the significance of liquidity in standard risk measures and its components. We will not discuss absolute risk levels in detail. The interested reader will find estimates of absolute price risk and absolute total risk in Appendix A.

i. Magnitude of liquidity impact

As a starting point, we look at the total impact of liquidity $\lambda(q)$ on risk in a standard 10-day, 99% confidence-level VaR setting according to equation (9). These parameters are typically used in a Basel II framework (cp. Dowd 2001, p. 51). Table 2 presents statistics on the overall liquidity impact $\lambda(q)$ by order size and index.

20 Less than 0.01% of all observations were available for connected periods of <7 days. We assume that the automatic calculation routine of the Xetra computer was extended to non-standard order sizes during trial periods.

Table 1 Market conditions during sample period

Market segment overview	II/2002	2003	2004	2005	2006	2007	1/2008	Total period[a]
Average continuous period return (%)[b]								
DAX	−52	24	6	27	20	22	−15	6
MDAX	−23	39	15	25	25	−1	−12	12
SDAX	−36	35	11	28	29	4	−14	10
TecDAX	NA	52	3	26	24	32	−25	23
Total	−35	24	10	26	24	11	−15	8
Average period return volatility (annualized) (%)[c]								
DAX	64	41	22	19	23	25	51	30
MDAX	54	39	28	26	30	35	59	35
SDAX	65	47	35	31	36	38	58	40
TecDAX	NA	54	42	31	38	44	71	42
Total	60	44	32	27	32	36	59	37
Average free-float market capitalization (in million euro)								
DAX	15,217	14,615	17,983	20,350	24,357	29,949	29,325	21,008
MDAX	1043	1330	1940	2537	3734	3797	3121	2453
SDAX	106	235	320	393	500	775	640	418
TecDAX	NA	725	863	898	995	1221	1204	955
Total	3639	3483	4319	4998	6154	7379	7009	5160
Average daily transaction volume (in thsd. euro)								
DAX	93,500	94,399	98,037	119,563	165,833	250,835	351,793	144,040
MDAX	1384	2297	4035	6242	11,034	18,243	22,351	7557
SDAX	36	160	237	514	958	2129	2081	780
TecDAX	NA	1813	2345	2308	4769	7946	11,430	4052
Total	20,431	19,543	20,268	25,206	35,797	54,891	75,739	31,020

Table shows per-stock averages. All values equal weighted
[a]Annualized.
[b]Includes dividend returns, because price series are adjusted for corporate capital actions.
[c]Volatility has been annualized with $\sqrt{250}$.

Table 2 Liquidity impact on risk (VaR, 10-day, 99%)

$\lambda(q)$, VaR(10-day, 99%) in % of price risk	Order size (in thsd. euro)															Size impact
	Minimum	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX																	
Mean (%)	1	NA	1	1	NA	1	NA	2	4	NA	9	16	21	24	26	10	0.78***
Median (%)	1	NA	1	1	NA	1	NA	1	3	NA	6	11	19	20	25	3	0.78***
Standard deviation (%)	1	NA	0	0	NA	1	NA	2	4	NA	8	14	16	16	18	14	0.84***
Observations	42,129	NA	42,710	42,710	NA	42,710	NA	42,710	42,706	NA	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX																	
Mean (%)	2	2	2	3	4	5	7	11	15	19	21	NA	NA	NA	NA	8	0.58***
Median (%)	2	1	2	2	3	4	5	9	11	14	17	NA	NA	NA	NA	4	0.62***
Standard deviation (%)	2	3	3	5	4	5	6	9	18	35	63	NA	NA	NA	NA	22	0.62***
Observations	69,578	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	NA	NA	NA	NA	670,108	
SDAX																	
Mean (%)	9	6	7	10	13	16	20	23	22	22	30	NA	NA	NA	NA	14	0.35***
Median (%)	3	3	4	7	8	9	11	14	19	20	23	NA	NA	NA	NA	8	0.44***
Standard deviation (%)	52	8	11	16	20	29	44	48	17	15	34	NA	NA	NA	NA	27	0.23*
Observation	69,988	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	NA	NA	NA	NA	441,361	
TecDAX																	
Mean (%)	3	1	2	3	4	5	8	11	16	18	18	NA	NA	NA	NA	7	0.64***
Median (%)	1	1	1	2	3	4	6	8	13	18	16	NA	NA	NA	NA	3	0.66***
Standard deviation (%)	5	1	1	2	4	8	7	10	18	12	13	NA	NA	NA	NA	10	0.66***
Observations	35,741	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	NA	NA	NA	NA	319,340	
All																	
Mean (%)	4	NA	3	5	NA	7	NA	12	13	NA	17	NA	NA	NA	NA	10	0.44***
Median (%)	2	NA	2	2	NA	3	NA	7	10	NA	13	NA	NA	NA	NA	4	0.62***
Standard deviation (%)	30	NA	7	10	NA	16	NA	25	16	NA	42	NA	NA	NA	NA	20	0.42**
Observations	217,436	NA	218,671	214,590	NA	207,201	NA	184,968	153,599	NA	113,798	NA	NA	NA	NA	1,843,272	

Table shows cross-sectional statistics of lambda, which is the impact of liquidity in percent of price risk according to equation (9); minimum column measures risk at minimum spread level; all column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient of log-size regressed on the log distribution statistic including an intercept.
*10%, **5% and ***1% confidence level of being different from zero based on a two-tailed test.

On average, over all stocks and across all order sizes, total risk – including liquidity risk – is 10% higher than price risk alone. DAX is generally the index with the lowest liquidity risk, while MDAX and TecDAX are second. SDAX consistently shows the highest liquidity impact levels across all order sizes. This finding is consistent with trading volumes and market values discussed in Section III.B.

There is significant variation in the liquidity impact between indices and within indices as indicated by standard deviations. Variation is of the same order of magnitude than the level. Impact is practically zero ($\leq$1%) in small order sizes of the DAX ($<$€250 thsd.). Liquidity impact can easily increases above 20% in large stock positions of the DAX or medium stock positions in small stocks. In an average €1 million SDAX positions, liquidity impact on risk increases to 30% of price risk at a 10-day horizon.

Especially interesting is the liquidity impact calculated with spread as revealed in the minimum column. This corresponds to the risk measurement approach suggested by Bangia et al. (1999). Impact remains rather small across all stocks and comparable to the liquidity impact measured with XLM(10) and XLM(25), respectively. In SDAX and TecDAX, it is slightly higher than in the smallest XLM bracket. Because median risk levels are comparable, this effect is probably due to few outliers as XLM and spread data come from two different databases.

Liquidity impact generally increases with order size.[21] To analyze this size effect more systematically, we separately estimated the impact of doubling order size on $\lambda(q)$ in percent in the last column. To do so, we regress the log row statistics on log order size including a constant intercept.[22] Size impact is the coefficient on log size and indicates the curvature of the price-impact function. It specifically investigates into the importance of price-impact data in contrast to spread data only and abstracts from the different levels in liquidity risk between indices. Generally, the estimated price-impact statistic is positive but smaller than one, which shows that the liquidity impact (risk) function is concave.[23] The price impact is larger in the DAX than in the other indices. Here, the difference between small, liquid and larger less-liquid positions is especially pronounced. With a size impact of 0.78, liquidity impact almost doubles in the DAX when doubling order size. In the other indices, liquidity impact is already large at small positions – hence the lower curvature. All size impacts are statistically significant at the 1% level. The economically large size-impact

21 The decrease in the average SDAX position between €250 thsd. and €500 thsd. results from a non-constant sample effect. Large SDAX positions were continuously tradable only in later years. Therefore, risk estimates for large SDAX positions are calculated on a more liquid period depressing liquidity impacts compared with more continuously traded small positions.

22 Ordinary least-squared regression equation is $\log(Stat(q)) = c + \log(q) + \varepsilon$, with *stat* being the row statistic and *c* being a constant intercept.

23 This is consistent as already the price impact cost function is empirically found to be concave (cp. Hasbrouck 1991; Hausman et al. 1992).

statistic underlines the importance of using order book information beyond the spread for risk estimation – even in the DAX.

These results have important consequences for risk estimation techniques. First, we find that liquidity is an important component in total risk, especially in larger order sizes, where the price-impact estimation error relative to price risk rises up to 30% at 10-day horizons. Second, estimating liquidity risk with spread data is no valid alternative, as liquidity risk impact in this size class is very small and increases considerably with size. Third, large variations indicate that the constant scaling of price risk across all stocks, 'hair cuts,' are probably insufficient and liquidity has to be accounted for specifically for each stock.

ii. Correlation effect

Next, we would like to specifically look into the tail correlation between mid-price return and liquidity cost. A correlation factor $\kappa(q)$ of zero corresponds to perfect tail correlation between liquidity and mid-price return. It mirrors the case that liquidity costs are highest when prices are lowest. Table 3 shows the results based on 10-day, 99% VaR according to (11). Mean correlation factors range between 40% and 60% of liquidity risk. On average, 60% of the liquidity cost risk is diversified away. The negative correlation factor reveals that large, illiquid positions are relatively more liquid in crises. Stock market crashes appear to attract liquidity, which allows to liquidate less-liquid positions more cost-efficiently, however, at lower prices. Because over half of the liquidity risk is diversified away, liquidity risk would be overestimated by about 100% at larger sizes when neglecting correlation (cp. equation (12)).

Correlation factors are quite uniform across order sizes and indices at around -55% to 65%. Only in the DAX it is slightly lower at about -40%. Correlation plays an even larger role at the spread level, where it is consistently higher than that in larger order sizes. This underlines the different dynamics between the spread, quoted by market makers, and weighted spread, which emerges from free market competition. The cross-sectional standard deviation is also quite constant. The size-independent nature is underlined by the statistically and economically insignificant price-impact statistic.[24]

The $\kappa(q)$ statistic should be treated with care. The effect of correlation on total risk is substantial only if the liquidity risk is also substantial (cp. equation (12)). As liquidity risk is quite low at small positions, the overall error remains small and the violation is less critical.

Overall, these empirical results refute the common assumption of a perfect tail correlation, i.e. that it is reasonable to simply add up the price and the liquidity risk. Doing so would overestimate the total risk, especially in large, more illiquid order sizes. These results resolve the discussion as to whether the perfect tail correlation assumption is valid or not. Our representative, empirical results are in line with the argument of Francois-Heude and Van Wynendaele (2001), who criticize the perfect correlation assumption of Bangia et al. (1999).

24 Estimated in a linear regression of the distribution statistic on size.

Table 3 Correlation factor (VaR, 10-day, 99%) by index and order size

$\kappa(q)$ VaR(10-day, 99%) in % of liquidity risk	Order size (in thsd. euro)															Size impact
	Minimum	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX																	
Mean (%)	−64	NA	−40	−41	NA	−43	NA	−45	−49	NA	−34	−41	−40	−40	−41	−41	0.00
Median (%)	−68	NA	−41	−41	NA	−41	NA	−44	−48	NA	−49	−47	−42	−41	−41	−44	0.00
Standard deviation (%)	21	NA	15	15	NA	16	NA	17	18	NA	87	47	30	24	19	36	0.00
Observations	42,129	NA	42,710	42,710	NA	42,710	NA	42,710	42,706	NA	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX																	
Mean (%)	−70	−62	−62	−61	−63	−64	−63	−63	−64	−60	−59	NA	NA	NA	NA	−62	0.00**
Median (%)	−72	−66	−66	−62	−63	−67	−64	−61	−62	−63	−60	NA	NA	NA	NA	−63	0.00**
Standard deviation (%)	15	17	17	18	16	16	17	16	18	24	17	NA	NA	NA	NA	18	0.00
Observations	69,578	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	NA	NA	NA	NA	670,108	
SDAX																	
Mean (%)	−67	−61	−65	−65	−66	−66	−64	−60	−57	−59	−54	NA	NA	NA	NA	−63	0.00***
Median (%)	−68	−61	−65	−65	−65	−66	−64	−59	−56	−61	−57	NA	NA	NA	NA	−64	0.00**
Standard deviation (%)	19	19	16	18	16	16	18	15	17	18	16	NA	NA	NA	NA	17	0.00
Observations	69,988	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	NA	NA	NA	NA	441,361	
TecDAX																	
Mean (%)	−72	−66	−66	−64	−66	−66	−67	−66	−63	−65	−62	NA	NA	NA	NA	−65	0.00**
Median (%)	−73	−66	−66	−66	−70	−66	−67	−69	−65	−63	−65	NA	NA	NA	NA	−66	0.00*
Standard deviation (%)	17	16	17	18	16	16	14	18	16	13	15	NA	NA	NA	NA	16	0.00
Observations	35,741	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	NA	NA	NA	NA	319,340	
All																	
Mean (%)	−68	NA	−59	−59	NA	−61	NA	−59	−59	NA	−50	NA	NA	NA	NA	−58	0.00**
Median (%)	−70	NA	−61	−61	NA	−61	NA	−58	−58	NA	−56	NA	NA	NA	NA	−60	0.00**
Standard deviation (%)	18	NA	19	19	NA	18	NA	18	19	NA	56	NA	NA	NA	NA	25	0.00**
Observations	217,436	NA	218,671	214,590	NA	207,201	NA	184,968	153,599	NA	113,798	NA	NA	NA	NA	1,843,272	

Table shows cross-sectional statistics of the correlation factor, which measures correlation between liquidity cost and mid-price return according to (11); minimum column measures the effect at minimum spread level; all column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of size in a linear regression of the distribution statistic on size including an intercept.
*10%, **5% and ***1% confidence level of being different from zero based on a two-tailed test.

However, the overall effect of this assumption remains small if the liquidity impact is small in total. It might also be different in other assets like currencies, which were analyzed by Bangia et al. (1999), but we see no *a priori* reason why this should be the case. We also hypothesize that correlation effects should be similar for other liquidity cost measures, because they proxy for the same phenomenon. Overall, our results indicate that tail correlation is important and should be taken into account in illiquid stock positions.

iii. Liquidity impact at shorter horizons

Risk on a 10-day horizon calculated above provides a comparable reference to the standard statistics usually requested by financial regulators. However, as noted already in Section II.B.ii, when correctly and directly accounting for liquidity risk, the 10-day horizon gets the notion of management reaction time instead of liquidation time. In order to stick to the original intention behind VaR, what a portfolio is worth in the worst case, we also calculate VaR at a 1-day horizon. This statistic is also more comparable to the intraday results available so far.

Table 4 shows the liquidity impact $\lambda(q)$ for a 1-day, 99% VaR according to equation (9). As expected, the relative liquidity impact magnifies when shortening horizons, because price risk is reduced while absolute liquidity risk remains unchanged. The structure between indices remains unchanged. While still being negligible in small DAX positions, total risk including liquidity is almost double the price risk for large positions. Average €1 million SDAX positions have a $>90\%$ liquidity risk impact. Even in some small positions, liquidity plays a substantial role, with liquidity impact surpassing 10% in the SDAX for small position sizes.

The size-impact statistic reveals a very similar curvature in magnitude in the daily compared with the 10-day case. All size impacts are statistically significant at the 1% level. Correlation effects are similar in structure but larger in magnitude when compared with the 10-day horizon.[25] Our results are comparable to the 2–30% range found in other studies (cp. Francois-Heude and Van Wynendaele 2001; Giot and Grammig 2005; Angelidis and Benos 2006).

D. Robustness tests

i. Effect of using the expected shortfall (ES) measure

Recently, literature has discussed coherent risk measures as an alternative to VaR to overcome the shortfalls of VaR like non-sub-additivity (cp. Artzner et al. 1997; Acerbi and Scandolo 2007). This raises the question as to whether our results would change significantly when switching to a different risk measure. To test whether our results are robust or specific to the VaR, we calculate ES, also called 'conditional VaR' or 'expected tail loss,' which is the expected loss in the

25 Results available on request.

Table 4 Liquidity impact on risk (VaR, 1-day, 99%)

$\lambda(q)$, VaR(1-day, 99%) in % of price risk	Order size (in thsd. euro)															Size impact
	Minimum	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX																	
Mean (%)	3	NA	2	2	NA	3	NA	5	11	NA	30	56	68	76	79	32	0.82***
Median (%)	3	NA	2	2	NA	2	NA	4	7	NA	20	32	42	61	69	9	0.79***
Standard deviation (%)	2	NA	1	1	NA	2	NA	4	11	NA	29	56	57	54	56	47	0.90***
Observations	42,129	NA	42,650	42,650	NA	42,650	NA	42,650	42,646	NA	42,604	41,716	39,970	38,225	36,343	412,104	
MDAX																	
Mean (%)	7	6	8	10	14	18	28	44	68	76	77	NA	NA	NA	NA	31	0.62***
Median (%)	6	5	6	7	11	13	22	37	56	66	61	NA	NA	NA	NA	15	0.66***
Standard deviation (%)	6	9	16	12	17	18	24	31	44	51	51	NA	NA	NA	NA	39	0.40***
Observations	69,578	73,902	73,697	73,053	72,414	71,794	70,480	67,645	59,972	52,777	46,239	NA	NA	NA	NA	661,973	
SDAX																	
Mean (%)	30	17	27	41	60	68	75	80	70	70	96	NA	NA	NA	NA	52	0.33***
Median (%)	12	13	18	32	43	49	58	56	62	51	73	NA	NA	NA	NA	33	0.34***
Standard deviation (%)	102	17	35	48	83	88	102	118	46	54	96	NA	NA	NA	NA	76	0.25*
Observations	69,988	68,497	64,068	60,824	57,733	54,871	49,291	39,714	23,114	13,985	8,363	NA	NA	NA	NA	440,460	
TecDAX																	
Mean (%)	11	6	8	11	17	23	32	47	62	67	66	NA	NA	NA	NA	29	0.60***
Median (%)	7	5	7	10	13	17	28	42	56	61	67	NA	NA	NA	NA	14	0.62***
Standard deviation (%)	28	3	5	8	15	25	26	36	64	47	34	NA	NA	NA	NA	37	0.62***
Observation	35,741	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	NA	NA	NA	NA	319,340	
All																	
Mean (%)	15	NA	13	18	NA	29	NA	43	52	NA	59	NA	NA	NA	NA	36	0.43***
Median (%)	7	NA	7	9	NA	13	NA	33	43	NA	53	NA	NA	NA	NA	17	0.61***
Standard deviation (%)	60	NA	23	31	NA	54	NA	65	50	NA	53	NA	NA	NA	NA	52	0.22*
Observations	217,436	NA	217,548	213,653	NA	206,264	NA	183,967	152,727	NA	113,237	NA	NA	NA	NA	1,833,877	

Table shows cross-sectional statistics of lambda, which is the liquidity impact on risk in percent of price risk according to equation (9); minimum column measures risk at minimum spread level; all column is average over all standardized order sizes, i.e. without minimum; size impact is the increase in risk in percentage points when doubling order size, measured as coefficient in 10^{-2} of log-size in a regression of the log distribution statistic on log-size including an intercept.
*10%, **5% and ***1% confidence level of being different from zero based on a two-tailed test.

worst α% of the cases. We continue to use our basic approach detailed in Section II.B.ii, but we replace VaR with ES defined as follows:

$$ES^{\alpha,\Delta t} = E(r|r < r^{\alpha}). \tag{13}$$

When we calculate risk based on ES instead of VaR using a 10-day horizon return and a 99% confidence interval of the empirical distribution, results are as displayed in Table 5; effects of order size get accentuated. Generally speaking, results are structurally similar when measuring risk as ES compared with VaR. While total risk estimates increase, the impact of liquidity is comparable even in the tail of the distribution. Our methodology and results are therefore quite robust to a change in the ES risk measure.

ii. Effects of time variation

As a further robustness test, we calculate monthly, rolling estimates of λ to counter concerns that our results are due to the long estimation period. Rolling total risk estimates are shown in Appendix A. This test also addresses any concerns for non-constant-sample bias, because we calculate risk estimates only on stocks included in the index due to data availability. Because empirical percentiles cannot be calculated on monthly samples of daily data, we chose a straightforward mean–variance estimation procedure. For each date, we calculate the 20-day backward variance σ_r of continuous price return and assume that the daily expected return is zero. The relative price risk on a 99% confidence level is then defined as

$$VaR^{1\%}_{\text{price}} = 1 - \exp(-2.33\sigma_r). \tag{14}$$

Similarly, we calculated liquidity-adjusted total risk with the mean μ_{rnet} and variance σ_{rnet} of a 20-day backward net-return distribution

$$VaR^{1\%}_{\text{total}}(q) = 1 - \exp(\mu_{\text{rnet}}(q) - 2.33\sigma_{\text{rnet}}(q)) \tag{15}$$

with net returns calculated according to equation (6). We then calculate the liquidity impact $\lambda(q)$ according to equation (9). Neglect of negative skewness and high kurtosis (fat tails) makes this procedure simple, but might underestimate risk. Because of the underestimation, absolute values need to be treated with care, but are still – as lower bound – a suitable indicator for the time variation of the liquidity impact on risk, especially if higher moments are fairly constant.

Results for $\lambda(q)$ on the basis of a 10-day, 99% VaR according to (9) and (15) are illustrated in Figure 3, details shown in Table 6. The impact of liquidity on risk has generally declined over time across all indices. In all years, the liquidity impact increased considerably with order size as the size-impact statistic reveals. Our previous finding of the index rank (DAX, MDAX/TecDAX and SDAX) is confirmed and stable over time. TecDAX, however, was shortly more liquid after its initiation in 2003 until 2004. Although to be interpreted with care, the liquidity impact probably remained non-negligible during the low-risk period

Table 5 Liquidity impact on risk (ES, 1-day, 99%)

$\lambda(q)$, ES(10-day, 99%) in % of price risk	Order size (in thsd. euro)															Size impact
	Minimum	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX																	
Mean (%)	1	NA	1	1	NA	1	NA	2	3	NA	12	17	22	23	26	9	0.82
Median (%)	1	NA	1	1	NA	1	NA	1	2	NA	4	8	12	13	16	2	0.70***
Standard deviation (%)	1	NA	0	0	NA	1	NA	1	3	NA	33	35	36	37	38	26	1.12***
Observations	423	NA	431	431	NA	430	NA	427	412	NA	388	354	335	314	289	3811	
MDAX																	
Mean (%)	2	2	2	3	4	5	7	9	13	13	15	NA	NA	NA	NA	6	0.51***
Median (%)	1	1	1	2	3	3	5	7	9	9	11	NA	NA	NA	NA	3	0.53***
Standard deviation (%)	2	4	3	4	7	7	13	9	13	11	11	NA	NA	NA	NA	9	0.30***
Observation	780	826	812	803	791	774	724	653	550	425	376	NA	NA	NA	NA	6734	
SDAX																	
Mean (%)	5	5	6	7	9	13	14	14	21	25	25	NA	NA	NA	NA	10	0.40***
Median (%)	2	2	3	4	5	7	9	12	17	22	22	NA	NA	NA	NA	6	0.54***
Standard deviation (%)	13	6	12	8	11	26	29	13	16	22	16	NA	NA	NA	NA	17	0.20*
Observations	608	564	499	431	397	370	319	234	154	98	55	NA	NA	NA	NA	3121	
TecDAX																	
Mean (%)	2	1	2	2	3	4	6	9	12	15	19	NA	NA	NA	NA	6	0.63***
Median (%)	1	1	2	2	3	3	4	6	9	14	14	NA	NA	NA	NA	3	0.59***
Standard deviation (%)	2	1	1	2	3	4	6	8	11	16	27	NA	NA	NA	NA	9	0.77***
Observations	169	347	333	329	334	313	300	261	196	153	107	NA	NA	NA	NA	2673	
All																	
Mean (%)	3	NA	3	3	NA	5	NA	8	11	NA	15	NA	NA	NA	NA	8	0.48***
Median (%)	1	NA	1	2	NA	3	NA	5	7	NA	10	NA	NA	NA	NA	3	0.52***
Standard deviation (%)	7	NA	7	5	NA	13	NA	9	12	NA	25	NA	NA	NA	NA	16	0.34**
Observations	1980	NA	2075	1994	NA	1887	NA	1575	1312	NA	926	NA	NA	NA	NA	16,339	

Table shows cross-sectional statistics of lambda, which is liquidity impact in percent of price risk according to equations (9) and (13); minimum column measures risk at minimum spread level; all column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of log-size in a regression of the log distribution statistic on log-size including an intercept.
*10%, **5% and ***1% confidence level of being different from zero based on a two-tailed test.

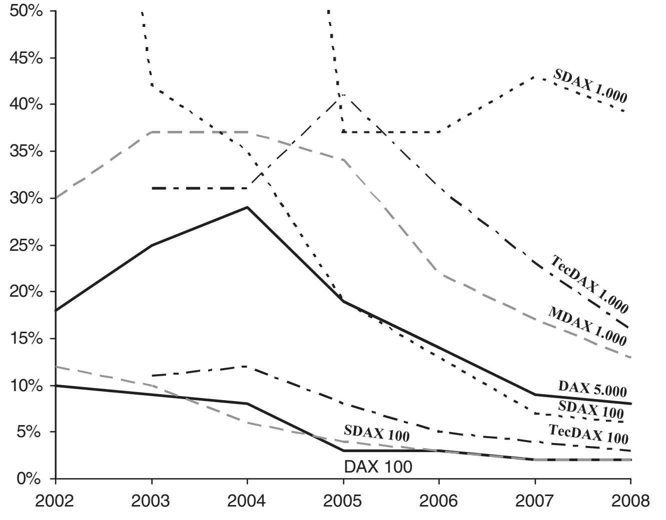

Figure 3 Liquidity impact on risk (rolling VaR, 10-day, 99%).
This figure shows mean λ, which is liquidity impact in percent of price risk by subsample calculated with a rolling mean–variance estimation of Value-at-Risk (10-day, 99%) according to (9) based on (15) for selected indices and volume classes.

from 2006 to 2007. The impact of liquidity on total risk was certainly economically significant in the crises periods of 2002–2003 and in 2008.

Results for the whole panel ('all') have to be treated with care, because they are distorted by the non-constant sample effect. Over the years, the liquidity of less-liquid stocks improved considerably, which made their liquidity cost data increasingly available. As consequence, less-liquid, high-cost stocks are increasingly included in the sample, which increases the average risk estimate. However, individual year estimates have almost no sample bias and underline the fact that liquidity impact is economically significant.

If skewness and kurtosis would be included, these findings are also likely to get confirmed, as the one-time liquidity cost deduction will probably introduce additional skewness, which keeps the relation between price and liquidity risk valid. Overall, this confirms the fact that liquidity price impact is economically significant enough to encourage integration into risk measurement systems.

iii. Effects of portfolio diversification

We have shown that liquidity risk is economically significant when looking at individual stocks in the different indices. But does this result persist when looking at portfolios of stocks? If diversification between mid-prices of different stocks is larger than that between liquidity of different stocks, the liquidity impact might be reduced substantially.

To test the robustness of our results against the effects of portfolio diversification, we calculated daily value-weighted index returns and determined the

Table 6 Liquidity impact on risk (rolling VaR, 10-day, 99%)

Avg. $\lambda(q)$, VaR(10-day, 99%) in % of price risk	Order size (in thsd. euro)															Size impact
	Minimum (%)	10 (%)	25 (%)	50 (%)	75 (%)	100 (%)	150 (%)	250 (%)	500 (%)	750 (%)	1000 (%)	2000 (%)	3000 (%)	4000 (%)	5000 (%)	All (%)	
DAX																	
2002	1	NA	1	1	NA	1	NA	2	4	NA	10	18	17	17	18	8	0.77
2003	1	NA	1	1	NA	1	NA	2	4	NA	9	19	23	25	25	10	0.82***
2004	1	NA	1	1	NA	1	NA	1	2	NA	8	15	21	25	29	10	0.83***
2005	1	NA	1	1	NA	1	NA	1	2	NA	3	7	11	14	19	6	0.70***
2006	0	NA	0	1	NA	1	NA	1	2	NA	3	5	7	10	14	4	0.65***
2007	0	NA	0	0	NA	1	NA	1	1	NA	2	4	5	7	9	3	0.60***
2008	0	NA	0	0	NA	0	NA	1	1	NA	2	3	5	7	8	3	0.63***
All	1	NA	1	1	NA	1	NA	1	2	NA	5	10	13	16	18	6	0.74***
Δ2002–2008[a]	−1	NA	0	0	NA	0	NA	−1	−1	NA	−4	−8	−4	−2	0	−1	
MDAX																	
2002	4	6	7	8	10	12	16	19	28	31	30	NA	NA	NA	NA	12	0.42***
2003	3	3	4	5	7	10	16	24	35	40	37	NA	NA	NA	NA	14	0.63***
2004	2	2	3	4	5	6	11	18	32	35	37	NA	NA	NA	NA	13	0.73***
2005	2	2	2	3	3	4	6	10	21	29	34	NA	NA	NA	NA	10	0.74***
2006	1	1	1	2	2	3	4	6	12	17	22	NA	NA	NA	NA	7	0.71***
2007	1	1	1	2	2	2	3	4	8	12	17	NA	NA	NA	NA	5	0.65***
2008	1	1	1	1	1	2	2	3	6	10	13	NA	NA	NA	NA	4	0.67***
All	2	2	3	3	4	6	8	12	20	24	26	NA	NA	NA	NA	10	0.61***
Δ2002–2008[a]	−2	−3	−4	−4	−5	−6	−7	−7	−8	−6	−4	NA	NA	NA	NA	−2	
SDAX																	
2002	13	12	27	61	91	120	153	115	79	NA	NA	NA	NA	NA	NA	36	0.56**
2003	10	12	17	26	36	42	40	41	40	51	64	NA	NA	NA	NA	27	0.32***
2004	6	7	11	20	27	35	40	52	46	38	120	NA	NA	NA	NA	24	0.49***
2005	6	5	7	11	15	19	24	29	32	41	37	NA	NA	NA	NA	17	0.46***
2006	3	4	5	7	10	13	19	28	37	36	37	NA	NA	NA	NA	15	0.56***
2007	2	2	3	4	5	7	10	17	31	38	43	NA	NA	NA	NA	13	0.71***
2008	2	2	2	3	5	6	10	16	26	32	39	NA	NA	NA	NA	10	0.75***
All	6	6	9	14	17	21	23	28	34	38	43	NA	NA	NA	NA	18	0.42***
Δ2002–2008[a]	−7	−6	−18	−47	−74	−99	−130	−86	−45	−13	−21	NA	NA	NA	NA	−18	

Table 6 (*continued*)

Avg. $\lambda(q)$, VaR(10-day, 99%) in % of price risk	Order size (in thsd. euro)																Size impact
	Minimum (%)	10 (%)	25 (%)	50 (%)	75 (%)	100 (%)	150 (%)	250 (%)	500 (%)	750 (%)	1000 (%)	2000 (%)	3000 (%)	4000 (%)	5000 (%)	All (%)	
TecDAX																	
2002	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
2003	2	3	4	5	8	11	15	21	25	28	31	NA	NA	NA	NA	11	0.57***
2004	2	3	4	5	8	12	18	26	26	29	31	NA	NA	NA	NA	13	0.60***
2005	2	2	3	5	6	8	11	17	27	38	41	NA	NA	NA	NA	12	0.67***
2006	2	2	2	3	4	5	7	13	22	27	31	NA	NA	NA	NA	10	0.68***
2007	1	1	2	2	3	4	5	8	15	19	23	NA	NA	NA	NA	8	0.66***
2008	1	1	1	2	2	3	4	6	11	15	16	NA	NA	NA	NA	5	0.65***
All	2	2	3	4	6	8	11	16	22	27	29	NA	NA	NA	NA	11	0.62***
Δ2002–2008[a]	0	−1	−1	−1	−2	−4	−4	−5	−3	−1	−1	NA	NA	NA	NA	−1	
All																	
2002	6	NA	8	10	NA	13	NA	12	14	NA	15	NA	NA	NA	NA	13	0.14***
2003	5	NA	7	9	NA	14	NA	19	21	NA	21	NA	NA	NA	NA	15	0.31***
2004	3	NA	5	8	NA	14	NA	20	22	NA	23	NA	NA	NA	NA	15	0.42***
2005	3	NA	4	5	NA	9	NA	14	19	NA	24	NA	NA	NA	NA	11	0.53***
2006	2	NA	2	3	NA	6	NA	12	16	NA	19	NA	NA	NA	NA	9	0.59***
2007	1	NA	2	2	NA	4	NA	8	14	NA	19	NA	NA	NA	NA	7	0.70***
2008	1	NA	1	2	NA	3	NA	7	11	NA	13	NA	NA	NA	NA	6	0.69***
All	3	NA	4	6	NA	9	NA	14	17	NA	20	NA	NA	NA	NA	11	0.45***
Δ2002–2008[a]	−3	NA	−4	−4	NA	−4	NA	2	3	NA	5	NA	NA	NA	NA	−2	

Table shows mean lambda, which is liquidity impact in percent of price risk by sub-sample calculated with a rolling mean-variance estimation of Value-at-Risk (10-day, 99%) according to (9) based on (15). Minimum column measures risk at minimum spread level; all column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of log-size in a regression of the log distribution statistic on log-size including an intercept.

[a]Statistic shows absolute change between 2003 and 2008 when 2002 number not available.

*10%, **5% and ***1% confidence level of being different from zero based on a two-tailed test.

liquidity impact $\lambda(q)$ based on a 10-day, 99% VaR according to (9). While our methodology does not use optimized position weights, a value-weighted portfolio should show effects of diversification if there are any. Results are presented in Table 7. Estimates demonstrate that the liquidity impact on the portfolio level is of a similar magnitude as that on the average individual stock level (cp. Table 2). Especially in larger sizes, liquidity impact is increased at the portfolio level, for example, it increases to 54% for the €1 million position in the SDAX portfolio compared with 30% for the average individual stock position. This must be driven by larger liquidity commonality in larger sizes, i.e. diversification in liquidity between stocks decreases with larger sizes. Even for the all-stock portfolio, liquidity impact levels are higher than for the average stock. Overall, our results are robust to diversification effects in stock portfolios.

IV. CONCLUSION AND OUTLOOK

In this paper, we modeled liquidity risk based on the weighted spread liquidity measure in a VaR framework. The main advantage over existing approaches is the higher precision of the weighted spread, which calculates liquidity cost differentiated by order size, i.e. the price impact, from the limit order book.

We argued that weighted spread is a valid liquidity measure from a risk perspective in a wide range of situations, which we defined clearly. If we look at limit order book markets, where this type of data is available and from the perspective of institutional investors, for whom other direct trading costs are negligible, two situational assumptions are critical.

First, our approach works most precise for continuously tradable asset positions, for example, for small- to medium-sized positions in developed stock markets. Positions cannot be too large, like block holdings, and markets have to be fairly liquid with few zero trading days. If this is not the case, forced execution delay can incur costs that we have neglected.

Second, we assume that deliberate, strategic delay has no significant benefit, which renders optimal trading strategies useless. This is a fair assumption in four possible cases. We can adopt a worst-case perspective, for example, because external restrictions require to close whole positions immediately. Any possible benefit from delay is then consciously ignored. From a risk perspective, strategic delay also remains with unrealizable benefit if the optimal trading strategy is non-stable in crises situations and can therefore not provide any benefit on an expected basis. Further, if liquidity prices are efficient in fairly liquid markets, strategic delay has per definition no marginal benefit. Finally, optimal trading strategies are also useless if the real or perceived (i.e. adverse) informational content of the trade is high and delay only increases the probability of adverse price movements.

This discussion shows that these cases cover a variety of situations. Overall, our approach is most valid for up to medium-sized positions in generally continuously trading markets.

Table 7 Liquidity impact $\lambda(q)$ (VaR, 10-day, 99%) by (index) portfolio and order size

$\lambda(q)$ VaR(10-day, 99%) in % of liquidity risk	Order size (in thsd. euro)															Size impact
	Minimum	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX																	
Estimate (%)	1	NA	1	1	NA	1	NA	1	2	NA	12	22	18	25	27	10	0.84***
Observations	42,129	NA	42,710	42,710	NA	42,710	NA	42,710	42,706	NA	42,663	41,759	40,002	38,294	36,388	412,652	
MDAX																	
Estimate (%)	2	2	2	2	3	5	9	5	13	21	20	NA	NA	NA	NA	7	0.60***
Observations	73,279	74,858	74,679	74,040	73,365	72,737	71,409	68,543	60,788	53,501	46,793	NA	NA	NA	NA	670,713	
SDAX																	
Estimate (%)	5	32	15	27	32	52	56	26	39	45	54	NA	NA	NA	NA	35	0.16*
Observations	70,048	69,197	64,614	61,119	57,938	55,000	49,410	39,920	23,442	14,435	9,112	NA	NA	NA	NA	444,187	
TecDAX																	
Estimate (%)	1	1	2	2	3	4	5	8	13	15	17	NA	NA	NA	NA	6	0.61***
Observations	36,980	37,157	37,157	37,150	37,099	36,973	36,323	34,028	27,077	20,868	16,291	NA	NA	NA	NA	320,123	
All stocks																	
Estimate (%)	3	NA	2	2	NA	3	NA	4	5	NA	20	NA	NA	NA	NA	6	0.55**
Observations	181,212	NA	219,160	215,019	NA	207,420	NA	185,201	154,013	NA	114,859	NA	NA	NA	NA	1,847,675	

Table shows portfolio statistics of lambda, which is the impact of liquidity in percent of price risk according to (9); minimum column measures risk at minimum spread level; all column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient of log-size regressed on the log distribution statistic including an intercept.
*10%, **5% and ***1% confidence level of being different from zero based on a two-tailed test.

We then defined liquidity-adjusted VaR of a specific position in a straightforward manner based on net return, i.e. mid-price return less the weighted spread of the position. This definition avoids any distortion through correlation assumptions between liquidity and price return, which we analyzed separately.

Empirically, we find that the impact of liquidity relative to price risk is small at small order sizes, especially at the spread level (<10% for 10-day, 99% VaR). However, it increases to 20–30% of price risk in larger sizes in illiquid indices as well as in the DAX. Results aggravate if we switch to daily VaR horizons.

We also took a detailed look at tail correlation between liquidity and mid-price returns and showed that it is non-negligible. Liquidity risk would be overestimated by 100% if correlations are ignored. In the cases we identified above, where liquidity risk is an economically significant component of total risk, total risk will be severely overestimated if liquidity cost risk is simply added to existing risk measures. Therefore, several common approaches should be adapted to avoid this distortion.

We find that results are structurally similar when using ES instead of VaR risk measures. Our results are therefore transferable. To check the time robustness of these findings, we use a monthly, rolling mean–variance estimation method. Results are confirmed. Results are also similar for portfolios of stocks, when portfolio diversification is accounted for.

Overall, we strongly advocate the use of weighted spread data such as XLM to improve risk estimates. Liquidity constitutes a large part of total risk, especially in larger positions and at short horizons – even in more liquid market segments.

Several venues are still open for future research. Because we have used empirical *ex-post* risk measurement to avoid any distortion by a specific choice of parametrization or forecasting, appropriate techniques will need to be selected. It will also be helpful to test the precision of our estimates against real transaction data. Future research can also address two assumptions to extend this approach to a larger realm of situations and assets. The empirical integration of delay risk is still unsolved as is the empirical questions when liquidity prices are efficient. Further insights into when and under which circumstances delay occurs will also help to advance this line of thinking. Another simplifying advance would be a method that directly integrates the liquidity–price correlation in a parametric approach when adding liquidity to price risk. Tackling these research areas will help to further advance the integration of liquidity into risk measurement.

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APPENDIX A

Table A1 Price risk (VaR, 10-day, 99%)

Index	Mean (%)	Median (%)	Standard deviation (%)	Observations
DAX	16.3	14.7	4.4	43,767
MDAX	17.2	15.8	8.1	76,750
SDAX	19.5	17.7	7.3	72,373
TecDAX	24.3	22.6	9.2	38,070
All	18.9	17.4	7.9	230,960

This table contains distribution statistics on price risk calculated as 10-day, 99% VaR according to equation (5).

Table A2 Price risk (VaR, 1-day, 99%)

Index	Mean (%)	Median (%)	Standard deviations (%)	Observations
DAX	5.6	5.5	1.1	43,710
MDAX	6.1	5.7	2.2	71,458
SDAX	7.2	6.5	3.0	72,313
TecDAX	8.2	7.9	1.8	36,801
All	6.7	6.0	2.5	224,282

This table contains distribution statistics on price risk calculated as 1-day, 99% VaR according to equation (5).

Table A3 Absolute liquidity-adjusted total risk (VaR, 10-day, 99%)

Total risk, VaR(10-day, 99%) abs., liquidity-adjusted in %	Order size (in thsd. euro)															Size impact
	Minimum	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX																	
Mean (%)	17	NA	17	17	NA	17	NA	17	17	NA	18	20	20	21	21	18	0.05***
Median (%)	15	NA	15	15	NA	15	NA	15	16	NA	17	18	18	17	18	17	0.04***
Standard deviation (%)	4	NA	5	5	NA	5	NA	5	5	NA	5	8	7	7	8	6	0.11***
Observations	42,129	NA	42,710	42,710	NA	42,710	NA	42,710	42,706	NA	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX																	
Mean (%)	18	19	20	20	20	20	20	22	23	22	24	NA	NA	NA	NA	21	0.05***
Median (%)	17	17	17	17	17	17	18	19	21	20	20	NA	NA	NA	NA	18	0.05***
Standard deviation (%)	8	9	9	9	9	9	9	9	10	10	11	NA	NA	NA	NA	9	0.03**
Observations	73,234	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	NA	NA	NA	NA	670,108	
SDAX																	
Mean (%)	21	20	21	22	24	24	25	27	28	30	35	NA	NA	NA	NA	24	0.11***
Median (%)	19	18	18	20	21	21	22	23	26	29	30	NA	NA	NA	NA	21	0.12***
Standard deviation (%)	8	7	7	8	9	9	9	10	9	10	16	NA	NA	NA	NA	9	0.12***
Observations	70,048	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8363	NA	NA	NA	NA	441,361	
TecDAX																	
Mean (%)	25	21	22	22	22	22	23	24	26	26	30	NA	NA	NA	NA	23	0.07***
Median (%)	23	19	20	20	20	20	21	22	24	24	28	NA	NA	NA	NA	20	0.08***
Standard deviation (%)	9	8	8	9	8	9	9	9	10	9	10	NA	NA	NA	NA	9	0.05***
Observations	36,980	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	NA	NA	NA	NA	319,340	
All																	
Mean (%)	20	NA	20	20	NA	21	NA	22	23	NA	23	NA	NA	NA	NA	21	0.04***
Median (%)	18	NA	18	18	NA	19	NA	20	21	NA	21	NA	NA	NA	NA	19	0.05***
Standard deviation (%)	8	NA	8	8	NA	9	NA	9	9	NA	11	NA	NA	NA	NA	9	0.08***
Observations	222,391	NA	218,671	214,590	NA	207,201	NA	184,968	153,599	NA	113,798	NA	NA	NA	NA	1,843,272	

This tables shows cross-sectional statistics on empirical, absolute total risk including a liquidity adjustment according to equation (8); minimum column measures risk at minimum spread level; all column is average over all standardized order sizes, i.e. without minimum; size impact is the increase in risk in percentage points when doubling order size, measured as coefficient in 10^{-2} of log-size in a regression of the distribution statistic on log-size including an intercept.
*10%, **5% and ***1% confidence level of being different from zero based on a two-tailed test.

Table A4 Absolute liquidity-adjusted total risk (VaR, 1-day, 99%)

Total risk, VaR(1-day, 99%) abs., liquidity-adjusted in %	Order size (in thsd. euro)															Size impact
	Minimum	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX																	
Mean (%)	6	NA	6	6	NA	6	NA	6	6	NA	7	9	10	10	10	7	0.12***
Median (%)	6	NA	6	6	NA	6	NA	6	6	NA	6	8	8	9	9	6	0.09***
Standard deviation (%)	1	NA	1	1	NA	1	NA	1	1	NA	2	5	4	4	4	3	0.30***
Observations	42,129	NA	42,710	42,710	NA	42,710	NA	42,710	42,706	NA	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX																	
Mean (%)	6	6	6	6	7	7	7	8	10	10	11	NA	NA	NA	NA	8	0.14***
Median (%)	6	6	6	6	6	7	7	8	10	9	9	NA	NA	NA	NA	7	0.12***
Standard deviation (%)	2	2	3	2	2	2	3	3	4	4	5	NA	NA	NA	NA	3	0.15***
Observations	73,234	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	NA	NA	NA	NA	670,108	
SDAX																	
Mean (%)	9	8	8	9	11	12	12	14	15	17	22	NA	NA	NA	NA	11	0.21***
Median (%)	7	7	8	9	9	10	11	11	13	15	16	NA	NA	NA	NA	9	0.18***
Standard deviation (%)	5	4	3	5	7	7	7	10	7	9	16	NA	NA	NA	NA	7	0.27***
Observations	70,048	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	NA	NA	NA	NA	441,361	
TecDAX																	
Mean (%)	9	7	7	8	8	9	9	10	12	13	15	NA	NA	NA	NA	9	0.16***
Median (%)	8	7	7	7	8	8	9	10	12	11	13	NA	NA	NA	NA	8	0.14***
Standard deviation (%)	3	2	2	2	2	3	4	4	5	5	6	NA	NA	NA	NA	4	0.28***
Observations	36,980	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	NA	NA	NA	NA	319,340	
All																	
Mean (%)	7	NA	7	7	NA	8	NA	9	10	NA	11	NA	NA	NA	NA	9	0.12***
Median (%)	7	NA	7	7	NA	7	NA	8	9	NA	9	NA	NA	NA	NA	7	0.10***
Standard deviation (%)	4	NA	3	3	NA	5	NA	6	5	NA	7	NA	NA	NA	NA	5	0.24***
Observations	222,391	NA	218,671	214,590	NA	207,201	NA	184,968	153,599	NA	113,798	NA	NA	NA	NA	1,843,272	

This tables shows cross-sectional statistics on empirical, absolute total risk including a liquidity adjustment according to equation (8); minimum column measures risk at minimum spread level; all column is average over all standardized order sizes, i.e. without minimum; size impact is the increase in risk in percentage points when doubling order size, measured as coefficient in 10^{-2} of log-size in a regression of the distribution statistic on log-size including an intercept.

*10%, **5% and ***1% confidence level of being different from zero based on a two-tailed test.

Table A5 Liquidity-adjusted total risk on 10 day horizon (rolling estimate, VaR, 10-day, 99%)

L-adj. VaR (10-day, 99%) in %	Order size (in thsd. euro)																Size impact
	Minimum (%)	10 (%)	25 (%)	50 (%)	75 (%)	100 (%)	150 (%)	250 (%)	500 (%)	750 (%)	1000 (%)	2000 (%)	3000 (%)	4000 (%)	5000 (%)	All (%)	
DAX																	
2002	24	NA	24	24	NA	24	NA	24	25	NA	26	28	29	30	31	26	1.32***
2003	16	NA	17	17	NA	17	NA	17	17	NA	18	20	21	21	22	18	0.94***
2004	9	NA	9	9	NA	9	NA	10	10	NA	10	11	12	12	13	10	0.70***
2005	8	NA	8	8	NA	8	NA	8	8	NA	8	9	9	9	10	9	0.29***
2006	9	NA	9	9	NA	9	NA	10	10	NA	10	10	10	10	11	10	0.29***
2007	10	NA	10	10	NA	10	NA	10	10	NA	10	11	11	11	11	11	0.22***
2008	16	NA	15	15	NA	15	NA	15	15	NA	15	15	16	16	16	15	0.18**
All	12	NA	12	12	NA	12	NA	12	12	NA	13	13	13	14	14	13	0.37***
Δ2002–2008[a]	−12	NA	−12	−12	NA	−12	NA	−12	−13	NA	−14	−15	−16	−16	−17	−14	−0.96
MDAX																	
2002	21	21	21	22	22	23	24	26	28	27	27	NA	NA	NA	NA	23	1.69***
2003	16	16	16	16	16	16	17	19	21	23	24	NA	NA	NA	NA	17	1.87***
2004	11	12	12	12	12	12	13	13	16	17	19	NA	NA	NA	NA	13	1.46***
2005	11	11	11	11	11	11	11	11	13	14	15	NA	NA	NA	NA	12	0.80***
2006	12	12	12	12	12	12	13	13	13	14	15	NA	NA	NA	NA	13	0.59***
2007	13	13	13	13	13	13	14	14	14	15	15	NA	NA	NA	NA	14	0.49***
2008	20	19	19	20	20	20	20	20	20	21	22	NA	NA	NA	NA	20	0.50***
All	14	14	14	14	14	14	14	15	16	16	17	NA	NA	NA	NA	15	0.65***
Δ2002–2008[a]	−7	−7	−7	−8	−8	−8	−9	−11	−13	−11	−10	NA	NA	NA	NA	−8	−1.12
SDAX																	
2002	24	23	27	35	41	47	47	32	4	NA	NA	NA	NA	NA	NA	28	−1.45
2003	19	19	20	21	23	25	28	31	35	28	29	NA	NA	NA	NA	22	2.98***
2004	14	15	15	16	18	20	21	25	31	31	33	NA	NA	NA	NA	18	4.49***
2005	13	13	13	13	14	15	16	18	24	27	30	NA	NA	NA	NA	15	3.83***
2006	14	14	14	15	15	15	16	18	23	27	31	NA	NA	NA	NA	17	3.53***
2007	15	15	15	15	15	16	16	17	20	23	26	NA	NA	NA	NA	17	2.22***
2008	20	22	22	22	21	21	22	24	28	33	35	NA	NA	NA	NA	23	2.80***
All	16	16	16	16	17	17	18	20	23	25	28	NA	NA	NA	NA	18	2.59***
Δ2002–2008[a]	−8	−7	−11	−18	−24	−29	−28	−13	19	−3	−1	NA	NA	NA	NA	−11	3.95

Table A5 (*continued*)

L-adj. VaR (10-day, 99%) in %	Order size (in thsd. euro)																Size impact
	Minimum (%)	10 (%)	25 (%)	50 (%)	75 (%)	100 (%)	150 (%)	250 (%)	500 (%)	750 (%)	1000 (%)	2000 (%)	3000 (%)	4000 (%)	5000 (%)	All (%)	
TecDAX																	
2002	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2003	21	21	21	22	22	23	24	25	27	29	28	NA	NA	NA	NA	23	1.87***
2004	17	17	17	17	17	18	19	20	22	24	25	NA	NA	NA	NA	18	1.86***
2005	13	13	13	13	13	14	14	15	17	20	21	NA	NA	NA	NA	15	1.76***
2006	15	15	15	15	15	15	16	16	18	20	23	NA	NA	NA	NA	16	1.49***
2007	16	16	16	16	16	16	17	17	18	20	21	NA	NA	NA	NA	17	1.03***
2008	24	23	23	23	23	23	23	24	26	26	29	NA	NA	NA	NA	24	1.11***
All	16	16	16	16	17	17	17	18	20	21	23	NA	NA	NA	NA	18	1.45***
Δ2002–2008[a]	−5	−5	−5	−5	−5	−6	−6	−7	−8	−7	−6	NA	NA	NA	NA	−5	−0.54
All																	
2002	22	NA	23	23	NA	24	NA	25	26	NA	26	NA	NA	NA	NA	24	0.96***
2003	18	NA	18	18	NA	19	NA	20	21	NA	21	NA	NA	NA	NA	20	0.96***
2004	13	NA	13	14	NA	15	NA	15	16	NA	15	NA	NA	NA	NA	15	0.61**
2005	11	NA	11	11	NA	12	NA	13	14	NA	14	NA	NA	NA	NA	12	0.96***
2006	13	NA	13	13	NA	13	NA	14	15	NA	16	NA	NA	NA	NA	14	0.84***
2007	14	NA	14	14	NA	14	NA	15	16	NA	17	NA	NA	NA	NA	15	0.84***
2008	20	NA	20	20	NA	20	NA	21	22	NA	23	NA	NA	NA	NA	21	0.84***
All	14	NA	14	15	NA	15	NA	16	16	NA	17	NA	NA	NA	NA	15	0.72***
Δ2002–2008[a]	−8	NA	−8	−9	NA	−9	NA	−9	−10	NA	−9	NA	NA	NA	NA	−9	−0.30

Table shows liquidity-adjusted total risk by sub-sample according to equation (9) calculated with a rolling mean-variance estimation. Minimum column measures risk at minimum spread level; all column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of log-size in a regression of the distribution statistic on log-size including an intercept.

[a]Statistic shows absolute change between 2003 and 2008 when 2002 number not available.

*10%, **5% and ***1% confidence level of being different from zero based on a two-tailed test.