

Asset Pricing and the Illiquidity Premium

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Abstract

In this paper, we examine the **asset-pricing role of liquidity (as proxied by share turnover) in the context of the Fama and French (1993) three-factor model**. Our analysis employs monthly **Australian data**, covering the sample period from 1990 to 1998. The key finding of our research is that the main test is unable to reject the test of **over-identifying restrictions, thus supporting the overall favorability of the liquidity-augmented Fama–French model**. In addition, we find **that the asset-pricing performance of the liquidity factor is generally very robust to a wide range of sensitivity checks**.

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1. Introduction

Does illiquidity attract a premium in equity markets? Our primary objective is to investigate the role of liquidity (proxied by share turnover) as an additional

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factor in asset pricing. We analyze the issue in the **context of the time-series version of the Fama–French three-factor model (Fama and French, 1993)** using Australian data.¹ Motivation for our study is provided by the growing interest in liquidity that has emerged in the asset-pricing literature over recent years (e.g., Brennan and Subrahmanyam, 1996; Brennan, Chordia, and Subrahmanyam, 1998; Datar, Naik, and Radcliffe, 1998; Chui and Wei, 1999; Rouwenhorst, 1999; Chordia, Subrahmanyam, and Anshuman, 2001; Pastor and Stambaugh, 2001; Amihud, 2002). The majority of the literature **uses U.S. data and investigates trading-volume proxies of liquidity in the context of the Fama–French model, with mixed results.**

Gathering evidence beyond the U.S. market (e.g., Chui and Wei, 1999) is essential to avoid the problem of data snooping (Lo and MacKinlay, 1990). In the current study, such a goal is best achieved by selecting a market in which illiquidity is likely to be an important factor for many of its listed stocks, yet is relatively large and globally important. Australia is an ideal choice. Its industrial structure ensures a considerable proportion of mining and resource sector stocks that are characterized by low liquidity. Nevertheless, from an aggregate perspective, the Australian market is visible and relatively important as reflected by its ranking in the top 10 world markets based on market capitalization.²

Accordingly, this paper has four related aims. First, using Australian data we examine the role of liquidity in a Fama and French (1993) framework through the formation of mimicking portfolios. Second, we use a generalized method of moments (GMM) system approach to overcome the errors-in-variables problem in the two-pass Fama and MacBeth (1973) common in Fama–French-based research. Third, we set up the experimental design so that direct estimates of risk premiums are obtained, thus providing a richer test of the models. Fourth, we use a more current data set than previous Australian studies of similar issues (see Anderson, Clarkson, and Moran, 1997; Halliwell, Heaney, and Sawicki, 1999).

2. Literature review

2.1. *Fama–French three-factor model*

The Fama and MacBeth (1973) two-pass approach, used in the influential paper by Fama and French (1992), constitutes the core method for many recent empirical papers on asset pricing. A common variant is to perform a set of time series

¹ Controversy reigns over the validity of the Fama–French model. Grinold (1993), Davis (1994), He and Ng (1994), and Fama and French (1993, 1995, 1996, 1998) have presented evidence that largely supports the original Fama and French (1992) conclusions. Studies on the other side of the debate include Black (1993), Amihud, Christensen, and Mendelson (1993), Kothari, Shanken, and Sloan (1995), Kim (1995), and Clare, Priestley, and Thomas (1997, 1998).

² With a market capitalization of USD 312 billion, Australia was ranked the 10th largest stock market, globally in 1996 (International Finance Corporation, 1997).

regressions in the first pass and, in the second pass, to conduct a cross-sectional test of variables including estimates from the first pass. Papers that use this approach include Chan, Hamao, and Lakonishok (1991), Jegadeesh (1992), Davis (1994), Kothari, Shanken, and Sloan (1995), Jagannathan and Wang (1996), and Daniel and Titman (1997). Although there is some debate about database problems and the correct interpretation of the results, generally the evidence shows that companies with higher book-to-market and smaller size appear to earn large risk-adjusted returns.

Empirical studies from the Asia-Pacific region (except Japan) that have used this method include Bryant and Eleswarapu (1997) on New Zealand shares and Chui and Wei (1998) on shares for Hong Kong, Korea, Malaysia, Taiwan, and Thailand. Bryant and Eleswarapu report that the coefficient on beta is significant but negative, but find limited significance of the book-to-market variable. However, their sample is small; on average, they have only 108 firms for each month over the sample period. Chui and Wei report that book-to-market is significant but varies across markets, while beta is not significant. A problem with the two-pass Fama–Macbeth (1973) method, regardless of how it is modified, is the errors-in-variables problem. This paper seeks to avoid this problem by taking a systems approach to estimation and testing.

Another common approach employed in the asset-pricing literature follows Fama and French (1993)—a time-series counterpart to their earlier method. First, mimicking portfolios are created on the size and book-to-market factors, and along with a market factor (excess returns on a market index), a three-factor model is produced. A time series regression is run on the factors to test whether they are able to explain a set of portfolio expected excess returns. Some examples of papers that use the method are Brennan and Subrahmanyam (1996), Fama and French (1996, 1997), and in the Australian context, Halliwell, Heaney, and Sawicki (1999).

Halliwell, Heaney, and Sawicki (1999) use Australian data from 1981 to 1991 to examine the explanatory power of book-to-market and size using the Fama and French (1993) method. They report that the size and the market risk premiums are statistically significant in explaining returns. However, although the book-to-market premium has similar magnitude as in the Fama–French paper, it is not statistically significant.³

2.2. Evidence on the role of a liquidity factor in asset pricing

One of the first studies to examine the role of liquidity in asset pricing is Amihud and Mendelson (1986). They seek to establish a link between the bid-ask spread and asset returns. They suggest that an investor with a long investment horizon, compared to an investor with a shorter horizon, will require a smaller premium for illiquidity as

³ Halliwell, Heaney, and Sawicki (1999) do not test a liquidity factor; their analysis focuses exclusively on the Fama and French (1993) three-factor model.

reflected in the bid-ask spread. Amihud and Mendelson use a Fama–MacBeth (1973) portfolio formation procedure that requires at least 11 years of data. They report a positive relation between annual portfolio return and bid-ask spread.

Eleswarapu and Reinganum (1993), using monthly bid-ask spreads, examine the effect of seasonality on the bid-ask spread and returns. They form portfolios based on the criteria of Amihud and Mendelson (1986) and a modified portfolio selection criterion that requires only 36 months of pre-formation data. The effect of the modified portfolio selection criterion is to reduce the bias in Amihud and Mendelson of excluding small firms. Eleswarapu and Reinganum report that the relation between the bid-ask spread and asset returns is mainly confined to January.

Brennan and Subrahmanyam (1996) separate the cost of transacting into a variable and a fixed cost. The variable component is related to both trade size and time horizon. The fixed component is related only to the time horizon. Brennan and Subrahmanyam conduct asset-pricing tests in a Fama and French (1993) framework augmented by a liquidity variable. They report that the variable component of transaction costs is strongly significant and the fixed component is insignificant, which is not consistent with Amihud and Mendelson (1986). Brennan and Subrahmanyam suggest that the results could be due to either a poor proxy for the variable and fixed transaction costs or the Fama and French model being an inadequate model for risk adjustment.

Several investigators (e.g., Brennan, Chordia, and Subrahmanyam, 1998; Datar, Naik, and Radcliffe, 1998; Chui and Wei, 1999; Rouwenhorst, 1999; Chordia, Subrahmanyam, and Anshuman, 2001) re-examine the relation between liquidity and asset returns using a proxy other than the bid-ask spread. In part, this has arisen due to concerns about the bid-ask spread proxy expressed by Brennan and Subrahmanyam (1996) and Peterson and Fialkowski (1994) and the difficulty of obtaining monthly spreads over long periods of time.

Brennan, Chordia, and Subrahmanyam (1998) also use the Fama and French (1993) factors of size and book-to-market ratio but extend the number of firm characteristics to include a measure of market liquidity. Their measure of liquidity is the dollar volume traded. They find that dollar volume of trading is statistically significant on the NYSE and AMEX but not on Nasdaq. However, the measure of liquidity used in their paper has two potential problems. First, it does not account for the number of shares on issue. Second, the use of the dollar volume would have a size bias.

Datar, Naik, and Radcliffe (1998) examine asset returns and liquidity using a share turnover ratio, defined as the number of shares traded divided by the number of shares outstanding, as a proxy for liquidity.⁴ They use a modified Fama and MacBeth

⁴ This is a more appropriate measure of liquidity than the raw number of shares traded as it controls for the number of shares outstanding.

(1973) approach in their analysis of cross-sectional returns. Size is measured as the (natural) log of the company's market value of equity in the prior month. Book-to-market is constructed along the lines of Fama and French (1992), and portfolio betas are constructed as in Amihud and Mendelson (1986). Datar, Naik, and Radcliffe find that liquidity plays a role in explaining the cross-sectional returns of stocks even after controlling for size, book-to-market, and beta. They also report that the liquidity effect is present throughout the year and is not restricted to January as Eleswarapu and Reinganum (1993) find.

A complementary paper to Datar, Naik, and Radcliffe (1998) is by Chui and Wei (1999). They test the liquidity hypothesis of Hopenhayn and Werner (1996) on NYSE, AMEX, and Nasdaq stocks and, using similar methods and variables to Datar, Naik, and Radcliffe (1998), report that the turnover ratio and book-to-market variable are significant in explaining cross-sectional returns. In contrast to Datar, Naik, and Radcliffe, they find that the liquidity effect is only significant in non-January months.

Rouwenhorst (1999) examines the cross-sectional returns of companies in emerging markets that were on the Emerging Markets Database of the International Finance Corporation.⁵ One difference from the earlier studies is that Rouwenhorst (1999) uses "turnover-sorted" portfolios ranked on a 30:40:30 split. He reports two main findings. First, the factors that explain cross-sectional stock returns in developed markets also explain emerging markets cross-sectional returns. Second, turnover is positively associated with the same factors that explain the cross-sectional returns of stocks.

Amihud (2002) and Pastor and Stambaugh (2001) use alternative empirical proxies to examine a different aspect of liquidity. Both papers examine the role of market liquidity risk. Amihud uses the daily ratio of the absolute stock return to its dollar volume as an approximate measure of the price impact of order flow. Amihud concludes that expected excess returns are in part a premium for stock illiquidity.

Pastor and Stambaugh (2001) use a "liquidity beta" that measures a stock's sensitivity to innovations in aggregate liquidity to examine expected stock returns. In a similar vein to Amihud (2002), the proxy relies on order flow and volume-related return reversals as its measure of liquidity. That is, the greater the expected stock return reversal for a given dollar volume, the more illiquid is the stock. In their analysis, stocks are assigned into one of 10 liquidity portfolios on a post-ranking basis. They report that smaller stocks are less liquid and are more sensitive to their measure of aggregate liquidity. The result is robust to the inclusion of a momentum factor. They also estimate a spread in expected returns of 7.5% per annum between low- and high-liquidity stocks.

⁵ Rouwenhorst (1999, p. 1442) acknowledges that this database is biased toward larger and more frequently traded shares.

Anderson, Clarkson, and Moran (1997) appears to be the only published study that seriously investigates the role of a liquidity factor using Australian data. For the period 1982 to 1989, they examine the role of size, seasonality, information, and liquidity risk in a multiple regression framework, but they do not have a formal asset-pricing foundation. Rather, they focus on explaining abnormal returns for a very limited sample—a set of 50 small stocks and 50 large stocks—using the average monthly dollar value of trading as their proxy for liquidity. They report that liquidity is not statistically significant.

As a follow-up Australian study, there is a range of major improvements and extensions that we make relative to Anderson, Clarkson, and Moran (1997). First, we conduct a more extensive analysis that goes considerably beyond analyzing 50 small and 50 large stocks. Second, we provide a more up-to-date analysis covering the decade of the 1990s. Third, we frame our analysis in a formal asset-pricing approach, the Fama and French (1993) framework. Finally, there is a concern that the use of monthly dollar value of trading by Anderson, Clarkson, and Moran does not take into account the number of shares *on issue* and this could bias the results. Therefore, we use a share turnover measure to represent liquidity.

3. Research method

The starting point for our experimental design is the Fama and French (1993) three-factor asset-pricing model augmented by the liquidity factor:

$$E(R_i) - R_f = b_i[E(R_m) - R_f] + s_i E(\text{SMB}) + h_i E(\text{HML}) + l_i E(\text{IMV}), \quad (1)$$

where R_i is the return on asset i , R_f is the return on the risk-free asset, R_m is the return on the market portfolio, SMB is the mimicking portfolio for the size factor, HML is the mimicking portfolio for the book-to-market factor, and IMV is a mimicking portfolio for the liquidity factor.⁶

It is useful to write the four-factor model in a form that parameterizes the premium for each factor:

$$E(r_i) = b_i \lambda_m + s_i \lambda_{\text{SMB}} + h_i \lambda_{\text{HML}} + l_i \lambda_{\text{IMV}}, \quad (2)$$

where $E(r_i)$ is the expected excess return on asset i , and λ denotes the respective factor premiums.

An empirical counterpart is:

$$r_{it} = \alpha_i + b_i(r_{mt} - \mu_m) + s_i(\text{SMB}_t - \mu_s) + h_i(\text{HML}_t - \mu_h) + l_i(\text{IMV}_t - \mu_l) + \text{error}_{it}, \quad (3)$$

⁶ As explained below, the liquidity mimicking factor is created by obtaining the mean return on a set of illiquid stock (I) portfolios minus (M) the mean return on a set of very (V) liquid stock portfolios hence the term IMV.

where $r_{it} = R_{it} - R_{ft}$, r_{mt} is the excess market return in month t , SMB_t is the return on a “small minus big” mimicking portfolio in month t , HML_t is the return on a “high minus low” mimicking portfolio in month t , and IMV_t is the return on an “illiquid minus very liquid” mimicking portfolio in month t . The parameters μ_m , μ_s , μ_h , and μ_l the means of each associated factor, which has the effect of mean-adjusting all of the independent variables.⁷ To allow identification of each mean parameter, we need to augment the system with mean equations for each factor as follows:

$$r_{mt} = \mu_m + \text{error}_{mt} \quad (4)$$

$$SMB_t = \mu_s + \text{error}_{st} \quad (5)$$

$$HML_t = \mu_h + \text{error}_{ht} \quad (6)$$

$$IMV_t = \mu_l + \text{error}_{lt} \quad (7)$$

After applying expectations to Equation (3) and comparing it to the asset-pricing model of Equation (2), we observe the cross-equation intercept restriction that constitutes the null hypothesis:

$$H_0: \alpha_i = b_i \lambda_m + s_i \lambda_{SMB} + h_i \lambda_{HML} + l_i \lambda_{IMV}; \quad i = 1, 2, \dots, N.$$

That is, the intercept term should capture the expected return predicted by the asset-pricing model. In the current setting, we convert Equation (3) into its restricted form of Equation (4) as follows:

$$r_{it} = b_i \lambda_m + s_i \lambda_{SMB} + h_i \lambda_{HML} + l_i \lambda_{IMV} + b_i (r_{mt} - \mu_m) + s_i (SMB_t - \mu_s) + h_i (HML_t - \mu_h) + l_i (IMV_t - \mu_l) + \text{error}_{it}. \quad (8)$$

This specification in the context of the system of Equations—(4) through (7) combined with (8) for each test portfolio—allows us to test the overall legitimacy of the four-factor model, and thus, by implication, it provides a test of the importance of a liquidity factor in that setting. We can test this four-factor model using a variety of different systems-based methods. In this paper, we choose the GMM approach of MacKinlay and Richardson (1991). Our estimation technique employs heteroskedasticity and autocorrelation consistent covariance matrices and, following Ferson and Foerster (1994), uses an iterated procedure.⁸ Further details of the testing procedure are as follows.

⁷ This empirical implementation comes at a cost, however, namely, that we are assuming the factor means are known. Such an assumption could lead to an overstatement of the statistical significance of the risk premiums (e.g., see Jagannathan and Wang, 2000; Cochrane, 2000) and the reader is cautioned accordingly. We thank an anonymous referee for alerting us to this concern.

⁸ An issue arises as to the reliability of iterated GMM. We address this in the context of our analysis later. We are grateful to an anonymous referee for bringing this to our attention.

In the case of the empirical model represented by Equations (4) through (8), there are $5N + 4$ sample moment equations. That is, there are five sample moment conditions for each of N test equations (see Equation (8)) as follows: (a) the mean regression error term is zero, and the regression error term is orthogonal to each regressor; namely, to (b) r_{mt} , (c) SMB_t , (d) HML_t , and (e) IMV_t and one sample moment condition that defines the mean of each factor—relating to each of Equations (4) to (7). Accordingly, the GMM test involves an evaluation of the $5N + 4$ sample moments, with $4N + 8$ unknown parameters to be estimated (i.e., $\phi = \mu_m, \mu_s, \mu_h, \mu_l, \lambda_m, \lambda_{SMB}, \lambda_{HML}, \lambda_{IMV}, b_1, b_2, \dots, b_N, s_1, s_2, \dots, s_N, h_1, h_2, \dots, h_N, l_1, l_2, \dots, l_N$). Hence, $N - 4$ over-identifying restrictions exist, and they can be tested using.⁹

$$J = (T - N - 4) \times g_T(\hat{\phi})' S_T^{-1} g_T(\hat{\phi}), \quad (9)$$

where $g_T(\hat{\phi}) = \frac{1}{T} \sum_{t=1}^T f_t(\hat{\phi})$, is the empirical moment condition vector and J is (asymptotically) distributed as a chi-square statistic with $N - 4$ degrees of freedom. All testing is conducted within this system of equations (multivariate) type framework.

4. Data and portfolio formation

4.1. Data issues

We use the monthly sampling interval for the period from January 1989 to December 1998, and all returns are measured as discrete periodic returns. In the formation of size, book-to-market, and liquidity portfolios outlined in Section 3, we collect data from two main sources. From the Integrated Real Time Equity System (IRESS) financial database, we collect for all currently listed companies, the volume of shares traded per month, the number of trades per month, the balance date, and the end of financial year balance sheet numbers to calculate a book value for each company.¹⁰ Companies without both a book value and trading activity data on IRESS are deleted from the sample.¹¹ The remaining companies are matched with the same companies from the Australian Graduate School of Management (AGSM) price relative file. From the AGSM price relative file, we extract the company share price data, the value-weighted market index, the risk-free return data, market capitalization, capital

⁹ This represents the small-sample adjusted version following MacKinlay and Richardson (1991).

¹⁰ IRESS is a real time database provided by Bridge News Service and Dunai Financial Service. From our discussion with IRESS, the financial statement data is originally sourced from the Australian Stock Exchange Financial Data (ASX's FINDATA) database.

¹¹ IRESS only retains currently listed companies in its financial statement database. For example, companies that were the subject of a takeover, bankruptcy, or that fail to meet the continuing listing requirements of the ASX are removed.

ranks, and the number of shares on issue for each company in each month of our sample period.¹²

4.2. *Mimicking portfolio formation*

For the size and book-to-market portfolio formation procedure, we follow Fama and French (1993). Size in (all months of) year t is proxied by the market value of equity from the AGSM price relative file at the end of December in year $t-1$. The companies are ranked on size and partitioned into small (S) and big (B) companies based on a 50:50 split. The book-to-market ratio in (all months of) year t is calculated at the end of December in year $t-1$ by dividing the book value by the market value at that date.¹³ Once more the sample companies are ranked—this time on the book-to-market figure—and the sample is partitioned into three groups, namely, high (H), medium (M), and low (L), based upon a 30:40:30 split.

The mimicking portfolio for liquidity is constructed as follows. First, we take the volume of shares traded per month and divide it by the quantity of shares on issue for that month, thereby producing a monthly “share turnover” ratio. The procedure is repeated for the whole calendar year $t-1$ to get 12 monthly liquidity (turnover) ratios. For each company and each calendar year, we then calculate an average monthly share turnover (liquidity).¹⁴ The ratio is used to rank companies into very liquid (V), moderately liquid (N) and illiquid (I) companies based on a 30:40:30 split.

Based on the independent sorts and ranking procedure in year $t-1$, we construct 18 portfolios (S/L/V, S/L/N, S/L/I, S/M/V, S/M/N, S/M/I, S/H/V, S/H/N, S/H/I, B/L/V, B/L/N, B/L/I, B/M/V, B/M/N, B/M/I, B/H/V, B/H/N, and B/H/I) from the intersection of the two size, three book-to-market, and three liquidity groups. The return for each of these 18 portfolios are value-weighted. Companies that have been sorted into these

¹² Capital rank represents a ranking of each company based on the size of their total capital or market capitalization relative to all other listed securities during that month. A rank of one is assigned to the company with the largest total capital, a rank of two is assigned to the next largest company on that date, and so on. The highest number rank corresponds to the company with the smallest total capital. The highest number rank will vary from month to month depending on the number of listed companies.

¹³ We use net tangible assets (NTA) as our book value. From the figures supplied by IRESS, we reconstruct the balance sheet to calculate the NTA. We check for accuracy of the data by comparing the reconstructed total assets less total liabilities with the reconstructed shareholders equity. The financial year-end for Australia is June, compared to the U.S. year-end of December. In our portfolio formation process, we use book value as of June and the market value in December to make it consistent with the reasoning behind the July portfolio formation date of Fama and French (1992, 1993).

¹⁴ We calculate a relative liquidity ratio because the use of an absolute volume of shares traded would be distorted by the number of shares on issue. Furthermore, we take the average of 12 monthly liquidity ratios as our measure of the company’s liquidity throughout the year. This is to avoid relying on a single monthly figure and the possible effect of seasonality. The annual averaging process also helps to control for any “glamour” effect (Lee and Swaminathan, 2000) that may be more evident in month-by-month share turnover data.

18 portfolios in year $t-1$ are then used to form the mimicking portfolios that proxy for the size, book-to-market, and liquidity factors in year t .

Extending the procedure developed in Fama and French (1993), the mimicking portfolios are created as follows. Our size mimicking portfolio, SMB (small minus big) is the difference in each month between the simple average of the returns on the nine small company portfolios (S/L/V, S/L/N, S/L/I, S/M/V, S/M/N, S/M/I, S/H/V, S/H/N, and S/H/I) and the simple average of the returns on the nine big company portfolios (B/L/V, B/L/N, B/L/I, B/M/V, B/M/N, B/M/I, B/H/V, B/H/N, and B/H/I).

The book-to-market mimicking portfolio, HML, (high minus low) is the difference in each month between the simple average of the returns on the six high book-to-market company portfolios (S/H/V, S/H/N, S/H/I, B/H/V, B/H/N, and B/H/I) and the simple average of the returns on the six low book-to-market company portfolios (S/L/V, S/L/N, S/L/I, B/L/V, B/L/N, and B/L/I).

The liquidity mimicking portfolio IMV (illiquid minus very liquid) is the difference in each month between the simple average of the returns on the six illiquid company portfolios (S/L/I, S/M/I, S/H/I, B/L/I, B/M/I, and B/H/I) and the simple average of the returns on the six very liquid company portfolios (S/L/V, S/M/V, S/H/V, B/L/V, B/M/V, and B/H/V).

One advantage of constructing the three mimicking portfolios—SMB, HML, and IMV—in this way is that they are all (approximately) orthogonalized with respect to each other. This orthogonality feature occurs because each mimicking portfolio is formed while controlling for the effect of the other two factors. For example, in forming the IMV mimicking portfolio every constituent illiquid portfolio has a matching very liquid counterpart (e.g., S/L/I vs. S/L/V). This is very important, in particular, with regard to the liquidity mimicking portfolio because there is legitimate concern that IMV could be proxying for either a glamour-value or size effect. For example, high (low) turnover stocks tend to behave like glamour (value) stocks (Lee and Swaminathan, 2000). The Fama–French (1993) type procedure for forming the mimicking portfolios ensures that the IMV effect is separate from the HML effect thereby controlling for possible contamination of the value effect.

4.3. *Dependent variable portfolio formation*

In the spirit of Fama and French (1993), we use excess returns on portfolios formed on the basis of size, book-to-market, and liquidity as our dependent variables. Specifically, we use a trivariate approach in the formation of 27 portfolios (as opposed to the $5 \times 5 = 25$ portfolios used by Fama and French).¹⁵ Accordingly, all sample

¹⁵ Ideally, we would prefer to construct portfolios using a three-way approach using the quintile split (i.e., $5 \times 5 \times 5 = 125$ portfolios). However, this outcome is infeasible in a systems framework. Furthermore, it would produce average portfolio sizes of approximately five stocks, which would inevitably lead to some portfolios having very few or even zero stocks.

Table 1

Number of companies in the sample for each year

This table shows the number of companies that satisfy various selection criteria. Because the methodology requires a liquidity ratio, the number of shares on issue per month is extracted from the Australian Graduate School of Management (AGSM) price relative file. This is matched with the price, trading volume, and accounting data extracted from Integrated Real Time Equity System (IRESS). Only companies in which there was trading volume for each of the 12 months of the calendar year were included in the final sample. Also, initial public offerings (IPOs) were not included in the final sample as the shares in these companies are heavily traded immediately following their listing. Thus, the omission of IPOs aims to avoid this potential distortion of the liquidity ratio. The percentage contained in parentheses is the sample size relative to the total number of stocks available.

Year	Total number of stocks available	Sample size (%)
1990	1,345	448 (33.31%)
1991	1,173	412 (35.12%)
1992	1,060	446 (42.08%)
1993	1,095	465 (42.47%)
1994	1,176	531 (45.15%)
1995	1,171	608 (51.92%)
1996	1,182	695 (58.80%)
1997	1,206	739 (61.28%)
1998	1,206	798 (66.17%)
Average	1,179	571 (48.45%)

stocks are independently sorted by size, book-to-market, and turnover. In the case of each sort, stocks are allocated into three groups according to a 30:40:30 partition, and 27 portfolios are then formed from the intersection of the three size, three book-to-market, and three share turnover groups. These 27 portfolios are formed in a similar way to the 18 size/book-to-market/turnover portfolios described earlier.

From the construction of the portfolios, we calculate value-weighted monthly returns from January to December of year t . The excess returns are the returns on the portfolios less a proxy for the risk free rate. The risk free proxy is an equivalent monthly return derived from 13-week treasury notes taken from the AGSM price relative file. The excess returns are the dependent variables used in our system based GMM regression.

Table 1 presents a year-by-year summary of the number of companies included in our analysis compared to the total number of available stocks. As shown in the table, the final sample averages 571 companies—a figure that constitutes just under half the average total number of available stocks. From the table, we observe a minimum sample of 412 companies (35.12% of available stocks) in 1991 and a maximum of 798 companies (66.17% of available stocks) in 1998. Any company that cannot be matched to the AGSM price relative file or that were initial public offerings is excluded from our final sample.

Table 2 gives a further perspective on the representativeness of our sampling procedure in the form of a size (market capitalization) comparison of our sample

Table 2

Size comparison of sample stocks versus all stocks

This table reports the size, as measured by market capitalization of equity (MCap) for our sample compared to the population of Australian listed stocks. Market capitalization is measured at calendar year-end for all companies in the sample. In this table, capital rank represents a ranking of each company based on the size of their total capital or market capitalization relative to all other listed securities during December month year-end. A rank of one is assigned to the company with the largest total capital, a rank of two is assigned to the next largest company on that date, and so on. The highest number rank corresponds to the company with the smallest total capital. Companies in the first quintile or top 20% represents those companies with capital ranks in the top 20% according to market capitalization.

Year	Maximum	Minimum	Median	Mean	Total (\$Abil)
<i>Panel A: Size (MCapA\$mil)—Current sample</i>					
1990	15,148.12	0.075	4.858	215.778	1,085
1991	21,510.49	0.108	9.484	348.955	1,563
1992	21,527.50	0.226	8.697	337.728	1,743
1993	29,127.80	0.350	19.814	475.239	2,709
1994	33,607.20	0.255	17.710	387.900	2,565
1995	37,039.01	0.393	18.408	410.780	3,002
1996	35,606.97	0.316	22.960	433.715	3,776
1997	30,266.97	0.658	21.030	474.953	4,338
1998	35,628.01	0.512	19.168	484.887	4,819
<i>Panel B: Size (MCapA\$mil)—All stocks</i>					
1990	15,148.12	0.032	3.000	125.580	1,863
1991	21,510.49	0.024	4.620	198.646	2,510
1992	21,527.50	0.020	6.403	234.592	3,791
1993	29,127.80	0.050	16.510	340.667	4,771
1994	33,607.20	0.097	17.011	292.265	4,291
1995	37,039.01	0.059	18.733	362.231	5,104
1996	35,606.97	0.028	23.985	423.526	5,798
1997	30,266.97	0.025	21.541	458.051	9,661
1998	35,628.01	0.059	18.988	499.843	9,891
<i>Panel C: Capital rank</i>					
Year	Top 20%	20–40%	40–60%	60–80%	Bottom 20%
1990	26.1	21.9	20.0	20.0	12.0
1991	28.2	22.1	21.1	20.1	8.5
1992	23.5	20.2	20.2	20.9	15.2
1993	22.6	20.4	20.6	22.6	13.8
1994	22.0	19.8	20.1	20.5	17.6
1995	19.1	18.7	22.4	23.5	16.3
1996	17.9	19.7	23.6	22.4	16.4
1997	18.8	19.1	22.3	22.3	17.4
1998	18.8	20.3	21.4	21.4	17.3

versus the total set of stocks prior to filtering. From the table we can see that, as expected, the sampling procedure is skewed toward the largest Australian stocks and thus suggests our analysis is biased against detecting a liquidity effect in asset pricing. Nevertheless, the table also clearly demonstrates that our sample does include

a number of very small stocks. For example, consider the minimum size of companies included in our sample. We see that in all years, companies of substantially less than A\$1 million market capitalization are present. Specifically, in 1990 a company of only A\$75,000 is included as compared to the counterpart minimum of A\$32,000 for the unfiltered sample. Further, a comparison of yearly median size figures is also quite instructive in that very minimal differences are evident between our sample and the full set of available companies.

As a final means of gauging the extent to which our sample of companies is representative of the total sample, capital ranks of each company are taken at year-end and grouped into quintiles. Companies in the first quintile are those in the top 20% according to market capitalization, the second quintile contains companies in the next 20% and so on. We compare the percentage of companies in our sample that fell within the above quintiles. Table 2 shows that apart from the top and bottom quintile for the years 1990 and 1991, most of our sample is broadly representative of the universe of all stocks in Australia. In short, based on the size comparisons, while our sample is skewed toward the larger and, hence, more liquid stocks, there are enough small, illiquid stocks for a meaningful investigation of a liquidity factor.¹⁶

5. Results

5.1. Background and descriptive statistics

Table 3 reports the average and minimum number of companies, and the number of months in which the portfolio comprises fewer than ten companies for each of the 27 dependent variable portfolios. The largest average number of companies is 35.3 for Portfolio 7 (small, high book-to-market, illiquid companies) and the smallest is 6.9 for Portfolio 27 (big, high book-to-market, very liquid companies). Indeed, Portfolio 27 is clearly the least popular, comprising just three stocks in one month and having 72 months (out of 108) in which fewer than 10 companies are included.

Table 4 reports some basic descriptive statistics and correlations for the (excess return) market index, for the Fama–French (1993) mimicking factors and for the liquidity (share turnover) mimicking factor. The average market risk premium is negative but very close to zero. The mean return for the size factor (SMB) is significantly positive with an average risk premium equivalent to over 20% per annum. The average return on the book-to-market (HML) and the liquidity (IMV) factors are positive and equivalent to 10% and 4% per annum, respectively. The HML mean return is significantly different from zero. The relatively modest figure for the turnover factor provides some confidence that we are not picking up a value-glamour effect (Lee and Swaminathan, 2000); the magnitude is more consistent with a liquidity premium.

¹⁶ We do not believe that survivorship is a major problem for our study. The measures described in the text suggest our sample is very representative of the population. We believe that, if anything, to the extent that we do have a survivorship bias it will go against finding results in favor of a liquidity factor. This belief rests on the view that less liquid stocks are more likely to be excluded, but they are the very stocks that should drive a liquidity premium.

Table 3

Descriptive statistics for the dependent variable portfolios

This table reports, for each of the 27 dependent variable portfolios over the 108 months from January 1990 to December 1998, the mean and standard deviation of returns, the average number of companies, the minimum number of companies, and the number of months in which the portfolio comprised fewer than 10 companies. Each sort involves a 30:40:30 partition of the companies based on the characteristic in question.

Portfolio	Size	Book-to-market	Liquidity	Average number of		Number of months
				stocks	minimum	
1	Small	Low	Illiquid	13.1	5	48
2	Small	Low	Medium	16.8	8	12
3	Small	Low	Very liquid	21.0	9	12
4	Small	Medium	Illiquid	14.6	6	12
5	Small	Medium	Medium	14.1	10	0
6	Small	Medium	Very liquid	21.8	12	0
7	Small	High	Illiquid	35.3	18	0
8	Small	High	Medium	26.1	15	0
9	Small	High	Very liquid	28.6	13	0
10	Medium	Low	Illiquid	12.6	5	24
11	Medium	Low	Medium	19.3	5	24
12	Medium	Low	Very liquid	29.2	10	0
13	Medium	Medium	Illiquid	24.3	13	0
14	Medium	Medium	Medium	20.4	15	0
15	Medium	Medium	Very liquid	18.1	10	0
16	Medium	High	Illiquid	33.8	12	0
17	Medium	High	Medium	19.7	15	0
18	Medium	High	Very liquid	13.2	4	36
19	Big	Low	Illiquid	21.2	10	0
20	Big	Low	Medium	32.3	20	0
21	Big	Low	Very liquid	28.3	16	0
22	Big	Medium	Illiquid	22.1	13	0
23	Big	Medium	Medium	26.7	16	0
24	Big	Medium	Very liquid	25.8	12	0
25	Big	High	Illiquid	11.6	6	36
26	Big	High	Medium	13.4	4	36
27	Big	High	Very liquid	6.9	3	72

Table 4 also reports a high negative correlation between the excess market return and the liquidity factor return. There also is a relatively small negative correlation between the returns on the mimicking portfolios (in the range of -0.1 to -0.2). Such low correlations support the earlier claim of approximate orthogonality among the three factors, in line with Fama and French, who report a correlation between their original SMB and HML factor returns of -0.08 (see French and Fama, 1993, Table 2, p. 14).

Table 4

Basic descriptive statistics for and correlations between excess market returns and Fama and French (1993) mimicking portfolio factor returns

Panel A reports the mean, median, maximum, minimum, standard deviation, skewness, and kurtosis for the excess market return and for the mimicking portfolio factor returns of size (SMB); book-to-market (HML), and liquidity (IMV). Panel B reports correlations between the excess market returns and the SMB, HML, and IMV factor returns. Data are monthly covering the period from January 1990 to December 1998. AGSM = Australian Graduate School of Management. VW = value weighted.

	Excess market return	Factor returns		
	VW AGSM	SMB	HML	IMV
<i>Panel A: Basic descriptive statistics</i>				
Mean	−0.0020	0.0176*	0.0082*	0.0031
Mean standard error	0.0036	0.0060	0.0040	0.0040
<i>t</i> -statistic (H_0 : mean = 0)	−0.57	2.91	2.08	0.76
Median	0.0029	0.0032	0.0088	0.0031
Maximum	0.0758	0.2832	0.1707	0.1035
Minimum	−0.1035	−0.1171	−0.1036	−0.1188
SD	0.0371	0.0628	0.0411	0.0418
Skewness	−0.3891	1.2644	0.3725	−0.1197
Kurtosis	2.9486	5.6010	4.7266	3.0329
<i>Panel B: Correlations</i>				
VW AGSM	1	—	—	—
SMB	0.117	1	—	—
HML	−0.047	−0.196	1	—
IMV	−0.494	−0.094	−0.158	1

* Indicates statistical significance at the 0.05 level.

5.2. Portfolio factor sensitivity estimates

In this section we aim to provide a general feel for the sign, magnitude, and significance of the factor loading (factor beta) estimates in our model. Accordingly, Table 5 presents the results for running a system of restricted FF three-factor models augmented with a liquidity factor for a subset of our 27 size, book-to-market and turnover sorted portfolios. Specifically, the subset is the eight portfolios that do not possess any medium characteristics in either size, book-to-market, or liquidity. We refer to this group as the “extreme” characteristic portfolios.

From Table 5 several key features are evident. First, with regard to the market betas, all cases are significantly positive. Second, four out of eight cases are significant and positive for the size (SMB) factor beta. As expected, all of these are portfolios that have a small stock focus. One portfolio has a significantly negative size beta, and this is one of the big stock portfolios (i.e., the portfolio with low book-to-market and very liquid). Third, all but two of the eight HML betas are significant of which three are positive (all high book-to-market portfolios) and three are negative (all low

Table 5

Generalized method of moments (GMM) system estimation of the liquidity-augmented Fama–French (1993) model factor betas for portfolios with “extreme” characteristics

This table reports the market, size (SMB), book-to-market (HML), and liquidity (IMV) beta estimates from a restricted GMM system estimation of the liquidity-augmented Fama–French model for eight test portfolios having “extreme” size, book-to-market, and liquidity characteristics. The estimated system of regressions is given by:

$$r_{mt} = \mu_m + \text{error}_{mt} \quad (4)$$

$$\text{SMB}_t = \mu_s + \text{error}_{st} \quad (5)$$

$$\text{HML}_t = \mu_h + \text{error}_{ht} \quad (6)$$

$$\text{IMV}_t = \mu_l + \text{error}_{lt} \quad (7)$$

$$r_{it} = b_i \lambda_m + s_i \lambda_{\text{SMB}} + h_i \lambda_{\text{HML}} + l_i \lambda_{\text{IMV}} + b_i(r_{mt} - \mu_m) + s_i(\text{SMB}_t - \mu_s) + h_i(\text{HML}_t - \mu_h) + l_i(\text{IMV}_t - \mu_l) + \text{error}_{it}, \quad \text{for } i = 1, 2, \dots, N \quad (8)$$

where r_{it} is the excess return for asset i in month t , r_{mt} is the excess market return in month t , SMB_t is the return on a “small minus big” mimicking portfolio in month t , HML_t is the return on a “high minus low” mimicking portfolio in month t , and IMV_t is the return on a “illiquid minus very liquid” mimicking portfolio in month t . The market index used is the Australian Graduate School of Management value-weighted market. The associated t -statistic is contained in parentheses below the coefficient estimate. Data are monthly covering the period from January 1990 to December 1998. DW = Durbin Watson statistic.

Portfolio	Size	Book-to-market	Liquidity	Factor beta estimate				R^2	DW
				b_i	s_i	h_i	l_i		
1	Small	Low	Illiquid	0.9159* (2.53)	2.0536* (6.57)	−1.1713* (−3.60)	1.0411* (2.05)	0.433	2.35
3	Small	Low	Very liquid	0.4178* (2.57)	1.1795* (14.13)	−0.4130* (−3.09)	−0.8343* (−6.32)	0.712	2.49
7	Small	High	Illiquid	0.6641* (4.78)	0.6843* (8.68)	0.2718* (2.99)	0.0681(0.45)	0.491	2.01
9	Small	High	Very liquid	0.8051* (5.16)	1.4618* (16.60)	0.2049* (1.98)	−1.0980* (−6.49)	0.742	1.93
19	Big	Low	Illiquid	0.7417* (8.32)	0.0860 (1.77)	−0.0495 (−0.90)	−0.0445 (−0.50)	0.496	2.01
21	Big	Low	Very liquid	0.9795* (12.44)	−0.0640* (−2.30)	−0.1316* (−2.56)	−0.0859(−1.42)	0.753	1.76
25	Big	High	Illiquid	0.8065* (6.10)	0.0316 (0.42)	0.0670 (0.70)	0.1945* (1.98)	0.259	1.68
27	Big	High	Very liquid	0.8041* (6.51)	0.0411 (0.81)	0.5914* (3.55)	−0.2765* (−3.04)	0.452	1.76

* Indicates statistical significance at the 0.05 level.

book-to-market portfolios). Fourth, Table 5 reveals that five of the eight betas are statistically significant for the liquidity (share turnover) factor. Notably, there is a strong pattern that the illiquid portfolios have positive or at least less negative IMV betas, and two of these cases are significantly positive at the 5% level. In contrast, three of the very liquid portfolios have significantly negative IMV betas.

The final observation worthy of mention from Table 5 relates to the regression R^2 's, which are lower than those in Fama and French (1993) (where all were around 0.9 or above). This no doubt reflects the fact that our portfolios contain fewer stocks and therefore have a higher ratio of noise to signal. This conjecture finds reasonable support when we reconsider Table 3, which reports some information regarding the number of stocks in each portfolio. Specifically, a comparison of Tables 3 and 5 reveals that some of the portfolios that contain low average stock numbers (e.g., Portfolio 1: $S/L/I = 13.1$ stocks; Portfolio 25: $B/H/I = 11.6$ stocks; Portfolio 27: $B/H/V = 6.9$ stocks) tend to have the lower R^2 's. However, the fact that the portfolio with the highest average stock count (Portfolio 7: $S/H/I = 35.3$) has a rather low R^2 of 0.491 suggests that other factors are also at play.¹⁷

5.3. Main asset-pricing test results

Table 6 presents results of restricted systems based GMM estimation and tests of the liquidity-augmented Fama–French (1993) model (as outlined in Section 2).¹⁸ The table also presents estimates of the premiums attaching to each of the four factors. Due to the fact that a 27-equation system is infeasible, initially we (arbitrarily) split the analysis into two parts. We analyze separately the odd- and the even-numbered portfolios.

The most outstanding and pervasive finding reported in Table 6 is that, based on the outcome of the GMM (J) test statistic, we cannot reject the test of the over-identifying restrictions at the 5% level of significance. Of additional interest are the findings with regard to the estimates of the factor premiums. First, we observe that the

¹⁷ In an effort to explore further the relation between the factor betas of different portfolios, we conduct various hypothesis tests of equality. Specifically, we conduct three sets of tests. The main set focuses on whether the liquidity (IMV) beta on the high turnover portfolio is significantly lower than the liquidity beta on the counterpart (after controlling for size and book-to-market) low turnover portfolio. Analogous tests of equality of SMB and HML betas are also conducted, and all such tests are performed allowing for cross-correlation effects. In short, the overwhelming finding is beyond dispute: The tests of beta equality are resoundingly rejected. Moreover, in all cases the rejection occurs in the expected direction. In only one instance, namely, for the big size/low book-to-market portfolios, can the equality of IMV betas not be rejected. Details are available from the authors on request.

¹⁸ All GMM estimation in this study was performed using EViews. We use the optimal weighting matrix, which is the inverse of the covariance matrix of the sample moments. Specifically, we impose a heteroskedasticity and autocorrelation consistent covariance matrix in all estimation, which involves a Bartlett kernel with a Newey–West fixed bandwidth and no pre-whitening. The initial weighting matrix is obtained using consistent two-stage least squares initial estimates of the parameter set.

Table 6

Generalized method of moments (GMM) system tests of the liquidity-augmented Fama–French (1993) model

The tests reported in this table are based on the following system:

$$r_{mt} = \mu_m + \text{error}_{mt} \quad (4)$$

$$\text{SMB}_t = \mu_s + \text{error}_{st} \quad (5)$$

$$\text{HML}_t = \mu_h + \text{error}_{ht} \quad (6)$$

$$\text{IMV}_t = \mu_l + \text{error}_{lt} \quad (7)$$

$$r_{it} = b_i \lambda_m + s_i \lambda_{\text{SMB}} + h_i \lambda_{\text{HML}} + l_i \lambda_{\text{IMV}} + b_i (r_{mt} - \mu_m) + s_i (\text{SMB}_t - \mu_s) + h_i (\text{HML}_t - \mu_h) + l_i (\text{IMV}_t - \mu_l) + \text{error}_{it}, \quad \text{for } i = 1, 2, \dots, N \quad (8)$$

where r_{it} is the excess return for asset i in month t , r_{mt} is the excess market return in month t , SMB_t is the return on a “small minus big” mimicking portfolio in month t , HML_t is the return on a “high minus low” mimicking portfolio in month t , and IMV_t is the return on a “illiquid minus very liquid” mimicking portfolio in month t . The GMM test statistic, J , has had the small sample size adjustment applied (following MacKinlay and Richardson, 1999) and is distributed as a chi-square with $N - 4$ degrees of freedom. The associated p -value is contained in parentheses. The associated t -statistic is contained in parentheses below the coefficient estimate. The likelihood ratio test (LRT) tests the significance of the IMV risk premium. The LRT test statistic has had the small sample size adjustment applied, is distributed as a chi-square with 1 degree of freedom and the associated p -value is contained in square brackets. Data are monthly covering the period from January 1990 to December 1998.

	Odd-numbered portfolios	Even-numbered portfolios
J	8.962 (0.44)	6.775 (0.66)
λ_m	0.0094* (2.22)	0.0113* (2.94)
λ_{SMB}	0.0094 (1.69)	0.0397* (4.49)
λ_{HML}	0.0083 (1.17)	0.0067 (1.40)
λ_{IMV}	0.0118* (3.33)	0.0291* (2.51)
LRT	0.760	7.363*
$H_0: \lambda_{\text{IMV}} = 0$	[0.383]	[0.007]

* Indicates statistical significance at the 0.05 level.

market premium is significant and positive in sign, averaging about 1% per month. Second, the estimated size premium is positive in both cases and highly significant for the even-numbered portfolio analysis. The average estimated value for the size premium is about 2.4% per month. Third, although insignificant, the book-to-market premium is positive in both cases. Fourth, with regard to the turnover premium, we see very strong evidence of a positive and significant estimate. However, although the magnitude of the turnover premium estimates produced in the odd-numbered portfolio analysis is quite plausible (around 1.2% per month), the even-numbered portfolios suggest much higher values. The average estimated turnover premium across the two sets of analysis is around 2% per month, annualizing well in excess of 20%.

A question arises regarding the plausibility of the premium estimates—particularly the liquidity premium. In this case, does an annualized figure of 20–25%

make economic sense? Unfortunately, there is no consensus answer to this question. Recent evidence in Pastor and Stambaugh (2001) suggests a premium in the order of 7.5%, which makes our estimate appear huge. However, it is important to bear in mind that our focus on a smaller, less liquid market would be expected to reveal higher illiquidity effects. Moreover, Amihud and Mendelson (1988) demonstrate a value impact in the order of 20% induced by the compounding nature of the transaction costs associated with repeated trading. As argued by the authors, this is a magnitude of considerable economic significance. In this context, our estimates appear more credible.

The preceding discussion needs to be cautiously revisited. Because we impose a nonlinear restriction in these tests, relying on the t -test to determine whether the IMV factor belongs is technically incorrect. To gain a more reliable assessment we calculate a likelihood ratio test (LRT) of the IMV risk premium. The outcome of this analysis is found in the bottom row labeled LRT $H_0: \lambda_{IMV} = 0$ in Table 6.¹⁹ Although the asset-pricing role of the IMV factor is strongly reinforced for the even-numbered portfolios, it is seriously questioned for their odd-numbered counterparts. Indeed, it generally seems that the odd-numbered portfolios are proving more troublesome for the four-factor asset-pricing model. This is an issue we investigate further in the following subsection.

Before moving on to this additional testing, one further issue is worthy of investigation. Because the factors are all “traded” and, hence, measured in returns form, an estimate of the risk premium is just the sample average of the factor. As such, in theory there ought to be a reasonably close alignment between the estimated risk premiums (from the systems analysis) and their counterpart sample factor means. Accordingly, consider a simple comparison of the sample means reported in Table 4 to the Table 6 premiums estimates. First, with regard to the excess market return, we observe a negative (albeit, statistically insignificant) sample average, whereas Table 6 reports a healthy and significant market risk premium of around 1% per month. Second, with regard to the SMB and HML factors, the average estimated premiums reported in Table 6 are highly comparable to their Table 4 sample averages. Third, similar to the case of the market factor, the average IMV premium estimate (around 2% per month) is considerably higher than the Table 4 sample mean (at 0.3% per month).

To explore these differences further, Table 7 reports the outcome of formally testing the pairwise equality of the factor means to their associated risk premiums in our three GMM systems-based experimental settings: (1) odd-numbered portfolios, (2) even-numbered portfolios, and (3) extreme characteristics portfolios.

From the table several interesting features are evident. First, it can be seen that the equality hypothesis is strongly rejected for the market factor in all cases. It seems

¹⁹ The square of the t -test is a Wald test, and it is not invariant to reparameterization in a nonlinear model. Also, it should be noted that the LRT tests use the same (one-step) weighting matrix for both the restricted and unrestricted estimates. We gratefully acknowledge an anonymous referee for drawing these issues to our attention.

Table 7

Tests of factor mean and risk premium equality

This table reports pairwise tests for the equality of each factor mean with its associated premium estimate. The tests are conducted within the following system:

$$r_{mt} = \mu_m + \text{error}_{mt} \quad (4)$$

$$\text{SMB}_t = \mu_s + \text{error}_{st} \quad (5)$$

$$\text{HML}_t = \mu_h + \text{error}_{ht} \quad (6)$$

$$\text{IMV}_t = \mu_l + \text{error}_{lt} \quad (7)$$

$$r_{it} = b_i \lambda_m + s_i \lambda_{\text{SMB}} + h_i \lambda_{\text{HML}} + l_i \lambda_{\text{IMV}} + b_i (r_{mt} - \mu_m) + s_i (\text{SMB}_t - \mu_s) + h_i (\text{HML}_t - \mu_h) + l_i (\text{IMV}_t - \mu_l) + \text{error}_{it}, \quad \text{for } i = 1, 2, \dots, N \quad (8)$$

where r_{it} is the excess return for asset i in month t , r_{mt} is the excess market return in month t , SMB_t is the return on a “small minus big” mimicking portfolio in month t , HML_t is the return on a “high minus low” mimicking portfolio in month t , and IMV_t is the return on a “illiquid minus very liquid” mimicking portfolio in month t . The test statistic is distributed as a chi-square with 1 degree of freedom. The associated p -value is contained in parentheses. Data are monthly covering the period from January 1990 to December 1998.

Factor	Portfolio group		
	Odd numbered	Even numbered	“Extreme” characteristics
Market	37.210* (0.000)	27.800* (0.000)	15.722* (0.000)
SMB	0.600 (0.439)	7.687* (0.006)	6.058* (0.014)
HML	0.035 (0.852)	0.010 (0.919)	4.762* (0.029)
IMV	4.588* (0.032)	5.236* (0.022)	3.135 (0.077)

* Indicates statistical significance at the 0.05 level.

the modeling is telling us that despite the fact that the realized sample mean market excess return is negative, an *ex ante* positive market risk premium is demanded by the market. Second, for both the SMB and IMV factors, the equality hypothesis is rejected in two out of three cases, but as noted above, the Table 4 versus Table 6 comparison for SMB is generally quite good. Third, for the HML factor the equality hypothesis is only rejected once. Taken together, the rejections of the equality hypothesis are consistent with the view that the factors are not truly “traded”; to an extent the returns are the outcome of hypothetical rather than implementable portfolio strategies (primarily due to binding short-selling constraints).

5.4. Additional asset-pricing test results

In summary, the results presented in Table 6 suggest that there is a reasonable degree of support for a four-factor model that incorporates a share turnover factor because the GMM tests fail to reject the over-identifying restrictions imposed by

the model. Moreover, the estimated premiums are generally positive and significant. Nevertheless, the outcome of our analysis is sufficiently mixed to caution us from being unduly enthusiastic about its merits.

Accordingly, the current section pursues the testing a little further. One observation highlighted above is that the arbitrary split into odd- and even-numbered portfolios produces some variation in results. Specifically, it was apparent that the odd-numbered portfolios produce evidence less favorable to the liquidity-augmented model. The question is: Is there some systematic difference between the two sets of portfolios that can potentially explain this?

On closer inspection, the answer is yes. It turns out that among the odd-numbered portfolios are all the portfolios that do not contain any of the “medium” characteristics, which earlier we referred to as the “extreme” characteristic portfolios. It is plausible that these portfolios contain a sufficiently diverse set of cases that, as a group, are more difficult to price. For example, Portfolio 7 comprises those sample stocks that are small in size, have a high book-to-market, and are illiquid. Traditionally, these are the very characteristics that have proven most puzzling in the empirical asset-pricing literature. To test this proposition more directly, we focus on that subset of our 27 test portfolios that do not have medium characteristics in either size, book-to-market, or liquidity. This group of portfolios numbers 8, as identified earlier, in the context of Table 5. The tests reported in Table 6 are repeated on this group of extreme characteristic portfolios, and the outcome of this analysis is reported in Table 8. Moreover, following on from the discussion at the end of the previous subsection, we present two versions of the tests. Panel A (Panel B), reports tests that (do not) impose the restriction that factor means must equal their counterpart factor risk premiums, labeled the “full restricted model” (“partially restricted model”).

Table 8 shows that the evidence in favor of the liquidity-augmented model is mixed. Specifically, based on the outcome of the GMM (J) test statistic, with p -values of 0.10 and 0.30, we again fail to reject the test of the over-identifying restrictions at conventional levels. These p -values are lower than those reported in the counterpart Table 6 tests. Moreover, the findings support the view that the liquidity-augmented asset-pricing model is more challenged in pricing these extreme characteristic portfolios. The one other result worthy of comment in Table 8 relates to the LRT test of the hypothesis, $H_0: \lambda_{IMV} = 0$. Again we see that the liquidity premium is finding it difficult to command a significant role in the model in the context of these hard-to-price portfolios.

Finally, we investigate whether the new model outperforms the Fama and French (1993) three-factor alternative. We retain our focus on the same set of eight hard-to-price portfolios considered in the first columns of Table 8, Panels A and B. Specifically, we rerun the same type of GMM tests on the model excluding the liquidity factor, and the outcome is reported in the second columns of Table 8, Panels A and B.

The most striking result, based on the outcome of the GMM (J) test statistic with a p -value of 0.03, is that we can reject the test of the over-identifying restrictions in the

Table 8

Generalized method of moments (GMM) system tests of the liquidity-augmented versus conventional Fama–French (1993) model—Portfolios with “extreme” size, book-to-market, and liquidity characteristics

The tests reported are based on GMM system estimation for eight test portfolios having “extreme” size, book-to-market, and liquidity characteristics. In the case of the partially restricted model (including IMV) the system is given by:

$$r_{mt} = \mu_m + \text{error}_{mt} \quad (4)$$

$$\text{SMB}_t = \mu_s + \text{error}_{st} \quad (5)$$

$$\text{HML}_t = \mu_h + \text{error}_{ht} \quad (6)$$

$$\text{IMV}_t = \mu_l + \text{error}_{lt} \quad (7)$$

$$r_{it} = b_i \lambda_m + s_i \lambda_{\text{SMB}} + h_i \lambda_{\text{HML}} + l_i \lambda_{\text{IMV}} + b_i (r_{mt} - \mu_m) + s_i (\text{SMB}_t - \mu_s) + h_i (\text{HML}_t - \mu_h) + l_i (\text{IMV}_t - \mu_l) + \text{error}_{it}, \quad \text{for } i = 1, 2, \dots, N \quad (8)$$

where r_{it} is the excess return for asset i in month t , r_{mt} is the excess market return in month t , SMB_t is the return on a “small minus big” mimicking portfolio in month t , HML_t is the return on a “high minus low” mimicking portfolio in month t , and IMV_t is the return on a “illiquid minus very liquid” mimicking portfolio in month t . The GMM test statistic, J , has had the small sample size adjustment applied (following MacKinlay and Richardson, 1999). The associated p -value is contained in parentheses. The associated t -statistic is contained in parentheses below the coefficient estimate. The likelihood ratio test (LRT) tests the significance of the IMV risk premium. The LRT test statistic has had the small sample size adjustment applied, is distributed as a chi-square with 1 degree of freedom, and the associated p -value is contained in square brackets. Data are monthly covering the period from January 1990 to December 1998.

	Panel A: Full restricted model (Factor means = Factor risk premiums)		Panel B: Partially restricted model (Factor means \neq Factor risk premiums)	
	Model including IMV	Model excluding IMV	Model including IMV	Model excluding IMV
J	13.263 (0.10)	15.198 (0.08)	4.910 (0.30)	12.585* (0.03)
λ_m	−0.0078* (−2.31)	−0.0088* (−2.65)	0.0056 (1.30)	0.0026 (0.60)
λ_{SMB}	0.0107 (1.73)	0.0167* (2.60)	0.0283* (3.15)	0.0293* (4.28)
λ_{HML}	0.0172* (6.14)	0.0149* (5.02)	0.0269* (3.56)	0.0080 (1.81)
λ_{IMV}	0.0110* (3.91)		0.0173* (2.75)	
LRT			0.937	
$H_0: \lambda_{\text{IMV}} = 0$			[0.335]	

* Indicates statistical significance at the 0.05 level.

case of the partially restricted three-factor Fama and French (1993) model (Table 8, Panel B). As such, we have some (but not overwhelming) evidence against this asset-pricing model in the context of these extreme characteristic portfolios. In contrast, the liquidity-augmented model copes much better (although not perfectly) with the pricing challenge of these particular portfolios, thereby leading us to reiterate the earlier conclusion that liquidity (as proxied by the turnover factor) has a potentially important role in asset pricing.

5.5. *Extended analysis and robustness checking*²⁰

We perform several extensions and variations of the earlier analysis to explore the robustness of our findings. In particular, we consider (1) pricing error diagnostics and the economic impact of our model, (2) the integrity of share turnover as a proxy for liquidity, and (3) the integrity of using iterated GMM techniques.

Until now we have been silent on the economic impact of the models tested and on the pricing errors implied by them. We now redress this situation in a number of ways. First, we report some pricing error diagnostics in Table 9. Specifically, the table reports mean and median measures of the differential pricing errors (expressed in basis points) produced by the liquidity-augmented Fama–French (1993) model versus the conventional Fama–French three-factor model for the eight test portfolios having extreme size, book-to-market, and liquidity characteristics. Monthly pricing errors for each model are captured by the absolute value of the residual term, and a differential pricing error is defined as the difference in the absolute residuals, namely, the four-factor model pricing error minus the three-factor model pricing error. Therefore, a negative differential pricing error indicates that the four-factor model has a smaller pricing error than its three-factor counterpart.

The most striking observation from Table 9 is the general pricing error superiority favoring the liquidity-augmented Fama–French (1993) model both in terms of mean and median differential pricing error. For example, seven of the eight portfolios produce a negative mean differential pricing error, and in two of these cases, the pricing error advantage is statistically significant at the 5% level. Specifically, for Portfolios 1 and 3, the mean pricing error superiority amounts to around 85 basis points. From the pricing error analysis we conclude that the four-factor model performs well, reinforcing the earlier statistical evidence favoring it over the three-factor counterpart.

Turning to the economic impact of the model, consider the following exercise. We can calculate an adjusted return by subtracting the expected return adjustments for the market, SMB, and HML (i.e., products of the relevant beta and the estimated risk premium) from the average return for each test portfolio. Given that this measures the average return left after controlling for the Fama–French (1993) factors, if the liquidity factor is important, it should explain this adjusted return. Accordingly, Figure 1 plots the adjusted return against the liquidity beta for our set of test portfolios. The plot shows that the liquidity betas do a pretty good job of explaining adjusted returns. The scatter plot of points cluster nicely around the positively sloped line that has a slope equal to the average of the liquidity premium estimated for odd- and even-numbered portfolios (see Table 6). Moreover, we can report that the cross-sectional correlation between the adjusted returns and the liquidity betas is +0.656. The analysis

²⁰ The authors gratefully acknowledge the detailed suggestions of an anonymous referee that largely shaped the analysis of this section.

Table 9

Pricing error diagnostics for the liquidity-augmented versus conventional Fama–French model—Portfolios with “extreme” size, book-to-market and liquidity characteristics

This table reports mean and median measures of the differential pricing errors (in basis points) produced by the liquidity-augmented Fama–French model versus the conventional Fama–French three-factor model for eight test portfolios having “extreme” size, book-to-market, and liquidity characteristics. Monthly pricing errors for each model are captured by the absolute value of the residual term and a differential pricing error is defined as the difference in the absolute residuals; that is, Differential pricing error = $|Res_{4faci}| - |Res_{3faci}|$, where $|Res_{4faci}|$ ($|Res_{3faci}|$) is the absolute value of the residual generated by the four-factor (three-factor) model obtained from iterated generalized method of moments estimation. For example, in the case of the liquidity-augmented Fama–French model, the residual terms (and, hence, the pricing errors) are obtained from the estimation of Equations (4)–(8) in the text, applied to the system of eight test portfolios having extreme size, book-to-market, and liquidity characteristics.

Portfolio	Size	Book-to-market	Liquidity	Mean differential pricing errors			Median differential pricing errors		
				Value (basis points)	<i>t</i> -Statistic	<i>p</i> -Value	Value (basis points)	Wilcoxon signed rank test statistic	<i>p</i> -Value
1	Small	Low	Illiquid	−85.36*	−2.15	0.034	−77.66*	2.21	0.027
3	Small	Low	Very liquid	−82.95*	−3.25	0.002	−88.66*	3.00	0.003
7	Small	High	Illiquid	−10.04	−0.66	0.510	−21.41	0.67	0.501
9	Small	High	Very liquid	−34.07	−0.95	0.347	−38.57	1.11	0.268
19	Big	Low	Illiquid	2.36	0.46	0.647	2.90	0.66	0.507
21	Big	Low	Very liquid	−2.83	−0.70	0.487	−1.81	0.93	0.354
25	Big	High	Illiquid	−3.02	−0.22	0.828	1.33	0.05	0.957
27	Big	High	Very liquid	−18.28	−1.72	0.089	−14.31	1.42	0.156

* Indicates statistical significance at the 0.05 level.

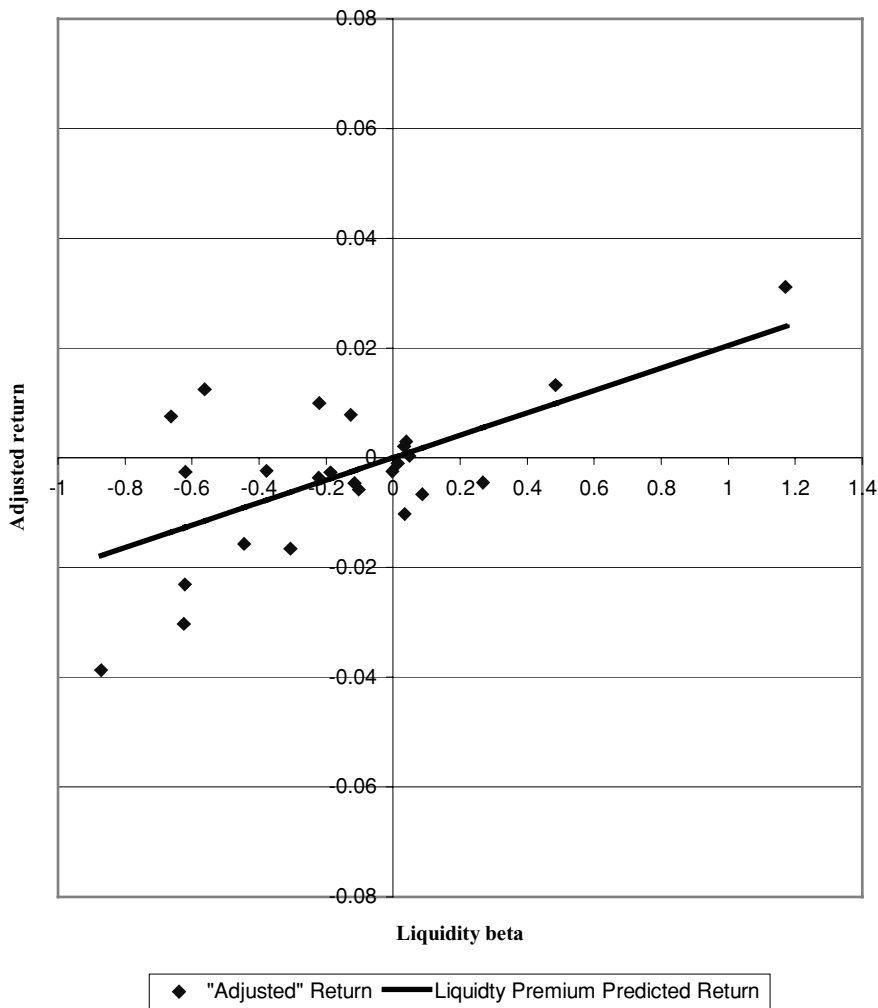


Figure 1

Plot of adjusted returns against liquidity betas

This figure provides a scatter plot of the adjusted return against the liquidity beta for our set of test portfolios. The “adjusted” return is obtained by subtracting the products of the relevant beta and the estimated risk premium (i.e., the expected return adjustments for the market, SMB, and HML) from the average return for each test portfolio. The straight line passes through the origin with a slope equal to the average liquidity premium estimated for the odd- and even-numbered portfolios.

suggests that, despite the concern expressed earlier about the reliability of this liquidity premium estimate, it does represent an economically meaningful quantity.

In the spirit of Jagannathan and Wang (1996), an alternative to help visually assess the economic performance of the four-factor versus three-factor models is to

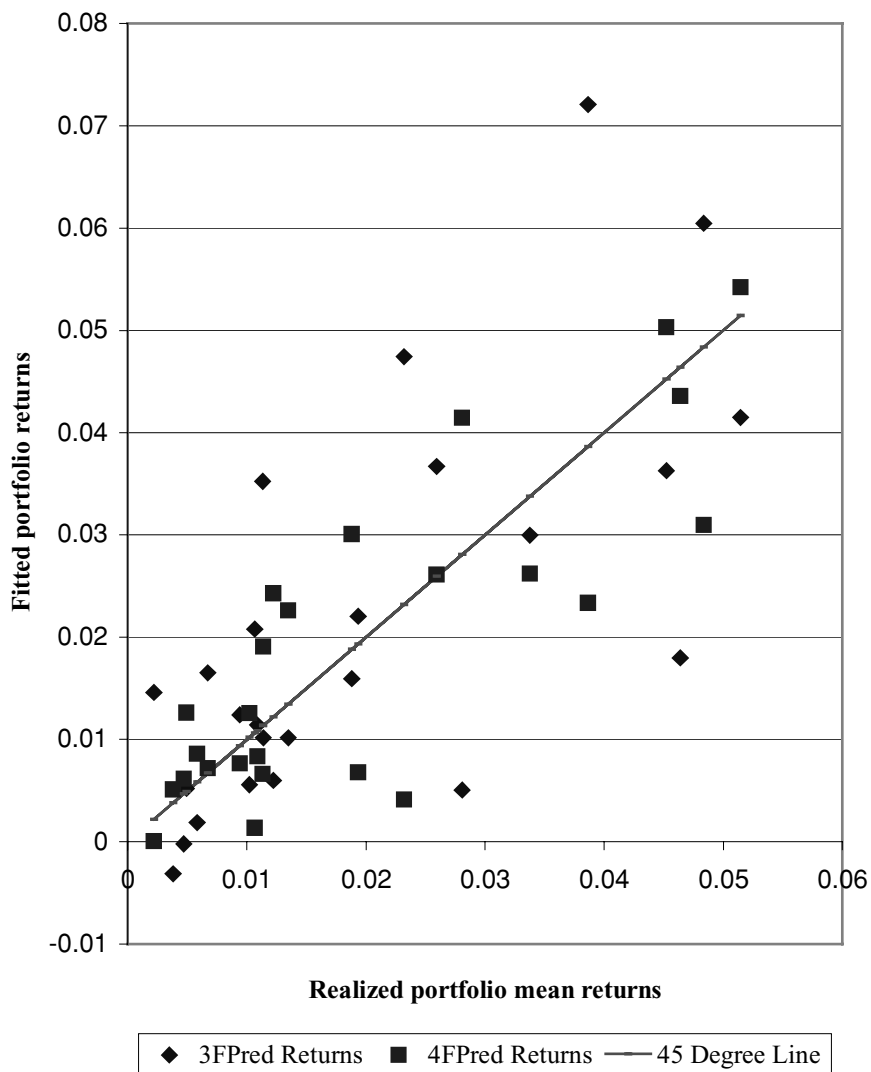


Figure 2

Plot of three- and four-factor model fitted returns against realized mean portfolio returns

This figure provides a scatter plot of the realized average returns against the fitted average returns for our set of test portfolios for the four-factor versus three-factor models. These plots are assessed against a 45-degree line.

plot realized average asset returns against the fitted average asset returns for each competing model. The plots can be assessed against a 45-degree line. The superior model should produce a set of points closer to the line. Accordingly, Figure 2 plots the realized average returns against the fitted average returns for our set of test portfolios.

Generally, the plot shows that the four-factor model produces a set of points that more closely congregate around the 45-degree line than the counterpart three-factor model. This visual appreciation is backed up by a comparison of cross-sectional correlations between fitted and realized returns, namely, a correlation of +0.706 relative to the three-factor model compared with a correlation of +0.834 for the four-factor model. Taken together, this and the previous analysis in this subsection increase our confidence that the turnover factor plays a real and meaningful role in asset pricing. The question is: To what extent is turnover actually proxying liquidity, or is it more a reflection of other phenomena like glamour or momentum? It is to this question that we now turn.

First, on the general question of the integrity of turnover as a liquidity proxy, it is important to note that existing aspects of the experimental design have been employed partly to help control for these other confounding effects (while recognizing the need to balance numerous research design tradeoffs). For example, as stated in Footnote 14, the fact that the turnover sorting variable is based on a monthly average over an annual period helps to partly control for monthly seasonality and the glamour effect. Also, as stated earlier, because we follow the Fama–French (1993) approach in forming the mimicking portfolios, this helps to control for value and other effects when creating the IMV mimicking portfolio.

However, the issue of potential confounding between liquidity and momentum is largely unaddressed thus far in our work. The fact that this is an important issue is clear from the literature. Lee and Swaminathan (2000) raised the question of whether a turnover premium can be interpreted as purely an illiquidity premium. Interestingly, in their study they found that high (low) turnover stocks tend to behave like glamour (value) stocks. Also, Pastor and Stambaugh (2001) included a consideration of momentum in their study of liquidity risk and found a distinct role for liquidity. To explore this issue, based on pre-ranking stocks according to their month -2 to month -6 performance, we create a momentum factor (MOM) by subtracting the month t return of the “loser” quintile from month t return of the “winner” quintile. The mean monthly return of this momentum factor is 0.5%, but of greater note is the fact that its correlation with IMV is only +0.154. Such a low correlation in and of itself strongly suggests that MOM and IMV are quite distinct factors and that IMV is unlikely to be a strong proxy for momentum.

As a further robustness check we rework our previous tests reported in Table 6 and 8 whereby the IMV factor is orthogonalized to MOM. This variation has no material impact on the outcome of the tests reported above, once again supporting a conclusion that turnover represents liquidity.²¹

Finally, as foreshadowed in Footnote 8, an issue arises as to the reliability of iterated GMM. We address this issue by re-estimating the partially restricted model for

²¹ Additionally, in unreported results beyond the scope of the current study, we conduct robustness checking in the cross-sectional setting of Fama and French (1992). The cross-sectional analysis confirms that the turnover measure is still significant even after controlling explicitly for momentum, thereby reinforcing the integrity of turnover as a liquidity proxy.

the extreme portfolios using two-step GMM (details suppressed to conserve space). With regard to the factor risk premiums, the point estimates and their statistical significance vary only marginally between the iterated and two-step GMM counterparts. Specifically, the two-step estimates are between 30 and 50 basis points less than the associated iterated cases. For example, the two-step (iterated) estimate of the liquidity premium is 0.0129 (0.173). Moreover, in both cases, all but the market premium estimate is statistically significant. Further, with regard to the pricing error analysis, the two-step approach produces very comparable measures to the iterated case. Mostly, the pricing error differences are less than two basis points, although for Portfolio 1 (4) the two-step pricing error is around 35 (10) basis points smaller than its iterated counterpart. Consequently, the comparison between iterated and two-step estimation does not reveal anything of material concern.

6. Summary and conclusion

In this paper, we examine the question: Does illiquidity attract a premium in equity markets? In doing so, we have four interrelated objectives. First, using Australian data we examined the role of liquidity in a Fama and French (1993) framework through the formation of a mimicking portfolio based on relative trading volume. Notably, this share turnover mimicking portfolio is created to be (approximately) orthogonal to the size and book-to-market factors. Second, we used a GMM system approach to overcome the errors-in-variables problem that is present in the two-pass Fama and MacBeth (1973) methodology and which forms the basis of most of the Fama–French papers. Third, we have produced results with a more current data set than that used by previous Australian research (Anderson, Clarkson, and Moran, 1997; Halliwell, Heaney, and Sawicki, 1999) that have investigated similar issues. Fourth, we set up the model so that direct estimates of risk premiums are obtained providing a richer test of the models.

Our analysis employs monthly data covering the sample period from 1990 to 1998. In summary, the key findings of our research are as follows. First, the main GMM tests fail to reject the over-identifying restrictions imposed by the liquidity-augmented Fama–French model. This suggests that there is strong support for a four-factor model that incorporates a share turnover factor. Second, the estimated premiums on the market, size, book-to-market, and turnover factors are positive in sign and generally significant—most notably so for turnover. Third, extended analysis and a battery of robustness checking considerably increases our confidence that turnover has an important role in pricing and that it is reliably capturing a liquidity effect. Nevertheless, the outcome of our analysis is sufficiently mixed to caution us from being unconditionally enthusiastic about its merits. Accordingly, we commend this area as one deserving of further research, especially in non-U.S. markets where a dearth of evidence exists, which is ironic given that a liquidity factor may be far more important in such contexts due to much thinner market activity.

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