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Asset Pricing with Downside Liquidity Risks

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Abstract. We develop a parsimonious liquidity-adjusted downside capital asset pricing model to investigate whether phenomena such as downward liquidity spirals and flights to liquidity impact expected asset returns. We find strong empirical support for the model. Downside liquidity risk (sensitivity of stock liquidity to negative market returns) has an economically meaningful return premium that is 10 times larger than its symmetric analogue. The expected liquidity level and downside market risk are also associated with meaningful return premiums. Downside liquidity risk and its associated premium are higher during periods of low marketwide liquidity and for stocks that are relatively small, illiquid, volatile, and have high book-to-market ratios. These results are consistent with investors requiring compensation for holding assets susceptible to adverse liquidity phenomena. Our findings suggest that mitigation of downside liquidity risk can lower firms' cost of capital.

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Keywords: liquidity risk • liquidity spiral • conditional moment • pricing kernel • downside risk

1. Introduction

Liquidity affects asset prices. The *level* of liquidity affects expected returns because investors know that in relatively less liquid stocks, transaction costs will erode more of the realized return. Thus, less liquid stocks are priced to include an illiquidity premium. Systematic variation in liquidity (liquidity *risk*) also affects expected returns because investors are concerned with how liquidity varies across states of the world. Consequently, capital asset pricing models have been developed in which traditional market return risk is augmented with liquidity level and liquidity risk (e.g., Jacoby et al. 2000, Acharya and Pedersen 2005, Liu 2006).

Several important aspects of how liquidity risk impacts asset prices are not well understood. First, liquidity behaves differently in good and bad states of the world. Pastor and Stambaugh (2003) are among the first to demonstrate this, reporting that the correlation between asset liquidity and market returns is around 0.5 in negative-return months and near zero in positive-return months. The recent financial crisis provides a well-documented illustration. Flights to liquidity (e.g., Acharya et al. 2013) and downward liquidity spirals (e.g., Brunnermeier and Pedersen 2009) create strong comovement between stock-level liquidity and market returns in bad states. Extreme cases can result in an "evaporation" of liquidity (e.g., Nagel 2012). Given the asymmetric behavior of liquidity, how

should liquidity risk be measured? Are symmetric liquidity risk measures reasonable approximations? Does asymmetry in the behavior of liquidity matter for asset pricing?

Second, investors are more concerned with losses of liquidity in future bad states where the marginal rate of substitution is high. Importantly, this is when flights to liquidity and downward liquidity spirals are most likely. Do differences in investor risk aversion across good and bad states affect how liquidity risk is priced?

Third, despite the intuitive appeal of theory articulating the effect of systematic liquidity risk on asset prices, the empirical support for the implied liquidity risk premium is mixed. For example, Acharya and Pedersen (2005) find some evidence of a liquidity risk premium, yet other more recent studies find no evidence of a liquidity risk premium (e.g., Hasbrouck 2009). Is the mixed empirical evidence the result of mischaracterization of the nature of liquidity risk? Is it a consequence of the symmetric treatment of both liquidity risk and investor risk aversion in existing models?

This paper develops and tests an asset pricing model to investigate these issues. We derive a parsimonious liquidity-adjusted downside capital asset pricing model (LD-CAPM). The model disentangles a stock's downside market risk from three downside liquidity risks. The first downside liquidity risk involves the comovement of a stock's liquidity with the market's excess return during market declines. The second and

third involve the comovement of a stock's excess return and liquidity with marketwide liquidity during market declines. The distinguishing feature of our model is that it does not impose symmetry in the behavior of liquidity or investor risk aversion in good and bad states.

We develop the LD-CAPM from the pricing kernel underpinning the downside CAPM (D-CAPM) of Hogan and Warren (1974) and Bawa and Lindenberg (1977). Our approach is analogous to that of Acharya and Pedersen (2005), who liquidity adjust the CAPM (Sharpe 1964, Lintner 1965). We adjust gross excess asset returns for liquidity costs, and account for the effects of marketwide liquidity by adjusting the pricing kernel of the D-CAPM. Our model bridges two strands of the asset pricing literature. The LD-CAPM is the downside analogue to the liquidity-adjusted CAPM (L-CAPM) of Acharya and Pedersen (2005), and it is the liquidity-adjusted version of the D-CAPM of Hogan and Warren (1974) and Bawa and Lindenberg (1977)—the last tile in a four-tile mosaic.

The LD-CAPM's risks are not subsumed by their symmetric counterparts in the L-CAPM, and thus downside liquidity risks contribute to explaining the cross section of returns. Our model fits the data better than other theory-driven asset pricing models and better than the empirically driven Fama and French (1993) three-factor model. With the addition of a medium-term momentum control, the fit of the LD-CAPM is comparable with the Fama–French–Carhart (Fama and French 1993, Carhart 1997) four-factor model. The LD-CAPM risks are unique and do not proxy for previously documented risk factors or stock characteristics.

We find that stocks with high downside liquidity risk (the comovement of a stock's liquidity with excess market returns during market declines) compensate investors with an economically meaningful expected return premium, 6.34% p.a. (per annum) (using the common approach of comparing the 10th and 1st deciles). This is consistent with investors disliking stocks that are more susceptible to liquidity spirals or abandonment during flights to liquidity. We show that subsuming downside liquidity risks by assuming symmetry (as is implicit in the existing theoretical and empirical literature) leads to underestimation of the importance of liquidity risk in explaining cross-sectional returns. Put succinctly, liquidity behaves asymmetrically and is priced asymmetrically.

We determine that downside market risk (the comovement of a stock's excess return with excess market returns during market declines) is associated with a statistically significant return premium of 5.47% p.a., whereas expected illiquidity level has a highly statistically significant premium of around 8.44% p.a. (comparing the 10th and 1st deciles). Using the more conservative comparison of 9th and 2nd deciles, we estimate

significant premiums of 1.26% p.a. for downside liquidity risk, 2.31% p.a. for illiquidity level, and 3.12% p.a. for downside market risk. These three return premiums are robust to controlling for a wide range of other characteristics and risks, including the market, size, and value factors; momentum and short-term reversal; coskewness; volatility and idiosyncratic volatility; turnover variance; and lottery-like features of stocks. The return premiums are also robust to a range of alternative liquidity metrics and different subperiods. Adding color to this picture, we find that the downside liquidity risk premium is typically higher when the market is in a relatively illiquid state. It is also typically larger for stocks that are relatively small, illiquid, volatile, and exhibit characteristics of value stocks (high book-to-market ratio).

This paper makes two main contributions to the literature. The first is in aligning the theory of how liquidity affects asset prices with the empirical evidence. Since the seminal theory on the asset pricing implications of liquidity risk (Jacoby et al. 2000, Acharya and Pedersen 2005, Liu 2006), a large body of empirical evidence has emerged, providing a richer description of the behavior of liquidity. There is extensive empirical evidence of asymmetry in the behavior of liquidity (e.g., Chordia et al. 2001, Pastor and Stambaugh 2003, Hameed et al. 2010, Roll and Subrahmanyam 2010) and on a range of mechanisms that give rise to the asymmetric behavior, including flights to liquidity (e.g., Acharya et al. 2013), feedback between market liquidity and funding liquidity (e.g., Brunnermeier and Pedersen 2009), and inventory effects and funding constraints among market makers (e.g., Comerton-Forde et al. 2010, Hameed et al. 2010, Nagel 2012). Whereas the underlying mechanisms and ex post effects of asymmetry in liquidity have been widely studied, our paper provides insights about the ex ante effects on asset prices. There is also empirical evidence on the influence of extreme downside liquidity events on asset prices (Menkveld and Wang 2012, Wu 2012, Ruenzi et al. 2013). To the extent of our knowledge, this is the first characterization of downside liquidity risks in a partial equilibrium model. Our paper extends the existing theory to bring it in line with the broader collection of evidence on the behavior of liquidity and asset prices.

The second main contribution is in offering a solution to the puzzle of why existing empirical evidence on the liquidity risk premium is mixed.¹ Our findings suggest misspecification of the form liquidity risk is part of the explanation. Subsuming downside liquidity risk through the symmetry imposed by the CAPM pricing kernel obscures the behavior of liquidity in bad states of the world and conceals the associated premium. To illustrate the consequences, we estimate the symmetric model of Acharya and Pedersen (2005) on

our sample. Whereas the downside liquidity risk premium is large (6.34% p.a.) and highly statistically significant, the equivalent *symmetric* liquidity risk is not statistically significant and of a considerably smaller magnitude (0.56% p.a., which is the same order of magnitude reported by Acharya and Pedersen 2005 and Hagströmer et al. 2013). Similarly, the conservative ninth minus second decile premium for downside liquidity risk is 1.26% p.a., whereas the corresponding symmetric liquidity risk premium is 0.08% p.a. Therefore, the return premium associated with downside liquidity risk is 10 to 15 times larger than suggested by symmetric liquidity risk measures. Although not a feature of our model, we explore the upside counterparts to our downside risks to further our understanding of the role asymmetry. In contrast to downside liquidity risk, upside liquidity "risk" attracts a negative premium. This finding reinforces the importance of asymmetry in characterizing liquidity risk and in accounting for its impact on asset prices.

Our results have implications for firm investments and the real economy. The large expected return premium associated with downside liquidity risk suggests that decision makers can lower their firm's cost of capital by adopting practices that moderate the loss of their equity's liquidity during market declines. Reducing information asymmetry through increased disclosure during such times is one example (Balakrishnan et al. 2014).

Delving deeper, we find that the downside liquidity risk premium is higher during periods of low marketwide liquidity. While not identifying the mechanism, this result is consistent with the notion that adverse liquidity phenomena, such as liquidity spirals, flights to liquidity, and feedback between market and funding liquidity, have a substantial negative impact on firms' cost of capital. Thus, an additional channel by which these phenomena can impact the real economy is through the level of investment undertaken by firms that are sensitive to the cost of capital. Marketwide illiquidity may be driven by low investor and consumer sentiment (Liu 2015) and the business cycle (Næs et al. 2011). This suggests that policy, regulation, and market design that mitigate downside liquidity risk and lower the cost of capital are likely to have greatest impact when the economy needs it most.

Competition and financing constraints in provision of market liquidity can also contribute to downside liquidity risk. Competition among liquidity providers increases the tendency for market liquidity to fall in bad states of the world. This is because competition from informal liquidity providers (such as high-frequency traders) in good states erodes profits and thus the ability of designated market makers to subsidize their activities in bad states (Roll and Subrahmanyam 2010). Comerton-Forde et al. (2010)

and Nagel (2012) provide evidence that the financing constraints of liquidity providers cause them to withdraw liquidity, particularly during market turmoil. Finally, recent efforts to incorporate liquidity-adjusted downside risk metrics into risk management procedures and capital standards reflect regulatory and practitioner concerns about downside liquidity risk (e.g., Angelidis and Benos 2006 and Bank for International Settlements 2012). These concerns are consistent with our finding of a significant downside liquidity risk premium.

This paper proceeds as follows. Section 2 reviews the theoretical and empirical findings that motivate our model. Section 3 defines the LD-CAPM and derives the model's expected return relation. We interpret the model in a state-pricing context to build some economic intuition. Section 4 describes the data and construction of variables for the empirical tests, which are presented and discussed in Section 5. Section 6 summarizes the conclusions on downside liquidity risk. Additional supporting material can be found in the online appendix.

2. Liquidity Spirals, Market Declines, and Risk Aversion

Several recent papers, in addition to Pastor and Stambaugh (2003), find evidence of asymmetry in the comovement of asset liquidity and returns. Chordia et al. (2001) find that liquidity plummets in down markets and recovers relatively slowly in up markets. Consistent with asymmetric comovement, return and liquidity distributions are left skewed (Ang and Chen 2002, Roll and Subrahmanyam 2010).²

One explanation for this asymmetry is found in the model of Brunnermeier and Pedersen (2009), which links the liquidity of an individual asset with the funding liquidity of intermediaries. Market liquidity and funding liquidity are mutually reinforcing and susceptible to negative "liquidity spirals"; deterioration in the balance sheets of financial intermediaries can induce collateral-driven security sales, which further exacerbate illiquidity. Nagel (2012) provides empirical evidence consistent with this mechanism. The feedback loop causes a nonlinear response in asset liquidity to declines in the market. Similarly, Hameed et al. (2010) find that changes in liquidity demand are negatively related to market returns with large declines having the greatest impact. They show empirically that the driver of the asymmetric relation is the deterioration of the balance sheets of financial intermediaries (see also Comerton-Forde et al. 2010).

Downward liquidity spirals can occur for other reasons. In the model of Garleanu and Pedersen (2007), a decrease in market liquidity leads to tighter risk management, which in turn leads to lower liquidity, and so

on. In the model of Carlin et al. (2007), traders cooperate and provide liquidity to one another most of the time, but occasionally, when stakes are high and a trader becomes distressed, they switch to predatory trading causing severe declines in liquidity across multiple assets. In the model of Morris and Shin (2004), when asset prices fall, some traders approach their loss limits and are induced to sell, which increases incentives for other traders to sell, causing behavior analogous to a bank run. The resulting "liquidity black holes," like the mechanisms in previously mentioned studies, imply asymmetric comovements in liquidity and returns. Similarly, asymmetry between good and bad states has been observed in the behavior of broader economic variables such as lending rates and defaults (Ordoñez 2013), and has been attributed to asymmetry in information diffusion speeds. There are many nonexclusive candidate causes of asymmetry; together they suggest that a symmetric treatment of liquidity risk in asset pricing will not adequately reflect the underlying

Periods of macroeconomic and financial stress make investors more averse to liquidity shocks and can cause them to abandon illiquid assets in favor of relatively more liquid assets (Acharya et al. 2013). Such "flights to liquidity" and "flights to quality" can take place across asset classes, such as stocks and bonds, but also within asset classes. In the cross section, some stocks become relatively more attractive to investors during a period of stress and thus experience a relative increase in liquidity. Investor "flights" are likely to coincide with sharp declines in the market and therefore, similar to liquidity spirals, cause asymmetric comovements between returns and liquidity, depending on the state of the market. The degree of asymmetry is likely to depend on whether a stock becomes relatively more or less attractive during a period of stress.

A second important source of asymmetry with regard to the pricing of liquidity risk presents in how investors view the uncertainty surrounding liquidity in future good and bad states. It is unlikely that investors view positive shocks in the same way they view negative shocks (e.g., Kahneman and Tversky 1979, Veld and Veld-Merkoulova 2008, Levy and Levy 2009). This means that a pricing kernel with the same level of risk aversion across good and bad states will not represent investor preferences well.³ Empirical work on the shape of the pricing kernel using gross returns supports the notion that risk aversion is high in bad states (e.g., Bakshi et al. 2010, De Giorgi and Post 2008, Rosenberg and Engle 2002).

3. The Model

We develop our model parsimoniously through two liquidity-related modifications of the D-CAPM of Hogan and Warren (1974) and Bawa and Lindenberg (1977). The first modification accounts for liquidity costs within net asset returns. The second adjusts the pricing kernel underpinning the D-CAPM for marketwide liquidity. In doing so we develop a direct downside analogue to the L-CAPM of Acharya and Pedersen (2005).

It is well known that the CAPM can be derived by either (i) assuming quadratic utility or (ii) assuming returns are elliptically symmetric and that investor's possess increasing and concave utility (Berk 1997). Acharya and Pedersen (2005) incorporate time-varying liquidity costs into the CAPM by following the second approach; however, there is strong evidence that the distributions of both liquidity costs and gross returns are left skewed (Ang and Chen 2002, Roll and Subrahmanyam 2010). The Acharya and Pedersen (2005) model can also be derived for arbitrary distributions under the assumption of quadratic utility. This approach replaces the conflict arising from evidence on skewed liquidity/return distributions with violations of the nonsatiation and no-arbitrage preference regularity conditions. Hogan and Warren (1974) and Bawa and Lindenberg (1977) address these issues through arguments underpinned by a utility function for the representative investor that is linear on the upside and quadratic on the downside.4

This utility function has several attractive features. First, its characterization of risk is consistent with several studies on investor preferences (Unser 2000, Veld and Veld-Merkoulova 2008, Levy and Levy 2009). Second, the change in the functional form around a reference point, such as the risk-free rate, is supported by a large body of literature (e.g., Hogan and Warren 1972, Fishburn 1977, Bawa 1978, Kahneman and Tversky 1979, Holthausen 1981, Post and Levy 2005). Third, this utility function is linked to the partial ordering of investment choices under stochastic dominance (Porter 1974). Fourth, it provides a pricing model free of distributional assumptions while maintaining consistency with basic preference regularity conditions (such as nonsatiation). Given these advantages, we develop our model from the pricing kernel, M_{t+1}^D , associated with the D-CAPM (Anthonisz 2012):

$$M_{t+1}^{D} = \kappa_t - \Theta_t \mathbf{1}_{t+1}^{-} R_{m,t+1}^{e}; \tag{1}$$

$$M_{t+1}^{D} = \kappa_{t} - \Theta_{t} \mathbf{1}_{t+1}^{-} R_{m,t+1}^{e};$$

$$\mathbf{1}_{t+1}^{-} = \begin{cases} 0 & \text{if } R_{m,t+1}^{e} \ge 0, \\ 1 & \text{if } R_{m,t+1}^{e} < 0. \end{cases}$$
(2)

Here κ_t is a constant, $\Theta_t > 0$ is the coefficient of risk aversion, and $R_{m,t+1}^e$ is the excess return of the market at time t + 1 ($R_{m,t+1}^e = R_{m,t+1} - R_{f,t+1}$). De Giorgi and Post (2008) find empirical support for the D-CAPM kernel. The shape of the kernel reported in many empirical studies (e.g., Bakshi et al. 2010, Rosenberg and Engle 2002) can be reasonably approximated by this kernel; however, this literature works with gross returns and

does not account for liquidity costs. Adjusting this kernel for marketwide liquidity ($C_{m,t+1}$) yields the pricing kernel for our base model:

$$M_{t+1}^{LD} = \kappa_t - \Theta_t \mathbf{1}_{t+1}^- (R_{m,t+1}^e - C_{m,t+1}); \tag{3}$$

$$\mathbf{1}_{t+1}^{-} = \begin{cases} 0 & \text{if } R_{m,t+1}^{e} - C_{m,t+1} \ge 0, \\ 1 & \text{if } R_{m,t+1}^{e} - C_{m,t+1} < 0. \end{cases}$$
 (4)

The indicator function, $\mathbf{1}_{t+1}^-$, partitions the state space, Ω , into a positive subspace, Ω^+ , where the liquidity-adjusted excess market return (in excess of the risk-free rate) is positive, and a negative subspace, Ω^- , where the liquidity-adjusted excess market return is negative. With this kernel, the representative investor is risk averse across Ω^- and risk neutral over Ω^+ .

In the absence of arbitrage, the pricing kernel framework of Ross (1978), Harrison and Kreps (1979), and Kreps (1981) is

$$\mathbb{E}_{t}[M_{t+1}R_{i,t+1} \mid I_{t}] = 1, \quad \forall i, t, \tag{5}$$

where M_{t+1} is the pricing kernel at time t+1, $R_{i,t+1}$ is the gross return of the ith asset at time t+1 ($R_{i,t+1}=1+r_{i,t+1}$), and I_t is the information set at time t. Assuming the risk-free asset has no liquidity cost, we reexpress Equation (5) in terms of excess returns ($R_{i,t+1}^e=R_{i,t+1}-R_{f,t+1}$) and incorporate the cost of liquidity:

$$\mathbb{E}_{t}[M_{t+1}(R_{i\ t+1}^{e} - C_{i\ t+1})] = 0, \tag{6}$$

where $C_{i,t+1}$ is the liquidity cost of the ith asset. Considering the covariance of the pricing kernel first with the liquidity-adjusted excess return of an individual stock and then that of the market leads to

$$\mathbb{E}_{t}[R_{i,t+1}^{e} - C_{i,t+1}] = \frac{\text{Cov}_{t}[M_{t+1}, R_{i,t+1}^{e} - C_{i,t+1}]}{\text{Cov}_{t}[M_{t+1}, R_{m,t+1}^{e} - C_{m,t+1}]} \cdot \mathbb{E}_{t}[R_{m,t+1}^{e} - C_{m,t+1}]. \tag{7}$$

The state pricing interpretation of Equations (6) and (7) is that the marginal rate of substitution is high in bad states of the world and thus assets that have relatively larger negative returns and/or relatively less liquidity in such states should, as compensation, provide a higher expected return.

Placing the LD-CAPM pricing kernel (Equation (3)) into the covariance identity (Equation (7)) gives the model in the beta framework for asset pricing. Using a conditional version of Theorem 1 in Anthonisz (2012), we reexpress the model in terms of conditional moments and comoments:

$$\mathbb{E}_{t}[R_{i,t+1}] = R_{f,t+1} + \mathbb{E}_{t}[C_{i,t+1}] + \gamma_{i,r}^{R} \lambda + \gamma_{i,l}^{R} \lambda + \gamma_{i,l}^{L} \lambda + \gamma_{i,l}^{L} \lambda,$$
(8a)

$$\gamma_{i,r}^{R} = \frac{\mathbb{E}_{t} [R_{i,t+1}^{e} R_{m,t+1}^{e} \mid R_{m,t+1}^{e} - C_{m,t+1} < 0]}{\mathbb{E}_{t} [(R_{m,t+1}^{e} - C_{m,t+1})^{2} \mid R_{m,t+1}^{e} - C_{m,t+1} < 0]},$$
(8b)

$$\gamma_{i,t}^{R} = \frac{\mathbb{E}_{t}[-C_{i,t+1}R_{m,t+1}^{e} \mid R_{m,t+1}^{e} - C_{m,t+1} < 0]}{\mathbb{E}_{t}[(R_{m,t+1}^{e} - C_{m,t+1})^{2} \mid R_{m,t+1}^{e} - C_{m,t+1} < 0]}, \quad (8c)$$

$$\gamma_{i,r}^{L} = \frac{\mathbb{E}_{t} \left[-R_{i,t+1}^{e} C_{m,t+1} \mid R_{m,t+1}^{e} - C_{m,t+1} < 0 \right]}{\mathbb{E}_{t} \left[(R_{m,t+1}^{e} - C_{m,t+1})^{2} \mid R_{m,t+1}^{e} - C_{m,t+1} < 0 \right]}, \tag{8d}$$

$$\gamma_{i,l}^{L} = \frac{\mathbb{E}_{t} [C_{i,t+1} C_{m,t+1} \mid R_{m,t+1}^{e} - C_{m,t+1} < 0]}{\mathbb{E}_{t} [(R_{m,t+1}^{e} - C_{m,t+1})^{2} \mid R_{m,t+1}^{e} - C_{m,t+1} < 0]}, \quad (8e)$$

$$\lambda = \mathbb{E}_t [R_{m,t+1}^e - C_{m,t+1}]. \tag{8f}$$

The theorem, a proof, and the derivation of the model are in the appendix. To avoid confusion and assist exposition, we represent the downside betas as downside gammas and refer to them as such. Downside gammas involve the asymmetric measures of conditional comoments and conditional moments. In contrast, traditional betas involve covariance and variance.

The LD-CAPM implies that the expected return of a stock is equal to the risk-free rate, $R_{f,t+1}$, plus a return premium for the expected level of liquidity costs, $\mathbb{E}_t[C_{i,t+1}]$, plus return premiums for the stock's return and liquidity risks. The gammas $(\gamma_{i,r}^R, \gamma_{i,l'}^R, \gamma_{i,r}^L, \gamma_{i,l}^L)$ measure the amount of each systematic risk in stock i. Lambda (λ) measures the "price" of each risk (the additional expected return per unit of the risk). Within the gammas, the superscripts "R" and "L" refer to the excess market return and marketwide liquidity, respectively. The subscripts "r" and "l" refer to stock i's excess return and liquidity, respectively.

The four (downside) gammas correspond to the four (symmetric) betas within the L-CAPM of Acharya and Pedersen (2005). The main difference is that gammas measure comovement between a stock's returns/ liquidity and the market returns/liquidity in bad states (when liquidity-adjusted excess market returns are negative), whereas the Acharya and Pedersen (2005) betas do not distinguish between good and bad states. This is an economically meaningful difference in the way risk is characterized. A second difference, which is less economically meaningful and more technical in nature, is that the gammas are conditional comoments (normalized by a conditional moment), whereas the Acharya and Pedersen (2005) betas are covariances (normalized by variance). Both betas and gammas measure the degree of comovement and are similar in scale.

The first gamma, downside market risk ($\gamma_{i,r}^R$), measures the sensitivity of a stock's excess returns to the market's excess returns within the negative subspace (Ω^-) defined by $R_{m,t+1}^e - C_{m,t+1} < 0$. This gamma is equivalent to the downside beta of the D-CAPM developed in Hogan and Warren (1974) and Bawa and Lindenberg (1977), but with a downside threshold involving marketwide liquidity. The other three gammas involve liquidity.

The second gamma, *downside liquidity risk* ($\gamma_{i,l}^R$), measures the sensitivity of a stock's liquidity to the market's excess returns within the negative subspace. For

example, stocks that tend to experience large declines in liquidity during falls in the market will have large positive values of $\gamma_{i,l}^R$. Thus, $\gamma_{i,l}^R$ can be interpreted as a measure of a stock's susceptibility to downward liquidity spirals and abandonment during a flight to liquidity or flight to quality. If investors demand a return premium to entice them to hold such stocks, we would expect to find a positive cross-sectional premium associated with $\gamma_{i,l}^R$. Acharya and Pedersen (2005) find that the corresponding (symmetric) beta of the L-CAPM accounts for 80% of a 1.1% p.a. liquidity risk premium. Therefore, a comparison of this premium with that of $\gamma_{i,l}^R$ provides a comparison of the L-CAPM's and LD-CAPM's characterizations of liquidity risk.

The third gamma, downside aggregate liquidity risk $(\gamma_{i,r}^L)$, measures the sensitivity of a stock's excess returns to marketwide liquidity within the negative subspace. This is a downside form of the risk termed "aggregate liquidity risk" by Pastor and Stambaugh (2003), who interpret marketwide liquidity as a state variable. Under this interpretation, investors will demand a return premium to hold stocks that tends to have large negative returns when marketwide liquidity falls (positive $\gamma_{i,r}^L$).

Similarly, the fourth gamma, downside liquidity commonality risk $(\gamma_{i,l}^L)$, measures the sensitivity of a stock's liquidity to marketwide liquidity within the negative subspace (Ω^-) . This gamma measures a downside form of the commonality in liquidity risk first explored by Chordia et al. (2001), Hasbrouck and Seppi (2001), and Huberman and Halka (2001).

4. Data and Method

4.1. Sample

Our sample spans the period January 1, 1962, to December 31, 2011, and consists of all common stocks listed on the New York Stock Exchange (NYSE) and the American Exchange (AMEX).⁷ Similar to Acharya and Pedersen (2005), we exclude stock-month observations in which the median stock price is below \$5 or above \$1,000. We exclude stock-year observations with negative book-to-market values. We use daily return, price, and volume data from the CRSP, balance sheet data (used to compute book values) from Compustat, and daily data on the Fama–French–Carhart factors from the Fama–French data library.⁸

4.2. Measures of Liquidity

Our primary liquidity measure is the price impact measure (*ILLIQ*) of Amihud (2002), transformed as per Acharya and Pedersen (2005) so that it approximates the dollar trading cost per dollar traded (in robustness tests we examine other measures). Amihud (2002) shows that this measure is strongly positively related to the bid–ask spread, price impact, and fixed trading costs. Goyenko et al. (2009) compare how closely

various daily and monthly liquidity proxies relate to high-frequency measures of transaction costs and find that *ILLIQ* is a relatively good proxy for liquidity. Similarly, Hasbrouck (2009) finds that among several daily proxies, *ILLIQ* is most strongly correlated with tradeand-quote-based measures of price impact.

At a daily frequency, ILLIQ is computed as

$$ILLIQ_{i,\tau} = \frac{|r_{i,\tau}|}{\$Vol_{i,\tau}},\tag{9}$$

where $r_{i,\tau}$ is the return on stock i on day τ , and V ol $_{i,\tau}$ is the dollar volume of trading in stock i on day τ measured in millions of dollars. Following Acharya and Pedersen (2005), we truncate and normalize $ILLIQ_{i,\tau}$ to obtain daily liquidity cost, $C_{i,\tau}$:

$$C_{i,\tau} = \min(0.25 + 0.30 ILLIQ_{i,\tau}P_{\tau-1}, 30.00)/100,$$
 (10)

where $P_{\tau-1}$ is the ratio of the aggregate market capitalizations on day $\tau-1$ and January 1, 1962, and the constants 0.25 and 0.30 are chosen by Acharya and Pedersen (2005) to give $C_{i,\tau}$ approximately the same level and variance as the effective half-spread. The truncation and normalization step serves several purposes: (i) it converts the liquidity cost into "dollar cost per dollar invested" consistent with the theoretical model; (ii) it removes the effects of inflation in nominal prices, which would otherwise affect $ILLIQ_{i,\tau}$; and (iii) it imposes realistic bounds on the liquidity cost. We aggregate to weekly and monthly frequencies by taking the simple average of $C_{i,\tau}$ across days in the week or month, as is common for $ILLIQ_{i,\tau}$.

One final step associated with the liquidity cost measure is decomposing it into an expected component and innovations. In the LD-CAPM, expected liquidity cost is a stock characteristic that affects expected returns, and innovations in liquidity costs are used to compute downside liquidity risks. Similar to Acharya and Pedersen (2005), we use the residuals from an autoregressive regression model (an AR(20) model of $C_{i,\tau}$ for each stock holding the value of $P_{\tau-1}$ fixed for all lags) as innovations in liquidity costs, $C_{i,\tau}^e$. We use two proxies for the expected liquidity cost: (i) the average of the past six months of normalized liquidity costs for that stock ($E_t[C_{i,t+1}^{Avg}]$) and (ii) the fitted value of an AR(2) model of monthly liquidity costs ($E_t[C_{i,t+1}^{AR}]$), as per Acharya and Pedersen (2005).

In robustness tests, we use various alternative measures of liquidity: (i) log of Amihud's *ILLIQ*; (ii) log of a modified *ILLIQ*, calculated using the daily high/low range instead of close-to-close returns; (iii) a turnoverbased price impact metric proposed by Brockman et al. (2008); and (iv) the actual quoted bid–ask spread using a more recent subperiod.

4.3. Estimation of Gammas

The gammas implied by the LD-CAPM are the conditional comoments defined in Equations (8b)–(8e). We estimate these four gammas using daily observations in rolling six-month windows ending at the end of the portfolio formation month:

$$\gamma_{i,r}^{R} \cong \frac{\sum_{\tau \in \Omega^{-}} R_{i,\tau}^{e} R_{m,\tau}^{e}}{\sum_{\tau \in \Omega^{-}} (R_{m,\tau}^{e} - C_{m,\tau}^{e})^{2}},$$
(11a)

$$\gamma_{i,l}^{R} \cong \frac{\sum_{\tau \in \Omega^{-}} -C_{i,\tau}^{e} R_{m,\tau}^{e}}{\sum_{\tau \in \Omega^{-}} (C_{m,\tau}^{e} - C_{m,\tau}^{e})^{2}},$$
(11b)

$$\gamma_{i,r}^{L} \cong \frac{\sum_{\tau \in \Omega^{-}} -R_{i,\tau}^{e} C_{m,\tau}^{e}}{\sum_{\tau \in \Omega^{-}} (R_{m,\tau}^{e} - C_{m,\tau}^{e})^{2}},$$
(11c)

$$\gamma_{i,l}^{L} \cong \frac{\sum_{\tau \in \Omega^{-}} C_{i,\tau}^{e} C_{m,\tau}^{e}}{\sum_{\tau \in \Omega^{-}} (R_{m,\tau}^{e} - C_{m,\tau}^{e})^{2}},$$
(11d)

where $R_{m,\tau}^e$ is the daily value-weighted market return in excess of the daily return implied by one-month Treasury bills, and $C_{m,\tau}^e$ is the daily innovation in market liquidity cost (innovations from an AR model of daily value-weighted mean normalized liquidity costs $(C_{i,\tau})$ across all stocks). Recall that the negative subspace (Ω^-) is defined by $R_{m,\tau}^e - C_{m,\tau} < 0$. In our sample, 5,461 days (43.4% of all days) are in the negative subspace, and 7,120 (56.6% of all days) are in the positive subspace. We require at least 15 negative subspace days in the estimation window for the gamma estimates to be valid.

The decision to use daily observations in rolling sixmonth windows to estimate the four gammas in our main analysis is driven by several considerations. First, using a higher sampling frequency (than say monthly observations) provides more precise estimates (e.g., Bollerslev and Zhang 2003, Barndorff-Nielsen and Shephard 2004). Second, there is considerable evidence that stocks' systematic risks and factor loadings are time varying, and using relatively short rolling windows allows us to capture this feature. Consequently, we are able to examine how downside liquidity risks vary across market conditions. In robustness tests, we examine different length windows and a lower (weekly) sampling frequency (which minimizes microstructure phenomena and nonsynchronous trading issues) and find similar results.

4.4. Control Variables and Other Asset Pricing Models

In examining the cross-sectional relation between the gammas emerging from our model and returns, we employ a large set of control variables, including stock characteristics and risk factor loadings found in previous studies to determine cross-sectional returns. Unless otherwise specified, the control variables, like the gammas, are measured at the end of the portfolio formation month. To maintain consistency with the

estimates of our model's gammas, we use six-month rolling windows of daily observations to estimate control variables that are betas.

We estimate the four betas from the Acharya and Pedersen (2005) L-CAPM in rolling six-month windows of daily observations:¹³

$$\begin{split} \beta_{i,1}^{AP} &= \frac{\text{Cov}_{t}[R_{i,t+1}^{e}, R_{m,t+1}^{e}]}{\text{Var}_{t}[R_{m,t+1}^{e} - C_{m,t+1}^{e}]} \\ &\cong \frac{\sum_{\tau \in \Omega} (R_{i,\tau}^{e} - \bar{R}_{i,\tau}^{e})(R_{m,\tau}^{e} - \bar{R}_{m,\tau}^{e})}{\sum_{\tau \in \Omega} (R_{m,\tau}^{e} - C_{m,\tau}^{e} - (\bar{R}_{m,\tau}^{e} - \bar{C}_{m,\tau}^{e}))^{2}}, \end{split} \tag{12a} \\ \beta_{i,2}^{AP} &= \frac{\text{Cov}_{t}[C_{i,t+1}^{e}, R_{m,t+1}^{e}]}{\text{Var}_{t}[R_{m,t+1}^{e} - C_{m,t+1}^{e}]} \\ &\cong \frac{\sum_{\tau \in \Omega} (C_{i,\tau}^{e} - \bar{C}_{i,\tau}^{e})(R_{m,\tau}^{e} - \bar{R}_{m,\tau}^{e})}{\sum_{\tau \in \Omega} (R_{m,\tau}^{e} - C_{m,\tau}^{e} - (\bar{R}_{m,\tau}^{e} - \bar{C}_{m,\tau}^{e}))^{2}}, \end{split} \tag{12b} \\ \beta_{i,3}^{AP} &= \frac{\text{Cov}_{t}[R_{i,t+1}^{e}, C_{m,t+1}^{e}]}{\text{Var}_{t}[R_{m,t+1}^{e} - C_{m,t+1}^{e}]} \\ &\cong \frac{\sum_{\tau \in \Omega} (R_{m,\tau}^{e} - \bar{R}_{i,\tau}^{e})(C_{m,\tau}^{e} - \bar{C}_{m,\tau}^{e})}{\sum_{\tau \in \Omega} (R_{m,\tau}^{e} - C_{m,\tau}^{e} - (\bar{R}_{m,\tau}^{e} - \bar{C}_{m,\tau}^{e}))^{2}}, \end{split} \tag{12c} \\ \beta_{i,4}^{AP} &= \frac{C_{i,1}^{e}(C_{i,t+1}^{e}, C_{m,t+1}^{e})}{\text{Var}_{t}[R_{m,t+1}^{e} - C_{m,t+1}^{e}]} \\ &\cong \frac{\sum_{\tau \in \Omega} (C_{i,\tau}^{e} - \bar{C}_{m,\tau}^{e})(C_{m,\tau}^{e} - \bar{C}_{m,\tau}^{e})}{\sum_{\tau \in \Omega} (C_{i,\tau}^{e} - \bar{C}_{m,\tau}^{e})(C_{m,\tau}^{e} - \bar{C}_{m,\tau}^{e})^{2}}. \tag{12d} \end{aligned}$$

In comparing the Acharya and Pedersen (2005) betas with the LD-CAPM betas, it is important to note the opposite signs for two of the four betas. Specifically, $\gamma_{i,l}^R$ measures comovement between a stock's *liquidity* and the market's excess returns, whereas the symmetric analogue, $\beta_{i,2}^{AP}$ measures comovement between a stock's *illiquidity* (liquidity cost) and the market's excess returns. Similarly, $\gamma_{i,r}^L$ measures comovement between a stock's excess return and marketwide *liquidity*, whereas the symmetric analogue, $\beta_{i,3}^{AP}$, measures comovement between a stock's excess return and marketwide *illiquidity* (liquidity cost).

We estimate the conventional CAPM market beta (β_i^{MKT}) from the following time-series regression for each stock i in a rolling six-month window of daily observations:

$$R_{i,\tau}^e = \alpha_i + \beta_i^{MKT} R_{m,\tau}^e + \varepsilon_{i,\tau}, \tag{13}$$

where $R_{i,\tau}^e$ is the daily return of stock i in excess of the daily return implied by one-month Treasury bills, and $R_{m,\tau}^e$ is the daily return on the CRSP value-weighted index in excess of the daily return implied by one-month Treasury bills. We also calculate market betas corrected for nonsynchronous trading using the Dimson (1979) method with four lags and a lead of the excess daily market return. The results are qualitatively similar so we report results using the simple market beta as our baseline.

We obtain loadings on the Fama–French–Carhart factors by estimating the following time-series regression for each stock *i* in a rolling six-month window of daily observations:

$$R_{i,\tau}^{e} = \alpha_{i} + \beta_{i}^{MKT} R_{m,\tau}^{e} + \beta_{i}^{SMB} SMB_{\tau} + \beta_{i}^{HML} HML_{\tau} + \beta_{i}^{IMMD} UMD_{\tau} + \varepsilon_{i,\tau};$$

$$(14)$$

 SMB_{τ} and HML_{τ} are the daily Fama and French (1993) size and value factors, UMD_{τ} is the Carhart (1997) momentum factor (winners minus losers portfolio), and β_i^{SMB} , β_i^{HML} , and β_i^{UMD} are stock i's loadings on the size, value, and momentum Factors, respectively. We estimate each stock's idiosyncratic volatility (IVOL) in the spirit of Ang et al. (2006) by taking the standard deviation of the residuals of the four-factor model above. A stock's total (realized) volatility (TVOL) is simply the standard deviation of daily returns, $R_{i,\tau}$, during the rolling past six-month interval.

We estimate coskewness (Rubinstein 1973, Kraus and Litzenberger 1976, Harvey and Siddique 2000), β_i^{COSKEW} , from the following time-series regression for each stock i in a rolling six-month window of daily observations:

$$R_{i,\tau}^e = \alpha_i + \beta_i^{MKT} R_{m,\tau}^e + \beta_i^{COSKEW} (R_{m,\tau}^e)^2 + \varepsilon_{i,\tau}.$$
 (15)

Other control variables are calculated following the conventions in the asset pricing literature. A firm's size (SIZE) is the natural log of its market capitalization (in \$ millions). The natural log of the equity book-to-market ratio (BM) is calculated as per Fama and French (1993). Short-term reversal (REV) is estimated following Jegadeesh (1990) as the stock's return over the prior month (i.e., the month ending at the point of portfolio formation). Following Jegadeesh and Titman (1993),

momentum (*MOM*) is the stock's cumulative return over an 11-month period ending one month prior to the point of portfolio formation. Following Chordia et al. (2001), the volatility of turnover (*SDTURN*) is calculated as the standard deviation of monthly turnover over the prior 12 months. Following Bali et al. (2011), a proxy for lottery features of a stock (*MAX*) is calculated as the stock's maximum daily return during the prior month.

5. Empirical Analysis

5.1. Descriptive Statistics and Characterization of Gammas

Table 1 reports descriptive statistics and correlations between the key variables in the LD-CAPM, as well as the Acharya and Pedersen (2005) L-CAPM, using the pooled sample of stock-month observations. The mean liquidity cost (0.019) is considerably larger than the median (0.003), consistent with a right-skewed distribution of liquidity costs (illiquidity) as documented by Roll and Subrahmanyam (2010). The gammas and betas involving liquidity have considerably smaller magnitude than those involving only returns ($\gamma_{i,r}^R$ and $\beta_{i,1}^{AP}$) due to the different scale of liquidity. Therefore, Table 1 provides a reference point for interpretation of later results. Downside market risk ($\gamma_{i,r}^R$), as well as its symmetric analogue ($\beta_{i,1}^{AP}$), have means and medians close to one, similar to beta in the CAPM.

The correlations between the LD-CAPM gammas are all close to zero, as are the correlations between the L-CAPM betas. Downside liquidity commonality risk, $\gamma_{i,l}^L$, has a moderate positive correlation (0.25) with the level of liquidity costs. This suggests that relatively illiquid stocks are more susceptible to becoming even more illiquid during declines in marketwide

Table 1.	Descriptive	Statistics and	Correlations
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Statistic		C_i	$\gamma_{i,r}^R$	$\gamma_{i,l}^R$	$\gamma_{i,r}^L$	$\gamma_{i,l}^L$	$eta_{i,1}^{AP}$	$eta_{i,2}^{AP}$	$eta_{i,3}^{AP}$	$eta_{i,4}^{AP}$
Mean		0.019	0.999	-0.038	-0.005	0.080	0.877	-0.031	-0.019	0.089
Median		0.003	0.926	-0.001	-0.006	0.000	0.812	-0.001	-0.016	0.001
Standard deviation		0.041	0.721	1.066	0.169	0.500	0.572	0.567	0.107	0.325
Correlation	C_i	1								
Correlation	$\gamma_{i,r}^{R}$	-0.27	1							
Correlation	$\gamma_{i,l}^{R}$	0.16	-0.03	1						
Correlation	$\gamma_{i,r}^{L}$	0.02	0.01	-0.02	1					
	$\gamma_{i,l}^{L}$	0.25	-0.10	0.09	0.01	1				
Correlation	$\beta_{i,1}^{AP}$	-0.31	0.88	-0.05	0.04	-0.12	1			
Correlation	$\beta_{i,2}^{AP}$	-0.04	0.00	-0.70	0.02	-0.07	0.01	1		
Correlation	$\beta_{i,3}^{AP}$	0.04	-0.01	-0.02	0.58	0.00	0.01	0.04	1	
Correlation	$\beta_{i,4}^{AP}$	0.37	-0.15	0.05	0.02	0.62	-0.18	-0.05	0.01	1

Notes. This table reports descriptive statistics and correlations between the key variables in the LD-CAPM: C_i (normalized liquidity cost), $\gamma_{i,r}^R$ (downside comoment of a stock's excess returns with the market's excess returns), $\gamma_{i,l}^R$ (downside comoment of a stock's liquidity with the market's excess returns), $\gamma_{i,r}^L$ (downside comoment of a stock's excess returns with the market's liquidity), and $\gamma_{i,l}^L$ (downside comoment of a stock's liquidity with the market's liquidity). It also reports statistics and correlations for the Acharya and Pedersen (2005) betas, $\beta_{i,1}^{AP}$ to $\beta_{i,4}^{AP}$ which are the symmetric analogues of the LD-CAPM downside risks. The statistics and correlations are calculated using the pooled sample of stock-month observations. The sample consists of NYSE and AMEX stocks between 1962 and 2011.

liquidity. The absolute correlations between the gammas and their symmetric analogues range between 0.58 and 0.88, indicating that symmetric betas capture some downside risk, but are far from perfect proxies.¹⁵

The interrelations between the LD-CAPM gammas and their relations with other stock-level characteristics may be nonlinear, in which case simple correlations will not adequately describe the relations. The online appendix uses decile sorts to allow nonlinearity to present and reaches similar conclusions while detailing somewhat U-shaped interrelations between some of the gammas and betas (e.g., downside liquidity commonality risk, $\gamma_{i,l}^L$, and its symmetric analogue).

5.2. Univariate Portfolio-Level Relation Between Gammas and Returns

We begin empirically analyzing the relation between the gammas and returns by examining returns for deciles of the LD-CAPM gammas. For comparison, we also report returns for deciles of the Acharya and Pedersen (2005) betas. Each month, t, we form deciles of the gammas/betas measured using a rolling six-month window [t-5, t]. For each decile portfolio in each month, we calculate equal-weighted and value-weighted means of the portfolio's next sixmonth return in excess of the risk-free rate, and alpha from a Fama-French-Carhart four-factor model estimated on the next six months of daily returns. To be clear, the gamma/beta estimation windows and the return/alpha measurement windows are sequential and nonoverlapping (gammas/betas are estimated using months [t-5,t], and returns/alphas are measured for the period [t+1,t+6]). For each month, we also compute the differences between decile 10 and decile 1 returns/alphas (D10 - D1). Finally, we calculate time-series means across all months in the sample period. These means are reported in Table 2 as annualized percentages together with the *t*-statistic

 Table 2. Univariate Decile-Sorted Returns and Four-Factor Alphas

	Realized excess return							Four-factor model alpha								
Decile	$\gamma_{i,r}^R$	$\gamma_{i,l}^R$	$\gamma_{i,r}^L$	$\gamma_{i,l}^L$	$\beta_{i,1}^{AP}$	$eta_{i,2}^{AP}$	$\beta_{i,3}^{AP}$	$eta_{i,4}^{AP}$	$\gamma_{i,r}^R$	$\gamma_{i,l}^R$	$\gamma_{i,r}^L$	$\gamma_{i,l}^L$	$eta_{i,1}^{AP}$	$eta_{i,2}^{AP}$	$\beta_{i,3}^{AP}$	$eta_{i,4}^{AP}$
Panel A: Equal weighted																
1 = low	11.92	10.25	13.80	13.72	11.84	17.52	13.71	13.57	9.13	7.12	7.49	9.58	9.70	13.65	6.60	9.10
2	10.73	9.45	11.85	10.48	10.65	15.55	11.63	8.39	6.68	4.18	5.23	4.03	6.96	9.02	4.90	2.94
3	10.98	8.52	10.55	7.94	10.38	12.44	11.20	7.12	6.18	2.23	4.32	1.91	5.99	4.98	4.69	1.63
4	11.26	6.93	10.37	7.64	11.01	10.03	10.87	8.28	5.68	1.50	4.56	1.96	5.75	2.73	4.54	1.65
5	11.71	7.08	10.79	8.38	11.62	8.71	10.63	9.13	5.71	2.03	4.60	2.58	5.54	2.10	4.45	1.96
6	11.62	8.51	10.98	10.00	11.67	7.56	10.94	10.72	5.03	2.75	4.90	2.75	5.12	1.77	4.94	3.01
7	11.87	11.51	10.92	11.69	11.78	7.79	11.08	12.29	4.79	3.76	4.86	3.81	4.79	1.79	5.30	4.52
8	12.32	14.92	11.46	13.80	12.43	10.07	11.68	14.59	4.57	5.99	5.40	6.52	4.51	3.06	5.76	7.97
9	12.32	19.73	12.42	16.72	12.84	12.39	11.83	16.50	4.09	11.52	6.39	10.93	3.98	6.55	6.24	11.51
10 = high	12.16	20.86	13.95	17.33	12.86	15.89	13.62	17.26	4.69	16.46	8.71	13.36	4.27	12.05	9.13	13.33
D10 - D1	0.25	10.61	0.15	3.61	1.02	-1.62	-0.09	3.69	-4.44	9.34	1.21	3.78	-5.43	-1.61	2.53	4.23
<i>t-</i> statistic	(0.09)	(10.88)	(0.07)	(3.41)	(0.37)	(-2.79)	(-0.05)	(5.13)	(-1.86)	(10.89)	(1.27)	(4.61)	(-2.19)	(-2.77)	(2.49)	(3.92)
						Pan	el B: Valu	ie weigł	nted							
1 = low	4.64	6.69	5.96	8.16	4.49	12.40	6.95	8.92	2.45	2.63	1.15	3.60	3.28	7.38	1.17	4.22
2	5.55	5.12	5.53	6.30	5.49	11.63	6.87	6.46	1.66	-0.10	1.26	0.78	2.21	5.13	1.51	2.44
3	5.97	4.88	6.05	5.05	5.54	9.68	5.74	5.55	1.63	-0.34	2.07	0.47	1.86	2.84	1.15	1.72
4	6.43	3.28	5.38	5.17	6.37	7.93	6.04	7.20	2.58	-0.29	1.39	1.30	2.74	1.74	2.04	1.32
5	6.43	3.66	5.98	6.21	5.48	6.84	5.66	7.49	1.84	0.82	1.75	2.37	1.36	1.39	1.53	1.27
6	6.16	6.27	6.96	7.66	6.05	5.81	6.49	9.39	2.09	2.07	2.38	2.01	1.92	1.60	2.09	2.41
7	6.71	9.49	7.05	9.48	6.45	6.13	6.55	10.26	1.74	3.13	2.23	2.41	1.64	1.48	2.29	2.95
8	6.44	11.69	7.35	10.71	6.28	7.97	6.39	10.98	1.80	3.74	2.28	3.73	1.69	2.16	1.87	4.41
9	6.34	15.07	6.55	12.23	6.95	9.07	6.10	11.91	0.85	7.29	1.74	5.85	0.69	3.57	2.02	6.29
10 = high	6.53	14.79	7.13	12.55	7.27	11.14	6.69	11.75	0.47	9.15	2.62	7.86	0.67	6.79	2.73	7.23
D10 - D1	1.89	8.10	1.18	4.39	2.78	-1.27	-0.26	2.83	-1.98	6.52	1.47	4.26	-2.62	-0.58	1.56	3.01
<i>t</i> -statistic	(0.66)	(7.04)	(0.49)	(4.26)	(0.94)	(-1.51)	(-0.11)	(2.42)	(-0.88)	(5.46)	(1.67)	(5.20)	(-1.07)	(-0.99)	(1.48)	(2.51)

Notes. This table reports mean realized returns (excess of risk-free rate) and four-factor model alphas during six-month periods [t+1,t+6] for deciles of the four LD-CAPM gammas and the four Acharya and Pedersen (2005) betas. Each month, t, we form deciles of the gammas/betas $(\gamma_{i,r}^R)$, downside comovement of a stock's excess returns with the market's excess returns; $\gamma_{i,r}^L$, downside comovement of a stock's liquidity with the market's excess returns; $\gamma_{i,r}^L$, downside comovement of a stock's liquidity; and $\gamma_{i,r}^L$ downside comovement of a stock's liquidity with the market's liquidity; $\beta_{i,1}^{AP}$ to $\beta_{i,1}^{AP}$ to $\beta_{i,1}^{AP}$ are the Acharya and Pedersen (2005) symmetric analogues of the downside risks). For each decile portfolio in each month, we calculate the equal-weighted (panel A) and value-weighted (panel B) means of the portfolio's next six-month return in excess of the risk-free rate, and alpha from a Fama—French—Carhart four-factor model estimated on the next six months of daily returns. We then calculate each decile's time-series mean across all months, as well as the mean for decile 10 minus decile 1 (D10-D1). The means are reported in the table in annualized percentages together with the t-statistic for D10-D1 using Newey–West standard errors. The sample consists of NYSE and AMEX stocks between 1962 and 2011, with an average of 151 stocks per decile.

for D10 - D1 using Newey–West standard errors with 36 lags.

We see no pronounced trends in the value-weighted or equal-weighted realized future returns across deciles of downside market return risk $(\gamma_{i,r}^R)$ nor deciles of downside aggregate liquidity risk $(\gamma_{i,r}^R)$. In contrast, returns for deciles of downside liquidity risk $(\gamma_{i,l}^R)$ and downside liquidity commonality risk $(\gamma_{i,l}^L)$ display strong (somewhat U- or J-shaped) patterns. Comparing the 1st and 10th deciles, the realized return for equalweighted (value-weighted) portfolios with high levels of downside liquidity risk is 10.61% (8.10%) above that of portfolios with low levels of this risk, with a t-statistic of 10.88 (7.04). For downside liquidity commonality risk, this return difference for equal (value) weighted portfolios is 3.61% (4.39%) with a *t*-statistic of 3.41 (4.26). The nonmonotonicity is likely to be the result of other priced factors (such as other gammas) being correlated with the gamma being used to form deciles.¹⁶

The Fama–French–Carhart four-factor model alphas assist the interpretation of these risk-return relations. The difference in the alphas across deciles of $\gamma_{i,l}^R$ and $\gamma_{i,l}^L$ represent the realized returns associated with these risks, purged of the premiums associated with stocks' exposure to the risks captured by the market, size, value, and momentum factors. The results in Table 2 indicate that downside liquidity risk $(\gamma_{i,l}^R)$ and downside liquidity commonality risk $(\gamma_{i,l}^R)$ both have a strong positive relation with future alphas. This result suggests the return premiums associated with these two gammas are not subsumed by risks associated with the Fama–French–Carhart factors.

The symmetric Acharya and Pedersen (2005) counterparts to the LD-CAPM gammas show smaller magnitude relations with returns and alphas, and also display nonlinearity in the relations. Of the four betas, the strongest relation with returns is for symmetric liquidity risk $(\beta_{i,2}^{AP})^{.17}$ For example, stocks in $\beta_{i,2}^{AP}$ decile 1 earn an average equal-weighted realized return that is 1.62% larger than stocks in decile 10, and the difference has a t-statistic of 2.79. The relatively stronger relation between the LD-CAPM gammas and returns/alphas suggests that the LD-CAPM more accurately characterizes how investors view liquidity risk.

5.3. Multivariate Firm-Level Cross-Sectional Tests of the LD-CAPM

The analysis of decile portfolios in the previous subsection has the advantages of being nonparametric and not imposing a functional form on the relation between risks and returns. It does not, however, lend itself to simultaneously controlling for multiple stock-level characteristics and risks. To do so, we turn to cross-sectional Fama–MacBeth regressions (Fama and MacBeth 1973) to test the equilibrium relations implied by

the LD-CAPM. Relaxing the model-implied constraints on the prices of the risks in Equation (24) gives

$$\mathbb{E}_{t}[R_{i,t+1}] = R_{f,t+1} + \lambda^{C} \mathbb{E}_{t}[C_{i,t+1}] + \lambda_{r}^{R} \gamma_{i,r}^{R} + \lambda_{l}^{R} \gamma_{i,l}^{R} + \lambda_{l}^{L} \gamma_{i,l}^{L}.$$
(16)

This step separates the four risks allowing the data to speak on how the associated premiums contribute to expected returns.¹⁸

We first test the expected return relations given by the LD-CAPM with and without the model-implied constraints, with different permutations of the gammas and with alternative methods of estimating the variables. We then compare the LD-CAPM to its symmetric counterpart and examine its incremental contribution to explaining returns. Next, we throw an extensive set of control variables and other model risk factors at the LD-CAPM tests to see which risks are unique and which are subsumed by other effects. Finally, we estimate other well-known asset pricing models for the sake of comparison.

We use a consistent approach through all of these tests. Each month, t, we estimate an equal-weighted stock-level cross-sectional regression of realized future six-month returns (return during the months [t + 1], [t+6]) on a set of characteristics, risk measures, and control variables.¹⁹ We then calculate and report the time-series averages of the intercepts and slope coefficients from the monthly cross-sectional regressions and test statistical significance using Newey-West standard errors with 36 lags. In robustness tests, we find very similar results when using nonoverlapping observations (estimating a cross-sectional regression every six months instead of every month). Like Da et al. (2012), Bali et al. (2011), and others, we use stocks as the units of observation in the cross-sectional regressions rather than forming portfolios. This avoids biasing the results in favor of (or against) a particular model as a result of the arbitrary but necessary choice of a sorting variable(s) in the portfolio formation. Furthermore, Lewellen et al. (2010) show that the use of the 25 Fama and French (1993) size/BM sorted portfolios gives a low hurdle in empirical asset pricing tests because of the strong factor structure created in the construction of the portfolios.

There is a large body of evidence on momentum effects in stock prices. If medium-term momentum is independent of return and liquidity risks, then failure to control for it could bias our estimates of the returns from bearing return and liquidity risks. This is particularly important in our setting because errors in estimation of the gammas could correlate with return momentum (e.g., stocks for which downside return risk is overestimated are likely to have had a negative average returns during the estimation window, which in the presence of medium-term momentum gives

Table 3. Cross-Sectional Fama–MacBeth Regressions

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Intercept	0.0299 (3.09)***	0.0311 (3.41)***	0.0339 (3.49)***	0.0325 (3.58)***	0.0301 (3.26)***	0.0294 (3.26)***	0.0298 (3.16)***	0.0294 (3.25)***	0.0291 (3.22)***
$R^e_{i,[t-5,t]}$	0.0209 (1.35)	0.0200 (1.31)	, ,	0.0202 (1.32)	,	0.0144 (0.92)	, ,	0.0052 (0.33)	0.0114 (0.75)
$E_t[C_{i,t+1}]$	0.341 (5.64)***	0.368 (4.91)***	0.361 (4.65)***	0.295 (4.55)***	0.368 (6.00)***	0.323 (4.52)***	0.329 (4.47)***	0.387 (5.45)***	0.326 (4.36)***
$\begin{array}{c} \gamma_{i,r}^R + \gamma_{i,l}^R \\ + \gamma_{i,r}^L + \gamma_{i,l}^L \end{array}$	0.0127 (3.62)***								
$\gamma_{i,r}^R$		0.0114 (1.87)*	0.0138 (1.78)*	0.0106 (1.75)*		0.0043 (0.54)	0.0099 (0.49)		0.007 (1.20)
$\gamma_{i,l}^R$		0.0099 (4.43)***	0.0106 (5.37)***	0.0107 (5.04)***		0.0209 (5.53)***	0.0208 (5.30)***		0.0105 (4.77)***
$\gamma_{i,r}^L$		-0.0097 (-0.20)	-0.0057 (-0.13)	-0.0094 (-0.19)		0.0892 (1.44)	0.0354 (0.85)		0.0066 (0.16)
$\gamma_{i,l}^L$		0.0016 (0.13)	-0.0044 (-0.27)	0.0155 (1.27)		-0.0800 (-1.41)	-0.0749 (-1.43)		0.009 (0.73)
$eta_{i,1}^{AP}$		(0.13)	(0.27)	(1.27)	0.0159 (2.08)**	0.0112 (1.40)	0.0116 (0.68)		(0.70)
$eta_{i,2}^{AP}$					-0.0016 (-0.55)	0.0233 (3.79)***	0.0228 (3.69)***		
$\beta_{i,3}^{AP}$					-0.150 (-0.95)	-0.159 (-1.22)	-0.0408 (-0.66)		
$eta_{i,4}^{AP}$					0.0655 (1.93)*	0.179 (1.60)	0.173 (1.63)		
$\zeta^R_{i,r}$					(=1, =)	(====)	(====)	0.011 (1.92)*	0.0075 (2.09)**
$\zeta_{i,l}^R$								-0.0106 (-3.34)**	-0.0115 (-3.62)**
$\zeta_{i,r}^L$								-0.107 (-1.13)	-0.0844 (-1.19)
$\zeta_{i,l}^L$								0.0863	0.0915
Adj. R ² (%)	2.99	4.22	3.08	4.19	3.12	4.86	4.08	(1.56) 4.00	(1.60) 4.91

Notes. Each month, t, we estimate an equal-weighted stock-level cross-sectional regression of realized future six-month excess returns (excess return during the months [t+1,t+6]) on various characteristics, risk measures, and control variables. The table reports the time-series averages of the intercepts and slope coefficients from the monthly cross-sectional regressions. The LD-CAPM gammas are $\gamma_{i,r}^R$ (downside comoment of a stock's excess returns with the market's excess returns), $\gamma_{i,r}^R$ (downside comoment of a stock's liquidity with the market's excess returns), $\gamma_{i,r}^R$ (downside comoment of a stock's liquidity with the market's liquidity), $\mathbb{E}_t[C_{i,t+1}]$ is expected liquidity cost, measured as the simple average of the stock's past six-month liquidity cost realizations in all models except Model 4, and the predicted value from an autoregressive model in Model 4; $R_{i,[t-5,t]}^e$ is the stock's excess return during the past six months; $\beta_{i,1}^{AP}$ to $\beta_{i,1}^{AP}$ are the Acharya and Pedersen (2005) L-CAPM betas (the symmetric analogues of the four downside risks); $\beta_{i,1}^{AP}$ measures the comovement of a stock's excess returns with the market's excess returns; $\beta_{i,2}^{AP}$ measures the comovement of a stock's liquidity; and $\beta_{i,3}^{AP}$ measures the comovement of a stock's liquidity with the market's liquidity. The zetas $(\zeta_{i,r}^R, \zeta_{i,r}^R, \zeta_{i,r}^L, \zeta_{i,r}^L)$ are the upside analogues of the downside gammas. t-Statistics using Newey–West standard errors are in parentheses. The sample consists of NYSE and AMEX stocks between 1962 and 2011, with an average of 1,508 stocks per cross-sectional regression.

***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

them an increased probability of low medium-term future returns). Therefore, in addition to the LD-CAPM explanatory variables, in most tests of the LD-CAPM, we include the stock's return during the six-month gamma estimation window as a control for medium-term momentum.

Table 3 reports estimates of various permutations of the LD-CAPM expected return relation. Model 1 imposes the constraint implied by the LD-CAPM that the prices of all downside risks are the same ($\lambda_{r}^{R} = \lambda_{l}^{R} =$

 $\lambda_r^L = \lambda_l^L$). The results indicate a statistically significant return premium for the total downside risk (coefficient of +0.0127 with t-statistic of 3.62). The coefficient on the expected liquidity cost level ($\mathbb{E}_t[C_{i,t+1}]$) is positive and statistically significant, suggesting that less liquid stocks earn a return premium for their liquidity cost *level*, independent of any premium associated with downside liquidity *risk*. These results seem intuitively sound and are consistent with existing evidence that (i) downside risks are associated with return premiums

(e.g., Ang et al. 2006) and (ii) the level of illiquidity is associated with a return premium (e.g., Amihud and Mendelson 1986, Amihud 2002).

To examine the contribution of the four risks to the downside risk premium, we relax the theoretical constraint on the prices of risk and allow them to take on distinct values. Model 2 in Table 3 shows that the return premium for total downside risk is driven mainly by downside liquidity risk, $\gamma_{i.l}^R$. This risk has a large and statistically significant coefficient (t-statistic greater than four). This is consistent with the notion that investors require a return premium to hold stocks that are more susceptible to the negative effects of liquidity spirals and flights to liquidity/quality. There is also a marginally statistically significant premium associated with downside market risk ($\gamma_{i,r}^R$). This supports the D-CAPM of Hogan and Warren (1974) and Bawa and Lindenberg (1977) and the empirical findings of Ang et al. (2006), and is consistent with the pricing kernel shapes seen in De Giorgi and Post (2008) and Rosenberg and Engle (2002). The coefficients of the other two gammas involving marketwide liquidity are not statistically distinguishable from zero. The results are similar when testing the effects of each of the gammas separately (reported in the online appendix). The return premium on the expected illiquidity level $(\mathbb{E}_t[C_{i,t+1}])$ remains positive and highly statistically significant in all of these specifications.

Model 3 in Table 3 examines the effect of excluding the medium-term momentum control variable. The coefficients on the four risks remain largely unchanged, as does the premium for the expected illiquidity. Model 4 in Table 3 replaces the expected illiquidity level calculated as the average of the liquidity cost during the past six months ($\mathbb{E}_t[C_{i,t+1}^{Avg}]$) with the predicted value from an autoregressive model ($\mathbb{E}_t[C_{i,t+1}^{AR}]$). The results are consistent across the two alternative measures of expected liquidity costs.

While the premium associated with downside liquidity risk $(\gamma_{i,l}^R)$ is large and statically significant, it is natural to question its economic meaningfulness.²⁰ To answer this, we compute the return premium (implied by Model 2 of Table 3) for stocks with high levels of $\gamma_{i,l}^R$ (decile 10) relative to stocks with low levels of $\gamma_{i,i}^{R}$ (decile 1). Measuring the implied 10th minus 1st decile return premium is a common approach in the empirical asset pricing literature. Using the decile results contained in the online appendix, we estimate the return premium for downside liquidity risk as $0.0099 \times (1.625 - (-1.528))$ per six months, or approximately 6.34% p.a. Using a similar approach, the premium for the expected liquidity cost level, $\mathbb{E}_{t}[C_{i,t+1}]$, is 8.44% p.a., and for downside market risk, $\gamma_{i,r}^{R}$, it is 5.47% p.a.

Given that outliers are likely to influence the means of characteristics/risks in the top and bottom deciles, a more robust and conservative way of computing the magnitude of the return premium is to compare ninth and second decile stocks. Using this method, the return premium for downside liquidity risk is approximately 1.26% p.a., for the expected liquidity cost level it is 2.31% p.a., and for downside market risk it is 3.12%. Another way to interpret the magnitudes is the implied return premium for a one standard deviation change in the gamma or characteristic. Using this approach, the premium for downside liquidity risk is 2.12% p.a., for the expected liquidity cost level it is 3.04% p.a., and for downside market risk it is 1.65%. While an evaluation of the risk premium depends on how one defines high and low levels of risk, the calculations above indicate that even conservative definitions result in economically meaningful premiums.

The adjusted R^2 for the LD-CAPM with disaggregated gammas (Model 2 in Table 3) is 4.22%. When interpreting this, one must keep in mind that our cross-sectional regressions use stocks not portfolios. This almost certainly leads to significantly lower R^2 because portfolios possess little idiosyncratic return variation. The adjusted R^2 for the LD-CAPM (4.22%) is comparable to the model fits seen in cross-sectional studies using stocks (e.g., Da et al. 2012 obtain adjusted R^2 s between 0.83% and 5.67% for the various models they examine, which include the Fama–French factors). We place the adjusted R^2 for the LD-CAPM into context by presenting estimates of alternative models below.

5.4. Comparisons of the LD-CAPM and the L-CAPM

Next, we compare the LD-CAPM to its symmetric analogue (the model developed by Acharya and Pedersen 2005) and examine the incremental contribution of the downside gammas in explaining returns. Model 5 in Table 3 estimates the Acharya and Pedersen (2005) L-CAPM, and Models 6 and 7 pool the symmetric betas and downside gammas (with and without the momentum control). Estimates in Model 5 are broadly consistent with Acharya and Pedersen (2005), despite the differences in estimation procedure (e.g., using daily rather than monthly observations, using individual stocks rather than portfolios). The signs of the return premiums correspond to those documented by Acharya and Pedersen (2005), and the premiums are statistically significant for the illiquidity level $(\mathbb{E}_t[C_{i,t+1}])$, symmetric market risk $(\beta_{i,1}^{AP})$, and symmetric liquidity commonality risk $(\beta_{i,4}^{AP})$. Interestingly, the model fit is approximately the same as for the LD-CAPM (3.12% compared to 3.08%, without momentum controls). Given the LD-CAPM gammas are estimated using only a subset (bad states) of the data used in the L-CAPM (good and bad states), the similarity in model fits suggests that most of the explanatory power stems from the relation between stock and market variables during bad states. Thus, the LD-CAPM is a parsimonious way to characterize the main effects of return and liquidity risks.

Combining the betas and downside gammas in Models 6 and 7 of Table 3 provides further insights about the characteristics of liquidity risks. In these models, the downside gammas measure the incremental premium associated with downside risks, above that captured by symmetric betas. Similarly, by controlling for the downside gammas, the premiums to the symmetric betas in these models are purged of the downside premium and therefore pick up the premium associated with comovement in *good* states. The results in Models 6 and 7 show an increase (doubling) of the premium associated with downside liquidity risk $(\gamma_{i,l}^R)$, which remains highly statistically significant (*t*-statistics above five). This result suggests investors view downside liquidity risk differently from symmetric or upside liquidity risk and require an additional return premium to hold stocks with high levels of this risk. Furthermore, the coefficient on symmetric liquidity risk ($\beta_{i,2}^{AP}$) flips sign, suggesting a return discount (negative premium) for symmetric liquidity risk, holding downside liquidity risk fixed.²¹ Given that the premium associated with symmetric liquidity risk in Models 6 and 7 is purged of the premium associated with downside liquidity risk, the return discount suggests investors view comovement between a stock's liquidity and excess market returns in good states as a desirable characteristic rather than a risk.

To more directly confirm the observation that investors view comovement in good states as a *desirable* characteristic rather than a risk, we estimate an upside version of our model, replacing the downside gammas with upside counterparts labeled zetas. The upside zetas are computed in the same way as the downside risks, but in good rather than bad states.²²

The results, reported in Models 8 and 9 of Table 3, confirm that upside liquidity risk $(\zeta_{i,l}^R)$ is associated with a significant premium that is opposite in sign to that of downside liquidity risk $(\gamma_{i,l}^R)$. The decile 10 minus decile 1 implied upside liquidity risk return premium is -6.14% p.a., which is similar in *magnitude* to the premium associated with downside liquidity risk. This finding reinforces the importance of asymmetry in characterizing liquidity risk and accounting for its impact on asset prices. Interestingly, the premium associated with the upside counterpart of market risk $(\zeta_{i,r}^R)$ is similar to that of downside market risk $(\gamma_{i,r}^R)$. The similarity in these upside and downside premiums suggests that, in contrast to liquidity risk, there is not a great deal of asymmetry in the characteristics of market risk or in how investors price this risk. The online appendix reports further tests of the upside zetas.

The joint tests of the LD-CAPM and L-CAPM and the tests of the upside counterpart to our model suggest an explanation for the mixed empirical findings in previous studies about the importance of liquidity risk;²³ namely, investors care differently about the degree of comovement between a stock's liquidity and excess market returns in bad states compared to that in good states—they require a positive return premium to hold stocks with downside liquidity risk, yet are willing to accept a lower rate of return for stocks with upside liquidity "risk." Symmetric liquidity risk measures obtain from imposing symmetry, effectively forcing the upside and downside premiums to be equal.²⁴ This restriction causes much of the upside and downside premiums to cancel out and leads to underestimation of the importance of liquidity risk. Imposing symmetry in estimating the liquidity risk premium is akin to regressing a dependent variable on two determinants (one positively related to the dependent variable and the other negatively related) and imposing the constraint that the two coefficients must take the same sign (are linearly related via a positive constant). While the upside and downside liquidity risk premiums offset each other in misspecified estimations that impose symmetry, they do not offset each other in asset returns because upside and downside liquidity risks are almost entirely uncorrelated (correlation of -0.025). Stocks with high downside liquidity risk do not tend to have a corresponding high level of upside risk, which would be required for the premiums to offset each other.

Our results support the hypothesis that symmetric liquidity risk models misestimate the liquidity risk premium through mischaracterization of liquidity risk. The premium associated with downside liquidity risk $(\gamma_{i,l}^R)$ is highly statistically significant with a *t*-statistic around five, whereas the premium for symmetric liquidity risk, $(\beta_{i,2}^{AP})$, is not statistically distinguishable from zero, with an absolute t-statistic around 0.6. The magnitudes tell the same story. Previously, we calculated that the 10th minus 1st decile cross-sectional return premium for downside liquidity risk is 6.34% p.a. Performing the same calculation on symmetric liquidity risk, $\beta_{i,2}^{AP}$, we get a cross-sectional return premium of only 0.56% p.a. (same order of magnitude as the 0.82% p.a. found by Acharya and Pedersen (2005) for the difference between portfolios 25 and 1), illustrating the misestimation caused by the assumption of symmetry. For the more conservative ninth minus second decile premiums, we get a downside liquidity risk premium of 1.26% p.a., whereas for $\beta_{i,2}^{AP}$, we get a cross-sectional return premium of only 0.08% p.a. Therefore, the return premium associated with downside liquidity risk is 10 to 15 times larger than suggested by symmetric liquidity risk measures.

5.5. Comparisons with Other Empirical Asset Pricing Models

It is natural to ask whether the LD-CAPM risks are unique or instead proxy for previously documented factors. To answer this question, we extend the previous

Table 4. Comparison with Other Asset Pricing Models Using Cross-Sectional Fama-MacBeth Regressions

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10
Intercept	0.0503	0.0320	0.0465	0.0313	0.0422	0.0441	0.0290	0.0496	0.0124	-0.0098
R^{ϱ}	(5.39)***	(3.50)*** 0.0282	(5.35)***	(3.40)*** 0.0180	(4.71)***	(5.12)***	(3.29)*** 0.0100	(5.34)***	(1.15) 0.0276	(-0.54) 0.0278
$R_{i,[t-5,t]}^e$		(1.98)**		(1.25)			(0.78)		(2.09)**	(2.14)**
$E_t[C_{i,t+1}]$		0.356		0.359			0.335		0.270	0.222
p		(5.12)***		(4.93)***			(4.95)***		(4.67)***	(3.75)***
$\gamma_{i,r}^R$		0.0322 (2.46)**		0.0081 (0.72)			0.0002 (0.02)		-0.0023 (-0.46)	-0.0010 (-0.21)
$\gamma_{i,l}^R$		0.0101		0.0102			0.0109		0.0106	0.0111
7 1,1		(4.68)***		(4.83)***			(5.85)***		(5.21)***	(6.08)***
$\gamma_{i,r}^L$		-0.0138		-0.0552			-0.0113		-0.0018	0.0010
ı		(-0.33)		(-0.67)			(-0.33)		(-0.06)	(0.03)
$\gamma_{i,l}^L$		-0.0010 (-0.08)		-0.0016 (-0.12)			-0.0199 (-0.82)		-0.0164 (-0.75)	-0.0197 (-0.83)
β_i^{MKT}	0.0035	-0.0207		(0.12)	0.0025	-0.0008	0.0083	0.0032	(0.75)	(0.00)
F 1	(0.50)	$(-1.82)^*$			(0.36)	(-0.13)	(0.90)	(0.45)		
β_i^+			-0.0002	-0.0003						
ρ-			(-0.11) 0.0085	(-0.10) 0.0038						
β_i^-			(1.89)*	(0.62)						
β_i^{SMB}			, ,	, ,	0.0125	0.0123	0.0127		0.0065	0.0063
					(3.71)***	(3.44)***	(3.59)***		(2.07)**	(2.07)**
β_i^{HML}					0.0040 (1.64)	0.0074 (3.87)***	0.0072 (3.75)***		0.0067 (3.61)***	0.0064 (3.37)***
β_i^{UMD}					(1.04)	-0.0043	-0.0028		-0.0035	-0.0031
						(-0.84)	(-0.80)		(-1.06)	(-0.96)
β_i^{COSKEW}								-0.0004	-0.0001	-0.0001
REV								(-1.46)	(-1.40) -0.0674	(-1.29) -0.0498
KEV									(-6.30)***	-0.0498 (-3.95)**
IVOL									1.42	1.92
									(3.44)***	(5.26)***
TVOL										-0.0578 (-0.52)
SDTURN										-0.0049
52 T 01711 V										(-2.18)**
MAX										-0.106
A 4: D2 (0/)	2.01	4.50	1.06	4.60	2.44	4 20	6 10	2.27	7.07	(-2.58)**
Adj. R ² (%)	2.01	4.59	1.96	4.62	3.44	4.38	6.12	2.37	7.07	7.58

Notes. Each month, t, we estimate an equal-weighted stock-level cross-sectional regression of realized future six-month excess returns (excess return during the months [t+1,t+6]) on various characteristics, risk measures, and control variables. The table reports the time-series averages of the intercepts and slope coefficients from the monthly cross-sectional regressions. The LD-CAPM gammas are $\gamma_{l,r}^R$ (downside comoment of a stock's excess returns with the market's excess returns), $\gamma_{l,l}^R$ (downside comoment of a stock's liquidity with the market's liquidity), and $\gamma_{l,l}^L$ (downside comoment of a stock's liquidity with the market's liquidity); $\mathbb{E}_{l}[C_{i,l+1}]$ is expected liquidity cost, measured as the simple average of the stock's past six-month liquidity cost realizations; $R_{i,[l-5,l]}^e$ is the stock's excess return during the past six months; β_{l}^{MKT} , β_{l}^{SMB} , β_{l}^{HML} , and β_{l}^{LMD} are loadings on the market, size, value, and momentum factors in a Fama–French–Carhart factor model; and β_{l}^+ are Ang et al. (2006) upside and downside betas. Other control variables are short-term reversal (REV), coskewness (β_{l}^{COSKEW}), idiosyncratic volatility (IVOL), total volatility (IVOL), turnover volatility (IVOL), and the stock's maximum return in the previous month (IVOL), with an average of 1,508 stocks per cross-sectional regression.

****,***, and * indicate statistical significance at the 1%, 5%, and 10% levels, repectively.

tests by taking the unconstrained LD-CAPM (Table 3, Model 2) as the base and progressively adding control variables and other known risk factors. In the process, we also estimate other asset pricing models on our sample for comparison. Table 4 reports these results.

Model 1 in Table 4 is the standard CAPM of Sharpe (1964) and Lintner (1965). Consistent with the findings

of other empirical studies, the market beta is not statistically significant. Model 2 in Table 4 augments the LD-CAPM with the addition of the (symmetric) CAPM market beta. Interestingly, the premium on downside market risk $(\gamma_{i,r}^R)$ increases sharply in both magnitude and statistical significance, whereas the premium on symmetric market beta becomes negative. This mirrors

our observations from the previous subsection when adding symmetric liquidity risk to our tests of downside liquidity risk and suggests that asymmetry also plays an important role in how investors view market risk.

Model 3 in Table 4 splits the market beta into upside and downside betas (β_i^+ and β_i^-) following the Ang et al. (2006) downside CAPM. Consistent with the findings of Ang et al. (2006) and the notion that asymmetry in risk measures matters, downside market beta is associated with a statistically significant return premium, but upside beta is not. When we combine the upside and downside market betas with the LD-CAPM gammas in Model 4 of Table 4, we find that the premium associated with downside liquidity risk remains large and significant, whereas the premium associated with downside market beta, β_i^- , drops by one-third in magnitude and is no longer statistically distinguishable from zero. The LD-CAPM subsumes the Ang et al. (2006) downside CAPM.

Models 5 and 6 in Table 4 are the Fama and French (1993) three-factor model and the Fama–French–Carhart (Carhart 1997) four-factor model, respectively. Model 7 in Table 4 adds the Fama–French–Carhart factors to the LD-CAPM. Consistent with other studies, we see that the coefficients of size and value factors are positive and statistically significant. The results on LD-CAPM variables are essentially the same as the base case in Model 1; the premiums associated with expected liquidity cost level and downside liquidity risk remain statistically significant and appear distinct to the risks captured by the Fama–French–Carhart factors.

A battery of additional control variables and risk factors tells a similar story. In Model 9 of Table 4, we augment the LD-CAPM with the Fama-French-Carhart factors, Jegadeesh (1990) short-term reversal (REV), Harvey and Siddique (2000) coskewness (β_i^{COSKEW}), and idiosyncratic volatility (*IVOL*) in the spirit of Ang et al. (2006). In Model 10 of Table 4 we further add total volatility (*TVOL*), the volatility of turnover (*SDTURN*) following Chordia et al. (2001), and a proxy for lottery features of a stock (MAX) following Bali et al. (2011). Regardless of how many and what combination of these additional variables we include, the LD-CAPM coefficients do not change considerably. In particular, expected liquidity cost level and downside liquidity risk maintain highly significant return premiums. The coefficients on most control variables and other risk factors are consistent with previous studies. For example, stocks with high size and value factor loadings and high past-12-month returns earn high future returns, whereas lottery stocks, recent winners (high past-onemonth return), and stocks with high turnover volatility tend to earn low future returns. The conclusion from these tests is that the LD-CAPM gammas are not

subsumed by previously documented risk factors and characteristics. Downside liquidity risk, along with the expected liquidity level, contributes incrementally to an explanation of the cross section of stock returns.

The use of stocks rather than portfolios as the crosssectional units of observation allows us to use adjusted R^2 (which accounts for differences in the number of explanatory variables) to compare the extent to which alternative asset pricing models explain the total cross-sectional return variation. The adjusted R^2 s of the existing asset pricing models estimated in Table 4 range from 1.96% for the Ang et al. (2006) downside CAPM (Model 3), to 2.01% for the CAPM (Model 1), to 2.37% for the Harvey and Siddique (2000) threemoment conditional CAPM (Model 8), to 3.44% for the Fama-French three-factor model (Model 5), and finally to 4.38% for the Fama-French-Carhart fourfactor model (Model 6). From Table 3, the unconstrained LD-CAPM has an adjusted R² of 4.22% (3.08%) omitting the medium-term momentum control), suggesting that the LD-CAPM explains a similar amount of the cross-sectional variation in stock returns as the leading existing empirical asset pricing models. The Fama-French-Carhart model is empirically motivated; however, the LD-CAPM, CAPM, downside CAPM, Harvey–Siddique three-moment conditional CAPM, and Acharya–Pedersen L-CAPM arise from economic theory. Of these theory-driven models, both the LD-CAPM and L-CAPM have equivalent adjusted R²s (3.08% and 3.12%), suggesting that the LD-CAPM performs well against its theory-driven peers.

In summary, the Fama–MacBeth tests of the crosssectional expected return relation implied by the LD-CAPM reveal the following. First, downside liquidity risk and liquidity level are associated with return premiums that are highly statistically significant and economically meaningful in magnitude. Second, downside liquidity risk is robust to controlling for the Fama-French market, size, and value factors; momentum; short-term reversal; coskewness; volatility; idiosyncratic volatility; turnover variance; and lottery-like features of stocks. Third, downside liquidity risk is not subsumed by symmetric liquidity risk (its premium increases when controlling for symmetric liquidity risk), suggesting investors care differently about liquidity "risk" in bad states compared to good states. Fourth, the LD-CAPM fits the data as well as the L-CAPM, and both these models explain a larger proportion of the total variation in cross-sectional returns than the other theory-driven models. Fifth, symmetric liquidity risk measures underestimate the premiums associated with liquidity risk.

5.6. Determinants of Downside Liquidity Risk

Given that downside liquidity risk is associated with a large and robust return premium, in this section we

Table 5. Characteristics of Downside Liquidity Risk

$\gamma_{i,l}^R$ decile	C_i	SIZE	ВМ	IVOL	TVOL	$eta_i^{ extit{MKT}}$	$eta_i^{ extit{COSKEW}}$	$\gamma^{\scriptscriptstyle R}_{\scriptscriptstyle i,l}$ liquid states	$\gamma^{\scriptscriptstyle R}_{\scriptscriptstyle i,l}$ illiquid states
1 = low	0.052	3.884	-0.297	0.023	0.023	0.590	-6.502	-1.059	-1.933
2	0.018	4.746	-0.443	0.023	0.024	0.862	-6.000	-0.315	-0.561
3	0.010	5.235	-0.530	0.023	0.024	0.968	-5.660	-0.144	-0.181
4	0.006	5.880	-0.587	0.021	0.022	0.967	-3.831	-0.072	-0.032
5	0.004	6.748	-0.652	0.018	0.020	0.952	-1.334	-0.028	-0.001
6	0.003	7.318	-0.631	0.017	0.019	0.987	-0.411	-0.007	0.008
7	0.004	7.094	-0.534	0.018	0.020	1.041	-1.731	-0.001	0.028
8	0.006	6.149	-0.444	0.020	0.022	1.033	-5.596	0.001	0.117
9	0.018	4.992	-0.324	0.023	0.024	0.925	-7.944	0.008	0.668
10 = high	0.072	3.789	-0.115	0.022	0.023	0.557	-6.120	0.574	2.917
D10 - D1	0.02	-0.09	0.18	0.00	0.00	-0.03	0.38	1.63	4.85
<i>t-</i> statistic	(5.87)	(-1.12)	(5.50)	(-1.22)	(-0.46)	(-1.11)	(0.73)	(4.35)	(4.76)

Notes. This table reports means of stock characteristics for deciles of downside liquidity risk $(\gamma_{i,l}^R)$. Each month, we sort stocks into deciles of $\gamma_{i,l}^R$. For each decile portfolio in each month, we calculate the equal-weighted mean of the portfolio's stock-level characteristics/factor loadings. We then calculate each decile's mean across all months as well as the mean for the 10th minus the 1st decile (D10-D1). t-Statistics for D10-D1 use Newey-West standard errors. The term C_i is normalized liquidity cost, SIZE is the natural log of market capitalization (in millions of dollars), BM is the natural log of the book-to-market ratio, IVOL is idiosyncratic volatility (standard deviation of residuals from a Fama-French-Carhart model), TVOL is total realized volatility (standard deviation of daily excess returns), β_i^{MKT} is the CAPM beta, and β_i^{COSKEW} is coskewness. The last two columns report means of downside liquidity risk for each decile portfolio, calculated during the most liquid six-month periods (lowest quintile of marketwide illiquidity, labeled "liquid states") and the least liquid (highest quintile of marketwide illiquidity, labeled "illiquid states"). The sample consists of NYSE and AMEX stocks between 1962 and 2011, with an average of 151 stocks per decile.

investigate what types of stocks possess high levels of downside liquidity risk and whether the premium associated with downside liquidity risk is amplified or attenuated by other stock characteristics. In Table 5, we sort stocks into deciles of downside liquidity risk each month. We compute the means of various characteristics (liquidity costs, size, book to market, idiosyncratic and total volatility, market beta, and coskewness) in each decile portfolio and then each decile's mean across all months.

Table 5 shows that stocks with high downside liquidity risk (the 10th decile) tend to be less liquid and smaller than the median, are more often value stocks (have higher book-to-market ratios), and have lower market beta and strong negative coskewness, but do not differ significantly in idiosyncratic or total volatility. Downside liquidity risk and coskewness may be driven by common sources of asymmetry, and thus a relation between them is not surprising. There is, however, fairly pronounced nonlinearity in how downside liquidity risk relates to other stock characteristics. Whereas stocks with high downside liquidity risk tend to be smaller, less liquid, and so on, so too are stocks with very low downside liquidity risk. In untabulated results, we find that this nonlinearity is not unique to downside liquidity risk; it is also observed for symmetric liquidity risk.

The last two columns of Table 5 examine the distribution of downside liquidity risk in different market states. As a first step, we take the time-series of marketwide liquidity costs and, in each decade, sort

six-month periods into quintiles by marketwide liquidity costs. We refer to six-month periods that fall within the top (bottom) quintile of marketwide liquidity costs as illiquid (liquid) "states." Next, for liquid and illiquid states separately, we form deciles of downside liquidity risk each month, compute the mean of downside liquidity risk in each decile, and take the time-series mean. This procedure gives an indication of the distribution of downside liquidity risk in times of high/low marketwide liquidity. The results show that downside liquidity risk is typically higher during illiquid states. Furthermore, there is greater cross-sectional dispersion in downside liquidity risk during illiquid states. For example, the difference in liquidity risk in the 10th and 1st deciles is three times greater during illiquid states compared to liquid states. Consequently, the largest values of downside liquidity risk in our sample are likely to occur during illiquid states. These results are consistent with the tendency for downward liquidity spirals and flights to liquidity to coincide with low marketwide liquidity (e.g., Acharya et al. 2013, Brunnermeier and Pedersen 2009, Nagel 2012). Recent work has found that marketwide illiquidity is driven by sentiment (Liu 2015) and is strongly related to the business cycle (Næs et al. 2011). This suggests that the impact of actions that lower the downside liquidity risk of a stock will be most beneficial during bad states of the world (periods of low sentiment, entering recessions, and during crises).

If the relation between downside liquidity risk and excess returns is a simple linear function, the foregoing characterization of the types of stocks and time periods that have the strongest downside liquidity risk also informs about where the premiums associated with downside liquidity risk are largest: small, illiquid value stocks with low CAPM betas during periods of marketwide illiquidity. However, it is possible that premiums associated with downside liquidity risk (the "price" of downside liquidity risk) are amplified or attenuated by other stock characteristics or the state of the market. To investigate this possibility, we use twoway sorts. As a first step, in each month, t, we sort stocks into quintiles of a stock characteristic (liquidity costs, size, book to market, idiosyncratic volatility, and CAPM beta). For each quintile in each month, we further sort stocks into quintiles of downside liquidity risk, giving us 25 portfolios each month.²⁵ We compute the equal-weighted average realized excess return for each portfolio over the six months after the portfolio formation month (i.e., the period [t + 1, t + 6]). Finally, we take the time-series means of the six-month realized returns for each portfolio, computing Newey-West standard errors on the fifth minus first quintile return differences.

Table 6 reports the results from the two-way sorts. Panel A shows that the premium associated with downside liquidity risk is present in all quintiles of liquidity level and is not only confined to illiquid stocks. The premium (measured by the return differential between the fifth and first downside liquidity risk quintiles, reported in the second to last column) is largest for the second to least liquid quintile of stocks (C_i quintile 4) and is larger on average for the least liquid two quintiles compared to the most liquid two, despite being relatively smaller for the very least liquid stocks. Therefore, there is a tendency for less liquid stocks to have a larger downside liquidity risk premium. The premium associated with the liquidity level is large and significant for all quintiles of liquidity risk.

Panel B of Table 6 shows that the premium associated with downside liquidity risk is larger in smaller stocks—the return differential between the fifth and first downside liquidity risk quintiles is twice as large in the smallest stocks (10.18% p.a.) as in the largest stocks (5.23% p.a.). The downside liquidity risk premium is nevertheless still present in all size quintiles. Smaller stocks earn higher returns on average than larger stocks in all quintiles of downside liquidity risk.

In panel C of Table 6, we see that that the premium associated with downside liquidity risk is present in all book-to-market quintiles and is considerably larger in value stocks, i.e., those with low book-to-market ratios. The value premium (higher average returns for stocks with high book-to-market ratios) is significant across all quintiles of downside liquidity risk.

The results in panel D of Table 6 are striking—they show that of all the characteristics examined, the premium associated with downside liquidity risk is

most closely related to the level of idiosyncratic volatility. Stocks with high levels of idiosyncratic volatility have substantially greater (by a factor of 10) return premiums associated with downside liquidity risk. Furthermore, the premium associated with high idiosyncratic volatility is small and not statistically distinguishable from zero for stocks with low downside liquidity risk, but in the top two quintiles of downside liquidity risk it becomes large and significant. It appears a combination of high idiosyncratic volatility and high downside liquidity risk attracts a very large return premium.

What about *systematic* risk as captured by CAPM beta? Panel E of Table 6 indicates that high CAPM beta stocks also have a larger downside liquidity risk premium (by a factor of four, comparing top and bottom beta quintiles), although the effect of *systematic* risk on the downside liquidity risk premium is not as strong as the effect of *idiosyncratic* volatility. Thus, volatile stocks, irrespective of whether the source is systematic or idiosyncratic, attract a larger downside liquidity risk premium.

Fama and French (1992) demonstrate that average stock returns are not related to CAPM beta. In panel E, the relation between CAPM beta and average returns is positive in stocks with high downside liquidity risk (consistent with an upward sloping security market line (SML)), but *negative* (as documented by Black et al. 1972 and Baker et al. 2011) for stocks with low downside liquidity risk. This result is consistent with the model of Frazzini and Pedersen (2014), in which leverage and margin constraints vary across investors and time. Their model attributes the relatively flat (or negative) relation between beta and average returns/alpha to funding constraints—investors unable to borrow tilt their portfolios toward high beta stocks, which flattens the SML. Sophisticated investors that are able to borrow offset this effect by buying low beta stocks on margin (and possibly shorting high beta stocks), thereby pushing the SML to have a positive slope. However, a side effect of margin buying is that when the market declines, leveraged investors receive margin calls and a contraction of funding liquidity, leading to forced liquidations, and amplified downside liquidity risk. Panels E and F of Table 6 indicate that downside liquidity risk (which tends to be higher at times of low marketwide liquidity), and its connection with leverage constraints, margin trading, and sentiment, may have some part in explaining the documented time-varying nature of the SML.

Finally, in panel F of Table 6, we examine how the downside liquidity risk premium varies with marketwide liquidity, or liquid and illiquid "states" of the market. Rather than initially sorting on a stock characteristic, we sort six-month periods into quintiles each decade based on marketwide liquidity and examine

Table 6. Two-Way Sorts

		Downsid	e liquidity risk (γ_i	$\binom{R}{l}$ quintile			
C_i quintile	1 = low	2	3	4	5 = high	Q5 - Q1	t-statistic
		P	anel A: Initial sort	on liquidity costs,	C_i		
1 = low	2.62	4.95	6.51	8.36	10.85	8.23	(7.04)
2	3.82	5.03	9.20	11.56	13.82	10.00	(6.52)
3	4.84	8.11	9.53	14.36	18.62	13.77	(9.73)
4	6.95	10.08	14.98	19.93	20.91	13.96	(11.05)
5 = high	11.61	16.01	18.95	21.06	19.30	7.69	(7.25)
Q5 - Q1	8.99	11.06	12.44	12.70	8.46		
t-statistic	(4.16)	(4.47)	(4.90)	(5.44)	(5.76)		
	, ,			arket capitalizatio			
1 = low	13.29	17.64	21.74	26.12	23.47	10.18	(9.05)
2	5.96	9.11	14.02	19.20	17.66	11.70	(11.33)
3	5.40	6.91	9.76	15.40	14.76	9.36	(8.15)
4	4.90	5.87	9.46	11.83	11.81	6.91	(7.27)
5 = high	3.55	4.91	6.69	8.15	8.78	5.23	(6.51)
Q5 - Q1	-9.74	-12.73	-15.05	-17.98	-14.70		
t-statistic	(-3.72)	(-4.16)	(-4.47)	(-6.72)	(-6.05)		
		Pa	nel C: Initial sort o	n book-to-market,	BM		
1 = low	5.58	5.06	6.62	8.57	18.04	12.47	(7.90)
2	7.94	6.60	5.44	9.84	15.35	7.41	(6.55)
3	8.84	7.90	6.95	11.49	15.74	6.89	(5.18)
4	11.77	8.92	8.73	13.93	17.71	5.95	(6.30)
5 = high	15.04	12.20	15.03	21.11	23.01	7.96	(9.42)
Q5 - Q1	9.47	7.14	8.41	12.54	4.96		
t-statistic	(4.31)	(3.32)	(3.20)	(4.65)	(2.26)		
		Panel I): Initial sort on id	iosyncratic volatili	ty, IVOL		
1 = low	8.87	6.85	6.43	8.45	10.74	1.87	(3.21)
2	9.50	7.51	8.22	10.63	13.10	3.60	(6.10)
3	9.21	8.31	8.50	12.05	17.19	7.98	(9.22)
4	9.80	8.71	9.48	14.92	21.50	11.70	(10.75)
5 = high	12.53	10.00	10.92	20.54	31.60	19.06	(10.23)
Q5 - Q1	3.66	3.15	4.49	12.09	20.85		
t-statistic	(1.49)	(1.01)	(1.28)	(3.33)	(7.63)		
				CAPM market beta			
1 = low	11.49	10.10	7.00	13.58	16.26	4.77	(5.76)
2	10.56	7.82	7.19	11.53	18.61	8.06	(10.38)
3	9.71	8.33	7.77	12.06	20.13	10.42	(9.09)
4	8.78	8.52	7.90	12.87	22.52	13.74	(11.60)
5 = high	6.78	7.16	8.48	14.63	24.33	17.56	(10.61)
Q5 - Q1	-4.72	-2.94	1.48	1.05	8.07		
t-statistic	(-2.03)	(-1.28)	(0.58)	(0.45)	(3.36)		
				rketwide liquidity			
1 = low	6.31	4.88	7.21	11.32	17.50	11.19	(10.80)
2	6.13	5.00	5.72	7.16	15.72	9.59	(5.21)
3	13.27	10.85	6.15	12.29	22.32	9.05	(6.80)
4	12.72	8.56	8.63	16.05	22.64	9.92	(19.99)
5 = high	10.59	8.99	11.34	19.24	23.01	12.41	(10.37)
Q5 - Q1	4.28	4.11	4.14	7.92	5.50		

Notes. This table reports mean realized returns during six-month periods [t+1,t+6] for portfolios of stocks. The portfolios are formed each month, t, by first sorting stocks into quintiles of a characteristic (liquidity costs (C_i), market capitalization (SIZE), book-to-market (BM), idiosyncratic volatility (IVOL), CAPM market beta (β_i^{MKT})) and then, within each quintile, sorting stocks into quintiles by downside liquidity risk ($\gamma_{i,1}^R$). For each of the 25 portfolios in each month, we calculate the equal-weighted mean of the portfolio's next six-month return in excess of the risk-free rate. We then calculate each portfolio's time-series mean across all months, as well as the mean for the fifth minus first quintile (Q5-Q1). The means are reported in the table in annualized percentages together with the t-statistic for Q5-Q1 using Newey-West standard errors. In panel F, rather than initially sorting on a stock characteristic, we sort six-month periods into quintiles based on marketwide liquidity and examine the returns to quintiles of downside liquidity risk in these five market "states." The sample consists of NYSE and AMEX stocks between 1962 and 2011, with an average of 60 stocks per portfolio.

the future six-month returns to quintiles of downside liquidity risk in these five market "states." The premium associated with downside liquidity risk is present in all market states and is largest during the most illiquid states (C_m quintile 5). These results provide further support for the notion that downward liquidity spirals and flights to liquidity tend to coincide with low marketwide liquidity (e.g., Acharya et al. 2013, Brunnermeier and Pedersen 2009, Nagel 2012). During such times, investors require a higher premium to hold stocks with high downside liquidity risk because of the increased risk that the stock is abandoned during a flight to liquidity or is affected by a downward liquidity spiral.

In summary, the evidence in this section shows that relatively illiquid, small value stocks have high levels of downside liquidity risk and a large downside liquidity risk premium. The downside liquidity risk premium is also much larger for volatile stocks, both high beta and high idiosyncratic volatility stocks. Finally, downside liquidity risk and the associated premium are larger during periods of low marketwide liquidity.

5.7. Robustness Tests

We subject our empirical analysis of the LD-CAPM to a battery of robustness tests, including (i) value weighting versus equal weighting, (ii) subperiod analysis, (iii) overlapping versus nonoverlapping estimation windows, (iv) alternative liquidity measures, (v) inclusion of nonlinear terms, (vi) different thresholds to define downside, (vii) demeaning the variables in gamma estimation, (viii) different rolling window lengths, and (ix) aggregation of observations to a weekly frequency to reduce the influence of microstructure phenomena and nonsynchronous trading issues. These tests all produce consistent results. For brevity, we report the results of tests (i)–(iv) for the main specification of our model in Table 7. The other robustness tests are reported in the online appendix.

Model 1 in Table 7 is the baseline specification with disaggregated gammas (same as Model 2 in Table 3) and is included for comparison. Model 2 uses value weighting instead of equal weighting (weighting stocks by their market capitalization in the cross-sectional Fama–MacBeth regressions). The value-weighted model fits

Table 7. Robustness Tests

	Model 1: Base case	Model 2: Value weighted	Model 3: Subsample 1962–1986	Model 4: Subsample 1987–2011	Model 5: Non- overlapping	Model 6: ln(<i>ILLIQ</i>)	Model 7: Range <i>ILLIQ</i>	Model 8: Turnover <i>ILLIQ</i>	Model 9: Quoted spread, 1993–2011
					11 0				
Intercept	0.0311	0.0171	0.0354	0.0269	0.0331	0.0250	0.0192	0.0195	0.0209
	$(3.41)^{***}$	(1.98)**	(2.61)***	(2.11)**	(5.50)***	(2.71)***	(2.03)**	$(1.67)^*$	(1.36)
$R_{i,[t-5,t]}^e$	0.0200	0.0363	0.0432	-0.0028	0.0096	0.0232	0.0247	0.0198	0.0125
1,[1 3,1]	(1.31)	(2.19)**	(2.53)**	(-0.12)	(1.11)	(1.52)	(1.62)	(1.35)	(0.42)
$E_t[C_{i,t+1}]$	0.368	0.280	0.540	0.198	0.379	0.0510	0.0507	0.0255	1.81
-11-1,1+13	(4.91)***	(1.99)**	(5.25)***	(3.17)***	(3.74)***	(4.90)***	(5.43)***	(3.41)***	(2.61)***
$\gamma_{i,r}^R$	0.0114	0.0042	0.0075	0.0153	0.0117	0.0128	0.0134	0.0104	0.0177
7 1,7	(1.87)*	(0.55)	(0.72)	(2.49)**	(2.84)***	(1.86)*	(2.13)**	(1.24)	(0.99)
$\gamma_{i,l}^R$	0.0099	0.0087	0.0104	0.0094	0.0127	0.0019	0.0017	0.0013	0.168
,.	(4.43)***	(2.08)**	(4.41)***	(2.59)**	(11.14)***	(7.31)***	(7.53)***	(2.95)***	$(1.88)^*$
$\gamma_{i,r}^L$	-0.0097	0.0405	-0.0067	-0.0126	0.0850	0.0040	-0.0079	0.0015	-0.0605
, 1,,	(-0.20)	(0.83)	(-0.62)	(-0.13)	(2.12)**	(0.87)	(-1.00)	(1.01)	(-0.21)
$\gamma_{i,l}^L$	0.0016	0.0004	0.0125	-0.0090	-0.0513	-0.0005	0.0002	-0.0004	-0.750
, 1,1	(0.13)	(0.02)	(2.23)**	(-0.37)	(-0.70)	(-1.58)	(1.00)	$(-1.71)^*$	(-1.01)
Adj. R ² (%)	4.22	8.17	4.79	3.66	4.24	4.72	4.90	4.11	5.54

Notes. This table reports cross-sectional Fama–MacBeth regression results for several variations of the LD-CAPM. Each month, t, we estimate an equal-weighted, stock-level, cross-sectional regression of realized future six-month excess returns (excess return during the months [t+1,t+6]) on various characteristics and risk measures. The table reports the time-series averages of the intercepts and slope coefficients from the monthly cross-sectional regressions. The LD-CAPM gammas are $\gamma_{i,r}^R$ (downside comoment of a stock's excess returns), $\gamma_{i,t}^L$ (downside comoment of a stock's liquidity with the market's excess returns), $\gamma_{i,t}^L$ (downside comoment of a stock's liquidity with the market's liquidity); $\mathbb{E}_t[C_{i,t+1}]$ is expected liquidity cost, measured as the simple average of the stock's past six-month liquidity cost realizations; and $R_{i,[t-5,t]}^e$ is the stock's excess return during the past six months. Model 1 is the full-sample equal-weighted base-case specification (Table 3, Model 2) and is included for comparison. Model 2 uses value weighting instead of equal weighting (weighting stocks by their market capitalization in the cross-sectional Fama–MacBeth regressions). Models 3 and 4 are estimated on subperiods corresponding to the first and second half of the sample. Model 5 uses nonoverlapping observations for the Fama–Macbeth regression instead of the rolling windows. Models 6–9 use different liquidity proxies: log of Amihud's ILLIQ, modified ILLIQ using the high–low range as the measure of price impact, modified ILLIQ using turnover in place of dollar volume, and the actual quoted bid–ask spreads from CRSP (expressed relative to the midquote). The actual quoted bid–ask spread is only available for the period to 1993–2011. t-Statistics using Newey–West standard errors are in parentheses.

 $^{^{***}}$, ** , and * indicate statistical significance at the 1%, 5%, and 10% levels, repectively.

the data better (R^2 of 8.17% compared to 4.22% under equal weighting), but the estimated effects of liquidity level and downside liquidity risk remain largely unchanged. Models 3 and 4 are estimated on subperiods corresponding to the first and second halves of the sample. The premium on the illiquidity level has decreased through time (the coefficient of $E_t[C_{i,t+1}]$ is 0.540 during 1962–1986 but only 0.198 during 1986– 2011), but the premiums associated with liquidity risks, in particular downside liquidity risk, are similar in magnitude and highly statistically significant in the first and second halves of the sample. Interestingly, the premium associated with downside market return risk $(\gamma_{i,r}^R)$ is about twice as large in the second half of the sample. Model 5 uses nonoverlapping observations in the Fama–MacBeth regressions rather than the rolling windows (a cross-sectional regression is estimated every six months, rather than every month). Compared to the baseline specification, the premium associated with $\gamma_{i,l}^R$ is greater, and its statistical significance is almost triple.

Models 6–9 of Table 7 use different liquidity proxies. Model 6 uses log of Amihud's ILLIQ as per Karolyi et al. (2012), winsorized at the 5th and 95th percentiles to limit the influence of outliers. In addition to the log transformation and winsorization, this proxy differs from our primary liquidity measure in that it does not use the normalization, truncation, and scaling applied to the primary measure. Model 7 uses log of a modified *ILLIQ* involving the high-low range as the measure of price impact. Model 8 uses log of a modified *ILLIQ* replacing dollar volume with turnover as per Brockman et al. (2008). Finally, Model 9 uses the actual quoted bid-ask spreads (expressed relative to the midquote) calculated from CRSP closing bid and ask quotes. Because the closing quotes for NYSE and AMEX stocks are available continuously only from 1993 onward, Model 10 of Table 7 is tested with a shorter sample period (1993-2011), and therefore the tests are likely to have lower statistical power. We find consistent results using all of the alternative liquidity measures, in many cases with improved model fit.

6. Conclusions

We develop and test a liquidity-adjusted downside capital asset pricing model. The key innovation of the model, the extraction of downside liquidity and return risks from their symmetric analogues, is motivated by (i) the growing evidence on asymmetries in the comovement between stock-level liquidity and market returns and (ii) the evidence suggesting investors view risk as the risk of loss.

The empirical evidence indicates that disentangling downside risk from symmetric risk is important—investors attach a significant expected return premium to stocks with high levels of downside liquidity risk.

This premium is not apparent in studies involving symmetric risk. Symmetric representations mischaracterize the role of liquidity risk in determining asset prices and consequently underestimate the importance of liquidity risk. The downside liquidity risk premium is consistent with investors requiring higher expected returns from stocks with greater susceptibility to downward liquidity spirals and greater likelihood of abandonment during a flight to quality/liquidity. Our findings suggest that adverse liquidity phenomena can affect the real economy via firms' cost of capital. Our model also confirms a premium for the *level* of illiquidity and downside market risk, consistent with the existing literature.

The LD-CAPM fits the data well compared to other asset pricing models and explains a larger proportion of the total variation in cross-sectional returns than other theory-driven models. The return premiums associated with downside liquidity risk and the liquidity level are robust to controlling for a wide range of characteristics and risks, including the market, size, and value factors; momentum; short-term reversal; coskewness; total volatility; idiosyncratic volatility; turnover variance; and lottery-like features of stocks.

There are many alternatives to the pricing kernel of the LD-CAPM; however, the size and statistical significance of the premium attached to the downside liquidity risk isolated by our kernel suggests that alternatives should not be chosen on the basis of pricing flexibility alone—they should be capable of representing downside liquidity effects. Taken together, the statistically insignificant premiums associated with the two downside risks involving marketwide liquidity ($\gamma_{i,r}^L$ and $\gamma_{i,l}^L$) suggest that marketwide liquidity may not be a feature of the pricing kernel (not a state variable as in Pastor and Stambaugh 2003). Investors care about a stock's downside liquidity risk because they are lossaverse and care about realizable after-cost returns. To this end, we extend the final question posed by Dittmar (2002): what sort of pricing kernel is globally decreasing, flexible enough to price assets well, and capable of correctly characterizing the asymmetry inherent in liquidity risk?

Our finding of a significant downside liquidity risk premium affirms recent efforts to develop liquidity-adjusted value-at-risk and expected shortfall metrics. These metrics incorporate downside liquidity risk into risk management procedures. Another practical implication is that a company's cost of capital can be reduced (and its value increased) by minimizing its stocks' downside liquidity risk. Recent work suggests that the structure of secondary markets, in particular, the degree of competition among liquidity providers and the nature of their funding arrangements, impacts downside liquidity risk (e.g., Roll and Subrahmanyam 2010, Comerton-Forde et al. 2010, Nagel 2012) and,

by extension of our findings, affects company valuations. Furthermore, companies may be able to "shape" their stocks' downside liquidity risk, for example, by increasing transparency during market downturns, thereby supporting liquidity when it matters most (Balakrishnan et al. 2014, Lang and Maffett 2011, Lang et al. 2012).

Our work also has implications for firm capital structure choices and financing decisions. Lipson and Mortal (2009) show that firms with more liquid equity adopt lower leverage and prefer equity over debt financing. McLean (2011) finds evidence that firms engage in an interesting form of precautionary saving—they issue equity and save cash during good times, thereby avoiding equity issuance during times of low liquidity. This is particularly so for small and unprofitable firms. We find that the downside liquidity risk premium is typically larger for stocks that are relatively small, illiquid, volatile, and have high bookto-market ratios. Policy or market design targeting this risk, therefore, has capital structure implications for such firms. More broadly, if marketwide illiquidity is driven by falling sentiment and the business cycle (Liu 2015, Næs et al. 2011), and, as we have found, downside liquidity risk and its associated premium are higher during these times, then policy or regulation addressing adverse liquidity phenomena will likely have the greatest impact when the economy needs it most.

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Appendix

In this appendix, we present a theorem on how asset pricing models involving conditional gammas arise from pricing kernels involving min functions. The theorem is proved using a result on conditional expectations. Last, the LD-CAPM is derived by substitution and mapping the coefficients of the kernel into the final result of the theorem.

Theorem. Let X, Y, and Z, be random variables and let ε and κ be constants. Let $X^* = X - \varepsilon$, $Y^* = Y - \varepsilon$, and $Z^* = Z - \varepsilon$. Last, let $\mathbf{1}^-$ be an indicator function on X^* defined as

$$\mathbf{1}^{-} = \begin{cases} 0 & \text{if } X^* \ge 0, \\ 1 & \text{if } X^* < 0. \end{cases}$$
 (17)

If the relation

$$\frac{\operatorname{Cov}[M^*, Y^*]}{\operatorname{Cov}[M^*, Z^*]} = \frac{\mathbb{E}[Y^*]}{\mathbb{E}[Z^*]}$$
(18)

holds, with

$$M^* = \kappa - \mathbf{1}^- \theta X^*, \tag{19}$$

then

$$\mathbb{E}[Y^*] = \gamma \mathbb{E}[Z^*],\tag{20}$$

where

$$\gamma = \frac{\mathbb{E}[Y^*X^* \mid X^* < 0]}{\mathbb{E}[Z^*X^* \mid X^* < 0]}.$$
 (21)

Proof. We first set down an important result on conditional expectations. Let g(x) be a function of $x \in \Omega$, and let $A \subseteq \Omega$. Now let $\mathbf{1}^A$ be an indicator function defined as

$$\mathbf{1}^{A} = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$
 (22)

The conditional expectation of g(x), where the condition is that $x \in A$, can be written as

$$\mathbb{E}[g(x) \mid x \in A] = \frac{\mathbb{E}[\mathbf{1}^A g(x)]}{\mathbb{E}[\mathbf{1}^A]},\tag{23}$$

and thus

$$\mathbb{E}[1^A g(x)] = \mathbb{E}[\mathbf{1}^A] E[g(x) \mid x \in A]. \tag{24}$$

In words, the expectation of the product of a function and an indicator function of x is equal to the product of the conditional expectation and the probability that the condition is met (e.g., see Winkler et al. 1972).

Substituting Equation (19) into Equation (18) followed by some rearrangement gives

$$\mathbb{E}[-Y^*\mathbf{1}^-\theta X^*]\mathbb{E}[Z^*] - \mathbb{E}[-\mathbf{1}^-\theta X^*]\mathbb{E}[Y^*]\mathbb{E}[Z^*]$$

$$= \mathbb{E}[-Z^*\mathbf{1}^-\theta X^*]\mathbb{E}[Y^*] - \mathbb{E}[-\mathbf{1}^-\theta X^*]\mathbb{E}[Z^*]\mathbb{E}[Y^*],$$

which after cancellation becomes

$$\mathbb{E}[Y^*\mathbf{1}^-\theta X^*]\mathbb{E}[Z^*] = \mathbb{E}[Z^*\mathbf{1}^-\theta X^*]\mathbb{E}[Y^*]. \tag{25}$$

This can be expressed as

$$\mathbb{E}[Y^*] = \frac{\mathbb{E}[\mathbf{1}^- Y^* X^*]}{\mathbb{E}[\mathbf{1}^- Z^* X^*]} \mathbb{E}[Z^*]. \tag{26}$$

This can be reexpressed in terms of conditional expectations using the result in Equation (24), and the expectation of the indicator function can be cancelled:

$$\mathbb{E}[Y^*] = \frac{\mathbb{E}[Y^*X^* \mid X^* < 0]}{\mathbb{E}[Z^*X^* \mid X^* < 0]} \mathbb{E}[Z^*]. \quad \Box$$

To apply the theorem and derive the LD-CAPM, we make the following replacements that represent the liquidity adjustments made to net asset returns and the D-CAPM pricing kernel:

$$\begin{split} X^* &= R^e_{m,\,t+1} - C_{m,\,t+1}, \quad Y^* = R^e_{i,\,t+1} - C_{i,\,t+1}, \\ Z^* &= R^e_{m,\,t+1} - C_{m,\,t+1}. \end{split}$$

Endnotes

- ¹ For example, Pastor and Stambaugh (2003) report a return premium of 7.5% p.a. (although part of this premium may be due to the liquidity level), Acharya and Pedersen (2005) report a premium of 1.1% p.a., and some studies find no evidence of a risk premium (e.g., Hasbrouck 2009).
- ²Roll and Subrahmanyam (2010) find that the distribution of bid–ask spreads (*illiquidity*) is right skewed, suggesting the distribution of *liquidity*, like returns, is left skewed.
- ³Markowitz (1959) acknowledges that it is natural to view risk as concern for adverse deviations. He proceeds with symmetric risk measures such as variance and covariance because of their mathematical convenience.
- ⁴To be precise, Hogan and Warren (1974) motivate the D-CAPM from Markowitz's (1959) observation that semivariance better represents risk than variance. In contrast, Bawa and Lindenberg (1977) motivate the D-CAPM through the connection between stochastic dominance and selection rules involving expected returns and lower-partial moments. The utility function underpinning the D-CAPM is $U_{t+1}(R_{t+1}^e) = a_t + b_t R_{t+1}^e c_t ((-R_{t+1}^e)^+)^2$, where R_{t+1}^e is the return in excess of the risk-free rate at time t+1 and $(x)^+ = \max(x,0)$.
- ⁵ To aid comparison of our model with that of Acharya and Pedersen (2005), we note that the pricing kernel for the L-CAPM (in the absence of strong distributional assumptions) can be written as $M_{t+1}^L = \nu_t K_t(R_{m,t+1}^e C_{m,t+1})$, where ν_t and K_t are constants. This L-CAPM kernel applies the same degree of risk aversion ($K_t > 0$) across good and bad states and violates the preference regularity conditions of nonsatiation and no arbitrage.
- ⁶To simplify notation, we drop the conditioning information (I_t) from subsequent expressions, treating it as implicit in the expectation operator, $\mathbb{E}_t[\cdot]$.
- ⁷Consistent with Acharya and Pedersen (2005), we do not include NASDAQ-listed stocks because their volume is overstated in the Centre for Research in Security Prices (CRSP) data because of the inclusion of interdealer trades (and only commences in 1982). The results are very similar when the crisis of 2008–2009 is omitted.
- ⁸ http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html (accessed June 2014).
- ⁹Similar to Amihud (2002), we reduce the influence of outliers by winsorizing $ILLIQ_{i,\tau}$ at the top 1% of the cross-sectional distribution on every day and top 1% of the time-series distribution for every stock.
- ¹⁰ Acharya and Pedersen (2005) use two approaches to make liquidity costs comparable in scale to returns. The first is freeing up the coefficients in their tests of the model, allowing them to absorb differences in scale. The second is scaling liquidity costs by a parameter κ , which is calibrated to average monthly turnover in their main specification. For consistency, we also use these two approaches. In most of our tests, coefficients are free to absorb differences in scale. When identifying the negative subspace ($R_{m,\tau}^e C_{m,\tau} < 0$), we follow the second approach and scale $C_{m,\tau}$ using the coefficient derived by Acharya and Pedersen (2005) from monthly turnover converted to daily units by diving by 22.
- ¹¹The use of innovations in liquidity when computing liquidity *risks* is important because liquidity is persistent and shocks rather than predictable changes contribute to risk. See, for example, Acharya and Pedersen (2005), Pastor and Stambaugh (2003), Sadka (2006), Liu (2006), and Bongaerts et al. (2011).
- ¹²The mean and standard deviation of $R_{m,\tau}^e$ are 5.43 bps and 97.75 bps, respectively. The mean and standard deviation of $C_{m,\tau}$ (in daily units) are 0.27 bps and 0.10 bps, respectively.
- ¹³Our estimation of $\beta_{i,1}^{AP}$ to $\beta_{i,1}^{AP}$ differs from that of Acharya and Pedersen (2005) in the following ways. We use a daily sampling frequency and rolling estimation window in contrast to their monthly

- observations and unconditional (full sample) estimation. We present the Acharya and Pedersen (2005) betas in a different order from that in their paper.
- ¹⁴The signs and magnitudes are similar to those reported by Acharya and Pedersen (2005), noting that the numbering of the betas differs.
- ¹⁵The negative correlation between downside liquidity risk and its symmetric analogue is explained by the fact that $\gamma_{i,l}^R$ is defined in terms of *liquidity*, whereas $\beta_{i,p}^{AP}$, following Acharya and Pedersen (2005), is defined in terms of *illiquidity*.
- ¹⁶Nonmonotonicity in these decile portfolio results is *not* inconsistent with the LD-CAPM, because the LD-CAPM implies linear *conditional* relations between gammas and expected excess returns (i.e., holding other gammas fixed), whereas the decile sorts test *unconditional* (univariate) relations. The online appendix provides examples of how nonmonotonicity can result from correlations among gammas (or correlations between gammas and the level of liquidity). In robustness tests, we find no evidence of nonlinearity in the conditional relations between gammas and returns in multivariate tests.
- ¹⁷This result is consistent with results of Acharya and Pedersen (2005), who report that 80% of the total premium associated with the three liquidity betas is due to $\beta_{i,2}^{AP}$. It is also consistent with the results of Hagströmer et al. (2013).
- ¹⁸ In the empirical tests of Acharya and Pedersen (2005), liquidity costs are scaled by a parameter κ . The model is then estimated using various values of κ : calibrated to average monthly turnover; a free parameter; $\kappa=0$; and, last, $\kappa=1$ to reflect the sale and repurchase of entire portfolios every holding period. We leave the equivalent parameter in our model, $\lambda^{\rm C}$, as a free parameter.
- ¹⁹The choice of window length for future returns (six months) is a compromise. If the window is made too short, we run the risk of predominantly measuring return *adjustments* to the recently observed characteristics/risks rather than equilibrium expected returns corresponding to those characteristics/risks, particularly if the adjustment process takes time (e.g., Hoberg and Welch 2009, Bali et al. 2014). Shorter return horizons also introduce more noise into realized returns. On the other hand, when betas/gammas are time varying, longer return windows introduce error in the risk measures relative to the returns to which they are related.
- ²⁰Harvey et al. (2016) argue that statistical significance criteria in cross-sectional asset pricing tests should be tightened owing to historical data mining by researchers. They suggest using a t-statistic critical value of three. Our main results clear this higher hurdle; for example, the return premiums associated with the liquidity level and with liquidity risk have t-statistics around four to six.
- ²¹Recall that, in contrast to downside liquidity risk, $\beta_{i,2}^{AP}$ is the comovement of a stock's liquidity *cost* (illiquidity) with the market's excess return. Stocks with high liquidity risk tend to increase in illiquidity when the market declines and consequently have negative $\beta_{i,2}^{AP}$. Therefore, a positive coefficient suggests a negative premium (return discount) to stocks that have a high level of symmetric liquidity risk.
- ²²This upside version of the LD-CAPM is inconsistent with conventional investor preferences; it requires a pricing kernel that is flat across losses and decreasing across gains, in other words, a representative investor possessing risk neutrality across losses and risk aversion across gains.
- ²³ For example, Hasbrouck (2009) finds no evidence of a liquidity risk premium, in contrast to earlier studies.
- ²⁴In the absence of strong distributional assumptions, the kernel underpinning the Acharya and Pedersen (2005) model (see Endnote 5) can be obtained from a combination of the LD-CAPM kernel and its upside counterpart. To recover the Acharya and Pedersen (2005) kernel from the upside and downside kernels without making strong distributional assumptions requires assuming equal risk aversion in good and bad states. In the beta/gamma representation

- of the models, this assumption translates to the restriction that the premium associated with upside risks is equal to that of downside risks.
- ²⁵Sorting on downside liquidity risk second (after a presort on a different characteristic) is a conservative way to investigate the premium associated with downside liquidity risk because it maximizes the variation in the other characteristic across its quintiles, at the expense of some variation in downside liquidity risk across its quintiles.
- ²⁶The magnitudes of the premiums reported in Table 6 are not directly comparable with those inferred from the earlier multivariate Fama–MacBeth tests of the LD-CAPM, which isolated the premium associated with downside liquidity risk, holding other factors constant.

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