

Explaining the Magnitude of Liquidity Premia: The Roles of Return Predictability, Wealth Shocks, and State-Dependent Transaction Costs

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ABSTRACT

Constantinides (1986) documents how the impact of transaction costs on per-annum liquidity premia in the standard dynamic allocation problem with i.i.d. returns is an order of magnitude smaller than the cost rate itself. Recent papers form portfolios sorted on liquidity measures and find spreads in expected per-annum return that are the same order of magnitude as the transaction cost spread. When we allow returns to be predictable and introduce wealth shocks calibrated to labor income, transaction costs are able to produce per-annum liquidity premia that are the same order of magnitude as the transaction cost spread.

A NUMBER OF RECENT PAPERS have found a difference in expected return across portfolios sorted on liquidity measures. While the finding of a difference is not surprising, the magnitude is, with expected return differences on the order of 6% to 7% per annum.¹ The magnitude of the difference seems too large to be explained by realistic transaction costs. In particular, the seminal work of Constantinides (1986) documents how investor utility is largely insensitive to transaction costs when the investor solves the canonical problem of i.i.d. returns calibrated to U.S. data, no nonfinancial income, and a constant proportional cost rate. For realistic proportional costs, Constantinides shows that the per-annum liquidity premium that must be offered to induce a constant relative risk aversion (CRRA) investor to hold the transaction-cost asset instead of an otherwise identical no-transaction-cost asset is an order of magnitude smaller than the transaction-cost rate itself. Constantinides also provides intuition for this result. Investors respond to transaction costs by turning over their portfolios much less frequently than annually because an investor's value function is insensitive to quite large deviations from the optimal no-transaction-cost portfolio allocation. Our paper bridges the gap between this theoretical result

*Lynch is with Stern School of Business, New York University, and Tan is with Fordham University, Graduate School of Business Administration. The authors would like to thank an anonymous referee; Editor Rob Stambaugh; Viral Acharya; George Constantinides; Ned Elton; Marty Gruber; Joel Hasbrouck; John Heaton; Lasse Pedersen; Hans Stoll; and participants of the NBER Spring Asset Pricing Group Meeting, the NYU Monday Finance Seminar, the NYU Macro-Finance Reading Group, the NHH Finance Seminar, the UCLA Finance Seminar, the Vanderbilt Finance Seminar, and the Yale Finance Seminar for helpful comments and suggestions. All remaining errors are of course the authors' responsibility.

¹ See, for example, Brennan and Subrahmanyam (1996) and Pástor and Stambaugh (2003).

and the empirical magnitude of the liquidity premium by examining dynamic portfolio choice with transaction costs in a variety of more elaborate settings that move the problem closer to the one solved by real-world investors. In particular, we allow returns to be predictable and we introduce wealth shocks, mainly labor income but also stationary multiplicative shocks. When returns are predictable, we also allow the wealth shocks and the transaction-cost rate to be state dependent.

We find that adding these real-world complications to the canonical problem can cause transaction costs to produce per-annum liquidity premia that are no longer an order of magnitude smaller than the rate, but are instead the same order of magnitude. In particular, return predictability and i.i.d. labor income growth calibrated to U.S. data are sufficient to obtain per-annum liquidity premia of the same order of magnitude as the cost rate, for realistic wealth-income ratios. For this reason, our results provide an important new insight into the effect of transaction costs on investor behavior. In particular, the effect of proportional transaction costs on the standard consumption and portfolio allocation problem with i.i.d. returns can be materially altered by reasonable perturbations that bring the problem closer to the one investors are actually solving. Moreover, since our base-case agent holds a positive amount of the low-liquidity assets, this agent can be regarded as a marginal or in-framarginal investor in these low-liquidity assets. Market clearing is an important consideration and we describe how heterogeneity in labor income and risk aversion, and heterogeneity induced by the presence of delegated portfolio management, can allow all assets to be held and net trades each month to sum to zero.

Our base-case agent has power utility with a relative risk aversion coefficient of six and has access to a low- and high-liquidity portfolio as well as a riskless asset. It is important that the agent has access to a high-liquidity asset in addition to the low-liquidity asset because otherwise the only asset available to trade off risk for expected return is the low-liquidity asset. The low-liquidity asset is calibrated to a portfolio of the 13 least liquid of 25 liquidity-sorted U.S. stock portfolios, while the high-liquidity asset is calibrated to the other 12 (see Acharya and Pedersen (2005)). Lesmond, Ogden, and Trzcinka (1999) quantify the transaction costs associated with trading individual stocks and find a 3% cost for the five smallest size deciles and a 1% cost for the five largest. While investors face transaction costs on both portfolios, intuition suggests that the spread in transaction costs across the two portfolios is what is critical for the spread in expected return across the two. Consequently, we set the transaction-cost rate on the high-liquidity asset to zero and keep the rate on the low-liquidity asset at 2%.

We find that return predictability calibrated to that in the data increases the liquidity premium on the low-liquidity portfolio by a factor of five from 0.08% per annum to 0.43% per annum. The reason for the increase is as follows. The usual motive for trading is to rebalance the portfolio back to the optimal weights after realized risky-asset returns alter the portfolio's composition from the optimal weights. Return predictability causes the optimal portfolio weights

to move around through time as the agent takes advantage of the time-varying expected returns. This time variation in the optimal weights creates an additional motive for trading that causes higher liquidity premia when returns are predictable.

Labor income is an important wealth shock that we calibrate to have a permanent component, as in Carroll (1996, 1997) and Viceira (1997). We ignore the temporary component discussed in these papers because we find that this temporary component is unimportant for liquidity premia. The parameter estimates we use are those obtained from U.S. PSID data by Gakidis (1997). With i.i.d. returns and a fixed transaction-cost rate, the inclusion of labor income uncorrelated with returns causes the liquidity premium on the low-liquidity portfolio to become 1.42% per annum for an agent with no financial wealth, an almost 18-fold increase relative to the canonical i.i.d. case. Once returns are allowed to be predictable (with the transaction-cost rate remaining a constant), i.i.d. labor income growth uncorrelated with returns causes the liquidity premium to be 1.63% and 1.08% for an agent with a wealth to monthly labor income ratio of 0 and 10, respectively. These represent 20-fold and 13-fold increases, respectively, relative to the standard i.i.d. case. So the introduction of i.i.d. labor income growth uncorrelated with returns is enough to greatly increase the liquidity premium relative to the canonical problem, especially for agents with wealth to monthly labor income ratios lower than 10.

Labor income causes liquidity premia to increase for two reasons. First, labor income is paid in cash and earns the riskless rate unless invested in stock. Thus, labor income lowers portfolio holdings in stock, distorting them away from the optimal weights and causing the agent to trade more to rebalance back to the optimal weights. Second, shocks to labor income are permanent and so have large effects on the agent's total wealth. These total wealth shocks can lead to large shifts in the optimal weights in the agent's financial wealth portfolio, particularly when the agent has a low wealth-income ratio. These shifts in the optimal weights provide a strong motive for the agent to trade, leading to much higher liquidity premia than in the case with no labor income. Since adding a temporary component to the monthly shock to labor income has almost no effect on liquidity premia, the implication is that the second of these two channels is much more important than the first. It is also not surprising that return predictability still has an incremental effect, further inflating the premium, in the presence of labor income. As discussed above, return predictability causes the optimal portfolio weights to move around with the stage in the business cycle. This additional variation in the optimal weights can provide a motive for trading over and above that provided by the effects of labor income on the optimal portfolio weights.

When returns are predictable, allowing labor income growth to be procyclical, as in the data, slightly reduces the liquidity premium at very low and very high wealth-income ratios, but actually increases the premium to 1.12% per annum at a wealth to monthly labor income ratio of 10. This increase occurs because the hedging demand generated by labor income growth being procyclical reduces the optimal holdings of stock and so causes the 100% stockholding

maximum to bind less often. The agent trades more. Reductions occur because expected returns are countercyclical but labor income growth is procyclical, and so expected stock returns and the optimal portfolio weights in stock are high (low) exactly when labor income's downward pressure on the portfolio weights in stocks is small (big). The agent trades less.

Finally, liquidity premia increase even further when the transaction-cost rate is allowed to be state dependent as observed in the data. The reason is as follows. In the data, the transaction-cost rate is countercyclical, which means it is high when future expected returns are high and so the optimal portfolio weights in stocks are high too. These are exactly the times when the agent trades the most since the effect of labor income is to lower the portfolio weights in stocks.

We find that removing access to the high-liquidity asset only slightly increases the liquidity premia, and that this occurs because the base-case agent holds very little of this asset. In the U.S. economy, the high-liquidity asset has the larger market capitalization so market clearing is not possible at observed prices if all agents in the economy are identical to the base-case agent. However, it is likely that there is considerable heterogeneity across agents regarding their risk aversions and the income processes they receive. We examine two other agents with plausible risk aversions and plausible labor income processes, showing that for initial wealth to monthly income ratios of 10 or less, both hold more of the high- than the low-liquidity asset early in life. For initial wealth to monthly income ratios of one or lower, one of these agents holds more of the high- than the low-liquidity asset for at least the first 11 years of her 20-year life and holds more than seven times more of the high- than the low-liquidity asset in her first month. Another important source of heterogeneity is participation by agents who care about something other than consumption from their labor incomes and financial wealth portfolios. Cuoco and Kaniel (2010) show how the existence of a delegated portfolio management industry in which managers receive a symmetric fulcrum fee that depends on performance relative to a benchmark can cause funds to tilt fund portfolios toward the stocks in the benchmark. The result is higher prices and lower Sharpe ratios in equilibrium for these benchmark stocks, which are typically high-liquidity stocks. Thus, funds managed on behalf of others may hold large amounts of the high-liquidity stocks while agents like our base-case agent are the inframarginal investors in the low-liquidity stocks.

Another variable of interest is turnover. Using simulations, we find that annual turnover goes from about 4% per annum in the standard case to 38% per annum for the base-case agent with a wealth to monthly income ratio of 10. This turnover number is in the ballpark of the monthly turnover numbers reported by Acharya and Pedersen (2005) for their low-liquidity portfolios, numbers that range from 3.25% up to 4.19%. Labor income drives a wedge between the liquidity premia and the direct cost of trading, which implies that labor income causes the agent to trade more often when poor than when rich in utility terms. For an agent with a wealth to monthly income ratio of one, the direct cost in the base case is 1.03% per annum compared to a liquidity premium

of 1.44% in the base case, while the direct cost in the case of i.i.d. returns and labor income growth is less than half the 1.38% per-annum liquidity premium.

We also consider multiplicative wealth shocks, for which comparative static analysis is easier because of the much lighter computational burden. It is difficult to calibrate these shocks, since there are a variety of possible reasons for their occurrence. Sources include shocks to the value of the agent's real estate holdings, proprietary income shocks, and health shocks. To make the multiplicative shock analysis more directly comparable to the labor income cases we consider, we calibrate the shock process to the base-case labor income process. We find that the liquidity premium exhibits a strong U-shaped pattern as a function of the unconditional correlation between a given month's wealth shock and the dividend yield at the beginning of that month, with the lowest premium occurring for a correlation close to zero. This U-shaped pattern suggests that the premia for the labor income cases likely would be higher if the correlation between the permanent labor income growth and beginning-of-month dividend yield was further away from zero in either direction. When the proportional transaction-cost rate is allowed to be state dependent, we find that, consistent with intuition, wealth shocks are especially painful and so generate higher liquidity premia if negative wealth shocks occur when the transaction-cost rate is high.

The paper is organized as follows. Section I discusses related literature while Section II describes the investor's dynamic optimization problem with predictable returns, transaction costs, and either labor income or multiplicative wealth shocks. Section III calibrates the investor problems considered to the U.S. economy. Section IV discusses the liquidity premia and turnover results, while Section V discusses equilibrium issues and market clearing. Section VI concludes.

I. Related Literature

A number of recent empirical papers examine how expected returns vary with measures of liquidity. Brennan and Subrahmanyam (1996) form 25 portfolios by forming quintiles on size and then, within each size quintile, forming quintiles on the Kyle (1985) inverse measure of market depth, λ , estimated as in Glosten and Harris (1988). They find a 6.6% per-annum spread in average abnormal return from the Fama–French (1993) three-factor model between the low- λ and the high- λ quintiles. Rather than sort on a measure of stock illiquidity, Pástor and Stambaugh (2003) form deciles based on the covariance between return and a measure of market liquidity, and find a spread in abnormal return between the two decile extremes of 7.5% per annum with respect to a four-factor model that accounts for sensitivities to the market, size, and book-to-market factors of Fama–French (1993) and a momentum factor. Easley, Hvidkjaer, and O'Hara (2002) examine how information-based trading affects asset returns and reports that a difference of 10 percentage points in the probability of information-based trading between two stocks leads to a difference in expected returns of 2.5% per annum. Other papers to examine how expected returns

vary with measures of liquidity include Brennan, Chordia, and Subrahmanyam (1998), Amihud (2002), and Hasbrouck (2003).

Several theoretical and numerical papers have considered how illiquidity and transaction costs affect asset prices. Early work by Stoll (1978) and Ho and Stoll (1981) examines how a dealer sets the spread given that she faces inventory carrying costs. In a single-period setting, Amihud and Mendelson (1986) show how transaction costs can affect expected returns on stocks. In an economy with two classes of agents, Heaton and Lucas (1996) examine numerically how idiosyncratic and uninsurable labor income affects equilibrium-expected returns both with and without transaction costs on the riskless and equity assets. In an overlapping generations economy, Vayanos (1998) shows how prices are affected by the presence of transaction costs. In his model, agents have a life cycle motive for trading and trading behavior is predetermined. Huang (2003) studies an equilibrium model in which agents receive unexpected liquidity shocks and can invest in liquid and illiquid riskless assets. Other papers to examine theoretically how illiquidity and transaction costs affect assets prices include Lo, Mamaysky, and Wang (2004), Aiyagari and Gertler (1991), and Acharya and Pedersen (2005). Recent papers also examine multiperiod portfolio choice in the presence of labor income, but few incorporate portfolio rebalancing costs (see Viceira (1997, 2001), Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2003)). Two exceptions are Balduzzi and Lynch (1999) and Lynch and Balduzzi (2000), who solve numerically the investor's multiperiod problem with transaction costs. However, neither of these papers incorporate labor income or multiplicative wealth shocks, and neither focus on liquidity premia, though Balduzzi and Lynch do report utility costs associated with ignoring transaction costs.

II. The Investor's Portfolio Allocation Problem

This section lays out the preferences of and constraints faced by the investor. We characterize the optimization problem for a dynamic investor who faces either i.i.d. or predictable returns. We first describe the problem when the agent receives labor income and then when the agent receives a multiplicative wealth shock. We also describe the solution technique for numerically solving the investor's problem and how the liquidity premia and turnover numbers are calculated.

A. Labor Income Problem

We consider the portfolio allocation between N risky assets and a riskless asset. In all the cases we consider, the investor has access to either one or two risky assets. The investor faces transaction costs that are proportional to the dollar amount traded.

Following Carroll (1996, 1997), labor income is specified to have both permanent and temporary components:

$$y_t = y_t^P + \epsilon_t, \quad (1)$$

$$g_t \equiv y_t^P - y_{t-1}^P = \bar{g} + b_g d_t + u_{t+1}, \quad (2)$$

where y_t is log labor income received at t , y_t^P is log permanent labor income at t , ϵ_t is log temporary labor income at t , $d_t \equiv \ln(1 + D_t)$, and ϵ_t and u_{t+1} are uncorrelated i.i.d. processes. All the cases reported in the paper switch off the temporary component because unreported results (available from the authors upon request) show that the presence of a temporary component calibrated to data has a negligible impact on liquidity premia. Thus, for all the cases reported, $y_t = y_t^P$ and $\epsilon_t = 0$ for all t .

With labor income, the law of motion for the investor's wealth, W , is given by

$$W_{t+1} = (W_t + Y_t - c_t)(1 - f_t)[\alpha_t(\mathbf{R}_{t+1} - R_t^f \mathbf{i}_N) + R_t^f] \quad \text{for } t = 1, \dots, T - 1, \quad (3)$$

where c is consumption, α is an $N \times 1$ vector of portfolio weights in the N risky assets, \mathbf{R} is an $N \times 1$ vector of returns on the N risky assets, R^f is the riskless rate, Y_t is labor income received at time t , and f is the transaction cost per dollar of portfolio value. At the terminal date T , $c_T = W_T$ so the investor does not receive labor income at the terminal date.

Let $\hat{\alpha}_t^i$ be the allocation to the i^{th} risky asset inherited from the previous period. Then

$$\hat{\alpha}_t^i \equiv \frac{\alpha_{t-1}^i (W_{t-1} + Y_{t-1} - c_{t-1})(1 - f_{t-1}) R_t^i}{W_t} = \frac{\alpha_{t-1}^i R_t^i}{\alpha_{t-1}'(\mathbf{R}_t - R_{t-1}^f \mathbf{i}_N) + R_{t-1}^f}, \quad (4)$$

where $\hat{\alpha}_t$ is the $N \times 1$ vector of these inherited portfolio weights. We assume that consumption at time t is obtained by liquidating costlessly the i^{th} risky asset and the riskless asset in the proportions $\hat{\alpha}_t^i$ and $(1 - \hat{\alpha}_t' \mathbf{i}_N)$. This assumption is not so onerous given the availability of money-market bank accounts and given that equities pay dividends. To the extent that the sum of the risky-assets' dividends exceeds the consumption out of the risky asset, c , a dividend reinvestment plan can be used to costlessly reinvest the excess dividend in the risky asset.

We allow returns to be predictable and assume that there exists a "predictive" variable D that affects the conditional mean of the risky-assets' return. We assume D follows a first-order Markov process. For simplicity, the riskless rate is assumed to be constant, and so $R_t^f = R^f$ for every t .

Labor income Y_t is assumed to be received as the riskless asset. Consequently, the vector of inherited risky-asset holdings becomes $\frac{\hat{\alpha}_t}{1 + Y_t/W_t}$ after the shock. Since Y_t is always positive, the shock is like a cash inflow. The investor sees the labor income at t before both the consumption and allocation decisions at t . However, neither u_t nor ϵ_t (if nonzero) contains any information about future returns $(\mathbf{R}_{t+1}, \mathbf{R}_{t+2}, \dots)$ or about future D values $(D_{t+1}, D_{t+2}, \dots)$.

The transaction-cost function f_t depends on the chosen portfolio weights α_t , the $N \times 1$ vector of portfolio weights inherited from the previous period $\hat{\alpha}_t$, and labor income Y_t :

$$f_t = \Phi_t' \left| \alpha_t - \frac{\hat{\alpha}_t \Gamma_t}{\Gamma_t + \exp\{g_t + \epsilon_t\}} \right|, \quad (5)$$

where $Y_{t-1}^P = \exp\{y_{t-1}^P\}$ is permanent labor income at $t-1$ and Γ_t is defined to be $\frac{W_t}{Y_{t-1}^P}$. For a given inherited risky-asset allocation at $t\hat{\alpha}_t$, the postlabor income inherited risky-asset allocation is decreasing in Y_t . This specification accommodates transaction costs on an asset that are proportional to the change in the value of the portfolio holding of that asset, as in Constantinides (1986). In general, the $N \times 1$ vector Φ has i^{th} element Φ^i , which gives the proportional cost rate associated with trading the i^{th} risky asset. However, in all applications considered here, only one asset has a nonzero transaction-cost rate and so Φ is used to denote that rate. It is straightforward to modify the transaction-cost function to accommodate more elaborate transaction-cost functions, like, for example, a cost that is the same fraction of portfolio value irrespective of how much of the asset is traded (see Lynch and Tan (2010)).

We also allow the cost parameters Φ_t to be random, with distributions that can be state dependent and thus depend on D_t when returns are predictable. The investor sees the cost parameter realizations at t , Φ_t , before both the consumption and allocation decisions at t . The cost parameter realizations for t do not contain any information about future returns or future D values.

We consider the optimal portfolio problem of an investor with a finite life of T periods and utility over intermediate consumption. Preferences are time separable and exhibit CRRA:

$$E \left[\sum_{t=1}^T \delta^t \frac{c_t^{1-\gamma}}{1-\gamma} \mid \Gamma_1, D_1, \hat{\alpha}_1 \right], \quad (6)$$

where γ is the coefficient of relative risk aversion, δ is the time-discount parameter, and $E[\cdot \mid \Gamma_t, D_t, \hat{\alpha}_t]$ denotes the expectation taken using the conditional distribution given Γ_t , D_t , and $\hat{\alpha}_t$. Note that the expected lifetime utility depends on the state of the economy at time 1. Further, the inherited portfolio weight for the i^{th} risky-asset $\hat{\alpha}_1^i$ is a state variable whenever the i^{th} element of Φ is greater than zero, since the value of this inherited portfolio weight determines the transaction costs to be paid at time 1. These preferences have been extensively used in empirical work by Grossman and Shiller (1981), Hansen and Singleton (1982), and many others.

Given this specification of the agent's problem with labor income, the value function at t is homogenous in Y_{t-1}^P and has an additional state variable, namely, the ratio of financial wealth at t to lagged permanent labor income Γ_t . The law

of motion for the investor's wealth, W , can be rewritten as

$$\Gamma_{t+1} = (\Gamma_t - \hat{\kappa}_t + \exp\{g_t + \epsilon_t\})(1 - f_t) \exp\{-g_t\} [\alpha_t(\mathbf{R}_{t+1} - R_t^f \mathbf{i}_N) + R_t^f] \quad \text{for } t = 1, \dots, T-1, \quad (7)$$

where $\hat{\kappa}_t \equiv \frac{c_t}{Y_{t-1}^P}$.

Given our parametric assumptions, the Bellman equation faced by the investor is given by

$$\begin{aligned} & \frac{\alpha(\Gamma_t, D_t, \hat{\alpha}_t, t)(Y_{t-1}^P)^{1-\gamma}}{1-\gamma} \\ &= E \left[\max_{\hat{\kappa}(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t), \alpha(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)} \left\{ \frac{\hat{\kappa}_t^{1-\gamma}(Y_{t-1}^P)^{1-\gamma}}{1-\gamma} + \delta \frac{(Y_{t-1}^P)^{1-\gamma}}{1-\gamma} \right. \right. \\ & \quad \times E \left[\alpha(\Gamma_{t+1}, D_{t+1}, \hat{\alpha}_{t+1}, t+1) (\exp\{g_t\})^{1-\gamma} | \Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t \right] \left. \right] \Bigg| \Gamma_t, D_t, \hat{\alpha}_t, \\ & \quad \text{for } t = 1, \dots, T-1, \end{aligned} \quad (8)$$

where $\alpha_t \equiv \alpha(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)$ and $\hat{\kappa}_t \equiv \hat{\kappa}(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)$, both time dependent since the time horizon T is finite. This form of the value function follows from the CRRA utility specification in equation (6). The Bellman equation (8) is solved by backward iteration, starting with $t = T-1$ and $\alpha(\Gamma, D, \hat{\alpha}, T) = \Gamma^{1-\gamma}$. If returns and permanent labor income growth are not predictable, D_t is no longer a state variable for the problem and so α is no longer a function of D_t . Moreover, $\hat{\kappa}$ and α are not functions of D_t either.

B. Multiplicative Wealth Shock Problem

We now describe the agent's problem when the shock to financial wealth is multiplicative and a stationary variable. This structure makes the wealth shock easy to handle, since its presence has no impact on the number of state variables for the agent's problem. The law of motion of the investor's wealth, W , is given by

$$W_{t+1} = [W_t(1 + L_t) - c_t](1 - f_t) [\alpha'_t(\mathbf{R}_{t+1} - R_t^f \mathbf{i}_N) + R_t^f], \quad \text{for } t = 1, \dots, T-1, \quad (9)$$

where L is the wealth shock expressed as the percentage change in wealth as a result of the shock. The wealth shock L is exogenous and assumed to follow a stationary process with age-dependent parameters. Letting $l_t = \ln(L_t)$ and $\bar{l}_t(D_t)$ be the age- and state-dependent mean of l_t , then $l_t - \bar{l}_t(D_t)$ is assumed to be i.i.d. with volatility σ_l . The dollar wealth shock at t is $W_t L_t$. At the terminal date T , $c_T = W_T$ so the investor does not receive a wealth shock at the terminal date. Dollar transaction costs at t are $[W_t(1 + L_t) - c_t]f_t$, and are paid by costlessly

liquidating the i^{th} risky asset and the riskless asset in the proportions α_t^i and $(1 - \alpha_t^i \mathbf{i}_N)$.

As with the labor income at t , the wealth shock $W_t L_t$ is like a cash inflow and is assumed to affect the riskless asset holding. Consequently, the vector of inherited risky-asset holdings becomes $\frac{\hat{\alpha}_t}{1+L_t}$ after the shock. When returns are predictable, we allow the distribution of the wealth shock L_t to be state dependent and thus depend on D_t . The transaction-cost function f becomes

$$f_t = \Phi_t' \left| \alpha_t - \frac{\hat{\alpha}_t}{1 + L_t} \right|. \quad (10)$$

The evolution equation for state variable $\hat{\alpha}_t$ remains (4). To make the multiplicative shock case as comparable to the labor income case as possible, consumption here is allowed to depend on the current date's wealth shock as well as the transaction-cost shock, and neither of these shocks contain any information about future returns ($\mathbf{R}_{t+1}, \mathbf{R}_{t+2}, \dots$) or future D values (D_{t+1}, D_{t+2}, \dots).

We define R_W as the rate of return on wealth, after the wealth shock and net of the transaction costs incurred. Given our parametric assumptions, the Bellman equation faced by the investor is given by

$$\begin{aligned} & \frac{a(D_t, \hat{\alpha}_t, t) W_t^{1-\gamma}}{1-\gamma} \\ &= \max_{\kappa(D_t, \hat{\alpha}_t, L_t, \Phi_t, t)} \left\{ \frac{\kappa_t^{1-\gamma} W_t^{1-\gamma}}{1-\gamma} + \delta \frac{(1-\kappa_t)^{1-\gamma} W_t^{1-\gamma}}{1-\gamma} \right. \\ & \quad \times E \left[\max_{\alpha(D_t, \hat{\alpha}_t, L_t, \Phi_t, t)} \left\{ E[a(D_{t+1}, \hat{\alpha}_{t+1}, t+1) R_{W,t+1}^{1-\gamma} | D_t, \hat{\alpha}_t, L_t, \Phi_t] \right\} | D_t, \hat{\alpha}_t \right] \Big\} \\ & \quad \text{for } t = 1, \dots, T-1, \end{aligned} \quad (11)$$

where $\kappa_t \equiv \frac{c_t}{W_t(1+L_t)}$, $\alpha_t \equiv \alpha(D_t, \hat{\alpha}_t, L_t, \Phi_t, t)$, and $\kappa_t \equiv \kappa(D_t, \hat{\alpha}_t, L_t, \Phi_t, t)$. As in the labor income case, the Bellman equation (11) is solved by backward iteration, starting with $t = T-1$ and $a(D, \hat{\alpha}, T) = 1$.

C. Solution Technique

The dynamic programming problems are solved by backward recursion. Irrespective of whether there is one or two risky assets, the state variable $\hat{\alpha}_1$ is discretized and the value function is linearly interpolated between $\hat{\alpha}_1$ points. This technique yields an approximate solution that converges to the actual solution as the $\hat{\alpha}_1$ grid becomes finer. In all the optimizations, the holdings of both the risky and the riskless assets are constrained to be nonnegative. In the two risky asset case, this restricts the action space with respect to allocation choice to the triangular region characterized by $\alpha_1 + \alpha_2 \leq 1$, $\alpha_1 \geq 0$, and $\alpha_2 \geq 0$. When actions are restricted to this set, implied inherited allocations for any return realization on the assets are again in the same region. We use a finer discretization for the action space of allocations than the state space of

inherited allocations. Allocation choices on each asset available to the investor always include the discrete grid $\{0.000, 0.001, \dots, 0.999, 1.000\}$.

For the labor income problem, the presence of an additional state variable, the wealth to lagged permanent income ratio, considerably complicates the methodology needed to obtain accurate solutions in a manageable time frame. Details of the methodology employed are provided in the Appendix.

D. Liquidity Premia

Each of the investor problems described above implies a policy function that, in turn, yields a particular level of expected lifetime utility. Specifically, for the labor income problem, the policy functions $\{\alpha(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)\}_{t=1}^{T-1}$ and $\{\hat{\kappa}(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)\}_{t=1}^{T-1}$ can be substituted into the actual law of motion for investor wealth (3) to obtain the consumption sequence $\{c_t = \hat{\kappa}(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)Y_{t-1}^P\}_{t=1}^T$. This consumption sequence is then substituted into (6) to obtain the investor's expected lifetime utility. Analogous substitutions can be performed for the multiplicative wealth shock problem to obtain the investor's expected lifetime utility.

Constantinides (1986) finds that, for a CRRA investor with access to a risky asset whose return is i.i.d., proportional transaction costs produce per-annum liquidity premia that are an order of magnitude smaller than the cost rate. We are interested in determining whether this result is robust to the introduction of real-world complications such as return predictability, state-dependent wealth shocks, and state-dependent transaction costs. Consequently, our definition of liquidity premium is in line with that adopted by Constantinides (1986). The liquidity premium is defined to be the decrease in the unconditional mean log return on the low-liquidity asset that the investor requires to be indifferent between having access to this risky asset with rather than without transaction costs. The mean is decreased by subtracting a constant from every state and the investor is assumed not to know the initial dividend yield value.

As mentioned above, the expected lifetime utility depends on the initial value of the inherited portfolio allocation $\hat{\alpha}_1$, the initial value of the wealth to lagged permanent labor income ratio Γ_1 , and the initial value of the vector characterizing the state of the economy, D_1 . For simplicity, we always take the inherited allocation for a given state to be the optimal allocation for the analogous no-transaction-cost problem.

E. Turnover and Direct Trading Cost

Another variable of interest is turnover. Turnover is calculated for the labor income problem by simulating one million paths and applying the optimal policies to each path. Per-annum turnover is defined to be

$$\frac{12}{240} \sum_{t=1}^{240} \frac{\text{av}(|A_t - \hat{A}_t|)}{\text{av}(\max(A_t, \hat{A}_t))}, \quad (12)$$

where $\text{av}(\cdot)$ is obtained by taking the average across the simulation paths, $A_t = [W_t - c_t + Y_t]\alpha_t$ and $\hat{A}_t = [W_t - c_t + Y_t](\hat{\alpha}_t W_t)/(W_t + Y_t)$. Thus, A_t is the dollar chosen risky-asset holdings at time t and \hat{A}_t is the dollar effective inherited risky-asset allocation at time t . The fraction being summed in (12) represents turnover for a given month of the life cycle. The denominator must be some measure of the dollar risky-asset holdings for the month. We take the maximum of the dollar chosen risky-asset allocation and the dollar effective inherited risky-asset allocation for the month.² This measure of turnover equally weighs the 240 months of the agent's life.³ Turnover is calculated analogously for the multiplicative wealth shock problem.

The turnover number can be multiplied by the average transaction-cost rate to get the direct effect of transaction costs on expected return. The extent to which the liquidity premium exceeds this direct cost can be attributed to some combination of risk premium for trading more when the agent is poor in utility terms, and in the case of a state-dependent cost rate, to the agent trading more when the cost rate is high.

III. Calibration

This section describes how the return, labor income, multiplicative wealth shock, transaction-cost rate, and predictive variable processes are calibrated to data. Parameter value choices are also described.

A. Return Calibration

We use a high- and a low-liquidity portfolio as the risky assets. The Acharya and Pedersen (2005) data set provides 25 value-weighted portfolios of NYSE and Amex stocks sorted on ILLIQ, a liquidity measure suggested by Amihud (2002). The high-liquidity asset is taken to be the value-weighted portfolio of the most liquid 12 portfolios and the low-liquidity asset is taken to be the value-weighted portfolio of the least liquid 13 portfolios. The stock return and riskless rate series are deflated using monthly CPI inflation. The continuously compounded riskless rate is estimated to be the mean of the continuously compounded real 1-month Treasury bill rate over this period, which gives a value for R^f of 0.110%. We use the 12-month dividend yield on the value-weighted NYSE as the predictive variable D .

Define \mathbf{R} to be an $N \times 1$ risky-asset return vector and let $\mathbf{r} \equiv \ln(\mathbf{1} + \mathbf{R})$ and $d \equiv \ln(1 + D)$. We estimate a vector autoregression (VAR) for the two risky-asset returns assuming that $[\mathbf{r}'d']'$ follows the VAR model:

$$\mathbf{r}_{t+1} = \mathbf{a}_r + \mathbf{b}_r d_t + \mathbf{e}_{t+1}, \quad (13)$$

$$d_{t+1} = a_d + b_d d_t + v_{t+1}, \quad (14)$$

² We also tried using a measure of holdings based on the average of the chosen risky-asset allocation and the effective inherited allocation. The results were virtually identical.

³ We tried other weighting schemes including one that weighs months according to the average dollar value of holdings over the months. The results were qualitatively similar to the ones we report.

where \mathbf{a}_r , an $N \times 1$ vector, and a_d are intercepts; \mathbf{b}_r , an $N \times 1$ vector and b_d are coefficients; and $[\mathbf{e}'v']$ is an i.i.d. vector of mean-zero multivariate normal disturbances with constant covariance matrix $\Sigma_{\mathbf{e}v, \mathbf{e}v}$. The covariance matrix of \mathbf{e} is $\Sigma_{\mathbf{e}, \mathbf{e}}$ and the variance of v is σ_v^2 . Similarly, the unconditional covariance matrix $[\mathbf{r}'d']$ is $\Sigma_{\mathbf{rd}, \mathbf{rd}}$, and the unconditional variance matrices for \mathbf{r} and d are $\Sigma_{r, r}$ and σ_d^2 , respectively. Without loss of generality, we normalize the mean of d , μ_d , to be zero and its variance, σ_d^2 , to be one. The specification in (13) and (14) assumes that d_t is the only state variable needed to forecast r_{t+1} , which is in line with other papers on optimal portfolio selection (e.g., Barberis (2000), Campbell and Viceira (1999)).

The data VAR is estimated using ordinary least squares (OLS) and discretized using a variation of Tauchen and Hussey's (1991) Gaussian quadrature method; the variation is designed to ensure that d is the only state variable (see Balduzzi and Lynch (1999) for details). However, following Lynch (2000), this study implements the discretization in a manner that produces exact matches for important moments for portfolio choice.⁴ We choose 19 quadrature points for the dividend yield and 3 points for the stock return innovations since Balduzzi and Lynch (1999) find that the resulting approximation is able to capture important dimensions of the return predictability in the data.

Table I presents data and quadrature VAR parameter values for the high- and low-liquidity returns. Panel A reports the slope coefficients \mathbf{b}_r and b_d as well as unconditional means for \mathbf{r} and d . Panel B reports the unconditional covariance matrix for $[\mathbf{r}'d']$ and the cross-correlations. Panel C reports the unconditional covariance matrix for $[\mathbf{e}'v']$ and the cross-correlations. The quadrature values almost always replicate the data values, which suggests that the discretization is capturing the important features of the data.

B. Transaction-Cost Rate, Labor Income, and Multiplicative Wealth Shock Calibrations

When calibrating the transaction-cost rates, it is the cost of trading the individual stocks and not the portfolio itself that is relevant. The transaction-cost rate on the low-liquidity asset is calibrated to the transaction-cost spread between stocks in a low-transaction cost portfolio and stocks in a high-transaction cost portfolio. Lesmond, Ogden, and Trzcinka (1999) form size deciles and then report the average round-trip transaction cost for the individual stocks in each decile. According to Table III of Lesmond, Ogden, and Trzcinka (1999), the average round-trip cost of trading a stock in the five largest portfolios less the average to trade one in the five smallest equals 4.01%, and so we take 2% to be the one-way transaction-cost rate for the low-liquidity asset. This number is likely to be a ballpark figure for the transaction-cost spread between the low- and high-liquidity portfolios and a lower bound for the cost of trading the low-liquidity portfolio. In practice, investors face transaction costs

⁴ In particular, the procedure matches both the conditional mean vector and the covariance matrix for log returns at all grid points of the predictive variables, as well as the unconditional volatilities of the predictive variables and the correlations of log returns with the predictive variables.

Table I

Sample Statistics, VAR Coefficients, and Quadrature Approximation: Low- and High-Liquidity Portfolios

The table reports moments and parameters for the high- and low-liquidity portfolios estimated from U.S. data and calculated for the quadrature approximation, based on a VAR that uses log dividend yield as the only state variable. *t*-statistics for the slope coefficient estimates, computed using exactly identified generalized method of moments (GMM) with 3 and 12 Newey–West lags, are also reported. The VAR is described in Section III.A. Acharya and Pedersen (2005) construct 25 value-weighted portfolios sorted on the ILLIQ variable of Amihud (2002) for the period from February 1964 to December 1996, and the low (high) liquidity portfolio is the value-weighted portfolio of the least (most) liquid 13 (12) portfolios. Panel A reports unconditional means, VAR slopes and R^2 s for the data and the quadrature approximation; *b* is the vector of VAR slopes, and R^2 denotes the regression R^2 . Panel B reports the unconditional covariance matrix for the data and for the quadrature approximation. Panel C reports the conditional covariance matrices for the data VAR and the quadrature VAR. All results are for continuously compounded returns except as noted in the table. Returns are expressed per month and in percent.

Data												Quadrature	
Panel A: Unconditional Sample Moments and VAR Coefficients													
Discretely Compounded					t-Stats								
Asset/Variable	Uncond. Mean	Sharpe Ratio	Uncond. Mean	b	3-month lag	12-month lag	R ²	Uncond. Mean	b	R ²			
	0.55	0.1023	0.45	0.37	1.66	1.68	0.73	0.45	0.37	0.73			
	0.98	0.1641	0.84	0.58	2.07	2.10	1.19	0.84	0.58	1.19			
			0.00	0.98	79.53	86.14	95.67	0.00	0.96	92.58			
Asset/Variable	High Liquidity	Low Liquidity	Dividend Yield										
Panel B: Unconditional Standard Deviations, Covariances (above diagonal), and Correlations (below)													
High liquidity	4.34	20.42	−0.49										
Low liquidity	0.88	5.33	−0.36										
Dividend yield	−0.11	−0.07	1.00										
Panel C: Conditional Standard Deviations, Covariances (above diagonal), and Correlations (below) for the VAR with Dividend Yield as Predictor													
High liquidity	4.32	20.20	−0.85										
Low liquidity	0.88	5.30	−0.92										
Dividend yield	−0.94	−0.84	0.21										
				4.32	20.20	−1.11		4.32	20.20	−1.11			
				0.88	5.30	−1.21		0.88	5.30	−1.21			
				−0.94	−0.84	0.27		−0.94	−0.84	0.27			

on both portfolios, but intuition suggests that the spread in transaction costs across the two portfolios is what is critical for the spread in expected return across the two. Because of this, the cost rate on the low-liquidity asset is taken to be this 2% while the cost rate on the high-liquidity asset (if available) is always taken to be zero. Keeping the transaction-cost rate on one of the risky assets equal to zero keeps the inherited allocation state-space one-dimensional, which keeps computation time manageable.

When we allow the cost rate Φ to be state dependent, the risky-asset returns are always predictable and the conditional expectation of $\phi = \ln(1 + \Phi)$ is linear in the d value. We always keep Φ 's unconditional mean μ_Φ equal to its value in the analogous case when it is a constant. We apply Gaussian quadrature rules to ϕ in such a way that the volatility of ϕ , σ_ϕ , and the mean of Φ match chosen values. In particular, for a given μ_Φ , the unconditional volatility of σ_ϕ is always chosen such that $\frac{\mu_\Phi}{\sigma_\phi}$ equals 2.63. The three values taken by the ϕ shock are always the same and 2.63 is chosen to ensure a large spread in Φ but without any negative values.

To calculate a volatility estimate for ϕ , we use a half-spread estimate developed by Hasbrouck (2003) using a Bayesian approach and daily data for all ordinary common equity issues on the Center for Research in Security Prices (CRSP) daily database from 1962 to 2002. Hasbrouck (2003) averages the daily numbers to obtain half-spread estimates at an annual frequency for each stock. Fixing the year, we sort the stocks on half-spread and equally weight the top and bottom 50% to obtain annual time-series estimates for two hypothetical portfolios, the liquid portfolio and illiquid portfolio, respectively. If the half-spread is greater than 30% for an observation, we set that observation to 30%, analogous to the treatment of outliers in Amihud's ILLIQ measure by Acharya and Pedersen (2005). We construct a third series by subtracting the liquid portfolio half-spread from the illiquid portfolio half-spread, which gives us our proxy for Φ .

Turning to the labor income process used in the base case, parameter values are chosen to be the baseline values in Viceira (1997, 2001), who describes these values as consistent with those obtained by Gakidis (1997) based on PSID data for professionals and managers not self-employed under age 45. Liquidity premia given these values are of interest since this is a group that holds stocks, and, in particular, low-liquidity stocks, as we will see. Viceira's baseline value for the standard deviation of the change in log permanent labor income is 15% per year, and his baseline value for the mean growth of permanent labor income is 3% per annum. These are used to back out values for the unconditional volatility of g , σ_g and the unconditional mean of g_t , \bar{g} . A number of papers (see, e.g., Chamberlain and Hirano (1997), Carroll and Samwick (1997)) estimate labor income parameters and a range of values are reported across these studies. The Gakidis (1997) values appear to lie within this range, which makes them reasonable to use.

We are also interested in calibrating to U.S. data the correlations between the beginning-of-month dividend yield, the transaction-cost rate, and the growth rate of labor income. The dividend yield series that we use is the one described

above. But we need proxies for the log growth in permanent labor income g , and the transaction-cost rate ϕ , since the proxy described above for ϕ is only available at an annual frequency.

Monthly aggregate labor income data are used to compute covariances between permanent labor income growth and both dividend yield and the transaction-cost rate. Correlations are then computed using the standard deviation of monthly individual permanent income growth calculated using Viceira (1997, 2001) as described above. It is reasonable to use aggregate data to estimate covariance if the idiosyncratic component of individual labor income growth is uncorrelated with these two series. Aggregate labor income data are from the Bureau of Labor Statistics' website. We use the Retail Trade income data series CEU4200000004, which is measured at a monthly frequency and is available from January 1972 until December 2003. The income data are deflated using a CPI measure, series CPIAUCNS, that's also available from the Bureau of Labor Statistics. Per capita income values are generated by dividing all income series by a population measure, series POP, available from the U.S. Department of Commerce's Census Bureau.

The proxy for ϕ available at a monthly frequency is a less direct measure of the per-trade cost than Hasbrouck's (2003) measure, which is why we only use it to calculate correlations. The Acharya and Pedersen (2005) data set, in addition to monthly returns on 25 value-weighted portfolios sorted on Amihud's ILLIQ measure, also provides a corresponding normalized ILLIQ for each portfolio, which they argue can be interpreted as a one-way transaction-cost rate. We value-weight to obtain monthly series of one-way proportional transaction-cost rates for the low- and the high-liquidity portfolios. Again, we construct a third series by subtracting the high-liquidity portfolio cost rate from the low-liquidity portfolio cost rate. Data are available from February 1964 to December 1996. We implement the transformation $\phi = \log(1 + \Phi)$ on this series and the two Hasbrouck (2003) series.

Gaussian quadrature is used to deliver the joint distributions for (g_t, ϕ_t, d_{t+1}) and (l_t, ϕ_t, d_{t+1}) . Three grid points are used for both the g and the l shocks. Unconditional moments for ϕ are chosen as described above. A number of labor income cases are considered. In the base case, the labor income process and its correlation with beginning-of-month dividend yield is calibrated to the data as described above, and the transaction-cost rate is constant. To gauge the importance of the business cycle variation in mean log permanent labor income growth found in the data, we consider a case (the i.i.d. labor income growth case) with predictable returns and i.i.d. labor income growth that has the same annual mean and volatility as base-case labor income. To assess whether labor income's ability to generate a sizeable liquidity premium depends on returns being predictable, we consider a case (the i.i.d. return case) in which both returns and labor income growth are i.i.d. To gauge the extent to which the availability of the high-liquidity asset reduces the liquidity premia, we also consider a case (the one risky asset case) that is exactly like the base case except there is no access to the high-liquidity asset. Finally, we assess the effect on liquidity premia of making the transaction-cost rate state dependent

as in the data by making the transaction-cost rate in the one risky asset case state dependent. We call this the state-dependent transaction-cost case.

We calibrate the multiplicative shock process to the labor income growth process that is used in the base case described above. In our base case, log labor income growth, Δy_t , is calibrated to be $N(\bar{g} + b_g d_t, \sigma_u^2)$. Given the wealth-income ratio at t , $\Gamma_t = W_t/Y_{t-1}$, $\ln(Y_t/W_t)$ equals $\Delta y_t - \ln(\Gamma_t)$ and is distributed $N(\bar{g} + b_g d_t - \ln(\Gamma_t), \sigma_u^2)$. Since the wealth shock L_t is multiplicative, it makes sense to calibrate the distribution of $\ln(L)$ to that of $\ln(Y_t/W_t)$. We allow its mean to be dependent on age and the dividend-yield state. The mean of $l \equiv \ln(L)$ is obtained by calculating $(\bar{g} + b_g d_t)$ using the base-case parameters, which are calibrated to the data, and using a Γ profile that depends on age and the dividend-yield state and is calculated by taking Γ at age 1 to be one, simulating and averaging at each age and each dividend yield state. With the correlation between l and d fixed at the data value for Δy and d 's correlation of -0.0372 , we adjust the volatility of l so as to match the liquidity premium for the base-case agent with a Γ at age 1 of one. Gaussian quadrature is applied to l and ϕ to obtain their joint distribution, with the conditional expectation of l linear in the d value. Three grid points are used for the l shock. Fixing the unconditional volatility of the wealth shock and the unconditional mean and volatility of the transaction-cost rate (if state dependent), the environment facing the investor depends on: (1) whether returns are i.i.d. or not, (2) whether the transaction-cost rate is state dependent or not, (3) the contemporaneous correlation between the wealth shock and the transaction-cost rate $\rho_{l,\phi} \equiv \rho[l_t, \phi_t]$, (4) the correlation between the wealth shock and the beginning-of-month dividend yield $\rho_{l,d} \equiv \rho[l_t, d_t]$, and (5) the correlation between the transaction-cost rate and the beginning-of-month dividend yield $\rho_{d,\phi} \equiv \rho[d_t, \phi_t]$. We focus on varying the last three of these determinants to get a sense of how each of these three correlations affects liquidity premia.

Panel A of Table II reports the unconditional means and the standard deviations (in %) of the three Hasbrouck (2003) series for the period 1962 to 2002. The volatility of the half-spread difference is 7.08%, which is much larger than the 0.76% used in the liquidity premia calculations. This result suggests that we are using a conservative volatility number for ϕ when we allow ϕ to be state dependent. Interestingly, the mean of 5.44% is also larger than the 2% used in the liquidity premia calculations.

Panel B of Table II reports the correlations between the ILLIQ transaction-cost measure and both the beginning-of-month dividend yield and the growth rate of labor income. Again we focus on the cost rate differential across the low- and high-liquidity portfolios.⁵ We find that ϕ is positively correlated with d , which is consistent with the idea that stocks are more expensive to trade in recessions. The correlation between ϕ and g is negative. We realize that the series we are using to proxy for g and ϕ are quite noisy and so the reported

⁵ When calculating a correlation, the same data period is used to calculate the standard deviations and the covariance, to ensure the correlation lies between -1 and 1 . For a given pair of variables, all dates with data for both are used in the calculation of their covariance.

Table II
Means and Standard Deviations for Transaction-Cost Rates,
and Their Correlations with Labor Income and Dividend Yield

The table presents empirical means and standard deviations for proportional transaction-cost rates, and their correlations with the change in log permanent labor income and the beginning-of-month dividend yield. Following Hasbrouck (2003), we sort stocks each year on half-spread and equally weight the top and bottom 50% to obtain annual estimates for a liquid and an illiquid portfolio from 1962 to 2002. We construct a third series, Illiquid–Liquid, by subtracting the half-spread of the liquid portfolio from that of the illiquid. Panel A reports the unconditional means and the standard deviations of these three series. Acharya and Pedersen (2005) construct 25 value-weighted portfolios sorted on the ILLIQ variable of Amihud (2002). We define the low (high) liquidity portfolio to be the value-weighted portfolio of the least (most) liquid 13 (12) portfolios. Normalized ILLIQ is used as the transaction-cost rate for these portfolios. We construct a third series, Low–High, by subtracting the transaction-cost rate for the high-liquidity portfolio from that of the low. Panel B reports the correlations of these three series with the beginning-of-month annual NYSE dividend yield, d , from February 1964 to December 1996, and with individual labor income growth, g , from January 1972 to December 1996. To obtain the correlations with individual labor income growth, we divide the covariance of the cost and aggregate income growth series (which is total U.S. Retail Trade income from the Bureau of Labor Statistics as described in Section III.B) with the appropriate monthly volatility of individual income and the monthly volatility of transaction costs. Φ denotes the proportional transaction-cost percentage, and $\phi = \ln(1 + \Phi)$. Section III.B contains further details about how the variables are constructed.

Panel A: Means and Standard Deviations (in percent)		
Φ Series	μ_Φ	σ_ϕ
Liquid	0.39	0.40
Illiquid	5.84	7.24
Illiquid–Liquid	5.44	7.08

Panel B: Correlations		
Φ Series	$\rho_{\Phi,d}$	$\rho_{\phi,g}$
High liquidity	13.49%	–3.79%
Low liquidity	11.40%	–1.55%
Low–High	11.16%	–1.46%

correlations are likely to be noisy estimates of the true correlations. Nonetheless, these numbers should at least be informative as to the direction of the relations and may even contain some information as to the magnitudes. Finally, we find that the correlation between g and d is negative and equal to -0.0372 , consistent with the intuition that labor income is lower during recessions.

C. Other Parameter Choices

The investor's risk aversion parameter, γ , is set to six for all the labor income cases described in Section III.B and for all the multiplicative shock cases considered. This γ choice is motivated by the Mehra and Prescott (1985)

argument that the existing evidence from macrostudies and microstudies provides a justification for restricting the value of γ to be less than 10. The horizon T of the young investor is 240 periods or 20 years, since the return processes are calibrated to monthly returns. A 20-year horizon is a realistic investment horizon for an investor who retires at time 1. We keep the horizon at 20 years for the labor income and multiplicative wealth shock cases to allow more direct comparisons to the canonical i.i.d. return problem with no labor income. The time preference parameter, δ is set equal to the inverse of the riskless return.

IV. Liquidity Premia and Turnover Results

This section discusses the liquidity premium, direct trading cost, and turnover numbers reported in Table III for the labor income cases and the liquidity premium numbers reported in Table IV and Figures 2 and 3 for the multiplicative wealth shock cases. Table III reports annual liquidity premia (in %) on the low-liquidity portfolio in Panel A, the associated direct trading costs in Panel B, and turnover values in Panel C, for financial wealth to monthly permanent income ratios (Γ) of 0, 1, 10, 100, 1,000, and ∞ . Each column contains results for one labor income case and the following cases are included: i.i.d. return, i.i.d. labor income growth, base, one risky asset, state-dependent transaction cost, moderate liquidity preferring, extreme liquidity preferring, and fixed point. All these cases are described in Section III.B except the two liquidity-preferring cases, which we describe in the market clearing section (Section V), and the fixed point case, which we describe in Section IV.C.

A. Return Predictability

As the financial wealth to monthly permanent income ratio Γ converges to ∞ , the labor income problem converges to the otherwise identical problem without labor income. Consequently, the $\Gamma = \infty$ rows in Table III report liquidity premia, direct trading costs, and turnover numbers for the otherwise identical problem without labor income. Thus, the i.i.d. return case with $\Gamma = \infty$ is the canonical allocation problem with i.i.d. returns, a constant transaction-cost rate, and no wealth shocks. The $\Gamma = \infty$ liquidity premium for this i.i.d. return case in Panel A of Table III is 0.08% per annum. Since the transaction-cost rate is 2%, this premium is an order of magnitude smaller than the rate, consistent with results in Constantinides (1986). Similarly, the i.i.d. labor income growth case with $\Gamma = \infty$ is the allocation problem with predictable returns calibrated to the data, a constant transaction-cost rate, and no wealth shocks. The reported premium for this problem is 0.43% per annum, which means that return predictability calibrated to that in the data increases the liquidity premium on the low-liquidity portfolio by a factor of five relative to the canonical case.

The reason for the increase is as follows. The usual motive for trading is to rebalance the portfolio back to the optimal weights after realized risky-asset returns alter the portfolio's composition from the optimal weights. Return predictability causes the optimal portfolio weights to move around through time

Table III
Per-Annum Liquidity Premia, Turnover, and Direct Trading Costs
on the Low-Liquidity Portfolio in the Presence of Labor Income

The table reports, for several cases, annual liquidity premia (in percent) on the low-liquidity portfolio and the associated direct trading costs and turnover values for financial wealth to monthly permanent income ratios (Γ) of 0, 1, 10, 100, 1,000, and ∞ . The low (high) liquidity portfolio is calibrated to the value-weighted portfolio of the least (most) liquid 13 (12) of the 25 value-weighted ILLIQ portfolios constructed by Acharya and Pedersen (2005) from February 1964 to December 1996. Unless otherwise stated, the agent has risk aversion of six and access to a riskless asset as well as the high- and low-liquidity portfolios, which trade costlessly and at a proportional transaction-cost rate of 2%, respectively, while the permanent labor income growth and return processes are calibrated to the data, as described in Section III. Returns are predictable and labor income growth is i.i.d. in the i.i.d. labor income growth case, while both are i.i.d. in the i.i.d. return case and both are predictable in the base case. The one risky asset and state-dependent transaction-cost cases are the same as the base case except there is no access to the high-liquidity portfolio, and in the latter case, the cost rate is allowed to be state dependent as described in Section III. In the two liquidity preferring cases, risk aversion is 12 and the parameters of the return and labor income processes are chosen so the young investor wants to hold more of the high-liquidity portfolio (see Section V.A for details). The fixed point case is the same as the base case except the portfolio Sharpe ratios net of any liquidity premium are equated as described in Section IV.C. Panel A reports the liquidity premium, which is defined in Section II.D. Panels B and C report the direct trading cost per annum and the per-annum turnover, which are defined in Section II.E.

Γ	i.i.d. Return	i.i.d. Labor Income Growth	Base	One Risky Asset	State-dep. Trans. Costs	Moderate Liquidity Preferring	Extreme Liquidity Preferring	Fixed Point
Panel A: Liquidity Premia per Annum								
0	1.42	1.63	1.48	1.54	1.71	1.53	1.67	1.42
1	1.38	1.56	1.44	1.50	1.63	1.48	1.59	1.36
10	0.94	1.08	1.12	1.13	1.24	1.22	1.31	1.06
100	0.27	0.54	0.53	0.58	0.68	0.57	0.62	0.48
1,000	0.11	0.50	0.47	0.48	0.51	0.53	0.54	0.42
∞	0.08	0.43	0.43	0.46	0.34	0.49	0.49	0.38
Panel B: Direct Cost per Annum								
0	0.65	1.04	1.06	1.18	0.92	0.97	1.15	1.05
1	0.64	1.01	1.03	1.14	0.90	0.95	1.10	0.96
10	0.46	0.71	0.76	0.85	0.69	0.76	0.84	0.69
100	0.23	0.42	0.40	0.44	0.48	0.43	0.46	0.35
1,000	0.10	0.39	0.40	0.42	0.44	0.40	0.41	0.32
∞	0.08	0.39	0.39	0.41	0.42	0.36	0.36	0.31
Panel C: Turnover per Annum								
0	32.58	52.14	53.14	58.95	46.09	48.64	57.32	52.59
1	32.02	50.38	51.40	56.87	45.03	47.57	55.12	47.83
10	23.12	35.49	38.14	42.28	34.62	37.88	41.83	34.27
100	11.44	20.84	20.13	22.15	23.80	21.73	22.76	17.41
1,000	4.94	19.74	19.93	20.78	22.12	19.84	20.28	16.01
∞	3.90	19.49	19.49	20.54	21.17	18.05	18.05	15.47

as the agent takes advantage of the time-varying expected returns. This time variation in the optimal weights creates an additional motive for trading that causes higher liquidity premia when returns are predictable.

B. i.i.d. Labor Income Growth

The i.i.d. return column in Panel A of Table III shows that with i.i.d. returns and a fixed transaction-cost rate, the inclusion of i.i.d. labor income growth uncorrelated with returns causes the liquidity premium on the low-liquidity portfolio to become 1.42% per annum for an agent with no financial wealth, an almost 18-fold increase relative to the canonical i.i.d. case. For an agent with a wealth to monthly labor income ratio of 10, the liquidity premium is still 0.94% per annum, a more than 11-fold increase. The i.i.d. labor income growth column of Panel A in Table III shows that once returns are allowed to be predictable (with the transaction-cost rate remaining a constant), i.i.d. labor income growth uncorrelated with returns causes the liquidity premium to be 1.63% and 1.08% for an agent with a wealth to monthly labor income ratio of 0 and 10, respectively. These represent 20-fold and 13-fold increases, respectively, relative to the canonical i.i.d. return case with no labor income. These results show that the introduction of i.i.d. labor income growth uncorrelated with returns is enough to drastically increase the liquidity premium relative to the canonical problem, especially for agents with wealth to monthly labor income ratios lower than 10. Interestingly, labor income still increases liquidity premia markedly when returns are i.i.d., which means that labor income still causes liquidity premia to increase considerably even if the sample estimates of b_g are higher than the true values. For a given wealth to monthly labor income ratio, the liquidity premium for i.i.d. returns can be regarded as a lower bound on the premium given the actual return predictability in the data.

Labor income causes liquidity premia to increase for two reasons. First, labor income is paid in cash and earns the riskless rate unless invested in stock. Thus, labor income distorts portfolio holdings away from the optimal weights, causing the agent to rebalance back to the optimal weights. Second, shocks to labor income are permanent and so have large effects on the agent's total wealth. These total wealth shocks can lead to large shifts in the optimal weights in the agent's financial wealth portfolio, particularly when the agent has a low wealth-income ratio. These shifts in the optimal weights provide a strong motive for the agent to trade, leading to much higher liquidity premia than in the case with no labor income. Adding a temporary component to the monthly shock to labor income has almost no effect on liquidity premia, which implies that the second of these two channels is much more important than the first. It is also not surprising that return predictability still has an incremental effect, further inflating the premium, in the presence of labor income. As discussed above, return predictability causes the optimal portfolio weights to move around over the business cycle. This additional variation in the optimal weights can provide a motive for trading over and above that provided by the effects of labor income on the optimal portfolio weights.

C. Procyclical Labor Income Growth

In the base case, returns are predictable and labor income growth is allowed to be procyclical, as in the data, but the transaction-cost rate is constant. The base-case column in Panel A of Table III shows that allowing labor income growth to be procyclical when returns are predictable slightly reduces the liquidity premium at very low and very high wealth-income ratios. The liquidity premium declines 0.15% to 1.48% when the agent has no financial wealth but this is still an 18-fold increase in the liquidity premium relative to the canonical i.i.d. return case with no labor income. The premium actually increases slightly to 1.12% per annum at a wealth to monthly labor income of 10.

The reductions in liquidity premia occur because expected returns are countercyclical but labor income growth is procyclical. When returns are predictable, the optimal stock portfolio weights vary positively with expected stock returns. When dividend yield is high, expected returns and the optimal stock portfolio weights are high but labor income growth is low and so labor income's downward pressure on the portfolio weights in stocks is small. Conversely, when dividend yield is low, expected returns and the optimal stock portfolio weights are low but labor income growth is high and so labor income's downward pressure on the portfolio weights in stocks is large. In both cases, the agent trades less relative to the i.i.d. labor income growth case and so liquidity premia are slightly lower too.

Labor income being procyclical causes liquidity premia to increase at moderate Γ values for the following reason. The hedging demand due to labor income growth being procyclical causes optimal stockholdings to decline (see Lynch and Tan (2011)). The result is that the 100% maximum for total holdings of stocks binds less often. Consequently, the agent trades more relative to the i.i.d. labor income growth case and the liquidity premium increases. We expect this increase to be largest for moderate Γ values, that is, for values not so small that the 100% stockholding maximum almost always binds even with labor income being procyclical nor so big that the 100% stockholding maximum rarely binds even when labor income is not procyclical.

An interesting question is how the magnitude of the spread in risk-adjusted returns across the low- and high-liquidity portfolios compares to the liquidity premium on the low-liquidity portfolio required by the base-case agent. Since the base-case agent is risk averse, we are interested in calculating the spread in volatility-adjusted returns across the two assets. To do this, we find the expected monthly discretely compounded return on the low-liquidity portfolio that would equate the monthly unconditional Sharpe ratio for the low-liquidity portfolio to the monthly unconditional Sharpe ratio for the high-liquidity portfolio and then we compare this expected return to the low-liquidity portfolio's unconditional mean discretely compounded monthly return in the data. The difference between the two is a measure of the difference in the volatility-adjusted returns across the two assets. Table I reports unconditional Sharpe ratios and unconditional mean monthly returns for the low- and high-liquidity portfolios based on monthly discretely compounded returns. The unconditional

Sharpe ratio for the high-liquidity portfolio is 0.1023, while the unconditional Sharpe ratio for the low-liquidity portfolio is higher, at 0.1641. Because the low-liquidity portfolio's return volatility is higher than that of the high-liquidity portfolio, the unconditional mean monthly return for the low-liquidity portfolio that equates the two unconditional Sharpe ratios, 0.65%, while lower than the portfolio's unconditional mean monthly return in the data of 0.98%, is still higher than the unconditional mean monthly return on the high-liquidity portfolio in the data of 0.55%. So the spread in volatility-adjusted returns across the two portfolios is the difference between 0.98% per month and 0.65% per month, with the low-liquidity portfolio having the higher volatility-adjusted return. This spread in the volatility-adjusted return of 0.33% per month, or 3.90% per annum, is much higher than the liquidity premium of 1.48% per annum required by the base-case agent with a wealth-income ratio of zero.

Since the volatility-adjusted return spread across the low and high portfolios is larger than the liquidity premium required by the base-case agent with a wealth-income ratio of zero, we are interested in finding out how much the spread must be reduced for this agent to require a liquidity premium equal to the spread. That is, what spread in volatility-adjusted returns across the low and high portfolios can cause this agent to demand a liquidity premium equal to the spread? Since any given return spread generates a liquidity premium required by this agent, we are looking for a fixed point such that the return spread generates a liquidity premium required by this agent that is exactly equal to the volatility-adjusted return spread. We have in mind an iterative procedure. At the second iteration, we reduce the mean return of the low-liquidity portfolio by 2.50% per annum to reduce the spread in mean return by 2.50% per annum. This also reduces the volatility-adjusted spread by 2.50% per annum, from the 3.90% per annum in the data to 1.40% per annum, which is lower than the 1.48% liquidity premium demanded by the base-case agent with a zero wealth-income ratio. Confronted with this reduced spread in volatility-adjusted returns, Table III shows (see the fixed point column) that this agent with a wealth-income ratio of zero demands a liquidity premium of 1.42% per annum, which is slightly higher but essentially the same as the spread in volatility-adjusted returns of 1.40% per annum. Since the liquidity premium number is essentially the same as the spread in volatility-adjusted returns, we do not iterate further. Thus, we have shown that a spread in volatility-adjusted returns of 1.40% per annum across the two portfolios can generate a liquidity premium of roughly the same magnitude.⁶

D. Only One Risky Asset Available: The Low-Liquidity Portfolio

Comparing the base case to the one risky asset case in Panel A of Table III, we find that removing access to the high-liquidity asset increases liquidity premia

⁶ Even if we ignore the differences in volatility across the two assets and just compare the spread in their expected returns to the liquidity premium required by the base-case agent with a wealth-income ratio of zero, we find that a spread in expected returns across the two assets of 1.02% per annum causes this agent to demand a liquidity premium of 1.05% per annum.

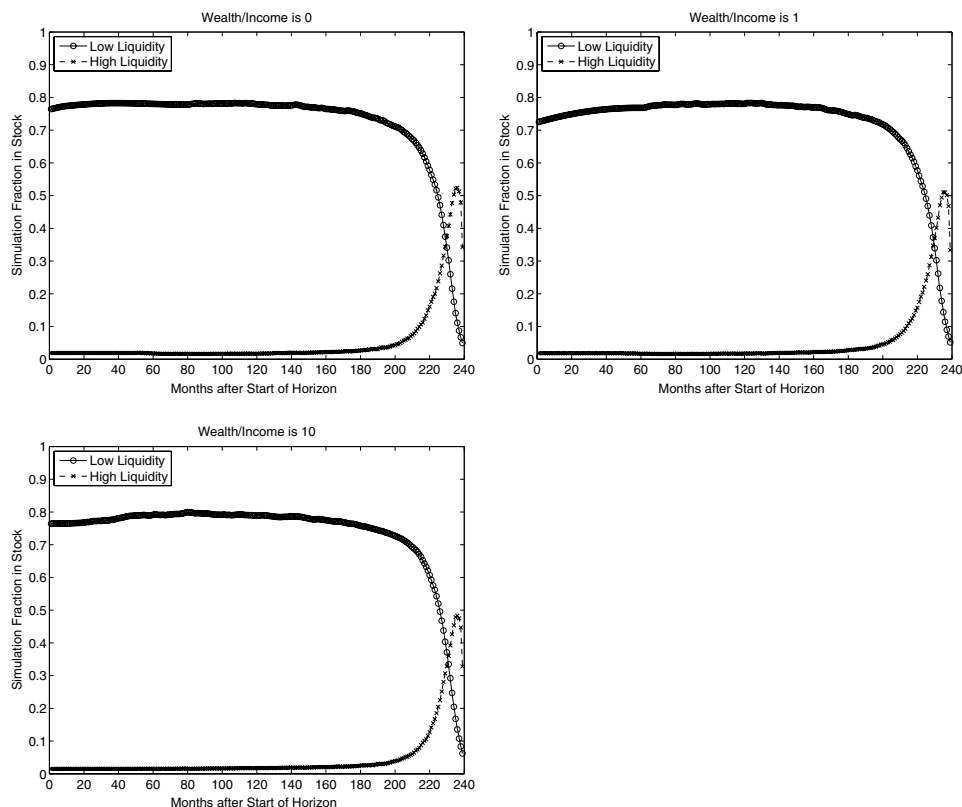


Figure 1. Simulation allocation results for the low- and high-liquidity portfolios: base case. The figures report simulation allocation fractions for the low- and high-liquidity portfolios for the base case for initial wealth to monthly permanent income ratios of 0, 1, and 10. The CRRA investor has risk aversion of six and access to the low-liquidity portfolio, which has a constant 2% proportional transaction cost; the high-liquidity portfolio, which trades costlessly; and a riskless asset. Returns and labor income growth are both predictable and are calibrated to the data as described in Sections III.A and III.B, respectively. Acharya and Pedersen (2005) construct 25 value-weighted portfolios sorted on the ILLIQ variable of Amihud (2002) for the period from February 1964 to December 1996, and the low (high) liquidity portfolio is the value-weighted portfolio of the least (most) liquid 13 (12) portfolios. For each simulation, the initial dividend yield value is drawn from its unconditional distribution, and one million paths are simulated for each wealth–income ratio.

only slightly. For a wealth to monthly labor income ratio of 10, the premium increases only 0.01% per annum while the increase is still only 0.06% when the agent has no financial wealth. Removing the high-liquidity risky asset would be expected to cause only small increases in liquidity premia if the base-case agent holds very little of this asset.

Figure 1 plots average holdings of the two risky assets by the base-case agent for initial wealth to monthly income values of 0, 1, and 10. Average holdings as a function of age are obtained by simulating a large number of return and

labor income paths, with the unconditional distribution of the dividend yield used to select an initial state for each path. Figure 1 shows that the agent's average holding of the high-liquidity asset is much smaller than the agent's average holding of the low-liquidity asset except in the last few months of life for all four initial Γ values. Moreover, the average holding of the high-liquidity asset is less than 10% for all but the last 3 years of life for all four initial Γ values.⁷ The implications for market clearing of the base-case agent holding much less of the high-liquidity asset than the low-liquidity asset are explored in Section V.

E. State-Dependent Transaction-Cost Rate

The effect on liquidity premia of allowing the transaction-cost rate to be state dependent with the dependence calibrated to the data is assessed by examining a case that is identical to the one risky asset case except for the state-dependent transaction-cost rate. The reason that the high-liquidity asset is not made available is the excessive computational burden associated with the two risky asset problem when the cost rate is state dependent. Comparing the state-dependent transaction-cost column to the one risky asset column in Panel A of Table III, we see that liquidity premia increase even further when the transaction-cost rate is allowed to be state dependent as observed in the data. In particular, the per-annum premium increases 0.17% for the agent with no financial wealth and 0.11% when the agent's wealth to monthly income ratio is 10. It seems reasonable to expect similar though slightly smaller increases when the transaction-cost rate in the base case with two risky assets is allowed to be state dependent.

The reason for the increase in the liquidity premia is as follows. In the data, the transaction-cost rate is countercyclical, which means it is high when future expected returns are high and so the optimal portfolio weights in stocks are high too. These are exactly the times when the agent trades the most since the effect of labor income (because it is paid in cash) is to lower the portfolio weights in stocks.

F. Multiplicative Wealth Shocks

Table IV reports per-annum liquidity premia on the low-liquidity portfolio in the presence of multiplicative wealth shocks for an agent with a risk aversion of six. Panel A tabulates liquidity premia for cases with a constant transaction-cost rate while Panel B tabulates them for cases with a state-dependent transaction-cost rate. Subpanels vary the wealth shock dynamics. Recall that to make the multiplicative shock analysis more directly comparable to the labor income cases we consider, the multiplicative shock process is calibrated to the base-case labor income process.

⁷ The base-case agent's risky-asset allocations wiggle in the last few months of life because the agent does not receive labor income at the terminal date.

Table IV
Per-Annum Liquidity Premia on the Low-Liquidity Portfolio in the Presence of Multiplicative Wealth Shocks

The table reports annual liquidity premia (in percent) on the low-liquidity portfolio in the presence of multiplicative wealth shocks. The CRRA investor has risk aversion of six and access to the low-liquidity portfolio, the high-liquidity portfolio (which trades costlessly), and a riskless asset. Panel A tabulates liquidity premia for cases with a constant transaction-cost rate Φ for the low-liquidity portfolio, while Panel B tabulates liquidity premia for cases with a state-dependent transaction-cost rate whose unconditional mean is always fixed at 2%. Subpanels vary the wealth shock L dynamics. The liquidity premium is defined to be the decrease in the unconditional mean log return on the low-liquidity asset that the investor requires to be indifferent between having access to the risky asset without rather than with transaction costs. The multiplicative shock process is designed to be similar to the labor income growth process that is used in the base case except that the wealth shocks are multiplicative rather than additive (see Section III.B for a description of the calibration). Returns can be predictable or i.i.d. and in either case are calibrated to U.S. data using a quadrature approximation. Acharya and Pedersen (2005) construct 25 value-weighted portfolios sorted on the ILLIQ variable of Amihud (2002) for the period from February 1964 to December 1996, and the low (high) liquidity portfolio is the value-weighted portfolio of the least (most) liquid 13 (12) portfolios. d denotes beginning-of-month log dividend yield and we have that $\phi = \ln(1 + \Phi)$ and $l = \ln(L)$. Gaussian quadrature is used to obtain the joint distribution of returns and $\{l_t, \phi_t, d_{t+1}\}$, as described in Sections III.A and III.B.

Panel A: $\Phi = 2\%$							
$\rho_{l,d}$		i.i.d.				Predictable	
−0.5		1.46				4.24	
−0.0372						1.44	
0						1.71	
0.5						3.62	
Panel B: $\mu_\Phi = 2\%$ $\sigma_\Phi = 0.76\%$							
		$\{\rho_{\phi,l} = -0.5\}$		$\{\rho_{\phi,l} = 0\}$		$\{\rho_{\phi,l} = 0.5\}$	
$\rho_{\phi,d}$	$\rho_{l,d}$	i.i.d.	Predictable	i.i.d.	Predictable	i.i.d.	Predictable
−0.5	−0.5	1.12	4.61	1.07	4.53	0.84	3.97
	0		2.39		2.07		1.65
	0.5		4.50		4.25		4.10
−0.5	4.99		4.86		4.62		
0	0		2.33		1.97		1.56
0.5	0.5		3.71		3.30		2.82
0.5	−0.5		5.84		5.46		4.99
	0		2.47		2.09		1.82
	0.5		3.19		2.82		2.48

Panel A indicates that when the transaction-cost rate is a constant, the liquidity premium exhibits a strong U-shaped pattern as a function of the unconditional correlation between a given month’s wealth shock and the dividend yield at the beginning of that month $\rho_{l,d}$. The table reports premia for $\rho_{l,d}$ of −0.5, 0, 0.5, and the data value of −0.0372. The liquidity premium is lowest

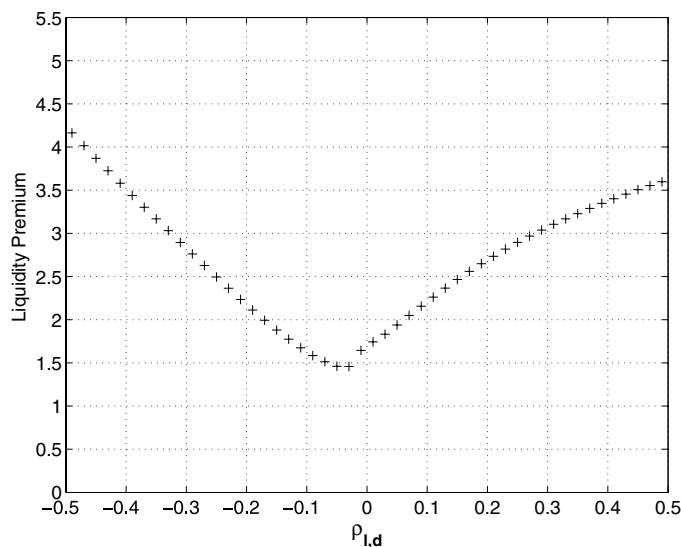


Figure 2. Liquidity premium as a function of the correlation between the multiplicative wealth shocks and beginning-of-month dividend yield. The figure plots the annual liquidity premium in percent on the low-liquidity portfolio against the unconditional correlation of the monthly log wealth shock with the log beginning-of-month dividend yield, $\rho_{l,d}$. The liquidity premium is the decrease in the unconditional mean log return on the low-liquidity asset that the investor requires to be indifferent between having access to the asset without rather than with transaction costs. The CRRA investor has risk aversion of six and access to the low-liquidity portfolio, which has a constant 2% proportional transaction cost; the high-liquidity portfolio, which trades costlessly; and a riskless asset. Returns and the multiplicative wealth shock are both predictable and are calibrated to the data as described in Sections III.A and III.B, respectively. Acharya and Pedersen (2005) construct 25 value-weighted portfolios sorted on the ILLIQ variable of Amihud (2002) for the period from February 1964 to December 1996, and the low (high) liquidity portfolio is the value-weighted portfolio of the least (most) liquid 13 (12) portfolios. Except for its correlation with the log beginning-of-month dividend yield, the multiplicative shock process is calibrated to the labor income growth process used in the base case: the distribution of the log wealth shock, l , is calibrated to that of $\ln(Y_t/W_t)$ in the base case.

for the data value. To ascertain exactly where the minimum liquidity premium value occurs, we plot the liquidity premium as a function of $\rho_{l,d}$ when the transaction-cost rate is a constant in Figure 2. The U-shaped pattern suggested by Table IV is confirmed by the graph, with the lowest premium occurring for a correlation of about -0.05 , which is close to the data value. Panel B of Table IV shows that this U-shaped pattern persists when the transaction-cost rate is state dependent, irrespective of the correlation between the cost rate and either beginning-of-month dividend yield or permanent labor income growth. The implication of the U-shaped pattern for the liquidity premium as a function of $\rho_{l,d}$ is that the premia for the labor income cases likely would be higher if the correlation between permanent labor income growth and beginning-of-month dividend yield was further away from zero in either direction than the

data correlation (obtained using the Retail Trade series as the aggregate labor income series).

The intuition for the large premium when the correlation is negative is as follows. The implied hedging demand with respect to future labor income is negative when this correlation is negative since return shocks are negatively correlated with dividend yield shocks. The resulting lower average risky asset holding means a higher premium for the same amount of trading. Turning to the large premium when this wealth shock–dividend yield correlation is positive, the dividend yield at the beginning of a month is positively related to the expected stock return over that month, so a positive wealth shock–dividend yield correlation implies a positive correlation between the wealth shock and expected stock return over the month. Since the wealth shock is paid in cash just like labor income, a higher wealth shock causes the inherited allocation to the risky assets to decrease more. So a positive unconditional relation between the wealth shock and conditional expected risky-asset return means that the inherited allocation to the risky assets is low because of the wealth shock precisely when the investor wants to hold the risky assets because of their high conditional expected returns. The result is a particularly large liquidity premium.

When the proportional transaction-cost rate is allowed to be state dependent, intuition suggests that wealth shocks are especially painful if negative wealth shocks occur when the transaction-cost rate is high. Panel B of Table IV confirms this intuition with the liquidity premium always decreasing in the unconditional correlation between wealth shocks and the transaction-cost rate, holding all else equal.

Another question of interest is how sensitive the liquidity premium is to changes in the volatility of the wealth shock. Intuition suggests the relation should be increasing and monotonic and Figure 3, which plots the liquidity premium as a function of the shock volatility, confirms this. Figure 3 plots liquidity premia for shock volatilities as high as 30% per annum, which generate premia as high as 2.21%. When the shock volatility is zero, the problem does not collapse to the base-case problem with an infinite wealth–income ratio because of the nonzero mean of the multiplicative shock. The liquidity premium is 0.63% per annum, which is slightly higher than the 0.43% in the base labor income case with $\Gamma = \infty$.

G. Turnover and Direct Trading Costs

Recall that turnover of the low-liquidity portfolio per annum and the associated direct cost per annum are reported in Panels C and B, respectively, of Table III for all the labor income cases considered in the paper. Panel C shows that annual turnover goes from about 4% per annum in the canonical case with i.i.d. returns and no labor income to 38% per annum for the base-case agent with a wealth to monthly income ratio of 10 and to 51% per annum for the base-case agent with a wealth to monthly income ratio of one. These turnover numbers in the presence of labor income are in the ballpark of the

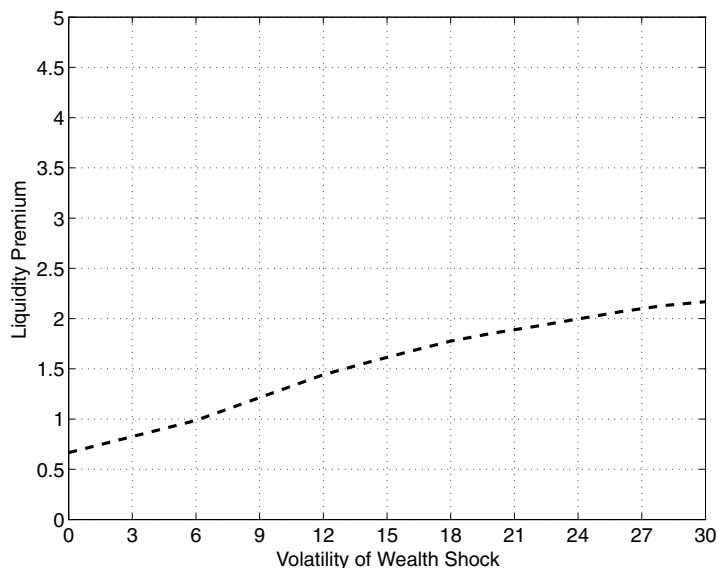


Figure 3. Liquidity premium as a function of the volatility of the multiplicative wealth shocks. The figure plots the annual liquidity premium in percent on the low-liquidity portfolio against the unconditional standard deviation of the monthly log wealth shock u . The liquidity premium is the decrease in the unconditional mean log return on the low-liquidity asset that the investor requires to be indifferent between having access to the asset without rather than with transaction costs. The CRRA investor has risk aversion of six and access to the low-liquidity portfolio, which has a constant 2% proportional transaction cost; the high-liquidity portfolio which trades costlessly; and a riskless asset. Returns and the multiplicative wealth shock are both predictable and are calibrated to data as described in Sections III.A and III.B, respectively. Acharya and Pedersen (2005) construct 25 value-weighted portfolios sorted on the ILLIQ variable of Amihud (2002) for the period from February 1964 to December 1996, and the low (high) liquidity portfolio is the value-weighted portfolio of the least (most) liquid 13 (12) portfolios. Except for its unconditional standard deviation, the multiplicative shock process is calibrated to the labor income growth process used in the base case: the distribution of the log wealth shock, l , is calibrated to that of $\ln(Y_t/W_t)$ in the base case.

monthly turnover numbers reported by Acharya and Pedersen (2005) for their low-liquidity portfolios, numbers that range from 3.25% to 4.19%. Consistent with return predictability alone increasing liquidity premia considerably and labor income alone increasing liquidity premia even more, turnover increases to 19% per annum when returns are predictable, and to 32% when the agent receives labor income whose growth is i.i.d. and the agent's wealth to monthly labor income ratio is one.

The turnover number can be multiplied by the average cost to get the direct effect of transaction costs on expected return. The extent to which the liquidity premium exceeds this direct cost can be attributed to some combination of risk premium for trading more when the agent is poor in utility terms, and in the case of a state-dependent cost rate, to the agent trading more when the cost rate is high. Not surprisingly, the direct cost is close to the liquidity premium when

there in no labor income and the cost rate is constant, irrespective of whether returns are predictable. Labor income drives a wedge between the liquidity premia and the direct costs. For the base-case agent with a wealth to monthly income ratio of one, Panel B reports a direct cost of 1.03% per annum while the liquidity premium in Panel A is 1.44%. For the case of i.i.d. returns and labor income growth and the same wealth–income ratio of one, the direct cost reported in Panel B is less than half the 1.38% per-annum liquidity premium reported in Panel A.

The reason for the wedge is as follows. Labor income causes the 100% upper bound on total stockholdings to bind often, which means that the agent is more likely to sell stock after a negative shock to permanent labor income than to buy stock after a positive shock. So the agent is trading more after negative labor income shocks, which tend to be times when the agent is poor in utility terms. Return predictability ameliorates this asymmetry because the 100% upper bound is likely not to bind when last month's dividend yield state is low.

V. Equilibrium Issues and Market Clearing

Market clearing is an important consideration when thinking about how prices and liquidity premia are determined in equilibrium. This section discusses how heterogeneity in labor income and risk aversion and heterogeneity induced by delegated portfolio management can allow all assets to be held and net trade each month to sum to zero.

A. All Assets Must Be Held

In the U.S. economy, the high-liquidity asset has a larger market capitalization than the low-liquidity asset, so market clearing is not possible at observed prices if all agents in the economy are identical to the base-case agent. However, it is likely that there is considerable heterogeneity across agents regarding their risk aversions and the income processes they receive. Some agents, like our base-case agent, invest large fractions of their portfolios in the low-liquidity asset and small fractions in the high-liquidity asset, while other agents invest small fractions of their portfolio in the low-liquidity asset and large fractions in the high-liquidity asset. They do so because they find the low-liquidity asset less attractive as compared to the high-liquidity asset than agents like our base-case agent. Aggregating across agents gives the aggregate values of the low- and high-liquidity portfolios. The aggregate value for the low-liquidity asset can be much lower than for the high-liquidity asset because most of the wealth in the economy is held by agents who hold small fractions of their portfolios in the low-liquidity asset. We focus on an agent who invests a large positive fraction of her portfolio in the low-liquidity asset because this is an agent who is definitely a marginal or inframarginal investor in the low-liquidity asset.

For this heterogeneity argument to be convincing, we need to demonstrate that agents with plausible preference parameters and plausible labor income streams hold much larger fractions of their portfolios in the high- than the

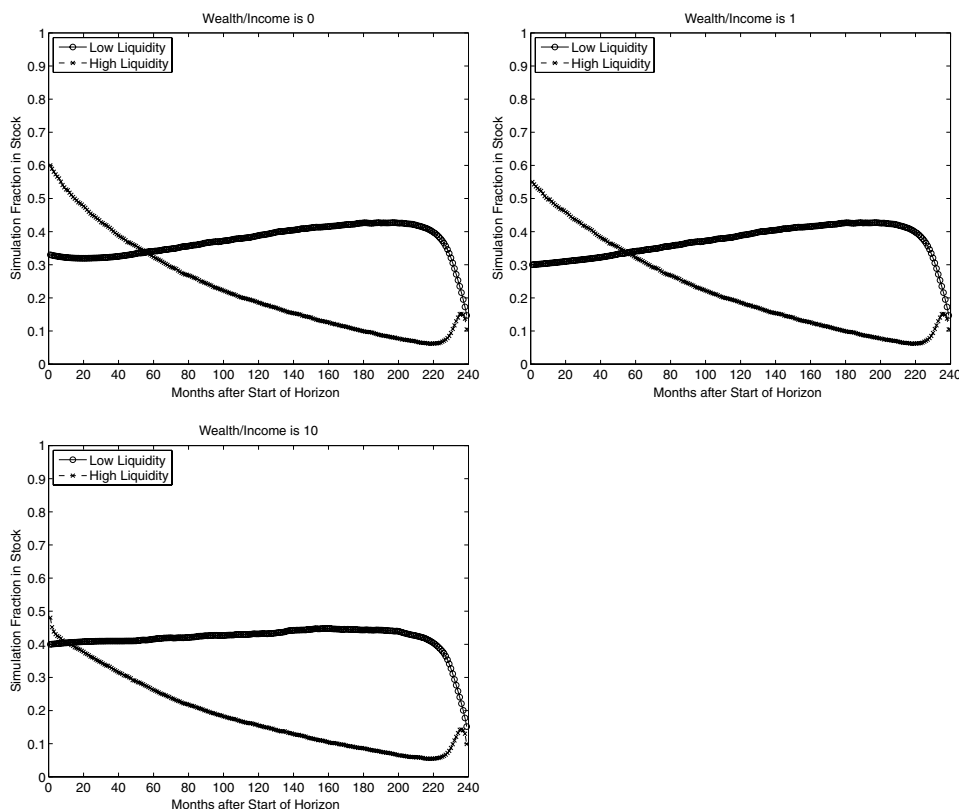


Figure 4. Simulation allocation results for the low- and high-liquidity portfolios: moderate liquidity-prefering case. The figures report simulation allocation fractions for the low- and high-liquidity portfolios for the moderate liquidity-prefering case for initial wealth to monthly permanent income ratios of 0, 1, and 10. The CRRA investor has risk aversion of 12 and access to the low-liquidity portfolio, which has a constant 2% proportional transaction cost; the high-liquidity portfolio, which trades costlessly; and a riskless asset. Returns and labor income growth are both predictable and are calibrated to the data as described in Section III.A, and in Sections III.B and V.A, respectively. Acharya and Pedersen (2005) construct 25 value-weighted portfolios sorted on the ILLIQ variable of Amihud (2002) for the period from February 1964 to December 1996, and the low (high) liquidity portfolio is the value-weighted portfolio of the least (most) liquid 13 (12) portfolios. For each simulation, the initial dividend yield value is drawn from its unconditional distribution, and one million paths are simulated for each wealth–income ratio.

low-liquidity asset, at least for considerable parts of their lives. Holdings for one such liquidity-prefering agent, the moderate liquidity-prefering agent, are reported in Figure 4. When her initial wealth to monthly income ratio is between zero and one, this agent holds more of the high- than the low-liquidity asset for at least the first 5 years of her 20-year life. At wealth to monthly income ratios as high as 10, this agent still holds more of the high- than the low-liquidity asset for some period of time early in life. This moderate liquidity-prefering agent has risk aversion of 12, which is higher than that for the agent

in the other cases but certainly is not unreasonably high. The permanent labor income growth process for this liquidity-preferring case is calibrated to that for young college-educated service workers in Gakidis (1997), who uses PSID data. The volatility of its log is 30% per annum, which matches the number in Gakidis (1997), and its mean is 4%, which lies between the 3% per annum for the base-case agent and the 7.42% per-annum number implied by Gakidis's (1997) results. Panel A of Table III reports liquidity premia for this moderate liquidity-preferring agent, and these premia are higher than those for the base-case agent, holding the wealth–income ratio fixed.

Holdings for another such liquidity-preferring agent, the extreme liquidity-preferring agent, are reported in Figure 5. This extreme liquidity-preferring agent also has risk aversion of 12 and receives the same permanent labor income growth process as the moderate liquidity-preferring agent, except that its conditional contemporaneous correlation with the low-liquidity portfolio's return is 30% and with high-liquidity portfolio's return is -20% (both correlations are zero for the moderate liquidity-preferring agent). When her initial wealth to monthly income ratio is between zero and one, this agent holds more of the high- than the low-liquidity asset for at least the first 11 years of her 20-year life and holds more than seven times more of the high- than the low-liquidity asset in her first month. In the last 9 years of life, the agent never holds more than 3.5 times more of the low- than the high-liquidity asset. At wealth to monthly income ratios as high as 10, this agent still holds more of the high- than the low-liquidity asset for the first 7 years of life, and holds more than twice the high- than the low-liquidity asset in her first month. Panel A of Table III reports liquidity premia for this extreme liquidity-preferring agent. These premia are higher than those for both the base-case and moderate liquidity-preferring agents, holding the wealth–income ratio fixed.

So far, we have taken as given that the agent lives for 20 years, even though most agents have working lives that are much longer than this. An agent who works from age 22 to age 65 has 43 years of working life. Holding the labor income and return processes fixed, the portfolio allocation and consumption rules we obtain for our 20-year agent can also be regarded as the portfolio allocation and consumption rules for the last 20 years of life for a 43-year agent. This agent's portfolio allocation rules for the first 23 years of her life are likely to look much more like her rule with 20 years until the terminal date than her rule with 1 month until the terminal date. But both liquidity-preferring agents tend to hold more of the high-liquidity asset relative to the low-liquidity asset as they get younger, which suggests that, for either agent, the 43-year version would on average allocate a much larger fraction of her portfolio to the high-liquidity asset as a multiple of the fraction allocated to the low-liquidity asset than the 20-year version. We cannot verify this directly due to computational constraints, but we do find that the 20-year version of either on average allocates a much larger fraction of her portfolio to the high-liquidity asset as a multiple of the fraction allocated to the low-liquidity asset than the 10-year version. So the intuition described above works going from a 10-year to a 20-year liquidity-preferring agent, which makes it likely that it also works going from a 20-year to a 43-year liquidity-preferring agent.

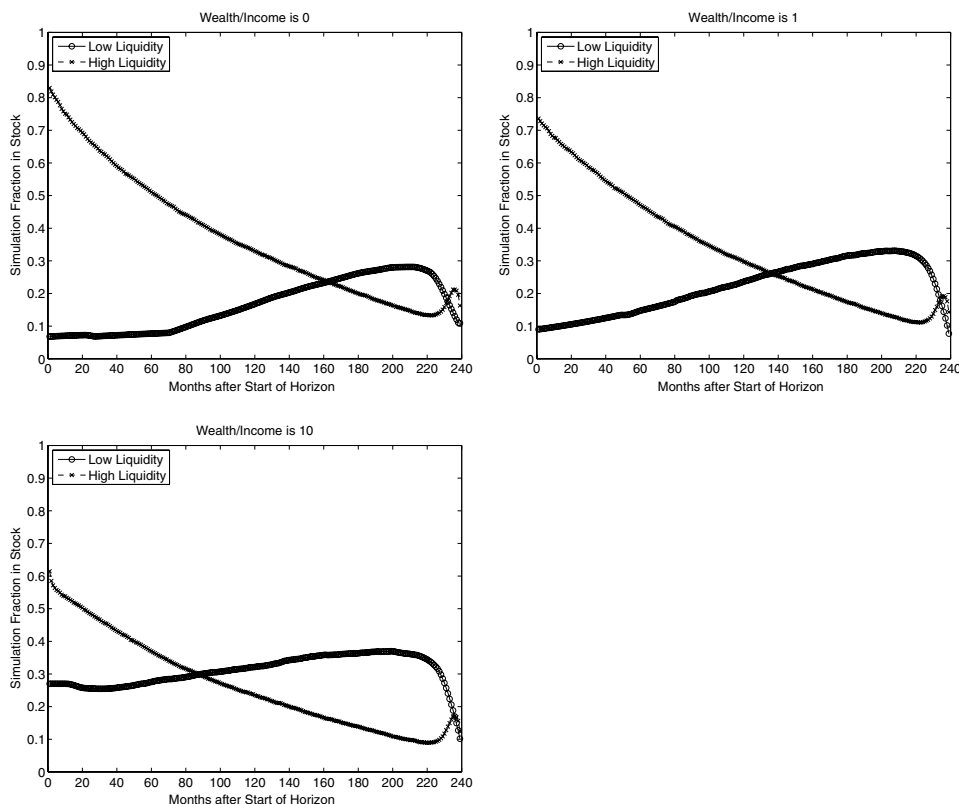


Figure 5. Simulation allocation results for the low- and high-liquidity portfolios: extreme liquidity-prefering case. The figures report simulation allocation fractions for the low- and high-liquidity portfolios for the extreme liquidity-prefering case for initial wealth to monthly permanent income ratios of 0, 1, and 10. The CRRA investor has risk aversion of 12 and access to the low-liquidity portfolio, which has a constant 2% proportional transaction cost; the high-liquidity portfolio, which trades costlessly; and a riskless asset. Returns and labor income growth are both predictable and are calibrated to the data as described in Section III.A, and in Sections III.B and V.A, respectively. Acharya and Pedersen (2005) construct 25 value-weighted portfolios sorted on the ILLIQ variable of Amihud (2002) for the period from February 1964 to December 1996, and the low (high) liquidity portfolio is the value-weighted portfolio of the least (most) liquid 13 (12) portfolios. For each simulation, the initial dividend yield value is drawn from its unconditional distribution, and one million paths are simulated for each wealth–income ratio.

There is another important source of heterogeneity that can help markets clear despite the presence of agents who hold more of the low-liquidity than the high-liquidity stocks. Heterogeneity may also come from participation by agents who care about something other than consumption from their labor income and financial wealth portfolios. In particular, Cuoco and Kaniel (2010) show how the existence of a delegated portfolio management industry in which managers receive a symmetric fulcrum fee that depends on performance relative to a benchmark can cause funds to tilt fund portfolios toward the stocks

in the benchmark. The result is higher prices and lower Sharpe ratios in equilibrium for these benchmark stocks, which are typically high-liquidity stocks. Thus, funds managed on behalf of others may hold disproportionate amounts of the high-liquidity stocks while agents like our base-case agent are the inframarginal investors in the low-liquidity stocks. Moreover, funds managed on behalf of others are likely to be playing a big role in market clearing in the U.S. because the delegated portfolio management industry holds such a large fraction of U.S. equities. As an illustration, 22% of U.S. corporate equities were held by U.S. mutual funds as of December 31, 2003 (see page 59 of the Investment Company Institute's *Mutual Fund Fact Book* (2004)). And the total fraction of U.S. equity held by the delegated portfolio management industry is likely to be much higher than this number once pension fund holdings are included.

B. Net Trades Must Sum to Zero

Labor income is able to generate large increases in turnover and hence the question arises as to whether it is reasonable to believe that this trading is sufficiently unsynchronized to allow the trades to sum to zero each period. What matters is the extent to which shocks to labor income are idiosyncratic rather than systematic. It is important to realize that this distinction is not the same as the distinction between permanent and temporary shocks to labor income. To get an idea of how much trading can be generated by the idiosyncratic component, we decompose the variance of permanent individual labor income shocks into a systematic component and an idiosyncratic component and show that almost all the variance is due to the idiosyncratic component. We ignore temporary shocks since their presence has a negligible impact on liquidity premia and their inclusion only makes the fraction due to the idiosyncratic component even larger.

To implement the decomposition, we compare aggregate and individual labor income growth volatility in the data. The wedge between the two can tell us something about the relative magnitudes of the idiosyncratic and systematic components. Specifically, we can decompose individual labor income growth into a systematic component and an idiosyncratic component as follows:

$$\Delta y^i = \Delta y^s + \Delta y^r, \quad (15)$$

where Δy^i is the log of individual labor income growth, Δy^s is the systematic component, and Δy^r is the idiosyncratic component. Since almost by definition the two components are orthogonal, it follows that

$$\sigma_{\Delta y^i}^2 = \sigma_{\Delta y^s}^2 + \sigma_{\Delta y^r}^2, \quad (16)$$

where $\sigma_{\Delta y^i}^2$ is the variance of Δy^i , $\sigma_{\Delta y^s}^2$ is the variance of Δy^s , and $\sigma_{\Delta y^r}^2$ is the variance of Δy^r . Assuming a large number of agents and sufficiently low cross-correlations between agents' idiosyncratic components, it is reasonable to use the variance of log aggregate labor income growth, $\sigma_{\Delta y^a}^2$, as a proxy for the

average variance of the systematic component. The variance of growth in per capita Retail Trade income is 0.000165 per month and we can use it as an estimate of $\sigma_{\Delta ya}^2$. Given the variance of the permanent component of individual labor income growth that we use in the base case of 0.001875 per month and ignoring the temporary component, we can calculate an estimate of the contribution of the idiosyncratic component to the total variation of individual labor income growth as follows:

$$\frac{\sigma_{\Delta yr}^2}{\sigma_{\Delta yi}^2} = 1 - \frac{\sigma_{\Delta ya}^2}{\sigma_{\Delta yi}^2} = 0.9107, \quad (17)$$

which is a very large fraction.

As described earlier, the variance of the permanent component of individual labor income growth is taken from Gakidis (1997) and is representative of the numbers that are obtained using data for U.S. households. The implication is that, because the idiosyncratic component constitutes such a large fraction of the variation in individual labor income growth, the idiosyncratic component of labor income can generate rebalancing demands that offset across agents and allow markets to clear. While this evidence is by no means conclusive, it is suggestive that markets can clear with agents rebalancing in response to labor income receipts. Moreover, to the extent that the idiosyncratic components are negatively correlated across agents due to economic growth being negatively correlated across geographic regions, rebalancing in response to the idiosyncratic component can also offset synchronized trading due to the systematic component and return predictability.

VI. Conclusions

The seminal work of Constantinides (1986) documents how, when the risky-asset return is calibrated to the U.S. market return, the impact of transaction costs on per-annum liquidity premia is an order of magnitude smaller than the cost rate itself. A number of recent papers form portfolios sorted on liquidity measures and find a spread in expected per-annum return that is not an order of magnitude smaller than the transaction cost spread: the expected per-annum return spread is found to be around 6% to 7% per annum. Our paper bridges the gap between Constantinides's theoretical result and the empirical magnitude of the liquidity premium by examining dynamic portfolio choice with transaction costs in a variety of more elaborate settings that move the problem closer to the one solved by real-world investors. In particular, we allow returns to be predictable and we introduce wealth shocks: mainly labor income shocks, but also stationary multiplicative shocks. With predictable returns, we also allow the wealth shocks and transaction costs to be state dependent.

We find that adding these real-world complications to the canonical problem can cause transaction costs to produce per-annum liquidity premia that are no longer an order of magnitude smaller than the cost rate, but are instead the same order of magnitude. For example, the presence of predictable returns and i.i.d. labor income growth uncorrelated with returns, both calibrated to the

data, causes the liquidity premium for agents with wealth to monthly labor income ratios of 0 and 10 to be 1.63% per annum and 1.08% per annum, respectively; these are 20-fold and 13-fold increases, respectively, relative to that in the standard i.i.d. return case with no labor income. Further, allowing labor income growth to exhibit the predictability observed in U.S. data causes very small reductions in these premia, even for very low wealth–income ratios. We conclude that the effect of proportional transaction costs on the standard consumption and portfolio allocation problem with i.i.d. returns can be materially altered by reasonable perturbations that bring the problem closer to the one investors are actually solving.

Clearly, our paper is but a first step toward bridging the gap between the theoretical literature to date and empirical work finding large spreads in expected returns for portfolios formed on the basis of liquidity. One important limitation of our analysis is that it is a partial equilibrium analysis. Therefore, it says nothing about how transaction costs affect equilibrium prices by limiting the ability of agents to share risk. More work is needed to understand how transaction costs affect prices and returns in a general equilibrium setting.

Appendix: Solution Technique for the Labor Income Problem

This appendix sketches the numerical procedures associated with computing liquidity premia in a dynamic savings and portfolio choice problem with predictable or i.i.d. returns and labor income. There are three key elements to the implementation. The first is to endogenize the discrete state representation of the value function to bound error propagation at each iteration. The second is to resort to extrapolation only when the problem at hand is economically sufficiently close (in a sense to be made clear below) to a problem for which the functional form of the value function is known. The third is to exploit a natural sense in which the algorithm can be parallelized across computational units to reduce the run-time linearly to a feasible time frame.

The concern that gives rise to the above elements is that the wealth to lagged permanent labor income ratio (wealth–income ratio, henceforth) state is unbounded on the nonnegative side of the real line. To represent the value function on this dimension, this range is partitioned into three disjoint nondegenerate intervals. A different algorithm is applied to obtain an approximation on each interval. At each iteration, the intermediate boundary points are each chosen to be the smallest wealth–income ratio such that an agent without labor income for all periods to the terminal date requires no more than a prespecified fraction of extra wealth (the boundary-point parameter) to be equally happy as an otherwise identical agent with labor income at that wealth–income ratio. The boundary-point parameter is chosen to be 10% for the lower intermediate boundary point and 1% for the upper intermediate boundary point. Over the lower end interval, the value function is approximated as a piecewise linear form or piecewise shape-preserving monotone cubic hermite interpolant of Fritsch and Carlson (1980) for the wealth–income ratio state and as a piecewise linear form for the inherited allocation state. Over the higher end interval, the

value function is taken to be that of an otherwise identical problem without labor income. Over the middle interval, the value function is approximated as a function of the form $V(W) = a \times (W - b)^c + d$, where a, b, c , and d are constants in \mathbb{R} , to match the function and the first derivative values at the upper and the lower intermediate boundary points. Further, at any given iteration and at any given grid node of the discretized state space, the objective function has to be jointly solved for consumption and portfolio policies subject to the short sales constraints on the T-bill and the risky assets. A recursive golden section algorithm is used to optimize the consumption policy, defined as the fraction of last period's permanent labor income consumed, accurate to the fourth decimal digit, and the portfolio policy, defined as the fraction of wealth invested in each of the risky assets, accurate to the third decimal digit.

We use a dynamic gridding algorithm to bound errors on policy functions at each iteration. This algorithm takes the value function representation for the previous iteration as given, computes the intermediate boundary points for the iteration at hand, and continues to add points in the lower end interval in a particular way until policies at no point on a representative grid for the next iteration differ by more than prespecified magnitudes across the increasingly denser grids being used for the current iteration.

The steps of the algorithm that produces the value function representation $V_t(\cdot)$ are as follows:

- (1) take the value function representation for the previous iteration, $V_{t+1}(\cdot)$, as given;
- (2) initialize a set of grid nodes, $X_t = \{\Gamma_{t,1}, \Gamma_{t,2}, \Gamma_{t,3}, \dots, \Gamma_{t,n}\}$, in increasing order, where $\Gamma_{t,1} = 0$, $\Gamma_{t,n} = LP_t$, n is a constant, and LP_t is the lower intermediate boundary point for iteration t with its boundary-point parameter set to 10%;
- (3) define $Y_t = \{(\Gamma_{t,1} + \Gamma_{t,2})/2, (\Gamma_{t,2} + \Gamma_{t,3})/2, \dots, (\Gamma_{t,n-1} + \Gamma_{t,n})/2\}$, in increasing order;
- (4) evaluate $V_t(x)$ for all $x \in X_t$ using the value function representation, $V_{t+1}(\cdot)$ from step (1);
- (5) evaluate $V_t(x)$ for all $x \in Y_t$ using the value function representation for $V_{t+1}(\cdot)$ from step (1);
- (6) calculate the lower intermediate point, LP_{t-1} , for iteration $t - 1$ using $V_t(\cdot)$ specified by appropriately interpolating between the $V_t(\cdot)$ values obtained in steps (4) and (5) for $X_t \cup Y_t$;
- (7) define $A_{t-1} = \{(i - 1) \times (LP_{t-1})/q\}_{i=1}^{q+1}$, where $q \geq 2$ is a constant;
- (8) evaluate $V_{t-1}(x)$ for all $x \in A_{t-1}$ using $V_t(\cdot)$ specified by appropriately interpolating between the $V_t(\cdot)$ values obtained in steps (4) for X_t ;
- (9) evaluate $V_{t-1}(x)$ for all $x \in A_{t-1}$ using $V_t(\cdot)$ specified by appropriately interpolating between the $V_t(\cdot)$ values obtained in steps (4) and (5) for $X_t \cup Y_t$;
- (10) define $Z_{t-1} \subseteq A_{t-1} = \{x \in A_{t-1} \text{ and the policy function } V_{t-1}(x) \text{ obtained using steps (8) and (9) differ by more than a prespecified magnitude}\}$;
- (11) if $Z_{t-1} = \emptyset$, accept $X_t \cup Y_t$ as a sufficiently fine grid to produce a usable V_t representation and exit the algorithm;

- (12) compute the range of wealth–income ratios, Γ_t , the system can possibly assume by starting at any point in Z_{t-1} under any realizations of the return and labor income shocks and any allowed policy; this range is generically of the form $[0 \ h_t]$, for some $h_t > 0$;
- (13) update X_t to be $X_t \cup Y_t$, in increasing order;
- (14) update Y_t as $\{(X_{t,1} + X_{t,2})/2, (X_{t,2} + X_{t,3})/2, \dots, (X_{t,k-1} + X_{t,k})/2\}$, where k is the smallest positive integer satisfying $X_{t,k} \geq \min(h_t, LP_t)$; and
- (15) return to step (5) and repeat.

Step (10) is implemented to ensure that the maximal absolute scaled deviation in the consumption policy (defined as the fraction of last period's permanent component of labor income consumed), $|(\hat{\kappa}_{\text{coarse}} - \hat{\kappa}_{\text{dense}})/\hat{\kappa}_{\text{dense}}|$, is bounded from above by 10^{-3} , and the maximal absolute deviation in portfolio policy (defined as the fraction of wealth invested in the risky asset), $|\alpha_{\text{coarse}} - \alpha_{\text{dense}}|$, is bounded from above by 10^{-2} , for each of the risky assets. q and n are set to 50. The initialization in step (2) uses the previous iteration's final representation of V_{t+1} .

This algorithm, given the choice of the interpolant, produces a set of grid nodes that can represent the value function at hand sufficiently well before one goes on to the next iteration. For the i.i.d. return case without the temporary shocks to labor income under the parametrization described in Section III.B, the choice of a piecewise linear interpolant leads, on average (over the life cycle), to approximately 400 grid nodes, and the choice of the piecewise shape-preserving monotone cubic hermite interpolant leads, on average (over the life cycle), to approximately 140 grid nodes. For the predictability cases, we always use the shape-preserving monotone cubic hermite interpolant. For the i.i.d. labor growth case with $\rho_{g,d} = 0$ and other parameters as described in Section III.B, this algorithm leads, on average (over the life cycle), to approximately 230 grid nodes.

For any distinct parameter specification and return-generating process, this dynamic gridding scheme is run without transaction costs, and each node on the resultant grid for the wealth–income ratio state is augmented with the 51-node uniform inherited allocation grid on $[0 \ 1]$ to get the state discretization for the transaction-cost problem. This procedure implies that the joint state for the i.i.d. labor growth case is on average (over the life cycle) represented by $19 \times 230 \times 51 \cong 222,000$ grid nodes.

To determine the reduction in unconditional mean returns required to offset elimination of the transaction costs and keep the agent's expected utility the same, a standard bisection algorithm is used. This algorithm is specified to produce liquidity premia accurate to the fourth decimal digit.

When reducing the unconditional mean return in the no-transaction-cost problem to calculate liquidity premia, we use the gridding scheme obtained for the otherwise similar case that calibrates the returns to the data. We perform the following robustness check. For the i.i.d. return case without the temporary shocks described in Section III.B and for the no financial wealth case, we recalculate the gridding scheme under the return processes with the unconditional

mean reduced exactly in the magnitude of the previously calculated liquidity premium for each case, and recompute the liquidity premium. The liquidity premium is unchanged up to the reported precision.

We also check that the results are robust to the parameters of the solution algorithm. For the i.i.d. return case without the temporary shocks described in Section III.B, we first change the boundary parameter for the lower intermediate boundary point from 10% to 9%, 8%, 7%, and 6%. The maximal $|(\hat{\kappa}_{10\%} - \hat{\kappa}_{x\%})/\hat{\kappa}_{x\%}|$ for an equally spaced 100-node grid on $[0, LP_t]$ is less than 10^{-3} for all $t = 1, \dots, 240$ and $x = 6, 7, 8, 9$. Similarly, the maximal $|\alpha_{10\%} - \alpha_{x\%}|$ is less than 10^{-2} for each of the risky assets for all $t = 1, \dots, 240$ and $x = 6, 7, 8, 9$. Fixing the boundary-point parameter for the lower boundary point at 10%, we also change the precision on the consumption policy, $\hat{\kappa}$, from 10^{-4} to 10^{-5} and 10^{-6} and find that the maximal absolute consumption policy difference scaled by the more accurate policy is always less than 10^{-3} over the life cycle.

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